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SIMPLIFIED THERMAL PARAMETERS: A MODEL OF THE DYNAMIC PERFORMANCE OF WALLS

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ABSTRACT

In-situ measurement of wall thermal performance entails two problems: 1) selecting a technique for measuring time-varying surface temperatures and heat fluxes on both sides of the test wall and 2) reducing this data set into a small number of parameters that characterize the wall.

The first problem is addressed by the Envelope Thermal Test Unit (ETTU), consisting of two four-foot by four-foot blankets placed on either side of the test wall that are used to both measure and control the surface heat fluxes and temperatures of the wall.

Dynamic measurements always require specifying a driving cycle to get the dynamic characteristics from the test wall. The choice of a preferred dynamic cycle is addressed, and a pink-noise driving cycle is chosen to maximize the amount of information from a given test.

To analyze the data gathered by ETTU we have developed a simplified dynamic model that describes the thermal performance of a wall by a small number of parameters: a steady-state conductance, a time constant, and some storage terms; we call these parameters Simplified Thermal Parameters (STPs). The ability of this model to simulate actual wall performance is demonstrated by comparison to results generated with conventional response-factor methods. The model is used to analyze the behavior of a theoretical multi-layer wall whose properties have been specified by a response-factor calculation.

Keywords: thermal performance, dynamic performance, field measurement, walls, building envelopes, semi-empirical modeling

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INTRODUCTION

Most wall performance measurements to date have been done in laboratories, typically with large hotboxes. In-situ performance is considerably more difficult to measure, for the experimenter usually has little control over temperature conditions or solar radiation, and wind may further complicate the measurement. Even when surface temperatures and heat fluxes are measured accurately and over sufficient lengths of time, the problem remains of how the data gathered should be analyzed. Most existing thermal wall performance models involve too many parameters to make them suitable for direct analysis. (A review of measurement techniques and models for assessing thermal performance of walls has been compiled by Carroll).[1]

At the Lawrence Berkeley Laboratory, we have designed and built an Envelope Thermal Test Unit (ETTU) which allows us to measure the surface temperatures and heat fluxes of a wall component while driving the wall with a known amount of heat. A great deal of discussion has centered around the desire for a preferred dynamic cycle that can be used to test the envelope component. We believe that the best driving flux is one that contains all the frequency components encountered by an envelope component; additionally, the optimal driving flux will have its frequency spectrum weighted toward the lower frequencies. A spectrum satisfying these criteria is called a pink-noise spectrum as defined for acoustics in ASTM standard C-634.

In the following sections we present a model of dynamic thermal performance that can be used to quantify the characteristics of a wall from measured surface temperatures and heat fluxes. The model uses a set of Simplified Thermal Parameters (STPs) to characterize the thermal performance of walls under an arbitrary temperature history. Although our model was designed to analyze the data from ETTU, it can be used on any set of data on surface temperatures and fluxes. To test our model, we will use the thermal properties of a synthetic wall as calculated from a standard response-factor method.

MEASUREMENT TECHNIQUE

Determining the thermal properties of an envelope component of unknown composition relies on the time histories of surface temperature. To measure the steady-state properties of the component (i.e. its U-value) all that is required is a long-term average of the temperature drop across it and one heat flux. However, for many applications (e.g. massive walls, passive solar applications, or mild climates) the steady-state conductance does not sufficiently describe the thermal behavior of that component — hence the need for determining the dynamic thermal properties of envelope components.

Measuring dynamic properties implies understanding the relation between time-varying heat fluxes and time-varying temperatures on the surfaces of the test component. Several laboratories (e.g. National Bureau of Standards, Owens Corning Fiberglas, Portland Cement Association), are using hotboxes to measure the dynamic thermal performance of walls. This technique generally affords a high degree of accuracy stemming from the high degree of experimental control over all boundary conditions (i.e. temperature and heat fluxes). No measurement tools and strategies of comparable scope have been developed for field applications, even though only field measurements can tell us about deterioration of walls with age, the role of construction quality in wall performance, and the heat losses associated with air leakage through walls. To this end, we have recently developed the Envelope Thermal Test Unit (ETTU) — as an effective means of performing dynamic measurements of wall thermal performance in the field, where the boundary conditions are much harder to control than in the laboratory. Because field applications carry constraints regarding weight, size, and control, we opted for a design in which heat flow is applied on one or both sides, resulting in changes to the surface temperatures. ETTU permits the interpretation of dynamic heat-transfer properties from the direct measurement of heat flux and surface temperature (cf. hotboxes where temperatures are regulated according to a preset schedule and the resultant heat fluxes are measured).

Description of ETTU 1.2

An earlier article[2] described a prototype of this same device (ETTU 1.1), which used plywood for support and thermocouples for temperature measurement. The device described in this article (ETTU 1.2) is lighter, stronger, and smaller and uses thermistors for the temperature measurement. The physical principles remain unchanged, but the method of data analysis has evolved considerably.

ETTU differs from a standard guarded hotbox in two respects: 1) it is portable and thus can be used for on-site testing of actual building walls, and 2) it measures the wall temperature response to known heat flows, as opposed to measuring heat flows in response to preset temperatures. The physical arrangement of ETTU is shown schematically in figure 1. Two identical "blankets" are placed in close thermal contact with the wall to be tested. Each blanket consists of a pair of 1.2-m by 1.2-m (4′ X 4′) electric heaters separated by a low-thermal-mass insulating layer. The heater in contact with the wall is called the "primary" heater, and the other one is called the "secondary" heater. Between each heater layer and the insulating layer is an array of temperature sensors (thermistors) that are used to measure the surface temperatures of the insulating layer in both the central measurement region as well as the bordering guard region of each blanket. The blankets are slightly flexible in order to conform to minor irregularities in the wall surfaces.

The ETTU blankets can be logically separated into three independent sections: a surface heater, a surface-temperature sensor, and a heat-flux meter. The heaters are effectively one-dimensional copper filaments deposited on thin sheets of mylar; the amount of heat generated by them is calculated from the voltage across and the current through them. Because the heaters are made from a single uniform filament of copper and the spacing between the filament windings is small compared to the dimensions of the test surface, the heat flux they generate is uniform over the surface of wall to be tested. The surface temperature sensors are thermolinear components made of thermistors and temperature-compensating resistors (i.e. the voltage output is proportional to the temperature). The heat-flux meters are composite components; they consist of a polystyrene slab embedded between two surface temperature sensors. By measuring the temperature across the polystyrene slabs and having previously measured the thermal properties of the blankets, we can calculate the heat flux through the polystyrene. The three sections (heater, temperature sensors, flux meter) share some components — the same temperature sensors are used to measure the temperatures of the wall and of the polystyrene slab (as part of the flux measurement technique).

Heat flow into one surface of the test wall is calculated from the output of the adjacent primary heater minus the heat lost through the polystyrene slabs (heat flow is defined as positive if it is into the wall). By suitably controlling the secondary heaters it is possible to maintain a zero temperature drop across one of the blankets — implying that the heat flux into the wall is equal to the heat flux of the primary heater. Regardless of the settings on the primary and secondary heaters, however, the flow of heat into the test wall can be calculated from the measured primary heat flux and the calculated flux loss through the blanket; the flux loss through the blanket is calculated from the temperature history on both sides of the blanket and the thermal properties of the blanket as previously determined from the blanket calibration.

Each ETTU blanket, then, allows a wide range of driving strategies. By employing suitable control algorithms, we could control the surface temperatures and measure the flux response, control the net flux and measure the surface temperatures, or use any control strategy in between. Thus, the design of ETTU leaves the choice of driving cycle relatively unincumbered. (See discussion below.)

Preferred Dynamic Cycle

The topic of dynamic measurement of thermal performance of walls implies a choice of driving cycle; that is, the dynamic characteristics of components rely on time-varying boundary conditions (i.e. surface temperature and heat flux). Invariably, the choice of driving cycle will be limited by the equipment used; that is, the response time, capacity, and range of the instrumentation will always constrain the cycle. For purposes of clarity, however, the discussion that follows will ignore the limitations imposed by a particular set of hardware.

Some general remarks should first be made about the thermal properties of an envelope component. For slowly-varying driving functions, the flux through the component will be proportional to the temperature difference across the component; the component is in near steady-state equilibrium and acts like a massless thermal resistance. For quickly-varying driving functions, the flux through one side depends only on the temperature on that side — it is independent of the temperature on the other side; the component acts like a semi-infinite solid. For driving functions varying at an intermediate rate the response of the component depends critically on its own internal structure. The quantity that separates these three regimes (slow, intermediate, fast) is the time constant of the wall. Defined more explicitly in later sections, the time constant is a measure of the time it takes a pulse of heat to travel from one side of the component to the other; it depends on the total amount of thermal mass and resistance contained in the component. For driving functions having characteristic times much longer than the time constant, the wall appears massless, and for driving functions having characteristic times much shorter than the time constant, the wall appears to be very thick.

Because the choice of driving cycle depends on the component being tested, we cannot specify the preferred dynamic cycle independent of the component; however, since the parameters of most envelope components are qualitatively similar, many general conclusions can be drawn. The information gained from a dynamic measurement is used to predict the behavior of that component in real-world conditions; therefore, the description and, hence, the dynamic cycle need not contain frequencies outside of the range of frequencies observed under normal circumstances. The lowest frequency generally of interest corresponds to the diurnal cycle;* hence, our dynamic cycle need not contain frequencies lower than once per day (except, of course, for steady-state terms). For many applications, we are not interested in predicting dynamic response more than about once an hour; therefore, we need not worry about frequencies faster than once every few minutes, suggesting that an upper frequency cut-off might be once every 15 minutes.

The time constant of the envelope component represents another constraint on the necessary frequency components. Frequencies much lower than the inverse of the time constant act as DC, or steady-state, terms; therefore, little additional information is gained by the presence of frequencies in that range. Time constants are typically in the range of one to five hours for typical residential wall construction.

Consideration of the above factors suggests that to retain all the information, a preferred dynamic cycle should contain all the frequencies between one cycle per day and one cycle every few minutes, suggesting that one possible driving cycle might consist of equal parts of all frequencies in that range, with random phase relationships to each other. This type of frequency spectrum is called broad-band noise or "white" noise. In real situations, lower frequency fluxes predominate (e.g. 6-hr, 12-hr, or 24-hr cycles) over higher frequency fluxes; furthermore, the most important thermal parameters (i.e. the conductance and time constant) are better determined from low-frequency data. Thus, a driving cycle that is weighted toward the low-frequency part of the spectrum is preferable; this type of spectrum is called a "pink"-noise spectrum because it contains more of the red (lower frequency) components than does a white-noise spectrum. Pink-noise spectra, which we have elected to use with ETTU, are sometimes called a 1/f ("one over f") noise spectra because the amplitude of each frequency is inversely proportional to the frequency. This spectrum occurs many places in the real world and is special because the amount of power in each octave (or

* For extremely massive structures, seasonal or even annual cycles may be important, but for most envelope components we can ignore these.

decade) is constant.

The pink-noise function we have chosen as our driving function has a fundamental frequency of once per day. Figure 2 shows typical pink-noise spectra for a 24-hour run, including the normalized heater values for each side of the wall. In virtually all analysis techniques used to calculate dynamic properties, it is necessary to have some information about the time history of the temperatures (or fluxes) for at least several time constants prior to the beginning of the calculation period; if the analysis involves a frequency analysis, the time history before the begining of the analysis should be identical to the time history before the end of the analysis. For these reasons, the 24-hour driving cycle should be repeated to make a 48-hour test wherein the first 24-hour flux and the second 24-hour flux is the same, and the analysis will be done only on the second 24-hour set of data.

DATA INTERPRETATION

Regardless of the source or character of the data, we must be able to use measured temperatures and fluxes to characterize the wall. This goal is the reverse of the more common problem of finding the flux response of a wall from the known properties of each component layer, in which one generally uses response factors, which are weighting factors used to calculate the flux at a particular time by summing temperatures at earlier times. A large body of knowledge exists on the subject of response factors[3–9] but, because of the large number of independent parameters required, response factors cannot be backed out from measured data. To overcome the problems of characterizing envelope components using measured heat fluxes and temperature data, we have developed a simplified model of wall behavior — one that expresses the performance of the wall in terms of a few parameters that describe the characteristics of the wall as a whole, rather than the make-up of individual layers within the wall. The complete derivation of simplified thermal parameters (STPs) is described elsewhere,[10] but its results will be used in the sections to follow.

QUALITATIVE WALL MODELS

Before actually presenting our model of simplified thermal parameters, we shall build up our model from qualitatively simple foundations until we have generalized it sufficiently to describe real walls. We will be quite general in order to show the most important features; that is, we will speak of temperature differences and fluxes without specifying exactly what system we are using or which temperatures and fluxes we are defining.

We begin with the simplest possible wall, that of a purely resistive wall (i.e. one that contains no thermal mass). In electronics such a system corresponds to a passive one containing only resistive elements.

$$J(t) \simeq U \Delta T(t) \tag{1}$$

where: $J(t)$ is the flux through the wall [W/m^2],
U is the conductance of the wall [W/m^2-K], and
$\Delta T(t)$ is the temperature difference across the wall [K].

This equation can also be applied when long-term average temperatures are used to calculate long-term average fluxes.

Because all real walls have thermal mass we must be able to add its effect into our model. The simplest way to add thermal mass is to include a lump of mass with a lump of resistance, the electrical analog of which is a lumped RC circuit (i.e. a circuit that includes resistors and a capacitor). Some of the heat will be stored in the thermal mass; therefore, the flux depends not only on the temperature difference across the wall, but also on the past history of the temperatures. Although the exact expression will depend on the relationship of the lumped parameters, a large number of the lumped models can be described by equations of this form:

$$J(t) \simeq U \Delta T(t) + U' F(t,tau) \tag{2}$$

where: τ is the time constant of the system,
U' is a (mass related) conductance term, and
$F(t,\tau)$ is a temperature filter [K].

The new term in this equation, the temperature filter, is a weighted sum of the past history of the temperature on the surface of the wall; the terms that are in the relatively recent past get weighted more heavily than those in the far past and, not unexpectedly, the time that separates the recent past from the distant past is the time constant.

The exact definition of our filters is,

$$F(t,\tau) = \frac{1}{\tau}\int_0^\infty e^{-\frac{t'}{\tau}} \left(T(t) - T(t-t') \right) dt' \tag{3.1}$$

This form is very similar to response factors; if we assume we have equally spaced data (e.g. hourly temperatures), we can break this up into a sum:

$$F_i \simeq T_i - \left(1 - e^{-\frac{\Delta t}{\tau}} \right) \sum_{j=0}^{\infty} \left[e^{-j\frac{\Delta t}{\tau}} \right] T_{i-j} \tag{3.2}$$

where: F_i is the $\underline{i}$th filter,
T_i is the $\underline{i}$th temperature, and
Δt is the interval between points.

Because real walls are distributed systems, they cannot easily be described by a finite lumped parameter equation. Real walls can be thought of as having an infinite number of infinitesimal lumps of resistance and thermal mass and, hence, the equation describing them would contain an infinite number of filters (one for every infinitesimal lump of thermal mass). Equivalently,

$$J(t) \simeq U\,\Delta T(t) + \sum_{n=0}^{\infty} \frac{U_n}{\tau_n} F(t,\tau_n) \tag{4}$$

where:

U_n is the nth (mass-related) conductance and
τ_n is the nth time constant.

This expression (4) is general enough to contain all the important features of the dynamic performance of envelope components. Although it leaves the impression that one requires an infinite number of conductances and time constants to describe a real wall, we shall see that this need not be the case.

WALL MODEL

The simplest distributed system is one in which the parameters are homogeneous — they are independent of position within the wall. The homogeneous wall has been solved exactly[11] but the results are not usually expressed in the form we have used. In our form all time constants and all conductances are related to one another:

$$\tau_n = \frac{\tau}{n^2} \qquad\qquad U_n = \pm 2U \tag{5}$$

The exact solution in our notation for the homogeneous wall still has an infinite number of filters in it but only two parameters (See Ref 10 for derivation):

$$\underline{J}^1(t) = U\,(T^1(t) - T^2(t)\,) + 2U \sum_{n=1}^{\infty} F_n^1(t) - (-1)^n\, F_n^2(t) \tag{6.1}$$

$$\underline{J}^2(t) = U\,(T^2(t) - T^1(t)\,) + 2U \sum_{n=1}^{\infty} F_n^2(t) - (-1)^n\, F_n^1(t) \tag{6.2}$$

where: $\underline{J}(t)$ are heat fluxes (W/m^2) of the homogeneous wall,
$T(t)$ are temperatures (K) at wall surface,
$F_n(t)$ are the normalized temperature filters (K) of degree n,
U is the conductance of the slab(W/m^2-K), and
τ is the time constant of the homogeneous wall.

Note that we have defined the surface heat fluxes to be positive when they flow into the wall, that the superscripts 1 and 2 refer to a specific side of the wall (e.g. $T^1(t)$ refers to the surface temperature on side 1 of the wall), and that the notation of the filters has been changed slightly to account for the fact that all time constants in the filters are related:

$$F_n(t) = F(t, \frac{\tau}{n^2}) \tag{7}$$

The time constant of a <u>homogeneous</u> wall can be calculated from the thermal and physical properties of the wall:

$$\tau = \frac{L^2}{\alpha \pi^2} \tag{8}$$

where: L is the thickness of the wall[m] and
α is the thermal diffusivity of the material [m^2/s].

Note the factor of π^2 ($\simeq$10) in the above expression; it may be different from the purely numerical constants in other definitions of the time constant.

The above derivation is an exact solution for a homogeneous wall; nevertheless, since few walls can be represented by a homogeneous wall, we must generalize our model yet further. Specifically, we must find a semi-empirical generalization of the model for the <u>inhomogeneous</u> wall, or we are reduced to finding an infinite number of parameters. We have elected to find this generalization by modifying the coefficients in front of the filters, $F_n^{(1,2)}$, and, therefore, by adding some new thermal parameters to the equation. Thus we assume that each of the filters keeps the same relationship to every other filter but we allow some of their coefficients to vary.

$$J^1(t) = \underline{J}^1(t) + \sum_{n=1}^{n_o} a_n F_n^1(t) \tag{9.1}$$

$$J^2(t) = \underline{J}^2(t) + \sum_{n=1}^{n_o} b_n F_n^2(t) \tag{9.2}$$

where: J^1, J^2 are predicted fluxes (W/m^2) for an inhomogeneous wall,
$\underline{J}^1$, $\underline{J}^2$ are fluxes (W/m^2) for the equivalent homogeneous wall,
a_n, b_n are the new thermal parameters (W/m^2-K), and
n_o is the order of the model.

An inhomogeneous wall is completely described by its conductance, time constant, and a small number of pairs (two or three) of correction terms (a´s and b´s) to express the deviation from homogeneity. These coefficients have a physical interpretation; for example, a large positive a_1 implies that the wall is very massive on side one, whereas a negative value would imply that the side was resistive; the b coefficients serve the same role for side two.

Frequency Representation

The above equations are in the time domain; that is, they represent the flux at the current time as combinations of the time histories of the surface temperatures. An equally valid representation, and one that makes analysis much more straightforward is a frequency, or Fourier, analysis. In a frequency analysis the flux components at a specified frequency are related to the temperature components at that frequency by these transfer functions. The set of equations below are the Fourier transforms of the equations for a homogeneous wall:

$$\underline{J}^1(w) = \underline{H}^1(w)T^1(w) - \underline{H}^0(w)T^2(w) \tag{10.1}$$

$$\underline{J}^2(w) = \underline{H}^2(w)T^2(w) - \underline{H}^0(w)T^1(w) \tag{10.2}$$

where: $\underline{H}^1$ is the transfer function for side 1,
$\underline{H}^2$ is the transfer function for side 2,
$\underline{H}^o$ is the transfer function across the wall, and
w is the frequency (rad/s).

These transfer functions can be found from a Fourier inversion of the time-series equations and some algebraic simplifications:

$$\underline{H}^o(w) = U \frac{\left[-iw\tau \right]^{1/2}}{\sinh\left(\left[-iw\tau \right]^{1/2}\right)} \qquad (11.1)$$

$$\underline{H}^2(w) = \underline{H}^1(w) = U \frac{\left[-iw\tau \right]^{1/2}}{\tanh\left(\left[-iw\tau \right]^{1/2}\right)} \qquad (11.2)$$

We can find the transfer functions for our generalization of the wall model in terms of the solutions for the homogeneous case and the additional STPs:

$$H^0(w) = \underline{H}^0(w) \qquad (12.1)$$

$$H^1(w) = \underline{H}^1(w) + \sum_{n=1}^{n_0} a_n \frac{-iw\tau}{n^2 - iw\tau} \qquad (12.2)$$

$$H^2(w) = \underline{H}^2(w) + \sum_{n=1}^{n_0} b_n \frac{-iw\tau}{n^2 - iw\tau} \qquad (12.3)$$

where: $\underline{H}(w)$ are the homogenous transfer functions and
$H(w)$ are the corrected transfer functions.

The transfer function formulation has some advantages over the time domain; by using a non-linear search-type fitting routine in the frequency domain, a set of $(2+2*n_o)$ Simplified Thermal Parameters[10] can be found from a given data set. We use a Chi-squared minimization/maximum likelihood maximization type algorithm to find the best set of STPs for a given data set.

VALIDATION

While the model was designed to reduce the data collected by ETTU, it can be appropriately used to reduce other sets of data — regardless of the means of collection. The best way to validate the model is to supply it with data from a wall whose thermal parameters are known precisely, and the best way to get known thermal parameters is to generate the response factors for a typical wall and calculate fluxes from a given set of temperatures.

Since we are primarily interested in testing the model's ability to describe the dynamic characteristics of a wall, we have chosen as our test wall an insulated masonry wall, which has a large thermal mass as well as resistance. The component layers and their thermal properties given below are as listed in the ASHRAE 1977 Handbook of Fundamentals:

No.	L^a	k^b	ρ^c	$C_p{}^d$	Name
					(Side 1)
1	.10(0.333)	2.36(0.417)	1922(120)	0.79(0.19)	CommonBrick
2	.06(0.208)	0.211(0.037)	112(7)	13.4(3.2)	Vermiculite
3	.10(0.333)	2.2(0.387)	891(56)	0.67(0.16)	CinderBlocks
4	.02(0.062)	2.36(0.417)	1922(120)	0.84(0.20)	Plaster
					(Side2)

All quantities are in SI (English) units.

a) m (ft)
b) W/m^2-K (BTU/ft^2-F)
c) kg/m^3 (lb/ft^3)
d) W/kg-K (BTU/lb-F)

The metric conductance for this assembly is 0.758 [W/m^2-K], and the time constant given from the common ratio is 5.5 hours.

We generated a set of data using our preferred dynamic cycling over a 24-hour period. As can be seen from figure 3, the fluxes are pink-noise functions and, as expected, the temperatures are much smoother functions of time. As shown in the figure, side 1 is the side closest to the brick (i.e. the nominal "outside"), and side 2 is the plaster side (i.e. the nominal "inside"). We generated the data at 90-second intervals to give us a very dense data set. Note that in the figures three through six, we refer to the data generated from the response-factors as the "measured" data, and the fluxes we calculate from our analysis as the "predicted" data.

We used this data set in four separate runs employing 4 to 10 STPs. As the number of parameters increased so did the degree to which the predicted fluxes matched the measured (generated) fluxes. In the table below we have listed the STPs along with the thermal parameters from the calculation of response factor; we have also included the rms deviation of each of our model calculations from the response-factor fluxes.

No. of STPs	rms	U	τ	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4
	—	0.758	5.5								
4	5.9	0.767	5.1	10.18	6.16						
6	4.9	0.760	5.3	2.32	-1.87	14.96	15.09				
8	4.2	0.758	5.4	4.78	1.42	-0.23	-4.58	20.38	26.05		
10	3.4	0.758	5.6	2.79	-0.29	16.49	12.18	-37.20	-33.66	52.75	55.00

The improvement of the fit with increasing numbers of STPs can be seen by comparing graphs of predicted and measured flux for a given number of parameters with that for another. Figure 4 is a graph of the predicted and measured flux under that assumption that the wall is homogeneous (i.e. there are only two STPs); it follows the overall trends of the data fairly well but does not reproduce the high-frequency components. Figures 5 and 6 are the data analyzed using 4 and 10 STPs respectively; as can be seen they both do better at high frequencies than does the homogeneous case — the 10-STP case doing the best of all.

Nevertheless, in all calculations the basic trends are reproduced, suggesting that the number of parameters required is determined by the highest frequency one wishes to model: the highest frequency that can be accurately represented is approximated by the expression,

$$w_{max} \simeq 2 \frac{n_o^2}{\tau} \qquad (13)$$

where: w_{max} is the maximum frequency and
n_o is the order of model.

We can use our example to calculate the maximum frequency:

Maximum Representable Frequency and Cycle Time					
Number of STPs	2	4	6	8	10
Frequency [h^{-1}]	<.18	.36	1.45	3.27	5.82
Cycle Time	>34.5	17.5	4.3	1.9	1.1

These maximum frequencies should be compared with the maximum frequency of the data which was 20 cycles/hr; it is, therefore, not surprising that the model did not predict all of the high frequencies exactly — it would need over three dozen STPs in order to have enough high-frequency capability to fit the data exactly. For most applications eight STPs would give more than enough information about the thermal behavior; the actual number of STPs required, however, will depend on the use to which the information is to be put.

DISCUSSION

Although finding a model that adequately describes the thermal performance of walls is necessary for the reduction of data, it is not sufficient to give us an intuitive understanding of the make-up of the wall. If the model is to be generally applicable, we must be able to give the model parameters some physical interpretation.

The first and most important of the Simplified Thermal Parameters is U, the conductance of the wall. The conductance, or its inverse the resistance, is familiar to most; it can be used to calculate the steady-state (or long-term average) flux from the steady-state (or long-term average) temperature difference. As soon as we wish to understand the behavior of the wall under non-steady conditions, we need to extend our parameter list to include information about time-varying phenomena.

The most important parameter for non-steady phenomena is τ, the time constant of the wall, because, as discussed earlier, the time constant is a measure of when the behavior of the wall can be considered steady and when it must be treated as time-varying. The time constant allows us to weight different frequencies differently; that is, the low frequencies are unaffected as they pass through the wall, but the higher frequencies become attenuated. For a certain class of wall, namely a homogeneous wall, the conductance and time constant are all that is needed to completely describe the dynamic properties of the wall.

Few walls, however, can be considered homogeneous; most have the thermal mass and resistance concentrated in layers within the wall. For this reason, the remaining STPs, the storage factors, are dedicated to describing the deviation from homogeneity of the wall; specifically, the storage factors indicate something about the relative distribution of the thermal mass of the wall. A large storage factor indicates a concentration of mass near that surface of the wall. The storage factors of large order (e.g. a_4 vs. a_1) are restricted to smaller depths within the wall. For example, the first storage factor, a_1, takes a weighted average of the thermal mass from the entire wall — the greater the mass near side 1 the larger will be the value of the storage factor; for a_2, on the other hand, we will see only 1/4 as far into the (side 1) wall as for a_1; for a_3, only 1/9 etc. In our example wall, where most of the mass is concentrated on the outside of the wall with the insulation in the center, we would expect the storage factors to be generally positive; and, in fact, it appears that the side 1 storage factors are mostly positive and the side 2 storage factors are evenly split.

Once a set of STPs has been calculated it can be used to calculate the fluxes from any temperature history in a manner similar to that used by the response-factor method. At each time step, a set of $2n_o$ filters is calculated (see Eqs 3,7,8), where n_o is the order of the model and corresponds to the highest frequency of interest. From those filters, the conductance, and the $2n_o$ storage factors, one can calculate the fluxes for both sides (See Eqs 6 and 9). Because each filter is the sum of many terms, our model would appear to be more work than the response-factor approach; however, this is not the case. The filters of the type shown in Eq 3 are called infinite impulse response filters, which means that the filter at one particular time can be calculated from the filter at the previous time step; thus, the filters can be easily calculated once the first one has been obtained.

Because walls display such complex behavior at frequencies higher than that given by their time constant, we must drive them in a way that provides all of these important frequencies. Furthermore, we must drive them in such a way that does not sacrifice a good determination of the important parameters (e.g. U and τ). Finally, the heat drive must not depend on the thermal properties of the test wall — which we do not know beforehand. The dynamic cycle that best fits these criteria is a pink-noise cycle, and, accordingly, we use a pink-noise driving function in all of our experiments.

SUMMARY

This report has described the three basic components of a dynamic thermal measurement — the measurement technique, the driving strategy, and the data analysis which form a complete system. We have described our hardware for making an in-situ dynamic measurement (ETTU), the dynamic cycle of heat flux with which the test wall should be driven (pink-noise), and the model and set of simplified thermal parameters (STPs) that describe a wall so measured.

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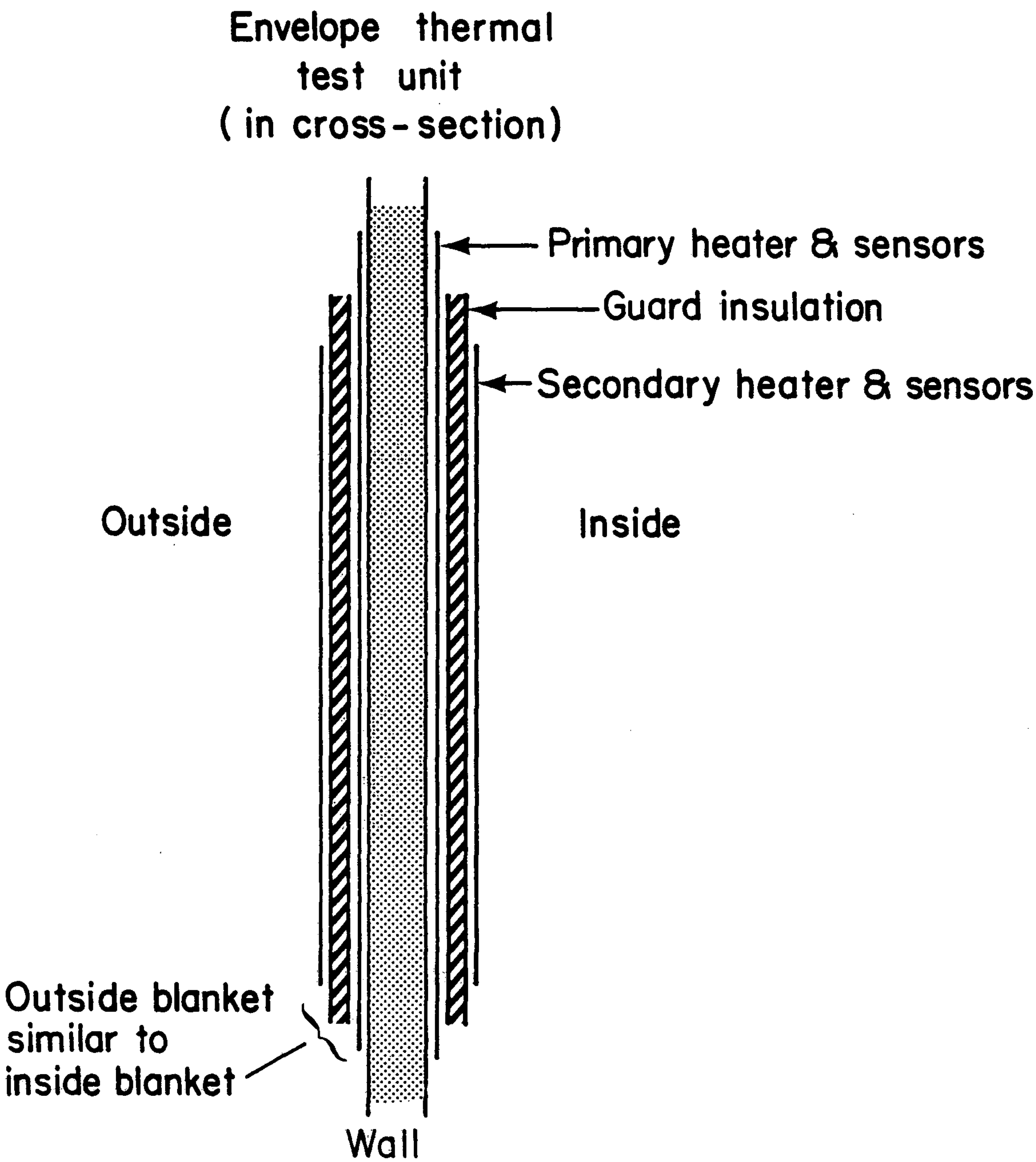

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Figure 1. Cross-section of Envelope Thermal Test Unit (ETTU).

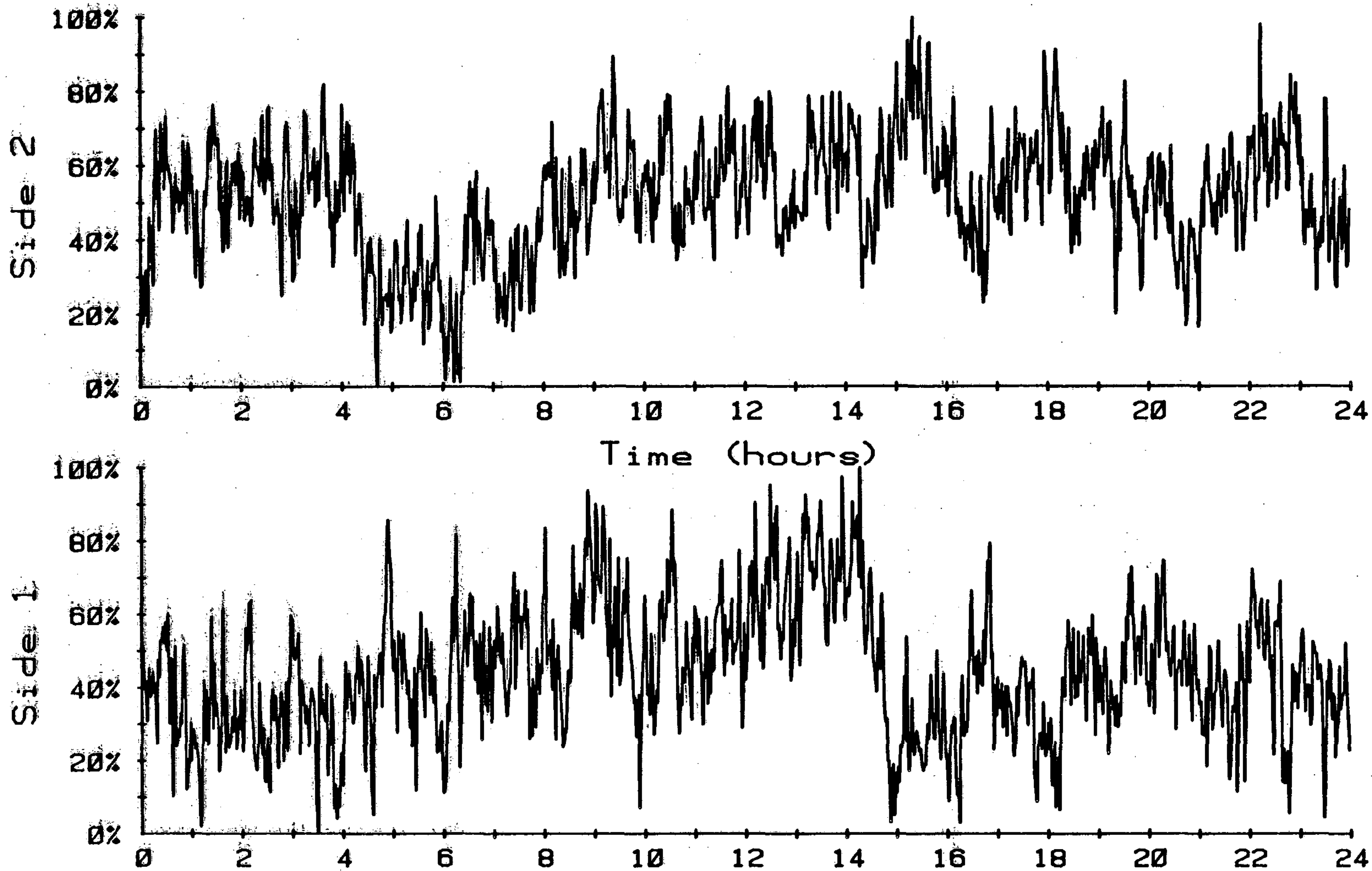

Figure 2. Normalized pink-noise spectra.

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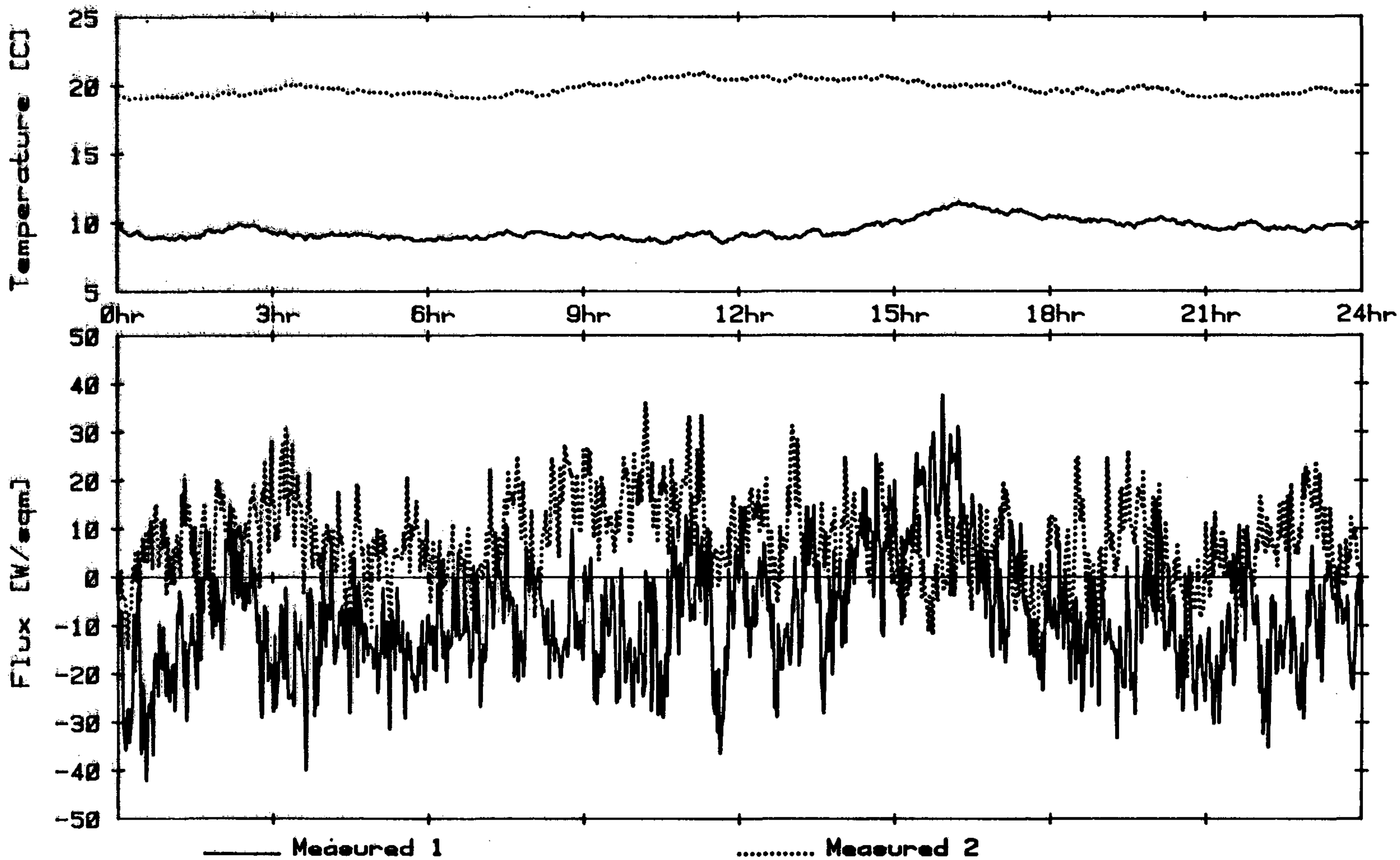

XBL 8111-12571

Figure 3. Measured data from the insulated cavity masonry wall.

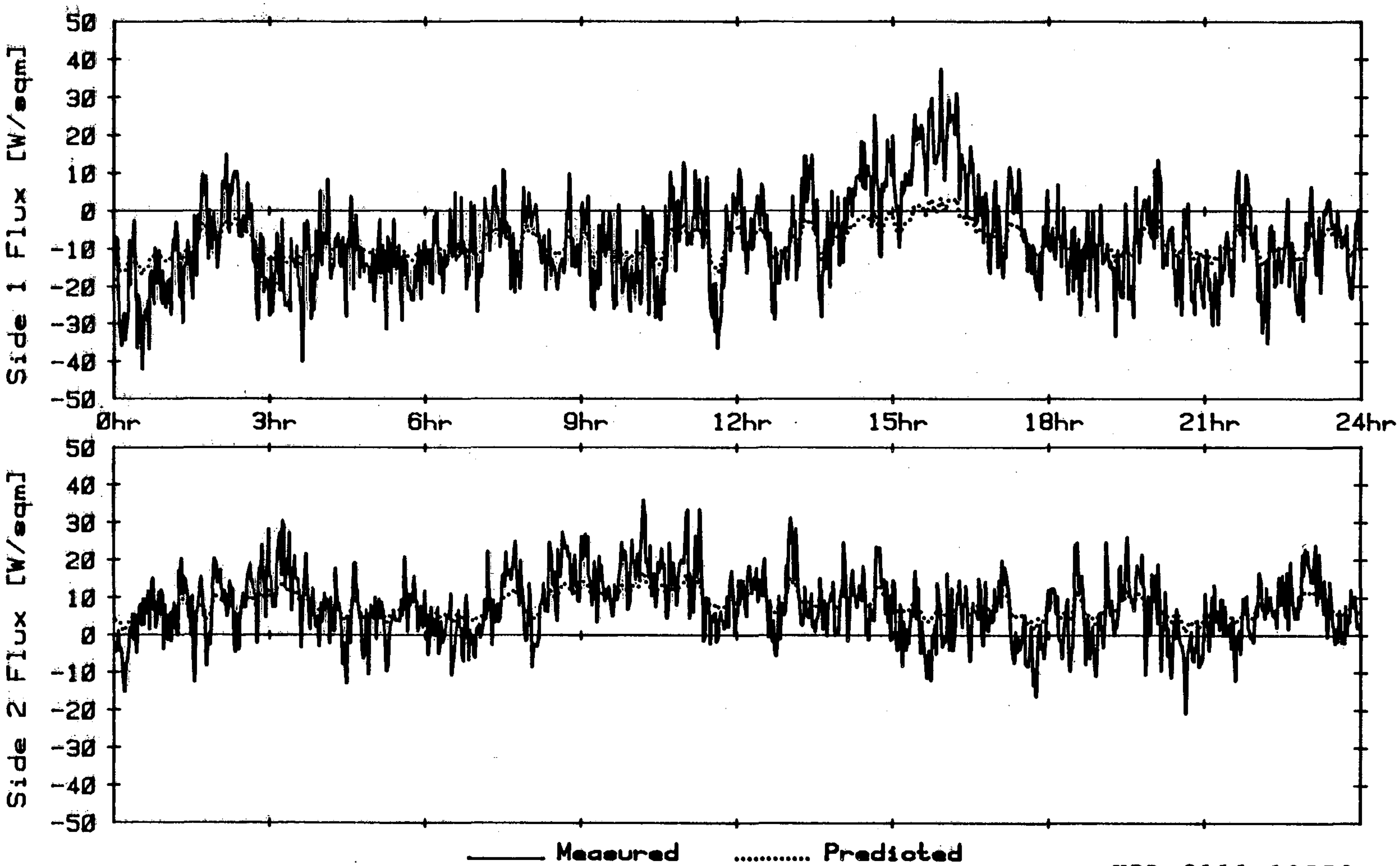

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Figure 4. Predicted and measured fluxes using two STPs

(i.e. a homogenous wall)

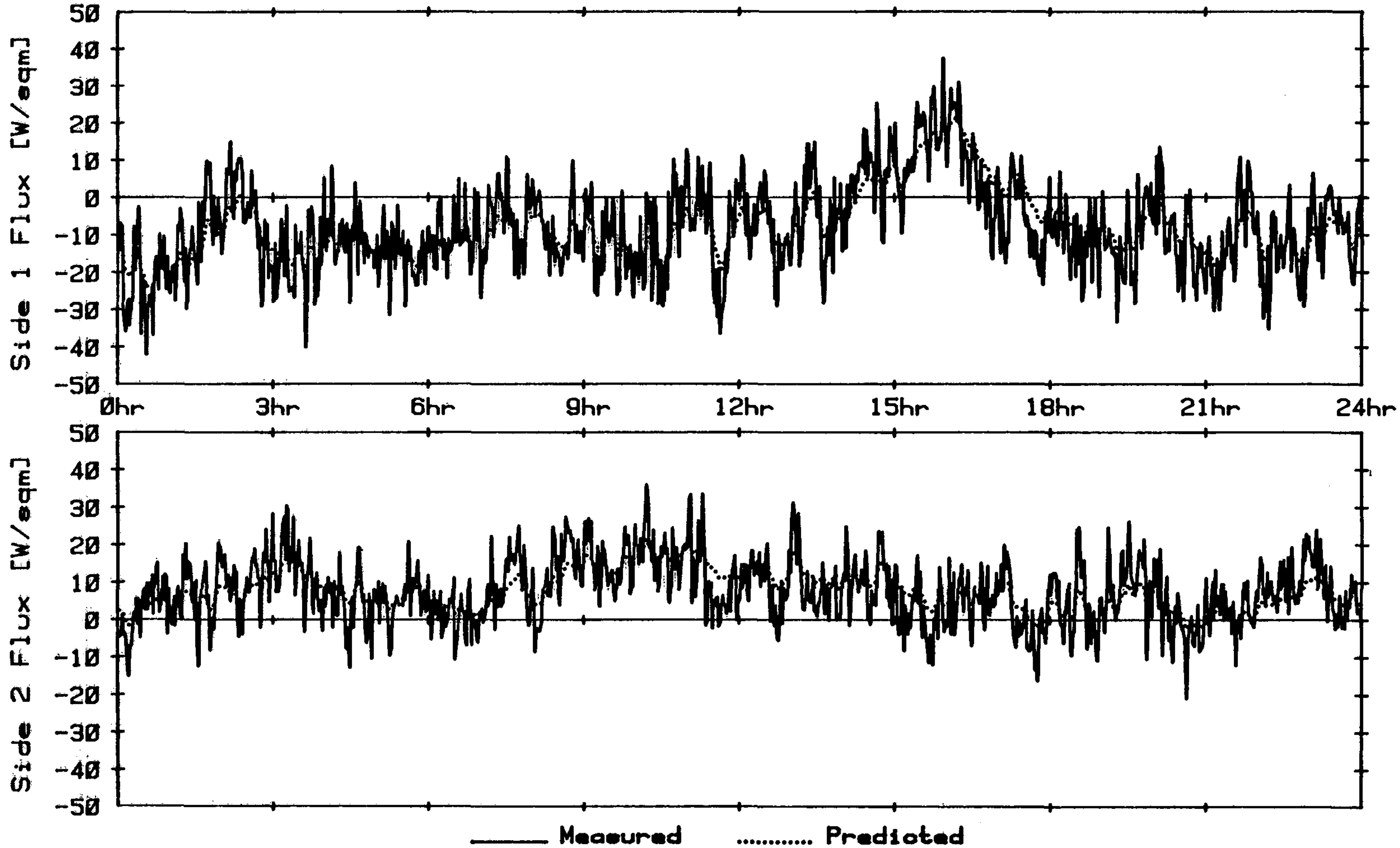

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Figure 5. Predicted and measured fluxes using four STPs.

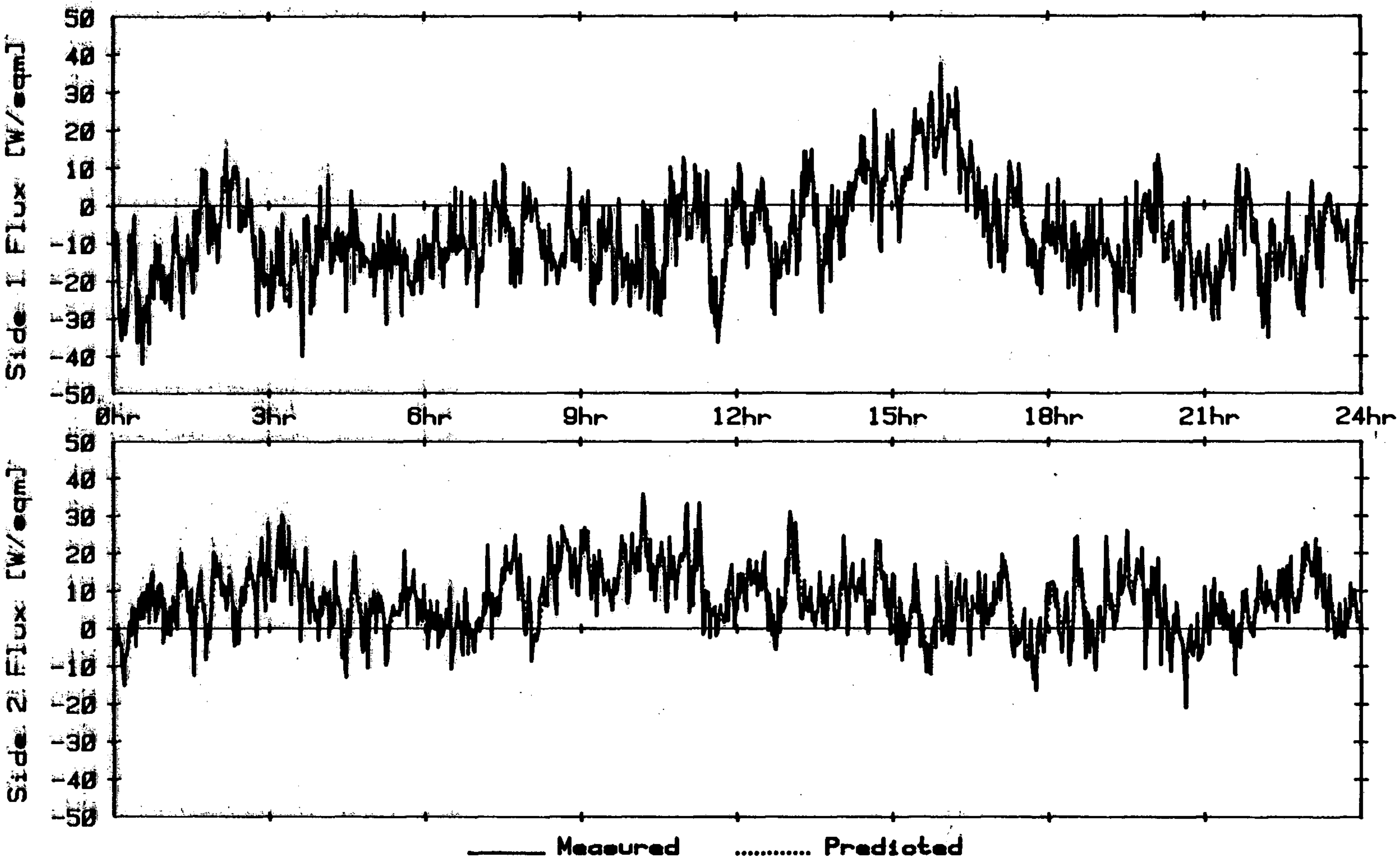

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Figure 6. Predicted and measured fluxes using ten STPs.

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