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Authors
CHEN, MU-CHUN
FALLBACHER, MAXIMILIAN
RATZ, MICHAEL

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SUPERSYMMETRIC UNIFICATION AND R SYMMETRIES*

MU-CHUN CHEN

Department of Physics and Astronomy, University of California, Irvine, CA 92697–4575, USA
muchunc@uci.edu

MAXIMILIAN FALLBACHER† and MICHAEL RATZ‡

Physik Department T30, Technische Universität München, James-Franck-Straße,
85748 Garching, Germany
†*maximilian.fallbacher@ph.tum.de*
‡*michael.ratz@tum.de*

We review the role of R symmetries in models of supersymmetric unification in four and more dimensions, and in string theory. We show that, if one demands anomaly freedom and fermion masses, only R symmetries can forbid the supersymmetric Higgs mass term μ. We then review the proof that R symmetries are not available in conventional grand unified theories (GUTs) and argue that this prevents natural solutions to the doublet–triplet splitting problem in four dimensions. On the other hand, higher-dimensional GUTs do not suffer from this problem. We briefly comment on an explicit string-derived model in which the μ and dimension-5 proton decay problems are solved *simultaneously* by an order four discrete R symmetry. We also comment on the higher-dimensional origin of this symmetry.

Keywords: Supersymmetry; grand unification; discrete symmetries; string theory.

1. Introduction and Outline

The minimal supersymmetric Standard Model (MSSM) provides an attractive scheme for physics beyond the Standard Model (SM) of particle physics. The MSSM has the following, attractive features:

- it is based on supersymmetry, which is, under certain modest assumptions, the maximal extension of the Poincaré symmetry of our four-dimensional Minkowski spacetime;

- it provides automatically a dark matter candidate, which is stable due to the $\mathbb{Z}_2^{\mathcal{M}}$ matter parity;
- supersymmetry allows us to stabilize the gauge hierarchy against radiative corrections.

In the context of unified theories, the perhaps most important property of the MSSM is that it provides us with the very compelling picture of precision gauge coupling unification.[1] That is, if one assumes that the superpartners have masses of the order TeV and extrapolates the gauge couplings g_i of the SM gauge factors SU(3), SU(2) and U(1) to higher energies, one finds that they meet with a high precision at the scale of a few $\times 10^{16}$ GeV. This property of the MSSM represents, given the still persisting lack of evidence for superpartners at the LHC, perhaps the greatest motivation for supersymmetry. Arguably, the most compelling explanation of this fact arises if the SM gauge group is embedded in a simple gauge group, specifically

$$G_{\mathrm{SM}} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1) \subset \mathrm{SU}(5) \tag{1.1}$$

or a group containing SU(5).

This brings us to the scheme of grand unified theories (GUTs). Specifically, GUTs based on the gauge groups SU(5) and SO(10) have many appealing features (for a review see, e.g., Ref. 2):

1. GUTs explain charge quantization;
2. They simplify the matter content. The five irreducible representations (irreps) forming one generation of SM matter can be grouped into two SU(5) irreps,[3]

$$\text{SM generation} = \mathbf{10} + \bar{\mathbf{5}} \,. \tag{1.2}$$

A further simplification of the matter sector happens in SO(10),[4] where

$$\mathbf{16} = \mathbf{10} \oplus \bar{\mathbf{5}} \oplus \mathbf{1}$$

$$= \text{SM generation with ``right-handed'' neutrino} \,. \tag{1.3}$$

One of the main assumptions of this review is that these features are not by accident.

In this review, we will specifically discuss the role of (discrete) R symmetries in supersymmetric models of unification. After a short review of some of the issues of the MSSM, we will discuss the importance of anomaly constraints and in particular "anomaly universality" for their resolution. Using these techniques, we will show that only R symmetries can forbid the μ term in the MSSM. Furthermore, as we will then argue, these R symmetries are already almost uniquely determined by the anomaly universality conditions. However, given certain general assumptions which we will specify, R symmetries are not available in four-dimensional models of grand unification. On the other hand, R symmetries are available in higher-dimensional and, in particular, in stringy settings, where they arise as discrete remnants of the Lorentz symmetry of compact space. We will comment on explicit models where

precisely the phenomenologically desired symmetries arise this way. Finally, we will provide a short summary.

2. The MSSM, Grand Unification and All That

We start by reviewing the problems of the MSSM in Sec. 2.1 and describe specifically the proton decay problems in Sec. 2.2. As we shall see, the conventional solutions to the MSSM problems are, arguably, not fully satisfactory.

2.1. *Problems of the MSSM*

As is well known, the MSSM has, besides many desired features, certain short-comings. Several of them are connected to the appearance of operators in the superpotential which are consistent with all symmetries of the MSSM but have phenomenologically undesired effects, or are plainly inconsistent with observation. The gauge invariant superpotential terms up to order four include

$$
\begin{aligned}
\mathscr{W} = {} & \mu H_d H_u + \kappa_i L_i H_u \\
& + Y_e^{ij} L_i H_d \bar{E}_j + Y_d^{ij} Q_i H_d \bar{D}_j + Y_u^{ij} Q_i H_u \bar{U}_j \\
& + \lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k \\
& + \kappa_{ij}^{(0)} H_u L_i H_u L_j + \kappa_{ijk\ell}^{(1)} Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)} \bar{U}_i \bar{U}_j \bar{D}_k \bar{E}_\ell \,.
\end{aligned} \tag{2.1}
$$

Here, in an obvious notation, H_u and H_d denote the MSSM Higgs doublets, and Q_i, $\bar{U}_i$, $\bar{D}_i$, L_i and $\bar{E}_i$ the three generations of MSSM matter. The μ term in the first line, for phenomenological reasons, has to be of order TeV, and the Yukawa couplings Y_e^{ij}, Y_u^{ij} and Y_d^{ij} are required in order to describe fermion masses. Moreover, the perhaps simplest explanation of small Majorana neutrino masses needs a nontrivial $\kappa_{ij}^{(0)}$ of the order $(10^{14} \text{ GeV})^{-1}$.

Unfortunately, there are various additional terms, which turn out to be very problematic. First of all, the so-called R-parity violating couplings κ_i, λ_{ijk}, λ'_{ijk} and λ''_{ijk} are strongly constrained by the experiments, i.e. they have to be either very small or completely absent (cf. e.g., Ref. 5). Second, there are strong bounds on the coefficients $\kappa_{ijk\ell}^{(1,2)}$ of the so-called dimension-5 proton decay operators. This shows that supersymmetry alone is not a viable theory. It has to be amended by some additional structure, preferably by symmetries which ensure that the phenomenological predictions of the extended model are in agreement with experimental data.

2.2. *Proton decay problems*

2.2.1. *The conventional approach to the proton decay problems*

Of course, these problems are well known and there are some standard solutions. Let us specifically discuss the traditional cure of proton decay problems. The R-parity

Table 1.   Matter parity $\mathbb{Z}_2^{\mathcal{M}}$, baryon triality B_3 and proton hexality P_6.

	Q	$\bar{U}$	$\bar{D}$	L	$\bar{E}$	H_u	H_d	$\bar{\nu}$
$\mathbb{Z}_2^{\mathcal{M}}$	1	1	1	1	1	0	0	1
B_3	0	−1	1	−1	2	1	−1	0
P_6	0	1	−1	−2	1	−1	1	3

violating terms can be forbidden by R- or matter parity $\mathbb{Z}_2^{\mathcal{M}}$ (Refs. 6 and 7), either of which is usually assumed to be part of the definition of the MSSM. Formally, these two symmetries differ by the transformation of the superpartners. However, there is an intrinsic symmetry in any supersymmetric theory which sends the superspace coordinate θ to minus itself. Using this ambiguity, one can easily convince oneself that the two symmetries are equivalent. After imposing this symmetry, there are still the dimension-5 proton decay operators, which can, however, be forbidden by baryon triality B_3 (Ref. 8) (see Table 1 for the charge assignment). The combination of both symmetries, i.e. $\mathbb{Z}_2^{\mathcal{M}}$ times B_3, is known as "proton hexality" P_6.[8–10] The P_6 symmetry has several very appealing features:

(1) it forbids dimension-4 and -5 proton decay operators;
(2) it allows the usual Yukawa couplings of the MSSM as well as the Weinberg's neutrino operator $\kappa_{ij}^{(0)} H_u L_i H_u L_j$;
(3) it is the unique anomaly-free symmetry with the above features assuming traditional anomaly cancellation.

Unfortunately, P_6 has also some disturbing aspects:

(1) it is not consistent with unification of matter, i.e. it is inconsistent with having universal discrete charges for all matter fields (cf. Ref. 11);
(2) it does not address the μ problem, i.e. it does not provide us with a solution to all the above-mentioned problems of the MSSM.

2.3. *Origin of proton decay operators in GUTs*

One may now wonder how serious the fact is that P_6 is not consistent with (SU(5) or SO(10)) unification. To this end, it is instructive to recall where the proton decay operators come from. One distinguishes between dimension-6 and -5 proton decay operators. While the dimension-6 operators can come from gauge boson exchange (cf. Fig. 1(a)),[3] the dimension-5 ones (Fig. 1(b)) may originate from the color-triplet Higgs exchange.[7,12] While the SUSY GUT predictions for the proton decay rates mediated by dimension-6 operators are still consistent with observation,[13] the dimension-5 proton decay and the associated doublet–triplet splitting problems cast some shadow on the scheme of (four-dimensional) SUSY GUT models (cf. e.g., Refs. 13 and 14). Some coefficients of the $QQQL$ operators have to be smaller than $10^{-8}/M_{\mathrm{P}}$,[15] which leads to a lower bound on the color-triplet Higgs mass far
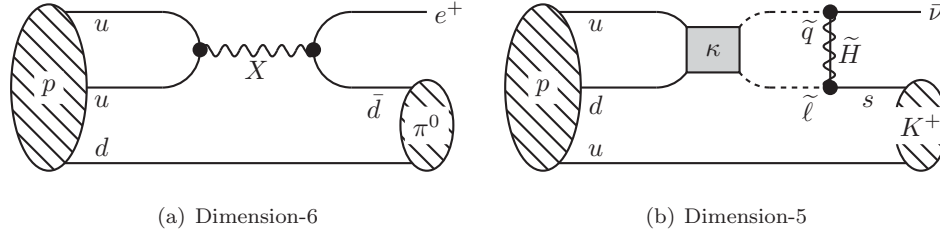
(a) Dimension-6   (b) Dimension-5

Fig. 1.   (a) Dimension-6 and (b) dimension-5 proton decay diagrams in GUTs leading to the proton decay modes (a) $p \to \pi^0 + e^+$ and (b) $p \to K^+ + \bar{\nu}$. While the SUSY GUT prediction for (a) is still consistent with experimental limits, the decay mode (b) often challenges explicit SUSY GUT models.

above M_{GUT} unless one arranges for very precise cancellations between unrelated couplings (see e.g., Refs. 16 and 17).

Given that P_6 is incompatible with grand unification, we see that this symmetry cannot be used to solve the most severe problems of GUT models. This is also in accordance with the fact that P_6 does not address the μ problem, i.e. it cannot help us to understand the doublet–triplet splitting.

Various other solutions to the dimension-5 proton decay problem of SUSY GUTs rely on intricate GUT breaking sectors.[18–20] The Higgs fields typically used for the GUT breaking and the generation of fermion masses are in representations as large as **75** of SU(5) or **126** of SO(10). The corresponding large amount of G_{SM} charged states typically induces large threshold corrections, which may clash with our basic assumption that gauge unification is not an accident.

In what follows, we will therefore discuss alternative discrete symmetries which do not suffer from these shortcomings. Specifically, we will identify anomaly-free discrete symmetries which are consistent with (precision) gauge unification and allow us to control the μ term.

3. Non-Anomalous Discrete Symmetries and Unification

In this section, we will first discuss (discrete) anomaly cancellation in general. Then we will focus on symmetries that are consistent with unification and forbid the μ term. In contrast to the traditional approach, we make use of the Green–Schwarz (GS) mechanism for anomaly cancellation.[21]

3.1. *Anomaly universality*

We begin our discussion with the observation that, in the framework of GUTs, once one allows for GS mechanism, the requirement of anomaly freedom is depleted to the demand of "anomaly universality", i.e. common anomaly coefficients of the SM gauge factors G_i.

Let us explain what that implies in practice. Consider, for example, the mixed $G_i - G_i - \mathbb{Z}_N$ anomaly coefficient for a $\mathbb{Z}_N$ symmetry,

$$A_{G_i-G_i-\mathbb{Z}_N} = \sum_f \ell(\mathbf{r}^{(f)}) \cdot q^{(f)}. \tag{3.1}$$

Here the sums extend over all fermion representations $\mathbf{r}^{(f)}$, while $\ell^{(f)}$ denotes the Dynkin index of the fermions f, w.r.t. the gauge group G_i and $q^{(f)}$ are the discrete $\mathbb{Z}_N$ charges. The traditional anomaly constraints[22,23] correspond to the condition that the $A_{G_i^2-\mathbb{Z}_N}$ coefficients[a] have to vanish for all G_i,

$$A_{G_i-G_i-\mathbb{Z}_N} = 0 \mod \eta \quad \forall\, G_i, \tag{3.2}$$

where

$$\eta := \begin{cases} N & \text{for } N \text{ odd}, \\ \dfrac{N}{2} & \text{for } N \text{ even}. \end{cases} \tag{3.3}$$

On the other hand, "anomaly universality" only amounts to the requirement that the anomaly coefficients be universal,

$$A_{G_i-G_i-\mathbb{Z}_N} = \rho \mod \eta \quad \forall\, G_i, \tag{3.4}$$

but that they do not necessarily have to vanish. Here ρ can be thought of as the contribution of a GS axion a, whose shift transformation under the $\mathbb{Z}_N$ symmetry cancels the anomaly.

Where does the "anomaly universality" come from? Although universality of anomaly coefficients is empirically found to be a property of most heterotic string models,[24,25] it is, as correctly pointed out in Ref. 26, in general not a necessary condition for anomaly freedom. This can most easily be seen in the path integral formulation[27,28] of the GS mechanism (see e.g., Refs. 29 and 30). The crucial ingredient is the coupling of the GS axion a to the $F\tilde{F}$ term of the gauge group G. The GS axion a is contained in the superfield S, $S|_{\theta=0} = s + \mathrm{i}a$, and shifts under the symmetry transformation. The GS anomaly cancellation requires the coupling

$$\int \mathrm{d}^2\theta\, f_S S W_\alpha W^\alpha \supset \mathscr{L}, \tag{3.5}$$

in the Lagrange density. Given this term, $s = \mathrm{Re}\,S|_{\theta=0}$ contributes to $\frac{1}{g^2}$, see Refs. 29 and 30 for more details. In general, different couplings of a to different SM gauge factors G_i would allow for different ρ constants for the different gauge factors of the SM. However, in general, the "saxion" s has a nontrivial vacuum expectation value (VEV), such that nonuniversal couplings imply nonuniversal contributions to $\frac{1}{g_i^2}$. This, in turn, would imply that precision gauge unification is spoilt. Since this would contradict our assumption that precision gauge unification is not an accident, we will require anomaly universality in the rest of our discussion.

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[a]Note that there are no meaningful $\mathbb{Z}_N^3$ anomaly constraints. This has been first shown in Ref. 23 and can be seen more directly in the path integral approach.[24]

3.2. *Non-R symmetries cannot forbid the μ term in the MSSM*

Let us now look at discrete anomalies of non-R symmetries in the MSSM. After imposing SU(5) relations for the matter charges, the relevant anomaly coefficients read

$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}(3q_{\mathbf{10}}^g + q_{\mathbf{\bar 5}}^g)\,, \tag{3.6}$$

$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}(3q_{\mathbf{10}}^g + q_{\mathbf{\bar 5}}^g) + \frac{1}{2}(q_{H_u} + q_{H_d})\,. \tag{3.7}$$

Here, in an obvious notation, $q_{\mathbf{10}}^g$ and $q_{\mathbf{\bar 5}}^g$ denote the discrete charges of the gth **10**- and $\mathbf{\bar 5}$-plet, respectively, with g playing the role of a generation index while q_{H_u} and q_{H_d} are the charges of the Higgs doublets. Now, imposing anomaly universality, i.e. demanding that

$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} - A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = 0 \mod \eta\,, \tag{3.8}$$

leads to a condition on the Higgs charges:

$$\frac{1}{2}(q_{H_u} + q_{H_d}) = 0 \mod \begin{cases} N & \text{for } N \text{ odd}\,, \\[2mm] \dfrac{N}{2} & \text{for } N \text{ even}\,. \end{cases} \tag{3.9}$$

It is easy to see that this implies that the $\mathbb{Z}_N$ symmetry does not forbid the Higgs bilinear. We hence see that ordinary, i.e. non-R, $\mathbb{Z}_N$ symmetries cannot forbid the μ term.

3.3. *Only discrete R symmetries may forbid the μ term*

It is also obvious that, if anomaly-free discrete non-R symmetries cannot forbid the μ term, this also applies to continuous non-R symmetries, for which the anomaly constraints are even stronger. We are hence left with R symmetries. Recalling that there are no anomaly-free continuous R symmetries in the MSSM,[31] the only remaining option is the discrete R symmetries.

3.4. *R symmetries and 't Hooft anomaly matching*

't Hooft's concept of anomaly matching is a powerful tool for analyzing symmetries,[32] which can also be used for discrete symmetries.[33] Let us spell this out for the case of discrete R symmetries in the MSSM, still assuming unification.[30] Trivially, at the SU(5) level, there is only one anomaly coefficient,

$$A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} = A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R}^{\mathrm{matter}} + A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R}^{\mathrm{extra}} + 5q_\theta\,, \tag{3.10}$$

which we have decomposed into the contribution from matter $A^{\mathrm{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}^R_M}$, extra states $A^{\mathrm{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}^R_M}$ and gauginos $5q_\theta$ with q_θ denoting the R charge of the superspace coordinate.[b] M is the order of the R symmetry transformation, which might be part of a larger symmetry. In addition to the anomaly constraint from the whole gauge group, we can also consider the SU(3) and SU(2) subgroups of SU(5). The corresponding anomaly coefficients read

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)^2-\mathbb{Z}^R_M} = A^{\mathrm{matter}}_{\mathrm{SU}(3)^2-\mathbb{Z}^R_M} + A^{\mathrm{extra}}_{\mathrm{SU}(3)^2-\mathbb{Z}^R_M} + 3q_\theta + \frac{1}{2}\cdot 2 \cdot 2 \cdot q_\theta\,, \quad (3.11\mathrm{a})$$

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)^2-\mathbb{Z}^R_M} = A^{\mathrm{matter}}_{\mathrm{SU}(2)^2-\mathbb{Z}^R_M} + A^{\mathrm{extra}}_{\mathrm{SU}(2)^2-\mathbb{Z}^R_M} + 2q_\theta + \frac{1}{2}\cdot 2 \cdot 3 \cdot q_\theta\,. \quad (3.11\mathrm{b})$$

Here we have decomposed the gaugino contributions into their SU(3) and SU(2) parts, respectively, and into the contributions from $\mathrm{SU}(5)/G_{\mathrm{SM}}$. Assume now that some mechanism eliminates the extra gauginos. This will lead to a nonuniversality of the anomaly coefficients, which will, given our assumption that matter charges commute with SU(5), have to be compensated for by the extra states. That is, the extra states have to come in split multiplets. In other words, 't Hooft anomaly matching for (discrete) R symmetries implies the presence of split multiplets below the GUT scale. The arguably simplest possibility to "repair" the gaugino mismatch is to assume that there is a pair of massless weak doublets, which is chiral w.r.t. $\mathbb{Z}^R_M$, but no corresponding triplets. From this one infers that, in the presence of an R symmetry, the same mechanism that breaks the GUT symmetry will also provide a mechanism for doublet–triplet splitting. However, as we will discuss later, it is impossible to construct a four-dimensional GUT with a low energy R symmetry without states beyond those of the MSSM. This is also consistent with the observation that there are no natural (in 't Hooft's sense) solutions to the doublet–triplet splitting problem in such schemes.

3.5. *A unique discrete R symmetry for the MSSM*

Let us now impose, instead of SU(5) relations, stronger SO(10) relations, i.e. that the charges q for matter fields are universal. That is, consider a $\mathbb{Z}^R_M$ symmetry under which quarks and leptons have the universal charge q. As we shall demonstrate, this implies a unique symmetry.[29,30] In the first step, we require that the symmetry allows for u- and d-type Yukawas, implying that

$$2q + q_{H_u} = 2q_\theta \mod M \quad \mathrm{and} \quad 2q + q_{H_d} = 2q_\theta \mod M\,. \quad (3.12)$$

---

[b]Note that there exists some confusion in the literature. It is often assumed that the superpotential $\mathscr{W}$ has R charge 2, corresponding to R charge 1 of the superspace coordinate, $q_\theta = 1$. However, as pointed out in Ref. 30, one cannot, in general, make this choice and, at the same time, demand that all discrete charges are integer. We follow the convention that all discrete charges are integer and keep q_θ variable.

Subtracting these equations from each other,

$$q_{H_u} - q_{H_d} = 0 \mod M \,, \tag{3.13}$$

shows that also the charges of the two Higgs fields coincide. The conditions for the presence of u-type Yukawa couplings and the Weinberg operator are

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + 2q_{H_u} = 2q_\theta \mod M \,, \tag{3.14}$$

implying that $q_{H_u} = 0 \mod M$. Altogether we see that

$$q_{H_u} = q_{H_d} = 0 \mod M \quad \text{and} \quad q = q_\theta \mod M \,. \tag{3.15}$$

From the conditions that the symmetry must be an R symmetry,

$$q_\theta \neq 0 \mod \eta \,, \tag{3.16}$$

and that it is "anomaly universal" in the MSSM,

$$A_{\mathrm{SU}(3)^2 - \mathbb{Z}_M^R} = 3q_\theta \mod \eta \stackrel{!}{=} q_\theta \mod \eta = A_{\mathrm{SU}(2)^2 - \mathbb{Z}_M^R} \,, \tag{3.17}$$

it follows that η is even which in turn implies that the order M of the symmetry is a multiple of 4,

$$M = 4m, \quad m \in \mathbb{N} \,. \tag{3.18}$$

Furthermore, given the ambiguity discussed on p. 4, Eqs. (3.16) and (3.17) fix the R charge of the superspace coordinate θ to $q_\theta = m$. As a result, the simplest nontrivial possibility is $M = 4$ and $q = q_\theta = 1$, i.e. a $\mathbb{Z}_4^R$ symmetry. As is straightforward to see, the extensions to $\mathbb{Z}_{4m}^R$ symmetries, $m > 1$, are trivial extensions as far as the MSSM is concerned. While it might certainly be worthwhile to study such symmetries in the context of (singlet) extensions of the MSSM, we can conclude that there is a unique symmetry for the MSSM: a $\mathbb{Z}_4^R$ with $q = q_\theta = 1$ and $q_{H_u} = q_{H_d} = 0$.

This symmetry was first discussed in Ref. 34. A version of the uniqueness proof appeared in Ref. 35. However, there it was assumed that the superpotential has charge 2 in a normalization in which all discrete charges are integer, which is, in general, not a valid assumption (cf. footnote b). The uniqueness proof has been completed in Ref. 30.

The $\mathbb{Z}_4^R$ anomaly coefficients are

$$A_{\mathrm{SU}(3)^2 - \mathbb{Z}_4^R} = 6q - 3q_\theta = q_\theta = 1 \mod 4/2 \,, \tag{3.19a}$$

$$A_{\mathrm{SU}(2)^2 - \mathbb{Z}_4^R} = 6q + \frac{1}{2}(q_{H_u} + q_{H_d}) - 5q_\theta = q_\theta = 1 \mod 4/2 \,. \tag{3.19b}$$

The fact that the coefficients are nontrivial implies that the $\mathbb{Z}_4^R$ is anomaly-free only via a nontrivial GS mechanism.

3.6. *GS anomaly cancellation and non-perturbative effects*

Let us briefly comment on the implications of GS anomaly cancellation. As discussed above, the GS axion a is contained in a superfield S, $S|_{\theta=0} = s + \mathrm{i}a$. Since $a = \mathrm{Im}\, S|_{\theta=0}$ shifts under the $\mathbb{Z}_M^R$ transformation, R non-covariant superpotential terms can be made invariant by multiplying them with e^{-bS}. To be specific, consider, as an example, the Higgs bilinear. The μ term is obviously forbidden by the $\mathbb{Z}_4^R$ symmetry, but the term

$$B\mathrm{e}^{-bS} H_u H_d \,, \tag{3.20}$$

will be allowed for appropriate values of b. In other words, the holomorphic e^{-bS} terms appear to violate the $\mathbb{Z}_M^R$ symmetry. Such terms have a well-known interpretation. Given the coupling (3.5), $s = \mathrm{Re}\, S|_{\theta=0}$ contributes to $\frac{1}{g^2}$, and the holomorphic $B\mathrm{e}^{-bS}$ terms can be interpreted as non-perturbative effects (cf. the "retrofitting" discussion[36]). Altogether we see that there is a unique symmetry of the MSSM that (i) forbids the μ term, (ii) is compatible with SO(10) and (iii) is anomaly-free; this symmetry has the feature that the μ term appears non-perturbatively and is naturally suppressed.

3.7. *Further implications of $\mathbb{Z}_4^R$*

The $\mathbb{Z}_4^R$ symmetry has important implications for the MSSM. Among the gauge invariant terms shown in (2.1), the μ term, the R-parity violating terms and the dimension-5 proton decay operators are forbidden at the perturbative level while, by construction, the Yukawa couplings and the Weinberg operator are allowed. As discussed above, μ and the dimension-5 proton decay operators appear at the non-perturbative level, whereas the R-parity violating terms are still forbidden at the non-perturbative level by a "non-anomalous" $\mathbb{Z}_2$ subgroup which is equivalent to matter parity. How can one determine the size of the non-perturbative terms? The order parameter for R symmetry breaking is the superpotential VEV $\langle \mathscr{W} \rangle$, or, in other words, the gravitino mass $m_{\frac{3}{2}}$. Hence

$$\mu \sim m_{\frac{3}{2}} \simeq \frac{\langle \mathscr{W} \rangle}{M_{\mathrm{P}}^2} \,, \tag{3.21}$$

with M_{P} denoting the Planck scale. The non-perturbatively generated dimension-5 proton decay operators are phenomenologically harmless,

$$\kappa_{ijk\ell}^{(1,2)} \sim \frac{m_{\frac{3}{2}}}{M_{\mathrm{P}}^2} \ll \frac{10^{-8}}{M_{\mathrm{P}}} \,, \tag{3.22}$$

where we compare the theoretical expectation with the experimental constraints.[15]

4. No R Symmetries in Conventional 4D GUTs

In the previous section, we have seen that only R symmetries can forbid the μ term in the MSSM. However, as we shall show now, R symmetries are not available in four-dimensional GUTs.[37] More specifically, if one assumes

  (i) a GUT model in four dimensions based on $G \supset \mathrm{SU}(5)$,

 (ii) that the GUT symmetry breaking is spontaneous, and

(iii) that there is only a finite number of fields,

one can prove that it is impossible to get a low-energy effective theory with both

1. just the MSSM field content, and
2. residual R symmetries.

For the purposes of this review, we will restrict ourselves to presenting the basic argument. Consider an SU(5) model with an (arbitrary) R symmetry and a chiral **24**-plet breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$. Recall the branching rule

$$\mathbf{24} \to (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{3},\mathbf{2})_{-\frac{5}{6}} \oplus (\bar{\mathbf{3}},\mathbf{2})_{\frac{5}{6}} \oplus (\mathbf{1},\mathbf{1})_0 \,. \tag{4.1}$$

Since the **24**-plet attains a VEV but may not break the R symmetry, it has to have R charge 0. In the course of GUT breaking, the multiplets $(\mathbf{3},\mathbf{2})_{-\frac{5}{6}} \oplus (\bar{\mathbf{3}},\mathbf{2})_{\frac{5}{6}}$ are absorbed by the extra gauge bosons from $\mathrm{SU}(5)/G_{\mathrm{SM}}$. Thus, there are extra massless states in the representations $(\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0$, whose masses are forbidden by the R symmetry.

    One can now ask the question whether it is possible to make these unwanted states massive. It is easy to see that the introduction of extra **24**-plets with R charge 2 only shifts the problem of massless states to different representations. In particular, in this case there would be massless states in the representation $(\mathbf{3},\mathbf{2})_{-\frac{5}{6}} \oplus (\bar{\mathbf{3}},\mathbf{2})_{\frac{5}{6}}$ representations. Repeating this argument inductively shows that with a finite number of **24**-plets one will always have massless exotics. The only way to circumvent this argument is to have infinitely many **24**-plets.

    It is possible to generalize the basic argument to

- arbitrary SU(5) representations;
- larger GUT groups $G \supset \mathrm{SU}(5)$;
- singlet extensions of the MSSM.

The proof can be found in Ref. 37. Here we shall only discuss the implications of these statements. A "natural" solution of the μ and/or doublet–triplet splitting problem requires a symmetry that forbids μ. So far we have learned that:

1. only R symmetries can forbid the μ term;
2. anomaly matching requires the existence of split multiplets;
3. R symmetries are not available in 4D GUTs.

This implies that "natural" solutions to the μ and/or doublet–triplet splitting problems are not available in four dimensions! This might be interpreted as the necessity to go to models with extra dimensions, such as string compactifications.

5. Higher-Dimensional and String Models

In this section, we will discuss how going to extra dimensions allows us to evade the no-go theorem presented in Sec. 4. In such settings it is moreover possible to answer the question of the origin of R symmetries and one has better control over the higher-dimensional operators such as the effective μ term.

5.1. *Grand unification in higher dimensions*

It is often stated that higher-dimensional GUTs appear more "appealing". This is because new possibilities of symmetry breaking arise.[38,39] In addition, the Kaluza–Klein towers provide us with, from a four-dimensional point of view, infinitely many states (cf. the discussion in Ref. 40), thus allowing us to evade the no-go theorem.

What is more, R symmetries have a clear geometric interpretation. They originate from the Lorentz symmetry of compact dimensions (cf. e.g., the discussion in Ref. 41) and are arguably on the same footing as the fundamental symmetries C, P and T.

5.2. *Extra-dimensional/stringy origin of $\mathbb{Z}_4^R$*

String models offer a geometric explanation of discrete symmetries (for a recent review see e.g., Ref. 41). Specifically, in stringy heterotic orbifolds, one obtains effective theories with residual discrete R symmetries. In particular, one can determine the R charges of the different states. Such models often exhibit a $\mathbb{Z}_4^R$ symmetry, under which localized fields have odd R charges while bulk fields have even R charges. This harmonizes nicely with the scheme of "local grand unification"[42] where matter fields are localized in regions with SO(10) symmetry and, therefore, come in complete SO(10) multiplets, while Higgs fields come from the bulk and, therefore, are split.[c]

Let us now discuss globally consistent string models with these features.[46,47] These are $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold models with the exact MSSM spectrum. They exhibit vacua, i.e. field configurations that preserve supersymmetry perturbatively, with various good features

(1) non-local GUT breaking;
(2) no "fractionally charged exotics";
(3) (most) exotics decouple at the linear level in SM singlets, i.e. just MSSM below GUT scale with masslessness of Higgs fields ensured by $\mathbb{Z}_4^R$;
(4) nontrivial full-rank Yukawa couplings;
(5) gauge-top unification;
(6) SU(5) relation $y_\tau \simeq y_b$.

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[c]In concrete models the third family comes partially from the bulk[43] (and is a so-called "patchwork family",[44] among other things giving rise to gauge-top unification[45]).

Note that these are, unfortunately, just toy models since they exhibit certain unrealistic features such as SU(5) Yukawa relations also for light generations. Nevertheless such models illustrate that a successful string embedding of the $\mathbb{Z}_4^R$ symmetry is possible.

6. Conclusions

In this review, we have discussed the role of R symmetries in supersymmetric models that give rise to (precision) gauge unification. Specifically, we have made the following assumptions:

(i) anomaly freedom (allowing for GS anomaly cancellation);
(ii) μ term forbidden at the perturbative level;
(iii) Yukawa couplings and Weinberg neutrino mass operator allowed;
(iv) SU(5) or SO(10) GUT relations for quarks and leptons.

We have then shown that

1. assuming (i) and SU(5) relations, only R symmetries can forbid the μ term in the MSSM;
2. assuming (i)–(iii) and SO(10) relations, there is a unique $\mathbb{Z}_4^R$ symmetry;
3. R symmetries are not available in 4D GUTs, implying that there is no "natural" solution to doublet–triplet splitting in four dimensions.

The simple anomaly-free $\mathbb{Z}_4^R$ symmetry turns out to provide a solution to the μ problem and, as a bonus, automatically suppresses proton decay operators. Models with this symmetry predict that proton decay proceeds via dimension-6 operators, i.e. via gauge boson exchange. Yet, since such settings cannot be embedded into four-dimensional GUTs, one will have to analyze higher-dimensional models in order to make more detailed predictions.

Deriving the $\mathbb{Z}_4^R$ symmetry from string theory allows us to understand where it comes from: it can arise as a discrete remnant of Lorentz symmetry in extra dimensions. Guided by this $\mathbb{Z}_4^R$ symmetry we have reported on a globally consistent string model with (i) the exact MSSM spectrum; (ii) nonlocal/Wilson line GUT breaking; (iii) nontrivial full-rank Yukawa couplings; (iv) exact matter parity; (v) $\mu \sim m_{\frac{3}{2}}$ and (vi) dimension-5 proton decay operators sufficiently suppressed.
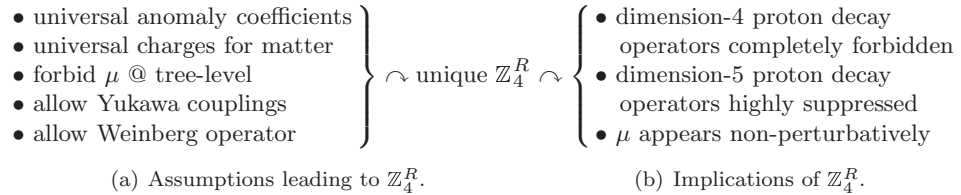
- universal anomaly coefficients
- universal charges for matter
- forbid μ @ tree-level
- allow Yukawa couplings
- allow Weinberg operator

$\curvearrowright$ unique $\mathbb{Z}_4^R$ $\curvearrowright$

- dimension-4 proton decay operators completely forbidden
- dimension-5 proton decay operators highly suppressed
- μ appears non-perturbatively

(a) Assumptions leading to $\mathbb{Z}_4^R$.

(b) Implications of $\mathbb{Z}_4^R$.

Fig. 2. (a) Assumptions leading to the unique $\mathbb{Z}_4^R$ and (b) implications.

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