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Pion Interferometry Studies of Relativistic Heavy-Ion Collisions Using the Intranuclear Cascade Model

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Abstract

A method is presented by which an intranuclear cascade (INC) model may be used to obtain pion source parameter predictions which can be directly compared with pion interferometry experiments. This method is applied with Cugnon's INC model to extract predictions for recent pion interferometry measurements, and generally good agreement is found.

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I. Introduction

Intranuclear cascade (INC) models have been widely used to understand various features of relativistic heavy-ion collisions. For example, they have had some success in predicting proton, pion, and kaon inclusive cross sections for laboratory bombarding energies in the range 0.4-2.1 A GeV, and for a variety of projectile-target combinations.[1-3] The basic assumption of INC models is that a relativistic heavy-ion collision can be approximated as a superposition of nucleon-nucleon interactions whose trajectories between interactions are described classically, while the interactions themselves are determined by experimental elastic and inelastic nucleon-nucleon cross sections. Because they are classical models, the positions and momenta of all particles taking part in the cascade are known as a function of time, so that the geometric aspects of the collision, such as the size of the interaction region and the duration of particle production, may be determined. In fact, this is just the kind of information obtained from pion interferometry measurements, where the radius, lifetime, and coherence of the pion source are extracted. Although some theoretical work has been carried out to study the geometry of the pion source with an INC model,[4,5] no study has yet been made which is directed towards understanding existing pion interferometry measurements with this model. Additional incentive is gained to perform such a study as a result of recent measurements which have become available for the systems ^{20}Ne + NaF[6] and ^{40}Ar + KCℓ[6-8]. Thus, the goal of the present work is to extract radius, lifetime, and coherence parameters from an INC model which can be directly compared with the results of these recent measurements.

II. Method

The procedure for obtaining these INC model predictions consists of three steps: (a) run the INC code for the system of interest to produce a set of "final" pions (pions which survive to the end of the calculation), recording their momenta and the space-time location where they were created; (b) impose Bose-Einstein symmetry on the pions produced in (a); and (c) form the two-pion correlation function from the symmetrized and unsymmetrized pions, and fit a Gaussian space-time pion distribution to the correlation function to extract R, τ, and λ, the radius, lifetime, and coherence parameters for the distribution.

Cugnon's CASCADE code [5,9] was used to make the INC model calculations. In this model, the Δ-isobars, which represent the mechanism for producing and absorbing pions, are given a Lorentzian mass distribution and an exponential lifetime distribution. Therefore, pions are produced and reabsorbed throughout the duration of the collision. Note that CASCADE does not take isospin into account, resulting in only one kind of nucleon, pion, and Δ-isobar in the calculation. At the end of the calculation, the momentum vector of each final pion is tagged with the position and time at which its "parent" Δ-isobar decayed and then is stored for later use. Calculations were made with fixed impact parameter so that the dependence of the pion source upon this variable could be studied.

The next step is to weight the pions produced by CASCADE so that they reflect the fact that they are bosons. This is done by using the expression of Yano and Koonin[10] for the symmetrized two-pion inclusive cross section:

$$\frac{d^6\sigma}{d^3k_1d^3k_2} = \int d^4X_1d^4X_2D(X_1,k_1)D(X_2,k_2)|\Psi^s_{k_1k_2}(X_1,X_2)|^2 \qquad (1)$$

where D(X,k) is the pion source distribution function which describes the production of a pion of 4-momentum k at space-time point X, and Ψ^S is the symmetrized two-pion wavefunction which, for plane waves, becomes

$$\Psi^S_{k_1k_2}(X_1,X_2) = e^{ik_1X_1}e^{ik_2X_2} + e^{ik_1X_2}e^{ik_2X_1} .$$

Gyulassy et.al.[11] have shown that in the limit appropriate for relativistic heavy ion collisions and if no dynamical correlations exist between X and k, Eq.1 follows from an exactly soluable field theoretic model, and that it is valid for collisions with $M_\pi \geq 2$, where M_π is the like-pion multiplicity. To apply Eq. 1, we identify the pion distribution produced by CASCADE in the first step with D(X,k) and, thus, randomly choose pairs of pions which are then weighted by $|\Psi^S|^2$, resulting in a list of symmetrized pion pairs.

The final step in the procedure is to form the two-pion correlation function from the symmetrized and unsymmetrized pions and to extract the R, τ, and λ predictions. The two-pion correlation function, $C(k_1, k_2)$, can be expressed as [12]

$$C(\bar{k}_1,\bar{k}_2) = A \frac{N_2(\bar{k}_1,\bar{k}_2)}{N(\bar{k}_1)N(\bar{k}_2)} , \qquad (2)$$

where $\bar{k}_1$ and $\bar{k}_2$ are the pion momenta, $N_2(\bar{k}_1,\bar{k}_2)$ is the two-pion count rate, $N(\bar{k})$ is the single pion count rate, and A is a normalization constant. $N_2(\bar{k}_1,\bar{k}_2)$ and $N(\bar{k})$ are obtained directly from the lists of symmetrized pion pairs and unsymmetrized single pions, respectively, described above. The pion source parameters R, τ, and λ are extracted by fitting the correlation function (Eq.2) with a Gaussian source distribution,[10]

$$C(q,q_o) = 1 + \lambda \exp(-q^2R^2/2 - q_o^2\tau^2/2),$$

using the principle of maximum likelihood[6] where $q = |\bar{k}_1-\bar{k}_2|$, $q_o = |E_1 - E_2|$, and $E = \sqrt{k^2+m_\pi^2}$. It is important to note that the computer codes used to form the correlation function and to carry out the fitting are identical to the ones used by Zajc et. al.[6] and similar to those of Beavis et. al.[8] in analyzing their two-pion correlation data. This should minimize programming biases and allow the CASCADE predictions for R, τ, and λ to be directly comparable with the experimental results.

III. Results and Discussion

A typical CASCADE generated correlation function and its Gaussian fit, both projected onto the q-axis, are shown in Fig. 1 for the system 1.5 A GeV $^{40}Ar + {}^{40}Ar$. This case was run with a 4π geometry (as is the case of the streamer chamber in Ref. 8), impact parameter, b, of 2 fm, minimum center-of-mass pion momentum, k_{MIN}, of 50 MeV/c ($k_{MIN} \leq k_1, k_2$), and about 270,000 two pion pairs. Error bars reflect statistical uncertainties only. Note the prominent Bose-Einstein enhancement at small q, which is a consequence of the pion wavefunction symmetrization carried out with Eq. 1. For this example, the extracted pion source parameters are $R = 3.6 \pm 0.1$ fm, $\tau = 3.2 \pm 0.5$ fm/c, and $\lambda = 0.94 \pm 0.05$.

Figure 2 shows a study of the dependence of the pion source parameters upon the pion momentum and the impact parameter using the prescription presented above. The study was performed for the system 1.5 A GeV $^{40}Ar + {}^{40}Ar$ assuming a 4π detection geometry for the pions. The impact parameter was fixed at 2 fm while varying k_{MIN}, and k_{MIN} was fixed at 50 MeV/c while varying b.

First consider the pion momentum dependence. Although there is little or no momentum dependence for τ and λ within the error bars shown in Fig. 2, R is seen to have a small but definite dependence, being an approximately linearly decreasing function of k_{MIN}. The explanation for this effect on R is seen in Fig. 3, which shows the time distribution for the creation of final pions for k_{MIN} values of 50 and 300 MeV/c. Clearly, for higher pion momenta the time distribution is weighted toward earlier times in the collision, for which, as the present and other INC studies[4] show, the size of the pion source is smaller. Thus, within the context of CASCADE, one probes the earlier stages of the collision by considering higher pion momenta.

Let us next consider the impact parameter dependence of the pion source parameters shown in Fig. 2. As would be expected by a simple geometrical overlap model, R and τ decrease with increasing impact parameter. However, it is seen that λ also decreases with increasing b, differing from unity at b = 6 fm by over 4 σ. This behavior for λ seems to disagree with the arguments presented by Gyulassy in Ref. 13 that, for $b > 0$, λ should become larger than unity due to the effects of final-state pion absorption by the cold spectator matter ("shadowing").

Figure 4 suggests the origin of this impact parameter dependence. In the figure, time-integrated pion source distirbutions from CASCADE, calculated in the nucleon-nucleon center-of-mass frame, are projected onto the reaction plane for b = 0 and b = 6 fm (the beam direction is along the z-axis). Whereas the pion source for b = 0 fm tends to be isotropic and well fit by a spherical Gaussian, in the b = 6 fm case it is extended along the z-direction and, as a result, is no longer approximated well by a simple product of Gaussians. These extended regions found for b = 6 fm are the result of pions which are absorbed and re-emitted in the spectator matter which streams by the interaction region along the beam axis. The λ-effect is thus an artifact of the fitting procedure which can be eliminated by choosing the appropriate functional form to fit (if one can guess what it should be). This effect gives another example[13] of how care must be taken in interpreting the physics behind λ-parameters which differ from unity. Note that for the present CASCADE study, events with $b \geq 5$ fm do not contribute significantly to the inclusive two-pion cross section since the pion multiplicity is relatively small (M_{π^-} - 1-2).

Comparisons between the CASCADE predictions of the present work and recent pion interferometry measurements are presented in Fig. 5 and Table I. The measurements of Zajc et.al.[6] were performed with a narrow acceptance magnetic spectrometer which was set to accept pions centered about 90^o in the center-of-mass with respect to the beam direction, whereas the measurements of Beavis et. al.[7,8] were performed with a streamer chamber having an almost 4π solid angle acceptance. In order to simulate, to some extent, the acceptances found in the two types of experiments, the CASCADE calculations were run with two different sets of center-of-mass windows: a) for the spectrometer experiments, $80^o \leq \theta \leq 100^o$, $0^o \leq \phi \leq 360^o$, and k_{MIN} = 150 MeV/c, and b) for the streamer chamber experiments, $0^o \leq \theta \leq 180^o$, $0^o \leq \phi \leq 360^o$, and k_{MIN} = 50 or 150 MeV/c (θ is the radial angle with respect to the beam direction, and ϕ is the azymuthal angle). In addition, all CASCADE predictions shown in Fig. 5 and Table I were run with b=2 fm. It was found that taking a weighted average over impact parameter for the system 1.5 A GeV ^{40}Ar + ^{40}Ar resulted in a 1σ or less difference in the pion source parameters from the b=2 fm case. This was judged to give acceptable accuracy for the CASCADE predictions, given the accuracy of the experimental results.

Generally speaking, the CASCADE predictions agree rather well with the experimental results. For the Zajc et. al.[6] measurements, agreement occurs within 1σ for R and τ for both 1.8 A GeV ^{20}Ne + NaF and ^{40}Ar + KCℓ, although CASCADE cannot account for the small measured values of λ, predicting λ to be close to unity for both systems. The CASCADE predictions for τ and λ are seen to agree with the measurements of Beavis et. al.[7] for 1.5 A GeV ^{40}Ar + KCℓ, while CASCADE slightly underpredicts the R values, both measurement and CASCADE showing a weak dependence of R on k_{MIN}. Finally, Fig. 5 shows that CASCADE agrees with the Beavis et. al. measurements [8] for 1.2 A Gev ^{40}Ar + KCℓ to within 1σ for all three source parameters.

Two other predictions extracted from CASCADE for the 1.5 A Gev ^{40}Ar + ^{40}Ar system are: a) R is not significantly different for the momentum cuts $150 > k_1, k_2 > 0$ MeV/c and $k_1, k_2 > 50$ MeV/c, and b) if the pion source is fit with longitudinal and transverse (with respect to the beam axis) radius parameters, $R_{||}$ and $R_{\perp}$, in a 4π geometry, it is found that the source is nearly spherical ($R_{||} \approx R_{\perp}$). These predictions are consistant with the experimental results in Ref. 7.

Recently, Pratt[14] has derived a more general expression for the two-pion cross section than Eq. 1. This expression more correctly takes into account dynamical correlations which may occur between X and k in the pion source distribution. Equation 1 can be shown to follow from Pratt's expression in the limit where the source distribution function is uncorrelated in x and k and is sufficiently wide in momentum. Since the results from CASCADE presented above suggest that only weak x-k correlations exist for the systems under study, Eq. 1 should be a reasonable approximation for the present work.

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FIGURE CAPTIONS

Fig. 1 Projected two-pion correlation function from CASCADE for the reaction 1.5 A GeV ^{40}Ar + ^{40}Ar, b=2 fm, k_{MIN} = 50 MeV/c

Fig. 2 Dependence of pion source parameters on k_{MIN} and b for the reaction 1.5 A GeV ^{40}Ar + ^{40}Ar. The lines are drawn to guide the eye.

Fig. 3. Comparison between final pion creation time distributions for k_{MIN} = 50 and 300 MeV/c, for the reaction 1.5 A GeV ^{40}Ar + ^{40}Ar, b = 2 fm.

Fig. 4. Time-integrated spacial pion source distribution projected onto the reaction plan for b = 0 and 6 fm, for the reaction 1.5 A GeV ^{40}Ar + ^{40}Ar, k_{MIN} = 50 MeV/c.

Fig. 5. Comparison between pion source parameters from CASCADE predictions and pion interferometry measurements: a) 1.8 A GeV ^{40}Ar + KCℓ and ^{20}Ne + NaF (Ref. 6); b) 1.5 A GeV ^{40}Ar + KCℓ (Ref. 7,8); and c) 1.2 A GeV ^{40}Ar + KCℓ (Ref. 8).

Table I. Comparison Between CASCADE

Predictions and Pion Interferometry Experiments

SYSTEM	k_{MIN}(MeV/c)	CASCADE R(FM)	CASCADE τ(FM/c)	CASCADE λ	EXPERIMENT R(FM)	EXPERIMENT τ(FM/c)	EXPERIMENT λ	COMMENT
1.8 A GeV Ne+NaF	150	2.84 ± .084	1.92 ± .31	.979 ± .032	$1.83^{+.8}_{-1.6}$	$2.96^{+.90}_{-1.00}$	0.59 ± .08	Zajc et al[6]
1.8 A GeV Ar+KCℓ	150	3.58 ± .11	2.83 ± .43	1.003 ± .045	$2.77^{+.6}_{-.9}$	$3.44^{+1.1}_{-1.5}$	0.63 ± .04	Zajc et al[6]
	150	3.58 ± .11	2.83 ± .43	1.003 ± .045	4.1 ± 0.4	$1.76^{+2.10}_{-1.76}$	0.73 ± .07	Zajc et al[6] (π^+DATA)
1.5 A GeV Ar+KCℓ	50	3.60 ± .12	3.19 ± .52	.939 ± .053	4.7 ± .5	$4.2^{+1.8}_{-4.2}$	1.2 ± .2	Beavis et al[7]
	150	3.24 ± .11	3.95 ± .43	.987 ± .058	4.1 ± .5		0.9 ± .2	Beavis et al[7]
1.2 A GeV Ar+KCℓ	50	3.33 ± .07	3.74 ± .35	.892 ± .039	3.8 ± .5	5.4 ± 1.8	0.74 ± .17	Beavis et al[8]

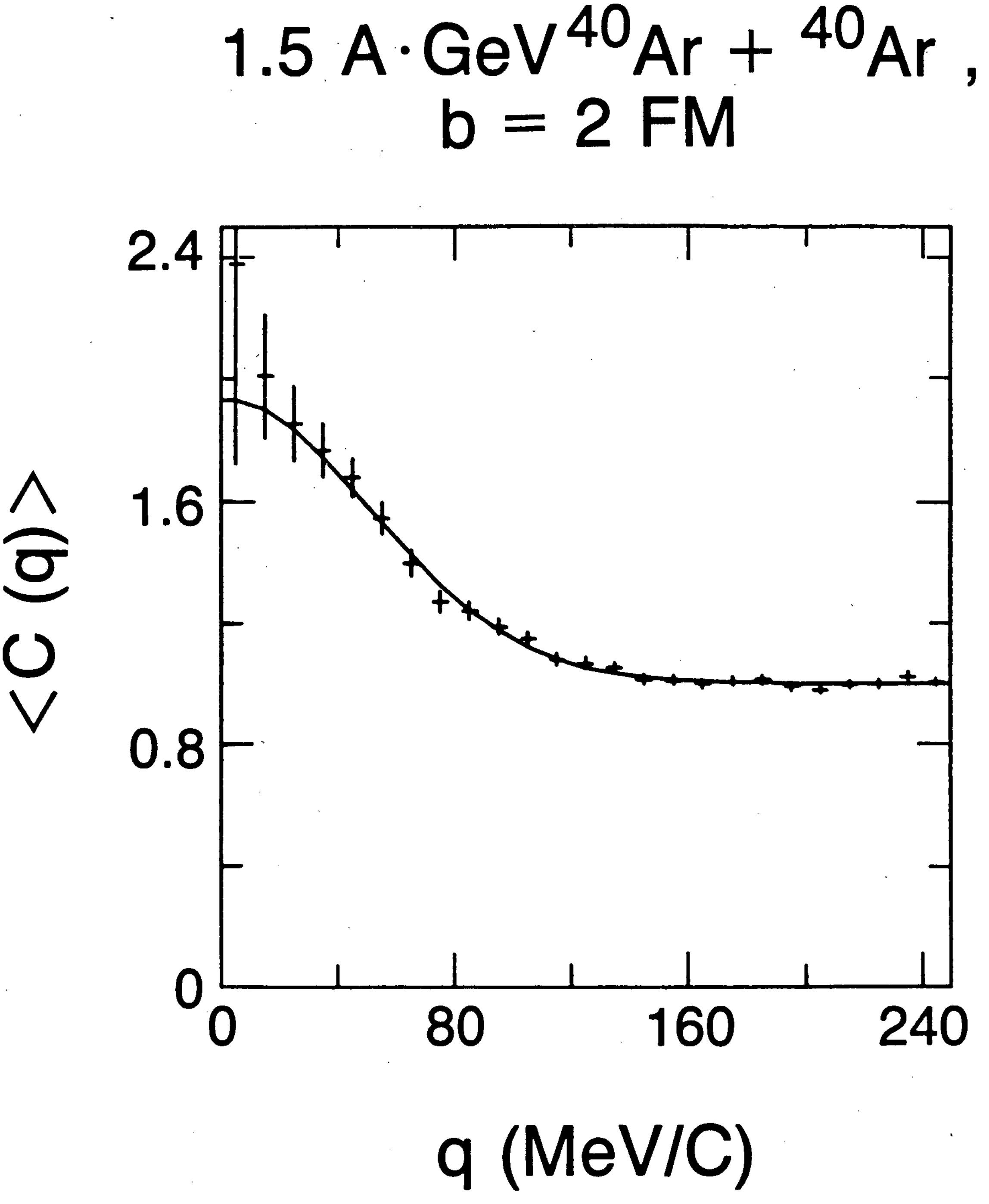

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Figure 1

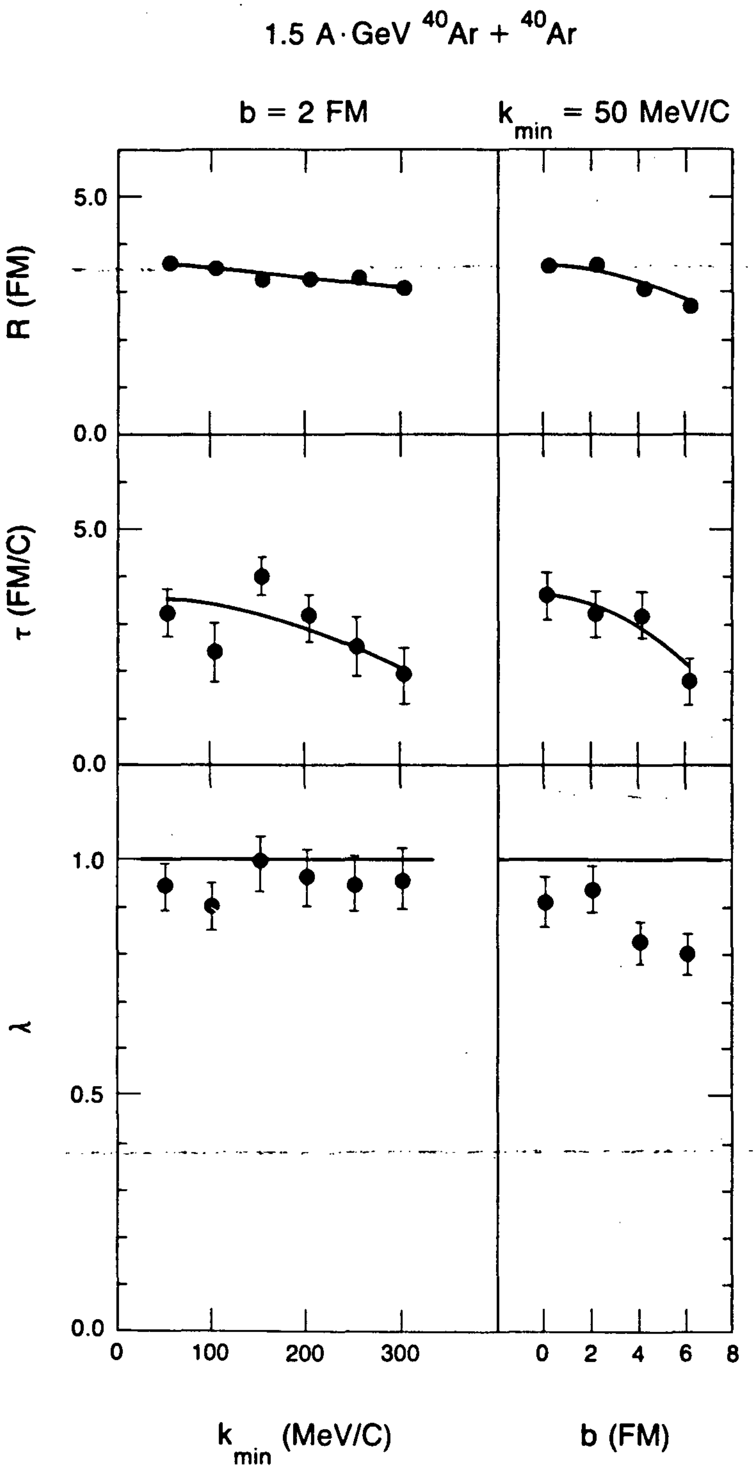

Figure 2

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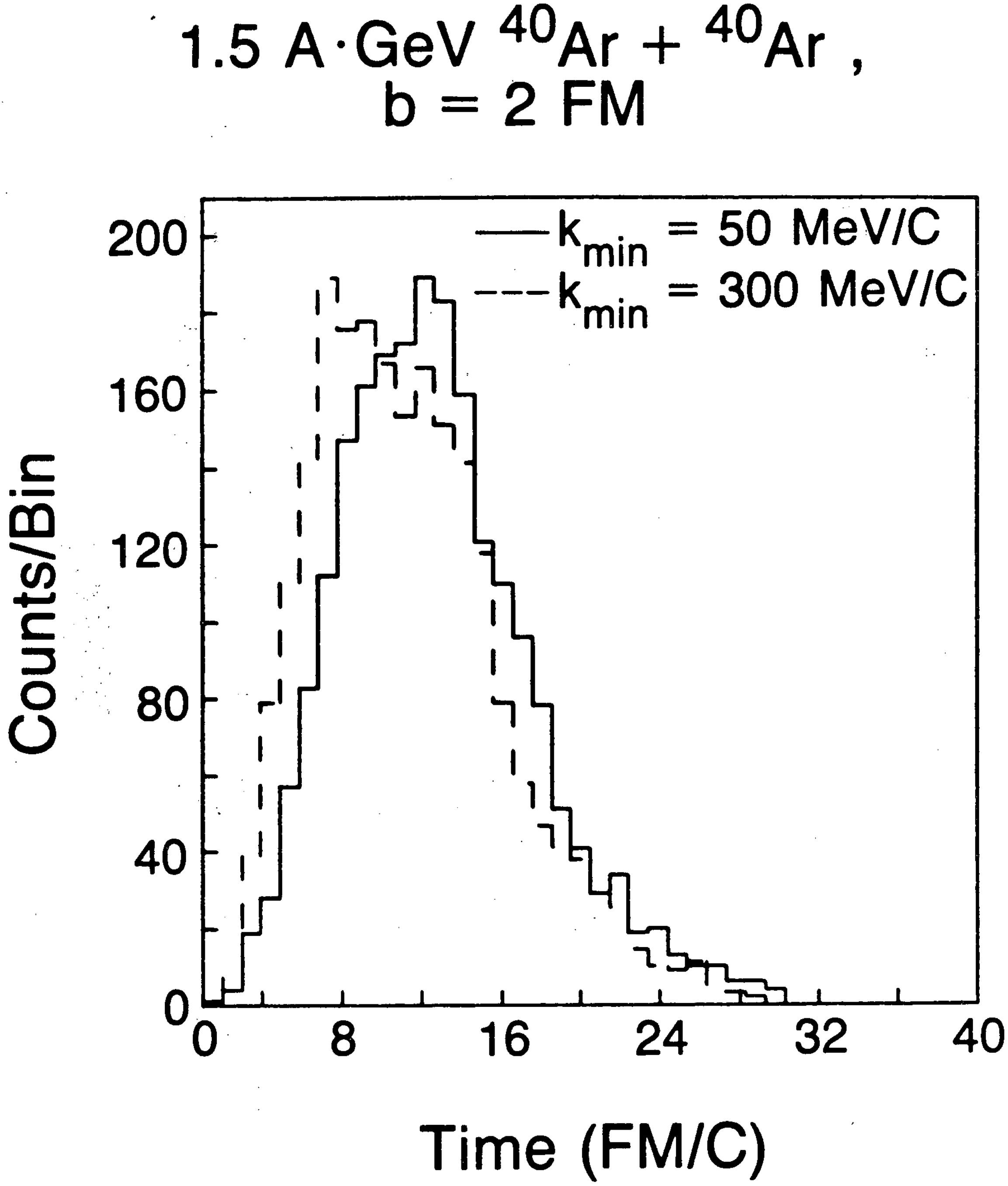

Figure 3

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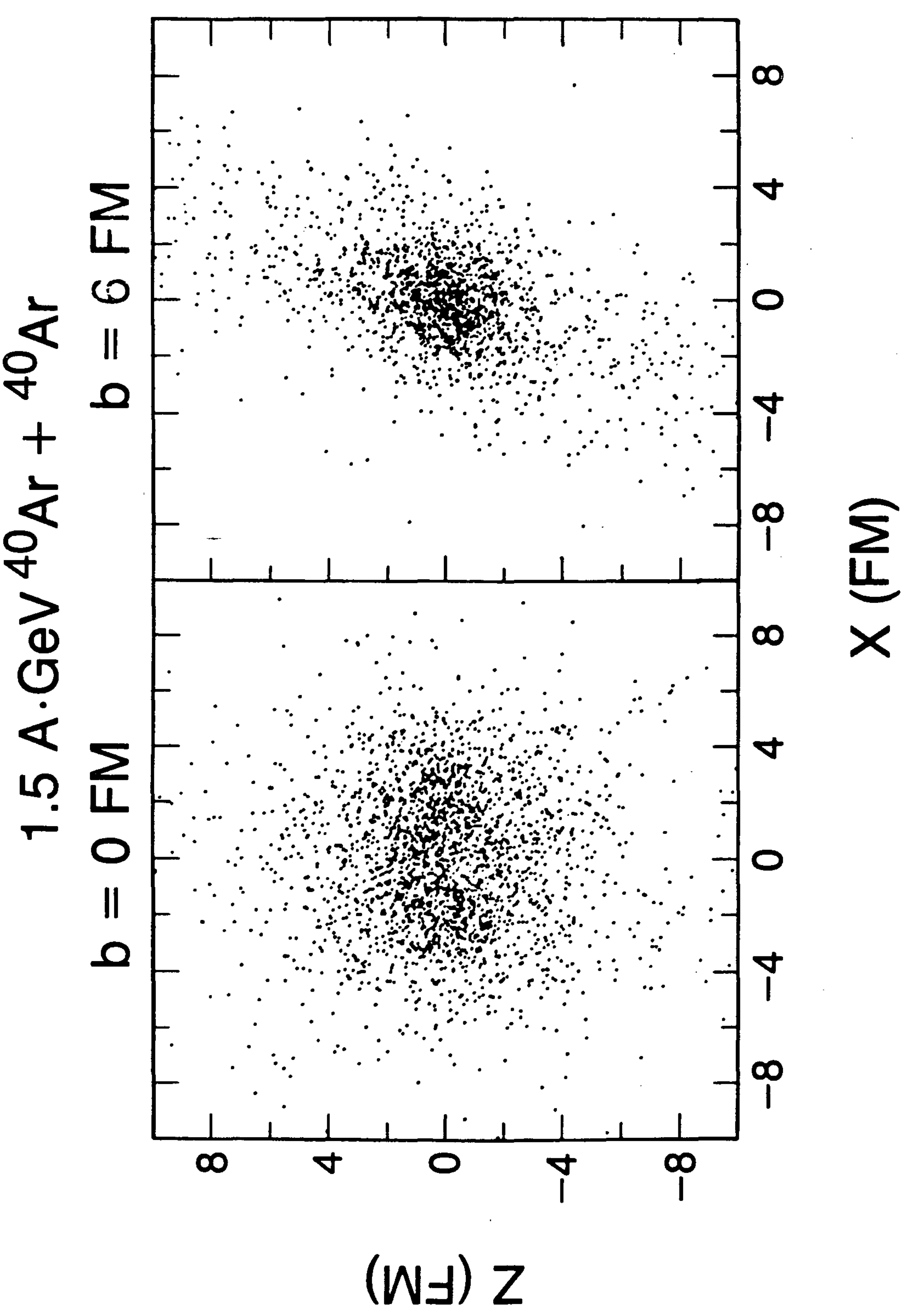

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Figure 4

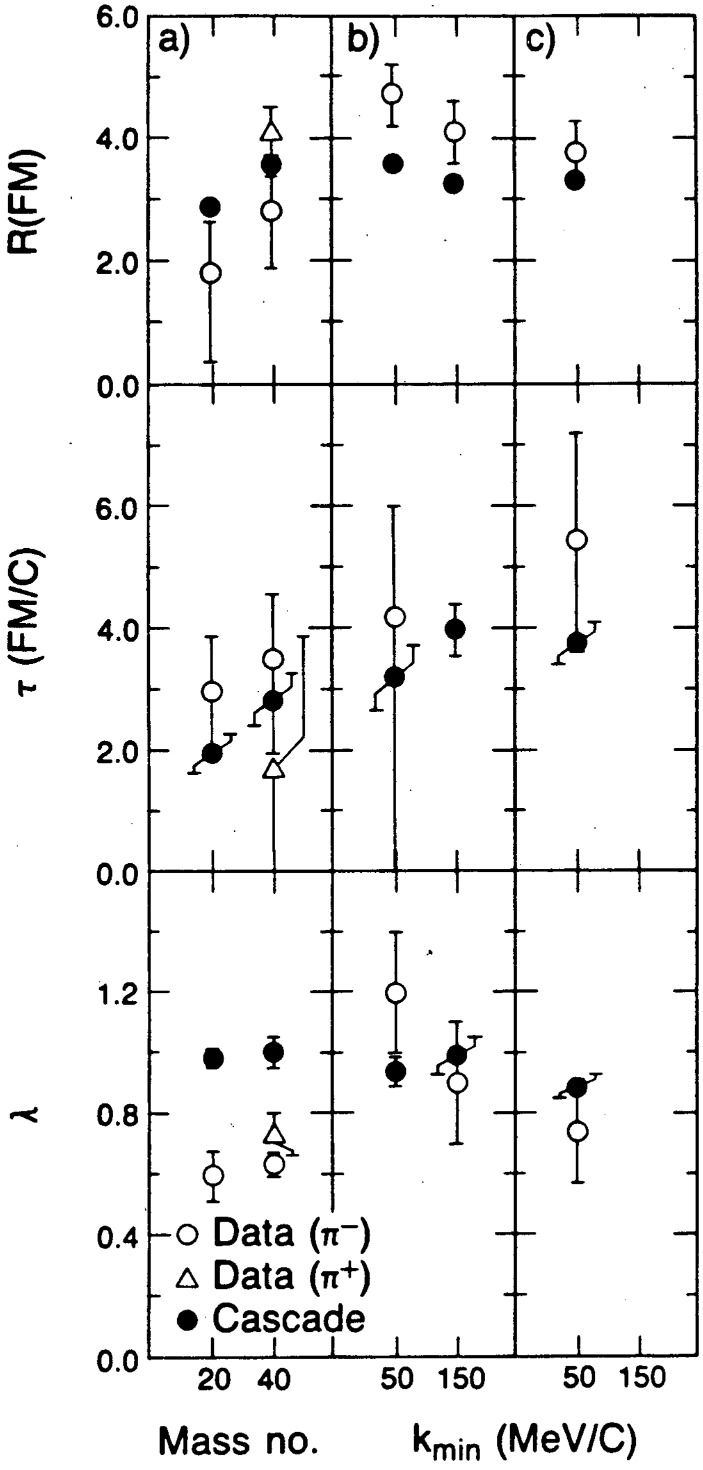

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Figure 5

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