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ABSTRACT

Assuming $(\omega_b \tau)^2 \gg 1$, equations that determine the electron energy distribution and drift velocity in the $\underset{\sim}{E} \times \underset{\sim}{B}$ drift frame of reference are derived, including a detailed treatment of inelastic collisions. These equations are valid when $\underset{\sim}{E}$, $\underset{\sim}{B}$, and the distribution function of the gas molecules are spacially uniform and constant in time, the component of $\underset{\sim}{E}$ along $\underset{\sim}{B}$ is negligible, and $E \ll B$ (in Gaussian units). The equations are suitable for numerical computations if the electron velocity distribution is assumed to be isotropic in the $\underset{\sim}{E} \times \underset{\sim}{B}$ drift frame. When the rms molecular speed in this frame is much smaller than the rms electron speed, the equations can be greatly simplified. Although the resulting equations are not new, this derivation clarifies their physical interpretation and limitations.

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I. INTRODUCTION

The distribution function of the electrons in a slightly ionized gas under the influence of external fields is determined by the Boltzmann equation. The electron distribution function in the gas frame, i. e., the reference frame in which the mean velocity of the gas molecules vanishes, is usually found by expanding the electron distribution function in spherical harmonics in velocity space.[1] When the rms electron speed is large in comparison with the electron drift speed and the rms speed of the gas molecules, the expansion converges rapidly and this "usual" method of solution works well. Under such conditions, elastic collisions are often much more probable than any other; this leads to further simplification of the calculations.

In most physical situations either the usual method of solution is adequate or the very difficult problem of runaway electrons must be faced.[2] However, when in the gas frame the electric field $\underset{\sim}{E}$ and the magnetic field $\underset{\sim}{B}$ are perpendicular with $E < B$ (in Gaussian units) and the electron cyclotron frequency ω_b is much greater than the electron collision frequency $1/\tau$, runaway electrons cannot occur even though the electron drift velocity, which is approximately $c[(\underset{\sim}{E} \times \underset{\sim}{B})/B^2] \equiv \underset{\sim}{v}_d$, can have any magnitude less than c. We will confine our attention to this problem, whose solution is possible because the electron motion in the applied fields is trivial and only the effects of relatively infrequent collisions need to be evaluated. We will assume that all speeds involved are non-relativistic.

We will show that elastic collisions tend (a) to heat the electrons until their mean kinetic energy is $\frac{1}{2} M v_d^2 + \mathcal{E}$, where M is the mass and $\mathcal{E}$ is the mean energy of a gas molecule in the gas frame, and (b) to spread the electron energy distribution until it is a Maxwellian distribution. The usual method of solution is clearly adequate except when inelastic processes are successful in holding the mean electron energy to a much smaller value.

When the usual method is not adequate, it is useful to use a reference frame moving at velocity $\underset{\sim}{v}_d$ with respect to the gas frame. In this frame, which we will call the drift frame, the electric field vanishes and the gas flows with velocity $-\underset{\sim}{v}_d$ perpendicular to the magnetic field. One advantage of this reference frame is that the apparent drift speed of the electrons is small so that an expansion in spherical harmonics in velocity space would converge rapidly. The problem thus reduces to evaluating the collision terms of the Boltzmann equation with a gas wind instead of an electric field. Since the results are relatively complicated, the usual approach should be used whenever it is adequate.

In this paper, the derivation of the equation governing the electron energy distribution in the drift frame is outlined. We also discuss the evaluation of quantities of physical interest, including the electron drift. These results are compared with those of the usual approach. Further discussion and details of the derivations as well as some numerical results of calculations applied to molecular hydrogen are given elsewhere.[3]

II. COLLISION PROCESSES, CROSS SECTIONS, AND COLLISION FREQUENCIES

The electron-molecule collision process j is distinguished by the energy $\epsilon_j = (\frac{1}{2}m)\alpha_j$ inelastically transferred to the molecule and by the number n_j of electrons leaving the collision. Considering a molecule initially at rest, we denote the velocity of the incident electron by $\underset{\sim}{v}'$ and the velocities of the outgoing electrons by $\underset{\sim}{v}_i$, where $i = 1, 2 \cdots n_j$, and we define $\cos\theta_i = (\underset{\sim}{v}' \cdot \underset{\sim}{v}_i)/v'v_i$. For each process, the collision frequency is $\nu^j(v') = n_g v' \sigma^j(v')$ where $\sigma^j(v')$ is the cross section for the process and n_g is the number of molecules per unit volume.

The momentum-transfer collision frequency $\nu_m{}^j(v')$ for process j is defined by

$$v' \nu_m{}^j(v') = \nu^j(v') \left\langle v' - \sum_{i=1}^{n_j} v_i \cos\theta_i \right\rangle_j , \tag{1}$$

where $\langle \, \rangle_j$ denotes the average effect of a collision process j for specified incident speed v'. The total momentum-transfer collision frequency is

$$\nu_m(v') = \sum_j \nu_m{}^j(v'). \tag{2}$$

Processes with $n_j = 0$ are electron attachment processes. From Eq. (1) we see that

$$\nu_m{}^j(v') = \nu^j(v'). \tag{3}$$

Processes with $n_j = 1$ are elastic if $\epsilon_j = 0$, and are inelastic otherwise. They are characterized by a differential cross section $\sigma_\theta{}^j(\theta;v')$ in terms of which

$$\sigma^j(v') = 2\pi \int_0^\pi \sigma_\theta{}^j(\theta;v') \sin\theta \, d\theta \tag{4}$$

and

$$\nu_m{}^j(v') = 2\pi n_g v' \int_0^{\pi} \sigma_\theta{}^j(\theta;v') \sin\theta \left[1 - \left(1 - \frac{a_j}{v'^2}\right)^{1/2} \cos\theta \right] d\theta, \quad (5)$$

if terms involving m/M are neglected.

Processes with $n_j \geq 2$ are ionization processes characterized by a differential cross section $\sigma_{\theta, v}{}^j(\theta, v; v')$ in terms of which

$$n_j \sigma^j(v') = 2\pi \int_0^{\infty} dv \int_0^{\pi} \sigma_{\theta, v}{}^j(\theta, v; v') \sin\theta \, d\theta \quad (6)$$

and

$$\nu_m{}^j(v') = 2\pi n_g v' \int_0^{\infty} dv \int_0^{\pi} \sigma_{\theta, v}{}^j(\theta, v; v') \sin\theta \left(\frac{1}{n_j} - \frac{v \cos\theta}{v'}\right) d\theta, \quad (7)$$

if terms involving m/M are neglected.

III. ELECTRON ENERGY DISTRIBUTION IN THE DRIFT FRAME

We assume that the applied fields and the distribution function of the gas molecules are spacially uniform and constant in time. We can then avoid considering the spacial dependence of the electron distribution function either by assuming that it has no spacial dependence or by considering the velocity distribution of a closed group of electrons--for example, the electrons in an avalanche.

In the drift frame the energy ϵ of a free electron is constant between collisions; therefore, the electron energy distribution $f(\epsilon, t)$ changes with time only because of collisions. Because we have assumed $\omega_b \tau \gg 1$, the effects of each collision process are independent of the other collision processes, and we have

$$\frac{\partial f(\epsilon, t)}{\partial t} = \sum_j \left(\frac{\partial f}{\partial t}\right)_j, \quad (8)$$

where $(\partial f/\partial t)_j$ is the effect of collisions of process j.[4]

In evaluating the collision terms in Eq. (8), we will simplify the algebra by neglecting molecular recoil and by assuming that the gas molecules are stationary in the gas frame. Although these assumptions are not necessary, they are often good when the usual method of solution is not adequate. We also assume that even with the strong fields, the scattering is independent of the azimuthal angle about the direction of the incident velocity.

We denote the probability distribution of a real quantity α for specified values of parameters $\beta, \gamma \cdots$ by $P(\alpha;\beta,\gamma\cdots)$, where $P(\alpha;\beta,\gamma\cdots) \geq 0$ and $\int P(\alpha;\beta,\gamma\cdots)d\alpha = 1$. The function $P(v;\epsilon)$, which is the distribution of speed v in the gas frame of the electrons with energy $\epsilon = \frac{1}{2}mV^2$ in the drift frame, will appear throughout our formulas. Notice that $P(v;\epsilon) = 0$ unless $|V - v_d| \leq v \leq V + v_d$. We will discuss the evaluation of $P(v;\epsilon)$ later.

The collision terms for each process j can be written in the form

$$\left.\frac{\partial f}{\partial t}\right|_j = \int_0^\infty G^j(\epsilon;\epsilon')f(\epsilon',t)d\epsilon' - N^j(\epsilon)f(\epsilon,t). \qquad (9)$$

Equation (9) states that electrons are removed from the distribution at a rate proportional to $N^j(\epsilon)$, and electrons are inserted into the distribution by the first term in a manner determined by $G^j(\epsilon;\epsilon')$.

Clearly, we have

$$N^j(\epsilon) = \int_0^\infty \nu^j(v)P(v;\epsilon)dv. \qquad (10)$$

The general properties of $G^j(\epsilon;\epsilon')$ are that

$$G^j(\epsilon;\epsilon') \geq 0 \text{ and } \int_0^\infty G^j(\epsilon;\epsilon')d\epsilon = n_j N^j(\epsilon'). \qquad (11)$$

For electron-attachment processes, $n_j = 0$, so we have

$$G^j(\epsilon;\epsilon') = 0. \qquad (12)$$

For ionization processes, we may write

$$G^j(\epsilon;\epsilon') = \int_{|V'-v_d|}^{v_d+V'} dv' \int_0^{\pi} d\theta \int_{|V-v_d|}^{V+v_d} dv P(v';\epsilon')\,[2\pi \sin\theta\, n_g v' \sigma_{\theta,v}^{j}(\theta,v;v')] \times P(\epsilon;v'v,\theta,\epsilon'), \tag{13}$$

where $P(\epsilon;v',v,\theta,\epsilon')$ is the distribution of energy ϵ of the electrons ejected at speed v and angle θ from collisions in which the incident electron speed is v'. The function $P(\epsilon;v',v,\theta,\epsilon')$ can be derived from the collision kinematics with the result[3]

$$P(\epsilon;v',v,\theta,\epsilon') = \frac{1}{\pi m v_d v}\,[-\cos^2\theta + 2b\cos\theta + c\,]^{-1/2}, \tag{14}$$

where
$$b = \left(\frac{v'^2 + v_d^2 - V'^2}{2v_d V'}\right)\left(\frac{v^2 + v_d^2 - V^2}{2v_d V}\right)$$

and
$$c = 1 - \left(\frac{v'^2 + v_d^2 - V'^2}{2v_d V'}\right)^2 - \left(\frac{v^2 + v_d^2 - V^2}{2v_d V}\right)^2 .$$

This function is defined to be zero except when

$$-1 \leq b - (b^2+c)^{1/2} \leq \cos\theta \leq b + (b^2+c)^{1/2} \leq 1 ,$$

and when $b^2 + c \geq 0$. The latter condition simply implies that $|V - v_d| \leq v \leq V + v_d$ must be satisfied whenever $|V' - v_d| \leq v' \leq V' + v_d$ is satisfied.

If the scattering is isotropic, the angular integration in Eq. (13) is trivial and yields

$$G^j(\epsilon;\epsilon') = \frac{1}{2mv_d}\int_{|V-v_d|}^{V+v_d} \frac{dv}{v} \int_{|V'-v_d|}^{V'+v_d} P(v';\epsilon') n_g v' \sigma_v^{\,j}(v;v') dv', \tag{15}$$

where $\sigma_v^j(v;v') \equiv 2\pi \int_0^\pi \sin\theta \, \sigma_{\theta,v}^j(\theta, v;v')d\theta$. For ionization process j, $G^j(\epsilon;\epsilon')$ is nonzero for values of ϵ between zero and approximately $\epsilon' - \epsilon_j$.

For elastic or inelastic processes, we may write

$$G^j(\epsilon;\epsilon') = \int_{|V'-v_d|}^{V'+v_d} dv' \int_0^\pi d\theta \, P(v';\epsilon') [2\pi \sin\theta \, n_g \, v'\sigma_\theta^j(\theta;v')] \, P^j(\epsilon;v',\theta,\epsilon'), \quad (16)$$

where $P^j(\epsilon;v',\theta,\epsilon')$ is just $P(\epsilon;v', v,\theta, \epsilon')$ from above evaluated at $v^2 = v'^2 - a_j$. If the scattering is isotropic, the angular integration in Eq. (16) yields

$$G^j(\epsilon;\epsilon') = \frac{1}{2mv_d} \int \frac{dv'}{v} P(v';\epsilon') \nu^j(v'), \quad (17)$$

where the limits are given by $|V - v_d| \leq (v'^2 - a_j)^{1/2} \leq V + v_d$. For elastic or inelastic process j, $G^j(\epsilon;\epsilon')$ is nonzero only for ϵ within a definite range about $\epsilon' - \epsilon_j$.

The function $P(v;\epsilon)$ that appears throughout the equations depends only upon the angular dependence of the electron velocity distribution in the drift frame. With large $\omega_b\tau$, the most general angular distribution of interest is independent of the azimuthal angle about $\underline{B}$, and is an even function of $\cos\xi$, where ξ is the polar angle measured from $\underline{B}$. Such a distribution can be expanded in the even Legendre polynomials of $\cos\xi$, and $P(v;\epsilon)$ can be written in terms of coefficients of the expansion. Since our procedure gives no way of calculating these coefficients we must make an assumption. We assume that the use of

$$P(v;\epsilon) = \frac{v}{2v_d V} \text{ for } |V - v_d| \leq v \leq V + v_d, \quad (18)$$

which is correct for an isotropic distribution, will give a good approximation to the correct physical results.

From physical arguments we expect the anisotropy of the velocity distribution to be largest when $\epsilon \approx \frac{1}{2} m v_d^2$ and to decrease rapidly as ϵ increases or decreases from this value. We also expect that anisotropy to be such that large values of $|\cos \xi|$ are less likely than with an isotropic distribution. To illustrate that the assumption of using Eq. (18) can be rather good even when the anisotropy is large, we calculate $G^j(\epsilon;\epsilon')$ for elastic collisions from Eq. (17) for the case $\epsilon' = \frac{1}{2} m v_d^2$—assuming that $\nu^j(v')$ is proportional to v'. Figure 1 shows the results for (1) an isotropic distribution (zero-order Legendre polynomial only), (2) one weighted by $\sin^2 \xi$ (the largest anisotropy using only the zero- and second-order Legendre polynomials), and (3) one containing only $\xi = \pi/2$ (the largest possible anisotropy).

Often the angular dependence of a scattering process is not known well enough for use in Eq. (13) or (16). In this case one can assume the scattering is isotropic and use Eq. (15) or (17). For elastic collisions, the elastic momentum-transfer collision frequency should then be used in evaluating Eqs.(10) and (17).

With the assumptions we have made, the electron energy distribution in the drift frame is determined by Eq. (8), which may be written symbolically as

$$\frac{\partial f(\epsilon, t)}{\partial t} = n_g \int \chi(\epsilon, \epsilon'; v_d) f(\epsilon', t) d\epsilon' , \tag{19}$$

where for a particular gas, $\chi(\epsilon, \epsilon'; v_d)$ is a kernel depending only upon v_d. Physically we expect that any initial energy distribution will quickly approach the separable form

$$f(\epsilon, t) = C f_0(\epsilon) e^{\beta t} \quad \text{with} \quad \int f_0(\epsilon) d\epsilon = 1 , \tag{20}$$

where C and β are constants, and $f_0(\epsilon) \geq 0$. Substitution of this important form of the solution into Eq. (19) shows that for a particular gas $f_0(\epsilon)$ and β/n_g depend only upon $v_d = cE/B$.

Once $f_0(\epsilon)$ is known, many quantities of interest can be calculated. The mean electron energy in the drift frame is $\bar{\epsilon} = \int f_0(\epsilon)\epsilon d\epsilon$. The distribution of electron speed in the gas frame is $\int P(v;\epsilon)f_0(\epsilon)d\epsilon$, and the mean electron energy in the gas frame is $\bar{\epsilon} + \frac{1}{2} m v_d^2$. The rate at which a collision process—such as ionization, dissociation, or excitation—proceeds is determined by

$$\int dv\, \nu^j(v) \int d\epsilon\, P(v;\epsilon)f_0(\epsilon) = \int d\epsilon\, N^j(\epsilon)f_0(\epsilon),$$

where $\nu^j(v)$ is the collision frequency for the process.

IV. DRIFT ALONG $-\underset{\sim}{E}$ IN THE GAS FRAME

Other quantities of interest are the diffusion tensor, which we will not discuss here, and the drift speed of the electrons along $-\underset{\sim}{E}$ in the gas frame. To calculate the latter we assume $\underset{\sim}{E} = E\hat{a}_y$ and $\underset{\sim}{B} = B\hat{a}_z$ so that $\underset{\sim}{v}_d = v_d\hat{a}_x$. From the electron equations of motion in the gas frame, the y-position of an electron is $y = Y + (v_d - v_x)/\omega_b$, where Y is the y-position of its guiding center. One can verify that the effect of a collision of process j at y upon the mean guiding-center position of the electrons is determined by

$$\sum_{i=1}^{n_j} (Y_i - Y') = \sum_{i=1}^{n_j} (v_{xi} - v_x')/\omega_b ,$$

where the guiding center of the incident electron is at Y', and those of the outgoing electrons are at Y_i, and where the speeds are those at the collision. The contribution of collision process j to the drift speed of the electrons with energy $\epsilon' = \frac{1}{2} m V'^2$ and speed v' is

$$\nu^j(v') \left\langle \sum_{i=1}^{n_j} (Y' - Y_i) \right\rangle_j = [\nu_m{}^j(v') + (n_j - 1)\nu^j(v')]\, v_x'/\omega_b,$$

where we have used the definition of the momentum-transfer collision frequency, and where $v_x' = (v'^2 + v_d^2 - V'^2)/2v_d$. Notice that electron attachment processes do not contribute, since for them $n_j = 0$ and $\nu_m{}^j = \nu^j$. For elastic and inelastic processes only the momentum-transfer collision frequency is important, since for them $n_j = 1$.

By averaging over ϵ' and v' and summing over the collision processes j, we find (dropping the primes)

$$v_E = \int_0^\infty d\epsilon\, f_0(\epsilon) \int_{|V-v_d|}^{V+v_d} dv\, P(v;\epsilon) \left(\frac{v^2 + v_d{}^2 - V^2}{2 v_d \omega_b} \right) \nu_m(v) + \sum_j (n_j - 1)\nu^j(v) \; . \tag{21}$$

In addition to the term involving ν_m, we have a term $\sum_j (n_j - 1)\nu^j$ that accounts for the changing number of electrons. This result does not depend upon the neglect of molecular recoil.

V. COMPARISON WITH THE USUAL METHOD OF SOLUTION

In the usual approach, the assumption of "large $\omega_b \tau$" consists of replacing $[1 + (\omega_b \tau)^2]^{1/2}$ by $\omega_b \tau$ throughout. When this approximation is good, the two methods of solution should agree in the limit of small v_d. This agreement can be demonstrated.

To illustrate the comparison, we consider v_E as given by Eq. (21). By assuming that $v_d \ll V$ and that $P(v;\epsilon)$ is given by Eq. (18), we can carry out the integral over v by expanding the integrand in a Taylor series about V. The result is a power series in $(v_d/V)^2$ with the first term being

$$\frac{v_d}{3V^2 \omega_b} \frac{\partial}{\partial V} \left\{ V^3 \left[\nu_m(V) + \sum_j (n_j - 1)\nu^j(V) \right] \right\} .$$

If most of the electrons in the distribution satisfy the condition $(v_d/V)^2 \ll 1$, we can use this term in Eq. (21) to find

$$v_E = v_d \int \frac{f_0(\epsilon)}{3V^2 \omega_b} \frac{\partial}{\partial V} \left\{ V^3 \left[\nu_m(V) + \sum_j (n_j - 1)\nu^j(V) \right] \right\} d\epsilon \; . \tag{22}$$

If we assume elastic collisions are much more probable than any other, this formula is identical to that found by the usual method.[1] We clearly see that the usual method is adequate only when $\bar{\epsilon} \gg \frac{1}{2} m v_d^2$ and when elastic collisions are much more probable than any other. The usual approach also shows that the drift speed along $\underset{\sim}{E} \times \underset{\sim}{B}$ in the gas frame is v_d, as expected.

In the same manner, we can demonstrate the equivalence of Eq. (8) to the equation in the usual approach that determines $F_0(v, t)$, the isotropic part of the electron velocity distribution in the lab frame. By using $P(v;\epsilon)$ as given by Eq. (18) and $G^j(\epsilon;\epsilon')$ as given by Eq. (15) or Eq. (17), we carry out the integrals in Eqs. (9) and (10) by assuming $V \gg v_d$ and proceeding as above. This yields $(\partial f/\partial t)_j$ in a power series in $(v_d/V)^2$. For inelastic, electron-attachment, and ionization processes the first term in this expansion is identical to that found by the usual approach.[5] For elastic collisions the first term vanishes, but because the usual approach assumes elastic collisions are much more probable than any other, we keep the second term. To facilitate comparison with the usual method we introduce an isotropic distribution of velocity in the drift frame by defining $g(\underset{\sim}{V}, t) = mf(\epsilon, t)/4\pi V$. Then, after a considerable amount of algebra, we find

$$\left(\frac{\partial g}{\partial t}\right)_{elas} = \nabla_V \cdot D_V \nabla_V g(\underset{\sim}{V}, t), \tag{23}$$

where

$$D_V = v_d^2 \nu_m^{elas}(V)/3.$$

Thus elastic collisions cause the isotropic velocity distribution $g(\underset{\sim}{V}, t)$ to diffuse in velocity space, the diffusion constant being that from kinetic theory with the mean free path replaced by v_d. This result follows from the usual approach only when $f(\epsilon, t)$ is near the form of Eq. (20); our analysis shows that such an assumption is not necessary.

In Eq. (19), v_d represents the molecular speed in the drift frame. If the molecules are not at rest in the gas frame, so that they have a distribution $P(u)$ of speed u in the drift frame, Eq. (19) is generalized to

$$\frac{\partial f(\epsilon, t)}{\partial t} = n_g \int f(\epsilon', t) d\epsilon' \int \chi(\epsilon, \epsilon'; u) P(u) du. \tag{24}$$

When $\bar{\epsilon} \gg \frac{1}{2} m v_d^2$ and elastic collisions are much more probable than any other, we have shown above that $\chi(\epsilon, \epsilon'; v_d)$ depends upon v_d only through D_V in Eq. (23). Thus the integral over u in Eq. (24) simply generalizes D_V from Eq. (23) to

$$D_V = \langle u^2 \rangle \nu_m^{elas}(V)/3 = \left(v_d^2 + \frac{2\mathcal{E}}{M} \right) \nu_m^{elas}(V)/3, \tag{25}$$

where $\mathcal{E}$ is the mean kinetic energy of the molecules in the gas frame. This result generalizes the result of the usual approach, which assumes the molecules have a Maxwellian velocity distribution in the drift frame.

The inclusion of the molecular recoil will lead to a term in $(\partial g/\partial t)_{elas}$ that is of zero order in (v_d^2/V^2). We can deduce this term from the following facts:

(a) This term cannot alter the number of electrons, and it must vanish when $m/M = 0$.

(b) When $v_d = 0$ and the gas molecules have a Maxwellian velocity distribution, $(\partial g/\partial t)_{elas}$ must vanish when $g(\underset{\sim}{V})$ is a Maxwellian velocity distribution with the same temperature.

The result is that when $\bar{\epsilon} \gg \frac{1}{2} m v_d^2$,

$$\left(\frac{\partial g}{\partial t} \right)_{elas} = \nabla_V \cdot [D_V \nabla_V g(\underset{\sim}{V}, t) + \frac{m}{M} \nu_m^{elas}(V) \underset{\sim}{V} g(\underset{\sim}{V}, t)] . \tag{26}$$

The term in m/M agrees with that found by the usual approach. From Eq. (26)

we conclude that elastic collisions always tend to make $g(\underset{\sim}{V}, t)$ a Maxwellian velocity distribution, in which the mean energy of an electron is $\frac{1}{2} M \langle u^2 \rangle = \frac{1}{2} M v_d^2 + \mathcal{E}$, since for this distribution $(\partial g/\partial t)_{elas}$ vanishes; this result is independent of the nature of $\nu_m^{elas}(V)$.

VI. CONCLUSION

By assuming the electron velocity distribution is isotropic in the drift frame, we derived an equation that determines the electron energy distribution in the drift frame. We then showed how to calculate quantities of physical interest, such as ionization rates and electron drift speeds, from this energy distribution. We expect the results calculated with this assumption to be a good approximation to the correct results, as we discussed briefly.

Comparison of our results with those of the usual method of solution in the "large $\omega_b \tau$" limit [when $(\omega_b \tau)^2 \gg 1$] showed that the usual method is adequate when the mean electron kinetic energy is large compared with $\frac{1}{2} m v_d^2$. When these conditions are satisfied, the effect of elastic collisions without recoil is a diffusion of the electron velocity distribution in velocity space, and the diffusion coefficient has a very simple form.

FOOTNOTES AND REFERENCES

* This work was performed under the auspices of the U. S. Atomic Energy Commission.

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FIGURE CAPTION

Fig. 1. The energy-scatter function for elastic scattering when $\epsilon' = \frac{1}{2} m v_d^2$ and when the scattering is isotropic and the cross section is constant. The electron velocity distribution in the drift frame is assumed to (1) be isotropic, (2) be weighted by $\sin^2 \xi$, or (3) contain only $\xi/2$, where ξ is the angle between $\underset{\sim}{B}$ and the electron velocity in the drift frame.

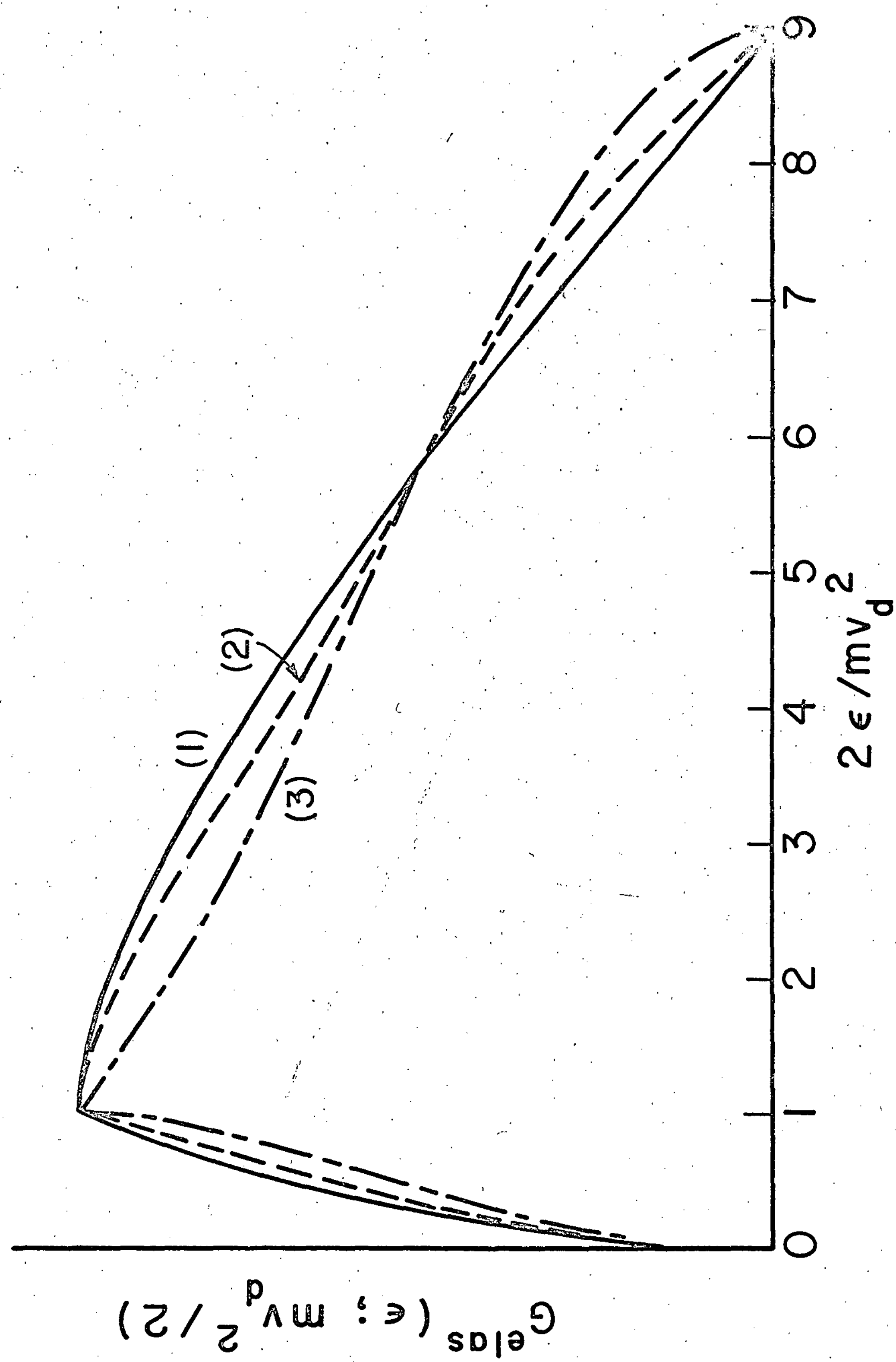

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Fig. 1.