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Use of Exact Solutions of Wave Propagation Problems to Guide Implementation of Nonlinear Seismic Ground Response Analysis Procedures

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Abstract: One-dimensional nonlinear ground response analyses provide a more accurate characterization of the true nonlinear soil behavior than equivalent-linear procedures, but the application of nonlinear codes in practice has been limited, which results in part from poorly documented and unclear parameter selection and code usage protocols. In this article, exact (linear frequency-domain) solutions for body wave propagation through an elastic medium are used to establish guidelines for two issues that have long been a source of confusion for users of nonlinear codes. The first issue concerns the specification of input motion as "outcropping" (i.e., equivalent free-surface motions) versus "within" (i.e., motions occurring at depth within a site profile). When the input motion is recorded at the ground surface (e.g., at a rock site), the full outcropping (rock) motion should be used along with an elastic base having a stiffness appropriate for the underlying rock. The second issue concerns the specification of viscous damping (used in most nonlinear codes) or small-strain hysteretic damping (used by one code considered herein), either of which is needed for a stable solution at small strains. For a viscous damping formulation, critical issues include the target value of the viscous damping ratio and the frequencies for which the viscous damping produced by the model matches the target. For codes that allow the use of "full" Rayleigh damping (which has two target frequencies), the target damping ratio should be the small-strain material damping, and the target frequencies should be established through a process by which linear time domain and frequency domain solutions are matched. As a first approximation, the first-mode site frequency and five times that frequency can be used. For codes with different damping models, alternative recommendations are developed.

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Introduction

Nonlinear seismic ground response analysis is seldom used in practice by nonexpert users because parameter selection and code usage protocols are often unclear and poorly documented, the effects of parametric variability on the analysis results are unknown, and the benefits of nonlinear analyses relative to equivalent-linear analyses are often unquantified. This article presents initial results of a broad study intended to resolve these issues so as to encourage appropriate applications of one-dimensional (1D) nonlinear seismic ground response analysis codes in engineering practice. The goals of the project are to provide clear and well documented code usage protocols and to verify the codes over a wide range of strain levels.

This paper considers five leading nonlinear seismic ground response analysis codes: DEEPSOIL (Hashash and Park 2001, 2002; Park and Hashash 2004; www.uiuc.edu/~deepsoil), D-MOD_2 (Matasovic 2006), a ground response module in the OpenSees simulation platform (Ragheb 1994; Parra 1996; Yang 2000; McKenna and Fenves 2001; opensees.berkeley.edu), SUMDES (Li et al. 1992) and TESS (Pyke 2000). The study focuses on two issues related to the application of nonlinear codes that can be resolved by comparing the results of such analyses (under linear condition) to known theoretical solutions. The first issue concerns the specification of input motions as "outcropping" (i.e., equivalent free-surface motions) versus "within" (i.e., mo-

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tion occurring at depth within a site profile). The second issue concerns the specification of the damping that occurs within a soil element at small strains, which is either accomplished using viscous damping or unload-reload rules that produce nonzero small-strain hysteretic damping.

This paper begins with a brief review of frequency-domain and time-domain ground response analysis procedures. This is followed by sections describing verification studies addressing the issues of input motion specification and modeling of small-strain damping.

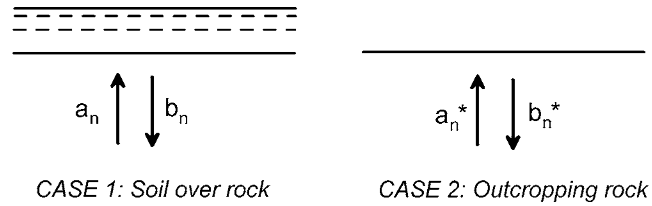

Fig. 1. Incident and reflected waves in base rock layer for case of soil overlying rock and outcropping rock (amplitudes shown are relative to unit amplitude in Case 1 surface layer)

One-Dimensional Ground Response Analysis Procedures

In 1D seismic ground response analyses, soil deposits are assumed to be horizontally layered over a uniform half-space. The incident wave is assumed to consist of vertically propagating shear waves. The response of a soil deposit to the incident motion can be modeled in the frequency or time domains, as described below.

Frequency-Domain Analysis

Frequency domain analyses are based on a closed form solution of the wave equation for shear wave propagation through a layered continuous medium, with each layer i having a specified density ρ_i, shear modulus G_i, and hysteretic damping β_i. The solution was presented by Roesset and Whitman (1969), Lysmer et al. (1971), Schnabel et al. (1972), and is also described in detail by Kramer (1996). In these frequency-domain methods, a control motion of frequency ω is specified at any layer j in the system. An exact solution of the system response can be expressed as a transfer function relating the sinusoidal displacement amplitude in any arbitrary layer i to the amplitude in layer j

$$F_{ij} = \frac{a_i(\omega) + b_i(\omega)}{a_j(\omega) + b_j(\omega)} \tag{1}$$

where F_{ij}=amplitude of transfer function between layers i and j; a_i and a_j=normalized amplitudes of upward propagating waves in layers i and j; and b_i and b_j=normalized amplitudes of downward propagating waves in layers i and j. The normalization of the wave amplitudes is generally taken relative to the amplitude in layer 1, for which $a_1 = b_1$ due to perfect wave reflection at the free surface. The normalized amplitudes a_i, a_j, b_i, and b_j can be computed from a closed-form solution of the wave equation, and depend only on profile characteristics (i.e., material properties ρ, G, and β for each layer and individual layer thicknesses).

The frequency domain solution operates by modifying, relative to the control motion, the wave amplitudes in any layer i for which results are required. These analyses are repeated across all the discrete frequencies for which a broadband control motion is sampled, using the fast Fourier transform. Once amplitudes a_i and b_i have been computed for a given layer at all those frequencies, time-domain displacement histories of layer i can be calculated by an inverse Fourier transformation.

Control motions for use in frequency domain analyses are most often recorded at the ground surface, and are referred to as "outcropping." As perfect wave reflection occurs at the ground surface, incident and reflected wave amplitudes are identical, and hence outcropping motions have double the amplitude of incident waves alone. Consider the example in Fig. 1. Rock layer n occurs at the base of a soil column in Case 1 and as outcropping rock in Case 2. In the outcropping rock case, incident and reflected waves are equivalent ($a_n^* = b_n^*$). The incident waves are identical in both cases ($a_n^* = a_n$), assuming equal rock moduli, but the reflected waves differ ($b_n^* \neq b_n$) because some of the incident wave transmits into the soil (nonperfect reflection) for Case 1, whereas perfect reflection occurs in Case 2. The motion at the base of the soil column in Case 1 (referred to as a "within" motion) can be evaluated from the outcropping motion using the transfer function

$$F_{nn^*} = \frac{u_n}{u_n^*} = \frac{a_n(\omega) + b_n(\omega)}{2a_n(\omega)} \tag{2}$$

As with any other transfer function, F_{nn^*} can be readily computed for any frequency ω and depends only on profile characteristics. Accordingly, through the use of Eq. (2), the within motion can be calculated for a given outcropping motion. The base-of-profile (within) motion can in turn be used to calculate motions at any other layer per Eq. (1).

The application of Eq. (2) results in a within motion that is reduced from an outcropping motion at the site (modal) frequencies. Consider, for example, a single soil layer with thickness= 30 m, $V_s = 300$ m/s [giving a fundamental mode site frequency of ($f_s = 300$ m/s)/(4×30 m)=2.5 Hz] overlying a half-space with shear wave velocity V_{s-H}. The results of the within/outcropping calculation (i.e., Eq. (2)) are shown in Fig. 2(a) for various values of equivalent viscous damping ratio (equal damping values are applied in both the soil layer and half-space) with $V_{s-H} = 2V_s$ and in Fig. 2(b) for zero damping and various levels of velocity contrast (V_s / V_{s-H}). As shown in Fig. 2(a and b), the transfer function amplitude (within/outcropping) drops below unity near the site frequencies, with the amplitudes at site frequencies decreasing only with decreasing amounts of equivalent viscous damping. At frequencies between the site frequencies, amplitudes decrease both with increasing damping and with decreasing velocity contrast.

At zero damping the transfer function amplitude goes to zero at site frequencies. To understand this phenomenon, consider that (1) control motion and response are in phase in this case because of the lack of damping, and (2) the site frequencies correspond to $2n+1$ quarter-wave lengths, where n=0, 1, 2, etc. (zero and positive integers). As shown in Fig. 2(c), at a depth below the surface of $2n+1$ quarter-wave lengths, the wave amplitude is zero (i.e., there is a "node" in the response at that depth), which in turn must produce a zero transfer function amplitude Fig. 2(c) shows mode shapes for the first and third modes, i.e., n=0 and 1. Additionally, as shown in Fig. 2(c) (lower right frame), as damping increases, the input and response are increasingly out of phase, and there are no true nodes in the site response.

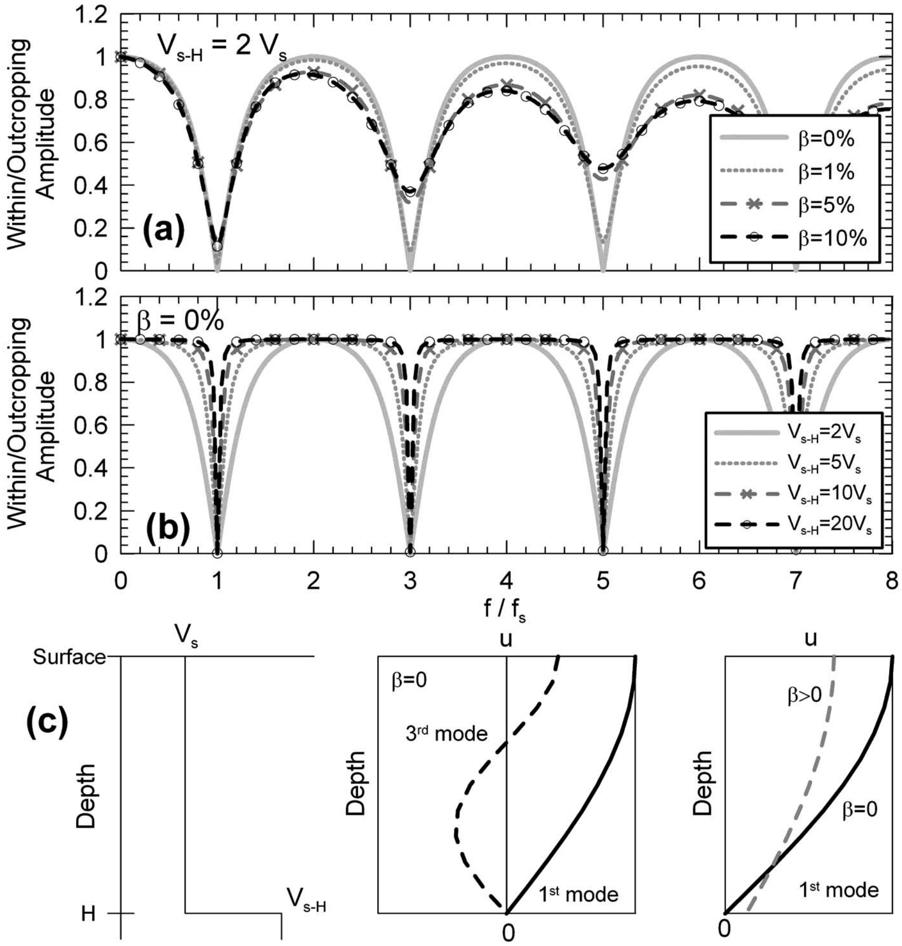

Fig. 2. Ratio of within to outcropping amplitudes for: (a) various equivalent viscous damping ratios; (b) various base layer velocities (V_{s-H}); and (c) mode shapes for various conditions

The trends shown in Figs. 2(a and b) at frequencies between the site frequencies can be explained as follows: (1) The decrease of within motion amplitude with increasing damping results from a reduction of reflected energy from the ground surface as damping increases, thus reducing the amplitude of within motions (that are the sum of incident and reflected waves); and (2) the decrease of within motion amplitude with decreasing V_{s-H} results from increased transmission of reflected (downward propagating) waves from the surface into the halfspace (i.e., less reflection), which causes energy loss from the system.

Time-Domain Analysis

The principal limitation of traditional frequency domain analysis methods is the assumption of constant soil properties (G and β) over the duration of earthquake shaking. Time-domain analysis methods allow soil properties within a given layer to change with time as the strains in that layer change. Modified frequency-domain methods have also been developed (Kausel and Assimaki 2002; Assimaki and Kausel 2002) in which soil properties in individual layers are adjusted on a frequency-to-frequency basis to account for the strong variation of shear strain amplitude with frequency. Since the frequencies present in a ground motion record vary with time, this can provide a reasonable approximation of the results that would be obtained from a truly nonlinear, time-stepping procedure. Nonetheless, the present focus is on true, time-stepping procedures.

The method of analysis employed in time-stepping procedures can, in some respects, be compared to the analysis of a structural response to input ground motion (Clough and Penzien 1993; Chopra 2000). Like a structure, the layered soil column is idealized either as a multiple degree of freedom lumped mass system [Fig. 3(a)] or a continuum discretized into finite elements with distributed mass [Fig. 3(b)]. Whereas frequency-domain methods are derived from the solution of the wave equation with specified boundary conditions, time-domain methods solve a system of coupled equations that are assembled from the equation of motion. The system is represented by a series of lumped masses or discretized into elements with appropriate boundary conditions. Table 1 summarizes the manner in which mass is distributed and nonlinear behavior is simulated for the five nonlinear codes considered here.

The system of coupled equations is discretized temporally and a time-stepping scheme such as the Newmark β method is employed to solve the system of equations and to obtain the response at each time step. TESS utilizes an explicit finite-difference solution of the wave propagation problem that is the same as the solution scheme used in FLAC developed by HCItasca. Unlike in

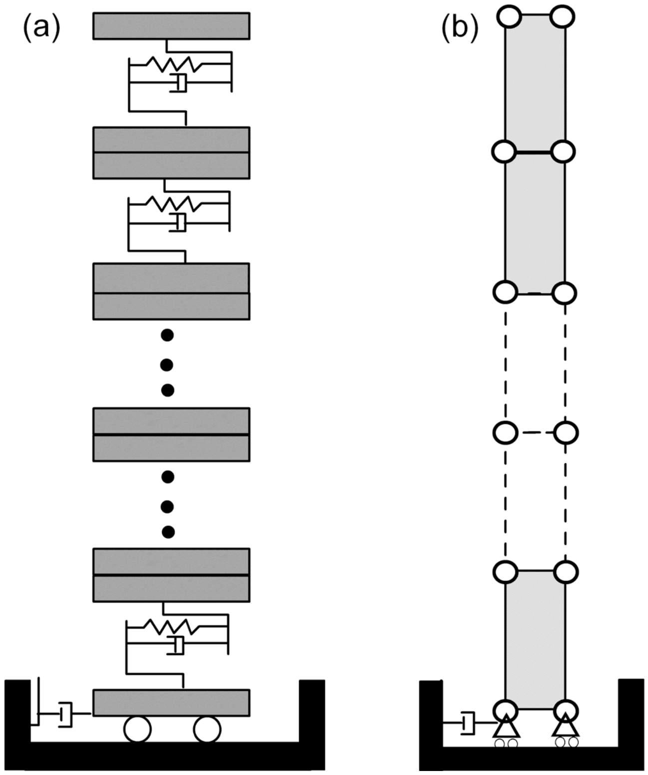

Fig. 3. (a) Lumped mass system; (b) distributed mass system

frequency-domain analysis where the control motion could be specified anywhere within the soil column, in time-domain analysis, the control motion must be specified at the bottom of the system of lumped masses or finite elements.

Specification of Input Motion

There has been confusion regarding the nature of the input motion that should be specified for time-domain analyses at the base of the profile. Consider the common case where the motion that is to be applied was recorded at the surface of a rock site (outcrop motion). One school of thought that has been applied in practice for many years is that the outcropping motion should be converted to a within motion using frequency-domain analysis [e.g., Eq. (2)], and that this within motion should then be specified for use at the base of the site profile for application in time-domain analysis. Most users of this approach were aware that the layer properties used in the outcropping-to-within conversion were a potentially crude approximation to the actual nonlinear soil properties. The approximation was accepted, however, due to the lack of a practical alternative for obtaining within motions. The second school of thought is that the outcropping rock motion should be applied directly at the base of the site profile without modification. Normally, this direct use of the outcropping motion is accompanied by the use of a compliant base in the site profile (the base stiffness being compatible with the character of the underlying rock), which allows some of the energy in the vibrating soil deposit to radiate down into the halfspace (Joyner and Chen 1975). Rigid base options are also available in all time-domain codes, but are seldom used because the conditions under which the rigid base should be applied are poorly understood.

Table 1. Mass Representation and Constitutive Models Used in Nonlinear Codes

Nonlinear code	Mass representation	Constitutive model
D-MOD_2	Lumped mass	MKZ (Matasovic and Vucetic 1993)
DEEPSOIL	Lumped mass	Extended MKZ (Hashash and Park 2001)
OpenSees	Distributed mass	Multiyield surface plasticity (Ragheb 1994; Parra 1996; Yang 2000)
SUMDES	Distributed mass	Bounding surface plasticity (Wang 1990) and other models
TESS	Distributed mass	HDCP (EPRI 1993)

To evaluate which of the two above approaches is correct, time-domain analyses with elastic material properties are exercised, for which frequency-domain analyses provide an exact solution. This can be investigated using linear analyses because the underlying issue involves the differences in linear wave propagation modeling with frequency-domain and time-domain analyses. Consider, for example, a single soil layer with thickness=30 m, shear wave velocity $V_s=300$ m/s (site frequency=2.5 Hz) that overlies an elastic half-space with $V_{s-H}=2V_s=600$ m/s. Equivalent viscous damping is assumed constant at 5%. A control motion is selected to represent an extreme scenario with respect to the variability between outcropping and within, which is a sine wave at the site frequency. As shown in Fig. 4(c), the particular motion selected has a frequency of 2.5 Hz, 12 cycles of shaking, and cosine tapers at the beginning and end of the signal with a four-cycle taper duration (the tapers have the shape of half a cosine wavelength). The control motion is specified for an outcropping condition. A large suppression of the within motion relative to the outcropping motion would be expected for this signal (e.g., as suggested by Fig. 2).

A frequency domain solution is exact because the material properties are elastic (i.e., strain invariant). The frequency domain calculations are performed with the computer program SHAKE04 (Youngs 2004), which is a modified version of the original SHAKE program (Schnabel et al. 1972). Both the within motion and the motion at the surface of the soil layer are calculated, with the results shown in Figs. 4(a and b) with the solid black lines.

Linear time-domain analyses are performed for this site using the "nonlinear" codes listed in Table 1 (the codes are implemented with linear backbone curves). Four combinations of control motion and base condition are considered:

1. Outcropping motion [Fig. 4(c)] with elastic base (V_{s-H} =600 m/s).
2. Within motion [which is extracted from frequency-domain analysis; see Fig. 4(b)] with elastic base.
3. Outcropping motion with rigid base (V_{s-H}=30,000 m/s or select the "rigid base" option in nonlinear code, if available).
4. Within motion with rigid base.

The results in Fig. 4(a) show that the surface acceleration histories for Cases (1) and (4) match the known solution from frequency-domain analysis. Using the within motion with an elastic base (Case 2) underestimates the surface motions, while using the outcropping motion with a rigid base (Case 3) overestimates the surface motions.

Based on the above, our recommendations are as follows: (*i*) For the common case in which the control motion is recorded as outcropping, the motion should be applied without modification for time-domain analyses with an elastic base; and (*ii*) if time-domain analyses are to be used to simulate the response of a

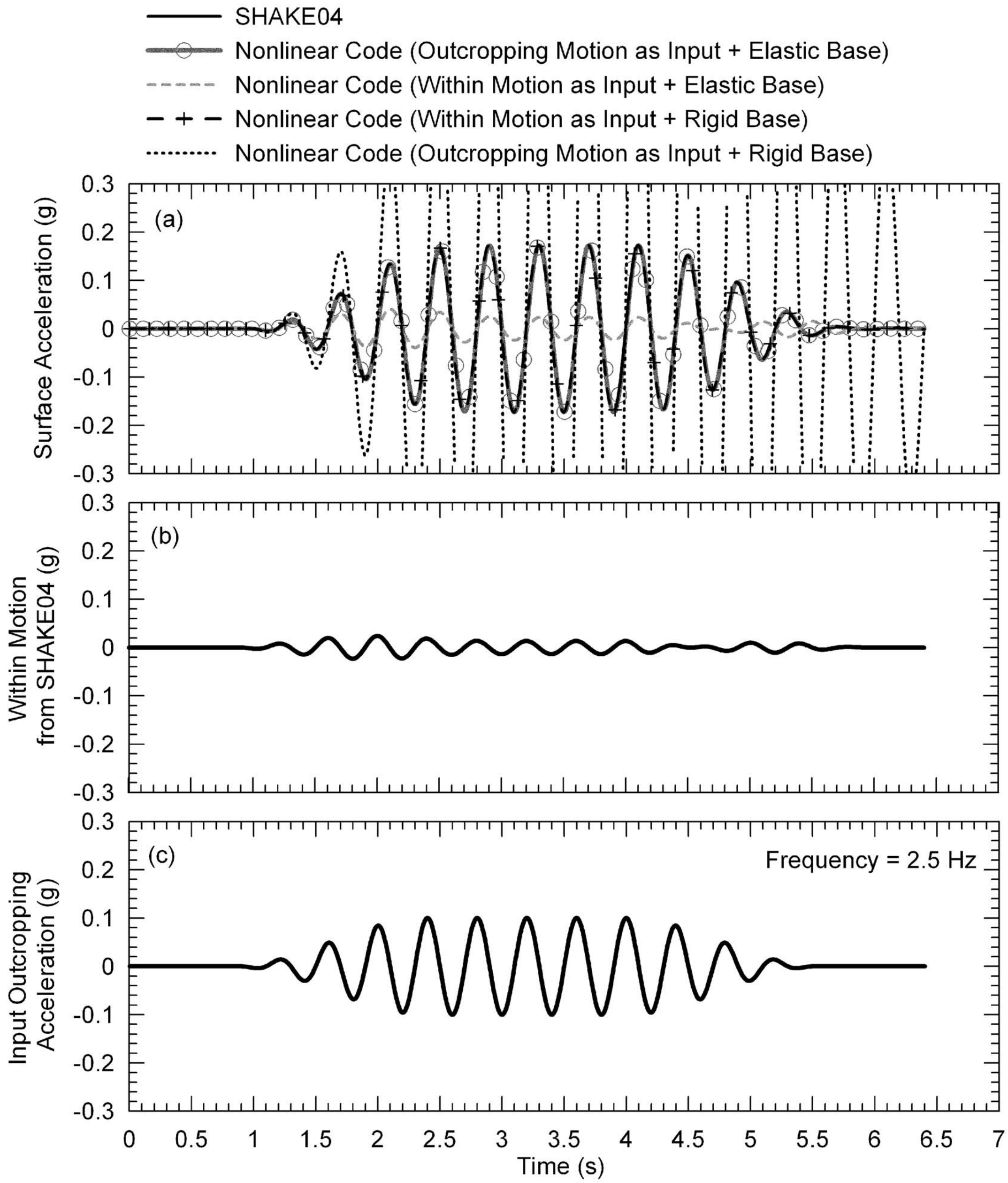

Fig. 4. Acceleration histories for the one-layer problem

vertical array using a control motion recorded at depth within the site, the "within" motion should be used without modification in conjunction with a rigid base.

Modeling of Damping in Nonlinear Time-Domain Analyses

In nonlinear time-domain response models, there are generally two sources of damping. One source is hysteretic damping (frequency independent) associated with the area bounded by hysteretic stress-strain loops. When Masing (Masing 1926) and extended Masing rules (Pyke 1979; Wang et al. 1980; Vucetic 1990) are used to represent the unload-reload behavior of soil, zero damping is encountered at small strains, where the backbone curve is linear. The zero damping condition is incompatible with soil behavior measured in the laboratory at small strains (e.g., Vucetic et al. 1998; Darendeli 2001) and can result in overestimation of propagated ground motion. One solution to this problem is to add velocity-proportional viscous damping in the form of dashpots embedded within the material elements depicted in Fig. 3 (this approach is used by D-MOD_2, DEEPSOIL, OpenSees, and SUMDES). An alternative approach is to introduce a scheme that produces nonzero hysteretic damping at small strains (e.g., TESS). It should be noted that the nature of soil damping at small strains is neither perfectly hysteretic nor perfectly viscous (Vucetic and Dobry 1986; Lanzo and Vucetic 1999). The incorporation of hysteretic or viscous damping schemes into nonlinear codes is merely a convenient approximation for simulation purposes, and is required to ensure numerical stability of lumped mass solutions.

Table 2. Available Viscous Damping Formulation for Nonlinear Codes and Summary of Analyses Discussed in Text

Nonlinear code	Rayleigh damping option	Rayleigh damping option considered in current analyses	Best match to frequency domain solution for all three sites
D-MOD_2	Simplified and full	Simplified (f_s[a]; f_m[a]; f_p[a]) ζ_{tar}=0.5% and 5% Full ($f_s+3\times f_s$; $f_s+5\times f_s$) ζ_{tar}=0.5% and 5%	Full ($5\times f_s$) at ζ_{tar}=5%
DEEPSOIL	Simplified, full and extended	Simplified (f_s; f_m; f_p) ζ_{tar}=0.5% and 5% Full ($f_s+3\times f_s$; $f_s+5\times f_s$) ζ_{tar}=0.5% and 5%	Full ($5\times f_s$) at ζ_{tar}=5%
OpenSees	Simplified and full	Simplified (f_s; f_m; f_p) ζ_{tar}=0.5% and 5% Full ($f_s+3\times f_s$; $f_s+5\times f_s$) ζ_{tar}=0.5% and 5%	Full ($5\times f_s$) at ζ_{tar}=5%
SUMDES	Simpified (assuming damping ratio given at 1 Hz)[b]	Simplified ($\zeta_{f1}=5\%/f_s$; $5\%/f_p$; $1\%/f_p$; 1%)	$\zeta_{f1}=1\%$
TESS	No viscous damping		

[a]f_s, f_m and f_p represent site frequency, mean frequency, and predominant frequency of motion, respectively.

[b]Any damping ratio at a desired frequency (e.g., f_s) can be converted to damping ratio at 1 Hz using simple proportionality (e.g., $\zeta_{f1}=\zeta_{fs}/f_s$).

Viscous Damping

There are a number of options for modeling viscous damping, which vary by code (Table 2). As illustrated in Fig. 5, there are three principal issues: (1) The form of the damping formulation (simplified versus full or extended Rayleigh damping; Hashash and Park 2002); (2) the target viscous damping ratio (labeled ζ_{tar} in Fig. 5) that is matched at specified target frequencies; and (3) the matching frequencies (one, two, and four for the cases of simplified, full, and extended Rayleigh damping, respectively).

Few formal protocols are available to guide users in the selection of the Rayleigh damping model type and parameters described above. With regard to the form of the damping formulation, most practitioners use simplified or full Rayleigh damping. Extended Rayleigh damping is seldom applied in practice. There are two schools of thought on the target damping level (ζ_{tar}), which in practice is either taken as the small-strain damping or as the smallest numerical value that appears to provide a stable solution in the judgment of the analyst (e.g., the SUMDES manual suggests 0.02% to 1%). With regard to matching frequencies, the lower target frequency is generally taken as the site fundamental frequency. The larger target frequency is generally taken as an odd-integer multiplier of the fundamental frequency (e.g., 3, 5, 7) (Hudson et al. 1994).

An alternative set of guidelines was presented by Park and Hashash (2004), in which the model parameters are selected through an iterative process, in which frequency and time domain elastic solutions are matched within a reasonable degree of tolerance over a frequency range of interest. The procedure is implemented through a user interface in the code DEEPSOIL, but is unavailable for other codes.

In the following, we develop recommended procedures for the specification of Rayleigh damping that are intended to resolve some of the ambiguities in current practice with respect to the three aforementioned issues (formulation, target damping, and target frequencies). Such recommendations are intended to meet two practical needs: (1) To form the basis for the specification of Rayleigh damping parameters for most time-domain codes, which lack a user interface to implement an iterative matching procedure such as Park and Hashash (2004); (2) for codes such as DEEP-

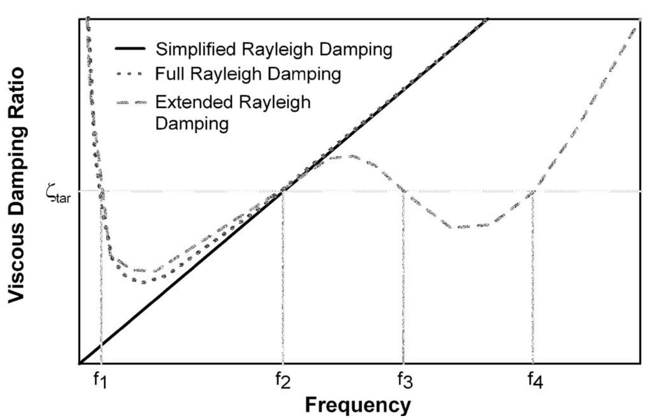

Fig. 5. Schematic illustration of viscous damping models and model parameters (adapted from Park and Hashash 2004)

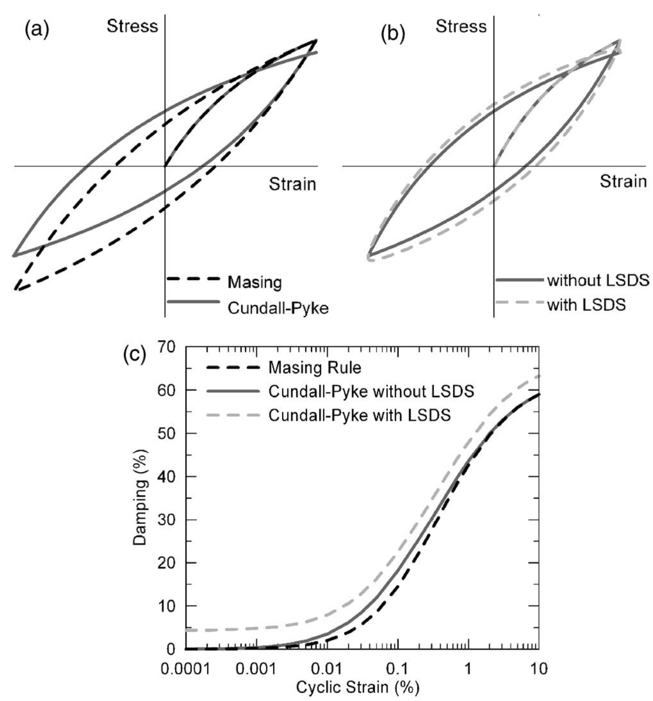

Fig. 6. (a) Comparison of stress-strain loops generated from Masing rules and Cundall-Pyke hypothesis; (b) comparison of stress-strain loops generated from Cundall-Pyke hypothesis with and without the low-strain damping scheme (LSDS); and (c) comparison of damping curves generated from different schemes

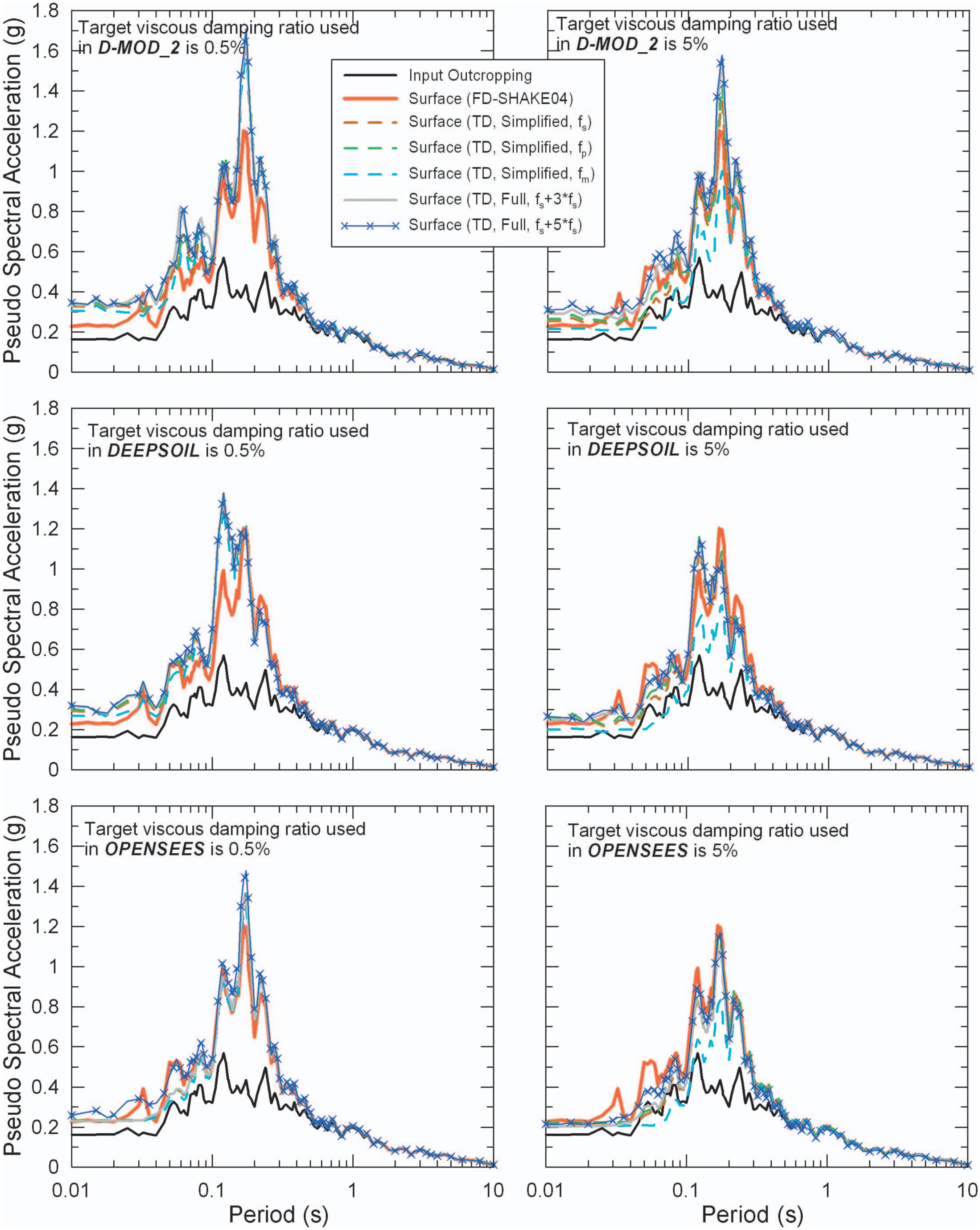

Fig. 7. (Color) Comparison of response spectra for shallow stiff site (Simi Valley Knolls School) for D-MOD_2, DEEPSOIL, and OpenSees

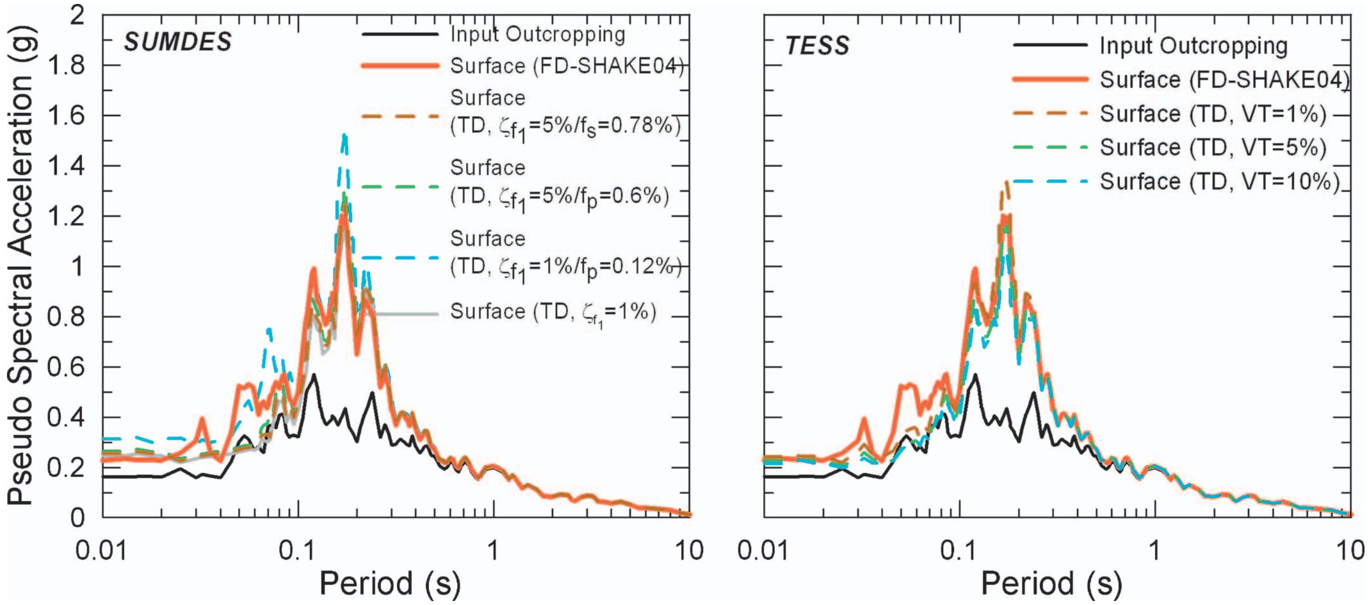

Fig. 8. (Color) Comparison of response spectra for shallow stiff site (Simi Valley Knolls School) for SUMDES and TESS

SOIL with a user interface, to provide a reasonable starting point for iterative analyses, which might be needed when dealing with deep soil profiles and very soft soils.

Hysteretic Damping

An alternative to viscous damping is the use of schemes that generate low-strain hysteretic damping (e.g., TESS). Such schemes produce damping that is additive to the hysteretic damping generated by nonlinear behavior at higher strains. Because TESS also employs an alternative hypothesis for controlling unloading and reloading behavior, the large-strain damping that results from this alternate hypothesis is first compared to that generated by the more conventional Masing hypothesis. When Masing rules are utilized, the unload and reload stress-strain curves have the same shape as the backbone curve, but are enlarged by a factor of two. This can be represented mathematically as

$$\frac{\tau - \tau_c}{n} = F_{bb}\left(\frac{\gamma - \gamma_c}{n}\right) \quad (3)$$

where $F_{bb}(\gamma)$=backbone function and (γ_c, τ_c)=strain/stress coordinates of the last reversal point. Masing rules fix n at 2. A byproduct of using Masing rules is that the tangent shear modulus upon load reversal matches the small-strain modulus of the backbone curve ($G_{\max}$). Pyke (1979) and Lo Presti et al. (2006) have suggested alternative unload-reload rules in which n in Eq. (3) can deviate from 2. Alternatively, Wang et al. (1980) introduced a damping correction factor to Eq. (3), which allows the damping to be corrected, based on the desired damping curve. All of these modifications to Masing rules produce a tangent shear modulus upon unloading (or reloading) that is not equal to $G_{\max}$.

Among the five nonlinear codes considered in this paper, only TESS has implemented non-Masing unload-reload rules. The scheme by Pyke (1979), also known as the Cundall–Pyke hypothesis, is used in which n is evaluated as follows:

$$n = \left| \pm 1 - \frac{\tau_c}{\tau_y} \right| \quad (4)$$

where τ_y=shear strength (always taken as positive). The first term in Eq. (4) is negative for unloading and positive for reloading. Figs. 6(a and c) compare the stress-strain loops and damping curves, respectively, generated by the original Masing rules and the Cundall–Pyke hypothesis. Note from Fig. 6(c) that the small-strain damping produced by the Cundall–Pyke hypothesis is still zero; hence, by itself this formulation does not solve the small strain damping problem.

TESS also uses a low-strain damping scheme (LSDS) to produce nonzero hysteretic damping at small strains (originally described in EPRI 1993 and recently updated). As shown in Fig. 6(b), the LSDS increases (in an absolute sense) the shear stress, relative to that produced by standard unload-reload rules (e.g., Cundall–Pyke). The stress increase is proportional to the normalized strain rate (i.e., current strain rate divided by the strain rate for the first time step following the last reversal). The constant of proportionality is termed VT. The parameter VT was initially based on the measured rate of strain effects on the shear modulus of young Bay Mud reported by Isenhower and Stokoe (1981), but as a practical matter is now set so that the model produces the desired low-strain damping. Note in Figs. 6(b and c) that LSDS produces nonzero low-strain damping (due to the fattening of the hysteresis curves).

Validation against Known Theoretical Elastic Solutions

Validation is performed by comparing results of linear time-domain analyses performed with alternative specifications of damping (viscous or LSDS) to an exact solution from linear frequency-domain analyses. The frequency-domain analyses are exact because of the use of linear soil properties and frequency-independent damping. This issue can be investigated using linear analyses because the problem is associated with small-strain conditions at which soil behavior is practically linear. The analyses

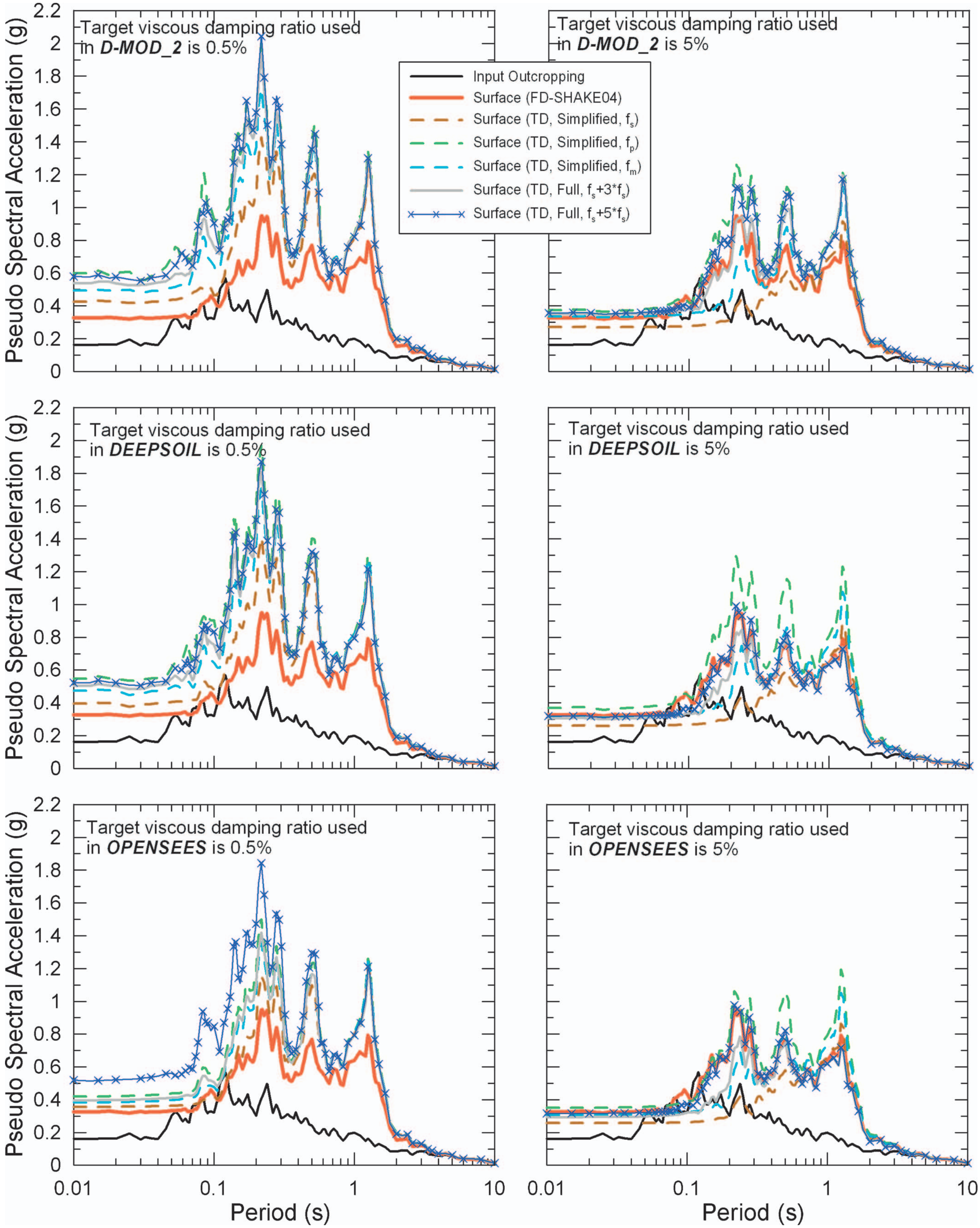

Fig. 9. (Color) Comparison of response spectra for midperiod site with large impedance contrast (Treasure Island) for D-MOD_2, DEEPSOIL, and OpenSees

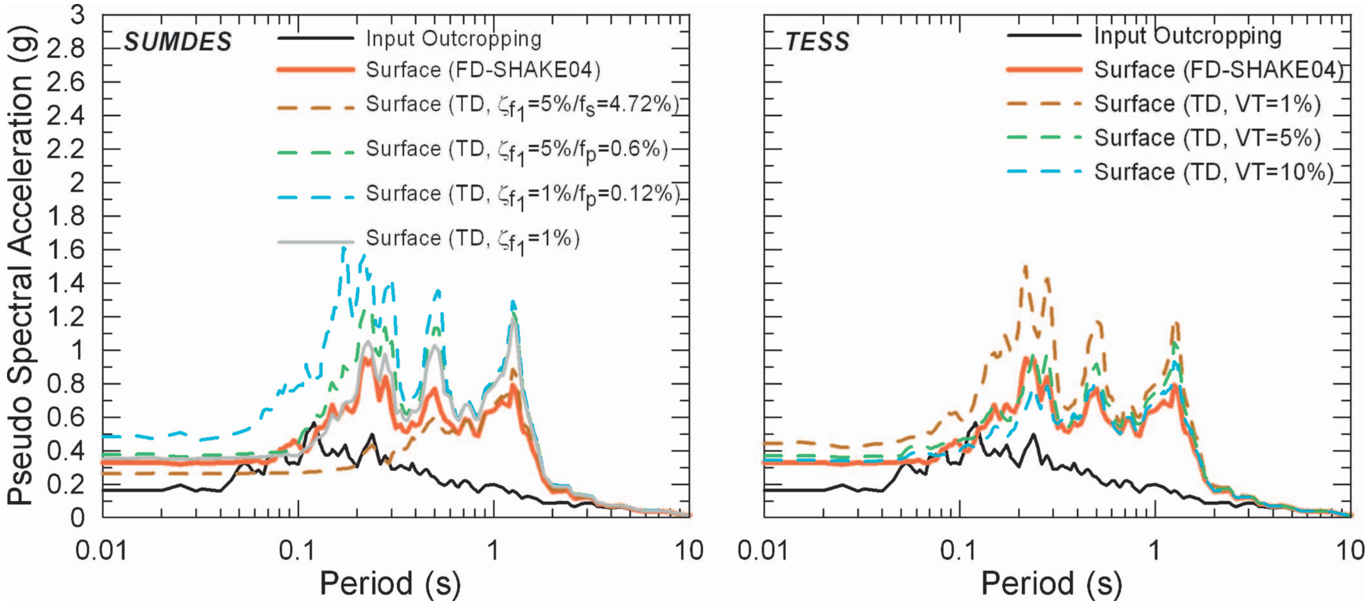

Fig. 10. (Color) Comparison of response spectra for midperiod site with large impedance contrast (Treasure Island) for SUMDES and TESS

are conducted for three selected sites that represent a broad range of site conditions: Shallow stiff soil over rock, soft clay overlying stiffer sediments and rock, and very deep stiff soils typical of the Los Angeles basin (site frequencies range from 0.45 Hz to 6.4 Hz). The control motion is a broadband synthetic acceleration history calculated for an outcropping rock site condition (motion provided by Dr. Walter Silva, personal communication, 2004). Similar results were obtained when other control motions were utilized. The equivalent viscous damping ratio used in the frequency-domain analysis is 5% for all layers. For the time-domain codes D-MOD_2, DEEPSOIL, and OpenSees, target damping ratios of 0.5% and 5% are used—the motivation being to evaluate whether the target viscous damping ratio should match the small strain material damping or a much smaller value. Both simplified and full Rayleigh damping formulations are used for these three codes (see Table 2 for details).

For SUMDES, only simplified Rayleigh damping was available at the time this article was written (implementaion of full Rayleigh damping is in progress), and the SUMDES manual calls for the target damping ratio to be specified at 1 Hz (Li et al. 1992). Past practice has been that the 1 Hz damping (ζ_{f1}) is scaled from the target damping level (ζ_{tar}, e.g., 5%) at some specified frequencies (often the predominant frequency of the input motion f_p) as

$$\zeta_{f1} = \zeta_{\text{tar}}/f_p \tag{5}$$

For SUMDES analyses, we use a 5% target damping level with matching frequencies at f_p (predominant frequency ≡ frequency having maximum spectral acceleration) and f_s (elastic site frequency). We also use a 1% target damping level with a matching frequency of f_p. Finally, we use a fixed damping ratio of ζ_{f1} =1%. These options are summarized in Table 2.

For TESS, it is recommended that the numerical value of the parameter VT be set to equal the desired low strain damping ratio (0.05) in this case. As indicated in Table 2, we also utilize values of 0.01 and 0.10 to test the sensitivity of the computed results to VT and hence to the amount of hysteretic damping that is introduced.

Shallow Stiff Site: Simi Valley Knolls School

The upper 14 m of Simi Valley Knolls School is composed of silty sand, which has shear wave velocities of about 300 m/s and is underlying by sandstone (site frequency f_s=6.4 Hz). Figs. 7 and 8 compare 5% damped acceleration response spectra of surface motions from the frequency domain solution (developed using SHAKE04; Youngs 2004) with time-domain results from the five codes listed in Table 2.

The trends in the D-MOD_2, DEEPSOIL, and OpenSees results are similar. Comparing the left and right frames, the results are somewhat better for the 5% target damping ratio, but are relatively insensitive to the damping for this shallow site. Both simplified and full Rayleigh damping formulations are reasonably effective, although simplified Rayleigh damping with the target frequency set to the mean frequency of the input motion overdamps the computed response at short periods (the site frequency is a preferred target).

For SUMDES, the results in Fig. 8 show that a target damping ratio of 5% produces overdamping at high frequencies, regardless of the matching frequency (f_p or f_s), whereas ζ_{tar}=1% provides a slightly improved fit. However, results for all the different damping formulations fall within a narrow range for this shallow, stiff site. For TESS, results are shown for three values of VT. The best fit is obtained by using values of VT that are one or two times the viscous damping ratio from frequency domain analysis.

Soft Clay Medium Depth Site: Treasure Island

The Treasure Island site has a 16-m-layer of San Francisco Bay Mud that overlies stiffer sands and clays. The site frequency is dominated by the soft clay layer, and is 1.06 Hz. The frequency-domain solution is developed using SHAKE04 (Youngs 2004). As shown in Fig. 9, analysis results for D-MOD_2, DEEPSOIL, and OpenSees indicate a much better match for ζ_{tar}=5% than for 0.5%. The greater sensitivity to ζ_{tar} (relative to the Simi Valley site) results from the thicker site profile relative the predominant wavelength. Simplified Rayleigh damping generally overdamps at low periods, although the results are reasonable when the target frequency is set at the mean frequency of the input motion. Full

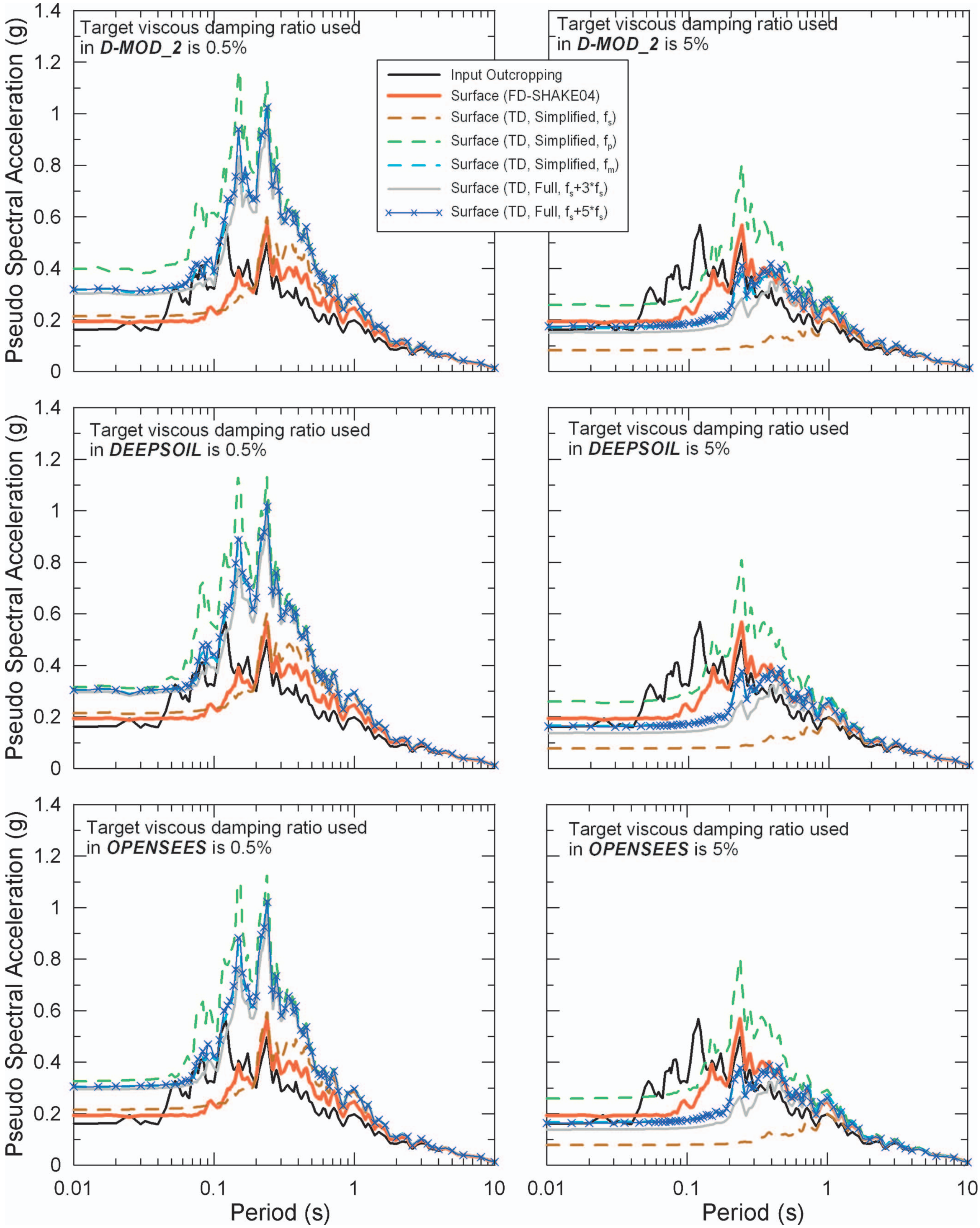

Fig. 11. (Color) Comparison of response spectra for long period site (La Cienega) for D-MOD_2, DEEPSOIL, and OpenSees

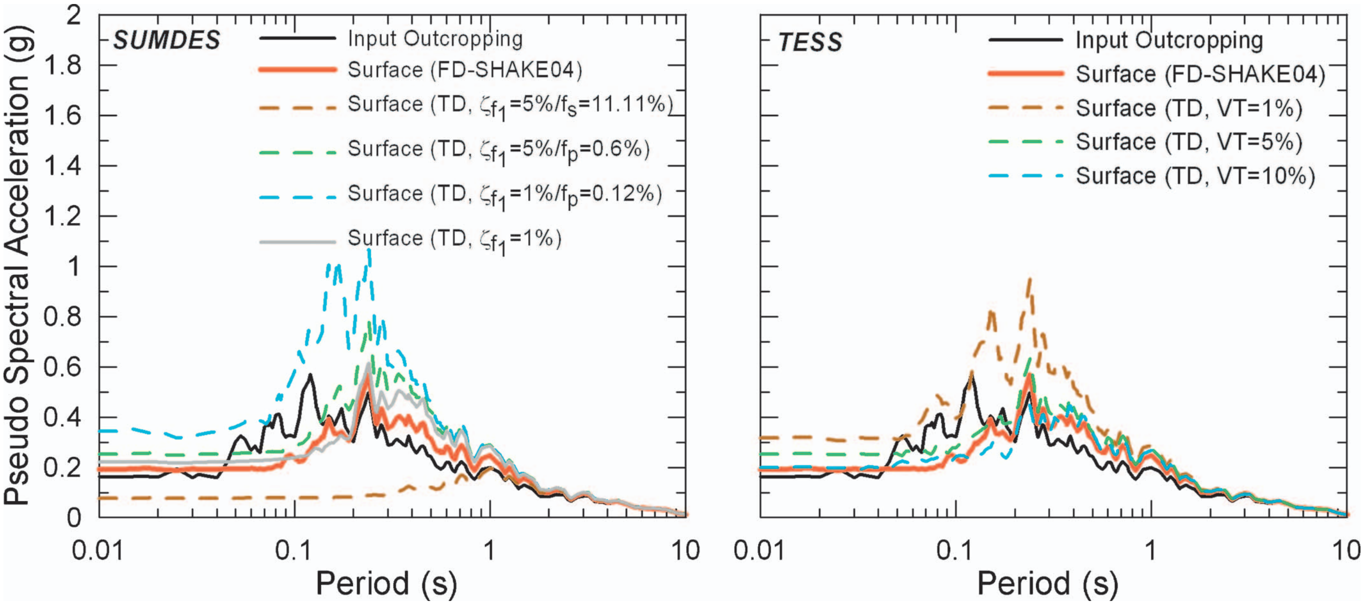

Fig. 12. (Color) Comparison of response spectra for long period site (La Cienega) for SUMDES and TESS

Rayleigh damping is preferred, with the results being fairly insensitive to the second target frequency ($3f_s$ or $5f_s$).

For SUMDES, the results in Fig. 10 show that the use of f_p as the matching frequency produces underdamping for $\zeta_{\mathrm{tar}}=1\%$ and 5%. Conversely, the use of f_s as the matching frequency produces overdamping. The best fit is obtained with viscous damping of 1% (i.e., $\zeta_{f1}=1\%$). For TESS, the best fit is again obtained when VT is set to either one or two times the viscous damping from frequency domain analysis.

Deep Stiff Site: La Cienega

The La Cienega site consists of bedded sands, silts, and clays that gradually increase in stiffness with depth. Only the upper 305 m of the profile is modeled, which has a site frequency of 0.45 Hz, although the true first mode site frequency is much lower because crystalline bedrock occurs at great depth.

The frequency-domain solution is developed using SHAKE04 (Youngs 2004). As shown in Fig. 11, analysis results for D-MOD_2, DEEPSOIL, and OpenSees show high sensitivity to ζ_{tar} (with 5% providing the better match). Simplified Rayleigh damping is most effective when the target frequency is set to the mean frequency of the input motion (f_m), and overdamps the computed response otherwise. Full Rayleigh damping generally provides an improved fit, with a slight preference towards the second frequency being $5f_s$.

For SUMDES, the results in Fig. 12 show similar trends to those for Treasure Island: the use of f_p at the matching frequency produces underdamping whereas the use of f_s produces overdamping. The best fit is again obtained for a viscous damping of $\zeta_{f1}=1\%$. For TESS the best fit is again obtained when VT is set to either one or two times the viscous damping from frequency domain analysis.

Recommendations

Where available, viscous damping should be estimated using the full Rayleigh damping formulation (available in DEEPSOIL, D-MOD_2 and OpenSees). The target damping ratio should be set to the small-strain material damping, and the two target frequencies should be set to the site frequency and five times the site frequency. For DEEPSOIL these frequencies would be a suitable starting point, and can be further refined using the matching procedure between linear frequency and time domain solutions available via a user interface. While simplified Rayleigh damping can produce reasonable results in limited circumstances (e.g., shallow site), in general, its use is discouraged. When simplified Rayleigh damping is applied, our current recommendation is to set the target damping value as described above and the target frequency as the site frequency when there is a strong impedance contrast in the profile (e.g., Simi Valley, Treasure Island), and the mean frequency of the input motion when a strong impedance contrast is not present (e.g., La Cienega).

The code SUMDES had only a simplified Rayleigh damping option at the time this article was written (a new version is in development with full Rayleigh damping). The past practice of scaling the 1 Hz damping based on a target damping at the predominant period does not appear to generally produce satisfactory results. The use of 1% damping at 1 Hz appears to provide improved performance, and is simpler to apply. For TESS, a good match to the SHAKE04 results is obtained when the parameter VT is set equal to the desired damping ratio, but the results obtained are not particularly sensitive to VT across the range of the desired damping ratio to two times that figure.

Conclusions

Frequency-domain, equivalent-linear methods of performing site response analysis remain significantly more popular in practice than time-domain, nonlinear methods (Kramer and Paulsen 2004). One reason this practice persists is that parameter selection for frequency-domain analysis is relatively straightforward, requiring only mass density, shear wave velocity, and nonlinear modulus reduction and damping versus shear strain curves. As a profession, we are generally well equipped to provide estimates of these quantities on a site-specific basis at reasonable cost.

In contrast, time-domain, nonlinear methods of analysis require the use of parameters that are less familiar to most engineers and/or relatively difficult to obtain (details below). Three major hurdles must be overcome before nonlinear analysis methods can be more widely adopted in practice. The first is clarification of the manner in which input motions should be specified. The second is the development of simple, practical guidelines for the specification of parameters that provide element damping at small strains. In this paper, these first two issues are addressed by comparing results of linear time-domain analyses to exact solutions from frequency domain analyses for elastic conditions. The third issue, which remains under investigation, is the development of practical and well-validated guidelines for estimating parameters that describe the backbone curve of soil and the unload/reload behavior given conventionally available data from a site investigation program (shear wave velocity and soil index properties).

Our finding on the input motion issue is that outcropping control motions should be used as recorded with an elastic base. Motions recorded at depth should also be used as recorded, but with a rigid base. In both cases, the motions are specified at the base of the site profile. For within motions, the depth at which the recording was made should match the depth of the profile base.

With respect to the viscous damping issue, when the option of using more than one target frequency is available (such as the full Rayleigh damping formulation), it should be applied in lieu of simplified Rayleigh damping because significant bias at high frequencies can occur with the latter. Target damping ratios should be set to the small strain material damping, and the two target frequencies in a full Rayleigh damping formulation should be set to the site frequency and five times the site frequency. For DEEPSOIL, these frequencies would be a suitable starting point, and can be further refined using the matching procedure between linear frequency and time domain solutions available via a user interface. Specialized recommendations were developed for SUMDES and its simplified Rayleigh damping formulation. Recommendations are also developed for relating parameter VT to small strain damping in the LSDS utilized in TESS. Whenever possible, it is recommended that a check be made that linear time domain and linear frequency domain solution provide similar results.

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