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Measurement of the B-mode power spectrum with POLARBEAR

by

Bryan Steinbach

A dissertation submitted in partial satisfaction of the

requirements for the degree of

Doctor of Philosophy

in

Physics

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Adrian T Lee, Chair Professor William L Holzapfel Professor Aaron Parsons

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Measurement of the B-mode power spectrum with POLARBEAR

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Abstract

Measurement of the B-mode power spectrum with POLARBEAR

by

Bryan Steinbach

Doctor of Philosophy in Physics

University of California, Berkeley

Professor Adrian T Lee, Chair

We present the POLARBEAR experiment and its measurement of the B-mode power spectrum. POLARBEAR is a millimeter wave telescope that is measuring the Cosmic Microwave Background (CMB) polarization, using large format arrays of photon noise limited Transition-Edge Sensor (TES) bolometers. The instrument observes from the Atacama Desert at 5.2km in elevation with a 2.5m primary aperture telescope. This telescope has sufficient angular resolution (3.5' FWHM) to resolve the gravitational lensing features of the CMB, a new channel for obtaining information on fundamental physics such as the sum of the mass of neutrinos. This dissertation describes the design, integration and results of the first season of POLARBEAR observations. The receiver observed for 1 year with a noise-equivalent temperature (NET) of $22.8\mu K\sqrt{s}$ and mapped a 30 square degree area of the CMB, obtaining evidence for gravitational lensing in the BB power spectrum at a significance of 2σ . In memory of Huan Tran

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Chapter 1

Cosmology

1.1 History of the Universe

Our creation story for the universe today begins with a hot big bang. A homogenous, isotropic and dense plasma filled space, and its energy drove a rapid expansion of the universe according to General Relativity. As the universe expanded, the plasma cooled, condensing nucleons into nuclei and later nuclei and electrons into neutral atoms. Slight perturbations in the density of the plasma collapsed gravitationally and grew to become the stars and galaxies we see today.

1.2 Neutrino mass and the matter power spectrum

Changing the contents of the universe changes its expansion history and the formation of structure. By measuring the amount of structure in the universe at different times, we can constrain the contents of the universe. Like other stable particles, neutrinos are produced in abundance in the early universe, and contain a significant fraction of the energy content of the universe, so measurements of structure constrain the properties of neutrinos.

1.2.1 Friedmann equations

The quantitative effect of neutrinos on the history of the universe is determined by General Relativity. In General Relativity, space time is a dynamic object described by a metric which obeys Einstein's equation. Einstein's equation relates the geometry of space $G_{\mu\nu}$ to the energy-momentum tensor $T_{\mu\nu}$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{1.1}$$

Observationally, our universe has been found to be flat[1], so on the largest scales it can be described by the Friedmann-Robertson-Walker (FRW) metric $g_{\mu\nu}$.

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0\\ 0 & a^2(t) & 0 & 0\\ 0 & 0 & a^2(t) & 0\\ 0 & 0 & 0 & a^2(t) \end{pmatrix}$$
(1.2)

With the metric of Eq. 1.2, the time-time component of Eq. 1.1, and assuming a homogeneous energy density ρ , the expansion history of the universe is determined

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho\tag{1.3}$$

This is written in terms of the Hubble rate $H(t) = \frac{\dot{a}}{a}$ and critical energy density $\rho_{cr} = \frac{3H_0^2}{8\pi G}$

$$\frac{H^2(t)}{H_0^2} = \frac{\rho}{\rho_{cr}}$$
(1.4)

Given an accounting for the energy density ρ in the universe, and the evolution of ρ with scale factor, we can determine the expansion history of the universe. To determine the effect of neutrino mass on the matter power spectrum, we need to know the expansion history during the matter dominated era. The energy density of matter scales proportionally to volume or scale factor cubed, so the Friedmann equations are:

$$H^2(t) = H_0^2 a^{-3} \tag{1.5}$$

1.2.2 Neutrino energy density

Early in the history of the universe, neutrinos are in thermal equilibrium at a temperature T_{ν} . As fermions in thermal equilibrium, they are distributed in momentum space according to the Fermi-Dirac distribution. As the universe expands and densities and scattering rates decrease, they eventually drop out of equilibrium with other species, but the distribution remains Fermi-Dirac. At very late times, because the neutrinos lose contact, the distribution has an effective energy given by the momentum, rather than the total neutrino energy including the rest mass. For one massive, left handed neutrino species with an anti particle, the energy density at late times is

$$2m_{\nu} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp(\beta p) + 1} \approx 0.182m_{\nu}T^3 \tag{1.6}$$

The neutrinos are slightly cooler than photons, because neutrinos lose contact before the universe drops below the temperature of the mass of the electron, so only the photons gain energy from the rest mass of the annihilating electrons and positrons. The relative energy of the neutrinos and photons at late times can be determined by counting the entropy before and after the annihilation of electrons, and the temperature today will be

$$T_{\nu} = T_{\gamma} \left(\frac{4}{11}\right)^{\frac{1}{3}} = 1.946K \tag{1.7}$$

With a critical energy density of $8.098 \times 10^{-11} h^2 eV^4$, the fraction of the universe's energy which will be in the form of neutrinos will be

$$\Omega_{nu}h^2 = \frac{m_\nu}{94eV} \tag{1.8}$$

Given the neutrino oscillation bound of $\sum m_{\nu} > 0.05 eV$ [2] for normal hierarchy, the minimum amount of energy in neutrinos in our universe is

$$\Omega_{\nu}h^2 > 0.00053 \tag{1.9}$$

This is a very small fraction of the total energy budget of the universe. Fortunately, a small change in the matter content of the universe can induce large changes to the matter power spectrum. Due to an exponential dependence of gravitational collapse on matter perturbations, neutrinos have a disproportionately large effect on the matter power spectrum today.

1.2.3 Growth of structure

To understand the effect of neutrino mass on the growth of structure, we need to trace the evolution of perturbations through the matter dominated era. At the epoch of radiation equality, models with and without neutrino mass are indistinguishable, so we will start with identical initial conditions from that point. During the epoch of matter domination, both the growth rate and the evolution time will differ due to the mass of neutrinos. To understand the basic physics we consider a simplified model without dark energy.

The continuity equation and Euler equation (analog to f=ma) for a general relativistic, pressureless fluid, for small scales where gradient of the potential dominates, are [3]

$$\dot{\delta}_{cdm} = \theta_{cdm} \tag{1.10}$$

$$\dot{\theta}_{cdm} = -\frac{\dot{a}}{a}\theta_{cdm} - k^2\phi \tag{1.11}$$

This describes the evolution of dark matter density perturbations δ_{cdm} and velocity divergence perturbations $\theta_c dm$, with space dimensions fourier transformed with wave number k. The dark matter perturbations are related to the metric perturbations ϕ and in the limit of no shear stress in the fluids, by the Poisson equation. Above the k where neutrinos can free stream, the neutrino perturbations are erased and the source term for the Poisson equation contains only dark matter perturbations.

$$-\frac{k^2}{a^2}\phi = 4\pi G\delta_{cdm}\rho_{cdm} \tag{1.12}$$

The growth of the universe during the matter dominated era is sourced by both the neutrinos and the cold dark matter.

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G a^2 \rho_m \tag{1.13}$$

Which gives the scale factor as a function of time $a \propto t^2$.

Combining Eqs. 1.10, 1.11, 1.12, and defining $f_{\nu} = \frac{\Omega_{\nu}}{\Omega_{cdm}}$ gives one second order equation for the growth rate of perturbations.

$$\ddot{\delta} + \frac{2}{\tau}\dot{\delta} - \frac{6}{\tau^2}(1 - f_{\nu})\delta = 0$$
(1.14)

This gives a growth rate of

$$\delta_{cdm} \propto a^{1-(3/5)f_{\nu}} \tag{1.15}$$

The two models reach the epoch of matter radiation equality at different times. The massive neutrinos delay matter radiation equality because there is less cold dark matter before neutrinos go non-relativistic. To relate the growth between the two models, we define $a_{nr}^{f_{\nu}=0}$, which is the scale factor at which if you had a massive neutrino in the massless neutrino universe, the massive neutrino would go non-relativistic. The difference between this and $a_{nr}^{f_{\nu}}$, the scale factor at which massive neutrinos in a massive neutrino universe go non-relativistic, along with the difference in perturbation growth rate, determines the effect neutrinos have on large scale structure.

The shift in scale factor at which the two models are equivalent before the nonrelativistic massive neutrino epoch is

$$a_{nr}^{f_{\nu}} = \frac{1}{1 - f_{\nu}} a_{nr}^{f_{\nu} = 0} \tag{1.16}$$

$$\frac{\delta_{cdm}^{f_{\nu}=0}(a_0)}{\delta_{cdm}^{f_{\nu}=0}(a_{nr}^{f_{\nu}=0})} = \frac{a_0}{a_{nr}^{f_{\nu}=0}} = \frac{a_0}{(1-f_{\nu})a_{nr}^{f_{\nu}}}$$
(1.17)

$$\frac{\delta_{cdm}^{f_{\nu}}(a_0)}{\delta_{cdm}^{f_{\nu}}(a_{nr}^{f_{\nu}})} = \left(\frac{a_0}{a_{nr}^{f_{\nu}}}\right)^{1-\frac{3}{5}f_{\nu}}$$
(1.18)

Because $\delta^{f_{\nu}}_{cdm}(a^{f_{\nu}}_{nr}) = \delta^{f_{\nu}=0}_{cdm}(a^{f_{\nu}=0}_{nr}),$

$$\frac{\delta_{cdm}^{f_{\nu}}(a_0)}{\delta_{cdm}^{f_{\nu}=0}(a_0)} = (1 - f_{\nu})(a_{nr}^{f_{\nu}})^{\frac{3}{5}f_{\nu}}$$
(1.19)

The matter power spectrum P(k) is defined as

$$P(k) = \left\langle \left(\frac{\delta \rho_{cdm} + \delta \rho_{\nu}}{\rho_{cdm} + \rho_{\nu}} \right)^2 \right\rangle \tag{1.20}$$

For small scales, the neutrinos contribute to the total energy density but not the perturbations, so the power spectrum is

$$P(k) = (1 - f_{\nu})^2 \langle \delta_{cdm}^2 \rangle \tag{1.21}$$

And the fractional change in the power spectrum between the massive and massless neutrino models is

$$\frac{\Delta P(k)}{P(k)} = (1 - f_{\nu})^4 (a_{nr}^{f_{\nu}})^{\frac{6}{5}f_{\nu}}$$
(1.22)

For a neutrino temperature today of 1.9K, the scale factor at which the thermal energy of a relativistic gas per particle 3kT and rest energy mc^2 are comparable is

$$a_{nr} = \frac{4.9 \times 10^{-4} eV}{m_{\nu}} = \frac{5.2 \times 10^{-6}}{f_{\nu}}$$
(1.23)

$$\frac{\Delta P(k)}{P(k)} = (1 - f_{\nu})^4 (1.9 \times 10^5 f_{\nu})^{-\frac{6}{5}f_{\nu}}$$
(1.24)

This approximates the suppression of the power spectrum at small scales due to neutrino masses past the neutrino free streaming length. The dominant component is the suppression of the gravitational collapse due to the slight erasure of matter perturbations on small scales. At large scales, there is no effect on the power spectrum, because the neutrinos behave like cold dark matter, and the modes do not begin to collapse until well into the matter dominated era when neutrinos are non-relativistic and the expansion history of the universe is unchanged. In an intermediate regime where modes are larger than the free streaming scale, and they enter the horizon before neutrinos go non-relativistic but after matter domination, there is a smaller suppression of the power spectrum.

The high sensitivity of the matter power spectrum, roughly an order of magnitude relative to the fractional changes in the energy budget of the universe, lets us make precision measurements of the neutrino mass. 1% measurements of the matter power spectrum provide neutrino mass constraints at the 0.1eV level.

1.2.4 Neutrino free streaming length

Travelling at the speed of light, the neutrinos can move one Hubble distance in a Hubble time. In comoving units, with a decreasing Hubble rate, this distance is dominated by the latest times. However, once the neutrinos go non-relativistic, their



Figure 1.1: Suppression of the matter power spectrum at small scales due to neutrino mass, in units of the fraction of the total energy budget which is neutrinos. The blue line shows the effect of the change on growth rate from neutrinos, the green line shows the effect from the change in expansion history, the red line shows the combined analytic model, and the cyan line is a linear approximation.

slow down in velocity will dominate over the slow down in Hubble rate, so the free streaming length of neutrinos is given by the Hubble distance at the relativistic to non-relativistic transition. Ignoring the effects of dark energy, which only affects the universe at very recent times, this gives a free streaming wavelength of approximately

$$k_{fs} \approx \frac{a_{nr}}{H(a_{nr})} = H_0 \sqrt{a_{nr}} = 0.016 \frac{0.001}{f_{\nu}} h M p c^{-1}$$
 (1.25)

1.3 Measuring the matter power spectrum

The observable effects of the massive neutrinos on the matter power spectrum are only visible at late times, well after neutrinos go non-relativistic. There are several techniques for measuring the matter power spectrum at late times, such as CMB lensing, galaxy weak lensing, intensity mapping and point source surveys in different bands. Here we describe the CMB lensing technique.

1.4 Cosmic Microwave Background

POLARBEAR uses gravitational lensing of the CMB to measure the matter power spectrum at intermediate redshifts. CMB lensing uses the CMB as a backlight of all the large-scale structure between us and the surface of last scattering. Intervening structure gravitationally distorts the paths of photons and introduces subtle effects into the statistics of the observed sky. The strength of the CMB lensing technique is the strength of its theoretical modelling and lack of unknown parameters. The primordial CMB perturbations are very well understood and Gaussian, homogenous and isotropic. Lensing introduces non-Gaussianities which are measurable in parity forbidden two-point channels and four-point statistics of the intensity and polarization maps of the CMB.

At recombination, the expansion of the universe cooled it to a temperature below which neutral atoms of hydrogen and helium became statistically mechanically preferable over ionized gas. The photon scattering cross section of neutral gas is negligible compared to an ionized gas, so the mean free path of the photons increased quickly to the point where photons could free stream from a virtual surface of last scattering to today's observers. The temperature at which neutral atoms forms is approximately 3000K, so from the FIRAS measurement of a CMB temperature of 2.725K[4][5] we know that the redshift at recombination is 1100.

The CMB is nearly uniform, but has small anisotropies at the 10^{-5} level in temperature. These anisotropies appear to be adiabatic scalar perturbations (density fluctuations) with a nearly scale invariant spectrum and Gaussian statistics.

1.5 Polarization of the Cosmic Microwave Background

The CMB is intrinsically polarized due to the Thomson scattering cross section. The Thomson scattering cross section for photons scattering off an electron at the surface of last scattering has a dependence on the dot product of the ingoing and outgoing polarizations. The Q and U Stokes polarization seen by an observer are integrals over the radiation field seen by the electron with a weighting from Thomson scattering that is proportional to a linear combination of spherical harmonics $Y_{2\pm 2}$ [6]. Only quadrupole moments in the radiation field at the surface of last scattering contribute to the CMB polarization anisotropies.

In the limit of a small patch of sky, the sphere appears flat, and the spherical harmonics can be approximated as fourier modes on a plane. A plane wave perturbation to the matter density intersected with the plane of the sky will produce a mode of CMB anisotropies projected onto the plane. The polarization produced will always be parallel or perpendicular to the wave vector due to the symmetry of the Thomson scattering cross section. This limits the polarization anisotropies to a vector space of even parity modes, called the E modes, orthogonal to a vector space of odd parity modes, called the B modes. Looking at the sky, for a universe that began with only scalar perturbations, we should expect to see only E modes and no B modes in the primary CMB anisotropies.

1.5.1 Gravitational Lensing

Later in the history of the universe, gravitational lensing can remap the polarization field. To lowest order, gravitational lensing changes the arrival direction of photons, without changing the orientation of their polarization. By bending the wave vector of what was initially a pure E mode, a mixture of E modes and B modes will be produced. The gravitational lensing field can be written as a gradient of a scalar field ϕ because the lenses only converge and diverge but do not twist the map.

$$Q'(x) = Q(x + \nabla\phi(x)) \approx Q(x) + \nabla Q \cdot \nabla\phi$$
(1.26)

$$U'(x) = U(x + \nabla\phi(x)) \approx U(x) + \nabla U \cdot \nabla\phi$$
(1.27)

In power spectrum space, this produces B modes following [7]

$$C_l^{BB} = \frac{1}{2} (W_{1l}^{l'} - W_{2l}^{l'}) C_l^{EE}$$
(1.28)

Where $W_{1l}^{l'}, W_{2l}^{l'}$ are certain weighted integrals over the lensing field correlation function. Therefore, by measuring the B field, we learn about the lensing field correlation function and large scale structure. In the next chapter, we describe the POLARBEAR experiment to measure the fluctuations in the B field.

Chapter 2

The POLARBEAR Experiment

2.1 POLARBEAR

The POLARBEAR experiment is designed to map the millimeter-wave sky in temperature and polarization with 4 arcminute resolution.

To reach a mapping speed necessary to measure gravitational lensing, POLAR-BEAR has a large 19cm diameter focal plane tiled with antenna coupled Transition-Edge Sensor (TES) bolometers. POLARBEAR uses double slot dipole antennae to couple 150GHz radiation from free space to microstrip. The simple structure and symmetry of the double slot dipole antenna guarantees good properties of the beams on the sky, with very low leakage from temperature to polarization. The TES bolometers are described in detail in Chapter 3

This chapter describes the design of the telescope, optics and cryogenics, and the observations of the first season.

2.2 Huan Tran Telescope

The Huan Tran Telescope is an offset Gregorian design, with angle between the central rays satisfying the Dragone condition[8][9]. The Gregorian design was chosen over a crossed Dragone configuration because we did not need the larger focal plane size of the crossed Dragone, and the smaller secondary of the Gregorian is much cheaper to implement. The offset Gregorian design has low cross polarization and a large diffraction limited field of view[10]. Aberrations, and therefore cross polarization, from a single reflecting mirror are minimized when the element is used on axis. Two reflecting elements both used off axis can be set an angle such that the cross polarization from each element cancels out at the center of the field, and low cross polarization is provided across the field. The advantage to this construction over a pair of on axis mirrors is the clear aperture. In an on axis telescope, the support structure for the secondary mirror interferes with the beam and scatters it to far

angles where it can see the ground, sun, moon, or galaxy.

The POLARBEAR primary aperture has a projected diameter of 2.5m. A cold 4K aperture stop inside the cryostat is imaged onto this surface. An additional lower precision guard ring of aluminum panel around the primary mirror extends to a projected diameter of 3.5m. The primary aperture is cast from a single piece of cast aluminum machined to a surface figure of 53um RMS. Using a single segment mirror for the primary aperture eliminates the possibility of far sidelobes from diffraction off of gaps between panels. The panels gaps between the precision primary and the guard ring are not illuminated by the main beam due to the cold aperture stop, and are additionally covered with aluminum tape.

2.2.1 Mirror Mount

The central single segment precision primary mirror surface is supported on a non kinematic ten point mount. In the piston direction, normal to the center of the mirror surface, the aluminum casting is supported by six steel rods. The rods fix the distance between the steel backing structure and the mirror, but are flexible in the direction perpendicular to the rod axes, the tangent to the mirror surface. In the tangent to the mirror surface direction, the mirror position and orientation is constrained by four steel balls in radially oriented steel slots. The slots allow for contraction and expansion, and minimize the extent to which the mirror is overconstrained. In the Cedar Flat deployment of POLARBEAR, the six rods were threaded, and nuts set the effective length of each rod. When the mirror was lowered onto the backing structure, the nuts were snugged up against the steel backing structure at the neutral position of the mirror. Because the mirror was manufactured separately from the backing structure and figured while not tensioned, and six points is kinematically overconstrained, this assembly procedure should leave the mirror in the correct figure once mounted, to the extent that the nuts on the threaded rod can be tightened relative to each other without moving. Imperfections in this technique may explain why the primary beam size was 4.0 arcminutes in Cedar Flat, rather than the ideal 3.5 arcminutes achievable from this aperture.

For the Chile deployment, the threaded rods were replaced with shoulder bolts. Given a precision machined backing structure, the shoulder bolts should repeatably and precisely constrain the primary mirror without deforming it. However, the steel backing structure is not precision machined or even ground flat; it is covered in weld spatter and not perpendicular to the axis of the shoulder bolt. This lead to a 1mm deformation of the primary mirror on initial assembly, degrading the spec 50um RMS surface sufficiently to widen the main beam to 10 arcminutes. We corrected this by shimming the shoulder bolts of the primary mirror. One at a time, we released one of the shoulder bolts, and measured with a dial micrometer the deflection of the primary mirror. Then the primary mirror was deflected an additional 1mm with a bottle jack to provide room to insert shims. Shims were inserted to compensate

for the primary mirror deflection, so that on retightening the bolt, the mirror is not under tension. As expected for a six point non-kinematic mount, we needed to insert shims in three positions, at 2, 6 and 12 o'clock when viewing the primary from the secondary mirror. The procedure was iterated twice, and reduced the primary mirror deformation from 1mm to 100μ m. The change in primary mirror surface measured with photogrammetry from before and after shimming is shown in Fig. 2.1, and the final mirror figure is shown in Fig. 2.2

2.2.2 Motor RFI

The motor drivers use high frequency pulse width modulation (PWM) to control the current in the motor windings. The controller rapidly switches the voltage across the motor windings to slew the current to the value commanded by the control computer. This rapid switching capacitively couples current to other components in the system and back to the motor controller through ground loops, interfering with sensitive electronics, in particular with the distribution of synchronization pulses and time stamps, because these signals travel long distances near the motor cables. We mitigated the interference by running the motor cables as far from other cables as possible. Between the telescope and the motor controllers, they run in separate cable trenches. In the telescope cable wrap, they are spaced one foot apart. We also added 2.7mH inductors to the output of the motor controller. These reduce the high inrush currents into the capacitance of the cables, and reduced interference seen on nearby components by a factor of two.

2.2.3 Lightning

In the Bolivian winter, lightning storms are common, and we experienced a couple instances of equipment damage to lightning. The microwave ethernet link between the high altitude and low altitude station failed several times after lightning storms, but was replaced due to its low cost. The timing pulse link between two containers failed once due after a lightning storm due to destruction of the RS-422 transceiver chip. All timing pulse links between containers were subsequently replaced with fiber optic cables.

The telescope and all shipping containers have lightning rods, running to a single ground point, a rod implanted into the concrete foundation of the telescope.

2.2.4 Photogrammetry

The primary and secondary mirror surfaces and their alignment to the receiver was measured using photogrammetry. Retroreflective targets are glued to the primary and secondary mirror, and a series of photographs from every possible angle using a consumer grade DSLR from a boom manlift. We use a prime, or fixed focal length, lens with very low distortion. The focal length is chosen to fill the field with the primary mirror. The aperture diameter is set small, with an F/# of 22, because of the large depth of field required when taking photographs on edge to the primary mirror. Flash is used with the shutter speed set as fast as possible while still maintaining sync with the flash. This maximizes the contrast between the background and the retroreflective targets. Only the green color channel from the photographs is used in analysis to reduce the effects of chromatic aberration. The photographs are analyzed using a commercial software package, PhotoModeler¹, to extract a point cloud for the primary mirror, by manually identifying a few targets from each photograph, and then letting the software automatically solve for the rest of the target identities, their locations, and the camera location and distortion parameters. Typical precision of the fit is $30\mu m$, or about 1/10th of a camera pixel. The extracted point clouds are then fit to the theoretical surface using a Matlab package written by Nils Halverson. Several sets of photographs are taken. To measure the primary or secondary mirror surface, dedicated sets of photographs are taken with the ideal focal length lens. To measure the relative alignment, a separate set of photographs is taken from the boom without the manlift, with additional targets placed along the boom to bootstrap the fit from primary mirror to secondary mirror to receiver. Because of the bootstrapping, errors accumulate and the procedure is substantially less precise. However, much less precision is required in the optics alignment than the mirror surface figures. We also used a theodolite to measure the alignment and mirror surface, however we found photogrammetry superior because it does not require the mount to be stable on long time scales and is much less labor intensive.

2.2.5 Beam size

The Huan Tran Telescope was first assembled in the Inyo Mountains in California for a test deployment. In this test assembly the full-width half-max (FWHM) beam size median was 4 arcminutes. The design illumination is a -10dB taper of a Gaussian at the projected 2.5m diameter of the primary mirror. The intensity in a Gaussian beam in cylindrical coordinates with z the distance along the beam from the primary mirror and r the distance from the central ray of the beam is

$$I(r,z) = I_0 \left(\frac{w_0}{w(z)}\right)^2 \exp\left(-\frac{2r^2}{w^2(z)}\right)$$
(2.1)

$$w(z)^2 = 1 + \left(\frac{z}{z_R}\right)^2 \tag{2.2}$$

$$z_R = \frac{\pi w_0^2}{\lambda} \tag{2.3}$$

¹Eos Systems, Inc. www.photomodeler.com



Figure 2.1: Difference in mirror surface height before and after shimming measured with photogrammetry. Each point is a photogrammetry target, and color indicates distance moved, with positive motion indicating motion towards the secondary mirror. The largest shims were added to the 2 o'clock and 8 o'clock positions visible as the red cluster of points at those positions. Figure courtesy Michael Myers.



Figure 2.2: Difference between final mirror figure measured with photogrammetry and design mirror surface. The final RMS, weighted by the truncated Gaussian beam illumination, is 51μ m. Weighting is shown by the diameter of the dot, while color shows the difference between measured and design surface. The measurement error of each point from photogrammetry is estimated at 30μ m. The 4 o'clock position of the mirror is bent at the edge, possibly due to damage in shipping, but due to the low power of the beam at the edge of the mirror this has little effect on the beam pattern on the sky.

For POLARBEAR , the -10dB taper at the edge of the primary mirror translates to $w_0 = 1.16m$.

The far field illumination pattern of a Gaussian beam is

$$I(\theta) \propto \exp\left(-\frac{2\theta^2 \pi^2 w_0^2}{\lambda^2}\right)$$
 (2.4)

For a 2mm wavelength, this gives an expected FWHM beam size of 2.23 arcminutes. This is overly optimistic because of the truncation of the beam, a more accurate approximation including the effects of truncation is [11]

$$\theta_{\rm FWHM} \approx \frac{1}{2} \sqrt{8 \log(2)} \frac{0.2428\lambda}{w_0} \sqrt{\frac{e}{1 - \exp\left(-\left(\frac{R}{1.0271w_0}\right)^2\right)} - 1}$$
(2.5)

This predicts a beam size of 3.0 arcminutes, slightly smaller than the measured median beam width of 3.5 arcminutes shown in Fig. 2.3.



Figure 2.3: Histogram of measured POLARBEAR beam widths.

2.2.6 Over the Primary Sidelobe

Scattering at the window of the receiver can produce sidelobes in an offset Gregorian design. In raytracing simulations, we found that light scattered by the top side of the zotefoam window of the receiver can reflect off the bottom of the secondary and over the top of the primary mirror. This sidelobe has structure on degree scales and is about 60dB down from the main beam. The sidelobe is 84° in the elevation direction away from the main lobe of the beam. We observed this in POLARBEAR season 1 when scanning our RA23 patch at an azimuth where the patch is directly opposed from the peak of Cerro Toco. At this position, the over the primary sidelobe scans the top of Cerro Toco, and is visible in maps in both ground centered and sky centered coordinates. This was subsequently confirmed with dedicated observations designed to scan the over the primary sidelobe on the sun.

We eliminated this sidelobe by adding additional baffling to the telescope six months into the first season. The additional baffling prevents the secondary mirror from seeing any sky directly; only primary mirror is visible from the secondary. We did this by adding two sets of baffles: A visor over the top of the primary, and a prime focus baffle. The visor extends 0.5m beyond the top of the primary mirror. The prime focus baffle extends completely around the receiver and secondary mirror and leaves only a hole at the focus of the primary mirror 27cm in diameter. A cone extends from the prime focus baffle 1m towards the edge of the primary. All the baffling is covered in metal-backed AN-72, a millimeter wave absorber with reflectivity of 5-15% at 150GHz for TE-polarized radiation incident at 45 degrees to the surface normal[12].

2.3 Observing Frequency

POLARBEAR is designed to observe in a spectral band centered at 148GHz with a bandwidth of 38GHz. This band is ideal for CMB observations from the ground for several reasons. The CMB is a black body with a temperature of approximately 2.7 Kelvin. In a single electromagnetic mode, the power per hertz from a black body source scales as

$$P(f) = \frac{hf}{\exp\left(\frac{hf}{kT}\right) - 1} \tag{2.6}$$

The CMB fluctuations are very small, and we are interested in measuring those fluctuations, so our sensitivity is determined by $\frac{dP}{dT}$.

$$\frac{dP}{dT} = \frac{h^2 f^2 e^{\frac{hf}{kT}}}{kT^2 \left(e^{\frac{hf}{kT}} - 1\right)^2}$$
(2.7)

For a ground based experiment, only windows of atmospheric emission are available for observing. Molecular lines from water vapor (e.g. 180GHz) and oxygen (e.g. 120GHz) limit the fractional bandwidth of the spectral band to about 25%. Additionally, at higher frequencies, there are more electromagnetic modes available to sample on a focal plane of a fixed diameter. Combined, these effects add a frequency cubed scaling to the quantity of CMB power that can be measured for an instrument of fixed physical size and fractional bandwidth. With this function, the sensitivity of an instrument peaks near 150GHz. As shown in Fig. 2.4, there is a convenient atmospheric window at this frequency. The atmosphere in this band with a 38GHz bandwidth contributes approximately $15K_{RJ}$ of loading.



Figure 2.4: Fractional transmission through the atmosphere and candidate locations of bands used to select POLARBEAR observing bands. This is at the CBI site 5km from the POLARBEAR site. Figure created by Kam Arnold.

This band is also fortuitous for occurring near a minimum of foreground contamination. The dominant foregrounds are galactic synchrotron radiation and polarized radiation spinning galactic dust grains. For a experiment observing in small relatively foreground clean areas of the sky, the 150GHz band is near the frequency of minimum foreground contamination.

2.4 Experiment Location

POLARBEAR observes on the Huan Tran Telescope at the James Ax Observatory, which is located at 5200m above sea level on Cerro Toco in the Atacama Desert in Northern Chile. This is an ideal site for observations due to its high altitude and dry weather. At an altitude of 5200m the air pressure, and therefore air mass above, is 50% what it is at sea level. As oxygen is a primary contributor of loading at 150GHz due to an absorption line at 117GHz, this decrease in air mass greatly reduces total atmospheric emission and additional photon noise. The dryness of the site, 1mm median precipitable water vapor (PWV), also reduces total atmospheric emission due to the water absorption line at 183GHz. Low water is particularly important because the water in the atmosphere is poorly mixed and fluctuates on short time scales. This introduces excess low frequency noise into the temperature (but not polarization) measurements of the experiment, and limits the gain stability of the detectors.

The POLARBEAR site is at latitude and longitude 22.958°S 67.786°W. This is near several other millimeter and submillimeter wave observatories. This allows us to share facilities and improve site safety for observing personnel. We also take advantage of the Atacama Pathfinder EXperiment (APEX) water vapor radiometer to measure PWV. Measurements of PWV are used to schedule and prioritize observations, predict weather, and cross check absolute calibration of the instrument through sky dips.

2.5 Cryogenics

2.6 Thermal Filtering

POLARBEAR has a large 30cm vacuum window which looks out to ambient 300K temperature. The power from infrared radiation incident on this aperture is 32W following the Stefan-Boltzmann law. This exceeds the $1\mu W$ cooling capacity of the sub-kelvin refrigerator by 75dB. Attenuating the power by this amount is done by with a thermal analog to a shunt-series resistive voltage divider network.

The first series resistor is the window of the cryostat. The window of the cryostat is a 15cm thick 46cm diameter cylinder composed of zotefoam, an expanded polypropylene foam. On the vacuum side, a flat plate of aluminum extends inwards to a 30cm diameter to support the zotefoam against 7000 newtons of atmospheric pressure. The zotefoam has a low thermal conductivity, 0.047W/(m K) at 300K, sufficiently low that the vacuum side cools to approximately 200K, reducing the power radiated to the 50K stage of the cryostat by a factor of 5. The refractive index of the zotefoam at 150GHz is low enough that no antireflection coating is required.

The thermal resistance is further increased with IR shaders, a thin plastic sheet patterned with metal squares which acts as a low pass filter. In POLARBEAR , for the higher temperature, shorter wavelength infrared radiation, it is unclear whether the IR shaders contribute to the total thermal resistance, or whether the low thermal conductivity of the zotefoam dominates.

Power is shunted to the 50K heater stage through a series of dielectric absorbers. First is a 3mm thick sheet of Mupor, a porous expanded teflon. Like the zotefoam, the index of the expanded teflon is also low enough to not need an antireflection coating. The mupor absorbs radiation in the infrared and conducts it out to the aluminum 50K shell which is sunk to the 50K refrigerator. This dramatically improves the overall attenuation of the thermal system, and its installation reduces the temperature of the next absorbing element from 170K to 120K. With out the absorbing shunt resistors, the thermal resistance scales only proportional to the number of reflecting elements in the system; with the shunt resistors we can get the geometric scaling necessary to reach 75dB of thermal attenuation.

After the mupor is a metal mesh filter, which is an antireflection coated stack

of thin metal patterned plastic sheets like the IR shaders. The metal mesh filter is reflective in the infrared and transmissive at 150GHz. It acts like both a reflective series resistance and absorbing shunt resistance sunk to 50K.

At 4K, the power has been dramatically reduced by the 50K filtering and the 4K aperture sees an effective temperature of 100K. Metal mesh filters and IR shaders are used at the entrance to the 4K tube, and the thick polyethylene lenses in the optics tube act as absorbing elements. A temperature of 6K is presented to the focal plane. This temperature would still overload the cooling capacity of the sub-kelvin refrigerator, because the lenslet array of POLARBEAR acts like a corrugated black absorber. This emissivity is reduced by a final low pass metal mesh filter mounted to the intermediate cooler of the sub-kelvin refrigerator at 350mK. The cutoff is chosen to be just above the high edge of the POLARBEAR 150GHz band.

2.7 Observing Strategy

2.7.1 Sky Patches

For its first season, POLARBEAR observed three patches of sky, each with an area of 9 square degrees. These patches are called RA23, RA12 and RA4p5, named for their right ascension in hours. The patch locations of RA23 and RA12 were chosen to overlap with patches from the Herschel ATLAS survey at submillimeter wavelengths, to cross correlate with CMB lensing reconstruction maps. RA23 was shifted by five degrees in right ascension after it was found it contained a bright 243 \pm 42mJy source, PLCKERC143 G013.80-62.93[13]. The RA4p5 patch was chosen to overlap with a patch observed by QUIET, but this was never used for cross correlation. The locations of all patches were chosen to lie in particularly low dust regions of the sky, as determined by the 100 μm FDS dust map. RA23 and RA4p5 lie in the southern hole, while RA12 is on the other side of the galaxy, because it is the best location to observe while RA23 and RA4p5 have set below the horizon.

The declination of the patches was chosen to maximize the time they are above the horizon, while not observing patches that go too high in elevation. On the high elevation range, the performance of the pulse tube cooler degrades, leading to marginal operation of the SQUIDs.

2.7.2 Constant Elevation Scan

The POLARBEAR observing strategy is built around Constant Elevation Scans (CES). For 15 minutes, the telescope scans at constant elevation over a constant range in azimuth, corresponding to 3° on the sky. It scans at a constant velocity of $0.75^{\circ}/s$ on the sky. The turnarounds where the telescope accelerates and switches direction are discarded from data analysis. Because of the Az/El mount, the speed in azimuth

and throw in azimuth increase with elevation to maintain a constant scan speed and throw on the sky. Each left or right sweep of the telescope is called a subscan, and there are 100-200 subscans in one CES, depending on the elevation.

The constant elevation and azimuth throw pattern is critical to POLARBEAR for rejection of scan synchronous signals. When scanning with the CES pattern, telescope vibrations and sidelobes repetitively inject the same signals into the bolometers. Any component which is non repetitive acts as noise which is rejected later in the analysis through day-day cross spectrum. Meanwhile, the CMB signal from the sky changes due to the rotation of the earth. This allows us to separate scan synchronous signals from sky signal. An exception is patterns on the sky which are constant along the right ascension direction; these modes can not be distinguished from scan synchronous signals and are assumed corrupted and filtered out in data analysis. In order to maximize the orthogonality of the scan synchronous modes and the sky modes, the CES duration is pushed to as long as possible such that as much of the sky patch is observed in one CES as possible.

The center of each CES is chosen so that the center of its coverage on the sky coincides with the location of the target patch. This is calculated as the location of the patch in 7.5 minutes from the start of the CES, half the duration of the CES. Because the field of view of the telescope, 2.4°, is very close to the throw of the telescope, this leads to a bell like spatial distribution of observing time on the sky, with very low noise in a CMB map at the center which rapidly increases outwards.

At a typical 60° observing elevation, the scan speed of the telescope in azimuth is $1.5^{\circ}/s$. A high scan speed is desirable because any excess low frequency noise which is independent of scan speed will be pushed to lower spatial frequencies in map space the faster the telescope scans. This is limited by the turnaround time, which in practice is limited by a combination of the maximum acceleration of the Huan Tran Telescope, $2^{\circ}/s^2$, and the bandwidth of the servo. Pushing the turnaround too aggressively in acceleration and rate of change excites oscillations in the servo and undesireable jerking motion of the telescope. The turnaround profile of the telescope uses linear ramps in acceleration up to the maximum acceleration, with a max rate of change of acceleration of $5^{\circ}/s^3$. Given this turnaround profile, the scan speed was chosen to compromise between low frequency noise and observing efficiency. The chosen speed leads to a relationship between time domain frequency and ℓ of 500/Hz and a constant velocity observing efficiency of 60%.

The azimuth scan pattern of the telescope is pregenerated as a set of 10 scans, each for a 5° wide band of elevation from 30° to 80°. Certain scan speeds generate vibrations, suspected to be from the azimuth motor speed reducer, which excite mechanical resonances in the telescope which show up in bolometer data as excess microphonic noise. These speeds were avoided by manually tweaking the scan speed away from the nominal $0.75^{\circ}/s$ sky scan speed for the bad elevation range. At very high elevation, where reaching the target sky scan speed would cause the azimuth scan speed to exceed the maximum azimuth speed of the telescope, the azimuth speed is clamped to $3.8^{\circ}/s$.

CES observations are performed on a patch down to 30° in elevation. This was initially 40° , but 30° was found to give identical noise performance. At lower elevations, the airmass seen by the telescope increases with the cosecant of the elevation, which increases detector loading and photon noise.

2.7.3 Science Observation

A continuous block of scans observing one patch for the day is called an observation. An observation is typically eight hours long, lasting from the time a patch rises above the 30° horizon, or scanning begins, until the patch sets below the horizon, or scanning ends. All interesting activities of the telescope during the observation are called scans. The critical scans during science observations are stimulator stares, elevation nods, detector tuning, and constant elevation scans. The scans are arranged into one hour blocks. At the beginning of the block, detectors are tuned, then stimulator stares and elevation nods are performed. Following this there are four CESes, and finally another elevation nod and stimulator stare. One hour between tuning was chosen to keep the detectors close to their optimal operating point, while not losing too much potential observing time retuning. The stimulator and elevation nod calibration scans are performed before and after the CES block to allow linear interpolation of detector gains across the hour.

2.8 Stimulator

The POLARBEAR stimulator is a chopped thermal source used to derive relative calibration. A 700C oven sits behind the secondary mirror. The receiver can see the oven through a 6.35mm light pipe passing through secondary mirror. The temperature of the oven and the ambient chopper temperature are measured with thermocouples and regulated for long term stability. The chopper is a two blade fan covered in AN-72 absorber, which can spin at 2-24Hz for 4-48Hz modulation of the chopped signal. The chopper is driven by a miniature stepper motor. The stimulator has an optional polarizer which can be mounted on the far end of the light pipe, and the light pipe can be rotated by a separate geared down stepper motor at 1.7Hz. The stimulator was built by Takayuki Tomaru at KEK.

In addition to the oven, the stimulator chopper interrupts a photosensor for synchronous demodulation of the bolometer timestreams. The photosensor is fed into a computer data acquisition card which samples at 40kHz. Edges are detected and the arrival time of the edge in 40kHz clock cycle counts are stored and transferred to the main control computer to be archived along with bolometer data. In data analysis, the speed of the chopper is assumed to be constant and a constant speed model is fit to the list of chopper edge arrival times by minimizing the least squares residual between predicted and measured edge arrival times. The RMS residual of the timing error is typically $30\mu s$. Much higher RMS residuals are indicative of stimulator failures. One observed failure mode was slipping of the chopper wheel on its shaft, which is secured with a set screw. The chopper continued to spin but with much larger speed instability, which would have caused large apparent gain variations in bolometer response. This was caught by setting a threshold on chopper RMS timing residuals.

The stimulator is observed at the beginning and end of every one hour block of CES scans. At frequencies of 4, 12, 20, 28, 36 and 44 Hz the stimulator is observed while the telescope is static for 30 seconds each for a total 2.5 minute observation time. The sweep of frequencies allows measurement of the bolometer time constant. While we have phase information from the chopper, this information is not used for the first season data. The apparent temperature of the stimulator at one frequency is measured by minimizing the squared residuals of a model consisting of sine plus cosine waves at the frequency of the chopper. Before fitting, the bolometer timestream is high pass filtered by fitting and subtracting a 10th order polynomial from the 30 second stimulator stare at one chopping frequency. The root mean squared amplitude of the sine and cosine components is used as the effective stimulator temperature. The uncertainty on the fit is estimated by assuming that the residual is entirely white detector noise, for which the expected RMS residual (i.e. reduced χ^2) is 1. An example of the fit bolometer response for a section of one bolometer's observation of the stimulator is shown in Fig. 2.5.

Given a collection of stimulator measurements at many frequencies for one bolometer, we fit a one pole time constant model. For m(f) the magnitude response at some frequency, g the detector gain, f_{3db} the bolometer time constant represented as 3dB roll off frequency, and n(f) the error on the measurement of the magnitude, the model is:

$$m(f) = \frac{g}{\sqrt{1 + \left(\frac{f}{f_{3dB}}\right)^2}} + n(f)$$
(2.8)

A typical time constant fit is shown in Fig. 2.6. At these frequencies, the bolometer response is fit well by a single pole model. Unfortunately, the stimulator does not spin slowly enough to measure inside the science band (0.25-3Hz) and look for additional poles in the response at low frequencies. We did not look for variation in bolometer response with frequency below 4Hz. A typical value for g, the effective temperature of the stimulator as seen by a bolometer, is $15-30mK_{RJ}$. The effective temperature is maximum at the center of the focal plane and tapers off towards the edges. There is a slight polarization to the stimulator, observable as a systematic difference in gain between the two bolometers in a pixel which varies slowly with focal plane position. This is confirmed by rotating the half wave plate and seeing a variation in stimulator response that varies sinusoidally as twice the half wave plate angle. Because of this intrinsic polarization of the stimulator, the apparent stimulator temperature is measured in many half wave plate orientations.

The effective stimulator temperature for each bolometer was measured by bootstrapping to planet observations, using the amplitude and size of the same elliptical Gaussian fits used to determine the beam centroids.



Figure 2.5: In blue, example response of a bolometer to the stimulator chopper. In green, the fit to the bolometer response. X axis is time and y axis is bolometer response. Chopping rate is 4Hz.

2.8.1 Stimulator measurement of demodulator phase

Because our bolometers are biased at 300-1200kHz, there are two possible phases of readout, called in phase (I) and quadrature (Q). We use the stimulator to determine the correct phase. The default procedure for phase selection is to measure the phase of the current in the bolometer while the bolometer is overbiased above the transition, before nulling. For an ideal bolometer with no series parasitics, this will be the correct phase to readout the bolometer once the bolometer is in the transition. Series parasitics will rotate the phase as the bolometer resistance changes. In the correct phase, we have maximum responsivity, while in the wrong phase, we have no responsivity and excess Johnson noise from the bolometer resistance, which is only suppressed in the phase with electrothermal feedback. The responsivity falls as $\cos(\theta)$ and the excess Johnson noise term grows as $\sin(\theta)$.

Under normal science operation, we only record one phase of current from each bolometer to reduce the volume of data collected. We took one dataset with both phases and stimulator running to calculate a demodulator phase correction. The correct phase is calculated by the relative amplitude and sign of the stimulator response in the two phases; in the ideal phase the amplitude is maximized. The distribution of


Figure 2.6: Example response of a bolometer to the stimulator chopper spinning at five frequencies. The fit to the chopper reference at 28Hz and 36Hz failed, and there is an extra measurement at 48Hz for this observation. Blue shows the measured magnitude response and error, and green is the single pole model fit. The 3dB frequency for this bolometer is 20Hz which is slow, a typical bolometer responds at 50Hz. Reduced chi squared for the fit is good at 0.94.

measured angles before phase correction is shown in Fig. 2.7. The tight distribution of angles about zero demonstrates that the overbiased current phase is a good default algorithm for phase selection.

2.9 Data Management

Data from the telescope, including bolometer and telescope encoder timestreams, is collected into a frame based archive format by GCP, the telescope control software. The data is split into 411MB archive files containing 500 frames of 1.049 seconds each (8.7 minutes, 0.8MB/s). The archive files are compressed with gzip by 75% down to 100-120MB each, for a compressed data rate of 200kB/s. An MD5 checksum is computed for each archive file and stored separately to verify integrity of the data. The archive files are recorded locally to solid state drives at the telescope to avoid the reduced lifetime of conventional spinning disks at high altitude. The solid state drive at the observatory has sufficient capacity to store data for two weeks. Data is transferred to San Pedro de Atacama (elevation 2400m) over a 150Mbit line of sight microwave link and replicated on three physically distinct hard drives in the low altitude control center. The MD5 checksums are verified after transmission, and if shown to match, the data is deleted from the observatory solid state disk.

The bandwidth of the internet link between the POLARBEAR low altitude



Figure 2.7: Histogram of measured ideal demodulator angles relative to overbiased current phase angle.

control center in San Pedro de Atacama and Berkeley is sufficient to transfer the compressed archive files directly over the internet. The files are stored on a server at Berkeley and then replicated to supercomputer facilities at NERSC (Oakland, California) and KEKCC (Tsukuba, Japan).

2.9.1 Archive Table

Rather than generating a catalog from executed schedule files, observations are cataloged by processing the archive files. This way the catalog consists of what we have rather than what we intended to have. First the archive files are parsed by PbArchive, a library written by Jacob Howard. For each frame in the archive file, or roughly once per second, data describing the current state of the telescope is extracted: Name of current scan file, name of the current field, the UTC time, feature bits set describing the current state of the telescope and intended observation, and the half wave plate position. This data is saved in plain text called the archive index.

The archive index contains all the information necessary to construct the archive table, the catalog of all observations performed by the telescope. The start of an observation is detected by the observation in progress bit in the features flag switching from false to true, and vice versa for the end of an observation. An observation is a block of related scans, such as an eight hour continuous observation of a single CMB patch at many elevations, including calibration. The scans are delineated by a change in the source being observed or a change in any of the features. The observations and scans are named by the time of the beginning of the observation or scan in a human readable text format: YYYYMMDD_HHMMSS. The name of all archive files contributing data to a given scan is also listed in the archive table.

This archive table can now be searched for data matching a pattern of interest, for example, all scans of the planet Jupiter for beam calibration, or all scans of a particular CMB patch between June 2012 and July 2013.

2.9.2 Repacking

The frame based archive file format is convenient for writing data, but inconvenient for reading. When reading data from disk, we often read data from an entire observation for a single bolometer, which requires many seeks to skip through each frame. This is particularly slow at the supercomputer where data is not stored locally on the computing machine, creating very high latencies for each disk operation. Furthermore, the data from a single scan can be spread across several archive files. To avoid this extra overhead during data analysis, the data is repacked to the HDF5 file format[14] using the Python h5py library. The HDF5 repacker loops through the archive table, finds all archive files contributing to a single scan, then creates a single HDF5 file from that data named by the scan date, creating a one to one map between scans in the archive table and HDF5 files. This simplifies the design of the map maker; it does not need to request any information directly from the archive table. Given a single HDF5 file, the map maker creates a single map for that CES.

During unpacking the data is left completely unchanged except in one respect. There are two types of data dropouts common in the archive files. One is a dropped UDP packet from a single DfMUX motherboard to the control computer. This loses 8 samples of data (0.04s) from all 32 bolometers read out by that motherboard. The second is a drop out in communication between the telescope encoders and the control computer. This creates gaps up to a second long in the data. During unpacking, the array of data is expanded using the recorded timestamps, and the gap is filled with a linear interpolation from one sample past either end of the gap. A mask vector is created to indicate which samples are interpolated, and the masked samples are not used in science analysis.

Chapter 3

Transition-Edge Sensors Bolometers

3.1 Bolometer tuning

The POLARBEAR bolometers are AC biased at a fixed RMS voltage which drives a current through the bolometer. Bolometer current varies with the precipitable water vapor in the atmosphere and the elevation of the telescope. At a fixed voltage, the changing optical power changes the bolometer operating temperature and resistance in the superconducting transition. The operating point has a strong effect on the bolometer responsivity and noise-effective temperature (NET) because of the changing steepness of the transition. For POLARBEAR bolometers, which operate at relatively low loopgain and effective transition steepness, this effect dominates over the voltage term in the responsivity. So, to improve the repeatability of bolometer NET, we bias to a fixed fraction of normal resistance.

The standard algorithm to bias the bolometers to a fixed fraction of normal resistance uses an IV curve. The IV curve starts from an overbiased point, with enough voltage applied to the bolometer to saturate it and warm it above the superconducting critical temperature. Then the voltage is stepped down in small incrments, the current is measured, and if the bolometer resistance has dropped to the target resistance, the algorithm is terminated. Finally, a second synthesizer generates a current which cancels out the non-varying component of the carrier current in the SQUID input coil. This nulling current reduces the dynamic range requirements of the SQUIDs when multiplexing many bolometers on one SQUID.

3.1.1 Ideal bias bolometer response

An ideally DC voltage biased bolometer is governed by an energy balance equation. The bolometer is operated out of thermal equilibrium at a temperature above the environment. Heat is injected through optical power P_o from the sky and electrical dissipation P_e from the bias voltage, and it leaves the bolometer P_t through a weak thermal link to the bath. The bolometer energy is E.

$$\frac{dE}{dt} = P_o + P_e - P_t \tag{3.1}$$

The electrical power is

$$P_{e} = \frac{V^{2}}{R} \approx P_{e0} + \delta P_{e} = \frac{V^{2}}{R_{0}} - \frac{V^{2}}{R_{0}^{2}} \delta R$$
(3.2)

Given a positive thermal coefficient of resistance to temperature, Eq. 3.2, the voltage bias stabilizes the bolometer temperature at the superconducting transition temperature T_c due to the negative feedback from the minus sign in the above equation.

The thermal power is linearized about the transition temperature

$$P_t = P_{t0} + G\delta T \tag{3.3}$$

The thermal energy in the bolometer is linearized about the transition temperature, and the bolometer balance equation for first order perturbations becomes

$$C\frac{\partial\delta T}{\partial t} = -\frac{V^2}{R_0^2}\delta R - G\delta T + \delta P_o \tag{3.4}$$

Resistance and temperature are related by the details of the superconducting transition

$$\delta R = \alpha R_0 \frac{\delta T}{T_0} \tag{3.5}$$

Fourier transforming 3.2, we find that the temperature excursions of the bolometer are described by

$$\frac{\delta T}{\delta P_o} = \frac{1}{G(1+\mathcal{L})(1+i\omega\tau)} \tag{3.6}$$

Where the temperature fluctuations are suppressed by the electrothermal loop gain ${\cal L}$

$$\mathcal{L} = \frac{\alpha V_0^2}{R_0 T_0 G} \tag{3.7}$$

And the electrothermal time constant τ is accelerated by the loopgain

$$\tau = \frac{C}{G} \frac{1}{1 + \mathcal{L}} \tag{3.8}$$

Finally, the responsivity from optical power fluctuations to readout current is

$$\frac{\partial I}{\partial P_o} = -\frac{1}{V} \frac{\mathcal{L}}{1 + \mathcal{L}} \frac{1}{1 + i\omega\tau}$$
(3.9)

Notably, the TES bolometer is a true power metering device, if the loopgain is high and the voltage bias is known accurately.

3.1.2 Bolometer bias impedance

In an AC biased system, the parasitics significantly degrade the voltage bias stiffness. When biased with a finite input impedance, rather than a perfect voltage bias, the TES bolometer responsivity becomes resistance dependent. The TES responsivity is given by the chain rule

$$\frac{\partial |I|}{\partial P_o} = \frac{\partial |I|}{\partial R} \frac{\partial R}{\partial T} \frac{\partial T}{\partial P_o}$$
(3.10)

For an imaginary parasitic impedance $X = i\eta R$, the bolometer current is

$$I = \frac{V}{R + iX} \tag{3.11}$$

$$|I| = \frac{V}{\sqrt{R^2 + X^2}}$$
(3.12)

$$\frac{\partial |I|}{\partial R} = \frac{-V}{R^2} \frac{1}{(1+\eta^2)^{\frac{3}{2}}}$$
(3.13)

The change in resistance with temperature is parameterized by α

$$\frac{\partial R}{\partial T} = \alpha \frac{R_0}{T_0} \tag{3.14}$$

The bolometer power balance is

$$P_t = P_e + P_o \tag{3.15}$$

Expand the thermal power to first order

$$P_t = \bar{G}T_0 + GT \tag{3.16}$$

Which influences the resistance by α

$$R = R_0 (1 + \alpha \frac{T}{T_0}) \tag{3.17}$$

Electrical power in the bolometer is

$$P_e = |I|^2 R = \frac{V^2 R}{R^2 + X^2} \tag{3.18}$$

Define the usual loopgain, parasitic free loopgain \mathcal{L}_0

$$\mathcal{L}_0 = \frac{\alpha V^2}{R_0 G T_0} \tag{3.19}$$

Then the modified loopgain of the bolometer with parasitics is

$$\mathcal{L} = \mathcal{L}_0 \frac{1 - \eta^2}{(1 + \eta^2)^2} \tag{3.20}$$

The temperature fluctuations of the bolometer to optical power is suppressed by the modified loopgain

$$\frac{\partial P_o}{\partial T} = G(1 + \mathcal{L}) \tag{3.21}$$

Finally, the responsivity contains resistance dependence due to the parasitics:

$$g = \frac{\partial |I|}{\partial P_o} = -\frac{1}{V} \frac{\sqrt{1+\eta^2}}{1-\eta^2} \frac{\mathcal{L}}{\mathcal{L}+1}$$
(3.22)

As $\eta \to 1$, the responsivity goes to infinity. This manifests as an increase in noise deep in the transition, and eventually, the bolometers latch superconducting.

3.1.3 Parasitic Gain Stability

This parasitic term causes variable gain with loading due to the changing bolometer resistance.

The fractional change in bolometer gain including parasitics due to a changing load, keeping loop gain and voltage bias fixed, is $\frac{1}{g} \frac{\partial g}{\partial P}$.

$$\frac{1}{g}\frac{\partial g}{\partial P} = \frac{1}{g}\frac{\partial g}{\partial \eta}\frac{\partial \eta}{\partial P}$$
(3.23)

Expand this about $\eta = 0$

$$\frac{1}{g}\frac{\partial g}{\partial P} = \frac{1}{P_e}3\eta^2 \tag{3.24}$$

Typical changes in atmosphere loading over an observation can be 0.5pW. For 10pW bolometers at a parasitic to bolometer resistance ratio of $\eta = 0.5$, this is only a 2% change in gain.



Figure 3.1: Enhancement of responsivity due to parasitic impedance as a function of bolometer resistance as described in Sec. 3.1.3.

3.1.4 Bias bandwidth

In either DC or AC biased system, even with no parasitics at DC or baseband DC, the bias will have a finite bandwidth. In either system this can be modelled as a series inductance to the bolometer at baseband. For the AC system, the impedance of the bias is given by the impedance of an RLC resonator.

$$Z(\omega) = i\omega L + \frac{1}{i\omega C} + R \tag{3.25}$$

The bolometer is biased at the frequency of the zero in the case of R = 0, which is $\omega_b = \frac{1}{LC}$. Expanding the impedance about this frequency, we find that the equivalent inductance determining the bias bandwidth of the AC biased system is double the inductance of the physical inductor.

$$Z(\omega_b + \omega) \approx R + 2i\omega L + O(\omega^2) \tag{3.26}$$

The electrical system has two elements, an inductor and a bolometer in series. The total voltage is V and the voltage across the two elements are V_L and V_b respectively. The entire system has two effective degrees of freedom. They can be chosen to be the bolometer current and temperature.

The temperature evolves according to the energy balance equation

$$C\partial_t \delta T = \delta P_e - \delta P_t \tag{3.27}$$

The current evolves as

$$L\partial_t \delta I = -\delta V_b \tag{3.28}$$

This reduces to a pair of differential equations. The stability can be analyzed by fourier transforming and solving for eigenvalues of the resulting linear system[15]. The resulting stability criterion is requires that the electrical time constant $\frac{R}{2L}$ of the bias circuit is smaller than the electrothermal time constant of the biased bolometer:

$$\frac{\frac{C}{G}}{1+\mathcal{L}} < \frac{R}{2L} \tag{3.29}$$

3.2 Sensitivity

Noise in bolometric measurements has three sources: Thermal, photon and readout. We express the noise in terms of NEP, noise-equivalent power. Usually we use units of $aW/\sqrt{(Hz)}$, or the power in attowatts that gives a signal to noise of one in a measurement that has a 1Hz bandwidth, or equivalently is two seconds long. Along with the optical responsivity of the detector, dP/dT, we can calculate NET, the ultimate mapping speed on the sky. In this section we describe the design targets and then report actual values.

Thermal noise is due to statistical fluctuations in phonons flowing through the heat link from the bolometer island to the bath. For POLARBEAR bolometers, which are substantially out of thermal equilibrium, the phonon noise is given by Mather[16].

$$NEP_{thermal}^2 = 4kG_c T_c^2 \gamma \tag{3.30}$$

 G_c is the change in power of the link due to changes in temperature of the hot end of the link. γ is a term that accounts for the reduction in thermal noise due to some parts of the link being colder than T_c . For POLARBEAR detectors, which are dominated by phonon conduction with a thermal conductivity proportional to T^3 and the critical temperature T_c at twice the bath temperature T_b , $\gamma \approx 0.5$. We designed for a 2x electrical to optical power ratio, or 18pW saturation power. This leads to $NEP_{phonon} = 32aW/\sqrt{Hz}$

Similar to phonon noise, photon noise is due to statistical fluctuations in photons arriving at the load resistor. Photon noise in a single mode is given by Zmuidzinas[17], and after translating photon occupation numbers to power, reads

$$NEP_{photon}^2 = 2h\nu P + 2\frac{P^2}{\Delta\nu}$$
(3.31)

Finally, readout noise is due to electrical noise, expressed as effective current noise at the input to the SQUID.

$$NEP_{readout} = NEI_{readout} \frac{\partial P}{\partial I}$$
(3.32)

The design value for this was $7pA/\sqrt{Hz}$. For an 18pW 1 ohm bolometer, the voltage bias (and inverse responsivity to power) is $4.24\mu V$. This gives $NEP_{readout} = 30aW/\sqrt{Hz}$.

The total design NEP of the bolometer is

$$NEP^{2} = NEP_{thermal}^{2} + NEP_{photon}^{2} + NEP_{readout}^{2}$$

$$(3.33)$$

And it is $NEP = 71aW/\sqrt{Hz}$.

The *NEP* is related to the mapping speed *NET* by the relationship between temperature on the sky to power dissipated on the island. *NET* is expressed in units of $T_{CMB}\sqrt{s}$, where the time units now refers to seconds of integration time rather than Hz of bandwidth. This carries with it a factor of $\sqrt{2}$.

$$NET = \frac{1}{\sqrt{2}} \frac{NEP}{\frac{\partial P}{\partial T}}$$
(3.34)

For a Rayleigh-Jeans source, the power in a single mode is

$$\frac{\partial P_{RJ}}{\partial T_{RJ}} = k_B \eta \Delta \nu \tag{3.35}$$

Where η is the optical efficiency of the telescope including truncation at the aperture stop.

The CMB is cold enough that at 150GHz, the spectrum has softened relative to the Rayleigh-Jeans. Integrating the power seen in fluctuations from Eq. 2.7 over the 38GHz band of the instrument, with a measured 30% optical efficiency, this is $0.10 \text{pW}/K_{CMB}$.

This gives the design sensitivity for a single detector $NET = 500 \mu K \sqrt{s}$ and a best possible array mapping speed of $14 \mu K \sqrt{s}$.

3.2.1 Measured Sensitivity

The actual sensitivity of POLARBEAR was $22.8\mu K\sqrt{s}$. This was due to less than 100% yield, 20% higher readout noise in the field, and higher than optimal saturation powers for many wafers. In addition to the additional phonon noise from

Wafer Name	P_{sat} (pW)	$NET_{CMB}(\mu K\sqrt{s})$	Warm Pixel Yield (%)
8.2.0	24	660	85
9.4	14	500	95
10.1	39	640	92
10.2	18	530	95
10.3	20	540	97
10.4	20	540	98
10.5	15	510	86

Table 3.1: POLARBEAR bolometer properties

high G_c , the high saturation power increases voltage bias, reduces responsivity, and increases the effective readout noise.

The measured median sensitivities of the wafers is in Table 3.2.1.

3.2.2 AC biased signal to noise

In the frequency multiplexing system, it is not immediately clear whether a system which can provide some readout noise in pA/\sqrt{Hz} will give the same NEP for DC and AC biased bolometers. Here we show that it is the same; there is no readout noise penalty to AC biased bolometers.

The current in the bolometer x(t) is a modulated function s(t) of the carrier tone $\cos(\omega t)$.

$$x(t) = s(t)\cos(\omega t) \tag{3.36}$$

The power dissipated by the bolometer is

$$x^{2}(t) = s^{2}(t)\frac{1}{2}(1 + \cos(2\omega t))$$
(3.37)

Time averaging to the low frequency portion, the RMS current is

$$RMS(x(t)) = \frac{1}{\sqrt{2}}s(t) \tag{3.38}$$

At the low frequencies where our science data appears, assuming an ideal infinite loopgain TES, the electrical power $P_e(t)$ will exactly make up for the difference between thermal $P_t(t)$ and optical power $P_o(t)$.

$$P_e(t) = P_t - P_o(t) = V_{RMS}I_{RMS} = V_{RMS}\frac{1}{\sqrt{2}}s(t)$$
(3.39)

$$s(t) = \frac{\sqrt{2}}{V_{RMS}} P_e(t) \tag{3.40}$$

$$x(t) = \frac{\sqrt{2}}{V_{RMS}} P_e(t) \cos(\omega t) + n(t)$$
(3.41)

The current is demodulated with carrier $\cos(\omega t)$.

$$y(t) = \cos(\omega t)x(t) = \frac{\sqrt{2}}{V_{RMS}}P_e(t)\frac{1}{2}(1+\cos(2\omega t)) + n(t)\cos(\omega t)$$
(3.42)

The high frequency components are eliminated with a low pass filter.

$$y(t) = \cos(\omega t)x(t) = \frac{1}{\sqrt{2}V_{RMS}}P_e(t) + n(t)\cos(\omega t)$$
(3.43)

The variance of the demodulated timestream σ_y^2 is a function of the variance of the power σ_p^2 and readout noise σ_n^2 .

$$\sigma_y^2 = \langle y^2(t) \rangle = \frac{1}{2V_{RMS}^2} \sigma_p^2 + \frac{1}{2} \sigma_n^2$$
(3.44)

In the demodulated timestream, the signal to noise is

$$SNR = \frac{1}{V_{RMS}} \frac{\sigma_p}{\sigma_n} \tag{3.45}$$

This is identical to a DC biased bolometer biased to the same resistance. For a SQUID with a white current noise, there is no noise penalty to AC biasing the bolometers.

3.3 TES Readout

The TES detector is biased with a constant voltage, and varies its resistance in response to varying optical power. The resistance variation can be read by sensing the current in the TES. This current sensing is done with a SQUID, a low noise ammeter with a low input impedance, placed at 4K. In order to reduce the wiring count, several TESes are multiplexed in the frequency domain.

TESes are frequency multiplexed by placing each in series with a resonant series LC filter. Each LC-bolometer set is then wired in parallel. Off the resonant frequency, the LC has high impedance, and on resonance, it has zero impedance. A voltage bias at a given frequency can then only bias a bolometer with an LC that has a matching resonant frequency. Power on the bolometer acts to amplitude modulate the current, which is summed for all the bolometers and fed into the SQUID. This requires two pairs of lines, the voltage bias and current sense. SQUID bias and voltage readout requires one more pair of lines going to 4K. Because the modulations are small, and the carrier or DC component of the modulation contains no science information, we

add one more pair of lines, called the nuller, which contains a sine wave that cancels out the non-amplitude modulated carrier current in the SQUID. This reduces the dynamic range requirements of the SQUID.

In total this is four pairs of lines running from room temperature to 4K for each SQUID. The current summing junctions which parallel the LC-bolometer sets are at milliKelvin, so only one differential pair is needed from 4K to milliKelvin for each comb, the set of bolometers read out by one SQUID. The name comb describes the appearance of a network analysis of the eight bolometer RLC circuit, with eight spikes, one for each resonance.

We are able to multiplex 8 bolometers on each SQUID. This is determined by the total bandwidth of the SQUID amplifier and the allowed channel spacing. Channel spacing is limited by the Q of the LC filters, because finite Q and frequency spacing leads to crosstalk, where current from one carrier leaks down the wrong bolometer and is modulated. This is slightly inflated by the tolerancing of the LC filters.

The L in the LC filters are lithographed flat spiral inductors fabricated at NIST. They have a vacuum inductance of $15.3\mu H$, and drop to $13\mu H$ on a ground plane. The inductors are made in a single layer process for cost and robustness reasons. This requires an extra wirebond to run from the center of the inductor back to the edge to avoid needing an extra layer and vias. Inductors are diced into chips with the eight inductors of one comb. The chip is rubber cemented on a standard fiber glass circuit board (the LC board) with bare copper pads for wire bonding. Under the chip is a strip of tinned copper. At millikelvin temperatures, the tin is superconducting. This provides the ground plane for the inductor. The ground plane provides a repeatable magnetic environment for the inductor to minimize variation in inductance, and crosstalk between nearby inductors. It must be superconducting because normal PCB copper has too much resistance and contributes too much loss or effective series resistance (ESR) to the resonant filter.

We screened all inductors used in POLARBEAR by installing them on a testing PC board with a standard set of capacitors. The board was designed to fit down the neck of a liquid helium storage dewar, and read out twelve chips simultaneously as twelve separate combs. The boards were rapidly dunked in liquid helium and characterized with an HP4195 network analyzer. Good inductor chips were selected on the basis of correct inductance, no dependence of inductance on drive amplitude, low ESR, and 8/8 inductors on the chip operational.

The capacitors in the LC filter were the component that was varied in order to vary the resonant frequency. To range the resonant frequencies from 300kHz to 1.1MHz, capacitance ranged from 1.6nF to 22nF. These capacitances were built out of stacks of commercial NP0 SMD capacitors in a 1206 package (120 mils x 60 mils). These capacitors are sufficiently stable in capacitance between 4K and 300K and can have sufficiently low ESR. Their initial tolerance is poor, so we purchase large quantities in several stock values and sort them to the nearest 10pF. The entire inventory is processed by a computer program to generate the minimal set of capacitor stacks that meet a list of design frequencies. These stacks are mostly two capacitors high and at most three high. Given the instructions from the computer program, the capacitors are then hand soldered onto the LC boards. Our design channel spacing is 75kHz, with a minimum acceptable frequency spacing of 50kHz. This gives us 12kHz of tolerance on the resonant frequencies. This was within the margin of error for the hand soldered, pre-sorted capacitors.

The LC boards, designed by Erin Quealy, have an identical hexagonal footprint to the detector tiles. This allowed them to be compactly stacked behind the focal plane. Each LC board has a flexible kapton layer which extends past the rigid fiberglass portion and carries a pair of wires for each bolometer. This acts as a cable which is routed up to the edge of the detector tile. Wirebonds connect the bolometers on the detector tile to the LC board flex section. The summing junction is on the LC board at the pin for the MDM connector that leaves to the SQUID. Placing the summing junction as close to the SQUIDs as possible minimizes parasitic inductance which increases electrical crosstalk between the bolometers. To reduce heat loads and the wiring count, the summing junction must be on the millikelyin side of the cables. The cables to the SQUIDs are a combination of differential microstrips in tinned copper on flexible kapton and niobium-titanium twisted pairs. The twisted pairs have very high inductance (25nH/inch) but low thermal conductance, so are used only as a thermal barrier. They run from 250mK to 4K, and are intercepted at the 350mK IC and 2K HEX. A 50 ohm resistor is soldered between the two MDM pins in a pair on the cold side of the pigtail. This resistor damps out a resonance in the flux locked loop.

3.3.1 SQUID amplifier

To reach background limited imaging performance, the first stage amplifier for the bolometer current must have a lower current noise and impedance than the bolometer. SQUIDs are the only candidate for this amplifier demonstrated to date. The SQUID used by POLARBEAR is fabricated at NIST with an input inductance of 150nH, an input current noise of 3.5pA/rtHz, and transimpedance of 600 ohms. In order to reach this high transimpedance, the SQUID is actually a series array of 100 SQUIDs, with input coils and SQUIDs in series. Using a series array opens up the possibility of trapped flux. If a single SQUID in the series array traps flux, then its response will be out of phase with the rest of the SQUIDs. This leads to spurious features in the voltage to flux curve which reduce linearity and dynamic range of the series array.

SQUID tuning

On the initial cooldown of the cryostat, the flux is trapped in the superconducting elements of the SQUID. This shifts the phase of the V-phi curves between the different SQUIDs in a series array, destroying their coherence and the amplification properties of the SQUID. We eliminate this by rapidly heating the SQUID above the T_c of

niobium (9K) to release the trapped flux and allowing them to cool again. This second cooldown of the SQUID happens very rapidly because the rest of the cryostat is very cold and the SQUIDs do not have time to trap flux.

After eliminating trapped flux, SQUIDs are tuned by taking a series of V-phi curves at different current biases. At very low current bias, the Josephson Junctions in the SQUID remain superconducting, and there is no voltage at the output of the SQUID as flux input is varied. At higher current biases, the SQUID voltage has a periodic response to the flux. The amplifier after the SQUID is very high gain, so to avoid railing the ADC, the measurement of V-phi curves is done in a nulling loop. Each step in the V-phi curve, software uses bisection to find the offset voltage to the first amplifier at the SQUID that gives zero volts in to the ADC. Once the V-phi curves are completed, the SQUID is biased at 10% higher current than the current that maximizes the peak to peak of the V-phi curve. The slight overbiasing is used to improve linearity, as the SQUID response becomes more sinusoidal at higher bias. Finally the flux bias is fixed at the point where the slope of the V-phi is maximized. The first amplifier after the SQUID is re-zeroed and the flux locked loop enabled to lock the SQUID at the operating flux point.

3.3.2 Flux locked loop

The 150nH input inductance of the SQUID input coil at a bolometer bias frequency of 1MHz has an impedance of $0.94j\Omega$, comparable to the $\approx 1\Omega$ bolometer impedance. This destroys the stiffness of the voltage bias. It also introduces a dependence on the system responsivity to the SQUID gain, which varies with SQUID temperature. To eliminate this impedance and the gain variability, the SQUID is placed in a feedback loop. The feedback loop consists of the SQUID, a 300K inverting amplifier, and a resistor leading from the amplifier output to the SQUID input coil. The open loop gain of the system is 12 at DC and is dominant pole compensated at 750kHz. This feedback loop limits the readout bandwidth available. Due to parasitics, such as the wire length between the SQUIDs and warm amplifier, there are physical limits to the stable bandwidth of the feedback. However, this loop gain and bandwidth was sufficient to read out 8 detectors on the comb with low parasitic impedance. Additionally, the feedback loop stabilizes the gain of the SQUID/amplifier system to the resistance of the feedback resistor. This eliminates dependence of the overall calibration of the system to the temperature of the 4K stage.

With the flux locked loop, the input impedance and gain stability of the SQUID is dramatically improved. There are three currents in the problem. I_i is the input current to the flux locked loop from the bolometers. I_f is the feedback current from the flux locked loop. I_l is the current in the SQUID input coil.

The current in the SQUID input coil is the sum of the feedback current and the bolometer current

$$I_l = I_i + I_f \tag{3.46}$$

The feedback current is given by the gain of the first stage amplifier g and the dominant pole compensation time constant τ

$$\frac{I_f}{I_i} = F = -\frac{g}{1+iw\tau} \tag{3.47}$$

Then the input impedance at the flux locked loop input Z_{fll} is the input coil impedance Z_l is suppressed by 1 - F

$$Z_{fll} = \frac{V_i}{I_i} = \frac{Z_l}{1 - F} = i\omega L \frac{1 + i\omega\tau}{(1 + g)\left(1 + i\omega\frac{\tau}{1 + g}\right)}$$
(3.48)

At low frequency, the flux locked loop input impedance has a negative real component which cancels out other parasitics and helps stabilize the bolometers.



Figure 3.2: Input impedance of a flux locked loop with dominant pole compensation at $f_{3db} = 750kHz$ and a DC loop gain of 12.

3.3.3 Rapid Biasing

The FIR filters used by the DfMUX have a long latency of about 1/2 second. With our initial algorithms for tuning detectors into the transition at the Cedar Flat test deployment, it took two hours to bias the entire array. This precluded changing the bias points to accomodate changing atmospheric conditions, which can be considerable even at the final deployment site. Our initial algorithm operated through serial IV, nulled, curves. A single bolometer would step down in voltage, and at each point the bolometer would be renulled, which takes a few measurements of current. The IV curve would be stopped when the bolometer reached the desired fractional resistance. To accelerate this, we parallelized and adapted the algorithm to take advantage of the constant power property of a TES in the transition. Parallelization was made possible by new DfMUX firmware with 16 channel readout. With 16 channels, in phase and quadrature current could be recorded for all detectors, allowing rapid nulling of all channel simultaneously.

The new algorithm dramatically reduced the number of steps required by noting that for an ideal, constant power TES, the voltage which will bring the TES to a target resistance is

$$V_{n+1} = \sqrt{P_n R_{target}} \tag{3.49}$$

This reduced the time to bias the array to a few minutes, and allowed us to rebias the detectors every hour, in response to changing atmospheric conditions.

Chapter 4

Half Wave Plate

4.1 Half Wave Plate

4.1.1 Half Wave Plate Theory

Rotating the polarization of a beam independently of the beam shape is the ideal way to modulate the beam response on the sky for the measurement of polarization anisotropies. It is superior to sky or instrument rotation because quadrupole components of the beam are spin 2, like polarization, so quadrupole beam non idealities (e.g. ellipticity) are not mitigated by sky rotation. It is superior to scanning because the half wave plate can be spun faster than the telescope can be moved. A fast spin of the half wave plate modulates all polarization data to high frequencies in the time domain away from excess low frequency noise.

An ideal half wave plate is a device which rotates the polarization of the beam with no other effects. A half wave plate is made from a birefringent material, with different indices of refraction along the two directions (\hat{x}, \hat{y}) in the plane of the plate. This provides a difference in phase delay for the electric field components E_x and E_y . For a normally incident electric field with polarization direction θ , the input electric field has components

$$E_x^i = E\cos(\theta) \tag{4.1}$$

$$E_u^i = E\sin(\theta) \tag{4.2}$$

The transmitted electric field acquires a phase delay $\exp(i\phi)$ for the \hat{y} component relative to the \hat{x} component.

$$E_x^t = E\cos(\theta) \tag{4.3}$$

$$E_y^t = E\sin(\theta)\exp(i\phi) \tag{4.4}$$

For an ideal half wave plate, the phase shift is $\phi = \pi$, so the effect is to mirror the orientation of the polarization about \hat{x} .

$$E_x^t = E_x^i \tag{4.5}$$

$$E_y^t = -E_y^i \tag{4.6}$$

This wave plate will map the Stokes components Q to Q and U to -U, i.e.

$$\begin{pmatrix} Q^t \\ U^t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} Q^i \\ U^i \end{pmatrix}$$
(4.7)

Because Q and U are spin 2 quantities, under rotation of the polarization states relative to the HWP, the action of a HWP rotated by angle θ is

$$\begin{pmatrix} Q^t \\ U^t \end{pmatrix} = \begin{pmatrix} \cos(4\theta) & -\sin(4\theta) \\ -\sin(4\theta) & -\cos(4\theta) \end{pmatrix} \begin{pmatrix} Q^i \\ U^i \end{pmatrix}$$
(4.8)

A polarized detector behind a spinning half wave plate will see the sky polarization modulated on sine waves with frequency four times the rotation rate of the half wave plate.

This is an idealized model which is exact only at a frequency where the waveplate has exactly π differential phase shift and the antireflection coating operates perfectly. In practice, this will not be the case. An on-axis electromagnetic calculation for a single layer quarter-wave antireflection coating and birefringent crystal HWP operating over a finite band can be better modelled as this Mueller matrix:

$$\begin{pmatrix}
t & r & 0 & 0 \\
r & t & 0 & 0 \\
0 & 0 & c & -s \\
0 & 0 & s & c
\end{pmatrix}$$
(4.9)

Calculation of this matrix is detailed in Appendix A. For POLARBEAR , with HWP physical properties

- Refractive index of sapphire ordinary axis 3.047
- Refractive index of sapphire extraordinary axis 3.361
- Thickness of Sapphire 3.16mm
- Refractive index of TMM3 antireflection coating 1.8197 (measured at 1.2K by Erin Quealy)
- Thickness of TMM3 antireflection coating 0.2748mm

The predicted t,r,c terms for the POLARBEAR HWP as a function of frequency for a monochromatic illumination source are shown in Fig. 4.1.



Figure 4.1: Predicted HWP mueller matrix elements as a function of frequency. From top to bottom, t, the I to I and Q to Q term, r, the I to Q and Q to I term, and c, the U to U term. For an ideal HWP, t=1, r=0, c=-1.

HWP Mueller matrix element	POLARBEAR design value	ideal value
\mathbf{t}	0.990	1
r	0.00081	0
с	-0.959	-1

Table 4.1: POLARBEAR HWP Mueller matrix elements

For the POLARBEAR design band, 148GHz \pm 19GHz, the effective t, r, c terms are given by an integral over the detector bandpass and the derivative of the CMB spectrum by the CMB temperature. The expected values of the t, r, c terms for the POLARBEAR design is shown in Table 4.1.1.

4.1.2 Half Wave Plate Prototype

At room temperature, sapphire has a large emissivity and a large differential emissivity, leading to about 1K and 2K additional loading in the two crystal axes of the wave plate. The temperature difference injects a signal into the bolometers at twice the rotation frequency (or 2f) of the wave plate, possibly driving the bolometers nonlinear. To eliminate this differential signal, we pursued development of a cryogenic half wave plate. Cooling sapphire drops its emissivity dramatically. This provides an additional potential benefit in reducing the total magnitude of 4f signals (which). We developed the POLARBEAR HWP in 1/3 small scale with a 3.8 inch aperture. This size was chosen because of the reduced cost of the components and in order to fit the mechanism into a standard 8 inch IR labs dewar. Most testing occurred at 77K, because the hold time of this dewar running under just liquid nitrogen was much longer, simplifying testing, and because 77K is cold enough to reach the low emissivity regime of sapphire.

Bearing development

The challenge to mechanical engineering at cryogenic temperature is the lack of fluid lubricants, the basis of modern mechanical engineering. The cold rotating half wave plate mechanism requires several rotating bearings to support moving elements. One large ball bearing supports the wave plate, with a clear aperture, and 4 miniature ball bearings support a belt drive wheel and an idler wheel. For the miniature ball bearings, we used a commercial, off the shelf technology from Champion Bearings Inc, part number SR2CBVRT7. These bearings have dimensions 1/8" ID 3/8" OD and 0.15" width. They are unsealed, and use stainless steel races, silicon nitride balls, a Vespel SP-1 retainer, and are tungsten disulphide (WS₂) dry coated for lubrication. Designed for vacuum operation, they use materials which work well at low temperatures. We tested them up to 100 million revolutions cold without failure.

The starting point for the large, clear aperture wave plate support bearing was similar. The base ball bearing is provided by Silverthin bearings, part number SC120CP0 (12" aperture) for the full scale half wave plate mechanism and SC040CP0 (4" aperture) for the small scale. These bearings have a 3/8" x 3/8" cross section and 3/16" diameter balls. The stock steel balls we replaced with silicon nitride balls to avoid cold welding of steel on steel in the absence of fluid lubrication. With 0.1875" diameter balls, the full scale mechanism seizes on cooling, though differential cryogenic contraction between steel and silicon nitride is too small to explain this. This problem was alleviated by undersizing the balls by 1 mil to 0.1865". The bearings are packed in oil from the manufacturer which we clean with 1,1,1-trichloroethane followed by acetone and methanol washes. Because the races of the bearing are 52100 steel, not a stainless steel, in each case, the solvent is blown off with dry nitrogen gas to prevent water from condensing and rusting the surface. Finally, the bearings are lubricated by dusting the balls with a paint brush dipped in a dry molydenum disulphide (MoS₂) powder, which has similar properties to WS₂.

The stock retainer of the bearing is brass. With out lubrication, the brass is gradually torn from the retainer by the motion of the balls and the races relative to the retainer. Rather than flowing, the brass appears to gall, and chips are torn off which gradually seize the bearing. In two tests, brass bearing retainers survived 7.5 million revolutions and 5 million revolutions, but produced copious chips which guaranteed eventual bearing failure. Based on research for oil free vacuum bearings[18], we solved this problem by plating 50 microinches of gold on the brass retainer following MIL-

G-45204C Type III. In one test, a bearing survived 3 million revolutions, where the test was ended due to a drive failure. This bearing showed none of the chipping of the plain brass retainer. The balls and ball contact area of the races acquired a light gold sheen shown in Fig. 4.2 and 4.3. The gold is much softer than the brass and flows rather than tear. In a second test, the gold plated brass retainer failed after 3.6 million revolutions, because the retainer was warped on installation. Current measurements of the drive motor indicated a 50% higher than normal run torque. This extra drag accelerated wear of the retainer, and the balls wore through the gold and chipped out the brass underneath, shown in Fig. 4.4.

Additionally, in the full scale, the differential cryogenic contraction between brass and steel is too large, so that on cooling a brass retainer would grip the inner race of the bearing. The warping retainer and cryogenic contraction problem were solved by machining a custom retainer from 304 stainless steel and gold plating it to the same specification, 50 microinches of MIL-G-45204C Type III. The final design is shown in Fig. 4.5. The ball pockets in the retainer are 0.2" diameter circles, but neck down to 0.182" at one end to allow assembly of the bearing. At 0.182" and 0.06" thick, the retainer can be snapped in to the bearing by hand, but can not spontaneously disassemble.

Drive mechanism

The drive mechanism for the half wave plate uses a motor outside the cryostat. Mechanical motion is coupled through the cryostat using a ferrofluidic coupler. We tried ferrofluidic couplers from a two manufacturers and found the lowest running torque available with a 1/4" shaft size was from Ferrotec model SS-250-SLCB part number 103532. Inside the cryostat, the shaft from the 50K plate where the half wave plate is mounted to the fluidic coupler can move 1/4" due to thermal contraction over 1 meter of distance. This contraction is accomodated with a loose slip fit keyed shaft in a socket. An 1/8" diameter hollow G10 fiberglass shaft provides the thermal contraction, and a flexible coupler attaches the G10 to the half wave plate drive pulley.

The first prototype tests of the half wave plate used a gear drive. In initial vacuum tests with no lubrication, the aluminum 2024 gears failed rapidly. After 50,000 revolutions the teeth disintegrated. A string drive worked much better, using kevlar or vectran string, but was plagued by failures after a few million revolutions, either at the knot, or due to wear on the pulleys. The pulleys were aluminum and hard anodized, but the kevlar wears through the anodization and then grinds the aluminum into dust contamination. This was fixed by switching to a belt, which spreads the load over a much larger surface of anodization and reduces the wear rate. The hard anodization used is MIL-A-8625F type III class 1. The final belt used was produced by Bally Ribbon Mills, a 1/2" wide pattern 1989 woven from Kevlar. This belt has a very tight mechanically robust weave. The belts were hand sewn into

continuous loops by Chase Shimmin.

The belt tension is maintained by an idler pulley. This is necessary because kevlar expands on cooling, and because the point where the belt is joined into the loop is thicker than the rest of the belt. The idler pulley spins freely on SR2CBVRT-7 miniature ball bearings, and the arm can rotate freely in plastic Rulon LR bushings. The idler tension is maintained by springs. The idler pulley determines the height of the belt through a 1/2" wide groove; a single groove in the drive system is sufficient to set the belt position and the other pulleys can be flat to loosen alignment tolerances.



Figure 4.2: Photograph of silicon nitride balls after test BG1, the first gold plated brass retainer, and 3 million revolutions. This test failed due to a broken knot for the drive string. From left to right, balls 1, 3, 4, 5 and 6 were in the bearing and show a light gold sheen. The second ball from the left is a control which was not used in the test and shows the raw color of silicon nitride.

4.1.3 Half-Wave Plate Characterization from Polarized Stimulator

With the optional polarizer installed on the stimulator, we can measure elements of the receiver Mueller matrix. In one block of observations, the stimulator polarizer was spun for thirty seconds at 1.7Hz at each of the thirty-two possible half-wave plate orientations. The stimulator chopper is spun at 12Hz. The detector response at each HWP orientation was modelled as:

$$d(t) = a\frac{1}{2}\cos(2\pi f_c t + \phi_c)(1 + \gamma\cos(2(\theta_p + \phi_p)))$$
(4.10)



Figure 4.3: Photograph of steel races after test BG1. In the center of the ball groove, a thin gold stripe is visible where gold has worn from the retainer to the balls to the race. A second stripe is visible on the flat to the right, where gold has worn directly from the retainer to the steel.



Figure 4.4: Photograph of gold plated brass retainer after test BG2. In this test, the retainer was warped and dragged on the bearing. The additional force of the balls against the retainer wore through the gold plating and chipped brass off the retainer.



Figure 4.5: Photograph of final prototype design of cryogenic bearing, before testing and MoS_2 lubrication. This bearing has stock 52100 steel races, custom silicon nitride balls and gold plated stainless steel retainer.

- d(t) Detector timestream
- $\bullet\,\,a$ Amplitude
- f_c Chopper frequency
- ϕ_c Chopper phase
- γ Apparent polarization efficiency
- + θ_p Stimulator polarizer orientation, measured by an encoder
- ϕ_p Apparent polarization angle

Error bars on the fitted parameters are estimated assuming that the noise is additive white Gaussian noise, such that the expected reduced χ^2 is 1. One element missing from the model is the stimulator source polarization term, however, this term is sufficiently orthogonal from the other parameters that it has no effect on their fits. Before fitting, the timestreams are high passed at 5Hz, and the error bars are compensated for this filtering.

The polarization efficiency and apparent detector angle vary with half wave plate orientation due to the non ideal t,c,r terms of the HWP. The expected model is:

$$\gamma = \gamma_0 \left(1 + \frac{c+t}{2} \cos(4(\theta_b - \theta_h))\right) \tag{4.11}$$

$$\phi = -\theta_s - \theta_b + 2\theta_h + \frac{c+t}{4}\sin(4(\theta_b - \theta_h)) + \frac{r}{2}\sin(2(\theta_b - \theta_h))$$
(4.12)

- θ_h HWP crystal axis orientation
- θ_b Detector polarization orientation
- θ_s Stimulator polarization orientation
- γ Measured polarization efficiency
- γ_0 Polarization efficiency of detector and stimulator
- c + t HWP c+t terms from Mueller matrix.
- ϕ Measured detector polarization angle

An example of a fit for one bolometer is shown in 4.6. The reduced χ^2 for the focal plane peaks at 1.6, indicating the presence of additional systematic effects at the noise level of the measurement. The distribution of c+t values measured is 0.05 ± 0.03 , consistent with the expectation of 0.031 from 4.1.1. The measurement of r is 0.001 ± 0.008 , also consistent with the expectation of 0.0008.



Figure 4.6: Measurement of polarization efficiency and detector angle as a function of half wave plate position using the stimulator for one bolometer. X axis is half wave plate physical orientation in degrees. After the test, it was noticed that the stimulator polarizer was visibly damaged, which can explain the low 91% average polarization efficiency.

Chapter 5

Data Analysis

5.1 Implementation

The POLARBEAR analysis pipeline was implemented in the Python programming language[19], and depends on Numpy[20], Scipy, SLALIB[21] and HDF5[14]. The code is pipelined, with separate tools to perform individual calibration tasks, make or simulate maps, estimate pseudospectra, transfer functions and the sky power spectra. The most computationally expensive task is simulating maps; this is parallelized by running a single instance of python for the n (typically n=100) simulations for a 15 minute stretch of data for the whole focal plane. This parallel task is run at a National Energy Research Scientific Computing Center (NERSC) supercomputer with typically 128-way parallelism; 16 nodes with 8 cores per node each running one instance of python. The entire set of simulations consumes a few thousand hours of CPU time.

5.2 Sky Model

The temperature on the sky $T(\hat{r})$ can be expanded in spherical harmonics $Y_{lm}(\hat{r})$ with coefficients a_{lm} .

$$T(\hat{r}) = \sum_{lm} a_{lm} Y_{lm}(\hat{r}) \tag{5.1}$$

Our theory for the CMB sky predict that it is homogenous, isotropic and nearly Gaussian such that all information constraining the underlying theory is encoded in the power spectrum C_l of the a_{lm} .

$$\langle a_{lm}a_{l'm'}^* \rangle = \delta_{ll'}\delta_{mm'}C_l \tag{5.2}$$

This defines the correlation function between two points on the sky.

$$\langle T(\hat{r})T(\hat{r}')\rangle = \sum_{lml'm'} \langle a_{lm}a_{l'm'}\rangle Y_{lm}(\hat{r})Y_{l'm'}(\hat{r}') = \sum_{lm} C_l Y_{lm}(\hat{r})Y_{lm}(\hat{r}') = \sum_l C_l P_l(\hat{r}\cdot\hat{r}')$$
(5.3)

5.3 Flat Sky Approximation

POLARBEAR observed very small patches of the sky, about 5° across. On this small of a patch, the sphere of the sky can be approximated as flat. At the equator, the spherical harmonics can be approximated with fourier modes.

$$Y_{lm}(\theta + \frac{\pi}{2}, \phi) \approx \begin{cases} \cos(k_y \theta) \exp(ik_x \phi), & \text{if } l \text{ even} \\ \sin(k_y \theta) \exp(ik_x \phi), & \text{if } l \text{ odd} \end{cases}$$
(5.4)

$$k_x = m \tag{5.5}$$

$$k_y = \sqrt{l^2 - m^2} \tag{5.6}$$

$$|k| = l \tag{5.7}$$

Given a patch length L the k are discretized

$$k = \frac{2\pi n}{L} \tag{5.8}$$

$$\Delta k = \frac{2\pi}{L} \tag{5.9}$$

$$T(x) = \sum_{n} a_k \exp\left(\frac{ikx}{L}\right)$$
(5.10)

The translation between power spectra of a_{lm} and a_k for simulation and power spectrum estimation is given by

$$\langle |a_k|^2 \rangle = C_l \frac{\Delta k^2}{(2\pi)^2} \tag{5.11}$$

5.4 Instrument Model

Our information about the sky comes from the sampled measurements of the bolometer current d_t . The current in the bolometers is a noisy, linear function of the temperature on the sky. The temperature on the sky is assumed to be smooth, such that it can be pixelized to a map s_p . Each pixel is a vector of the three Stokes components I,Q and U. The noise current is n_t .

$$d_t = A_{tp}s_p + n_t \tag{5.12}$$

The pointing matrix A_{tp} encodes the pixel the bolometer i_t is pointed at and the orientation of the sky relative to the detector polarization angle α_t . The smoothing of the sky due to the beams is assumed to be rotationally symmetric and time invariant such that it can be absorbed into the sky map and corrected for later.

$$A_{tp} = \begin{cases} [1\cos(\alpha_t)\sin(\alpha_t)], & \text{if } i_t = p\\ 0, & \text{otherwise} \end{cases}$$
(5.13)

The noise current n_t is Gaussian distributed and zero mean, so it can be described by a covariance matrix N.

$$\langle n_t n_{t'} \rangle = N_{tt'} \tag{5.14}$$

We apply a simple transformation to the timestreams to help diagonalize the noise covariance. Phonon, photon and readout noise are nearly ideal additive white Gaussian noise, but atmospheric noise is highly correlated in time and in space, but unpolarized[22]. Differencing the two orthogonally polarized bolometer current timestreams from one focal plane pixel creates polarization channel timestreams which are free from atmospheric contamination. The two bolometers in one focal plane pixel are labelled "t" and "b".

$$d^{\pm} = d^{\mathrm{t}} \pm d^{\mathrm{b}} \tag{5.15}$$

5.5 Downsampling

The bolometer currents are sampled at 25MHz by ADCs in the digital frequency multiplexing readout[23]. They are downsampled to 190.73Hz by a series of CIC and FIR filters designed for zero phase distortion and flat response in the pass band. The 190.73Hz sample rate was chosen to allow for rapid scanning of the telescope with small beams. For season 1, POLARBEAR scanned at only $0.75^{\circ}/s$, so a lower sample rate is adequate. The sample rate is further divided by 6 to 31.78Hz to accelerate map making. The final antialiasing filter before the downsampling step is a 257-tap FIR filter constructed by windowing a brick wall filter at the nyquist frequency of the new sample rate with a kaiser $\beta = 6$ window. The filter ripple is less than 0.01% in the passband.

Before downsampling, the bolometer samples are represented with 16b signed integers, with a gain near the quantization noise limit. Quantization noise contributes an RMS noise of $12^{-1/2}$ of a sample, or in noise effective current (NEI) with a sampling rate of $f_s = 190.7Hz$ and a gain from current to ADC counts of g = 91.3pA/ADC,



Figure 5.1: Transfer function of downsampling antialiasing filter.

$$NEI_q = \frac{g}{\sqrt{6f_s}} \approx 2.7pA/\sqrt{Hz} \tag{5.16}$$

Decreasing the sampling rate by a factor of 6 increases the quantization noise by $\sqrt{6}$ to $6.6pA/\sqrt{Hz}$, which would increase the noise by 9% for a nominal $15pA/\sqrt{Hz}$ bolometer. To avoid this, the bolometer sample bit depth is increased to 32 bit signed integers and prescaled by 4 bits (multiplied by 16). The high bits in this format will always be zero, but are compressed away when stored on disk in the HDF5 file.

5.6 Map making

The compressed archives from the POLARBEAR observing campaign contain 5TB of Time-Ordered Data (TOD). The TOD can be compressed into maps which contain the same information about the sky with many fewer bits[24].

The optimal, unbiased solution for the map can be constructed from a likelihood principle.

$$\mathcal{L} = \frac{1}{\sqrt{|N|}} \exp\left(-\frac{1}{2}(d - As)^T N^{-1}(d - As)\right)$$
(5.17)

Maximizing this likelihood over s gives the optimal unbiased map \hat{s}

$$\hat{s} = (A^T N^{-1} A)^{-1} A^T N^{-1} d \tag{5.18}$$

This can be shown to be unbiased by plugging in the instrument model Eq. 5.12

$$\langle \hat{s} \rangle = (A^T N^{-1} A)^{-1} A^T N^{-1} A s = s$$
 (5.19)

The map noise covariance matrix $N_{pp'}$ is calculated from substituting Eq. 5.14 into Eq. 5.18

$$N_{pp'} = \langle \hat{s}\hat{s}^T \rangle_{s=0} = (A^T N^{-1} A)^{-1} A^T N^{-1} N N^{-1} A (A^T N^{-1} A)^{-1} = (A^T N^{-1} A)^{-1}$$
(5.20)

In practice we need noise weighted maps m for optimal B-mode power spectrum estimation. The covariance matrix fortuitously cancels out of the map maker, greatly reducing the computational cost to making maps.

$$m = (A^T N^{-1} A)\hat{s} = A^T N^{-1} d \tag{5.21}$$

Maps are made from eight hours of contiguous scans per day.

5.7 Time domain noise model

In the bolometer difference timestreams, the noise is nearly white. However, some modes are corrupted, which we wish to project out of the timestream. We remove two sets of corrupted modes: Slow drifts in each telescope scan, and scan position synchronous signals. The slow drifts are modelled as linear polynomials for each 3° scan of the telescope. The scan synchronous signal is pixelized with 0.08° pixels, independent for each bolometer CES. First the polynomials are estimated and subtracted, then the scan synchronous signal. The filters are constructed with a least squares procedure, like map making[25]. For corrupted mode templates v, an analog to the pointing matrix, and assuming white noise, the corrupted mode amplitudes yare estimated by

$$(v^T v)y = v^T d \tag{5.22}$$

The operator F projects out all power in the corrupted modes v.

$$F = I - v(v^T v)^{-1} v^T (5.23)$$

This construction has two useful properties. It is symmetric $F = F^T$ and idempotent F = FF. This allows it to play the role of an inverse noise covariance matrix, if the non-corrupted modes have a covariance matrix proportional to the identity matrix, i.e. white noise.

The modes for the polynomial F^p and scan synchronous signal filters F^s are estimated and removed sequentially.

$$F = F^s F^p \tag{5.24}$$

This construction is now only approximately idempotent and symmetric, so the map will only be approximately inverse noise covariance weighted.

The noise weighting additionally includes a frequency domain low pass filter. This filter eliminates aliasing from high frequencies in the timestream to low frequencies in the map domain, which occur because the map is a resampling operation. The filter used has a transfer function G(f) with $f_0 = 7.5Hz$.

$$G(f) = \exp\left(-\left(\frac{f}{f_0}\right)^6\right) \tag{5.25}$$

5.8 Power spectrum estimation

We construct a frequentist estimator for the power spectrum through analytical calculations and monte carlo simulations. First we estimate pseudospectra, which are quadratic functions of the map with an unknown relationship to the true spectra on the sky. Abstractly, we can consider the sky maps as random realizations of a set of C_l . With a vector of unit Gaussians ξ , S the fourier transform operator, and C the diagonal matrix of C_l , a realization of a map is

$$m = S\sqrt{C}\xi \tag{5.26}$$

Our estimator for the pseudospectra is a linear operator E_b

$$\tilde{C}_b = m^T E_b m = \xi^T \sqrt{C} S^T E_b S \sqrt{C} \xi$$
(5.27)

The expectation value for the pseudospectra is

$$\langle \tilde{C}_b \rangle = Tr[S^T E_b SC] \tag{5.28}$$

This is a linear relationship Z_{bl} between \tilde{C}_b and C_l , with

$$\tilde{C}_b = Z_{bl} C_l \tag{5.29}$$

$$Z_{bl} = \sum_{i,j} S_{li} E^b_{ij} S_{jl} \tag{5.30}$$

If we knew Z_{bl} , then given our observed \tilde{C}_b we could construct an unbiased estimator for the true sky spectra C_l .

5.8.1 Pseudospectrum estimator

The pseudospectrum estimator is an approximation to the optimal quadratic estimator. The optimal quadratic estimator for a parameter p given signal and noise covariance S and N is[24]

$$E = (S+N)^{-1} \frac{dS}{dp} (S+N)^{-1}$$
(5.31)

The first step in this estimator is to inverse covariance weight the data by applying $(S+N)^{-1}$. For the case of BB, with very low signal to noise, this is fortuitously taken care of by the biased map making procedure. The second step is to weight the modes by $\frac{dS}{dp}$, or by how sensitive the covariance is to the parameter. Our parameters are bins in power spectrum space, so we approximate this step by fourier transforming the inverse covariance weighted map, band pass filtering by applying a mask in fourier space, and then measuring the remaining RMS power.

We eliminate the noise bias by splitting our data by day into one day maps, and removing elements from E which connect maps from like days. Only cross spectra from different days are included in the pseudospectrum estimator, the autospectrum from one day is not used. A pseudospectrum estimate \tilde{C}_l^{XY} is calculated from the apodized and fourier transformed maps \tilde{m}_{ik}^X , noise weight w_i^X , where *i* is the day and *k* is the fourier mode

$$\tilde{C}_l^{XY} = \frac{1}{\sum_{i,j\neq i,k\in bin_l} w_i^X w_j^Y} \sum_{i,j\neq i,k\in bin_l} w_i^X \tilde{m}_{ik}^X w_j^Y \tilde{m}_{jk}^{Y*}$$
(5.32)

The noise weights w_i^X are calculated from the sum of the pixel inverse noise covariance estimates.

It would be prohibitively expensive to directly compute Z_{bl} , so we use an adhoc model introduced by Hivon et al[26]. This model notes that the two primary effects of estimator E_b is a suppression of power from filtering and beam smoothing, and mode mixing due to the apodization of the finite sky cut.

$$\tilde{C}_{l} = \sum_{l'} M_{ll'} F_{l'} B_{l'}^2 C_{l'} = \sum_{l'} K_{ll'} C_{l'}$$
(5.33)

5.8.2 Pure EB transform

Cosmological EE is much brighter than the expected BB from lensing, because the magnitude of lensing ($\approx 2am$) is small compared to the angular scales where there is substantial EE power. Applying an apodization to the Q and U Stokes maps acts like lensing, breaks statistical isotropy, and mixes E into B. With a naive conversion of Q and U into E and B, the leaked E to B power substantially exceeds the lensing B power. The sample variance from the leakage exceeds the noise variance. We work

around this by using the pure transform introduced by Smith[27]. The pure transform exactly projects out from the B mode map all apodized E modes. On the flat sky, the pure estimator for the fourier transformed B mode map is

$$\tilde{B}_{l} = -\sin(2\phi_{l})F_{l}[Qw] + \cos(2\phi_{l})F_{l}[Uw] -\frac{2i}{|l|}(\sin(\phi_{l})(F_{l}[Q\partial_{x}w + U\partial_{y}w]) + \cos(\phi_{l})F_{l}[Q\partial_{y}w - U\partial_{x}w]) +\frac{1}{l^{2}}F_{l}[2Q\partial_{xy}w + U(\partial_{yy}w - \partial_{xx}w)]$$
(5.34)

where F_l is the fourier transform, $\partial_x w$ denotes the x derivative of the apodization map calculated by symmetric finite difference, $\phi_l = \arctan(ly/lx)$ is the angle of the fourier mode, and Q and U are the Stokes Q and U maps in real space.

This estimator eliminates the leakage sample variance, but increases noise variance due to adding Q and U where the window changes, which is at the edge of the map where the variance is higher. Because of this increase in noise variance, and because the cosmological B signal is negligible, the pure transform is only used for B, and not for E.

5.8.3 Mode mixing matrix

We calculate the mode mixing matrix following Louis et al[28]. This is Z_{bl} if there were no filtering. The mode mixing matrix when using the pure B mode transform is

$$M_{bb'}^{XY} = \sum_{ll'} P_{bl} |W^{XY}(l-l')|^2 \left(\frac{l'}{l}\right)^{\beta_{XY}} Q_{l'b'}$$
(5.35)

Where P_{bl} and $Q_{l'b'}$ are binning and interpolation operators, and $\beta_{XY} = 2(\delta_{BX} + \delta_{YB})$, and the W is the fourier transform of the map apodization. M is calculated at a resolution of $\Delta \ell = 40$.

5.8.4 Filter transfer function

The effect of the time domain filtering on the data is modelled as a transfer function in the ℓ domain that induces no mode mixing. This is only an approximation, which we account for by measuring the effective transfer function at a fiducial cosmology, and showing that changing the cosmology has negligible effect on the transfer function. The transfer function is measured in monte carlo simulations. We create simulated CMB skies with a fiducial cosmology. The CMB skies are scanned to create simulated timestreams using the real detector pointing, and the simulated timestreams are processed into pseudospectra identically to how they are for real data. Following Eq. 5.33, the filter transfer function can be solved for from these simulations

$$F_l = \frac{\sum_{l'} M_{ll'}^{-1} \tilde{C}_{l'}}{B_l^2 C_l}$$
(5.36)

Because the mode mixing matrix is a smoothing operator on power as a function of angular scale, the inverse operation is a sharpening operator that enhances high frequency noise. This leads to large ringing in the filter transfer function. The ringing has no effect on the final binned results due to the smoothing of the binning operation. This was verified by smoothing the filter transfer function and confirming no significant change in the C_l^{BB} power spectrum amplitude.

The filter transfer function is measured separately for T, E and B, due to the difference in filtering for each case. We use two sets of simulations to measure the filter transfer function. One has power in TT,EE and TE and one has power in TT and BB. The TT+EE simulations are used to measure the transfer function for TT and EE, while the TT+BB simulations are used for the BB transfer function. The TT+EE simulations are also used to measure the leakage modes from EE into BB due to filtering. This leakage transfer function is estimated from Eq. 5.36 by using the TT+EE simulations with C_l from EE, \tilde{C}_l from BB, and M from BB. The leakage and EE transfer functions are used to estimate the leakage in pseudospectrum space from the measured EE pseudospectrum.

$$\tilde{C}_{l}^{EE \to BB} = \frac{F_{l}^{EE \to BB}}{F_{l}^{EE \to EE}} \tilde{C}_{l}^{EE}$$
(5.37)

For the TE, TB, and EB power spectra, the filter transfer function is estimated as the geometric mean of the auto spectra filter transfer functions

$$F_l^{XY \to XY} = \sqrt{F_l^{XX \to XX} F_l^{YY \to YY}} \tag{5.38}$$

5.8.5 Binned solution

Because of the small size of our patch, we can't distinguish between power at nearby ℓ . This is expressed in the mode mixing matrix as a high condition number and an explosion in error bars if we tried to solve for each ℓ individually. Instead, we only try to solve for the spectrum binned to low resolution, without significant loss of information. Because there is expected to be little information in the shape of the BB spectrum, we pushed this to a bin size of $\Delta \ell = 400$. This is wide enough to make the covariance between bins negligible and simplify the calculation of error bars and estimation of cosmological parameters. The binning used is flat in D_l space, where

$$D_l = \frac{l(l+1)}{2\pi} C_l \tag{5.39}$$

The binning operator is P_{bl} and the interpolation operator that undoes it is Q_{bl} . Assuming that the true sky spectrum is flat in a bin modifies Eq. 5.33 to

$$\tilde{C}_b = \sum_{b'} K_{bb'} C_{b'} \tag{5.40}$$

$$K_{bb'} = \sum_{ll'} P_{bl} K_{ll'} Q_{l'b'}$$
(5.41)

Given an observed set of \tilde{C}_b , the solution for the unbiased binned sky power spectrum is

$$\hat{C}_{b} = \sum_{b'} K_{bb'}^{-1} \tilde{C}_{b}$$
(5.42)

5.8.6 Band power window functions

Band power window functions w_{bl} describe the response of a binned measurement \hat{C}_b to a change in theory power spectrum C_l .

$$\hat{C}_b = \sum_l w_{bl} C_l \tag{5.43}$$

This is obtained by combining Eq. 5.33 and Eq. 5.42

$$w_{bl} = \sum_{b'l'} K_{bb'}^{-1} P_{b'l'} K_{l'l}$$
(5.44)

The band power window functions for the season one power spectrum analysis are shown in Fig. 5.2.



Figure 5.2: Band power window functions w_{bl} describing the transfer function for TT, EE and BB power to binned band powers.
5.8.7 Power spectrum uncertainty

We use analytical estimates of the power spectrum uncertainty, verified with Monte Carlo simulations. For auto spectra, the analytical error bars where XX = TT, EE or BB,

$$\Delta \hat{C}_{b}^{XX} = \sqrt{\frac{2}{\nu_{b}^{XX}}} (C_{b}^{XX} + N_{b}^{XX})$$
(5.45)

For the cross spectra XY = TE, TB or EB,

$$\Delta \hat{C}_b^{XY} = \frac{1}{(\nu_b^{XX} \nu_b^{YY})^{1/4}} \sqrt{(C_b^{XY})^2 + (C_b^{XX} + N_b^{XX})(C_b^{YY} + N_b^{YY})}$$
(5.46)

The sample variance terms C_b are estimated from the WMAP-9 Λ CDM spectra. Noise spectra are estimated by measuring the noise bias of auto spectra.

5.8.8 Optimality of the Power Spectrum Estimator

The estimator used by POLARBEAR for the Season 1 data analysis was an approximation to the optimal quadratic estimator of Eq. 5.31. The approximation was necessary for computational efficiency. To test the statistical efficiency of this estimator, we performed simulations comparing the optimal quadratic estimator to the approximation. In order to make this simulation computationally feasible, the map pixel size was increased to 10 arcminutes and the map width decreased to 6 degrees. At this resolution, it is possible to directly construct the signal and noise covariance matrices needed for the optimal quadratic estimator. The simulations used 100 monte carlo realizations with white noise in the input, and the same modes filtered as used for the real analysis. A single CES of pointing, chosen as a typical example, was used. Using a single CES is a pessimistic case for the approximate estimators because without sky rotation, the noise covariance is more anisotropic. Matrices were assembled from the "outside in" as described in [25], i.e.:

$$N_{pp'}^{-1} = \sum_{i} \sigma_i^{-2} (A^T A - (A^T v)(v^T v)^{-1}(A^T v)^T)$$
(5.47)

where σ_i is the RMS noise for pixel *i* and *v* is the matrix of modes to be projected out of the timestream by the filter.

The parametrization used in the optimal quadratic estimator was the band power in the ell bins used for science. Two ell bins were looked at, $\ell = 100 - 300$ and $\ell = 300 - 500$. We looked at these low ℓ ranges to investigate the suitability of this estimator for extension of results to larger angular scales.

Estimator	$\Delta C_l^{BB} \ \ell = 300 - 500$	$\Delta C_l^{BB} \ \ell = 100 - 300$
POLARBEAR Season 1	56	45
Matrix Projection	52	31
Optimal Quadratic Estimator	55	32

Table 5.1: Power spectrum estimator comparison

A third estimator tested was the matrix projection method used by BICEP2 to remove E to B leaked modes from the map, described in [29]. The eigenvalue threshold used was 1000.

Noise for the single CES was scaled to a map depth of $\Delta Q = 6.5 \mu K - arcmin$. Scaling down to lower noise increases the penalty of a bad estimator, because the sample variance of the E modes starts to dominate.

These results are limited by the statistical precision of only 100 monte carlo realizations, but already show that the simple estimator used by POLARBEAR saturates the optimal estimator even for a bin lower than the lowest reported in the real results. The estimator is 50% worse than optimal at the lowest bin; here the matrix projection method is necessary to saturate the optimal quadratic estimator bound.

5.9 Pointing Model

If perfectly constructed, the POLARBEAR telescope can be modelled as an elevation bearing sitting on top of an azimuth bearing. The origin of the telescope, when both azimuth and elevation encoders read zero, the center of the field of view (or boresight) points due north along the horizon. When the azimuth or elevation drive is moved, the action on the boresight is a sequence of rotation matrices. Because the azimuth bearing is on bottom, the orientation of its rotation axis never changes. The action it has on the boresight never changes, so it must be the last operation in the sequence. The elevation bearing has an effect on the boresight dependent on azimuth, so it is the first operation. These rotation operators describe how to map the position of a pixel slightly offset from boresight to azimuth and elevation on the sky. For a coordinate system where \hat{x} is north, \hat{y} is east, and \hat{z} is zenith, the action of the telescope on the receiver field of view is

$$\hat{v}_f = R_z(az)R_y(el)\hat{v}_i \tag{5.48}$$

This is an idealized model of the telescope. In practice, there are several geometric non idealities: the axes of the rotations is not known exactly, the zeroes of the rotations is not known exactly, and the orientation of the detectors relative to boresight is not known exactly. Additionally there are non-geometric terms due to flexure of the boom and refraction in the atmosphere. Some of these are degenerate,

Name	Description
AN	Azimuth axis tilted north of vertical
AW	Azimuth axis tilted east of vertical
NPAE	Elevation axis not perpendicular to azimuth axis
CA	Telescope beam not perpendicular to elevation axis
IA	Azimuth encoder zero
IE	Elevation encoder zero
TF	Flexure

Table 5.2: Pointing model parameters

and in practice we need seven parameters to describe the departures from the ideal telescope. We name the parameters following the TPOINT¹ and JCMT² conventions, reproduced in Table 5.9. The effect on the azimuth and elevation to first order of the pointing terms is described in Eq. 5.49.

$$az' = az - AN\sin(az)\sin(el) - AW\cos(az)\sin(el) + NPAE\sin(el) - CA + IA\cos(el)$$

$$el' = el + AN\cos(az) - AW\sin(az) - IE + TF\cos(el)$$
(5.50)

Using this first order expansion adequately describes the pointing of the telescope because all the terms which would have a nonlinear effect are very small. It is also necessary that we do not observe near the zenith where gimbal lock would increase errors. While the telescope can point this high, that elevation is outside of the range at which the pulse tube cooler cools effectively.

The first order expansion conveniently gives us a linear model for the pointing, so given a set of measurements of the actual azimuth and elevation vs. observation. Abstractly this is modelled in the same matrix equation as CMB map making:

$$d_t = A_{tp}s_p + n_t \tag{5.51}$$

Here d_t is difference between observed and actual azimuth and elevation. The observed azimuth and elevation is defined as the value of the telescope encoders when a detectors response is maximized on a source. The actual azimuth and elevation are computed from ephemeris obtained from JPL Horizons³ and converted to Azimuth and Elevation using SLALIB. A_{tp} is a matrix describing how the pointing terms affect azimuth and elevation. It has 7 columns in pairs of rows to include the effects of Eq. 5.49 on both azimuth and elevation. s_p is a 7 element vector of the pointing model parameters of Table 5.9. Finally, n_t is noise with covariance N.

¹http://www.jach.hawaii.edu/JCMT/telescope/pointing/tpoint.ps

²http://www.jach.hawaii.edu/JCMT/telescope/pointing/parameters.html

³http://ssd.jpl.nasa.gov/horizons.cgi

This has the usual maximum likelihood solution

$$\hat{s} = (A^T N^{-1} A)^{-1} A^T N^{-1} d \tag{5.52}$$

With covariance matrix for the solution

$$\langle (\hat{s} - s)(\hat{s} - s)^T \rangle = (A^T N^{-1} A)^{-1}$$
 (5.53)

One of the reasons we chose the seven parameters of Table 5.9 was that they are non degenerate. For a given set of pointing observations, degeneracies show up as large eigenvalues in the covariance matrix of Eq. 5.53. The most degenerate combination of parameters can be read off of the eigenvector of the largest eigenvalue. For example, with radio pointing observations which may observe only a narrow band of elevations, it's not possible to tell the difference between the telescope tube flexing and refraction in the atmosphere $(\cot(el))$, because both are approximately linear over a small range of elevation.

5.9.1 Time offset

One more parameter was found to be necessary to explain our initial pointing at Cedar Flat - absolute timing error. If the time of day is different than expected, the rotation of the earth is different than expected, and coordinates will be incorrectly converted between right ascension/declination and azimuth/elevation. This has the same effect as a longitude error, and is similar to a tilt, but has an effect dependent on declination rather than elevation. This can be seen by noticing that tilt has an effect on the apparent location of the north star, while a timing error will have no effect on this point. The motion of the sky is only 15 arcseconds per second, and it is trivial to reach millisecond absolute timing accuracy with a GPS receiver. We found timing errors up to 40 seconds which varied over time of day, and reset at UTC midnight, at which time the apparent position of the source would jump by 10 arcminutes, a very noticeable 2.5 beams. This was traced to a bug in the control software in a conversion between floating and fixed point, where the constant for fixed point full scale was mismatched between conversion to and conversion from floating point. The remaining timing and latitude error from GPS are both negligible.

5.9.2 Star Camera

The POLARBEAR star camera is an optical camera attached to the boom of the Huan Tran Telescope used to measure the non idealities in the geometry of the Huan Tran Telescope. The star camera has a 0.4 degree field of view with a 350 x 250 pixel CMOS camera. It has a single 50mm diameter lens with a 1018.6m focal length and an 8 arcsecond diffraction limited diameter spot size. The camera is angled to point in approximately the same direction as the radio beam. The camera is readout with a PCI card in PBDaq, a computer in one of the telescope saddlebags. The frame readout is not synchronized with the motion of the telescope, so photographs are only taken with the telescope at rest. The star camera was constructed by Ian Schanning at UC San Diego. A photo is shown in Fig. 5.3.

Star camera data was taken only while the telescope was deployed at Cedar Flat. Because of the many bright stars available in the sky, the star camera was used to measure the initial azimuth offset of the telescope. When the telescope is initially constructed, the physical zero of the azimuth encoder is aligned approximately with true north by using a compass and compensating for the difference between true and magnetic north. This procedure is accurate only at the degree level. The rest of the geometry of the telescope, for example tilt or non perpendicularity of axes, is constructed to a precision of about 1 arcminute. In order to have good enough rough pointing to command the telescope to a target on the sky, the azimuth zero is the only parameter we need to measure. Measuring the azimuth zero with the star camera requires observing only two stars at different elevations. Two elevations are needed to separate the different effects of azimuth zero and the pixel direction relative to the boresight; the error induced in sky position from azimuth zero will decrease with the cosine in elevation, while the pixel direction will give error independent of elevation.

The physical model for the star camera is a map from positions relative to boresight to pixels in a star camera photograph. This has a scale factor and orientation. We measure this by taking a series of photographs of a single source with slight offsets in the azimuth and elevation directions.

An example of fitting a pointing model using Eq. 5.52 to star camera data is shown in Fig. 5.4.



Figure 5.3: The POLARBEAR star camera as installed at Cedar Flat. Figure courtesy Ian Schanning.



Figure 5.4: Example results from one day of star camera observations. Blue points are the measured data, and black points are the predictions from the best fit pointing model. The azimuth zero component has been removed to show the other effects. RMS residual is 17 arcseconds.

5.9.3 Tiltmeter

The Huan Tran Telescope has a Applied Geomechanics D711-2A (4x) tiltmeter installed above its azimuth bearing. The tiltmeter measures the tilt of the azimuth bearing axis relative to gravity. The mount of the tiltmeter relative to the azimuth bearing is not perfectly flat, but this effect can be separated from the tilt of the foundation by rotating the azimuth bearing. When the azimuth bearing is rotated, the base tilt component stays static while the foundation component varies sinusoidally. To measure the tilt of the base, we rotate the telescope in 45 degree steps from 0 to 315 degrees, and read out both axes of the tiltmeter with a pair of voltmeters. The voltage is converted to angle using a calibration from the manual, 0.0005 arcminutes/volt. The tilt parameters AN, AW and the mount errors x_0 , y_0 are fit to the data from the two axes x(az) and y(az) using linear least squares and a model:

$$x(az) = aw\cos(az) + an\sin(az) + x_0 \tag{5.54}$$

$$y(az) = aw\sin(az) - an\cos(az) + y_0 \tag{5.55}$$

An example result of this fit is shown in Fig. 5.5

Once the tilt is measured, we level the base of the telescope. Zeroing out the tilt makes it easier to confirm that there are no effects that mimic tilt in our radio pointing measurements, because we don't need to know how to transfer the sign of the tiltmeter tilt to the sign of the tilt in the radio pointing model. The telescope rests on a three point base. Each point is a threaded rod sunk into a concrete foundation with two large nuts which clamp around a clearance hole in the telescope base. We



Figure 5.5: Tiltmeter data fit to an observation from October 26, 2012. Blue shows the x axis output of the tiltmeter and green the y axis output. Error bars are estimated from scatter of repeated measurements. T1 is the AN tilt and T2 the AW tilt. The principal measured tilt is 0.58 ± 0.05 arcminutes.

release the top nut and lift the telescope with a hydraulic jack. The telescope weighs 30 tons, so 10 tons is needed to lift one point. The lower nut is then rotated according to the pitch of the threaded rod to eliminate the tilt. Two points need to be adjusted to eliminate two tilt degrees of freedom. Using this method we eliminate the tilt at the 0.1 arcminute level, confirmed through radio pointing measurements.

5.9.4 Coordinate systems

POLARBEAR pointing is built around two coordinate systems: Local horizontal coordinates, or azimuth (az) and elevation (el), which are fixed to the ground and sometimes referred to as the instrument frame, and celestial coordinates, or right ascension (RA) and declination (Dec), which are fixed to the Cosmic Microwave Background and sometimes referred to as the sky frame. We convert between these coordinate systems using the SLALIB software package and measurements of the earth orientation called UT1UTC. UT1UTC is the difference in time between the orientation of the earth (UT1) and a standard time format which can be calculated from gps time (UTC). The orientation of the earth must be measured like other ephemerides; we use measurements provided by the USNO⁴.

Other planets in the solar system are very bright at millimeter wave length and make excellent sources for developing pointing and beam models. However, because they are so close to the earth, parallax effects are important in computing their apparent position on the sky. It is insufficient to predict where we will see them

⁴http://toshi.nofs.navy.mil/

based only on their apparent RA/Dec as seen from the center of the earth. We use ephemerides from JPL Horizons which give the apparent positions on the sky in celestial, CMB fixed coordinates for the latitude and longitude of the POLARBEAR site in Chile.

We also refer occasionally to offset coordinate systems. The offset coordinate system is aligned with the original coordinate system, but is rotated so that offset distances equal distances in angle in the frame. This is convenient for constructing square projections for maps.

5.9.5 Radio Pointing

The primary pointing model we use for science data is derived from measurements of radio sources on the sky. The brightest radio sources available are planets, quasars and galactic H II regions. H II regions are not ideal for pointing because they are extended in the POLARBEAR beam, so it is difficult to estimate where the apparent centroid position should be. For the POLARBEAR instantaneous sensitivity and beam size, we need sources of about 3Jy or brighter to be useful for radio pointing. We observe several of these sources by scanning a central pixel on the focal plane in a rectangle 0.5° elevation by 4° in azimuth on the sky. The rectangle is larger than necessary in the azimuth direction because we want to scan fast to suppress 1/f noise, and scanning a shorter distance would not save time due to the time spent accelerating.

We make maps for diagnostic purposes, but the data is fit in the time domain. The response of one bolometer B(x) to the source is modelled as an elliptical Gaussian in azimuth and elevation offset.

$$B(x) = A \exp\left(-\frac{1}{2}(x - x_0)^T R^T S R(x - x_0)\right)$$
(5.56)

- A: Amplitude of beam at peak
- x: Azimuth and elevation offset between telescope and sky source for a point in the timestream
- x_0 : Azimuth and elevation offset of pixel beam relative to telescope boresight
- R: Rotation by θ , the angle of the major axis of the ellipse relative to the azimuth axis
- S: Diagonal matrix with elements the major and minor axis beam widths

The timestream for the central pixel is fit to this model for a single raster scan of one source. After observing a large collection of sources at a range of azimuth and elevation, the location parameters x_0 are gathered and combined in a fit to the pointing model Eq. 5.49.

The same fitting procedure is used to measure x_0 for the rest of the pixels on the focal plane. This uses a separate scan with the same azimuth throw and velocity, but rasters over a planet 4 degrees in elevation to cover the entire focal plane. Once the pointing model is constrained by one pixel, x_0 is the only free information describing where the other beams are on the sky. Because the elliptical Gaussian fit also includes the largest contributions to non idealities in beam shape, we also use the fits to simulate temperature to polarization leakage.

5.10 Beam Model

The effect of the experiment beam on CMB observations must be characterized precisely to measure the shape of the B-mode power spectrum. The POLARBEAR beam is approximately rotationally symmetric. The rotational symmetry is enhanced by the rotation of the sky, which smoothes out azimuthal features in the beam in the final coadded measurements. With an assumption that the beam is constant wherever we observe in the sky, the action of the beam is a rotationally symmetric convolution, which can be expressed directly in power spectrum space as a wavelength dependent scale factor B_{ℓ} .

$$C_{\ell}^{observed} = B_{\ell}^2 C_l^{sky} \tag{5.57}$$

In order to measure B_{ℓ} , we make a map of a source with a known power spectrum, and then measure the observed power spectrum, and finally solve Eq. 5.57 for B_{ℓ} . We use Jupiter as the source, using the same type of observations used to measure the locations of all the beams. Once the beam locations are measured, the maps for individual detectors can be coadded into one map using the same algorithm used for CMB map making. First order polynomial filtering is used for this map making to suppress atmospheric noise, with the planet masked off in a 50 arcminute radius. The beam has negligible power outside of this radius, so masking eliminates bias on B_{ℓ} due to the filtering. When coadding the individual detector maps, the maps are weighted by the inverse variance of the individual detector map outside the mask radius. This is a proxy for the method used for science map making, which is an integral over the time domain power spectrum in the science frequency band. The power spectrum method would not be practical for the planet due to the bright planet signal.

The millimeter wave emission from a planet is approximately a flat disc of emission. For a temperature T_p and radius R, the power spectrum is

$$C_l^{sky} = 2\pi T_p R^2 \frac{J_1(\ell R)}{\ell R}$$
(5.58)

Where J is a Bessel function of the first kind.

The square pixels used in map making add additional smoothing, called the pixel window function W_{ℓ} . For a pixel of size Δ , we use this estimate for the pixel window function:[30]

$$W_l = \exp(-(\ell\Delta)^{2.04}/18.1)(1 - (0.0272)(\ell\Delta)^2)$$
(5.59)

The observed power spectrum is calculated in two ways. In either case, the first step is to fourier transform the map and average in azimuthal bins of width $\Delta \ell = 80$. The magnitude of this function is a noise biased estimate of C_l^{sky} . Alternatively, we can take the real part of the fourier transform, then square. This suffers from no noise bias, but ignores asymmetric power, for example due to beam shifted off the center of the map. The two methods are consistent for POLARBEAR , and the real part method is adopted for the beam model.

We use the average of dozens of observations of Jupiter over the season for the science beam model. The average is inverse noise covariance weighted based on the same noise weights used for coadding individual detector maps. Errors on B_{ℓ} are estimated from the noise weighted sample covariance matrix of B_{ℓ} from the set of Jupiter observations. Observations of Saturn give a consistent measurement of B_{ℓ} .

5.11 Gain Calibration

The relative gain of bolometers is directly derived from the planet measurements. During a planet observation, the beam filling response of the bolometers is measured to produce a calibration in ADC units per Kelvin. Simultaneously, the stimulator described in Section 2.8 is observed, and its effective temperature in the bolometer timestreams is recorded. Subsequently during CMB observations, only the stimulator is observed, which now provides a reference signal with a known temperature.

Due to the systematic uncertainties associated with the planet brightness and detector band pass, the most accurate absolute calibration available is the CMB itself. We use the entire first season dataset to perform a final absolute calibration by comparing our C_l^{TT} spectrum and fitting it to the WMAP9 measurement.

5.12 Polarization Angle Calibration

5.12.1 Self calibration

Errors in polarization angle calibration leak E modes to B modes. Under rotation of polarization by an angle θ , E and B modes transform like a spin-2 vector.

$$E' = E\cos(2\theta) + B\sin(2\theta) \tag{5.60}$$

$$B' = E\sin(2\theta) + B\cos(2\theta) \tag{5.61}$$

To lowest order in θ , the systematic error introduced into C_l^{BB} is

$$C_l^{BB} = 4\theta^2 C_l^{EE} \tag{5.62}$$

Similarly, a correlation between E and B is introduced

$$C_l^{EB} = 2\theta C_l^{EE} \tag{5.63}$$

Because C_l^{EB} is zero in the simplest ACDM, we can use Eq. 5.63 to measure the polarization orientation of the receiver. Because the leakage is proportional to θ for C_l^{EB} and θ^2 for C_l^{BB} , we have sufficient sensitivity from C_l^{EB} to measure the angle precisely enough that negligible bias in C_l^{BB} from angle calibration remains. The error bar on C_l^{EB} is approximately

$$\Delta C_l^{EB} = \sqrt{\frac{1}{\nu} \left((C_l^{EB})^2 + (C_l^{EE} + N_l^{EE}) (C_l^{BB} + N_l^{BB}) \right)}$$
(5.64)

In practice, the variance from EE noise and BB signal will be negligible, so the errors simplify to

$$\Delta C_l^{EB} = \sqrt{\frac{1}{\nu} C_l^{EE} N_l^{BB}} \tag{5.65}$$

From the fisher matrix, this gives an angle error of

$$\Delta \theta = \sqrt{\frac{N_l^{BB}}{4\nu C_l^{EE}}} \tag{5.66}$$

Combining this with Eq. 5.62 the expected bias on C_l^{BB} is independent of C_l^{EE}

$$C_l^{BB} = \frac{N_l^{BB}}{\nu} \tag{5.67}$$

The error bar on BB in the same limit is

$$\Delta C_l^{BB} = \sqrt{\frac{2}{\nu}} N_l^{BB} \tag{5.68}$$

The relative size of the systematic bias and C_l^{BB} error bar is

$$\frac{C_l^{BB}}{\Delta C_l^{BB}} = \frac{1}{\sqrt{2\nu}} \tag{5.69}$$

In the limit where the C_l^{EE} and C_l^{BB} spectrum are white, this expression says that the bias is suppressed relative to the statistical error by the square root of twice the number of modes measured on the sky. POLARBEAR measures hundreds of degrees of freedom over its patch at the angular scales of interest, so the bias is heavily suppressed, and the calibration method is very effective. One key caveat to this method though is that a primordial EB spectrum could be produced by a background of magnetic fields or new physics, which we can not measure with this method. In some cases, such as cosmic birefringence, applying this calibration corrects for the effect in the sense that it helps us look for only our primary science targets such as B-modes from gravitational lensing.

The C_l^{TB} spectrum can also be used for self calibration[31], but is not effective for POLARBEAR due to the low ℓ resolution. The C_l^{TB} spectrum is oscillating about zero, so a large map is needed to determine the wavenumber of a mode and weight it appropriately. Our maps were too small to measure the wavenumber precisely enough, so the C_l^{TB} spectrum oscillations are washed out.

The precision of the C_l^{EB} angle measurement is $\Delta \theta = 0.2^{\circ}$. The fit to the C_l^{EB} spectrum is shown in Fig. 5.6.



Figure 5.6: (a) Black points: The C_l^{EB} band powers measured using an absolute angle derived from Tau A. Red line and points: Expected C_l^{EB} spectrum from the best fit EB self calibration. The measured angle is -1.08° from the Tau A calibration with a probability to exceed (PTE) of 42.8%. (b) C_l^{EB} spectrum consistent with zero after self calibration. Figure courtesy Yuji Chinoney.

5.12.2 Tau A

The signal to noise of POLARBEAR is not high enough to use the CMB as the calibration source for relative angle calibration of pixels. Tau A is one of the brightest

polarized sources on the sky, so high signal to noise measurements of polarization can be made much more quickly on it than can be made on the CMB. We observed Tau A several times a week to relatively calibrate the polarization angle of the pixels, except when the angle between Tau A and the sun was less than 30°.

5.13 Systematic Errors

5.14 T to QU leakage test

Given leakage from temperature into polarization that is constant across the map, i.e.

$$Q = \gamma T \tag{5.70}$$

Correlation between T and Q is created, and the power spectrum C_l^{TQ} acquires a non zero expectation value. The usual C_l^{TE} correlation expected in λ CDM does not create C_l^{TQ} due to rotational symmetry.

Given the transforms between QU and EB in flat sky fourier space,

$$Q_l = E_l \cos(2\phi_l) + B_l \sin(2\phi_l) \tag{5.71}$$

$$U_l = E_l \sin(2\phi_l) - B_l \cos(2\phi_l) \tag{5.72}$$

Once azimuthally integrating over the two point function $\langle T_l Q_l \rangle$, the power spectrum C_l^{TQ} will be zero, due to the $\cos(2\phi_l)$ azimuthal dependence of Q_l . Therefore detection of C_l^{TQ} can only come from leakage of T to Q, e.g. of the form given in Eq. 5.70.

However, POLARBEAR makes high signal to noise maps of both T and E, so C_l^{TE} will be limited by sample variance. Because the azimuthal integration does not null out the noise, C_l^{TQ} will also be limited by sample variance. To eliminate this sample variance we would like to use something like a C_l^{TB} spectrum to look for the leakage, but the same rotational symmetry argument implies that C_l^{TB} can not be produced by T to Q leakage. Instead we need to decompose Q and U into just those Q and U modes which are produced by B modes. These are the modes

$$Q_l^B = B_l \sin(2\phi_l) \tag{5.73}$$

$$U_l^B = B_l \cos(2\phi_l) \tag{5.74}$$

Finally, we can construct azimuthally averaged power spectra like $\langle T_l B_l \sin(2\phi_l) \rangle$ which are free of E mode sample variance.

This test is limited in an assumption that the leakage from T to Q is constant across the map, and the assumption that the leakage is proportional to T, while some leakages can produce more complicated patterns, e.g. the gradient leakage of differential pointing. While tests designed to overcome these limitations are possible we did not explore this for POLARBEAR season one.

5.15 T to QU leakage simulations

The beams of POLARBEAR detectors not perfectly symmetric and identical. These asymmetries cause leakage from CMB temperature anisotropies to polarization anisotropies. This is easiest to see in a model of the experiment that has one focal plane pixel. In this model, all the polarization information is contained in the difference between the two orthogonally polarized bolometers in that pixel. Any mismatch between the shape of the beam of those two bolometers which is ignored in map making leads to temperature anisotropies leaking to polarization anisotropies in the measured map.

5.15.1 Crosstalk

Electrical crosstalk can leak temperature anisotropies to polarization if it is not perfectly balanced from one pixel to the two bolometers in another pixel. If only one bolometer in a pixel receives crosstalk from another pixel, a temperature sidelobe is created in the polarization differenced beam. Crosstalk is inherent in the frequency multiplexing system due to the finite spacing of the resonances and finite resonance width. Detailed calculations of crosstalk are in Appendix B.

The POLARBEAR frequency schedule was designed to partially cancel out crosstalk. There are four pixels, two bolometers each, frequency multiplexed to one SQUID. The frequencies are arranged such that the lowest four frequencies have one bolometer from each pixel, and the highest four frequencies have one bolometer from each pixel in the same order. The two bolometers in one pixel are named t and b. Pixel 1t crosstalks to pixel 2t, and pixel 1b crosstalks to pixel 2b, but in the polarization difference channel 2t - 2b the temperature component of the crosstalk cancels out. This eliminates temperature to polarization leakage due to crosstalk for two of the pixels on complete combs. For pixels 1 and 4 on the edge of the comb, or pixels on combs with failed detectors, there is no crosstalk cancellation. There is also leakage from mismatch between crosstalk levels, but this is subdominant to the expected leakage into pixels 1 and 4.

We simulate the dominant temperature to polarization leakage from crosstalk by introducing -2% crosstalk from bolometer i on a comb to bolometer $i\pm 1$. While the actual crosstalk will vary from -2% depending on the precise frequency spacing of the two channels, this calculation establishes whether crosstalk at this order of magnitude can affect the measured BB power spectrum. Maps from a single simulation realization are shown in Fig. 5.7. The crosstalk in this simulation is worst at the edges because edge pixels see the least number of observations from different bolometers and sky orientations; the crosstalk leakage averages down over the many different pixel configurations and sky orientations. Fortuitously, the edges of the map are also downweighted in power spectrum estimation as the least observed and noisiest pixels. For the central region, where many bolometers are averaged over, we expect the crosstalk to be a large spatial scale effect, because the distance between crosstalking bolometers on the focal plane can be up to 30 arcminutes different, so that the effective coadded crosstalk leakage beam is large and does not see small scale temperature anisotropies. The leakage power spectrum is shown in Fig. 5.8 and is only 0.3% of the theory spectrum at the lowest season one science bin, l=500-900.



Figure 5.7: Simulations of one day of data with a TT only ΛCDM input cosmology including crosstalk induced temperature to polarization leakage. From top to bottom are T, Q and U maps. X and Y axis coordinates are in number of 2 arcminute pixels. Simulation has no noise and maps are not apodized.

5.16 Null tests

We test the internal consistency of our data by splitting it in two in ways that amplify possible systematic errors, constructing band powers for each set, and checking that the difference is consistent with zero. The construction we use is adopted from QUIET[32].

$$\hat{C}_{b}^{null} = \sum_{b'l} \left((K_{bb'}^{A})^{-1} P_{b'l} \tilde{C}_{l}^{A} + (K_{bb'}^{B})^{-1} P_{b'l} \tilde{C}_{l}^{B} - 2 (K_{bb'}^{AB})^{-1} P_{b'l} \tilde{C}_{l}^{AB} \right)$$
(5.75)



Figure 5.8: Simulation of TT to BB leakage from crosstalk, using five days of real telescope pointing and the average of ten monte carlo realizations. HWP orientation rotation is disabled to make the simulation maximally pessimistic. Blue line shows the theoretical BB power spectrum from gravitational lensing, green points show the expected leakage of TT to BB due to crosstalk.

$$K_{bb'}^{AB} = \sum_{ll'} P_{bl} M_{ll'}^{AB} F_{l'}^{AB} B_{l'}^2 Q_{l'b'}$$
(5.76)

 $M_{ll'}^{AB}$ is computed analytically from the overlapping sky region. \tilde{C}_l^A is the pseudo power spectrum from one data set and F_l^A is the filter transfer function simulated for that one data set. C_l^{AB} and F_l^{AB} are the equivalent quantities computed for the cross power spectrum between the two data sets. The cross power spectra are calculated by crossing each one day map from the first set with each one day map from the second set.

This construction is free from signal bias by construction. Because the filtering between two maps will in general be different, differencing two maps and taking the power spectrum will leak some signal power, which requires knowledge of the signal power spectrum. Our construction is designed to be free from this bias.

One caveat to null tests is that for many possible systematic errors, the null tests provide constraints only at the level of a few times the statistical errors. For example, a null test designed to look for sidelobe ground contamination (g) may see pickup at low elevation (x) and not in high elevation (y) maps, in addition to the usual signal and noise components.

The x and y maps will have components

$$x = s + n_1 + g \tag{5.77}$$

$$y = s + n_2 \tag{5.78}$$

The power spectra of these maps X and Y will be

$$X = S + N + G \tag{5.79}$$

$$Y = S + N \tag{5.80}$$

The null test power spectrum Z sees only the ground contamination power G.

$$Z = X - Y = G \tag{5.81}$$

The error bar on the null test power spectrum for the case of G = 0 will be

$$\Delta Z = \sqrt{\frac{2}{\nu}}\sqrt{2}N\tag{5.82}$$

So the signal to noise on the ground contamination is

$$\frac{Z}{\Delta Z} = \frac{1}{\sqrt{2}} \frac{G}{f} \tag{5.83}$$

Where $f = \sqrt{\frac{2}{\nu}}N$.

This should be compared to the signal to noise on the ground contamination in the coadded map. The coadded map is

$$c = \frac{1}{2}(x+y) = s + \frac{1}{2}(g+n_1+n_2)$$
(5.84)

The coadded power spectrum is

$$C = S + \frac{1}{2}N + \frac{1}{4}G \tag{5.85}$$

Now there are two important cases for the error bars on the coadded power spectrum, signal dominated and noise dominated. In the noise dominated case, relevant for the POLARBEAR season 1 BB power spectrum, the error bars are

$$\Delta C = \sqrt{\frac{2}{\nu}} \frac{1}{2} N = \frac{1}{2} f$$
 (5.86)

And the signal to noise of the ground contamination in the coadded power spectrum is

$$\frac{C_G}{\Delta C} = \frac{1}{2} \frac{G}{f} \tag{5.87}$$

This is $\sqrt{2}$ less significance than the statistical significance in the null test. If we limit the signal to noise of the systematic error in the null test to *n* sigma, then the systematic error in the real data will be limited to $\frac{n}{\sqrt{2}}$. This is diluted by the number of null tests included in the suite. For 100 tests, a probability to exceed (PTE) criterion for the entire suite of 5% requires a 0.05% PTE per test, or 3.3 sigma. This implies a 2.3 sigma bias is allowed for in the science power spectrum, where our actual cited systematic error bars as determined from direct instrumental measurements is only a 0.2 sigma effect.

Sources of systematic errors which produce correlated effects in the split can have more favorable limits. For example, differential pointing cancels out if the sky is observed in two orientations 180 degrees apart in a coadded map, but sums constructively in the null test.

If the coadded power spectrum is signal dominated, for example TT or EE at low enough ℓ , then the error bars of the coadded power spectrum will be much larger than the error bars of the null test, and the null tests can be very constraining. These null tests are hard to pass due to the high signal to noise amplifying the effect of calibration errors beyond where they are significant for the BB measurement.

Calculating PTEs for the null tests is challenging in practice due to inexact overlap in modes from filtering and scan coverage between the two datasets. To calculate PTEs, we constructed two levels of test statistic. Using a suite of 500 monte carlos, first we estimated the error bars on the null test for each data division. This was combined into a single test statistic designed to look for extreme systematic errors by taking the most extreme test from the suite. To calculate the correct PTE for the suite, including the effects of correlations between tests, we used the 500 monte carlo realizations to construct a PTE distribution with approximately 0.2% resolution, sufficient to determine if the entire suite passes at the 5% level.

Our choice of null tests was motivated by picking divisions which were reasonably uncorrelated with each other and could detect a hypothetical physically motivated systematic error. The splits were:

- First six months vs. second six months. This test is generically sensitive to long term drifts. It was particularly motivated by the change to the optical baffling to eliminate the over the primary sidelobe which scanned Toco. This change was made almost coincident with the mid point of data taking.
- Rising vs. setting. This test splits the data by whether the patch was rising or setting, or equivalently by left vs. right azimuth range. This test is sensitive to the sidelobe scanning Toco because that occurred only in one range of azimuth. It is also sensitive to effects which vary with sky orientation, like differential beam systematics.
- High elevation vs. low elevation. In addition to being sensitive to the same effects as the rising vs. setting test, we found an elevation dependent source of

microphonics. If this had an unanticipated systematic effect, this test could see it.

- High responsivity (ADC counts per Kelvin) vs. low responsivity detectors. This test probes for nonlinearity effects, because responsivity is correlated with linearity. It could also be sensitive to calibration errors in responsivity.
- Good weather vs. bad weather. This probes for problems due to bad weather. Bad weather can cause the detectors to saturate or latch.
- Pixel orientation. Pixels are fabricated on the wafer in two orientations. Common systematic errors between the two orientations will be averaged down as if the orientation were sky rotation, which this null test can detect.
- Left vs right side of the focal plane. The areas of the map which are in common between the two halves of the focal plane are scanned by those areas in different sky orientations, which can induce different beam systematic errors.
- Left going vs right going subscans. This test probes for residual scan correlated microphonic effects, which were found to be different in left and right going subscans.
- Moon distance. This test looks for contamination from sidelobes scanning the moon.

Chapter 6

Results

6.0.1 BB power spectrum



Figure 6.1: Black points: Measured BB power spectrum from all three patches (\approx 30deg²). Red line: Theoretical WMAP-9 ACDM best fit cosmology prediction for BB. Error bars show the diagonal of the covariance matrix.

ℓ	D_{ℓ}^{BB}	ΔD_{ℓ}^{BB}
700	0.093	0.056
1100	0.149	0.117
1500	-0.317	0.236
1900	0.487	0.482

Table 6.1: POLARBEAR season 1 band powers

6.0.2 Interpretation of negative bandpowers

Our third BB bandpower is negative, at $-0.317\mu K^2$ for the l=1300-1700 band, but the power spectrum must be a positive quantity. This occurs because the bandpowers are produced by an unbiased estimator which is approximately normally distributed. If the true BB spectrum were zero, then half of the bandpowers would be expected to be negative. Only the total signal plus noise spectrum is required to be positive; when we calculate the noise bias-free cross spectra, the noise fluctuations can by chance anti-correlate.

The negative bandpower possibility forces us to be careful in the construction of confidence intervals. The frequentist definition of confidence intervals stands, i.e. if we construct 95% confidence intervals and repeated the experiment many times, 95% of the intervals would contain the true bandpower of the BB spectrum. For a negative bandpower, this doesn't seem very useful, because the negative region of the confidence interval is not a possible value for the true bandpowers. One way around this is to use upper limits for negative bandpowers and confidence intervals only when the entire interval would be positive, however, because the confidence interval construction depends on the data, the intervals will no longer contain the true parameter at the designed rate[33]. We chose avoid these issues by distributing our results only in the form of a likelihood which describes the probability of measuring our results given some theoretical model. The model for our bandpowers \hat{C}_b is a function of the true bandpowers C_b and Gaussian noise n_b with covariance matrix $N_{bb'} = \langle n_b n_{b'} \rangle$.

$$\hat{C}_b = C_b + n_b \tag{6.1}$$

The likelihood for \hat{C}_b is

$$\mathcal{L}(\hat{C}_b|C_b) = \frac{1}{\sqrt{|N|}} \exp\left(-\frac{1}{2}(\hat{C} - C)^T N^{-1}(\hat{C} - C)\right)$$
(6.2)

For the theory model C, we use the best fit WMAP-9 Λ CDM power spectrum scaled by an amplitude A_{BB} , where $A_{BB} = 1$ is the predicted BB from gravitational lensing power spectrum amplitude. We find $A_{BB} = 1.12 \pm 0.61(stat)^{+0.04}_{-0.10}(sys)$. The

rejection of the null hypothesis that there is no BB from gravitational lensing, or $A_{BB} = 0$, is 97.5%. This is calculated by integrating the likelihood for $A_{BB} > 0$, using a uniform prior on A_{BB} .

Consistency with ΛCDM is also tested by calculating $\chi^2 = -2\log\mathcal{L}$ for $A_{BB} = 1$. The probability to exceed for the χ^2 is 42%.

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Appendix A Half-Wave Plate Model

Here we derive a Mueller matrix for the single layer antireflection coated, single birefringent crystal half-wave plate at normal incidence. We calculate in the configuration where the crystal axes are aligned vertical and horizontal with the electromagnetic basis, so that there is no rotation, and no coupling between vertical and horizontal electromagnetic fields. This allows us to only keep track of two components in each medium, left going and right going electric field amplitude, and separately solve the problem for the vertical and horizontal field.

An electromagnetic field with angular frequency ω is incident on the plate from the right. At each interface, some fraction of the electromagnetic field reflects, and some transmits. In the bulk, the phase of the wave advances or retards depending on direction of travel. Because Maxwell's equations are linear, this can be modelled as a linear system of equations on the left going and right going electromagnetic field amplitudes at each interface. The transmission through the entire stack of the wave plate can be calculated by multiplying a sequence of matrices describing the properties of each medium B or interface I together with the incident and reflected electromagnetic field.

$$o = I_{va}B_a I_{as}B_s I_{sa}B_a I_{av}i = Ai \tag{A.1}$$

- *o* Transmitted electric field vector. The far side reflected component is zero, because there is no light shined on that side of the wave plate.
- i Incident and reflected electric field vector.
- $I_{av} = I_{va}^T$ Interface matrix of vacuum to antireflection coating and vice versa
- $I_{as} = I_{sa}^{T}$ Interface matrix of sapphire to antireflection coating and vice versa
- B_a Bulk matrix of antireflection coating
- B_s Bulk matrix of sapphire, the birefringent crystal

The interface matrix is given by the Fresnel equations at normal incidence. For indices n_1 and n_2 in regions 1 and 2, the fresnel equations give a relationship between the left and right going electromagnetic fields in each region.

$$E_l^1 - E_r^1 r_{12} = E_l^2 t_{21} \tag{A.2}$$

$$E_r^1 t_{12} = E_r^2 - E_l^2 r_{21} \tag{A.3}$$

$$r_{12} = \frac{n_1 - n_2}{n_1 + n_2} \tag{A.4}$$

$$t_{12} = \frac{2n_1}{n_1 + n_2} \tag{A.5}$$

Solving these equations for the interface matrix gives

$$I_{12} = \frac{1}{2n_2} \begin{pmatrix} n_1 + n_2 & n_2 - n_1 \\ n_2 - n_1 & n_1 + n_2 \end{pmatrix}$$
(A.6)

The bulk propagation matrix is determined by the wavelength of the electromagnetic field in the medium, and for a thickness x is

$$B_1 = \begin{pmatrix} e^{ik_1x_1} & 0\\ 0 & e^{-ik_1x_1} \end{pmatrix}$$
(A.7)

$$k_1 = \frac{n_1 \omega}{c} \tag{A.8}$$

Because one component of Eq. A.1 is known in o as zero, the left going component incident from the right side of the HWP, it is easier to solve for i in terms of the transmitted component in o by inverting A. Choosing an output electric field magnitude of 1, o is

$$o = \begin{pmatrix} 0\\1 \end{pmatrix} \tag{A.9}$$

Then the transfer function from incident electromagnetic field to transmitted is

$$\frac{E_t}{E_i} = \frac{1}{(A^{-1})_{01}} \tag{A.10}$$

Finally, we need to calculate the Mueller matrix, which relates Stokes components on the inputs to Stokes components on the outputs. This can be done inputting some electric field and calculating the output electric field, then calculating the Stokes components of the input and output. With four choices of input electric field, all sixteen Mueller matrix components can be extracted. Given an electric field, the Stokes components are

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} |E_x|^2 + |E_y|^2 \\ |E_x|^2 - |E_y|^2 \\ 2Re[E_x E_y^*] \\ 2Im[E_x E_y^*] \end{pmatrix}$$
(A.11)

Using these expressions to calculate the HWP mueller matrix elements, we find that it can be written

$$M_{HWP} = \begin{pmatrix} t & r & 0 & 0 \\ r & t & 0 & 0 \\ 0 & 0 & c & -s \\ 0 & 0 & s & c \end{pmatrix}$$
(A.12)

For the ideal HWP, t = 1, c = -1, s = 0, and r = 0. In practice, for a sky with no circularly polarized power, we can ignore s and only the three elements c,t,r are important.

The polarization efficiency is determined by the ratio of input polarized power to output polarized power. Total polarized power is given by the vector length of Q,U, or

$$P = \sqrt{Q^2 + U^2} \tag{A.13}$$

In an arbitrarily rotated frame, a Stokes vector transforms by

$$S' = R_{\theta}S = \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & \cos(2\theta) & -\sin(2\theta) & 0\\ 0 & \sin(2\theta) & \cos(2\theta) & 0\\ 0 & 0 & 0 & 1 \end{pmatrix} S$$
(A.14)

The mueller matrix M transforms by

$$M' = R_{\theta}^T M R_{\theta} \tag{A.15}$$

Appendix B Electrical crosstalk

Here we calculate the responsivity of current in the SQUID I^{ω} biasing the frequency ω of bolometer to optical power deposited on a second bolometer m. The current through a single bolometer is I_k^{ω} . One source of crosstalk occurs even in an ideal circuit where the voltage bias of the comb is perfectly stiff. In this case, because of the finite width of the resonances, current at one frequency flows in small amounts through bolometers nominally biased at other frequencies. The leakage current is modulated by the resistance fluctuations of the

$$\frac{dI^{\omega}}{dP_m} = \frac{\partial I_m^{\omega}}{\partial R_m} \frac{\partial R_m}{\partial P_m} \tag{B.1}$$

For an infinite loopgain bolometer, the total electrical and optical power is fixed, so the change in bolometer resistance is determined by the derivative of the electrical power with resistance.

$$P = \frac{V^2}{R} \tag{B.2}$$

$$\frac{\partial R_m}{\partial P_m} = -\frac{R^2}{V^2} \tag{B.3}$$

The change in current through the leaked bolometer is determined by Ohm's law, including the impedance of the off resonance series RLC Z_m^{ω} .

$$I_m^{\omega} = \frac{V^{\omega}}{Z_m^{\omega}} \tag{B.4}$$

$$\frac{\partial I_m^{\omega}}{\partial R_m} = -\frac{V^{\omega}}{(Z_m^{\omega})^2} \tag{B.5}$$

This gives the crosstalk in phase with the carrier current on the receiving bolometer as

$$\mathcal{R}\left[\left(\frac{R_m}{Z_m^{\omega}}\right)^2\right]\frac{1}{V^{\omega}}\tag{B.6}$$

The $(V^{\omega})^{-1}$ is the usual bolometer responsivity, so the crosstalk is suppressed by the squared ratio of the bolometer resistance to the off resonance impedance of the LC filter. For the nominal channel spacing of 75kHz and $L = 13\mu H$, and a bolometer resistance of 1 Ω , the off resonance impedance is 8 Ω , and the crosstalk is suppressed to 1.6% of the usual responsivity. The sign of the crosstalk is negative such that when scanning a planet the mainlobe is a positive peak and the crosstalk sidelobes are negative peaks.