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March 1988

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Abstract

Vacuum renormalization of relativistic nuclear field theory is studied for nuclear and neutron star matter. It is found that when the coupling constants of the theory, with or without vacuum renormalization, are adjusted so that the five saturation properties of nuclear matter, binding, density, compression modulus, symmetry energy and effective nucleon mass, are reproduced, that the equation of state in the two cases differ by only several percent over the entire density range of interest. If the effective mass and compression are not controlled, as in some works, the high density behavior is markedly different.

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Contents

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1 Introduction

So far the only known effective relativistic field theory that can describe nuclear matter *and* finite nuclear properties is the scalar-vector-isovector (σ, ω, ρ) theory. Although it is known how to incorporate vacuum renormalization effects [1,2], and this has been done in several recent works[3,4], it so far has not been studied systematically in a way that preserves the five important properties of nuclear matter at saturation, the binding, density, compression modulus, effective mass and symmetry energy. Therefore it has not been possible to disentangle the renormalization effects from those produced by shifting nuclear matter properties. Moreover, vacuum polarization in neutron star matter that is in generalized beta equilibrium has not been investigated previously, except in the chiral-sigma model[5], which seems incapable of describing the normal ground state of finite nuclei, producing instead a bubble configuration [6]. In this paper we undertake such a systematic investigation of nuclear and neutron star matter for the σ, ω, ρ theory. This requires the form of the theory in which cubic and quartic self-interactions of the scalar field are incorporated, and renormalized, for they, together with the nucleon interaction with the scalar, vector and vector-isovector mesons, permit the five saturation properties to be controlled.

2 Theory

The Lagrangian of the σ, ω, ρ theory is,

$$
\begin{aligned}
\mathcal{L} =& \sum_B \overline{\psi}_B (i\gamma_\mu \partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu \omega^\mu - \tfrac{1}{2}g_{\rho B}\gamma_\mu \tau_3 \rho_3^\mu)\psi_B \\
&+ \tfrac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \tfrac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \tfrac{1}{2}m_\omega^2 \omega_\mu \omega^\mu \\
&- \tfrac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \tfrac{1}{2}m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu - \tfrac{1}{3}bm_n(g_\sigma \sigma)^3 - \tfrac{1}{4}c(g_\sigma \sigma)^4
\end{aligned}
\tag{1}
$$

The scalar meson is Yukawa coupled to the baryon scalar density and the vector and vector-isovector mesons to the baryon current and the isospin current respec-

tively. In preparation for application to dense matter we have included, in addition to the nucleons, other baryon species, denoted by B, where the sum is over all *charge* states of $N, \Lambda, \Sigma, \Xi, \Delta$, etc. [7,8]. When the corresponding Euler-Lagrange equations are solved by replacing the meson fields by their mean values, and the nucleon currents by those generated in the presence of the mean meson fields, one obtains the so called mean field approximation (MFA). It is in this approximation that nuclear field theory has been typically solved and applied. However, as is well known, the presence of matter alters the vacuum, by altering the masses of antiparticles. The energy of the filled sea therefore shifts with density. There are well known procedures for renormalizing the theory with respect to nucleon, and scalar and vector mesons[2]. So far it is not known how to renormalize the vector-isovector meson, and we shall regard as phenomenological the energy contributed to asymmetric matter by the coupling of this meson to the isospin current, with baryon-ρ coupling chosen to reproduce the empirical symmetry energy coefficient.

With the inclusion of vacuum renormalization energies, the energy density is given by

$$\epsilon_{RHA} = \epsilon_{MFA} + V_N + V_\sigma \tag{2}$$

where ϵ_{MFA} is the mean field energy and the last two terms represent the contributions from renormalization of the scalar meson and nucleon[2], and are given by,

$$
\begin{aligned}
V_\sigma &= \frac{m_\sigma^4}{(8\pi)^2}\Big[(1 + \phi_1 + \phi_2)^2 \ln(1 + \phi_1 + \phi_2) \\
&\quad -(\phi_1 + \phi_2) - \frac{3}{2}(\phi_1 + \phi_2)^2 \\
&\quad -\frac{1}{3}\phi_1^2(\phi_1 + 3\phi_2) + \frac{1}{12}\phi_1^4\Big]
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
V_N &= -\frac{m_n^4}{4\pi^2}\Big[(1 - \chi)^4 \ln(1 - \chi) + \chi \\
&\quad -\frac{7}{2}\chi^2 + \frac{13}{3}\chi^3 - \frac{25}{12}\chi^4\Big]
\end{aligned}
\tag{4}
$$

where

$$\phi_1 = \frac{2bm_n g_\sigma^3 \sigma}{m_\sigma^2}, \qquad \phi_2 = \frac{3cg_\sigma^4 \sigma^2}{m_\sigma^2}, \qquad \chi = \frac{g_\sigma \sigma}{m_n} \tag{5}$$

and m_n and m_σ are the nucleon and σ mass. The approximation which includes the vacuum renormalization is known as the relativistic Hartree approximation (RHA)[2].

When the field equations, obtained by minimizing the energy density at fixed baryon density, are solved subject to the subsidiary constraint of zero isospin, one obtains the solution corresponding to uniform nuclear matter. When they are solved subject to the constraints of charge neutrality and general equilibrium, one obtains the solution corresponding to neutron star matter. We shall characterize both solutions by the corresponding properties of symmetric matter.

2

Table 1: Nuclear Matter Properties at Saturation

	ρ_0 (fm^{-3})	B/A (MeV)	a_{sym} (MeV)	K (MeV)	$m^\star/m$
expt.	.153	-16.3	32.5	300	.78
this work					
MFA and RHA	.153	-16.3	32.5	300	.78
Serot - Uechi					
MFA	.193	-15.75	22.1	540	.557
RHA	.193	-15.75	17.9	471	.718

3 Nuclear and neutron star matter

The five important properties of nuclear matter, mentioned in the introduction, can be used to fix the coupling constants $g_\sigma/m_\sigma, g_\omega/m_\omega, g_\rho/m_\rho$, and the parameters of the scalar self-interactions, b and c. In uniform matter, it is only the ratio of coupling constant to mass on which the theory depends, aside from the scalar mass, which appears independently in the vacuum renormalization energy. For that mass we take $m_\sigma = 600$ MeV. The binding, saturation density and symmetry energy coefficient are relatively well known[9]. The compression modulus has been the subject of considerable debate in the last several years. However a recent analysis of a broad body of evidence[10], and recent new experiments on the giant monopole resonance[11] both suggest that $K \approx 300$ MeV. The Landau effective nucleon mass, $m_L^\star/m = 0.83$, has recently been obtained through a careful study of the mean field of heavy nuclei, and we fix this property in accord with those findings[12]. The scalar effective mass of this theory, $m^\star = m - g_\sigma\sigma$, is related at saturation by $m_L^\star = (m_{sat.}^{\star 2} + k_F^2)^{\frac{1}{2}}$ which yields $m_{sat.}^\star/m = 0.78$. The nuclear properties are listed in Table 1.

We first assess the effect of the vacuum polarization on the binding energy of normal nuclear matter, by adjusting the coupling constants so that the saturation properties listed in Table 1 are reproduced in both the mean field (MFA) and the relativistic Hartree approximation (RHA). The corresponding coupling constants are given in Table 2, and the comparison of the two approximations can be seen in Fig. 1, for both nuclear matter and pure neutron matter. The equation of state in both approximations are surprisingly alike, differing by at most by about three percent even at ten times nuclear density. This is a very encouraging result, since in the many applications of the theory to finite nuclei and neutron stars, the MFA has been employed up till now. Next we show in Fig. 2 the separate contributions to the equation of state arising from the two-body, as well as the three and four-body terms in the energy and the two contributions V_N and V_σ of the vacuum renormalization. Aside from the region near saturation, the three and four-body terms, and the

Table 2: Coupling constants

	$(g_\sigma/m_\sigma)^2$ fm^2	$(g_\omega/m_\omega)^2$ fm^2	$(g_\rho/m_\rho)^2$ fm^2	b	c
this work (m_σ= 600 MeV)					
MFA	9.031	4.733	4.825	.003305	.01529
RHA	9.249	4.732	4.823	.005723	.000601
Serot - Uechi (m_σ= 550 MeV)					
MFA	11.805	8.6359	0	0	0
RHA	8.094	5.067	0	0	0
K = 471 MeV, $m^\star/m$ = .557, m_σ= 550 MeV					
MFA	7.697	5.0589	0	-.00427	.08756

vacuum renormalization energies are all rather independent of density. The scalar renormalization energy, V_σ, is particularly small. It is noteworthy that in the chiral-sigma model all of the corresponding terms are much larger, by a factor of five or so, vary more drastically with density and play a decisive role in that theory[5,13]. It can be regarded as an advantage of the σ, ω, ρ model that these terms are so small and relatively constant. The implication of the above result is that the neglect of vacuum renormalization, which in principle could produce drastic changes in the nuclear properties, is unlikely to be very important in many applications to finite nuclei and to neutron star structure.

In a recent paper Serot and Uechi [4] also investigated the effects of vacuum renormalization, but they fixed only the saturation energy and density, the compression and effective mass being different in the two cases (see table 1). As a consequence of this, the two approximations yield very different results for the equation of state at higher density, as shown in Fig. 3. On the other hand, when the comparison is made in the case that all five nuclear properties are identical, the two approximations again yield equations of state that are insignificantly different. The coupling constants in these three cases are shown as the last three entries respectively in table 2. The two MFA calculations shown in Fig. 3 are so different from each other because both K and $m^\star_{sat.}$ are different. The latter quantity, for given binding and saturation density, uniquely specifies the vector coupling constant. For fixed K, the equation of state becomes stiffer at high density as $m^\star_{sat.}$ decreases. For fixed $m^\star_{sat.}$, it becomes stiffer as K increases. These are the reasons why it is important to bring both of these parameters under control, through the freedom afforded by the scalar self-interaction terms in Eq.(1). Without this control, the application of the theory to neutron star properties or other high density phenomena can be misleading.

The above conclusion is all the more reinforced by an examination of Fig. 4, where the effective mass as a function of density is shown for the three sets of

nuclear properties shown in table 1.

Finally we study the composition of neutron star matter, which is charge neutral and in chemical equilibrium. We include all baryon states to convergence in the density range studied. The populations as a function of total baryon density are shown in Fig. 5. It is seen that the lowest hyperon threshold is little over $2\rho_0$ and that neutron star matter has a complex composition. This calculation, which incorporates the effects of vacuum polarization, agrees well with the earlier one[8], just as does the equation of state, as shown above.
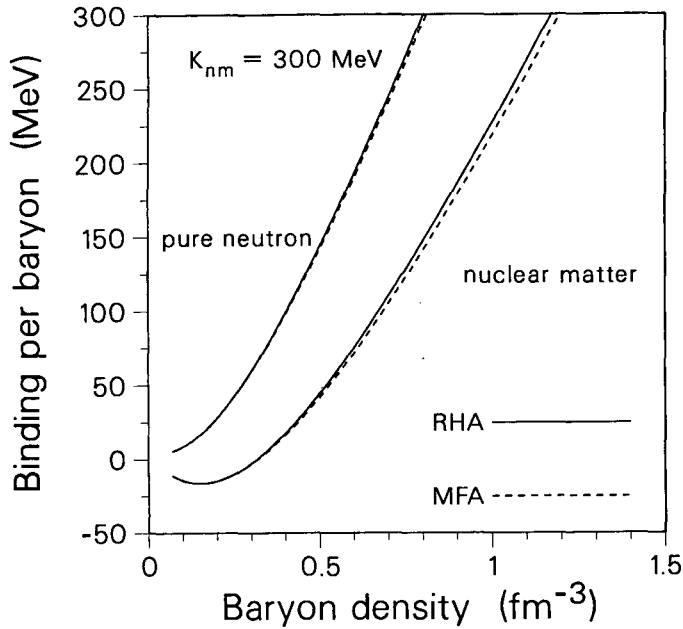
4   Summary

We evaluated the vacuum polarization effects on the equation of state in the σ, ω, ρ theory. It was found that, although not small, when the coupling constants are chosen so as to reproduce the five saturation properties of nuclear matter, the effect on the equation of state is negligible. It might be expected therefore that the limiting mass computed for neutron stars is insensitive to vacuum polarization. We have confirmed that this is so. However when only the saturation density and binding are controlled as in [4], the equation of state with and without vacuum polarization are different. Failure to adequately constrain the equation of state at saturation can therefore lead to spurious conclusions in applications to dense matter as in neutron stars, as well perhaps in applications to nuclear structure, especially for properties that depend on K or $m^\star$.

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Figure 1: Binding energy, $\epsilon/\rho - m$, of nuclear and pure neutron matter as a function of density, computed with and without vacuum renormalization, denoted as RHA and MFA respectively. The corresponding nuclear matter properties are listed in Table 1.
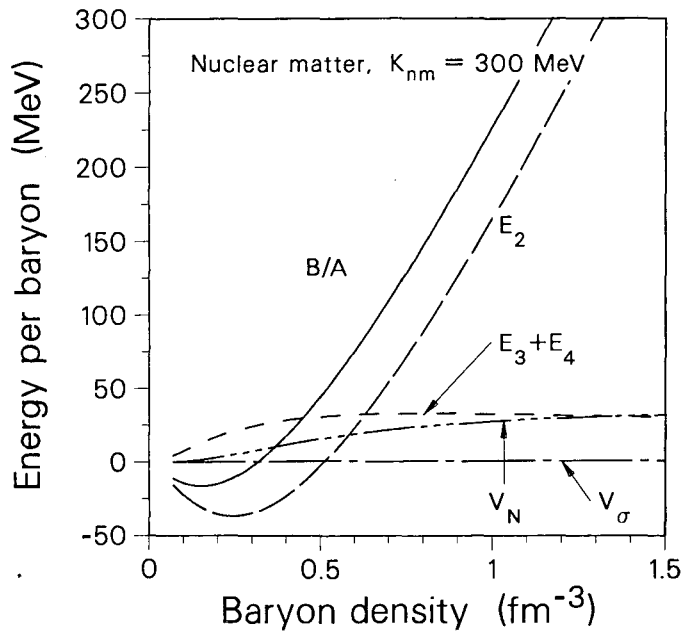
XBL 883-816

Figure 2: For nuclear matter, the separate contributions of the two-, three- and four-body terms and the vacuum polarization energies.
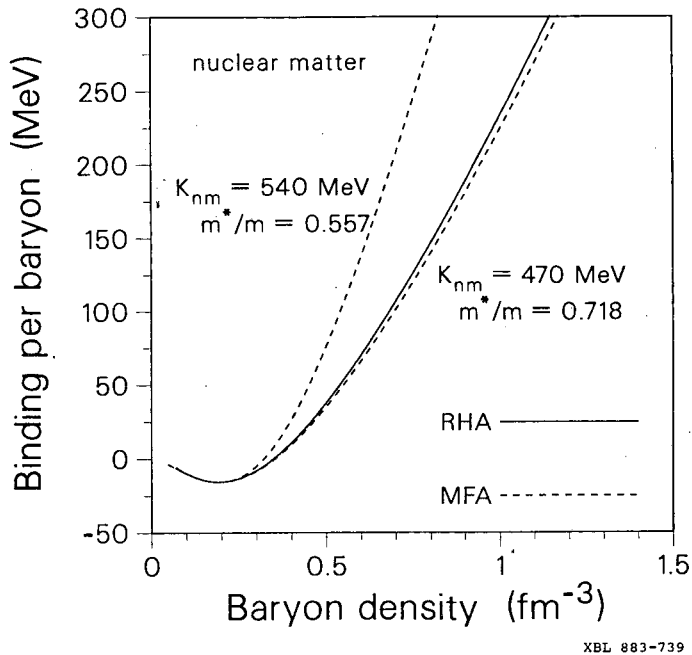
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Figure 3: Binding energy with and without vacuum renormalization (RHA and MFA respectively) for the two sets of nuclear matter properties used by Serot and Uechi (see table 1). Also shown is the MFA for their compression and effective mass.
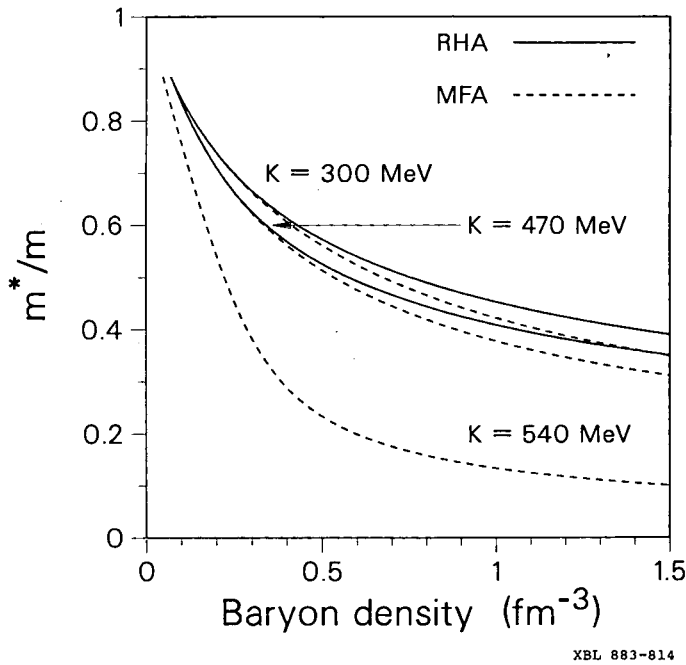
XBL 883-739

Figure 4: Effective nucleon mass as a function of density with and without vacuum polarization (RHA and MFA respectively), for the three sets of nuclear properties of table 1 which can be identified through K.
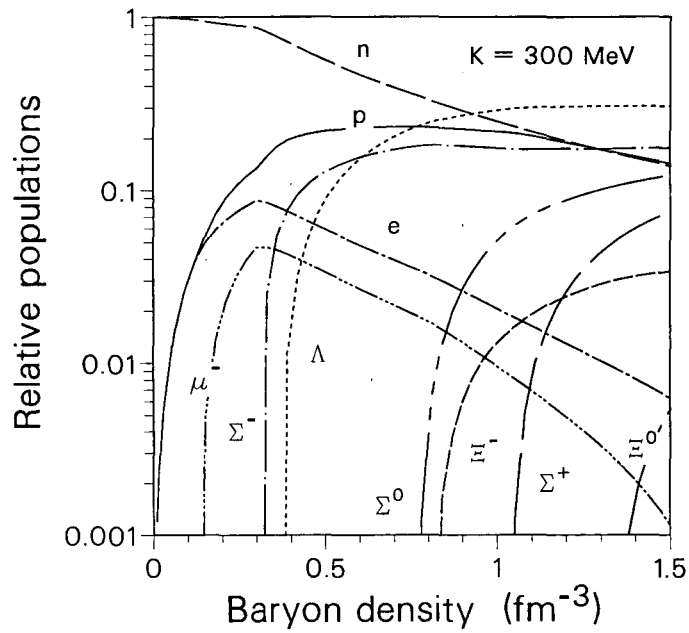
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Figure 5: Populations as a function of baryon density for neutron star matter in RHA.

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