Title
Sales and Consumer Inventory

Authors
Hendel, Igal
Nevo, Aviv

Publication Date
2001-09-01

Working Paper No. E01-307

Sales and Consumer Inventory

Igal Hendel

University of Wisconsin, Madison and NBER

and

Aviv Nevo

University of California, Berkeley and NBER

September 2001

Abstract

Temporary price reductions (sales) are quite common for many goods and usually result in an increase in the quantity sold. We explore whether the data support the hypothesis that these increases are, at least partly, due to dynamic consumer behavior: at low prices consumers stockpile for future consumption. This effect, if present, has broad implications for interpretation of demand estimates. We construct a dynamic model of consumer choice and use it to derive testable predictions. We test the implications of the model using two years of store-level scanner data and data on the purchases of a panel of households over the same time. The results support the existence of household stockpiling behavior.

We wish to thank David Bell for the data, Iain Cockburn, Ken Hendricks, John Kennan, Ariel Pakes, Rob Porter, Joel Wladfogel and seminar participants in several seminars for comments and suggestions. The second author wishes to thank the Center for the Study of Industrial Organization at Northwestern University, for hospitality and support, and gratefully acknowledges support from the NSF (SES-0093967).
This paper is available on-line at http://www.haas.berkeley.edu/groups/iber/wps/econwp.html or contact University of California Berkeley, IBER, Berkeley CA 94720-1922.

1. Introduction

For many non-durable consumer products prices tend to be at a modal level with occasional short-lived price reductions, namely, sales. Unsurprisingly, during sales the quantity sold is higher. A price reduction may have two effects on quantity bought: first, a consumption effect if consumption is price elastic, second, a stockpiling effect if dynamic considerations lead consumers to inventory for future consumption. For example, in our sample the quantity of laundry detergents sold is 4.7 times higher during sales than during non-sale periods (provided there was no sale the previous week). Instead if there was a sale in the previous week, then the quantity sold is only 2.0 times higher. This pattern suggests the relevance of consumer stockpiling, which leads to several questions: Do consumers stockpile? Does stockpiling respond to price variations? Is the observed behavior consistent with dynamic forward looking behavior? In order to address these questions we derive and test the implications of a consumer inventory model.

There are several reasons to carefully model and quantify consumers' stockpiling behavior. First, suppose the data available for demand estimation presents frequent price reductions (as is the case with scanner data). In principle, the presence of frequent sales is a blessing for demand estimation, as they provide price variability needed to identify price sensitivities. However, when the good in question is storable, there is a distinction between the short run and long run reactions to a price change. Short run reactions reflect both the consumption and stockpiling effects. In contrast, for most demand applications (e.g., merger analysis or computation of welfare gains from introduction of new goods) we want to measure long run responses.

Second, detailed data, such as the household-level sample we describe below, present an opportunity to study whether a dynamic model of forward looking agents fits household behavior. The pronounced price changes, observed in some of the products we study, create incentives for consumers to stockpile. Our analysis will focus on grocery products, in particular, laundry detergents, yogurt and soft-drinks. From our data we can compute the potential gains from dynamic behavior. One such measure is given by comparing the actual amount paid by the household to what they would have paid if the price was drawn at random from the distribution of prices for the same

product at the same location over time. This is a lower bound on the potential gross gains from optimizing behavior. In our data the average household pays 12.7 percent less than if they were to buy the exact same bundle at the average price for each product.[2] Some households save little, i.e., they are essentially drawing prices at random, while others save more (the 90th percentile save 23 percent). Assuming savings in these 24 categories represent saving in groceries in general, the total amount saved by the average household, over two years in the stores we observe, is 500 dollars (with 10th and 90th percentiles of 150 and 860 dollars, respectively). Hence, the price movements provide incentives for storage and dynamic behavior.

Third, consumer stockpiling has implications for how sales should be treated in the consumer price index. If consumers stockpile, then ignoring the fact that consumers can substitute over time will yield a bias similar to the bias generated by ignoring substitution between goods as relative prices change (Feenstra and Shapiro, 2001).

A final motivation to study stockpiling behavior, is to understand sellers' incentives when products are storable. Although this paper does not answer this question, our findings provide some guidance on how to model the problem.[3]

Determining whether consumers stockpile in response to price movements would be straightforward if we observed the consumers' inventory. However, our data includes information on purchases and not consumption (therefore inventory is unknown). We could proceed in one of two ways. We could make an assumption about consumption (for example that it is constant). This would be a reasonable approach for products such that consumption effects are absent. However, such an approach would not help disentangle long run from short run effects for those products for which the distinction really matters. Indeed the results in Section 4 show that the consumption effect is important for some products. Thus, this approach will not provide any insights about the decisions

[2] This is for the 24 products we have in our data set. For the households we observe these products account for 22 percent of their total grocery expenditure.

[3] In ongoing work we study the behavior of a storable good monopolist. Most of the literature on sales is based on the Sobel (1984, 1991) model, which is a model of durable goods. Preliminary results show the main forces at play are quite different when the good is instead storable.

of how much to buy, which is crucial in order to quantify the magnitude of these effects.

We, therefore, take an alternative route. We propose a dynamic model of consumer choice and use it to derive implications about observed variables. We concentrate on those predictions of the model that stem exclusively from the stockpiling effect, but would not be expected under static behavior (only the consumption effect is present). In our model the consumer maximizes the discounted expected stream of utility by choosing in each period how much to buy and how much to consume. She faces uncertain future prices and in any period decides how much to purchase for inventory and current consumption. Optimal behavior follows a (conditional) S-s type behavior: if inventory is low enough the consumer buys and fills her inventory to a target level. In this model the consumer will purchase for two reasons, for current consumption and to build inventories.

In order to test the model we use store and household-level data. The data were collected using scanning devices in nine supermarkets, belonging to different chains, in two sub-markets of a large mid-west city. The store level data includes weekly prices, quantities, and promotional activities. The household-level data set follows the purchases of about 1,000 households over two years. We know when each household visited a supermarket, how much was spent in each visit, which product was bought, where it was bought, how much was paid and whether a coupon was used.

We test the implications of the model regarding both household and aggregate behavior. Our results support the model's predictions in the following ways. First, using the aggregate data, we find that duration since previous sale has a positive effect on the aggregate quantity purchased, both during sale and non-sale periods. Both these effects are predicted by the model since the longer the duration from the previous sale, on average, the lower the inventory each household currently has, making purchase more likely. Second, we find that indirect measures of storage costs are correlated with households' tendency to buy on sale. Third, both for a given household over time, and across households, we find a significant difference, between sale and non-sale purchases, in both duration from previous purchase and duration to next purchase. The duration effects are a consequence of the dependence of the trigger and target inventory levels (s and S) on current prices. In order to take

advantage of the low price, during a sale a household will buy at higher levels of current inventory, i.e., s is higher. Furthermore, during a sale a household will buy more and therefore, on average, it will take more time till the next time inventory crosses the threshold for purchase. Fourth, even though we do not observe the household inventory, by assuming constant consumption over time we can construct a measure of implied inventory. We find that it is negatively correlated with the quantity purchased and with the probability of buying. Finally, we find that the pattern of sales and purchases during sales across different product categories is consistent with the variation in storage costs across these categories. All these finding are consistent with the predictions of the model.

The rest of this paper is organized as follows. We next survey the relevant literature and explain how it relates to our paper. In section 2 we describe our data and display some preliminary analysis describing the three categories we focus on. Next, we present a formal model of consumer inventory and use it to derive testable predictions. Section 4 presents the results of the tests. We conclude by discussing how the findings relate to our motivation.

1.1 Literature Review

Several theoretical papers offer models of random price dispersion (Varian,1980; Salop and Stiglitz,1982; Narasimhan, 1988; and Rao, 1991), interpreted as sales, however, most of those models do not capture the dynamics of demand for a storable good in the presence of sales.

Models with cyclical pricing, in the context of a durable good, are presented by Sobel (1984), Conlisk, Gerstner and Sobel (1984), and Sobel (1991). They assume that two types of consumers –high valuation and low valuation– arrive in the market over time, where they stay until they find a price they are willing to pay, and leave the market forever once they purchased. In equilibrium sellers periodically find it optimal to lower prices to clear out low valuation consumers. Although relevant to our problem, the storable good problem is different than the durable good one, in that buyers do not leave the market forever.

Hong, McAfee and Nayyar (2000) present a storable good model that generates demand dynamics, namely, a (negative) link between current prices and future demand. They characterize

suppliers' behavior in a competitive context. It is shown that there exist equilibria where firms use random pricing. Moreover, prices are negatively correlated over time. Relative to Hong et al. (2000), our focus is on consumer behavior while their interest is to characterize prices. Therefore, we model demand more generally, which allows us to derive predictions relevant to our data.

Pesendorfer (forthcoming) studies sales of ketchup. He proposes a model in which a fixed number of consumers appears every period. These consumers differ in the willingness to pay (as in the Sobel-type models mentioned above) and in their store loyalty. He proves that in equilibrium the decision to hold a sale is a function of the duration since the last sale. His empirical analysis shows that both the aggregate quantity sold and the decision to hold a sale are a function of the duration since the last sale. We instead model the behavior of a consumer who can store the product, and derive the specific implications of the model regarding inventory behavior and its sensitivity to sales at the household level. Our model explicitly captures consumers' endogenous decision to return to the market, rather than having an exogenously given number of consumers arrive each period. Empirically, we also show that quantity sold is a function of the duration since the pervious sale, however, in our model this is true for both sale and non-sale periods. Furthermore, we provide evidence using household level data.

Boizot et al (forthcoming) study dynamic consumer choice with inventory. Unlike us, they assume that consumption does not respond to prices and more importantly due to data limitations can not apply any of the tests that we examine. Erdem et al (2000), look at sales from the inventory perspective, constructing a structural model, closer to Hendel and Nevo (2001). Besides several modeling assumptions we differ from them in focus. Our main goal is primally descriptive, testing the most general predictions of the theory, while their starting point is a dynamic forward looking model, which they structurally estimate. Hosken et al. (2000) study the probability of a product being put on sale as a function of its attributes. They report that sales are more likely for more popular products and in periods of high demand. Warner and Barsky (1995), Chevalier, et al. (2000) and MacDonald (2000), study the relation between seasonality and sales. The effect we study complements the seasonality they focus on. The same is also true for Aguirregabiria (1999), who

studies retail inventory behavior. His paper is about firm's inventory policy and its effect on prices, while our focus is on consumers' inventory policies given the prices they face.

There are numerous studies in the marketing literature that examine the effects of sales, or more generally the effects of promotions (see Blatteberg and Neslin, 1990, and references therein). Closest to our approach are the papers that examine the effect of sales on household stockpiling. Several papers[4] use household-level data to show that when purchasing during a promotion households tend to buy more units, larger sizes and in shorter duration to their previous purchase. We re-examine some of these questions. Using the panel structure of the data we decompose the differences between sales and non-sales into two effects: between household differences and within household differences. An important distinction that has been mostly neglected in the marketing literature.

Based on the results from the household-level data, there have been some attempts to find a dip in the (aggregate) quantity sold following a sale. The difficulty is finding this effect is noted in Blattberg, Briesch and Fox (1995) and termed as the "post-promotion dip puzzle." Neslin and Schneider Stone (1996) discuss eight possible arguments for why this might be the case. van Heerde, Leeflang and Wittink (2000) empirically examine the importance of these arguments. Our results directly shed light on this puzzle.

We discuss, in somewhat more detail, how our findings relate to this literature in Section 4 (as we present our results).

2. The Data and Categories

2.1 *Data*

We use data collected by IRI using scanning devices in nine supermarkets, belonging to different chains, in two separate sub-markets in a large mid-west city. The data has two components, one with store and the other with household-level information. The first contains prices, quantities

[4]For example, see Ward and Davis (1978), Shoemaker (1979), Wilson, Newman and Hastak (1979), Blattberg, Eppen and Lieberman (1981), Neslin, Henderson and Quelch (1985), Gupta (1988), Chiang (1991), Grover and Srinivasan (1992) and Bell, Chiang and Padmanabhan (1999).

sold and promotional activities, for each product (brand-size) in each store, in each week. The second component of the data set is a household-level sample of the purchases of 1,039 households over a period of 104 weeks. We know when a household visited a supermarket and how much they spent each visit. The data includes detailed information on 24 different product categories about which we know exactly which product each household bought, where it was bought, how much was paid, and whether or not a coupon was used.

2.2 *The Product Categories*

We focus here on three product categories available in the data: laundry detergents, soft-drinks and yogurt. We focus on only three categories due to space limitations, however we believe each of these categories represents a class of similar products. In addition, the differences between the characteristics of these products allow us to examine cross-category implications.

Laundry detergents come in two main forms: liquid and powder. Liquid detergents account for 70 percent of the quantity sold. Unlike many other consumer goods there are a limited number of brands offered. The top eight (six) brands account for 75 percent of the liquid (powder) volume sold. The leading firms are Procter and Gamble (which produces Tide and Cheer) and Unilever (All, Wisk and Surf). Detergents can be stored for a long time before and after they are initially used. However, they probably require a designated area for storage.

The yogurt category is very concentrated at the brand level with the top two brands, Dannon and Yogplait, accounting for roughly 78 percent of the quantity sold. These brands are offered in many different varieties, which are differentiated along two main dimensions: fat contents and flavor (plain, vanilla and various fruit flavors, which can be blended or on the bottom). Unlike detergent, yogurt can be stored for a limited time only (several weeks). Nevertheless, for the relevant time horizon, which is a function of the frequency of consumption and visits to the store (at least once a week for most of the households in the sample), yogurt is still a storable product. Once the container is initially opened yogurt can still be stored although for a shorter period.

The soft-drinks category combines several sub-categories: cola, flavored soda and club

soda/mixer, all of which can be divided into regular and low calorie. The club soda/mixer sub-category is the smallest and for much of the analysis below will be excluded. The cola and low-calorie cola sub categories are dominated by Coke, Pepsi and Rite, which have a combined market share of roughly 95 percent. The flavored soda sub categories are much less concentrated with both more national brands and also a larger share of generic and private labels.

In all these categories, the prices for brand-size combinations have a clear pattern: they are steady at a "regular" price, which might vary by store, with occasional temporary reductions. While this pattern is easy to spot it is less easy to define exactly what is a sale price. The first possibility we explore is to define the regular price as the modal price for each brand-size-store over the entire period, and a sale as any price below this level. This definition can miss changes in the regular, non-sale, price and therefore mis-classify sale and non-sale periods. We check the robustness of the analysis to the definition of sales in two ways. First, we explore defining a sale as any price at least 5, 10, 25 or 50 percent below the regular price (defined as above). Second, we define the regular price as the max price in the pervious three weeks, and a sale as any price at least 0, 5, 10, 25 or 50 percent below this price.

None of these definitions is perfect. For the purpose of this section, which is purely descriptive, the exact definition is less important. As we show in the next section the theory provides an exact definition of what is a sale from the consumer's perspective, but this provides little insight as to what definition we should use. Although for the most part all quantitative results reported below are robust to the different definitions, we must keep in mind that none of the definitions is perfect, hence any of them will introduce measurement error into the analysis in Section 4.

Using these different definitions of a sale we display in Table 1 for each category the percent of weeks the product was on sale and the percent of the quantity sold during those weeks. The figures are averaged across all products at all stores. It is not surprising that for any definition of a sale the percent of quantity sold on sale is larger than the percent of weeks the sale price is available. Already in this table we can foresee some of the support for our theory. The comparison between

the figures for detergent and yogurt provide support for stockpiling behavior. For any definition of a sale, despite the fact that sales are less frequent for laundry detergents the quantity sold on sale is higher than that sold for yogurt. Since laundry detergent is more storable than yogurt this is consistent with stockpiling behavior. Furthermore, the main alternative explanation is that consumers simply increase their consumption in response to a price reduction (i.e., they buy more because they consume more, not in order to stockpile). If anything it is more likely that the response of consumption to price is higher in yogurt, which makes this result even stronger. We return to this point in Section 4.

The products we examine come in different sizes. Consumers can stockpile by buying more units or by buying larger sizes. Indeed, size discounts are consistent with price-discrimination based on consumer storage costs.[5] In Table 2 we display statistics for the major sizes in each category. The sizes displayed account for 97 and 99 percent of the quantity sold of liquid detergent and yogurt, respectively. Soft-drinks are sold in either cans or various sizes bottles (the main size bottles are 16 oz. 1, 2 and 3 liter). For the purpose of this table we focus on cans, which can be sold as singles or bundled into 6, 12 or 24 packs.

The first column in Table 2 displays the quantity discounts. Since not all sizes of all brands are sold in all stores reporting the average price per unit for each size could potentially be mis-leading. Instead we report the ratio of the size dummy variables to the constant, from a regression of the price per 16 ounce regressed on size, brand and store dummy variables. The results show that there are quantity discounts in all three categories, but more so in detergents and soft-drinks.

The next three columns document the frequency of a sale, quantity sold on sale and average discount during a sale, for each size. We define a sale as any price at least 5 percent below the modal price. In all three categories there is an interaction between size and both the frequency of a sale and the quantity sold. The figures suggest that for both detergents and soft-drinks the larger sizes have more sales, and more quantity is sold on sale in the larger sizes. For yogurt, however, the pattern is

[5]Size discounts are also consistent with varying costs by size, and therefore one could claim are not at all due to price discrimination (Lott and Roberts, 1991).

opposite. There are more sales, and a larger fraction sold on sale, for the smaller sizes. In Section 4 we discuss this finding.

Our data records two types of promotional activities: *feature* and *display*. The *feature* variable measures if the product was advertised by the retailer (e.g., in a retailer bulletin sent to consumers that week.) The *display* variable captures if the product was displayed differently than usual within the store that week.[6] Defining as a sale any price at least 5 percent below the modal price we find that conditional on being on sale, the probability of being featured (displayed) is 19 (18), 31 (7) and 30 (14) percent for detergents, yogurt and soft-drinks, respectively. While conditional on being featured (displayed) the probability of a sale is 88 (47), 87 (83) and 78 (53) percent, respectively. The probabilities of being featured\displayed conditional on a sale increase as we increase the percent cutoff that defines the sale.

3. The Model

We present a simple inventory model, which we use to generate testable predictions about both observable household purchasing patterns and aggregate (store level) demand patterns. In order to derive analytic predictions, the model abstracts from important dimensions of the problem, like non-linear pricing and brand choice. Our goal here is to test the fundamental implications of stockpiling in the simplest possible set up. In a companion paper, Hendel and Nevo (2001), we impose more structure in order to deal with the additional dimensions ignored here.

3.1 The Basic Setup

Consumer *i* obtains the following (per period) utility in period *t*

$$u(c_{it}, \nu_{it}; \theta_i) + \alpha m_{it}$$

where c_{it} is the quantity consumed, ν_{it} is a shock to utility, θ_i is a consumer-specific vector of taste parameters and m_{it} is the utility from consumption of the outside good. The stochastic shock, ν_{it},

[6] These variables both have several categories (for example, type of display: end, middle or front of aisle). We treat these variables as dummy variables.

captures demand shocks unobserved to the researcher. For simplicity we assume the shock is additive in consumption, $u(c_{it}, \nu_{it}; \theta_i) = u(c_{it} + \nu_{it}; \theta_i)$, affecting the marginal utility from consumption. Low realizations of ν_{it} increase the household's need, increasing demand and making it more inelastic. We also assume $\frac{\partial u(c_{it} + \nu_{it}; \theta_i)}{\partial c} \geq \alpha p \ \ \forall p$ and $\forall \nu$, which is sufficient for positive consumption every period. This assumption has no major impact on the predictions of the model, while it avoids having to deal with corner solutions.

Facing random prices, p_t, the consumer at each period has to decide how much to buy, denoted by x_{it}, and how much to consume. Since the good is storable, quantity not consumed is kept as inventory for future consumption. We could assume consumption is exogenously determined, either at a fixed rate or randomly distributed (independently of prices), instead of endogenously determined. Both these alternative assumptions, which have been made by previous work, are nested within our framework. All the results below hold, indeed the proofs are simpler. We feel it is important to allow consumption to vary in response to prices since this is the main alternative explanation to why consumers buy more during sales, and we want to make sure that our results are not driven by assuming it away.

We assume the consumer visits one store at an exogenously given frequency, i.e., the timing of shopping is assumed to be determined by overall household needs (a bundle). Each of these products is assumed to be a minor component of the bundle, hence, need for these products does not generate a visit to the store.

After dropping the subscript *i*, to simplify notation, the consumer's problem can be represented as

$$V(I(0)) = \max_{\{c_t, x_t\}} \sum_{t=0}^{\infty} \delta^t E\left[u(c_t + \nu_t; \theta) - C(i_t) - \alpha p_t x_t | I(t)\right]$$
$$s.t. \quad 0 \leq i_t, \quad 0 \leq x_t \qquad \textbf{(1)}$$
$$i_t = i_{t-1} + x_t - c_t$$

where α is the marginal utility from income, δ is the discount factor, and $C(i_t)$ is the cost of storing

inventory, with $C(0) = 0$, $C' > 0$ and $C'' > 0$.[7]

The information set at time *t*, I(*t*), consists of the current inventory, i_{t-1}, current prices, and the current shock to utility from consumption, ν_t.[8] Consumers face two sources of uncertainty: utility shocks and future prices. We assume that shocks to utility, ν_t, are independently distributed over time. Prices are set according to a first-order Markov process.[9] We assume $F(p_{t+1}|p_t)$ first order stochastically dominates $F(p_{t+1}|p'_t)$ for all $p'_t > p_t$. Namely, the probability of a sale at time $t+1$ is higher if there is no sale at time *t*.

3.2 Consumer Behavior

In each period a consumer weights the costs of holding inventory against the (potential) benefits from buying at the current price instead of future expected prices. She will buy for storage only if the current price and her inventory are sufficiently low. At high prices the consumer might purchase for immediate consumption, depending on her inventory and the realization of the random shock to utility.

The solution of the consumer's inventory problem is characterized by the following Lagrangian

$$\max_{\{c_t, x_t, i_t\}} E\Big(\sum_{t=0}^{\infty} \delta^t \{ u(c_t + \nu_t; \theta) - C(i_t) - \alpha p_t x_t + \lambda_t (i_{t-1} + x_t - c_t - i_t) + \psi_t x_t + \mu_t i_t \} \mid I(t)\Big) \qquad \textbf{(2)}$$

where μ_t, ψ_t, and λ_t are the Lagrange multipliers of the constraints in equation (1). From equation (2) we derive the first order conditions with respect to consumption,

$$u'(c_t + \nu_t; \theta) = \lambda_t, \qquad \textbf{(3)}$$

purchase,

---

[7] Notice we do not need to impose $c \geq 0$ since we assumed $\partial u/\partial c$ is such that there is always positive consumption.

[8] It is quite reasonable to assume that at the time of purchase the current utility shock has still not been realized. This will generate an additional incentive to accumulate inventory – the cost of a stock out. Since this is not our focus, we ignore this effect, but it can be easily incorporated.

[9] A Markov process fits the observed prices reasonably well. Qualitatively, we believe our results continue to hold but the analysis is significantly more complicated.

$$\alpha p_t = \lambda_t + \psi_t, \tag{4}$$

and inventory,

$$C'(i_t) + \lambda_t = \delta E(\lambda_{t+1} | I(t)) + \mu_t. \tag{5}$$

Using these conditions we derive the basic predictions of the model. We show that consumers follow a (conditional) S-s type behavior, where the target inventory level is a function of current price only, $S(p)$, and the trigger inventory level depends both on prices and the utility shock, $s(p,v)$.

Let $c^*(p_t,\nu_t)$ be the consumption level such $u'(c^*(p_t,\nu_t) + \nu_t) = \alpha p_t$ and let $S(p)$ be the inventory level such $C'(S(p)) + \alpha p_t = \delta E(\lambda_{t+1} | I(t))$.

Proposition 1 In periods with purchases, $x_t > 0$, the target level of inventory, i_t, equals $S(p_t)$, a decreasing function of p_t, independent of the other state variables i_{t-1} and ν_t. Moreover, the inventory level that triggers a purchase is $s(p_t,\nu_t) = S(p_t) + c^*(p_t,\nu_t)$, which is decreasing in both arguments.

Proof: All proofs are provided in the Appendix.

Remark: If only discrete quantities are available or prices are non-linear in quantities then the target inventory $S(\cdot)$ becomes a function of i_{t-1} and ν_t.

Corollary 1: Holding p_t and ν_t constant the end of the period inventory level, i_t, is an increasing function of i_{t-1}.

Since we do not observe inventories in our data, our next step is to derive predictions regarding information we observe, namely, purchases, prices and duration between purchases.

Proposition 2 The quantity purchased, $x(i_{t-1},p_t,\nu_t)$, declines in the three arguments.

Corollary 2: There is a price $p^r < p^m$, where p^m is the highest (non-sale) price, such that at any

price $p \geq p^r$ if consumers buy they do so for current consumption exclusively.

3.3 Testable Implications

In this section we present the testable implications of the model. We focus on those predictions that help us distinguish the model from a static one, where all the reactions to sales stem purely from consumption effect. We first present the implication of Propositions 1 and 2 regarding the impact of inventories on purchases.

Implication I1: *Quantity purchased and the probability of purchase decline in inventories*.

As we do not observe consumer inventories we cannot directly test implication 1, hence, we use two different strategies. In Section 4.4 we assume that consumption is fixed, which allows us to compute a proxy for the unobserved inventory. As we mentioned above this is not an attractive assumption (and seems to be inconsistent with some of our findings) since it assumes away a main alternative. Therefore for most of the paper we resort to predictions on other aspects of consumer purchase behavior, which indirectly inform us about the stockpiling behavior. The following predictions follow this approach.

From Proposition 2 we know that during sales quantity purchased is bigger. Quantity purchased can increase simply because consumption increases when the price decreases (see equation (3)) or because of stockpiling (Corollary 2).[10] Since we do not know the size of the consumption effects, showing that quantity purchased increases during sales does not necessarily imply stockpiling. Therefore, we concentrate our attention on those implications that help us distinguish the model from a pure increase in consumption.

The consequence of stockpiling is a higher end of period inventory, which, all else equal,

[10]Corollary 2 describes the stockpiling effect for every price less than p^r, which we have implicitly equated with a sale. In the empirical analysis we do not observe p^r and therefore we will experiment with several definitions of a sale.

implies a longer duration until the next time the consumer's inventory crosses the threshold for purchase, $s(\cdot)$. Absent stockpiling duration would be unaffected by sales. This gives the following implication, which indirectly testifies to the presence of stockpiling.

Implication I2: *Duration until next purchase is longer during a sale.*

From Proposition 1 we know that the inventory that triggers a purchase, $s(\cdot)$, is lower at non-sale prices. Hence, according to our model, since during a non-sale purchase initial inventory is lower, on average the duration from previous purchase will be longer.

Implication I3: *Duration from previous purchase is shorter at sale periods.*

Furthermore, if the previous purchase was on sale then, all else equal, end of period inventory would have been higher. Then by Corollary 1 the consumer's inventory would be higher today, relative to their inventory if the previous purchase was not during a sale. Therefore, conditional on purchasing on non-sale today, it is more likely that the previous purchase was not during a sale.

Implication I4: *Non-sale purchases have a higher probability that the previous purchase was not on a sale, namely:* $Pr(NS_{t-1}|S_t)<Pr(NS_{t-1}|NS_t)$, *where S =sale purchase and NS=non-sale purchase.*

We now turn to implications on aggregate demand. The aggregation of implication I2 over a population of buyers who visit the supermarket at different periods leads to implication I5, namely, that store level demand increases with duration since the last sale. Moreover, since at non-sale periods consumers only demand for current consumption, while, on sale they hoard inventories (Corollary 2) we expect duration to have stronger effects during sales.

Implication I5: *Aggregate demand increases in the duration from the previous sale. Furthermore, duration effects are stronger during sale.*

4. Results

In this section we test the implications derived in the previous section. Using the store level data we show that, as predicted, the quantity purchased increases with duration from previous sale, once we control for promotional activity. Next we turn our attention to household data and use it to (1) study which household characteristics determine proneness to buy on sale; (2) characterize the difference between sale and non-sale purchases, both across households and for a given household over time; and (3) examine the purchase decision conditional on being in a store and the decision of how much to buy conditional on a purchase. We conclude this section by comparing the results across product categories.

For these tests we will need to define a sale and as a result we have to deal with two issues. First, sales have to be separated from changes in the "regular" price, for example, due to seasonality. In addition we have to define what is highest price which the consumers treat as a sale. Corollary 2 defines this cutoff but gives us little guidance as to how we should define a sale in the non-structural tests we perform below. Both these problems suggest that a correct definition of a sale will vary across households and across products, which implies that a definition that is held constant across households and products will introduce noise into the analysis (and most likely bias towards zero). Nevertheless, for the sake of consistency all the tests below were conducted defining a sale as any price at least 5 percent below the modal price, for that UPC in that store over the two years. We checked the robustness of the results to this definition by looking at different definitions of the "regular" price (e.g., the max over 3 or 4 previous weeks) and by varying the cutoff for a sale (from 0 to 25 percent below the regular price). Qualitatively the results are robust to the different definitions we examined.

4.1 Aggregate data: the effect of duration from previous sales

According to implication I5 demand should increase with the duration from the previous sale (i.e., as consumers run out of the inventory stockpiled during the last sale). Table 4 presents the results of regressing the log of quantity sold, measured in 16 ounce units, as a function of price, measured in dollars per 16 ounce, current promotional activity and duration since previous promotional activity. Different columns present the results for the different product categories.

The results in the second column, for each category, show the coefficient on duration since pervious sale is positive and significant as predicted by implication I5, for all three categories.[11] This result depends on controlling for duration since previous feature/display. In the first column, for each category, we display the results prior to controlling for these effects. For laundry detergents and yogurt the effect of duration is negative, which is driven by the correlation between sales and other promotional activities. These activities have a lasting effect, which implies a positive spillover into following weeks and therefore a negative effect of duration. Without controlling for duration from these promotional activities the coefficient on duration from previous sale captures both the effects.

We also ran the regressions in Table 4 separately for sale and non-sale periods. The results show that the negative effect of duration from previous sale, which we find in the first column, is driven by non-sale periods. During sale periods even prior to controlling for duration from previous feature/display the quantity increases the longer the duration from previous sale. This also means that, consistent with implication I5, the effect of duration is stronger during sales periods.

There is a literature in marketing that attempts to find a dip in the (aggregate) quantity sold following a sale. The difficulty in finding this effect is noted in Blattberg, Briesch and Fox (1995) as the "post-promotion dip puzzle." One exception is van Heerde, Leeflang and Wittink (2000), which use a complicated distributed lag of past sales, price, and other promotional activities to find the dip. Our numbers show that the dip is apparent once we control for duration from feature\display, in particular during non-sale periods, suggesting the lasting effect of feature\display that coincides with sales is hiding the expected dip.

[11] Duration is measured in weeks/100. In all the columns, even in cases where the coefficient on duration squared is significant, the implied marginal effect will be of the same sign as the linear term for the range of duration values mostly observed in the data. Therefore, we limit the discussion to the linear coefficient on duration.

4.2 Household sales proneness

We now turn our attention to the household data, described in Section 2. In this section we study the factors that impact a household's fraction of purchases on sale. For the 1,039 households we regress the fraction of times the household bought on sale, in any of the three categories we study, during the observed period on various household characteristics.[12] The results suggest that demographics have little explanatory power. We found that households without a male tend to buy more on sale, as do households with a female working less than 35 hours a week. Households with higher per person income are less likely to buy on sale, and so are households with a female with post high school education. These effects are just barely statistically significant, and some not significant, at standard significance levels. Overall observed demographics explain less than 3 percent of the variation, across households, in the fraction of purchases on sale. Both the direction and lack of significance of these results is consist with previous findings (Blattetberg and Neslin, 1990).

While the frequency a household buys on sale is not strongly correlated with standard household demographics it is correlated with two other household characteristics, relevant from the theory perspective. First, households that live in market 1 tend to buy less on sale. This is true even after controlling for demographic variables including income, family size, work hours, age and race, as seen in column (i) of Table 3. Market 1 has smaller homes with less rooms and bedrooms, relative to the other market. Under the assumption that home size proxies for storage costs, this finding is consist with our model that predicts lower storage costs are correlated with purchasing more frequently on sale. Second, though we know nothing about each households' house, we know the number of dogs they own. Columns (ii) shows that the having a dog is positively, and significantly, correlated with purchasing on sale, even after we control for other household characteristics. At the same time owning a cat is not. Assuming that dog owners have larger homes, while cat owners do not, this further supports our theory. Dog ownership is not just a proxy for the market since the effects persist once we also include a market dummy variable, as seen in column

[12]We also looked at the fraction of quantity purchased on sale. The results are essentially identical.

(iii).[13]

In the last three columns we explore the correlation between frequency of purchasing on sale and other shopping characteristics. The results in column (iv) show that households who bought in more than one store tend to buy more on sale. This finding relates to Pesendorfer (fortcoming) who reports that consumers that buy at low prices tend to shop in more stores. Column (v) shows that households who shop more frequently tend to buy more on sale. These effects also hold once we control for the characteristics used in columns (i) - (iii).

4.3 Sale vs. non-sale purchases

In Table 5, we compare, for each product category, the averages of several variables between sale and non-sale purchases. The first column, in each category, displays the average during non-sale purchases. The next three columns display the averages during sale purchases minus the average during non-sale purchases. The columns labeled *Total* display the difference between the mean of all sale purchases and the number in the first column. The *Total* difference averages purchases over time and across households. Hence, it reflects two different components: (i) a given household's sale purchases are likely to differ from non-sale ones (a within effect), and (ii) the profiles of households purchasing more frequently on sale is likely to differ from those not purchasing on sale (a between effect).[14] Actually, our theory has predictions regarding both the within and between effects and therefore in some cases also regarding the total effect. However, since each effect has a different interpretation we believe that in order to rigorously test the theory one has to separate these effects.[15] In order to do so, the next column, labeled *Within*, displays the difference between each household's

[13]The dog dummy variable might, alternatively, be a proxy for spare time, which may reflect a higher propensity to search. However, if the dog dummy variable was capturing propensity to search it would lose importance once we control for measures that proxy for the propensity to search (e.g., frequency of visits and number of stores). In fact dog ownership is uncorrelated with those proxies, moreover, the significance of the dog dummy variable is not affected by controlling for search proxies (see column (vi)).

[14]For a precise definition of within and between estimates see Greene (1997).

[15]Previous work was not always careful in separating these effects (see, for example, Neslin and Schneider Stone, 1996; van Heerde H., Leeflang P. and D. Wittink, 2000; and references therein.)

sale and non-sale purchases, averaged across households. Finally, the last column, labeled *Between*, displays the coefficient, from a cross household regression, of the mean of the variable in question for each household, on the proportion of purchases on sale (namely, the mean of the sale dummy across purchases of that household).

The results in the first row of Table 5 suggest that when purchasing on sale households buy more quantity (size times number of units). This is true both when comparing between households (households that make a larger fraction of their purchases during sales tend to buy more quantity) and within a household over time (when buying during a sale a household will tend to buy more), as predicted by Proposition 2. There is a difference across the three categories in how the additional quantity is bought. When buying laundry detergents households buy both more units and larger sizes. When buying yogurt households buy smaller units, but more of them, while when buying soft-drinks households buy less units but of larger size (e.g., a single 24 pack instead of 2 six packs). This relates to Table 2, which highlights the interaction of sales and non-linear prices.

While the effect that households buy more on sale is consistent with our theory it is also consistent with the main alternative theory: when prices go down households will buy and consume more of the product. If one is willing to assume that increased consumption is less relevant for some of our products, then we could use the increased quantity as proof for stockpiling. Instead of making such an assumption we turn to predictions that allow us to separate the two theories (and indirectly say something about this assumption). Rows 4 and 5 of Table 5 show that duration to next purchase is larger for purchases on sale, while duration from previous purchase is shorter for sale purchases. These finding match the within household duration predictions of implications I2 and I3. The alternative, of a pure increase in consumption, cannot explain these results. Furthermore, a back of the envelope comparison of the quantity and duration effects suggests that consumption goes up after sales. The consumption effect is particularly clear for sodas where the within increase in quantity purchased is 60% while the duration forward increases roughly 15%.

Notice that both implications I2 and I3 are within household implications. However, they have between households counterparts, namely, those households that consume more buy more on

sale. Indeed all the between affects are positive and quite large in economic terms. Households more prone to buy on sale buy larger quantities and less frequently. Although these figures do not rule out alternative theories, they are consistent with stockpiling, and possibly generated by heterogeneity in storage costs. We believe that the alternative explanations, of the between differences, are likely to involve a stockpiling component and therefore the between differences also point to the relevance of stockpiling. For example, a possible alternative explanation is that households with large demand are likely to have a larger incentive to search, as they spend a higher budget on the item, and also a higher incentive to store for future consumption once they find a low price; making them more prone to buy on sale, buy larger quantities, store and hence buy less frequently.

Since the large between effects suggest substantial heterogeneity across households in how responsive they are to sales, and perhaps in how much they store. Such heterogeneity provides sellers incentives to hold sales as a way of discriminating across types with different abilities to store or responsiveness to sales.

The magnitude of within effects might seem small, especially compared to the magnitude of the between effects. This could be driven by several factors. First, the between effects imply heterogeneity in the sensitivity to sales. Therefore the within effects, which average responses across all households, are likely to be more pronounced for households who are sensitive to sales, while close to absent (zero) for non-deal prone households. Hence, the within effect probably understate the responses of those households aware of sales. Second, the definition of sale probably introduces measurement error and biases the effect towards zero.[16] Third, there might be consumption and (potentially) stockpiling of several products. For example, a household might buy diet colas for the parents and a flavored soda for the kids. These could be two separate processes or there could be substitution between them. The results in Table 5 implicitly assume that these are perfect substitutes, since duration is measured to any purchase in the category. If we take the other extreme and compute the results for the sub categories separately the effects generally increase.

[16]For example, if we use a higher cutoff for the definition of a sale (i.e., a sale is any price at least 10, instead of 5, percent below the regular price) then the effects increase both in statistical significance and in economic magnitude.

Finally, we find that the probability the previous purchase was not on sale, given that current purchase was not on sale is higher (implication I4). The reasoning behind the prediction is that since non-sale purchases have a lower inventory threshold (namely, inventories have to be low for the buyer not to be willing to wait for a sale) a non-sale purchase informs us that inventories are low, which in turn means, other things equal, that the last purchase was not on sale. As before, the large between effects suggest a large cross-household heterogeneity in sales proneness, as those households buying today on sale, are a lot more likely to have purchased last time on sale as well.

4.4 Inventories, purchases and promotional activities

Up to now the results focused on testing the implications of our model assuming we cannot observe inventories. In this section we take an alternative approach. We assume constant consumption, compute a proxy for inventory and use it to study, in Table 6, (i) the decision to purchase conditional on being in a store and (ii) the quantity purchased by a household conditional on a purchase, as a function of the price paid and promotional activities. The dependent variable in the first set of regressions is equal to one if the household purchased the product and zero if they visited the store but did not purchase. In the second set of regressions, the dependent variable is the quantity purchased, measured in 16 ounce units. The independent variables include the price and promotional variables for the brand-size purchased, household-specific dummy variables (as well as dummy variables for each store and for each, broadly-defined, product).

We approximate the unobserved inventory in the following way. For each household we sum the total quantity purchased over the two year period. We divide this quantity by 104 weeks to get the average weekly consumption for each household. Assuming the initial inventory for each household is zero, we use the consumption variable and observed purchases to construct the inventory for each household at the beginning of each week. Since we include a household-specific dummy variable in the regressions assuming a zero initial inventory does not matter (as long as the inventory variable enters the regression linearly).

The results, presented in Table 6, are consistent with implication I1: the higher the inventory

a household holds the lower the probability they buy and the less they buy, conditional on a purchase. To get an idea of the magnitude of the coefficients consider the following. The average purchase of soft-drinks is roughly 7.25 units (116 oz.). Increasing the inventory by this amount, holding everything else constant, the probability of purchase conditional on being in a store decreases by 2.2 percentage points (relative to roughly 3 percent if inventory is zero). The effects for detergents and yogurt are 2.4 percentage points and 1.1, respectively. In the quantity regression the estimated coefficients suggest that each unit of (16 ounce) inventory reduces the quantity purchased by 0.72, 0.19 and 0.47 ounces, for the three categories respectively.

While the effect of inventories on quantity purchased is statistically different than zero, the magnitude of the effect is low. From Proposition 1 we know that assuming continuous quantities and linear prices , the model predicts a slope of minus one: conditional on a purchase the target inventory is not a function of current inventory and therefore every additional unit of inventory reduces the quantity purchased by one. Since in practice both these assumptions fail purchases should be less sensitive to inventories, in particular during non-sales, where no matter what the initial inventory is, consumers are predicted to buy only for consumption. Furthermore, the way we approximate inventory creates measurement error. For example, we ignore differentiation in the definition of inventory. Once a quantity is bought we just add it to inventory. In reality, however, consumers might be using different brands for different tasks. Finally, the inventory variable was constructed under the assumption of constant consumption, which might be right on average but will yield classical measurement error, which will bias the coefficient towards zero. As we noted in the previous section there is support in the data that consumption is not constant but reacts to prices. Indeed we claimed, in Section 2, that modeling this effect is important.

The model does not incorporate other promotional activities, but naturally they affect purchasing behavior. Columns two, four and six in Table 6 include promotional variables in the quantity regression and for the most part the effects of the promotional variables are as expected. We also allow the price sensitivity to vary with promotional activity. We find that sales tend to increase the quantity purchased and increase the price sensitivity.

4.5 A Cross-Category Comparison

The last set of tests of our theory involve a comparison across products. Unfortunately, none of the categories in our data is completely perishable. We were able to obtain data comparable to ours, but from a different city, on milk.[17] The retail price exhibits a very different pattern than the one we find in the categories in our data set. Prices tend to change every 6-7 weeks and stay constant till the next change. There are essentially no temporary price reductions. Assuming that milk is not storable (and that the only reason for sales is to exploit consumer heterogeneity in storage costs), then according to our model there should be no sales for milk. Indeed, that seems to be the case.

Another cross-category comparison involves the difference between laundry detergents and yogurt. Since the average duration between supermarkets visits is less than a week both these products are storable. However, there is a key difference between how one would store them. It is reasonable to assume that the storage costs for yogurt, holding total quantity fixed, are lower for smaller sizes of yogurt. It is easier to store in the refrigerator four small 8 ounce containers rather than one large 32 ounce container. Furthermore, unlike detergents, the storability of yogurt decreases once a container is opened. This suggests that for detergents we should see more sales for larger sizes and when consumers purchase on sales they buy larger units. For yogurt we should see the opposite: more sales for smaller sizes and purchase of smaller units on sale. Both these predictions hold and can be seen in Table 2 columns two and three and Table 5 second row .

Further evidence linking the relation between the easier-to-store size and sales is presented in the last column of Table 2, where we show the potential gains from stockpiling (defined in the Introduction) for the different sizes. Bigger savings are associated with the containers easier to store, namely larger sizes of detergents and soda, while small yogurt containers.

5. Conclusions and Extensions

In this paper we propose a model of consumer inventory holding. We use the model to derive

[17]We wish to thank Sachin Gupta, Tim Conley and Jean-Pierre Dube for providing us with these data.

several implications, which we take to the data. Our data consists of an aggregate detailed scanner data and a household-level data set. Using these data sets we find several pieces of evidence consistent with our model. (1) Aggregate demand increases as a function of duration from previous sale, and this effect differs between sale and non-sale periods. (2) Fraction of purchases on sale are higher in one market (the market that on average has larger houses) and if there is a dog in the house. Both of these could potentially be correlated with lower storage costs. (3) When buying on sale households tend to buy more quantity (either by buying more units or by buying larger sizes), buy earlier and postpone their next purchase. (4) Inventory constructed under the assumption of fixed consumption over time, is negatively correlated with quantity purchased and the probability of purchase. (5) The patterns of sales across different product categories is consistent with the variation in storage costs across these products.

The main negative result is that while the effects (e.g., the effect of inventory in Table 6) are consistent with the theory and statistically significant, they seem to be economically small, relative to the model's prediction. We discuss several causes for this result, including measurement error in the construction of the inventory and sales variables and aspects of reality ignored by the model. Both of these are handled, at least partly, by the structural model in Hendel and Nevo (2001).

We are currently exploring extensions along several dimensions. First, in Hendel and Nevo (2001) we estimate a structural model that addresses some of the issues we ignore here. Obviously, the cost is that we have to impose more structure on the data. The benefits, however, are substantial. The structural model provides interpretable estimates and enables us to perform counterfactual experiments. Both are these are crucial in order to address our motivation. Preliminary results, using data from the laundry detergents category, suggest that ignoring the dynamic effects can substantially bias the estimates of own- and cross-price elasticities and have profound effects on their implications.

Second, we are extending our theoretical analysis to include the supply side. This, jointly with the structural estimates, will allow us to examine questions like what are the optimal patterns of sales and why are they profitable to sellers.

References

Aguirregabiria, V. (1999) "The Dynamics of Markups and Inventories in Retailing Firms," *The Review of Economic Studies*, 66, 275-308.

Bell D., J. Chiang and V. Padmanabhan (1999), "The Decomposition of Promotional Response: An Empirical Generalization," *Marketing Science*, 18, 504-26.

Blattberg R., G. Eppen and J. Lieberman (1981), "A Theoretical and Empirical Evaluation of Price Deals in Consumer Nondurables," *Journal of Marketing*, 45, 116-129.

Blattberg, R. and S. Neslin (1990), *Sales Promotions*, Prentice Hall.

Blattberg R., R. Briesch and E. Fox (1995), "How Promotions Work," *Marketing Science*, 14, G122-132.

Boizot, C., J.-M. Robin, M. Visser (forthcoming), "The Demand for Food Products. An Analysis of Interpurchase Times and Purchased Quantities," *Economic Journal*.

Chevalier, J., A. Kashyap and P. Rossi (2000), "Why Don't Prices Rise During Peak Demand Periods? Evidence from Scanner Data," NBER Working Paper no. 7981.

Chiang J. (1991), "A Simultaneous Approach to the Whether, What and How Much to Buy Questions," *Marketing Science*, 10, 297-315.

Conlisk, J., E. Gerstner and J. Sobel (1984), "Cyclic Pricing by a Durable Goods Monopolist," *Quarterly Journal of Economics*; 99(3), 489-505.

Erdem, T., M. Keane and S. Imai (2000), "Consumer Price and Promotion Expectations: Capturing Consumer Brand and Quantity Choice Dynamics under Price Uncertainty," University of California at Berkeley, mimeo.

Feenstra, R. and M. Shapiro (2001), "High Frequency Substitution and the Measurement of Price Indexes," NBER Working Paper no. 8176.

van Heerde H., Leeflang P. and D. Wittink (2000), "The Estimation of Pre- and Postpromotion Dips with Store-Level Scanner Data," *Journal of Marketing Science*, 37, 383-395.

Hendel, I. and A. Nevo (2001), "Measuring the Implications of Sales and Consumer Stockpiling Behavior", working paper.

Hong, P., P. McAfee and A. Nayyar (2000), "Equilibrium Price Dispersion with Consumer Inventories," working paper.

Hosken, D., D. Matsa, and D. Reiffen (2000) "How do Retailers Adjust Prices: Evidence from Store-Level Data," working paper.

Grover R. and V. Srinivasan (1992), "Evaluating the Multiple Effects of Retail Promotions on Brand Loyal and Brand Switching Segments," *Journal of Marketing Research*, 29, 76-89.

Gupta S. (1988), "Impact of Sales Promotions on When, What, and How much to Buy," *Journal of Marketing Research*, 24, 342-55.

Lott Jr., J. and R. Roberts (1991), "A Guide to the Pitfalls of Identifying Price Discrimination," *Economic Inquiry* 29: 14-23.

MacDonald, J. (2000), "Demand, Information, and Competition: Why Do Food Prices Fall At Seasonal Demand Peaks?," *Journal of Industrial Economics*, 48 (1), 27-45.

Narasimhan, C. (1988), "Competitive Promotional Strategies," *Journal of Business*, 61 (4), 427-49.

Neslin S., C. Henderson and J. Quelch (1985), "Consumer Promotions and the Acceleration of Product Purchases," *Marketing Science*, 4(2), 147-165.

Neslin, S. and L. Schneider Stone (1996), "Consumer Inventory Sensitivity and the Postpromotion Dip," *Marketing Letters*, 7, 77-94.

Pesendorfer, M. (forthcomin), "Retail Sales. A Study of Pricing Behavior in Supermarkets," *Journal of Business*.

Rao, R. (1991), "Pricing and Promotions in Asymmetric Duopolies," *Marketing Science*,10(2), 131-44.

Salop, S. and J. Stiglitz (1982) "The Theory of Sales: A Simple Model of Equilibrium Price Dispersion with Identical," *American Economic Review*; 72(5),1121-30.

Shoemaker R. (1979), "An Analysis of Consumer Reactions to Product Promotions," in *Educators' Conference Proceedings*, Chicago: American Marketing Association, 244-248.

Sobel, J. (1984), "The Timing of Sales," *Review of Economic Studies*, 51, 353-368.

Sobel, J. (1991), "Durable Goods Monopoly with Entry of New Consumers," *Econometrica*, 59(5),

1455-85.

Varian, H. (1980), "A Model of Sales," *American Economic Review*; 70(4),651-59.

Ward, R. and J. Davis (1978), "A Pooled Cross-Section Time Series Model of Coupon Promotions," *American Journal of Agricultural Economics*, 60, 393-401.

Warner, E. and R. Barsky (1995), "The Timing and Magnitude of Retail Store Markdowns: Evidence from Weekends and Holidays,"*Quarterly Journal of Economics*;110(2), 321-52.

Wilson D., L. Newman and M. Hastak (1979), "On the Validity of Research Methods in Consumer Dealing Activity: An Analysis of Timing Issues," in *Educators' Conference Proceedings*, Chicago: American Marketing Association, 41-46.

Appendix

Proof of Proposition 1: If $x_t > 0$ then $\psi_t = 0$. If $i_t = 0$, there is nothing to show, simply $S(p_t) = 0$. In the complementary case, $i_t > 0$, we know $\mu_t = 0$. Using equation (4) and $\mu_t = \psi_t = 0$, equation (5) becomes: $C'(i_t) + \alpha p_t = \delta E(\lambda_{t+1} | I(t))$, which shows the end-of period inventory, i_t, is independent of the states variables i_{t-1} and ν_t. Moreover, since $F(p_{t+1}|p_t)$ increases in p_t, by equation (3) we get that the right hand side of the last equality declines in p_t. Hence, since $C'' > 0$ the end of period inventory, i_t, declines in price.

To show that the inventory level that triggers a purchase is $S(p_t) + c^*(p_t, \nu_t)$, assume first that the consumer is willing to buy when she has an initial inventory $i_{t-1} > S(p_t) + c^*(p_t, \nu_t)$. In such a case, $i_t > S(p_t)$, which violates equation (5) since it would hold with equality for $i_t = S(p_t)$, but the left-hand side is bigger and the right-hand side smaller for $i_t > S(p_t)$. Now suppose the consumer does not want to purchase when $i_{t-1} < S(p_t) + c^*(p_t, \nu_t)$. Since $x_t = 0$ we know $\psi_t > 0$, which in turn, by equation (3), implies $c_t > c^*(p_t, \nu_t)$. Hence, $i_t < S(p_t)$, which implies equation (5) cannot hold. By definition, it holds for $S(p_t)$, but for $i_t < S(p_t)$ the left-hand side is lower than the right-hand side. We conclude that the inventory $i_{t-1} = S(p_t) + c^*(p_t, \nu_t)$ triggers purchases. ■

Proof of Corollary 1: There are three cases to consider. Case 1: Both levels of inventory trigger purchase. By Proposition 1 the target level of inventory is $S(p)$, which is independent of initial inventory, and therefore the result holds. Case 2: i''_{t-1} triggers purchase but i'_{t-1} does not. By the second part of Proposition 1this implies that $i'_{t-1} > S(p) + c^*(p_t, \nu_t)$. Since no purchase was made optimal consumption will be (weakly) less than $c^*(p_t, \nu_t)$. Therefore, $i_t(i'_{t-1}, p_t, \nu_t) > S(p) = i_t(i''_{t-1}, p_t, \nu_t)$.

Case 3: Neither inventory level triggers purchase. If the optimal consumption $c(i_{t-1}, p_t, \nu_t)$ is decreasing in i_{t-1} then since there is no purchase the result trivially holds. Consider the case where the optimal consumption is increasing in i_{t-1}. Suppose that i_t decreases in i_{t-1}. Plugging equation (3) into equation (5), we see that the left-hand side of equation (5) declines in i_{t-1}. However, the right hand side increases in i_{t-1}. Since we supposed that consumers with higher i_{t-1} have a lower

i_t, as i_t decreases the consumer will have a higher expected future marginal utility from consumption. Moreover, if the non-negativity constraint binds, it adds another positive term in the right hand side. This leads to a contradiction, which implies i_t increases in i_{t-1}, during non-purchase periods. ■

Proof of Proposition 2: There are two cases to consider. Case 1: $x_t > 0$ and $i_t = 0$. In this case purchases equal consumption minus initial inventories: $x(i_{t-1}, p_t, \nu_t) = c(i_{t-1}, p_t, \nu_t) - i_{t-1}$. Since $x_t > 0$ we can combine equations (3) and (4) to get $u'(c_t + v_t; \theta) = \alpha p_t$, which implies that $c(i_{t-1}, p_t, \nu_t)$ declines in ν_t and p_t, and is independent of i_{t-1}. Thus, $x(i_{t-1}, p_t, \nu_t)$ declines in ν_t, p_t, and i_{t-1}.

Case 2: $x_t > 0$ and $i_t > 0$. From Proposition 1 we know $x(i_{t-1}, p_t, \nu_t) = S(p_t) + c(i_{t-1}, p_t, \nu_t) - i_{t-1}$. The result follows from Case 1 and Proposition 1, which showed $S(p_t)$ declines in p_t. ■

Proof of Corollary 2*:* At p^m if $x_t = 0$ there is nothing to show. If $x_t > 0$ we can combine equations (4) and (5) to get $C'(i_t) + \alpha p_t = \delta E(\lambda_{t+1} | I(t)) + \mu_t$. Moreover, from equation (4) we know $E(\lambda_{t+1} | I(t)) \le \alpha E(p_{t+1} | I(t))$. The right hand side of the last inequality is strictly lower than αp^m (as long as prices lower than p^m, arise with positive probability). Hence, since $\delta \le 1$ we know $C'(i_t) + \alpha p^m > \delta E(\lambda_{t+1} | I(t))$ for any $i_t \ge 0$. Therefore, equation (5) can hold with equality only if $\mu_t > 0$, i.e., when $i_t = 0$. Since the inequality is strict it holds also for some $p^r < p^m$. Concluding the proof that if any quantity is purchased, it is for consumption only, since no inventories will be left at the end of the period. ■

Table 1
Percent of Weeks on Sale and Quantity Sold on Sale,
by Category for Different Definitions of Sale

	Laundry Detergents		Yogurt		Soft-drinks	
	weeks on sale	quantity sold	weeks on sale	quantity sold	weeks on sale	quantity sold
regular price equals modal price and a sale is any price less than:						
< regular price	18.6	39.0	22.8	33.2	30.9	63.2
< .95*regular price	12.6	32.3	16.9	25.2	28.0	60.9
< .9*regular price	7.5	26.9	13.0	20.5	23.1	54.8
< .75*regular price	1.8	14.9	4.4	10.4	12.0	36.6
< .5*regular price	0.04	1.4	0.4	1.8	2.0	4.3
regular price equals max in previous 3 periods and a sale is any price less than:						
< regular price	12.9	33.8	16.2	26.2	29.9	61.1
< .95*regular price	8.9	28.6	13.4	21.5	26.3	58.5
< .9*regular price	5.9	24.8	10.0	17.4	22.4	54.5
< .75*regular price	1.7	13.9	4.0	9.7	11.5	36.1
< .5*regular price	0.05	1.4	0.5	1.9	1.8	3.8

Table 2
Quantity Discounts and Sales

	price/ discount ($ / %)	quantity sold on sale (%)	weeks on sale (%)	average sale discount (%)	quantity share (%)	saving (%)
Detergents						
32 oz.	1.08	2.6	2.0	11.0	1.6	4.3
64 oz.	18.1	27.6	11.5	15.7	30.9	1.3
96 oz.	22.5	16.3	7.6	14.4	7.8	10.0
128 oz.	22.8	45.6	16.6	18.1	54.7	18.6
256 oz.	29.0	20.0	9.3	11.8	1.6	–
Yogurt						
6 oz.	1.39	37.8	23.6	19.7	27.4	13.7
6*4.4 oz.	7.8	19.4	15.2	18.5	12.4	8.9
8 oz.	9.3	25.3	14.4	21.9	40.4	7.2
16 oz.	9.9	1.1	1.8	16.6	5.7	1.3
32 oz	28.3	15.9	10.8	13.0	12.9	3.0
Soft-drinks						
1 can	1.07	24.3	19.4	21.9	6.8	6.3
6 cans	2.3	59.5	34.3	35.4	16.8	21.8
12 cans	14.7	72.8	43.9	22.0	21.8	17.2
24 cans	34.4	78.3	41.7	20.8	54.5	17.6

All cells are based on data from all brands in all stores. The column labeled *price/discount* presents the price per 16 oz. for the smallest size and the percent quantity discount (per unit) for the larger sizes, after correcting for differences across stores and brands (see text for details). The columns labeled *quantity sold on sale, weeks on sale* and *average sale discount* present, respectively, the percent quantity sold on sale, percent of weeks a sale was offered and average percent discount during a sale, for each size. A sale is defined as any price at least 5 percent below the modal. The column labeled *quantity share* is the share of the total quantity (measured in ounces) sold in each size. The column labeled *savings* is the average percent increase in the amount consumers would pay if instead of the actual price they paid the average price for each product they bought.

Table 3
Correlation Between Households Fraction of Purchases on Sale and Household Characteristics

Variable	(i)	(ii)	(iii)	(iv)	(v)	(vi)
constant	0.50 (0.02)	0.50 (0.02)	0.49 (0.02)	0.39 (0.02)	0.51 (0.02)	0.44 (0.03)
market 1	-0.05 (0.01)	–	-0.05 (0.01)	–	–	-0.04 (0.01)
dog dummy variable	–	0.04 (0.01)	0.04 (0.01)	–	–	0.04 (0.01)
cat dummy variable	–	-0.001 (0.016)	0.005 (0.016)	–	–	0.005 (0.016)
# of stores	–	–	–	0.033 (0.006)	–	0.027 (0.006)
avg days b/ shopping	–	–	–	–	-0.008 (0.002)	-0.005 (0.002)
R-squared	0.045	0.037	0.051	0.059	0.042	0.080

The dependent variable is the fraction of purchases made during a sale averaged across the three categories: laundry detergents, yogurt, and soft-drinks. A sale is defined as a price at least 5 percent below the modal price. There are 1039 observations, where each household is an observation. All regressions also include per person HH income and dummy variables for a male head of HH, female works less than 35 hours and if she works more than 35 hours (excluded category is retired/unemployed), female post high school education and if head of HH is Latino. See text for discussion of the effect of these variables.

Table 4
Demand as a Function of Duration from Previous Promotional Activity

Variable	Laundry Detergents		Yogurt		Soft-drinks	
log(price per 16 oz)	-2.51 (0.56)	-2.46 (0.02)	-1.60 (0.02)	-1.61 (0.02)	-1.83 (0.03)	-1.83 (0.02)
duration from previous sale	-0.25 (0.10)	0.50 (0.12)	-0.88 (0.19)	0.93 (0.23)	1.23 (0.26)	2.27 (0.25)
(duration from previous sale)^2	-0.22 (0.22)	-1.05 (0.25)	2.05 (0.92)	-3.50 (1.00)	2.32 (0.35)	-3.58 (0.36)
feature	0.44 (0.03)	0.46 (0.02)	0.37 (0.02)	0.37 (0.02)	0.12 (0.02)	0.13 (0.02)
display	1.22 (0.06)	1.19 (0.02)	0.74 (0.03)	0.73 (0.03)	1.54 (0.02)	1.52 (0.02)
duration from previous feature	–	-0.71 (0.09)	–	-2.31 (0.19)	–	-0.14 (0.13)
(duration from previous feature)^2	–	1.16 (0.12)	–	7.36 (0.64)	–	0.36 (0.17)
duration from previous display	–	-0.47 (0.08)	–	-0.35 (0.08)	–	-1.73 (0.12)
(duration from previous display)^2	–	0.13 (0.11)	–	0.64 (0.16)	–	1.37 (0.18)
N =	41,995	41,995	50,523	50,523	37,024	37,024

The dependent variable in all regressions is the natural logarithm of quantity purchased (measured in 16 ounce units). Each observation is a brand-size combination in a particular store. Duration from previous sale/feature/display is measured as number of weeks, divided by 100, from previous sale/feature/display for that brand in that store for any size. All regressions include brand and store dummy variables. The regressions in the soft-drinks category are for the sub-sample of cans and include a dummy variables for high demand holiday weeks (July 4, labor day, Thanksgiving and Christmas).

Table 5
Differences in Purchasing Patterns Between Sale and Non-Sale Purchases

	Laundry Detergents				Yogurt				Soft-drinks			
	Avg during non-sale	Difference during sale			Avg during non-sale	Difference during sale			Avg during non-sale	Difference during sale		
Variable:		Total	Within households	Between households		Total	Within households	Between households		Total	Within households	Between households
Quantity (16 oz.)	4.79 (0.04)	1.55 (0.07)	1.14 (0.07)	2.22 (0.27)	1.60 (0.01)	0.16 (0.02)	0.20 (0.02)	0.22 (0.08)	5.00 (0.26)	5.04 (0.31)	3.01 (0.34)	6.44 (0.61)
Units	1.07 (0.01)	0.09 (0.01)	0.08 (0.01)	0.12 (0.03	2.63 (0.03)	0.99 (0.05)	0.80 (0.04)	1.24 (0.16)	4.18 (0.17)	-2.34 (0.20)	-1.75 (0.26)	-1.70 (0.29)
Size (16 oz.)	4.50 (0.03)	0.91 (005)	0.63 (0.05)	1.28 (0.20)	0.80 (0.01)	-0.19 (0.01)	-0.11 (0.01)	-0.23 (0.04)	2.82 (0.13)	4.31 (0.15)	2.73 (0.16)	5.05 (0.27)
Days from previous	44.38 (0.68)	6.70 (1.12)	-2.01 (1.03)	29.85 (8.11)	27.35 (0.59)	6.25 (1.10)	-1.27 (1.03)	6.87 (8.85)	24.71 (2.30)	8.85 (2.75)	-2.47 (2.07)	23.64 (7.66)
Days to next	43.75 (0.67)	8.56 (1.14)	1.95 (1.04)	28.91 (8.46)	26.08 (0.59)	9.87 (1.09)	2.78 (1.03)	21.64 (8.53)	21.49 (2.31)	12.89 (2.77)	2.50 (1.99)	29.74 (8.00)
Previous purchase not on sale	0.75 (0.01)	-0.29 (0.01)	-0.05 (0.01)	-0.77 (0.02)	0.78 (0.01)	-0.31 (0.01)	-0.13 (0.01)	-0.66 (0.03)	0.53 (0.02)	-0.26 (0.02)	-0.07 (0.03)	-0.36 (0.04)

A sale is defined as any price at least 5 percent below the modal price, of a UPC in a store over the observed period. The column labeled *Within households* controls for a household fixed effect, while the column labeled *Between households* is the regression of household means. Standard errors are provided in parentheses.

Table 6
Purchase Conditional on Store Visit and Quantity Purchased Conditional on Purchase by Household as a Function of Price and Promotional Activities

	Laundry Detergents		Yogurt		Soft-drinks	
Dep variable: variable	= 1 if purchase	quantity	= 1 if purchase	quantity	= 1 if purchase	quantity
constant	0.08	8.98	0.06	0.94	0.03	4.96
	(0.001)	(0.64)	(0.0005)	(0.44)	(0.0004)	(1.14)
inventory/100	-0.43	-4.45	-0.63	-1.16	-0.30	-2.91
	(0.01)	(0.27)	(0.0002)	(0.16)	(0.007)	(0.43)
price	–	-3.79	–	-0.27	–	-5.09
		(0.15)		(0.08)		(0.55)
price*sale	–	-1.53	–	0.70	–	-5.83
		(0.15)		(0.09)		(0.78)
sale	–	1.39	–	-0.08	–	3.22
		(0.16)		(0.12)		(0.62)
feature	–	0.14	–	0.11	–	-0.76
		(0.09)		(0.03)		(0.16)
display	–	0.18	–	0.14	–	0.38
		(0.08)		(0.04)		(0.15)
N =	149,802	12,731	149,802	10,457	149,802	4,768

All results are from linear regressions. The dependent variable in the regressions in columns 1, 3, and 5 is equal to one if the HH bought and zero if visited the store and did not buy. In all other regressions the dependent variable is the quantity purchased (measured in 16 oz units), conditional on purchasing a strictly positive quantity. All regressions also include household, product and store dummy variables. Prices ($/16 oz) and promotional variables are for the product purchased. A sale is defined as any price at least 5 percent below the modal price. The sample for soft-drinks includes only purchases of cans of low calorie colas. Standard errors are provided in parentheses.