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Abstract

This is a model of quality ladder of machines in an endogenous growth context, when some machines are scrapped while others coexist with the latest variety, with the economic life of machines endogenously determined as in old vintage-capital models. Policies that affect this economic life of machines (for example, those influencing the gross savings or investment rate, or trade policy in a two-sector open economy) will have an effect on the long-run growth rate.

ENDOGENOUS GROWTH THEORY IN A VINTAGE CAPITAL MODEL[1]

by

Pranab Bardhan and Rodrigo Priale

I INTRODUCTION

In the development of the endogenous growth models over the last decade, the idea of a continual introduction of new inputs as an embodiment of technological progress has played a central role. In particular, the original formulations by Ethier (1982) and Romer (1990) in terms of an expanding variety of intermediate goods contributing to final good productivity have been very popular. In the "old" growth theory literature, the vintage capital models also carried the idea of a continual introduction of new inputs, with the latest technology embodied in the newest machine that comes into operation. Some of the early formulations of this type of models are associated with Johansen (1959), Solow (1962), Phelps (1963) and Solow, Tobin, von Weizsacker, and Yaari (1966).[2] However, in all these vintage models, the ultimate source of technical progress embodied in the latest machine was exogenous; in particular, the steady-state growth rate was not amenable to policy influence. In contrast, in the more recent models, growth is driven by endogenous processes of research and development (R & D) or learning, and these processes can be influenced by policy. Furthermore, most of the earlier models assumed competitive markets, whereas growth theory in the 1990's has formalized endogenous technical progress in terms of a tractable imperfect-competition framework, in which

[1] We are grateful to Paul Romer for valuable discussion at an early stage of formulation of the model. Of course, all errors remain ours alone.

[2] For extensions of these models, see Bliss (1968) and Bardhan (1969).

temporary monopoly power sometimes acts as a motivating force for private innovators and there are scale economies.

On the other hand, in many of the recent models -- for example, in that of Romer (1990) -- technical progress works through a horizontal expansion of the range of inputs, and the wider the range the better for productivity. In these models, the old abacus goes on being utilized for calculation jobs side by side with the latest desktop computer. In most of the vintage models, as new inputs arrive, some of the old varieties are no longer profitable to use, and a central focus of these models was the endogenous obsolescence of machines.

In this case the economic life of machines becomes an important determinant of productivity. Different countries with different wage rates (that one has to pay to operators of machines) will thus have a different economic life of machines, even if "best-practice" technology (or technology on the newest machine) is the same everywhere.[3] One of the earliest economists to have a serious empirical treatment of this issue was Salter (1960). To quote him for an example:

> "In fact there is some evidence to suggest that one of the chief reasons for Anglo-American productivity differences lies in standards of obsolescence. It is a common theme in Productivity Mission Reports that the productivity of the best plants in the United Kingdom is comparable with that of the best plants in the United States, and that the difference lies in a much higher proportion of plants employing outmoded methods in the United Kingdom -- a much greater 'tail' of low-productivity plants. Such a situation is consistent with a higher standard of obsolescence in the United States which follows from a higher level of real wages. (pp.72-73)"

Clearly, productivity differences on such grounds will be much sharper between

[3] For an early model of international trade patterns based on differences in comparative advantage following from differences in the economic life of machines, see Bardhan (1966).

rich and poor countries.

Of course, obsolescence of inputs is captured in part of the recent growth literature, most notably in the Schumpeterian models of Aghion and Howitt (1992) and Segerstrom *et al.* (1990), and in the quality ladder models of Grossman and Helpman (1991). But in those models obsolescence of inputs takes place in an extreme fashion: the state-of-the-art product completely and instantaneously displaces all the old varieties. So the richness of the vintage models in this respect, with some old inputs being scrapped while some others coexisting with the latest variety, is missing in the recent literature, and accordingly the differential productivity effects of the different length of the 'tail' that Salter talks about is not analyzed.

In this paper we combine this aspect of richness of the old vintage models with endogeneity of technical progress and monopolistic competition and dynamic economies of scale (aspects in which the recent models are richer). In particular, we show how the endogenously determined economic life of machine, denoted by T in our model, affects not just the level of productivity, as in the old vintage models, but also the steady-state growth rate. Policies can affect the equilibrium value of T (for example, those influencing the gross savings rate in the economy, accelerated depreciation allowance in tax laws, etc.), and that will have an effect on the long-run growth rate. We also present a two-sector extension of the basic model where we show that trade policy, by influencing the relative price of goods, can affect T and thus the long-run growth rate of a trading economy.[4]

While our model is abstract, oversimplified and extremely limited in applicability, it may not be entirely out of place to link the results with some of the issues that have come up in the empirical and policy literature on comparative economic growth and

[4] In Bardhan and Kletzer (1984) there is an analysis of the dynamic effect of protection on the time path of productivity in a partial equilibrium model of learning.

economic history. For example, in comparisons of fast economic growth in East Asia with relative stagnation in Latin America in recent history, it is commonplace to point to the significant differences in the rates of saving in the two areas. Our model may suggest an additional dimension of the effects of higher saving and investment on the growth rate in terms of modernization of the capital stock. Our result is also consistent with the significant correlation observed by de Long and Summers (1991) between equipment investment and growth. We may also venture to suggest that embodied technical progress in new machines may have some implications for the widely noted and remarkable empirical work of Young (1995) which shows that factor accumulation largely explains the high growth in East Asia. If a high rate of capital accumulation also leads to a modernization of the capital stock and if embodiment matters, it may be statistically difficult to disentangle the effects of factor accumulation from those of technical progress.[5]

In the economic history literature, the relationship between trade policy and modernization of capital stock has sometimes been commented upon. For example, Williamson (1971) has emphasized the importance of highly protective tariffs in early nineteenth-century United States in encouraging a faster scrapping of capital in favor of technologically superior equipment in the textile industry and thus fostering rapid productivity growth in that industry. Temin (1966), in interpreting the relative decline of the British steel industry in the period from the 1880's to World War I, has referred to the tariff-induced adoption of superior capital equipment in the United States and Germany. For all its limitations, our two-sector model may provide a simple framework for analyzing such questions of the impact of trade policy on the economic life of capital and thus on the rate of growth, even though our particular results about the growth-retarding effects of protection may be model-specific.

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[5] There is some old empirical literature showing that embodiment may not have mattered in the productivity growth over some periods in the United States. But no one has yet shown this for East Asia.

The plan of the rest of the paper is as follows. In section II we enumerate the basic vintage capital model with endogenous technical progress and economies of scale, both external and internal, in the production of capital goods. In section III we derive some of the comparative-dynamic results. In section IV we present a two-sector extension of the basic model to focus on the effect of trade policy and relative price changes. Section V provides some conclusions.

II. THE BASIC MODEL

Let us consider an economy in which a single final good is produced, with a fixed-coefficient production function, using labor and capital goods of different vintages.[6] If factors of production are fully employed, the final good is produced with the following production function:

$$Y(v\ ,\ t)\ =\ a(v)\ K(v)\ =\ b(v)\ L_Y\ (v\ ,\ t) \tag{1}$$

where Y(v,t) is output of the final good produced at time t using capital goods of vintage v, L_Y(v,t) is labor used to produce that output, and K(v) is the composite of capital goods of vintage v.

[6] In this respect, our vintage model is akin to that of Solow, Tobin, von Weizsacker, and Yaari (1966).

Since the final good can be produced using capital goods of different vintages, total production in the final good sector at time t, Y(t), is given by

$$Y(t) = \int_{t-T}^{t} Y(v,t)\, dv \qquad (2)$$

where t - T is the vintage of the oldest machines in use, and T is the economic life of the machines.

In this economy, at any time t, n(v=t) differentiated capital goods are produced by n(v=t) monopolistic competitive firms, each of them producing $I_i(v)$ units of capital good i of vintage v. The composite of those capital goods of vintage v, K(v), is

$$K(v) = \left[\int_{0}^{n(v)} I_i\,(v)^{\alpha}\ di \right]^{\frac{1}{\alpha}} \qquad 0 < \alpha < 1 \qquad (3)$$

where I_i (v) stands for units of capital good i of vintage v, and n(v) is the number of differentiated capital goods of vintage v.

In the production of these capital goods we assume increasing returns to scale, both external and internal; this has family resemblance to the internal and external scale economies assumed in the production, through the R & D process, of intermediate goods in recent growth models, like that of Romer (1990). To produce each capital good of vintage v, $F/K_H(v)$ units of labor has to be invested first, where $K_H(v)$ is the stock of general knowledge capital available at the moment of producing the new machines of vintage v. Once the fixed cost in terms of labor is invested, it is possible to produce $\beta(S(v))$ units of I_i (v) with one unit of labor, where S(v) is the quality index of machines of vintage v. This implies that

$$I_i(v) = \beta\ (S(v))\ L_i\ (v) \qquad \textbf{(4)}$$

That quality index increases as new machines are produced, according to the following equation

$$\dot{S}\ (v) = n(v)\ I(v) \qquad (5)$$

where, as we shall see later, $I(v) = I_i(v)$ for all i. Thus, we have internal scale economies through the fixed cost and external scale economies through learning effects. The learning effect operates both on reducing the fixed cost as general knowledge capital improves and on the variable cost through the cumulated quality index, S(v). This index has a learning effect akin to that of the "serial number of machines" used in Arrow's (1962) original learning by doing model.

With these assumptions, the variable cost of producing capital goods of vintage v is w(t) L_i (v), where w(t) is the wage rate at time t. Using (4), we can write the marginal cost of producing capital good i of vintage v at time t, for all i, as

$$MC(v,t) = \frac{w(t)}{\beta\ (S(v))} \qquad \textbf{(6)}$$

Because of the symmetric way in which each capital good i of a given vintage v enters in the sub-production function (equation (3)) and also because the cost of producing each capital good of a given vintage is the same, it is optimal to produce the same quantity of capital goods of the same vintage, that is, I_i (v) = I (v) for all i. Then, from (3) it is easy to obtain that

$$K(v) = l(v)\, n(v)^{\frac{1}{\alpha}} \tag{7}$$

Let $X(v) = \int_0^{n(v)} l_i(v)di$ be the aggregate demand for capital goods of vintage v. Since $l_i(v) = l(v)$ for all i, then

$$X(v) = n(v)\, l(v) \tag{8}$$

Using equation (8), equation (7) can be rewritten as follows:

$$K(v) = n(v)^{\frac{1-\alpha}{\alpha}} X(v) \tag{9}$$

Let us assume that in the final good sector technological progress is purely labor-augmenting. Additionally, to simplify the model's solution, we are going to assume that labor productivity in the production of final goods is also a function of past experience and depends on the quality index, that is,

$$b(v) = b(S(v)) \tag{10}$$

and

$$a(v) = a \tag{11}$$

Firms in the final good sector are competitive and maximize profits. They maximize

$\pi_f(t) = P_Y(t) \int_{t-T}^{t} Y(v,t)dv - \int_{t-T}^{t} w(t)\, L_Y(v,t)dv - \int_{t-T}^{t} [\int_0^{n(v)} R_i(v,t)\, I_i(v)\, di]dv$, where $P_Y(t)$ is the price of the final good, and $R_i(v,t)$ is the price of the capital good i of vintage v, at time t. Taking the price of the final good as the numeraire, so that this price is unity, the first order conditions of profit maximization are:

$$b(S(v)) = w(t) \tag{12}$$

$$I_i(v) = \left[\frac{a\, K(v)^{1-\alpha}}{R_i(v\ ,\ t)} \right]^{\frac{1}{1-\alpha}} \tag{13}$$

In the capital good sectors, to maximize profits, each monopolistic competitive firm equates marginal revenue to marginal cost. Marginal revenue is equal to $MR_i(v,t) = (1 - 1/\eta)R_i(v,t)$, where η is the price elasticity of the demand for $I_i(v)$. From equation (13) we have that $\eta = 1/(1-\alpha)$. Therefore, $MR_i(v,t) = \alpha R_i(v,t)$, whereas the marginal cost is given by equation (6). Thus, each firm in the capital good sector maximizes profits setting $\alpha R_i(v,t) = w(t)/\beta(S(v))$, which implies that[7]

$$R_i(v\ ,\ t) = \frac{w(t)}{\alpha\ \ \beta\ (S(v))} \tag{14}$$

In the final good sector, a machine is scrapped when the wage bill paid to operate

[7] Equation (14) states that profit maximization in the capital good sectors implies that $R_i(v,t) = R(v,t)$, for all i, which according to equation (13) allows us to conclude that $I_i(v) = I(v) = [\ aK(v)^{(1-\alpha)}/R(v,t)]^{1/(1-\alpha)}$, for all i.

it exhausts the value of total output.[8] Then, from equation (1) the following scrapping condition is obtained

$$b(t - T) = b(S(t - T)) = w(t) \tag{15}$$

Let $P_K(v,t)$ be the price of the composite of capital goods K(v). Then the following relationship must hold: $P_K(v,t)\,K(v) = \int_0^{n(v)} R_i(v,t)I_i(v)di$. But since $I_i(v) = I(v)$ and $R_i(v,t) = R(v,t)$ for all i, from this last relationship we obtain

$$P_K(v,t)\,K(v) = n(v)\,R(v,t)\,I(v) \tag{16}$$

Combining equations (7) and (16), the price of the composite of capital goods is determined by

$$P_K(v,t) = R(v,t)\,n(v)^{\frac{\alpha-1}{\alpha}} \tag{17}$$

In the final good sector, wages plus quasi-rents must exhaust the value of output, that is,

$$Y(v,t) = L_Y(v,t)\,w(t) + K(v)\,\rho_K(v,t) \tag{18}$$

[8] Machines of vintage v are scrapped when $w(t)L_Y(v,t) = Y(v,t)$, that is, when $w(t) = Y(v,t)/L_Y(v,t)$.

where $\rho_K(v,t)$ is the quasi-rents of the composite of capital goods of vintage v at time t. Using equations (1), (10), (11) and (15), from equation (18) we find that the quasi-rents are given by

$$\rho_K(v, t) = a\left[1 - \frac{b(S(t-T))}{b(S(v))}\right] \tag{19}$$

In equilibrium, the price at time t of the composite of capital goods K(v) must be equal to the present value of the expected quasi-rents, discounted at the market rate of interest, r(t). Since $P_K(v,t)$ is the price of K(v) at time t, then

$$P_K(v, t) = \int_t^T \rho_K(v, u)\, e^{-\int_t^u r(z)dz}\, du \tag{20}$$

As v = t, equation (20) gives $P_K(t,t)$, which is the market price of the composite of capital goods at the moment of their construction. Since in steady-state the interest rate, r, is constant, setting v = t equation (20) becomes

$$P_K(t, t) = \int_t^{t+T} \rho_K(t, u)\, e^{-r(u-t)}\, du \tag{21}$$

Labor market equilibrium requires that the supply of labor at time t be equal to the sum of labor used to produce final goods and capital goods. Assuming that labor supply is constant and equals L, this implies that

$$L = L_Y(t) + \int_{t-T}^{t} n(v)\, L_i(v)\, dv + \frac{F}{T} \qquad (22)$$

where $L_Y(t)$ is labor used to produce final goods, $\int_{t-T}^{t} n(v)L_i(v)\, dv$ is labor used to produce capital goods, and since $F/K_H(v)$ units of labor are required to develop a new capital good, at time t, $n(v = t)F/K_H(v = t)$ is the amount of labor devoted to research and development. Let us assume that the stock of capital knowledge is proportional to the economy's cumulative experience in research and development. With that assumption and by an appropriate choice of units, the factor of proportionality may be set to one, so that $K_H(v) = n(v)T$.[9] Therefore, at time t, labor employed in the development of new capital goods is given by $n(v{=}t)F/K_H(v{=}t) = F/T$, which is the third term of the right hand side of equation (22).

Labor employed in the production of final goods is equal to $L_Y(t) = \int_{t-T}^{t} L_Y(v, t)\, dv$. But from equation (1) we have that $L_Y(v, t) = a(v)\, K(v)/b(v)$, and since from (7), (8), and (11) we know that $K(v) = l(v)\, n^{1/\alpha}$, $X(v) = n(v)\, l(v)$, and $a(v) = a$, then $L_Y(t) = \int_{t-T}^{t} n^{(1-\alpha)/\alpha} X(v)/b(v)\, dv$. Therefore, using this last expression together with equations (4) and (8), the labor market equilibrium condition becomes

$$L = a \int_{t-T}^{t} n(v)^{\frac{1-\alpha}{\alpha}} \frac{X(v)}{b(S(v))}\, dv + \int_{t-T}^{t} \frac{X(v)}{\beta\, (S(v))}\, dv + \frac{F}{T} \qquad \textbf{(23)}$$

The operative profits at time t of a typical firm that produces capital good i of

[9] Formally, the stock of knowledge capital should be proportional to $K_H(t) = \int_{t-T}^{t} n(v)dv$. We will see later that due to the assumption that technical progress in both the final good sector and the capital good sectors depend on the quality index, and as a result both grow at the same rate, it is obtained that $n(v) = n$ for all v. This implies that $K_H(t) = n(t)T$.

vintage v are equal to $\pi(v,t) = R(v,t)\ I(v) - w(t)\ L_I(v)$. But since from (4) we know that $L_I(v) = I(v)/\beta(S(v))$, then $\pi(v,t) = [\ R(v,t) - w(t)/\beta(S(v))\ ]\ I(v)$. Using equation (14), this last expression can be rewritten as $\pi(v,t) = (1 - \alpha)\ R(v,t)\ I(v)$.

There is free entry in the capital good sectors. Therefore, firms in these sectors are going to enter until the operative profits are equal to the entry costs, that is, until $\pi(v,t) = (1 - \alpha)\ R(v,t)\ I(v) = w(t)\ F/n(v)T$.[10] Recalling that $I(v) = X(v)/n(v)$, and using (14), the following zero-profit condition is obtained

$$X(v) = \frac{\alpha}{(1 - \alpha)}\ \beta\ (S(v))\ \frac{F}{T} \tag{24}$$

To close the model it remains to specify the equilibrium in the final good sector. To this end, let us assume that consumers save a constant fraction, s, of their income, so that total investment at time t, $I_T(t)$, is given by

$$I_T(t) = s\ Q_T(t) \tag{25}$$

where $Q_T(t)$ is the economy's income (or total output).

III COMPARATIVE DYNAMICS

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[10] This is a simplifying assumption. Formally, at time t, the entry costs $w(t)F/n(v)T$ should be equal to the present value of the stream of profits, that is, $\int_t^{t+T} \pi(v, u)\ e^{-\int r(z)dz}\ du$. The assumption that the entry costs are just equal to the current profits is a simplification that makes the model more tractable, and does not alter the results.

Having specified the basic equations of the model, in this section we are going to show how the economic life of the machines, denoted by T, can become an important determinant of the rate of growth of the economy.

To simplify the model's solution, let us now assume that $\beta(S(v)) = \beta\ S(v)$, and $b(S(v)) = b\ S(v)$, where β and b are positive constants. Since from (8) we know that $X(v) = n(v)\ I(v)$, equation (5) implies that the quality index, S, accumulates according to $dS(v)/dt = n(v)\ I(v) = X(v)$, which in turn implies that

$$\frac{\dot{S}(v)}{S(v)} = \frac{I(v)\ n(v)}{S(v))} = \frac{X(v)}{S(v))} \tag{26}$$

Substituting equation (24) in (26), and then integrating both sides of the resulting equation, it is obtained that $\beta(S(v)) = \beta\ e^{\lambda v}$ and $b(S(v)) = b\ e^{\lambda v}$, where $\lambda = \beta\alpha F/(1-\alpha)T$. With these results, equation (19) becomes

$$\rho_K(v\ ,\ t) = a\ [\ 1\ -\ e^{\lambda\ (t\ -\ T\ -\ v)}\ ] \tag{27}$$

Substituting equation (27) in (21) and then integrating, after some algebraic manipulations, we get that the price of the composite of capital goods at the moment of their construction is equal to

$$P_K(t\ ,\ t) = a\ [\frac{(r - \lambda) - r\,e^{-\lambda T} + \lambda\ e^{-rT}}{r\ (r - \lambda)}] \qquad (28)$$

From equation (17) we have that as v = t, $n(v{=}t)^{(1-\alpha)/\alpha} = R(t,t)/P_K(t,t)$ and from (14) we know that $R(t,t) = w(t)/\alpha\beta(S(t))$. Therefore, $n(v{=}t)^{(1-\alpha)/\alpha} = w(t)/[\alpha\beta(S(t))P_K(t,t)]$. Since $\beta(S(v)) = \beta\ e^{\lambda v}$, substituting equation (28) in this last expression, for the case in which v = t, and using the scrapping condition, equation (15), it is obtained that the number of differentiated capital goods of vintage v, when they are first produced, that is, as v = t, is given by

$$n(v)^{\frac{1-\alpha}{\alpha}} = \frac{b\ e^{-[\beta \frac{\alpha}{1-\alpha} F]}}{\beta\ a\ \alpha}\ \frac{r\ (r-\lambda)}{[\ (r-\lambda)\ -\ r e^{-\lambda T} + \lambda\ e^{-rT}]} \qquad (29)$$

Equation (29) implies that if the rate of interest and the economic life of the machines are constants, the number of differentiated capital goods is the same for all vintages.[11] Replacing equation (24) in the resource constraint equation (23)), after integrating we get

$$L(t) = \frac{F}{(1 - \alpha)}\ [\ \frac{\beta\ \alpha\ a\ n^{\frac{1-\alpha}{\alpha}}}{b} + \alpha + \frac{(1 - \alpha)}{T}\ ] \qquad \mathbf{(30)}$$

To be able to solve the model, it remains to determine the economy's income. To

[11] This result is mainly obtained due to the assumption that technical progress in both the final good sector and the capital good sectors is a function of the quality index S(v). Without this assumption, n(v) would not be the same across vintages and it would be much more complicated to derive an analytical solution for the model.

do so, let us define the value of the economy's stock of capital, $K_T(t)$, as follows:

$$K_T(t) = \int_{t-T}^{t} n(v)\ R(v,t)\ I(v)\ dv = \int_{t-T}^{t} P_K(v,t)\ K(v)\ dv \tag{31}$$

Differentiating this last equation with respect to time we obtain that

$$\dot{K}_T(t) = K(t)\ P_K(t,t) + \int_{t-T}^{t} \dot{K}(v)\ P_K(v,t)\ dv + \int_{t-T}^{t} K(v)\ \dot{P}_K(v,t)\ dv \tag{32}$$

But $K(t)P_K(v,t) = n(v{=}t)R(t,t)I(t)$ is the economy's gross investment at time t, $I_T(t)$, and since $dK(v)/dt = 0$, equation (32) becomes

$$\dot{K}_T(t) = I_T(t) + \int_{t-T}^{t} K(v)\ \dot{P}_K(v,t)\ dv \tag{33}$$

In turn, if we differentiate equation (20) with respect to time we find that

$$\frac{\rho_K(v,t)}{P_K(v,t)} + \frac{\dot{P}_K(v,t)}{P_K(v,t)} = r \tag{34}$$

which implies that $dP_K(v,t)/dt = rP_K(v,t) - \rho_K(v,t)$. Substituting this expression in equation (33) we have that

$$\dot{K}_T(t) = I_T(t) + r\, K_T(t) - \int_{t-T}^{t} K(v)\, \rho_K(v,t)\, dv \tag{35}$$

Equation (35) can be rewritten as follows

$$I_T(t) - \dot{K}_T(t) = \int_{t-T}^{t} K(v)\, \rho_K(v,t)\, dv - r\, K_T(t) \tag{36}$$

This last equation states that gross investment, $I_T(t)$, minus net investment, $dK_T(t)/dt$, is equal to gross quasi-rents, $\int_{t-T}^{t} K(v)\rho_K(v, t)\, dv$, minus net profits, $rK_T(t)$. Both terms of equation (36) can be identified as "true depreciation." Since we are ignoring physical depreciation, as in Solow, Tobin, von Weizsacker and Yaari (1966), only "obsolescence" of the machines account for "true depreciation." Knowing that, the economy's GDP, from the income side, must be equal to wages plus capitalists' gross income, which in turn must be equal to net profits plus "true depreciation", that is, capitalists' income is equal to $rK_T(t) + I_T(t) - dK_T(t)/dt$. This implies, using equation (36), that GDP from the income side, $Q_T(t)$, is given by

$$Q_T(t) = w(t)\, L + \int_{t-T}^{t} K(v)\, \rho_K(v,t)\, dv \tag{37}$$

where, as was mentioned before, L is the amount of labor available in the economy. Then, substituting equations (9) and (27) in (37), and then using (24), after integrating, the following equation is obtained

$$Q_T(t) = w(t)L + a\, n^{\frac{1-\alpha}{\alpha}} e^{\lambda t} [1 - e^{-\lambda T}] - a \frac{\alpha}{1-\alpha} \beta\, F n^{\frac{1-\alpha}{\alpha}} e^{\lambda (t-T)} \quad (38)$$

In turn, total investment at time t, $I_T(t) = n(v{=}t)R(t,t)I(t)$. But $n(v{=}t)I(t) = X(t)$, and from equations (14) and (24) we know that $R(t,t) = w(t)/\alpha\beta(S(t))$ and $X(t) = \alpha\, \beta(S(t))\, F/(1 - \alpha)T$, which implies that $I_T = w(t)F/(1-\alpha)T$. Replacing equation (15) in this last expression, we obtain that

$$I_T(t) = \frac{1}{(1 - \alpha)} \frac{b\, F\, e^{\lambda (t-T)}}{T} \quad (39)$$

And, finally, replacing equations (38) and (39) in equation (25), after some algebraic manipulations, we get that

$$\frac{1}{(1 - \alpha)}\, b\, \frac{F}{T} = s[\, b\, L + a\, n^{\frac{1-\alpha}{\alpha}} [e^{\lambda T} - 1] - a \frac{\alpha}{1-\alpha} \beta\, F n^{\frac{1-\alpha}{\alpha}} \,] \quad (40)$$

The economy can be described by a system of three equations ((29), (30) and (40)) with three unknowns: the number of differentiated capital goods of a given vintage, n, the rate of interest, r, and the economic life of the machines, T. Combining those three equations, the following expression can be obtained, through which the economic life of the machines, T, is determined

$$[(1 - \alpha) \frac{L}{F} - \alpha - \frac{(1 - \alpha)}{T}] = \frac{\alpha \beta [\frac{1}{1-\alpha} \frac{F}{T} - s L]}{s [e^{\beta \frac{\alpha}{1-\alpha} F} - 1 - \frac{\alpha}{1-\alpha} \beta F]} \quad (41)$$

Equation (41) can be solved graphically in a plane in which the left and right hand side of that equation are measured along the vertical axis, and T is measured along the horizontal axis. As shown in figure 1, in that plane the LHS curve tends to minus infinity as T approaches zero, and tends to $[(1-\alpha)L/F - \alpha]$ as T goes to infinity. Therefore, in that figure, the LHS curve can be represented by an upward sloping curve, which intersects the horizontal axis as $T = (1-\alpha)F/[(1-\alpha)L - \alpha F]$. In turn, the RHS curve tends to infinity as T approaches zero and tends to zero as T tends to infinity. Therefore, the RHS can be represented by a downward sloping curve. The economic life of the machines, T, is determined at the intersection point of both schedules, point E. As the labor force, L, increases, the LHS curve would shift upwards, to LHS', whereas the RHS curve would shift downwards, to RHS'. Therefore, in the new equilibrium point, point E', the economic life of the machines,T, is smaller.

Recalling that $w(t) = be^{\lambda(t-T)}$, from equation (38) it is easy to see that the growth rate of income, $Q_T(t)$, and income per-capita, due to the assumption that L is constant, is equal to $\lambda = \beta\alpha F/(1-\alpha)T$. Therefore, this vintage capital model predicts that as the labor force, L, increases, the economic life of the machines,T, decreases and as result the rate of growth of the economy rises. This growth effect of a larger labor force is similar to the size or scale effect noted in the new growth literature.

Figure 2 illustrates the impact of an increase of the saving rate on T. In that figure, as the saving rate, s, increases, the LHS curve would remain unchanged, but the RHS schedule would shift downwards, to RHS'. Thus, in this case, also in the new intersection point, E', the economic life of the machines is smaller and consequently the growth rate of income increases. Again, this growth-promoting effect of higher saving rates is similar to that obtained in the new growth models.

While both of these results are comparable to those obtained in other growth models, in this vintage capital model they are obtained through a different mechanism. In this model, both the size effect of a larger labor force and higher savings rates reduce the time of profitable operability of machines, and thus modernizes the economy's stock of capital sooner, whereby the productivity of the economy grows faster.

III THE TWO-SECTOR EXTENSION

In this section we are going to analyze how trade policies, by influencing the relative price of goods, can affect the economic life of the machines and thus the long-run rate of growth of the economy.

To this end, let us now consider an economy in which two final goods are produced: good Y and good Z. For simplification, good Z is produced using only labor, whereas good Y is produced using labor and capital goods of different vintages. The production function used to produce good Y is the same as in section II, and is given by equation (1). Thus, equations (1) to (21) of the previous section remain unaltered and describe the production side of the capital intensive sector. In this two-sector economy, the price of good Y is taken as the numeraire. On the other hand, output in sector Z, the labor intensive one, is produced with a diminishing returns production function, which is given by

$$Z(t) = A(t)\, L_Z(t)^{\gamma} \qquad \gamma < 1 \tag{42}$$

where Z(t) is output of good Z, $L_Z(t)$ is labor employed in the production of good Z, and A(t) is a productivity parameter.

With this production function, profit maximization in sector Z implies that

$$\gamma \; P_Z(t) \, \frac{Z(t)}{L_Z(t)} = w(t) \tag{43}$$

where $P_Z(t)$ is the relative price of good Z at time t. Since it is assumed that labor market is competitive, equations (15) and (43) imply that

$$b(t - T) = b\, S(t - T) = \gamma \; P_Z(t) \, \frac{Z(t)}{L_Z(t)} = w(t) \tag{44}$$

In turn, labor market equilibrium requires that the supply of labor, L, be equal to the sum of labor used to produce both final goods and capital goods, that is,

where L_Y (t) and $L_Z(t)$ stand for labor employed in the production of final goods Y and Z,

$$L = L_Y(t) + \int_{t-T}^{t} n(v)\, L_i(v)\, dv + \frac{F}{T} + L_Z(t) \qquad (45)$$

respectively, and as was explained in section II, $\int_{t-T}^{t} n(v)L_i(v)\,dv$ is labor used to produce capital goods, and F/T is labor devoted to R & D.

Following the same procedure as in the previous section (see equation (23), (24) and from (26) to (29)), equation (45) becomes

$$L = \frac{F}{(1-\alpha)} \left[\frac{\beta\ \alpha\ a\ n^{\frac{1-\alpha}{\alpha}}}{b} + \alpha + \frac{(1-\alpha)}{T} + \frac{(1-\alpha)}{F} L_Z(t) \right] \qquad (46)$$

Then, let us assume that it is the case of a two-sector small open economy in which both final goods can be traded internationally. In the absence of barriers to trade, and under the assumption that the economy is small, the "law of one price" applies, and consequently the relative price of the labor intensive good (Z) is determined in the world economy and is taken as given by the small open economy. From equation (44), this implies that $P_Z(t)$ is given by

$$P_Z = P_Z^* = \frac{b\ S(t-T)\ L_Z(t)}{\gamma\ Z(t)} \qquad (47)$$

To close the model we need to specify the equilibrium in the final good sectors. Let us assume that consumers spend a constant fraction of their income on the consumption of the two final goods, that is,

$$C_Y(t) = c_1\ Q_T(t) \tag{48}$$

$$P_Z(t)\ C_Z(t) = c_2\ Q_T(t) \tag{49}$$

where c_1 and c_2 are the fractions of income that are spent on the consumption of good Y and good Z, respectively, and as before $Q_T(t)$ is the economy's income (or total output). In this case also the economy's income, $Q_T(t)$, is obtained adding labor income and capitalists' income, as was done in section II, and is given by equation (38).

Also in this small open economy, at any time t, each household devotes its income to consume the homogeneous final goods or to save. Therefore, the equilibrium in the product markets requires that total income be equal to total expenditure, that is,

$$C_Y\ (t) + P_Z(t)\ C_Z(t) + I_T(t) = Q_T(t) \tag{50}$$

where, $I_T(t)$ is total investment at time t. Implicitly, in equation (50) it is assumed that trade is balanced, so that $NX_T(t) = 0$, where NX_T stand for net exports. This implies that

$$NX_T(t) = NX_Y(t) + P_Z(t)\ NX_Z(t) = 0 \tag{51}$$

where $NX_Y(t)$ and $NX_Z(t)$ stand for net exports of good Y and good Z, at time t, respectively.

Substituting equations (48) and (49) in (50) we have that

$$I_T(t) = (1 - c_1 - c_2)\, Q_T(t) = s\, Q_T(t) \tag{52}$$

where s is the saving rate, and $I_T(t) = n(v{=}t)R(t,t)I(t)$ is gross total investment at time t, which is given by equation (39). Replacing equations (38) and (39) in equation (52), after some algebraic manipulations, we get that

$$\frac{b}{(1-\alpha)}\frac{F}{T} = s\,[b\,L + a\,n^{\frac{1-\alpha}{\alpha}}[e^{\lambda T} - 1] - a\frac{\alpha}{1-\alpha}\beta\,Fn^{\frac{1-\alpha}{\alpha}}\,] \tag{53}$$

Since it is the case of a small open economy, in sector Z total consumption of good Z plus net exports of that good must equal total output, that is, $C_Z(t) + NX_Z(t) = Z(t) = A(t)\, L_Z(t)^{\gamma}$. From equation (49) we know that $P_Z(t)C_Z(t) = c_2Q_T(t)$, which implies that $c_2Q_T(t) + P_Z(t)NX_Z(t) = P_Z(t)Z(t) = P_Z(t)A(t)\, L_Z(t)^{\gamma}$. But from equation (44) we know that $w(t) = \gamma P_Z(t)\, Z(t)/L_Z(t)$. Using these expressions, we obtain that

$$L_Z(t) = \frac{\gamma\;[\,c_2\,Q_T(t) + P_Z(t)\,NX_Z(t)\,]}{w(t)} \tag{54}$$

Using equation (38), and since $w(t) = be^{\lambda(t-T)}$, where $\lambda = \beta\alpha F/(1-\alpha)T$, labor employed in the production of final good Z can be rewritten as follows

$$L_Z(t) = \gamma \quad c_2 \, [L + \frac{a \, n^{\frac{1-\alpha}{\alpha}}}{b}[\, e^{\lambda\,T} - 1] - a \, \frac{\alpha}{1-\alpha}\frac{\beta}{b}\frac{F}{}n^{\frac{1-\alpha}{\alpha}} \,] + \frac{\gamma \, P_Z(t) N X_Z(t)}{b \, e^{\lambda\,(t-T)}} \qquad (55)$$

If in steady-state technical progress in sector Z grows at the same rate as that of in sector Y, that is, if $A(t) = A_0 e^{\lambda t}$, where A_0 is a constant, from equations (29), (46), (47), (53) and (55), the following relationship can be obtained, through which the economic life of capital is determined

$$(1-\alpha)\frac{L}{F} - \alpha \quad -\frac{(1-\alpha)}{T} - \frac{(1-\alpha)}{F}\,[\frac{\gamma \quad P_Z \, A_0 \, e^{\beta \frac{\alpha}{1-\alpha}F}}{b}]^{\frac{1}{1-\gamma}} = \frac{\alpha \;\; \beta \;\; [\frac{1}{1-\alpha}\frac{F}{T} - s \, L \,]}{s \, [\, e^{\beta \frac{\alpha}{1-\alpha}F} - 1 - \frac{\alpha}{1-\alpha}\beta \;\; F]} \qquad \textbf{(56)}$$

As shown in figure 3, equation (56) can be solved graphically, in a plane in which the left and right hand side of that equation are measured along the vertical axis, and the economic life of the machines, T, is measured along the horizontal axis. In that plane, the LHS curve tends to minus infinity as T approaches zero, and tends to $[(1-\alpha)L/F - \alpha - (1-\alpha)/F[\gamma P_Z A_0 e^{\beta\alpha F/(1-\alpha)}/b]^{1/(1-\gamma)}\,]$ as T goes to infinity. Therefore, in figure 3, the LHS schedule can be represented by an upward sloping curve, which intersects the horizontal axis as

$T = (1-\alpha)F / [(1 - \alpha) L - \alpha F - (1-\alpha)[\gamma P_Z A_0 e^{\beta\alpha F/(1-\alpha)}/b]^{1/(1-\gamma)}]$. In turn, the RHS curve tends to infinity as T approaches zero, and tends to $- \alpha\beta L/[e^{\beta\alpha F/(1-\alpha)} - 1 - \alpha\beta F/(1-\alpha)]$ as T tends to infinity. Therefore, in figure 3, the RHS can be represented by a downward sloping curve, which intersects the horizontal axis as $T = F/[(1-\alpha)sL]$. The economic life of the machines, T, is determined at the intersection point of both schedules. As the labor force, L, increases, the LHS curve shifts upwards, whereas the RHS curve shifts downwards. Therefore, in the new equilibrium point, E', the economic life of the machines is smaller. In this case, as in the one-sector model, the economy's income is given by equation (38), whereby the growth rate of income (and income per-capita) is equal to $\lambda = \beta\alpha F/(1-\alpha)T$. Thus, this two-sector extension of the model predicts also that as the size of the labor force increases, T falls, and as a result income grows faster.

Figure 4 depicts the case of an increase of the saving rate. As in the previous section, in this case if the saving rate, s, increases, the LHS curve would remain the same, but the RHS schedule would shift downwards. Thus, at the new intersection point, E', the economic life of the machines would be smaller and consequently the growth rate of income would increase.

Finally, in this open economy, if the relative price of good Z, P_Z, decreases, perhaps due to the adoption of trade policy aimed at protecting the more capital intensive industry (sector Y), the LHS curve shifts downwards, whereas the RHS curve remains unaltered. In that case, as shown in figure 5, in the new equilibrium point, E', the economic life of the machines, T, increases and consequently the growth rate of the economy $\lambda = \beta\alpha F/(1-\alpha)T$ falls.

IV CONCLUSIONS

In this paper we have tried to add an extra dimension to the standard endogenous growth story. This is provided by the endogenously determined economic life of capital, T, when machines embodying the different technology of their different dates of construction coexist as is usual in the old vintage capital models. But unlike in the latter models, T will now have an effect on the long-run growth rate of the economy. Policies that affect T will thus change this growth rate. For example, we show in section III that an increase in the savings rate will lower T and raise the growth rate. Similarly, in the two-sector version of the basic model presented in section IV, we show that if the two final goods are tradeable, protection of the capital-intensive good may lower the growth rate by lengthening T.

We should add that the purpose of this paper is to illustrate a richer variety of questions that may open up in the endogenous growth theory literature if we borrow some aspects of the vintage capital models, but our answers to them should not be interpreted as conclusive. This is particularly because some of those answers may be model-specific, as is not uncommon in much of the endogenous growth literature. For example, there is a whole slew of assumptions to fit the model into the strait-jacket of the steady state. Those who have dabbled in the old vintage models are aware that these models can become too cumbersome once we are off the steady state. Yet this area of transitional dynamics is an obvious area that needs to be explored; to begin with, some numerical simulation exercises may be useful.

We have also made a number of other assumptions to keep our analysis tractable, for example about the fixed-coefficients production function with each vintage of machines, no machines used in producing one of the final goods in the two-sector version, similarity of rates of technical progress in final goods and in capital goods production, about the measure of the stock of knowledge capital, and so on. The fixed-coefficients production function assumption makes our model akin to what used to be known as a "clay-clay" model in the vintage capital literature, the best example of which is that of Solow, Tobin, von Weizsacker, and Yaari (1966). It is well-known in the vintage literature -- see, for example, Bardhan (1969) -- how introducing some elasticity of substitution between capital and labor ex ante (i.e., prior to the installation of the machinery), but not ex post (or what used to be called a "putty-clay" model), can have a significant effect on the comparative-dynamic results with respect to T, the economic life of capital.

Another assumption that needs to be relaxed is that of tradeability only of final goods, but not of the capital goods, in our two-sector version where we discuss the impact of trade policy on the growth rate. This may have a particular bearing on the discussion relating to East Asian growth where trade allowing for imports of machines from abroad embodying the latest technology is supposed to have played a very important role.

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Figure 1

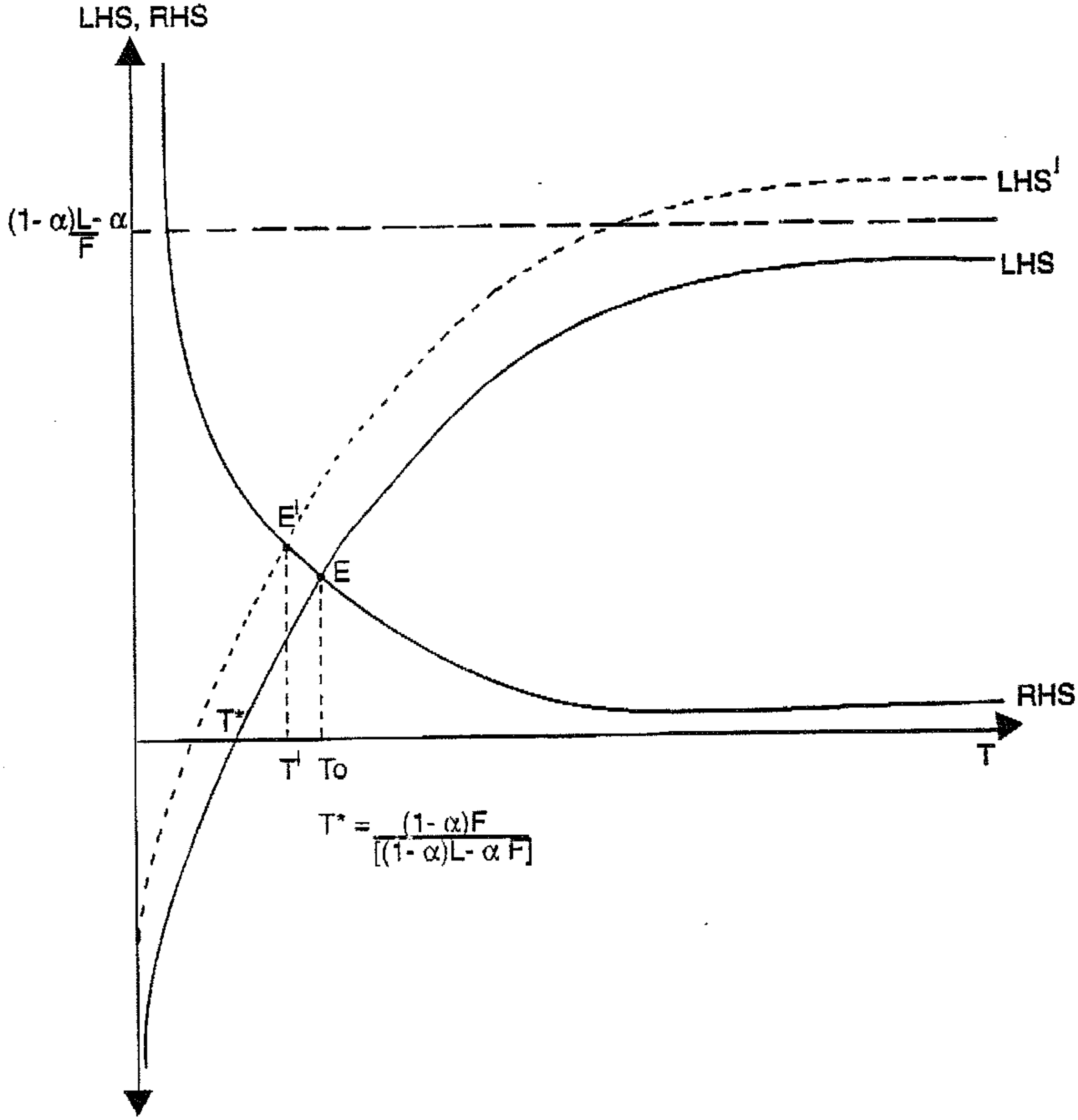
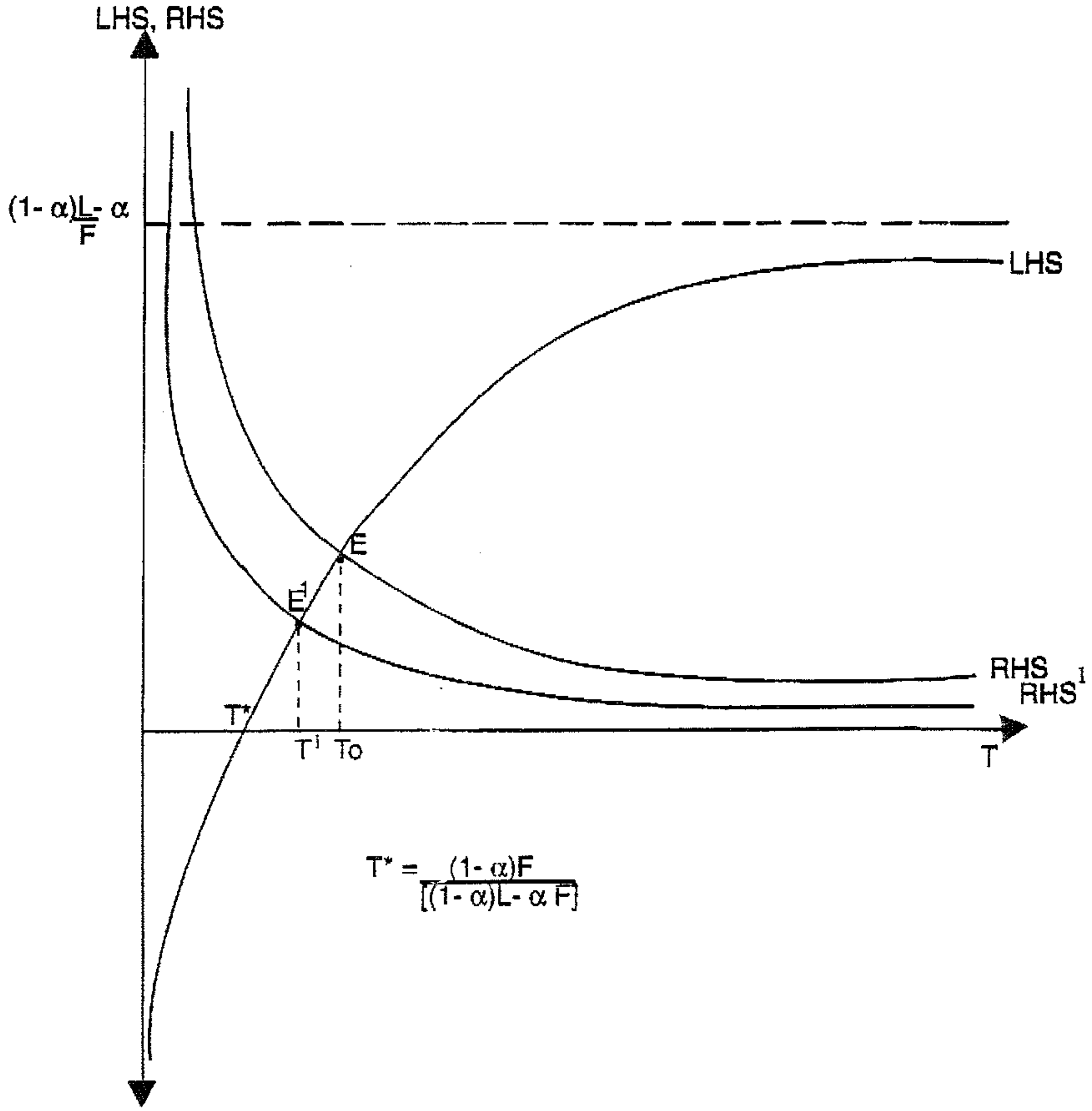
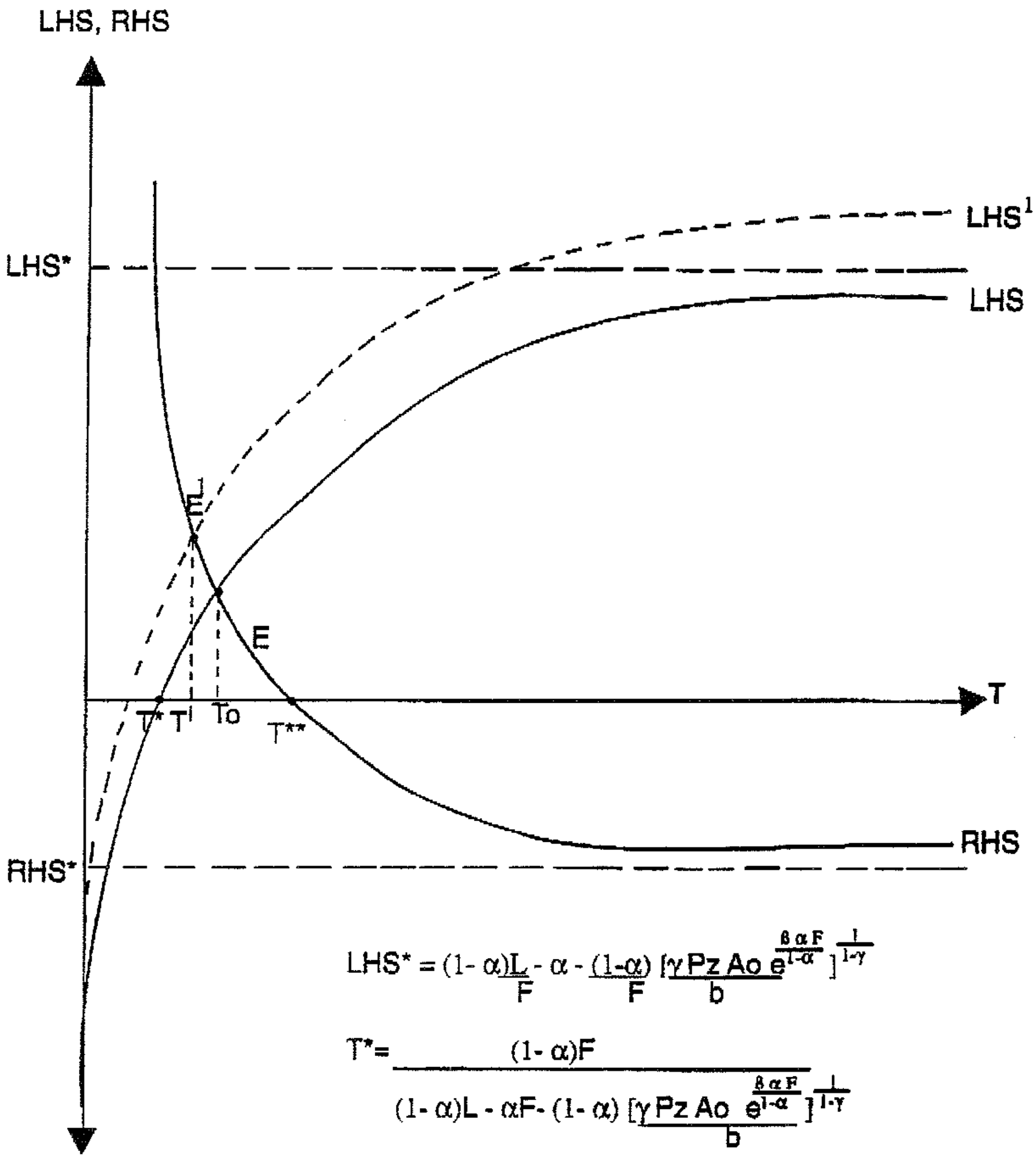


Figure 2



$$T^* = \frac{(1-\alpha)F}{[(1-\alpha)L - \alpha F]}$$

Figure 3



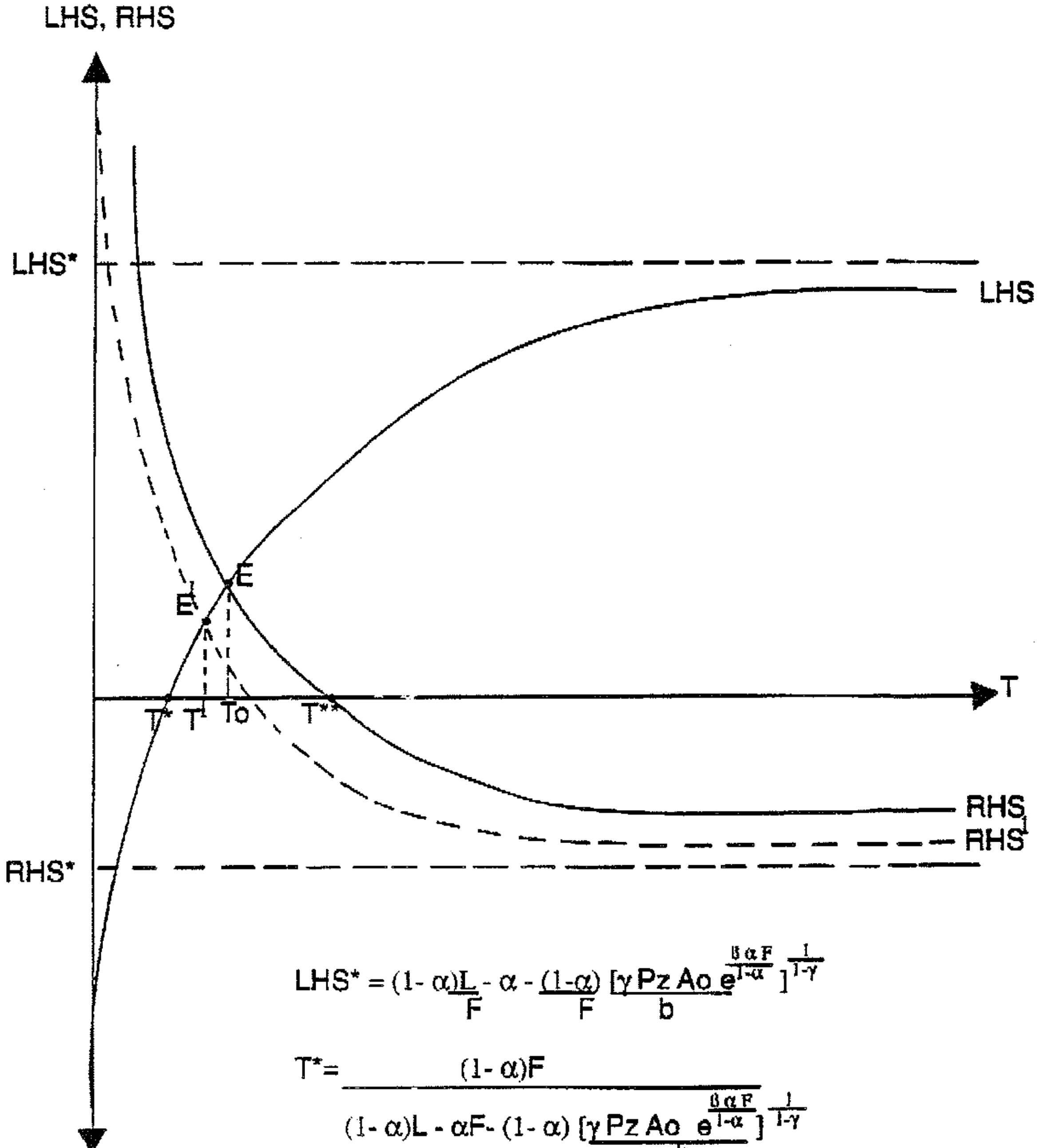
$$LHS^* = (1-\alpha)\frac{L}{F} - \alpha - \frac{(1-\alpha)}{F}\left[\frac{\gamma Pz\,Ao\,e^{\frac{\beta\alpha F}{1-\alpha}}}{b}\right]^{\frac{1}{1-\gamma}}$$

$$T^* = \frac{(1-\alpha)F}{(1-\alpha)L - \alpha F - (1-\alpha)\left[\frac{\gamma Pz\,Ao\,e^{\frac{\beta\alpha F}{1-\alpha}}}{b}\right]^{\frac{1}{1-\gamma}}}$$

$$RHS^* = \frac{-\alpha\beta L}{\left[e^{\frac{\beta\alpha F}{1-\alpha}} - 1 - \frac{\alpha\beta F}{1-\alpha}\right]}$$

$$T^{**} = \frac{F}{(1-\alpha)sL}$$

Figure 4



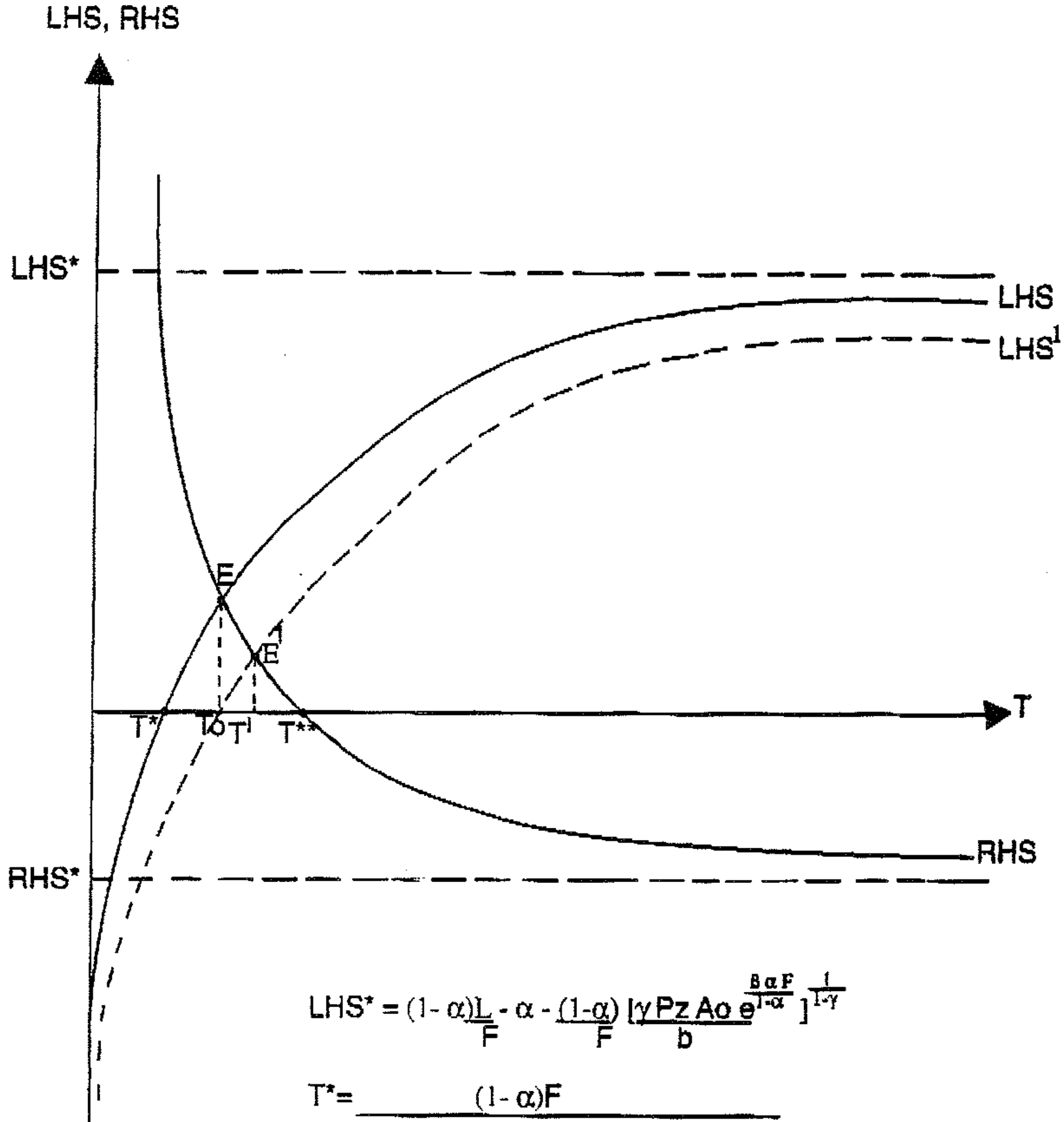
$$LHS^* = (1-\alpha)\frac{L}{F} - \alpha - \frac{(1-\alpha)}{F}\left[\frac{\gamma\, Pz\, Ao\, e^{\frac{\beta\alpha F}{1-\alpha}}}{b}\right]^{\frac{1}{1-\gamma}}$$

$$T^* = \frac{(1-\alpha)F}{(1-\alpha)L - \alpha F - (1-\alpha)\left[\frac{\gamma\, Pz\, Ao\, e^{\frac{\beta\alpha F}{1-\alpha}}}{b}\right]^{\frac{1}{1-\gamma}}}$$

$$RHS^* = \frac{-\alpha\beta L}{\left[e^{\frac{\beta\alpha F}{1-\alpha}} - 1 - \frac{\alpha\beta F}{1-\alpha}\right]}$$

$$T^{**} = \frac{F}{(1-\alpha)sL}$$

Figure 5



$$LHS^* = (1-\alpha)\frac{L}{F} - \alpha - \frac{(1-\alpha)}{F}\left[\frac{\gamma Pz\, Ao\, e^{\frac{\beta\alpha F}{1-\alpha}}}{b}\right]^{\frac{1}{1-\gamma}}$$

$$T^* = \frac{(1-\alpha)F}{(1-\alpha)L - \alpha F - (1-\alpha)\left[\frac{\gamma Pz\, Ao\; e^{\frac{\beta\alpha F}{1-\alpha}}}{b}\right]^{\frac{1}{1-\gamma}}}$$

$$RHS^* = \frac{-\alpha\beta L}{\left[e^{\frac{\beta\alpha F}{1-\alpha}} - 1 - \frac{\alpha\beta F}{1-\alpha}\right]}$$

$$T^{**} = \frac{F}{(1-\alpha)sL}$$