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DESIRABLE EXCITATION PATTERNS FOR TAPERED WIGGLERS

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DESIRABLE EXCITATION PATTERNS FOR TAPERED WIGGLERS

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ABSTRACT

Angular and displacement errors associated with excitation patterns of tapered wigglers will be discussed, and patterns will be described that do not lead to such errors even when the energy of the electrons changes along the length of the wiggler, and when the real excitation is different from the ideal excitation because of systematic iron saturation.

1. Introduction

Even though our understanding of free electron lasers (FEL's) has improved enormously over the last few years, it is clearly prudent to design a large range of adjustability into a tapered wiggler (W), at least during the development phase of a FEL that requires a tapered W. It is also clear that the preferable method of tuning such a W would be electromagnetic (em). Unfortunately, incorporating em tuning into the presently strongest W's with moderate period λ, namely permanent magnet (PM) W's, leads to either performance reduction or other undesirable effects (see ref. 1 and 2). For that reason, PM assisted em W's have been developed (ref. 1 and 2). For the purpose of the discussion in this paper, such a W is the same as a plain em W, consisting of soft iron poles that are excited by current in coils.

It is the purpose of this paper to describe a method to connect power supplies to coils such that power supply settings can be changed at will without causing cumulative displacement and steering errors of the electron trajectory. This has to hold even if the ratio γ of total energy to rest energy of the electrons changes as the electrons travel downstream. In order to allow a simple derivation and formulation of the proper excitation patterns of the poles of the W, a number of simplifications and approximations are made.

The period λ of the W is assumed to be constant, and it is assumed that there is no focusing of the motion of the electrons in the midplane.

It is assumed throughout that the W has perfect midplane symmetry, and when the term "pole" is used, what is meant is a pole in one half of the W, and it is implied that the mirror symmetrically located pole in the other

half of the W is excited such that perfect midplane symmetry of the fields is maintained. It is also assumed that no field errors due to construction errors or finite manufacturing tolerances exist; some of these errors have been discussed elsewhere (ref. 2), and their correction will be discussed in tne future. One class of deviation from ideal conditions is saturation of the iron. While random saturation effects, due to non-uniform iron properties, are ignored, systematic saturation effects, associated with the systematic tapering of the field, will be taken into account in section 3.

In writing down the equation of motion for an electron, the interaction of tne electron with the fields of the em wave is ignored, and the change of transverse momentum due to the emission of photons is ignored as well (see ref. 3). Assuming in addition that the square of the angle between the electron trajectory and the ideal forward direction (the z-axis), is small compared to one, the equation of motion of the electron in the transverse direction becomes

$$x''(z) = g(z) \cdot B(z); \quad g(z) = \frac{e}{m_0 c} \cdot \frac{1}{\gamma(z)} \cdot \tag{1}$$

In this equation $B(z)$ represents the magnetic field in the direction perpendicular to the midplane, and c, m_0, e are the velocity of light in vacuo, and rest mass and charge of the electron.

2. Displacement for Simple Excitation Patterns with γ = Const.

Assuming that $B(z) = 0$ for $z < z_{min}$, and that $x = 0$, $x' = 0$ there, it follows from eqn. (1) that

$$x(Z) = \int_{z_{min}}^{Z} g(z) \, B(z) \, (Z-z) dz. \tag{2}$$

If one looks at the displacement caused by the field produced by a small number of poles, giving essentially $B(z) = 0$ for $z > z_{max}$, and if $Z > z_{max}$, then eqn. (2) can be written as

$$x(Z) = \int_{-\infty}^{\infty} g(z)\ B(z)\ (Z-z)dz. \qquad (3)$$

If a symmetrical pole is centered at $z = 0$, and $b(z) = b(-z)$ is the field produced when that pole is excited with unity strength, then a pole centered at $z = z_1$ with relative excitation strength v_1 will produce, for $g = $ const.:

$$x(Z) = g \int_{-\infty}^{\infty} v_1\ b(z-z_1)(Z-z)dz = g \int_{-\infty}^{\infty} v_1\ b(z)\ (Z-z_1-z)dz.$$

With

$$\int_{-\infty}^{\infty} b(z)\ dz = A_1, \qquad (4)$$

the displacement x caused by that pole becomes

$$x(Z) = gv_1 A_1 (Z-z_1). \qquad (5)$$

The term Z in the parenthesis of eqn. (5) indicates the obvious and expected steering effect.

If the adjacent pole, located at $z = z_1 + D_2$,

$$D_2 = \lambda/2, \qquad (6)$$

is excited with strength $-v_1$, then the total displacement caused by both poles is given by

$$x(Z) = gv_1 A_1 D_2. \qquad (7)$$

Even though this equation (and its derivation) is trivially simple, it is worthwhile to emphasize two points:

1) If one has injected into an originally flat W such that the trajectory is symmetrical with respect to the z-axis, and one changes beyond a certain point all excitations by the same relative amount, each pair of poles, having its excitation changed by $\pm v_1$, contributes the displacement given by eqn. (7), i.e. the displacements given by eqn. (7) are cumulative, and therefore disastrous for the operation of the FEL.

2) A short distance downstream from the location where the excitation change has started, the magnetic field is again perfectly periodic. Assuming for simplicity that higher harmonics are not significant, the field change there would have the form

$$B(z) = v_1 B_0 \cos kz; \quad k = 2\pi/\lambda. \tag{8}$$

One might be tempted to assume that the quantity A_1 equals the area under the normalized $B(z)$ curve between $z = \pm \lambda/4$, i.e.,

$$A_0 = \int_{-\lambda/4}^{\lambda/4} B_0 \cos kz \, dz = 2 B_0/k. \tag{9}$$

However, since $B_0 \cos kz$ is produced by linear superposition of the fields produced by all poles i.e.,

$$B_0 \cos kz = \sum_{n=-\infty}^{\infty} b(z-nD_2) \cdot (-1)^n,$$

it is clear from Fig. 1 that A_1 will always be larger than A_0: the "tails" of $b(z)$ in the region $\lambda/4 < |z| < 3\lambda/4$ are not only missing in A_0, but are contributed from the neighboring poles with opposite

polarity. We therefore write:

$$A_1 = \rho A_0.$$

ρ can be computed with analytical methods or magnetic field analysis codes and is typically 1.5–5.

It is instructive to compare the displacement given by eqn. (7) to the wiggle amplitude

$$x_W = gv_1 B_0/k^2 \qquad (11)$$

associated with the periodic field given by eqn. (8). From eqn's. (7), (9), (10), (11) follows immediately

$$x = x_W \cdot 2\pi \cdot \rho. \qquad (12)$$

It is clear from the above that energizing each pair of poles with equal absolute strength and opposite polarity will lead to unacceptably large cumulative displacements. We therefore ask the next question: What should the relative excitations v_0, v_1, v_2 of these poles at locations $z = 0$, D_2, $2D_2$ be to avoid cumulative displacement and steering? From eqn. (5) follows for that case

$$x(Z) = g A_1 (Z(v_0 + v_1 + v_2) - D_2(v_1 + 2v_2)).$$

Setting $v_0 = 1$, $x(Z)$ vanishes for

$$v_0 = 1; v_1 = -2; v_2 = 1. \qquad (13)$$

Table 1 shows schematically the use of this excitation pattern. If each coil has 2N turns, power supply No. 1 excites N turns on pole No. 1, -2N turns on pole No. 2, and N turns on pole No. 3 with the same current. Power supply No. 2 excites the remaining N turns on pole No. 3, -2N turns on pole No. 4, and N turns on pole No. 5, etc. etc.

When all power supplies carry the same current, this pattern gives the well known rule to start a flat W with "1/2 pole". A disadvantage of this pattern, even for the case γ = const, will be discussed in section 4.

3. Displacement-free Excitation Pattern for Variable γ

A tapered W is used to maintain the FEL resonance condition when γ changes. Consequently, if one wants to avoid cumulative displacement and steering errors, the variation of γ needs to be taken into account in the design of the excitation pattern. Representing g(z) over the range of a particular excitation pattern, energized by one power supply, by

$$g(z) = g_0 + g_1 z + g_2 z^2 + \ldots\ldots, \tag{14}$$

and using that expression in eqn. (3), yields

$$x(Z) = \int_{-\infty}^{\infty} (g_0 + g_1 z + g_2 z^2 + \ldots) B(z) (Z-z)dz. \tag{15}$$

B(z) produced by poles at locations $z_n = nD_2$ excited by strengths v_n can be written as

$$B(z) = \sum_n v_n b(z-nD_2). \tag{16}$$

Using this in eqn. (15), exchanging the order of integration and summation, and introduction in each term of the sum $z-nD_2$ as new variable gives

$$x(Z) = \sum_n v_n \int_{-\infty}^{\infty} (g_0 + g_1 (z+nD_2) + g_2 (z+nD_2)^2 + \ldots)$$

$$\times (Z-nD_2-z) b(z)dz. \tag{17}$$

Executing the integration, one obtains

$$x(Z) = \sum_n v_n \left(a_0 + a_1 n + a_2 n^2 + a_3 n^3 + \ldots\right). \tag{18}$$

The coefficients a_m contain D_2, the moments $\int_{-\infty}^{\infty} b(z) \, z^p \, dz$ of $b(z), Z, g_0, g_1, g_2$.

To make $x(Z) = 0$ independent of Z, g_1/g_0, g_2/g_0, $\ldots$, one clearly needs to satisfy

$$\sum_{n=0}^{M+2} v_n \, n^m = 0, \quad m = 0, 1, 2, \, M + 1, \tag{19}$$

where M is the order of the highest derivative of g whose effect one wants to eliminate. The v_n for the case $M = 0$ were given by the binomial co-efficients of order 2 with alternating signs, and it is verified in the Appendix that

$$v_n = (-1)^n \cdot \binom{M+2}{n} \quad ; \quad n = 0, 1 \ldots\ldots, M+2 \tag{20}$$

satisfy eqn's. (19).

Table 2 shows the implementation of this pattern for the case M=2. This table has been "constructed" for the case where one wants to use one power supply per period. If one wants to have only one power supply for two peri-ods, the excitation pattern for each such power supply would be 1,-4, 7,-8, 7,-4, 1.

If one needs magnetic field strengths in the W that are large enough that saturation of iron becomes noticeable, the real excitation of the pole

will be less than the ideal excitation. These saturation effects can obviously be represented by

$$v_{n,real} = v_{n,ideal} \, S(n), \tag{21}$$

where $v_{n,ideal}$ represents the v_n given by eqn. (20). If the field level in a tapered W is changed systematically, it is reasonable to expect that $S(n)$ can be well represented by

$$S(n) = S_0 + S_1 n + S_2 n^2 + S_3 n^3 + \ldots . \tag{22}$$

Using the such modified v_n in eqn. (17), one sees that these systematic saturation effects are very effectively compensated by the pattern given by eqn. (20), provided that the pattern is of sufficiently high order when measured against the significant terms in eqn. (22).

4. Displacement of a Trajectory in Flat Part of W with γ = Const.

While it seems not possible to develop simple formulas that characterize the trajectory in a region where the pole excitation and/or γ changes, it is easy to make some general statements about a trajectory in a region where the trajectory is periodic. By "periodic trajectory" is meant that γ is constant, that the excitation pattern in that region is periodic, and that one is sufficiently far away from the regions where these conditions are not satisfied, so that the "tails" from these regions have essentially disappeared. We assume that an excitation pattern is used to compensate for all significant terms in the Taylor series expansion of $g(z)$.

The average value of x' of the trajectory must be zero, since a non-zero average of x' would be equivalent to cumulative displacements, which is not possible if a proper excitation pattern is used.

Since the electron "sees" at any point the fields from at least one excitation patterns, a (non-cumulative) displacement of the trajectory from the z axis is possible. To calculate this displacement, we calculate x at a point half way between two adjacent poles, and we normalize z = 0 there. With

$$D_4 = \lambda/4, \tag{23}$$

we get for B(z)

$$B(z) = \sum_{n=odd} v_n \, b(z-nD_4). \tag{24}$$

Using that in eqn. (2) yields

$$x(0) = -g \cdot \int_{-\infty}^{0} \sum_{n=odd} v_n \, b(z-nD_4) z dz. \tag{25}$$

The sum has to include as many terms to the right (i.e. z > 0) as b(z) contributes to locations z ≤ 0. To the left. one has to include at least as many terms as one has on the right, but one can exclude all complete patterns to the left of that point since complete excitation patterns cannot contribute to x(0).

Since we are dealing with a flat part of a W, we can calculate the contributions of each pair of poles at locations ± nD_4 together:

$$J = v_{-n} \cdot \int_{-\infty}^{0} (b(z+nD_4) - b(z-nD_4)) z dz.$$

Introducing new integration variables gives

$$J = v_{-n} \cdot \left(\int_{-\infty}^{nD_4} b(z) \ (z-nD_4)dz - \int_{-\infty}^{-nD_4} b(z) \ (z+nD_4)dz \right).$$

Introducing in the second integral $-z$ as new a variable, and using

$b(-z) = b(z)$, the second integral becomes

$$\int_{\infty}^{nD_4} b(z) \ (z-nD_4)dz,$$

yielding

$$J = v_{-n} \cdot \int_{-\infty}^{\infty} b(z) \ (z-nD_4)dz = - A_1 D_4 n \ v_{-n}.$$

Using this in eqn. (25) gives

$$x(0) = g \ A_1 D_4 \sum_{n>0, \text{odd}} n \ v_{-n}. \tag{26}$$

For a pattern with binomial coefficients of order 2, one obtains for an

excitation strength $v_{-1} = |v_n| = 1$ in the flat part of the W

$$\sum_{n>0, \text{odd}} n \ v_n = .5.$$

Using this in eqn. (26), and relating that answer to the wiggle amplitude

x_W in the flat part of the W, one obtains

$$x_2(0) = g \ A_1 \lambda/8 = x_W \cdot \rho \pi/2. \tag{27}$$

Notice that again, it is A_1, and not A_0, that determines directly $x_2(0)$,
and that, depending on ρ, x_2/x_W can be remarkably large.

A calculation of $\sum n v_{-n}$ for the case of an excitation pattern with
binomial coefficients of order 3 gives zero for that sum. Since an exci-
tation pattern of order 4 can be constructed by superposition of two pat-
terns of order 3 shifted by $\lambda/2$, and one having the opposite signs as the
other, a pattern of order 4 will also give zero displacement in the flat
part of a W, and this argument can, of course, be repeated at infinitum,
giving

$$x_m(0) = 0 \quad \text{for } m > 2. \tag{28}$$

5. Comments and Generalizations

The concept of using excitation patterns that allow independent control
of power supplies without causing steering or displacement even for the case
of variable γ has been described here for the specific case of an electro-
magnet with iron poles. The reason for choosing that particular system to
demonstrate the concept in detail is the likelihood that most systems to
which this method will be applied in the near future will be such W's. From
the descriptions given above, it is clear that this method is applicable to
other systems as well. For instance, the basic element, equivalent to the
iron pole used here, used in the ELF W (ref. 4) is a coil with its axis
parallel to the z axis. This leads to an equivalent function $b(z)$ that
describes the field that has the property $b(-z) = - b(z)$. Using this
properly in eqn. (17) leads to the conclusion that in order to be inde-
pendent of the M'th derivative of g, one needs a binomial coefficient pat-
tern of alternating sign of only order M+1. All the other formulas are

also very easily rewritten. Similarly, work is in progress to apply this concept to a helical em W, and application to the design of a hybrid W is fairly straightforward.

It is also clear that this system of exciting a W or an undulator (U) may be very beneficial when designing an U that should produce a very good synchrotron radiation spectrum when used in an electron storage ring. It looks as if the field quality necessary to produce a high quality spectrum cannot be obtained with present manufacturing techniques, so that correction coils will have to be used. However, it also appears that magnetic field measurements will not be of sufficient accuracy, leaving only a measurement of the synchrotron radiation spectrum itself as the diagnostic tool to determine the correction coil settings. It seems, however, hopeless to try to determine the correction coil settings from the whole spectrum produced by a very long U. If it is possible to look at the spectrum emitted by a small section (of the order of ten periods) of the U, then it will be possible to correct each such section in turn by setting displacement, steering, and phase shift correctors. If one energizes these sections with a binomial pattern of order 3, one can change the field level of any section for this purpose without displacement of the trajectory. Operating different parts of an U at different field levels in this manner could also be used to generate simultaneously photons of different energy in the same U, such as one might, for instance, want to do in multiple ionization studies.

To correct a W in a FEL, one can use essentially the same procedure. If the binomial pattern is of sufficiently high order, this can be done without operation of the FEL, i.e. with constant γ. If the W is strongly tapered,

one may want to use beam position monitors instead of the synchrotron radiation spectrum, particularly since the former would probably be of sufficient accuracy and the latter would be more difficult to interpret.

Some care has to be taken at the entrance and end of a W or U: as stated in section 3, the coefficients a_m in eqn. (18) contain the moments of $b(z)$, and eqn. (18) is valid in that form only if the moments of $b(z)$ of all poles are identical. This will clearly not be the case for the poles at the very ends of the W or U. The simplest solution to that problem is probably to have not only one un-excited pole at each end (namely the field clamp), but to have two or three poles at each end at zero scalar potential.

Acknowledgment

Appendix

With

$$M' = M+2, \quad\quad\quad (A1)$$

eqn's. (19) and (20) become

$$\sum_{n=0}^{M'} v_n \, n^m = 0; \quad m = 0, 1, \ldots M'-1 \quad\quad\quad (A2)$$

$$v_n = (-1)^n \binom{M'}{n} \; ; \quad n = 0,1, \ldots M' \quad\quad\quad (A3)$$

To verify that eqn's. (A3) satisfy eqn's. (A2), we consider the function

$$F(x) = (1-x)^{M'} = \sum_{n=0}^{M'} (-1)^n \binom{M'}{n} \cdot x^n = \sum_{n=0}^{M'} v_n \, x^n \; . \quad\quad (A4)$$

Clearly,

$$F(1) = \sum_{n=0}^{M'} v_n = 0.$$

Differentiation of eqn. (A4) gives

$$F'(1) = - \left(M'(1-x)^{M'-1}\right)_{x=1} = 0 = \left(\sum_{n=0}^{M'} n \, v_n \, x^{n-1}\right)_{x=1} = \sum_{n=0}^{M'} n v_n \; . \quad (A6)$$

Differentiating eqn. (A4) twice yields

$$F''(1) = M'(M'-1)\left((1-x)^{M'-2}\right)_{x=1} = 0 = \left(\sum_{n=0}^{M'} n(n-1) \, v_n \, x^{n-2}\right)_{x=1} . \quad (A7)$$

Since eqn. (A6) shows that eqn. (A2) is correct for m = 1, eqn. (A7) proves that eqn. (A2) is correct also for m = 2. This procedure can be repeated until one has differentiated F(x) M'-1 times, verifying that eqn's. (A3) satisfy all eqn's. (A2)

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Figure Caption

Fig. 1. Field Produced by Individual Poles.
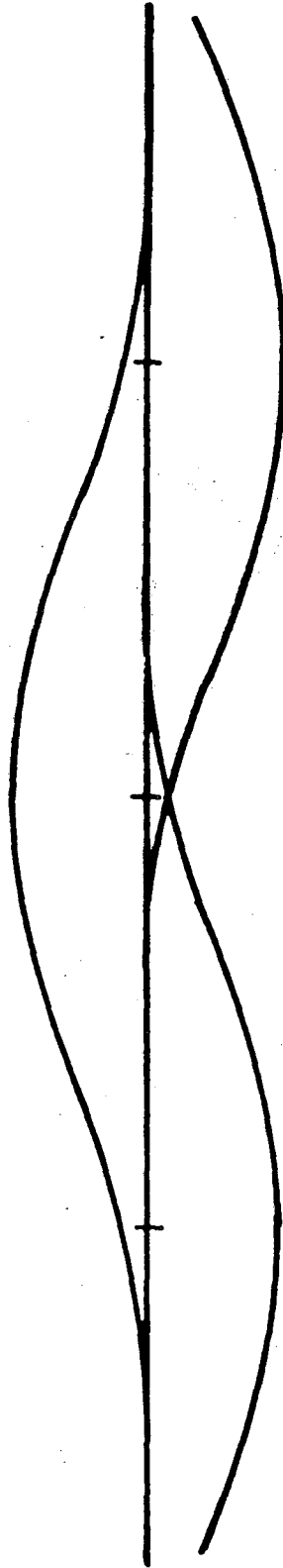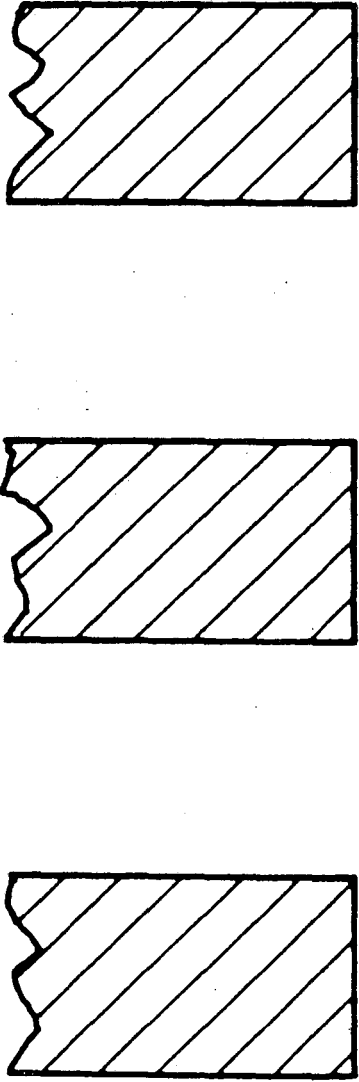
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Fig. 1

Table 1

Power Supply No.

Pole No.

	1	2	3	4	5	6	7
1	1	-2	1				
2			1	-2	1		
3					1	-2	1

Table 2

Power Supply No.

Pole No.

	1	2	3	4	5	6	7	8	9	10	11
1	1	-4	6	-4	1						
2			1	-4	6	-4	1				
3					1	-4	6	-4	1		
4							1	-4	6	-4	1

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