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## Field-dependent collective ESR mode in YbRh<sub>2</sub>Si<sub>2</sub>

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### ABSTRACT

Electron spin resonance (ESR) experiments in YbRh<sub>2</sub>Si<sub>2</sub> Kondo lattice ( $T_K \approx 25$  K) at different field/frequencies ( $4.1 \leq \nu \leq 34.4$  GHz) and  $H_{\perp c}$  revealed: (i) a strong field dependent Yb<sup>3+</sup> spin–lattice relaxation, (ii) a weak field and  $T$ -dependent *effective g*-value, (iii) a suppression of the ESR intensity beyond 15% of Lu-doping, and (iv) a strong sample and Lu-doping ( $\leq 15\%$ ) dependence of the ESR data. These results suggest that the ESR signal in YbRh<sub>2</sub>Si<sub>2</sub> may be due to a coupled Yb<sup>3+</sup>-conduction electron *resonant collective mode* with a subtle field-dependent spins dynamic.

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### 1. Introduction

The heavy-fermion (HF) Kondo lattice YbRh<sub>2</sub>Si<sub>2</sub> ( $T_K \approx 25$  K) is an antiferromagnetic (AF,  $T_N = 70$  mK) tetragonal (I4/mmm) intermetallic compound. At low- $T$  ( $T \lesssim T_K$ ) the magnetic susceptibility exhibits a HF behavior and at high- $T$  ( $T \gtrsim 200$  K) an anisotropic Curie–Weiss with a full Yb<sup>3+</sup> magnetic moment ( $\mu_{\text{eff}} \approx 4.5\mu_B$ ) is observed [1–3]. The AF ordering of YbRh<sub>2</sub>Si<sub>2</sub> may be driven to  $T_N \approx 0$  by fields of  $H_{\perp c} \sim 650$  Oe and  $H_{\parallel c} \sim 7$  kOe [4]. At these fields a quantum critical point (QCP) is observed with non-Fermi-liquid (NFL) behavior [1,2]. Therefore, among other systems [5,6], YbRh<sub>2</sub>Si<sub>2</sub> is particularly an interesting system to study quantum criticality and NFL behavior.

Electron spin resonance (ESR) experiments at low- $T$  ( $T \lesssim 20$  K) in YbRh<sub>2</sub>Si<sub>2</sub> by Sichelschmidt et al. [7] have reported on a narrow (100–200 Oe) single dysonian resonance with no hyperfine components,  $T$ -dependence of the linewidth,  $\Delta H$ , and a  $g$ -value anisotropy consistent with Yb<sup>3+</sup> in a metallic host of tetragonal symmetry. As in the early work of Tien et al. [8] the observation of a narrow Yb<sup>3+</sup> ESR in the intermediate valence compound of YbCuAl and, recently, in a dense Kondo system below  $T_K$  were unexpected results [7]. Nevertheless, various reports were already published on the ESR of Yb<sup>3+</sup> in stoichiometric YbRh<sub>2</sub>Si<sub>2</sub> [9],

YbRh<sub>2</sub>Si<sub>2</sub> [10], and YbRh<sub>2</sub>Si<sub>2</sub> doped with non-magnetic impurities as Ge [11] and La [12]. Also, the ESR of Ce<sup>3+</sup> in dense Kondo systems was communicated [13]. However, it is not clear yet what mechanism allows the observation of the Yb<sup>3+</sup> ESR line with local magnetic moment features in these highly correlated electron systems.

The main purpose of this work is to investigate the  $H$ -dependent ESR data in the NFL phase of YbRh<sub>2</sub>Si<sub>2</sub> ( $4.2 \lesssim T \lesssim 10$  K;  $0 < H \lesssim 10$  kOe). We found an unexpected  $H$ -dependence behavior of the Yb<sup>3+</sup> ESR data in YbRh<sub>2</sub>Si<sub>2</sub> that, we hope, will contribute to the understanding of the observed ESR signal in this system.

### 2. Experiment

Single crystals of Yb<sub>1-x</sub>Lu<sub>x</sub>Rh<sub>2</sub>Si<sub>2</sub> ( $0 \leq x \leq 1.00$ ) were grown from In and Zn-fluxes as reported elsewhere [14–16]. The structure and phase purity were checked by X-ray powder diffraction. The high quality of our undoped crystals was confirmed by X-rays rocking curves which revealed a mosaic structure of maximum  $c$ -axis angular spread of  $\lesssim 0.015^\circ$ . The electrical residual resistivity ratio,  $\rho_{300\text{K}}/\rho_{1.9\text{K}}$ , for the In and Zn-flux grown crystals were 35 and 10, respectively [14–16]. For the ESR spectra  $\sim 2 \times 2 \times 0.5$  mm<sup>3</sup> single crystals were used. The ESR experiments were carried out in a Bruker S, X, and Q-bands (4.1, 9.5 and 33.8 GHz) spectrometer using appropriated

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resonators and  $T$ -controller systems. A single anisotropic resonance, with no hyperfine components, from the Kramer doublet ground state was observed at all bands. The dysonian lineshape ( $A/B \approx 2.5$ ) corresponds to a microwave skin depth smaller than the size of the crystals [17].

### 3. Results and discussions

Fig. 1 shows the  $\text{YbRh}_2\text{Si}_2$  ESR X-band spectra at 4.2 K and  $H_{\perp c}$  for single crystals grown in In and Zn-fluxes. From the anisotropy of the field for resonance,  $H_r(\theta)$  (not shown), one can obtain the angular-dependence of the effective  $g$ -value which is given by  $h\nu/\mu_B H_r(\theta) = g(\theta) = [g_{\perp c}^2 \cos^2 \theta + g_{\parallel c}^2 \sin^2 \theta]^{1/2}$ . From the fitting of the experimental  $H_r(\theta)$  we obtain  $g_{\parallel c} \lesssim 0.6(4)$  and  $g_{\perp c} = 3.60(7)$ . The inset of Fig. 1 displays, for  $H_{\perp c}$ , the Korringa-like [18,19] linear thermal broadening of the linewidth,  $\Delta H(T) = a + bT$ , and the fitting parameters for the crystal grown in Zn-flux at X and Q-bands. As it will be shown below, the large values measured for  $a$  and  $b$  indicates that Zn impurities were incorporated in this crystal.

Fig. 2 presents the relative normalized X-band integrated ESR intensities for  $\text{Yb}_{1-x}\text{Lu}_x\text{Rh}_2\text{Si}_2$  at 4.2 K and  $H_{\perp c}$  as a function of  $x$ ,  $I_{4.2}(x)/I_{4.2}(0)$ . The ESR intensities were determined taking into consideration the crystal exposed area, skin depth and spectrometer conditions. These results show that, while for  $x \leq 0.15$  the  $\text{Yb}^{3+}$  ESR intensity is nearly constant, for  $0.15 < x \leq 1.00$  the intensity vanish completely and no ESR could be detected. It is worth mention that for  $x > 0.15$  and  $T \gtrsim 200$  K,  $\chi_{\perp c}(T)$  follows an Curie–Weiss law with a full  $\text{Yb}^{3+}$  magnetic moment and that for  $x \leq 0.15$  there is no appreciable changes in the thermodynamic properties of these compounds [14,15]. The absence of resonance for  $x > 0.15$  strongly suggests that the observed ESR for  $x < 0.15$  cannot be associated to a single  $\text{Yb}^{3+}$  ion resonance but rather to a resonant collective mode of exchange coupled  $\text{Yb}^{3+}$ – $ce$  (conduction electrons) magnetic moments. We argue that a strong  $\text{Yb}^{3+}$ – $ce$  exchange coupling may broadens and

shifts the  $ce$  resonance toward the  $\text{Yb}^{3+}$  resonance allowing their overlap and building up a  $\text{Yb}^{3+}$ – $ce$  coupled mode with possibly bottleneck/dynamic-like features. Evidences for bottleneck/dynamic-like features in the Lu-doped crystals will be published elsewhere. An internal field caused by the  $\text{Yb}^{3+}$  local moments may be responsible for the shift of the  $ce$  resonance [21]. Moreover, the Lu-doping may disrupt the collective mode coherence and, probably, may also opens the bottleneck/dynamic regime [22].

Figs. 3a and b show, respectively, the low- $T$  dependence of  $\Delta H$  and effective  $g$ -value of the  $\text{Yb}^{3+}$  ESR in In-flux grown  $\text{YbRh}_2\text{Si}_2$ ,

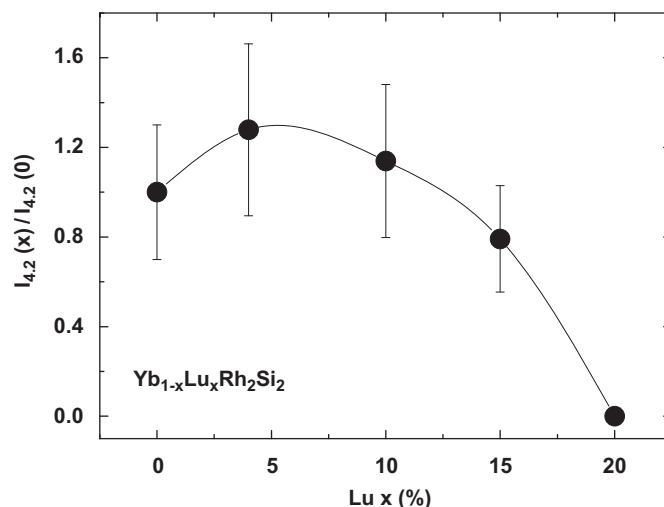


Fig. 2. X-band  $\text{Yb}^{3+}$  relative normalized integrated ESR intensities,  $I_{4.2}(x)/I_{4.2}(0)$ , at 4.2 K and  $H_{\perp c}$  for  $\text{Yb}_{1-x}\text{Lu}_x\text{Rh}_2\text{Si}_2$ .

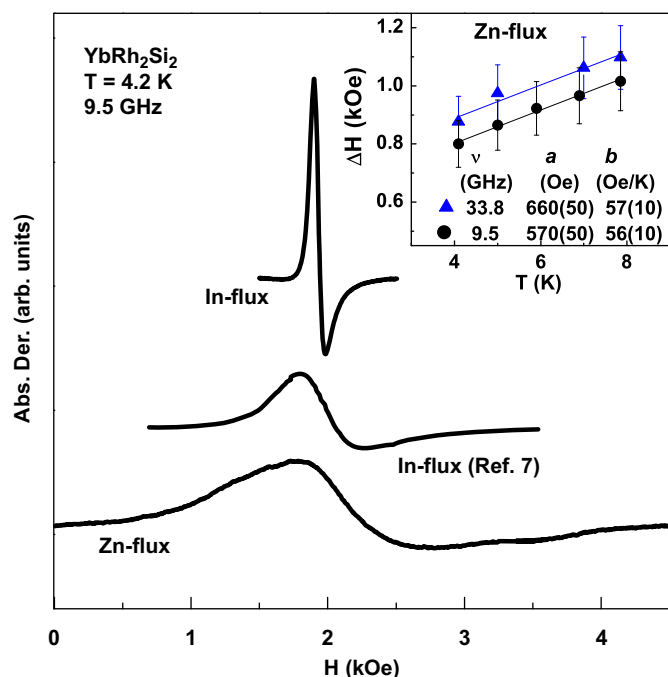


Fig. 1. (Color online) ESR spectra at 4.2 K and  $H_{\perp c}$  for single crystals grown in In and Zn-fluxes and for the In-flux crystals of Ref. [7]. Inset:  $\Delta H(T) = a + bT$  for Zn-flux crystals and  $H_{\perp c}$  at X and Q-bands.

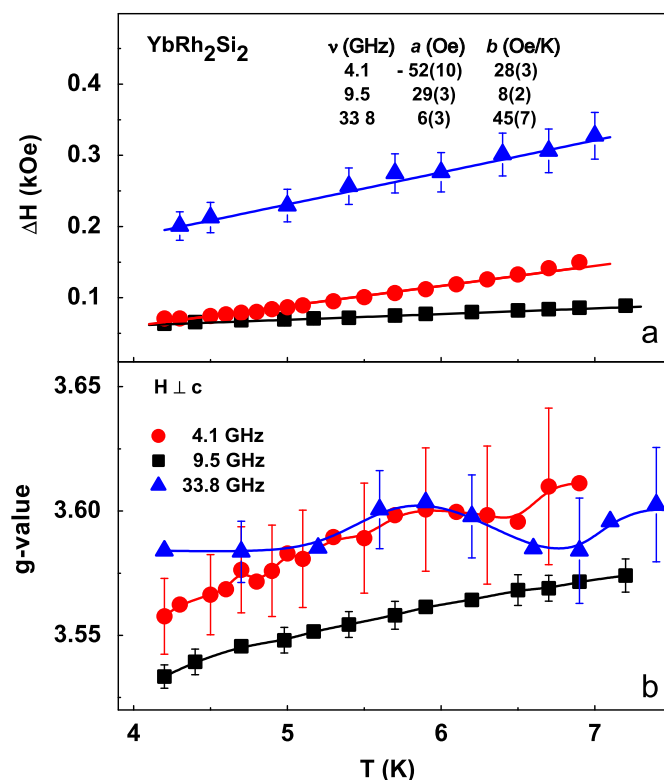


Fig. 3. (Color online) X, S and Q-bands low- $T$  dependence of the  $\text{Yb}^{3+}$  ESR for  $H_{\perp c}$ : (a)  $\Delta H(T)$  and (b) effective  $g(T)$ -value. The lines represent the linear fit in (a) and a guide to the eyes in (b).

measured at S, X and Q-bands for  $H_{\perp c}$ . In this  $T$ -interval, and within the error bars, it is also found that  $\Delta H = a + bT$  for the three bands. This suggests a Korringa-type of mechanism for the  $\text{Yb}^{3+}$  spin–lattice relaxation (SLR), i.e. the  $\text{Yb}^{3+}$  local moment is exchange coupled to the conduction electrons [18]. The residual linewidth,  $a$ , and relaxation-rate,  $b = \Delta H/\Delta T$ , are given in Fig. 3a. Notice that the minimum relaxation rate,  $b$ , is found at X-band,  $H \approx 1900$  Oe. The actual determination of the residual linewidth,  $a = \Delta H(T = 0)$ , would require measurements at lower- $T$ , therefore, the obtained values should be considered just as fitting parameters. A  $H$ -dependent SLR-rate  $b$  is not expected for a normal local magnetic moment- $ce$  exchange coupled system, where the Korringa-rate is frequency/field independent [19]. However, since the  $\text{Yb}^{3+}$  and  $ce$  magnetic moments carry unlike spins and different  $g$ -values, the  $H$ -dependence of  $b$  may be an anomalous manifestation of a bottleneck-like behavior. Fig. 3b shows that the  $T$ -dependence of the effective  $g$ -values are slightly different in the three bands, with minimum effective  $g_{4,2}$ -values also at the X-band. The effective  $g$ -value accuracy is much higher than that obtained from  $h\nu/\mu_B H_r(\theta) = g(\theta) = [g_{\perp c}^2 \cos^2 \theta + g_{\parallel c}^2 \sin^2 \theta]^{1/2}$  because proper experimental conditions were chosen for these  $H_{\perp c}$  measurements.

Fig. 4 displays the  $H$ -dependence of  $b$  and  $g_{4,2}$ -values. Notice that both parameters have minimum values at the X-band field,  $H \approx 1900$  Oe.

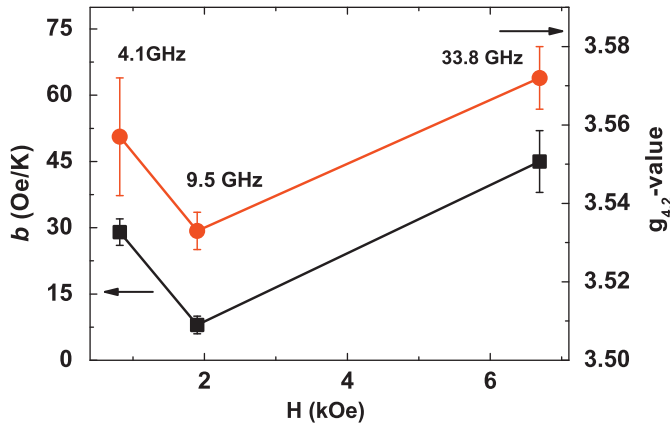


Fig. 4. (Color online)  $\text{Yb}^{3+}$  ESR  $H$ -dependence (X, S and Q-bands) of  $b$  and effective  $g_{4,2}$ -value for  $H_{\perp c}$ .

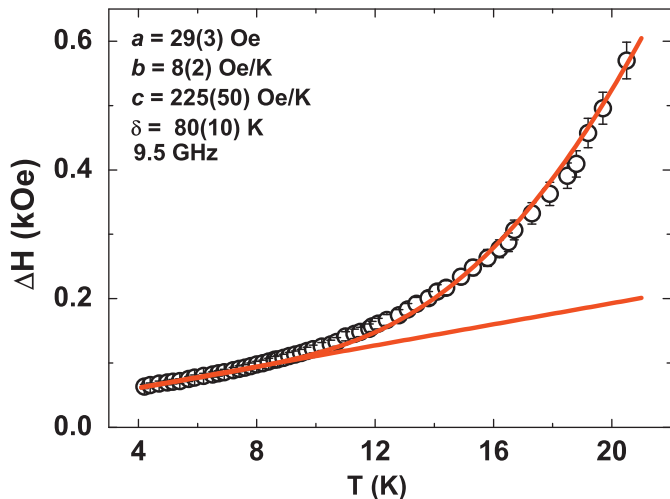


Fig. 5. (Color online)  $\text{Yb}^{3+}$  ESR X-band  $\Delta H(T)$  for  $H_{\perp c}$  and  $4.2 \leq T \leq 21$  K. The fitting parameters correspond to the best fit to  $\Delta H(T) = a + bT + c\delta/[\exp(\delta/T) - 1]$ .

Fig. 5 shows the X-band  $\Delta H(T)$  for  $4.2 \leq T \leq 21$  K and  $H_{\perp c}$ . The data were fitted to  $\Delta H(T) = a + bT + c\delta/[\exp(\delta/T) - 1]$  taking into consideration all the contributions to  $\Delta H$  in a metallic host. The 1st and 2nd terms are the same as above. The 3rd is the relaxation, also via an exchange interaction with the  $ce$ , of a thermally populated  $\text{Yb}^{3+}$  excited crystal field state at  $\delta$  K above the ground state [23]. The fitting parameters are in the inset of Fig. 5. This analysis does not consider any direct  $\text{Yb}^{3+}$  spin–phonon contribution [23]. The S and Q-band  $\Delta H(T)$  data for  $7 \leq T \leq 20$  K also show exponential behaviors with  $c \approx 200(70)$  Oe/K and  $\delta \approx 75(20)$  K.

Within a molecular field approximation the effective  $g(T)$  may be written as  $g_{\text{eff}} = g(1 + \lambda\chi_{\perp c}(T))$ . Fig. 6 presents a plot of  $\Delta g/g = (g_{\text{eff}}(T) - g(15))/g(15) \propto \lambda\chi_{\perp c}(T)$  for  $T \leq 15$  K and X-band for our  $x = 0$  crystals [15] and that from Refs. [7,24]. A linear correlation is obtained with  $\lambda$  values in the interval of  $-2 \text{ kOe}/\mu_B > \lambda > -3 \text{ kOe}/\mu_B$ . In the Appendix, it is shown that the shift that gives the temperature dependence of  $g_{\text{eff}}$  arises from anisotropic exchange interactions between the  $\text{Yb}^{3+}$  ions. Fig. 7 shows the comparison between theory and experiment. Therefore, these results definitely indicate that the  $T$ -dependence of the effective  $g(x, T)$  is nothing but a consequence of the shift of the

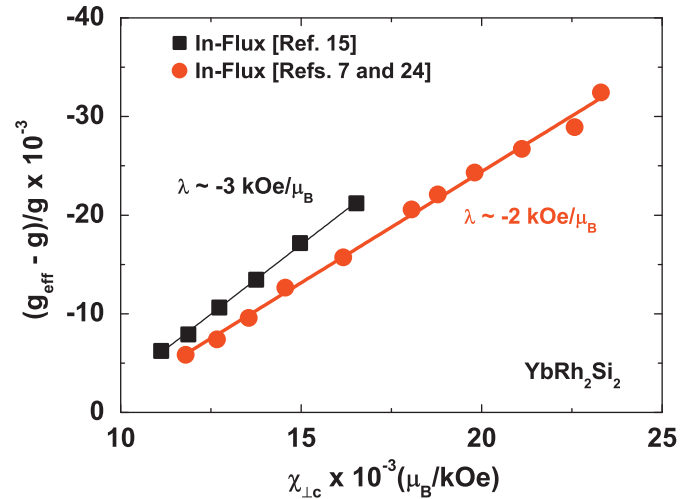


Fig. 6. (Color online) effective  $g$ -shift and magnetic susceptibility correlation,  $(g_{\text{eff}} - g)/g \propto \lambda\chi_{\perp c}(T)$ , for  $T \leq 15$  K and X-band for  $x = 0$  crystals (see text).

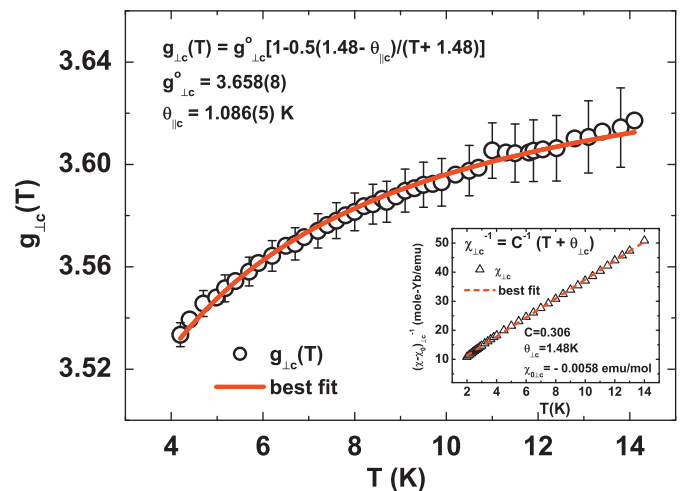


Fig. 7.  $g_{\perp c}(T)$  vs.  $T$ . The solid line is the theoretical curve obtained from Eq. (4). The inset shows the fit of  $\chi_{\perp c}^{-1}(T)$  to a Curie–Weiss law.

field for resonance toward higher fields due to an AF internal molecular field and has nothing to do with a  $\Delta g = c/\ln(T_K/T)$  divergence [7]. Moreover, the expected  $g$ -shift caused by the exchange interaction between the  $\text{Yb}^{3+}$  and  $ce$  local moments,  $J_{fce}$ , can be estimated from the largest Korringa-rate value measured at Q-band in our In-flux crystal,  $b \cong 45 \text{ Oe/K}$ . Within a single band approximation [23] and absence of  $q$ -dependence of the  $\text{Yb}^{3+}$ - $ce$  exchange interaction,  $J_{fce}(\mathbf{q}) \cong J_{fce}(\mathbf{0})$ , [26] one can write  $(\Delta g/g)^2 = \mu_B b / \pi g k_B$ , which gives  $|\Delta g/g| \lesssim 2\%$ . This values is far much smaller than that estimated in Ref. [7] using as a reference the ESR of  $\text{Yb}^{3+}$  in the insulator  $\text{PbMoO}_4$ . On the other hand, from the Korringa relation [23] using  $b \cong 45 \text{ Oe/K}$  and assuming a maximum *bare* density of state per one spin direction at the Fermi level,  $\eta_F$ , given by the Sommerfeld coefficient of the specific heat measurements ( $\gamma \cong 900 \text{ mJ/mol K}^2$ ) [27] we extract a lower limit for  $|J_{fce}| \gtrsim 3 \text{ meV}$ , which is about two order of magnitude larger than the value found for the  $\text{Yb}^{3+}$ - $\text{Yb}^{3+}$  exchange interaction,  $|J_{ff}|$ , inferred from the molecular field parameters  $\theta_{lc}$  and  $\theta_{\perp c}(T)$  in a nearest-neighbor approximation (see Appendix) [25].

Our experiments confirm the ESR results of Sichelschmidt et al., in  $\text{YbRh}_2\text{Si}_2$  below  $T_K \simeq 25 \text{ K}$  [7]. However, our results suggest that this ESR corresponds to a strong exchange coupled  $\text{Yb}^{3+}$ - $ce$  resonant collective mode. The features of this resonant collective mode resembles the bottleneck/dynamic scenario for diluted magnetic moments exchange coupled to the  $ce$ , both with  $g \cong 2$ , where in a normal metal the SLR-rate,  $b$ , and  $g$ -value depend on the competition between the Korringa/Overhauser relaxation and the  $ce$  SLR [18,29,19,28]. Then, the increase of  $b$  by the addition of non-magnetic impurities to  $\text{YbRh}_2\text{Si}_2$  (Lu, Zn in Fig. 1 and La in Ref. [12]), may be associated to “opening” the bottleneck regime due to the increase in the  $ce$  spin-flip scattering [19,28]. The results and discussion about the non-magnetic Lu impurities effects and bottleneck behavior will be the subject of a forthcoming publication.

Another striking result reported in Fig. 4 is the non-monotonic  $H$ -dependence of  $b$  and effective  $g$ -value of the  $\text{Yb}^{3+}$ - $ce$  resonant collective mode. Admixtures via Van Vleck terms [30] may be disregarded because this contribution should scale with  $H$ . Therefore, we believe that the low  $H$ -tunability [1–3] of the ESR parameters in  $\text{YbRh}_2\text{Si}_2$  is an “intrinsic” property of the NFL state near a QCP, where the strength of the  $\text{Yb}^{3+}$ - $ce$  magnetic coupling may be subtly tuned and allows the formation of the resonant collective mode. We attribute the absence of low  $H$ -dependent ESR results in previous reports [7] to the presence of “extrinsic” impurities and/or Rh/Si defects [15,31] that increase the SLR,  $b$ , and residual linewidth,  $a$ . Thus, as for our Zn-flux crystals, hiding the low  $H$ -dependence of the ESR parameters in the NFL phase.

In the bottleneck scenario, the  $\text{Yb}^{3+}$ - $ce$  resonant collective mode presents the strongest bottleneck regime (smallest  $b$ ) at  $H \approx 1900 \text{ Oe}$ . However, due to the subtle details of the coupling between the Kondo ions and the  $ce$  in a Kondo lattice and to strong impurity effects, these resonant collective modes may not be always observable, unless extreme bottleneck regime is achieved. The proximity to a QCP and/or the presence of enhanced spin susceptibility may favor this condition [13,32]. The bottleneck scenario for the  $\text{Yb}^{3+}$ - $ce$  resonant collective mode may also explain the absence of  $\text{Yb}^{3+}$  hyperfine ESR structure [33].

Recent calculations by Abrahams and Wolfe have suggested that the ESR linewidth may be strongly reduced by a factor involving the heavy fermion mass and quasiparticle ferromagnetic (FM) exchange interactions  $(m/m^*)(1 - U\chi_{ff,H}^+(0))$  [34]. These results indicate that the estimation of the linewidth from the Kondo temperature,  $T_K$  ( $\Delta H = k_B T_K / g\mu_B$ ) is an over estimation. However, these calculations may not be contemplating all the

possibilities and have to be taken with care when applied to the dynamic of the ESR of  $\text{YbRh}_2\text{Si}_2$  compound because, (i) it presents an AF  $\text{Yb}^{3+}$ - $\text{Yb}^{3+}$  exchange interaction (although other works in literatures have claimed in favor of the existence of FM fluctuation in  $\text{YbRh}_2\text{Si}_2$  [35,36] and (ii) samples with the same thermodynamic properties present quit different linewidths (see Fig. 1). Furthermore, the anisotropy in the ESR in  $\text{YbRh}_2\text{Si}_2$  reflects both single-ion crystal field effects and the  $\text{Yb}^{3+}$ - $\text{Yb}^{3+}$  and  $\text{Yb}^{3+}$ - $ce$  interactions. In principle, the analysis of crystal field effects is straightforward although somewhat hindered by the inability to detect a signal when the field is along the  $c$ -axis. The anisotropy of the  $\text{Yb}^{3+}$ - $\text{Yb}^{3+}$  and  $\text{Yb}^{3+}$ - $ce$  is more difficult to determine and in the latter case more critical. The application of the resonant collective mode model is based on the assumption that the  $\text{Yb}^{3+}$ - $ce$  coupling is dominated by a scalar interaction between the  $\text{Yb}^{3+}$  ground state doublet pseudo-spins  $S_{ps}$ , and the spins of the conduction electrons,  $s$ , with the consequence that the total spin  $s + S_{ps}$  is (approximately) a constant of the motion [37,38]. In the presence of uniaxial anisotropy, only the component of the total spin along the symmetry axis is a constant of the motion. How the lower symmetry affects the formation of the collective mode is an unsolved problem requiring further study [34].

Finally, we hope that our results motivate new theoretical approaches to understand the dynamics of strong exchange coupled magnetic moments of unlike spins and  $g$ -values, as  $\text{Yb}^{3+}$  and  $ce$ , and explore the general existence of a resonant collective mode with a bottleneck/dynamic-like behavior.

#### 4. Summary

In summary, this work reports low  $H$ -dependent ESR, below  $T_K \simeq 25 \text{ K}$ , in the NFL phase of  $\text{YbRh}_2\text{Si}_2$  ( $T \lesssim 10 \text{ K}$ ). It is suggested that the observed ESR in  $\text{YbRh}_2\text{Si}_2$  corresponds to a  $\text{Yb}^{3+}$ - $ce$  resonant collective mode in a strong bottleneck-like regime, which is highly affected by the presence of impurities and/or defects. The analysis of our data allowed us to give estimations for the  $\text{Yb}^{3+}$ - $\text{Yb}^{3+}$  exchange parameter,  $J_{ff}$ , and a lower limit for the  $\text{Yb}^{3+}$ - $ce$  exchange parameter,  $J_{fce}$ .

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We thank FAPESP and CNPq (Brazil) and NSF (USA) for financial support; and P. Coleman, E. Miranda, D.J. Garcia and M. Continentino for fruitful discussions.

#### Appendix

In Fig. 6, it was noted that  $g_{eff}$  depends linearly on the transverse susceptibility. In this appendix we show the origin of the linear dependence. The starting point is a general theory for magnetic resonance in anisotropic magnets [39] which leads to an expression for the effective  $g$ -factor in an orthorhombic crystal that can be written in the form:

$$g_{eff} = \frac{g_b^M g_c^M \chi_a}{g_a^M (\chi_a \chi_b)^{1/2}} \quad (1)$$

when the static field is along the  $a$ -axis. Here the microscopic  $g$ -factors are denoted by  $g_{a,b,c}^M$  and the corresponding static susceptibilities by  $\chi_{a,b,c}$ . In the case of a uniaxial system with the static field perpendicular to the  $c$ -axis, (1) reduces to

$$g_{eff \perp c} = g_c^M \left[ \frac{\chi_{\perp c}}{\chi_{lc}} \right]^{1/2} \quad (2)$$

In applying this equation to the resonance in  $\text{YbRh}_2\text{Si}_2$  it must be kept in mind that the  $g$ -factors and susceptibilities refer to the ground state doublet. We assume that the relevant doublet susceptibilities take the form  $C_{\perp,\parallel}/(T + \theta_{\perp,\parallel})$  where  $C_{\perp}$  and  $C_{\parallel}$  are constants and fit the experimental data for  $\chi_{\perp,c}$  to the form  $\chi_0 + C_{\perp,c}/(T + \theta_{\perp,c})$  in the temperature range of the resonance experiment (see inset of Fig. 7) with the result  $\theta_{\perp,c} = 1.48$  K. We chose  $\theta_{\parallel,c}$  and the overall amplitude  $g_{\perp,c}^0$  as adjustable parameters. The measured values  $g_{\perp,c}(T)$  were then fit to the function:

$$g_{\perp,c}(T) = g_{\perp,c}^0 \left[ 1 - \left( \frac{\theta_{\perp,c} - \theta_{\parallel,c}}{T + \theta_{\perp,c}} \right) \right]^{1/2} \quad (3)$$

$$\approx g_{\perp,c}^0 \left[ 1 - 0.5 \left( \frac{\theta_{\perp,c} - \theta_{\parallel,c}}{T + \theta_{\perp,c}} \right) \right] \quad (4)$$

with the best fit given by  $g_{\perp,c}^0 = 3.66$  and  $\theta_{\parallel,c} = 1.09$  K (see Fig. 7). The result shown in Fig. 6 is in accord with Eq. (4) and Fig. 7 since the  $T$ -dependent part of  $\chi_{\perp,c}(T)$  is proportional to  $(T + \theta_{\perp,c})^{-1}$ , the same factor that is present in Eq. (4).

The results outlined in the preceding paragraph show that the  $T$ -dependence of  $g_{\perp,c}(T)$  is associated with the difference in the longitudinal and transverse exchange interaction which is reflected in the difference between  $\theta_{\perp,c}$  and  $\theta_{\parallel,c}$ . Although it has not been possible to observe the resonance with the static field along the  $c$ -axis, there is a corresponding shift there as well, which takes the form:

$$g_{\parallel,c}(T) = g_{\parallel,c}^0 \left[ 1 + \left( \frac{\theta_{\perp,c} - \theta_{\parallel,c}}{T + \theta_{\parallel,c}} \right) \right] \quad (5)$$

Note that the shift in  $g_{\parallel,c}(T)$  is in the opposite direction from the shift in  $g_{\perp,c}(T)$ .

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