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Affirmative Action in Hierarchies

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Abstract

There is considerable evidence that males are more prone to take risks than females. This difference has implications for rates of promotion in hierarchies where promotion is based on random signals of ability. I explore the promotion consequences of three types of performance standards: gender-blind standards, standards designed to promote agents of equal ability on average, and standards designed to promote equal numbers of both genders. These three objectives lead to different promotion standards, which highlights among other things that the goal of affirmative action is not well defined. Lower promotion standards for females can be necessary to ensure either equal abilities or equal numbers in the promoted populations.

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1. Introduction

Affirmative action remains a contentious subject in the regulation of labor markets. Policies that give preferential treatment to women or minorities can be seen as addressing inequities by creating other inequities. For the most part, economists have tried to evaluate affirmative action policies by their effects on efficiency rather than equity, especially productive efficiency. See for example, Holzer and Neumark (2000), who argue from an extensive empirical literature that "affirmative action offers significant redistribution toward women and minorities, with relatively small efficiency consequences" (page 559).

At the theoretical level, authors have argued that affirmative action policies can enhance efficiency rather that undermine it. Lundberg and Startz (1983) and Lundberg (1991) consider a model of statistical discrimination where wages depend on imperfect signals of ability, and show, among other things, that if workers with different signalling ability are pooled, there is more incentive to invest in human capital. Milgrom and Oster (1987) argue that affirmative action policies can efficiently prevent employers from underpromoting women and minorities. The incentive to underpromote derives from a fear of revealing the worth of their employees to rival firms, a threat which is higher for the more "invisible" workers, such as women and minorities.

In this paper I take a different view of both labor markets and affirmative action. I consider labor market hierarchies, in which promotion to stage t requires prior promotion to stage $t-1$. I take investments in human capital as exogenous, and assume that wages at each stage of the hierarchy are immutable. My focus is entirely on rates of promotion, and how they are affected by discrimination of various types. Examples of such hierarchies might be

- law, where law students are promoted to associates in law firms, associates are promoted to partner, and some partners eventually become judges;
- corporate life, where there is a well-defined executive hierarchy;
- and academic life where undergraduates are promoted to graduate student, graduate students are promoted to assistant professor, and assistant professors are promoted to full professor.

The behavioral premise of the model presented here, which leads to different promotion rates in hierarchies, is that males are more inclined to take risks than females. There is considerable evidence that this is so. An excellent summary can be found in Eckel and Grossman (forthcoming 2003), who report on experiments that demonstrate, for example, that males and females have different gambling behavior. The evidence is also strong from "field studies" (natural experiments such as observing behavior in placing bets), but less conclusive in "contextual environmental" experiments such as experiments involving insurance choices. One of the most interesting risk-taking contexts is investment. In a study that used measures of risk tolerance reported in the Wall Street Journal, and measures of personality traits developed by psychologists, Stanford and Vallenga (2002) found that males have much higher risk tolerance than females. Jianakoplos and Bernasek (1998) came to the same conclusion by observing investment portfolios. Much of the experimental evidence comes from disciplines other than economics. For example, psychologists Ginsburg et al (2002) observed children at a zoo in contexts where the children could choose to engage in a risky activity or not. They concluded strongly that young boys were much more inclined to put themselves at risk than young girls.

I do not wish to leave the impression that this list is exhaustive or even representative, but only to argue that it is much easier to find papers that support a gender difference in risk-taking than to find papers that reject it. Many scholars have suggested evolutionary arguments for why it might be so. Dekel and Scotchmer (1999) postulated that males play "winner-take-all" games, and explored a precise sense in which such games do (or do not) lead to riskier behavior. The premise in that paper, which is also adopted here, is that such behavior is genetically *coded.* The premise that risk-taking is genetic, rather than a rational response to incentives, or a product of "nurture" rather than "nature," seems consistent with other genetic evidence, such as the fact that males have higher variance than females on dimensions such as longevity, size, and vulnerability to disease.

In this paper, I do not try to explain why males are more risk-taking than females, but simply explore the consequences for promotion in hierarchies. Agents are promoted based on signals of ability that can be noisy. The random process is determined by their genetic coding. This is obviously an extreme and stylized assumption, but one worth exploring if there is any element of truth to it.

I explore the promotion consequences of three alternative types of performance standards: equal promotion standards for both genders, standards designed to promote agents of equal average ability, and standards designed to promote equal numbers of both genders. The intuition for the consequences of these policies are explained graphically in the next section, at least with respect to the first stage of the hierarchy. Perhaps the most important implication of this discussion is that, in such a model, "affirmative action" has no clearly defined meaning. It cannot be defined without an objective in mind, and the following objectives are pairwise inconsistent:

- equal promotion standards
- equal numbers of promotions
- promotion of a pool of agents with equal average ability.

The next section gives a graphical discussion of how the difference in risk-taking matters for promotion. This is followed in Section 3 by a more formal discussion that extends to an inifinite hierarchy. Section 4 explores alternative interpretations of riak-taking in hierarchies, and in Section 5, I point out some implications for efficiency.

2. A Graphical Discussion

Figure 1 shows the distribution of true ability a, denoted G with density g, in each of two populations, a risk-taking population (males) and a risk-averse population (females). The density $\tilde{g}$ represents the distribution of signals that the risk-taking population will generate, when their true ability a is confounded by noise. The signal of a random male will be $\sigma = a + u$, where a is his true ability, and u is distributed according to a cumulative distribution function ϕ with mean zero.

Consider the first round of a promotion hierarchy. Suppose that the promotion standard for males is c. That is, every male who generates a signal above c is promoted. The other promotion standards are for females: The promotion standard f^e will ensure that females are promoted with the same probability as males, and the promotion standard f^a will ensure that the expected ability of promoted females is the same as that of promoted males. If the promotion policy is gender blind, then females are also promoted according to the standard c.

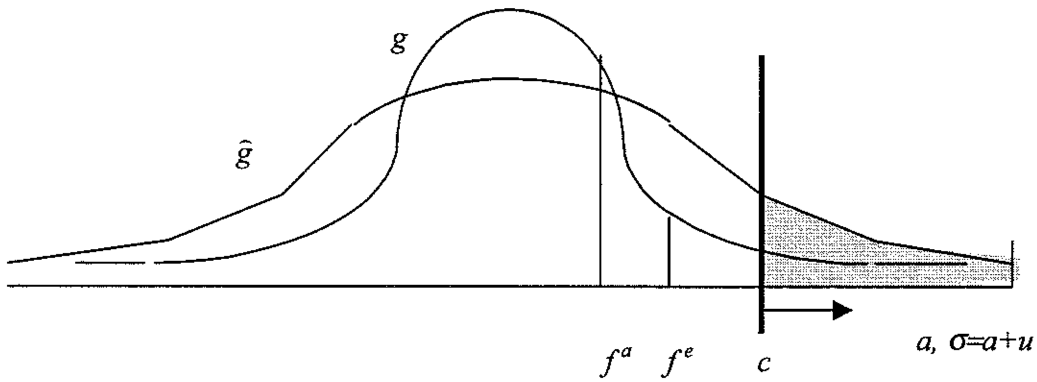

Figure 2.1:

The first thing to notice is the consequence of a gender-blind policy. If males and females are treated equally in the sense of being promoted according to the same standard c, then (provided that fewer than half are promoted)

- more males than females are promoted; and
- the females have higher ability on average.

The latter is for two reasons: more men than women are promoted, and some of them are mistakes.

The gender-blind policy is clearly inhospitable to females at the first stage, however reasonable it may seem from a procedural point of view. Consider instead an "affirmative action" policy to promote equal numbers of males and females. Then the promotion standard for females must clearly be lower than for males, in particular, f^e. Even so,

- under an affirmative action program to promote equal numbers of males and females, promoted females will on average have higher ability than promoted males.

Is this "fair?" An affirmative action policy aimed at equal numbers is still inhospitable to females in the sense that, on average, promoted females have higher ability than promoted males. Their superior ability is due to the fact that, in promoting males, mistakes are made in both directions. Low-ability males are promoted, and high-ability males are excluded. Females could reasonably argue that the system should impose an even *lower* bar for females, in order to remedy the discrepancy in average (and marginal) ability.

Consider then an affirmative action policy aimed at ensuring equal *ability* of both promoted groups, instead of equal *numbers*. Then

- under an affirmative action goal of promoting females with the same expected ability as males, fewer males than females will be promoted; and
- the standard for female promotion should be lower than for males, and even lower than the one than ensures equal numbers.

The much lower promotion standard for females is a bit paradoxical: it appears to favor females of lower ability than males, but in fact the females have higher ability on average. A higher standard must be applied to males in order to compensate for the mistakes.

The graphical discussion only illuminates the first stage of promotion. At the second stage, the pools of surviving males and females are different. High-ability males have been eliminated due to randomness, and low-ability males remain. How many agents get promoted at the second stage depends again on the objective. Consider, for example, the gender-blind policy of a common standard. At stage two, the males may still have an advantage due to the new draw of noise that will boost some of them above the bar. However, at stage two, there is a countervailing effect. The boost due to noise must be strong enough to overcome the higher ability of the remaining females. At some point in the hierarchy, ability will dominate noise, and males will no longer be promoted in higher numbers. Fewer and fewer males are promoted, but in yet another switch-around, at much later stages of the hierarchy, the only males that remain are those with very high ability who survived their many opportunities to be eliminated.

These issues are considered formally below.

3. The Hierarchy

Let G be a distribution from which each agent's ability, denoted $a \in \mathbf{R}$, is drawn independently. Index the agents by $i = 1, \ldots$. An agent i generates a signal of ability $\sigma_t^i \in \mathbf{R}$ in period t. If the agent i is female, we assume that $\sigma_t^i = a^i$ (the signal is nonrandom). If the agent i is a male, $\sigma_t^i = a^i + u_t^i$, where the random noise u_t^i is distributed according to a cumulative distribution function ϕ with mean zero and positive variance, and the random draws of noise in different stages of the hierarchy are independent. The designations "male" and "female" refer to the riskiness of the signals that are produced. This analysis would obviously apply to any two groups that differ in the randomness of their signals. In that sense, the designations male and female are only illustrative, and can even be reversed (see below).

Promotion standards are sequences $\{m_t\}_{t=1\ldots}, \{f_t\}_{t=1\ldots}$. A male agent i *survives* to stage t if $\sigma_d^i \geq m_d$ for each $d \leq t$, and a female agent i *survives* to stage t if $a^i \geq f_d$ for each $d \leq t$. We say that the promotion standards are *gender-blind* if there is a sequence $\{c_t\}$ such that $m_t = f_t = c_t$ for each t.

For females, we can assume without loss of generality that the promotion standards are nondecreasing. If at any point a higher cutoff is followed by a lower cutoff, that is, $f_{t+1} < f_t$, then f_{t+1} can be replaced by f_t with no consequence. All the agents with ability between f_{t+1} and f_t have in any case been eliminated at stage t. We will thus assume that $\{f_t\}$ is nondecreasing. Then a female survives to stage t if $a \geq f_t$ and does not survive otherwise. Hence the probability that a random female survives to stage t is

$$\int_{f_t}^{\infty} g(a)da \tag{3.1}$$

A male with ability a survives to stage t if $a + u_d > m_d$ for all $d \leq t$. Hence the probability that a random male survives to stage t is

$$\int_{-\infty}^{\infty} g(a)\Pi_{d=1}^{t}(1 - \phi(m_d - a))da \tag{3.2}$$

The expected ability of a random female who survives to stage t is

$$\int_{f_t}^{\infty} a \; \frac{g(a)}{\int_{f_t}^{\infty} g(a)da} da = \int_{f_t}^{\infty} a \; \frac{g(a)}{1 - G(f_t)} da \tag{3.3}$$

and the expected ability of a random male who survives to stage t is

$$\int_{-\infty}^{\infty} a \; \frac{g(a)\Pi_{d=1}^{t}(1 - \phi(m_d - a))}{\int_{-\infty}^{\infty} g(a)\Pi_{d=1}^{t}(1 - \phi(m_d - a))da} da \tag{3.4}$$

We use the following assumptions, which are assumed throughout.

1. The distribution G is symmetric,[1] strictly increasing, has a density g that is strictly quasiconcave and continuous, and has the real line as support.

2. The distribution ϕ is symmetric and strictly increasing with zero mean and support the real line.

[1] For all x in the support, $G(x) = 1 - G(-x)$.

We begin with two lemmas. The intuition for the first lemma is that the promoted males include mistakes in both directions. Lower-ability males are promoted by mistake, and higher-ability males are excluded by mistake. Since no mistakes are made in promoting females, the only way to ensure that promoted males have as high ability as females is to promote fewer of them. At the first stage, promoting fewer of them will require that females have a lower promotion standard. At later stages, after males have been eliminated in previous promotion stages, a lower promotion standard for males can still be consistent with fewer promotions or higher ability.

Lemma 3.1. *Let $\{m_t\}$,$\{f_t\}$ be the promotion standards. The expected ability of a random surviving male is lower than the expected ability of a random surviving female at any stage t at which males have at least as high a probability of survival.*

Proof: With a change of variables, $y = a - f_t$, the females' expected ability conditional on survival to t, (3.3), can be written:

$$\int_0^{\infty} (f_t + y) \frac{g(f_t + y)}{\int_0^{\infty} g(f_t + y)\, dy} dy = f_t + \int_0^{\infty} y \frac{g(f_t + y)}{\int_0^{\infty} g(f_t + y)\, dy} dy \tag{3.5}$$

For males, with a change of variables $y = a - f_t$, the expected ability conditional on survival to t, (3.4), can be written:

$$\begin{aligned}&\int_{-\infty}^{\infty} (f_t + y) \frac{g(f_t + y)\Pi_{d=1}^{t}(1 - \phi(m_d - f_t - y))}{\int_{-\infty}^{\infty} g(f_t + y)\Pi_{d=1}^{t}(1 - \phi(m_d - f_t - y))dy} dy \\ &= f_t + \int_{-\infty}^{\infty} y \frac{g(f_t + y)\Pi_{d=1}^{t}(1 - \phi(m_d - f_t - y))}{\int_{-\infty}^{\infty} g(f_t + y)\Pi_{d=1}^{t}(1 - \phi(m_d - f_t - y))dy} dy \end{aligned} \tag{3.6}$$

It holds that (3.5) is greater than (3.6) if the following inequality holds for $y \geq 0$:

$$\frac{g(f_t + y)\Pi_{d=1}^{t}(1 - \phi(m_d - f_t - y))}{\int_{-\infty}^{\infty} g(f_t + y)\Pi_{d=1}^{t}(1 - \phi(m_d - f_t - y))dy} < \frac{g(f_t + y)}{\int_0^{\infty} g(f_t + y)\, dy}$$

Since $g(f_t+y)\Pi_{d=1}^{t}(1-\phi(m_d-f_t-y)) \leq g(f_t+y)$, (3.5) is greater than (3.6) if the denominator of the lefthand side is no smaller than the denominator of the righthand side. The denominators are the probabilities that a male or female survives, respectively. $\square$

In the next lemma, the first part reflects the fact that, regardless of the promotion standards, each male has positive probability of being eliminated at each stage. Since excluded agents cannot re-enter the pool, only few males will survive in the long run.

The second part reflects the fact that, regardless of the promotion standards, only the males with very high ability will survive many opportunities to be eliminated. Thus, in the "long run", it does not matter very much what the promotion standards are, as long as there is a possibility to be eliminated at each stage. Males that survive will likely have very high ability. In contrast, a female will survive with probability one if her ability is above the maximum promotion standard. This means that more females survive in the long run, even without extraordinary ability.

Lemma 3.2. *Let $\{m_t\}$,$\{f_t\}$ be promotion standards that are bounded above and below. Then*
(1) Given $\varepsilon > 0$, there exists $\tilde{t}$ such that for $t > \tilde{t}$, the probability that a male survives to stage t is less than ε; and
(2) There exists $\hat{t}$ such that for $t > \hat{t}$, the expected ability of a surviving male is larger than the expected ability of a surviving female.

Proof: Let $\underline{m} = \inf\{m_t\}$, $\bar{m} = \sup\{m_t\}$, $\underline{f} = \inf\{f_t\}$, $\bar{f} = \sup\{f_t\}$.
(1) Let $\varepsilon > 0$. Let $\tilde{a} > 0$ satisfy $0 < 1 - G(\tilde{a}) < \varepsilon/2$ and let $\tilde{t}$ satisfy

$\phi(a - \underline{m})^{\tilde{t}} < \varepsilon/2$ for all $a \leq \tilde{a}$. Then for $t \geq \tilde{t}$,

$$\begin{aligned}
& \int_{-\infty}^{\infty} g(a)\Pi_{d=1}^{t}(1 - \phi(m_d - a))da \\
= & \int_{-\infty}^{\tilde{a}} g(a)\Pi_{d=1}^{t}(1 - \phi(m_d - a))da + \int_{\tilde{a}}^{\infty} g(a)\Pi_{d=1}^{t}(1 - \phi(m_d - a))da \\
\leq & \int_{-\infty}^{\tilde{a}} g(a)\phi(a - \underline{m})^t da + \int_{\tilde{a}}^{\infty} g(a)\Pi_{d=1}^{t}(1 - \phi(m_d - a))da \\
< & \ G(\tilde{a})\varepsilon/2 + (1 - G(\tilde{a}) < \varepsilon
\end{aligned}$$

(2) Let $\bar{a}^f$ be an upper bound on the expected ability (3.3) of surviving females at each stage:

$$\bar{a}^f = \int_{\bar{f}}^{\infty} a \, \frac{g(a)}{1 - G(\bar{f})} da$$

Let $1 > \delta > 0$. Let $\tilde{a} > 0$ satisfiy $-\tilde{a} - \underline{m} < \tilde{a} - \bar{m}$ and $\frac{\bar{a}^f}{1-\delta} < \tilde{a}$. Let $\hat{a}$ satisfy $\tilde{a} - \underline{m} < \hat{a} - \bar{m}$ Let $\hat{t}$ be such that for $t > \hat{t}$

$$\frac{\bar{a}^f}{1-\delta} \, G(\tilde{a}) \, \phi(\tilde{a} - \underline{m})^t < (\tilde{a} - \frac{\bar{a}^f}{1-\delta}) \, (1 - G(\hat{a})) \, \phi(\hat{a} - \bar{m})^t$$

$$\text{and} \qquad \left(\frac{\phi(-\tilde{a} - \underline{m})}{\phi(\tilde{a} - \bar{m})} \right)^t < \delta$$

To give a lower bound on the expected ability (3.4) of surviving males we will use the following inequality:

$$\begin{aligned}
& [1 - \left(\frac{\phi(-\tilde{a} - \underline{m})}{\phi(\tilde{a} - \bar{m})} \right)^t] \ \tilde{a} \int_{\tilde{a}}^{\infty} g(a)\Pi_{d=1}^{t}\phi(a - m_d)da \\
< & \int_{\tilde{a}}^{\infty} ag(a) \ \ \Pi_{d=1}^{t}\phi(a - m_d) \ \ [1 - \frac{\Pi_{d=1}^{t}\phi(-a - m_d)}{\Pi_{d=1}^{t}\phi(a - m_d)}] \ \ da \\
< & \int_{0}^{\infty} ag(a) \ \ [\Pi_{d=1}^{t}\phi(a - m_d) - \Pi_{d=1}^{t}\phi(-a - m_d)] \ \ da \\
= & \int_{0}^{\infty} ag(a) \ \Pi_{d=1}^{t}\phi(a - m_d)da + \int_{0}^{\infty} (-a)g(a)\Pi_{d=1}^{t}\phi(-a - m_d) \ da \\
= & \int_{-\infty}^{\infty} ag(a) \ \ \Pi_{d=1}^{t}\phi(a - m_d) \ \ da
\end{aligned} \qquad (3.7)$$

Then

$$\begin{aligned}
\frac{\bar{a}^f}{1-\delta}\int_{-\infty}^{\tilde{a}} g(a)\Pi_{d=1}^t\phi(a-m_d)da &< \frac{\bar{a}^f}{1-\delta}\, G(\tilde{a})\ \phi(\tilde{a}-\underline{m})^t < \\
(\tilde{a}-\frac{\bar{a}^f}{1-\delta})\ (1-G(\hat{a}))\phi(\hat{a}-\bar{m})^t &\leq (\tilde{a}-\frac{\bar{a}^f}{1-\delta})\int_{\hat{a}}^{\infty} g(a)\Pi_{d=1}^t\phi(a-m_d)da \\
&< (\tilde{a}-\frac{\bar{a}^f}{1-\delta})\int_{\tilde{a}}^{\infty} g(a)\Pi_{d=1}^t\phi(a-m_d)da
\end{aligned}$$

which implies

$$\frac{\bar{a}^f}{1-\delta}\int_{-\infty}^{\infty} g(a)\Pi_{d=1}^t\phi(a-m_d)da < \tilde{a}\int_{\tilde{a}}^{\infty} g(a)\Pi_{d=1}^t\phi(a-m_d)da$$

Hence, combining with (3.7):

$$\begin{aligned}
\frac{\bar{a}^f}{1-\delta}\int_{-\infty}^{\infty} g(a)\Pi_{d=1}^t\phi(a-m_d)da &< \tilde{a}\int_{\tilde{a}}^{\infty} g(a)\Pi_{d=1}^t\phi(a-m_d)da \\
&< \frac{1}{(1-\left(\frac{\phi(-\tilde{a}-\underline{m})}{\phi(\tilde{a}-\bar{m})}\right)^t)}\int_{-\infty}^{\infty} ag(a)\Pi_{d=1}^t\phi(a-m_d)da
\end{aligned}$$

Since $1 < (1-\left(\frac{\phi(-\tilde{a}-\underline{m})}{\phi(\tilde{a}-\bar{m})}\right)^t)/(1-\delta)$, the result follows:

$$\bar{a}^f < \frac{(1-\left(\frac{\phi(-\tilde{a}-\underline{m})}{\phi(\tilde{a}-\bar{m})}\right)^t)}{1-\delta}\bar{a}^f < \int_{-\infty}^{\infty} a\frac{g(a)\Pi_{d=1}^t\phi(a-m_d)}{\int_{-\infty}^{\infty} g(a)\Pi_{d=1}^t\phi(a-m_d)da}da$$

For $t > \hat{t}$, female ability (3.3) is less than male ability (3.4). □

I use these lemmas to characterize the consequences of gender-blind promotion standards.

Proposition 3.3. *(Gender Blind Promotions) Suppose that the promotion standards are gender blind and that $c_1 > E_G(a)$, $G(c_t) < 1$ for all t. Then*
(1) At the first stage, if $c_1 > E_G(a)$, a random male has a higher probability of

survival than a random female, and a random surviving female will have higher expected ability than a random surviving male.
(2) At later stages, $t > \tilde{t}$ for some appropriate $\tilde{t}$, the probability that a random male survives is smaller than the probability a random female survives, but the expected ability of surviving males is larger than the expected ability of surviving females.

Proof: (1) At stage 1, the probability (3.2) that a male survives can be written as follows with a change of variables $x = a - c_1$, and using symmetry of ϕ :

$$\int_{-\infty}^{\infty} g(a)(1-\phi(c_1-a))da = \int_{-\infty}^{\infty} g(c_1+x)\phi(x)dx$$

$$\begin{aligned}
&= \int_{-\infty}^{0} g(c_1+x)\phi(x)dx + \int_{0}^{\infty} g(c_1+x)\phi(x)dx \\
&= \int_{0}^{\infty} g(c_1-x)\phi(-x)dx + \int_{0}^{\infty} g(c_1+x)(1-\phi(-x))dx \\
&= \int_{0}^{\infty} [g(c_1-x) - g(c_1+x)\phi(-x)dx + \int_{0}^{\infty} g(c_1+x) \;\; dx \\
&> \int_{c_1}^{\infty} g(a) \;\; da
\end{aligned}$$

The inequality holds because $\int_0^\infty [g(c_1-x) - g(c_1+x)\phi(-x)dx > 0$ due to the strict quasiconcavity and symmetry of g and $c_1 > E_g(a)$. Hence (3.2) is larger than (3.1) at $t = 1$. Using Lemma 3.1, the expected ability of a surviving male is lower than the expected ability of a surviving female.

(2) follows directly from Lemma 3.2 by choosing $\varepsilon > 0$ such that $(1-G(\underline{f})) > \varepsilon$.
□

We now turn to alternative policy goals. We first consider the goal of equalizing the probabilities of promotion at each stage, and then consider the goal of equalizing the average ability of the survivors at each stage.

It follows directly from Lemma 3.2(1) that bounded sequences $\{m_t\}$,$\{f_t\}$ cannot have the property that males and females have the same probability of promotion at all stages. Part (2) of the following proposition points out that it is impossible to equalize promotion rates with a nondecreasing sequence of promotion standards for males, and in fact, the sequence cannot be bounded below. A nondecreasing sequence of promotion standards would be the natural interpretation of a promotion hierarchy. In order to promote equal numbers of males and females, females must be favored at early stages of the hierarchy, and males must be favored at later stages of the hierarchy, in terms of the promotion standard.

Proposition 3.4. *(Promoting Equal Numbers) Let $\{m_t\}$,$\{f_t\}$ be promotion standards such that males and females have the same probability of promotion at each stage t.*
(1) If $f_1, m_1 > E_G(a)$, then $f_1 < m_1$ (the promotion standard for females is lower than for males at stage 1).
(2) If the sequence $\{f_t\}$ converges to a finite limit, then the sequence $\{m_t\}$ is not bounded below.

Proof: (1) follows from Proposition 3.3(1), which implies that if $m_1 = f_1$, males have a higher probability of survival than females. Since the probability of survival is decreasing in m_1, the probabilities can only be equal if $m_1 > f_1$.

(2) Since $\{f_t\}$ converges, the sequence of female survival rates $\{1-G(f_t)\}_{t=1,...}$ also converges, and the sequence of male survival rates $\{\int_{-\infty}^{\infty} g(a)\Pi_{d=1}^{t}(1-\phi(m_d - a))da\}_{t=1,...}$ converges to the same limit, say L. Choose an $\varepsilon > 0$ such that $\varepsilon < L$. Suppose, contrary to the proposition, that $\{m_t\}$ is bounded below by $\underline{m}$. The ,male survival rate at stage t satisfies

$$\int_{-\infty}^{\infty} g(a)\Pi_{d=1}^{t}(1-\phi(m_d - a))da$$

$$\leq \int_{-\infty}^{\infty} g(a)(1-\phi(\underline{m}-a))^t da \tag{3.8}$$

Choose $\tilde{a}, \hat{a}$ such that $\hat{a} < \tilde{a}$ and

$$\begin{aligned} 1 - G(\tilde{a}) &< \varepsilon/3 \\ G(\hat{a}) &< \varepsilon/3 \end{aligned}$$

Choose $\hat{t}$ such that $(1-\phi(\underline{m}-\tilde{a}))^{\hat{t}} < \varepsilon/3$. Then if $t > \hat{t}$, the upper bound on the male survival rate at stage t, (3.8), can be written

$$\int_{-\infty}^{\hat{a}} g(a)(1-\phi(\underline{m}-a))^t da + \int_{\hat{a}}^{\tilde{a}} g(a)(1-\phi(\underline{m}-a))^t da + \int_{\tilde{a}}^{\infty} g(a)(1-\phi(\underline{m}-a))^t da$$

$$\begin{aligned} &< \int_{-\infty}^{\hat{a}} g(a)da + [G(\tilde{a}) - G(\hat{a})](1-\phi(\underline{m}-\tilde{a}))^t + \int_{\tilde{a}}^{\infty} g(a)da \\ &< \varepsilon/3 + (1-\phi(\underline{m}-\tilde{a}))^t + \varepsilon/3 < \varepsilon < L \end{aligned}$$

□

Proposition 3.5. *(Promoting Equal Average Ability) (1) Suppose that the expected abilities of surviving males and females are the same at stage t under the promotion standards $\{m_t\}$,$\{f_t\}$. Then the survival rate of females at stage t must be greater than that of males. (2) There are no bounded sequences of promotion standards $\{m_t\}$,$\{f_t\}$ for which promoted males have the same average ability as promoted females at each t.*

Proof: The probability densities of females' and males abilities, conditional on surviving to stage t, are respectively

$$\frac{g(a)}{1-G(f_t)} \tag{3.9}$$

$$\frac{g(a)\Pi_{d=1}^{t}(1-\phi(m_d-a))}{\int_{-\infty}^{\infty} g(a)\Pi_{d=1}^{t}(1-\phi(m_d-a))da} \tag{3.10}$$

(1) Suppose to the contrary that (3.2) is at least as great as (3.1). Thus the denominator of (3.10) is at least as great as the denominator of (3.9). Since $\Pi_{d=1}^{t}(1-\phi(m_d-a)) < 1$ at each t, it follows that the density (3.10) is smaller than the female density (3.9) at each $a \in (f_t, \infty)$. The remaining density for males is on abilities lower than the minimum ability for females, f_t. Hence the expected ability for females is higher than that for males, a contradiction.

(2) Lemma 3.2(2) shows that, for any bounded sequences, the average ability of surviving males is higher than the average ability of surviving females for late stages of the hierarchy (large t). □

4. Interpretations

Some of these conclusions can be noticed empirically and others cannot. At most we can observe promotion rules, signals, and proportions promoted, but we cannot in general observe true abilities.

Of course there is the additional problem of identifying hierarchies that have adhered to a particular promotion policy despite the legal and political challenges of the past several decades. It is also hard to identify hierarchies where the same proportions of women and men have wanted to stay in the pool. Instead, women and men drop out at different rates for self-motivated reasons such as child bearing. Nevertheless, I point out two conclusions that would be empirically consistent with this model if data were available:

1. Under a gender-blind promotion policy, the ratio of surviving females to surviving males at early stages of the hierarchy should be smaller than in the original population, but should be larger at later stages of the hierarchy.

The proportion of females that survive in the limit should exceed their proportion in the original population.

2. Under an equal-abilities promotion policy, the ratio of surviving females to surviving males should be increasing with time, and should be greater at every t than in the original population.

The hypotheses that males generate riskier signals than females, and that the two groups start from identical distributions of ability, can both be challenged. It is thus worth commenting on how this model changes under alternative hypotheses.

First, instead of assuming that males and females have the same distribution of abilities, assume that males have the same mean ability as females, but greater variance. This is also a "riskiness" hypothesis, but one that characterizes the populations rather than behavior. The model can be thought of as one in which males get a single draw of random noise, which persists throughout their working lives. Or, instead of being independent, the draws of random noise in successive periods are perfectly correlated.

With independent draws of random noise, a promoted male is always in jeopardy of being excluded by a subsequent draw, and that is why the survival rate of males is smaller than that of females in the long run. With perfectly correlated noise, the promoted male has no such fear. Like females, he can only drop out at a subsequent stage if the promotion standard is raised. As a consequence, the initial advantage described by Proposition 3.3(1) for gender-blind standards will persist, and there will always be disproportionately many males in the pool, with lower average ability than females.

This discrepancy could be remedied with a sequence of standards $\{f_t\}, \{m_t\}$

that favor females, $f_t < m_t$ for all t. If the higher signal generated by males is interpreted as persistent noise, then such a program of affirmative action would have the dual benefits of increasing the promotion of women and increasing the average ability of people who are promoted. However if the higher signal generated by males is due to the fact that males have higher variance in ability, and signals accurately reflect ability, then the policy of affirmative action would reduce the average ability of people who are promoted.

The second alternative interpretation reverses the hypothesis about which group generates risky signals. Again assume that the distributions of abilities are the same, but instead of assuming that males generate risky signals, assume that females generate risky signals. An explanation for this reversal might lie in a variant on the Milgrom and Oster (1987) "invisibility" hypothesis: Neither the ability of males nor of females is observable, but males generate more evidence about their true ability than females. Thus when an observer views the signal at any stage of the hierarchy, interpreted as some type of mean performance, he believes that he is observing a random variable which is an unbiased estimator of the mean, but has higher variance for females than males. For reasons that we will leave aside here, males may generate more evidence in each hierarchical stage than females. Their abilities may be fully observable, whereas the abilities of females are observable with noise.

If the hypothesis on riskiness of signals is reversed, then the interpretation of the above propositions is reversed. Instead of being disfavored at the early stages of the gender-blind hierarchy and favored in later stages, females are favored in early stages and disfavored in later stages. In fact, Proposition 3.3(2) can then be interpreted as the formalization of a 1970's slogan: Women have to be "twice as good to get half as far."

5. Efficiency

The analysis above has been positive and not normative. I have described the paths of promotions that would follow from various promotion standards. Of course the motive behind affirmative action is a normative one, namely, to redress the apparent inequity of promoting more males than females. We now turn to whether there is an "efficiency/equity" tradeoff.

Efficiency is hard to define in a partial model of a labor market such as this. In fact, since affirmative action has many faces, its efficiency effects are hard to identify in general, as discussed by Holzer and Neumark (2000). I will think of efficiency as being served by the promotion of the most able agents.

If the males' signals were so random that the truth was mostly obscured, it would probably be better to promote only females, for whom the ability is more observable. This wisdom is particularly compelling if the number of agents required at the next level of promotion is small relative to the pool, so that ability is not compromised by promoting enough females to fill the slots. The main prescription in this regard is given by Proposition 3.5, which points out that, if equal abilities are desired in the promoted pool, more females than males must survive at every stage. At early stages of the hierarchy, this should be accomplished by giving females an affirmative-action boost (Proposition 3.3(1)), and at later stages of the hierarchy, equal abilities require that males get an affirmative-action boost (Proposition 3.3(2)).

When the initial winnowing process promotes less than half the pool – captured in the hypothesis that the promotion standard is on the downward sloping part of the density function – females will initially be disadvantaged under a gender-blind policy. However their disadvantage will be overcome at later stages. The disadvantage is self-rectifying. However, both the early-stage inequities and late-stage inequities are inefficient. A better policy would be to increase the promotion of females at the early stages, e..g, by giving them a lower promotion standard ("affirmative action"), and to increase the promotion of males at later stages, also by tinkering with the promotion standard. This remedy will not be implemented by promoting equal numbers. With equal numbers, according to Lemma 3.1, promoted males are less able than promoted females at all stages.

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