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UNIVERSITY OF CALIFORNIA  
SANTA CRUZ

**ESSAYS IN MARKET DYNAMICS**

A dissertation submitted in partial satisfaction of the  
requirements for the degree of

DOCTOR OF PHILOSOPHY

in

ECONOMICS

by

**Curtis Matthew Kephart**

June 2015

The Dissertation of Curtis Matthew Kephart  
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Dean Tyrus Miller  
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## Abstract

Essays in Market Dynamics

by

Curtis Matthew Kephart

### In **Hotelling Revisits the Lab: Equilibration in Continuous and Discrete Time**

we investigate experimentally the impact of continuous time on a four-player Hotelling location game. The static pure strategy Nash equilibrium (NE) consists of firms paired-up at the first and third quartiles of the linear city. In a repeated simultaneous move (discrete time) treatment, we largely replicate previous findings in which subjects fail to converge to the NE. However, in asynchronous move (continuous time) treatments we see clear convergence towards the NE.

In **Stability in Competition? Hotelling in Continuous Time** we study Hotelling's classic location duopoly model in continuous time with flow payoffs accumulated over time and the price dimension made explicit. In an experimental setting, subjects chose price and location in treatments varying only by the speed of adjustment. We find that the principle of minimum differentiation generally holds, with little distance between subjects' location decisions. Price decisions, however, tend to be volatile, which is arguably consistent with theory. Our data also support recent literature that the ability to respond quickly increases cooperation.

**Aggregate Dynamics in a Large Virtual Economy: Prices and Real Activity in Team Fortress.** Virtual economies are growing as internet technology continues

to advance. We analyze a large and complete set of transaction data from the Team Fortress 2 virtual economy, which was designed to allow decentralized barter as the sole exchange institution. A small subset of goods emerges endogenously to act as media of exchange. Taking one of these money goods as numeraire, we generate daily prices for thousands of goods. We then generate macroeconomic indicators, including nominal growth and inflation. We find evidence of a particular sort of nominal rigidity related to the circulation of multiple types of currency goods, and also find some localized asset price bubbles associated with announcements by the game designers.

To my parents.

## Acknowledgments

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entitle Valve Corporation to review work before it is submitted for publication. The authors would also like to thank Danny Oberhaus at Statistics New Zealand and Anya Stockburger at the Bureau of Labor Statistics for offering expertise in the development of our price index. Matthew Browne, Matthew Jee, Sergio Ortiz, and Zachary Petersen provided excellent programming support.

## Part I

# Hotelling Model Experiments

# Chapter 1

## Hotelling Revisits the Lab:

## Equilibration in Continuous and

## Discrete Time

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### 1.1 Introduction

In his seminal model of spatial competition, Hotelling [38] analyzed the behavior of two sellers of a homogenous product choosing price and location in a bounded, one-dimensional marketplace. The model has since been expanded to allow numerous sellers to interact strategically in more general marketplaces. As the preeminent model of



spatial competition, it has been widely applied, e.g., in industrial organization to analyze geographic competition and product differentiation [50], and in political economy as a tool to analyze voting dynamics [24].

Here we investigate the dynamic foundations of this static model. How does the ability to quickly adjust product characteristics, and quickly respond to competitors' repositioning, affect firm behavior? The question is theoretical but quite relevant to 21st century applications as firms adopt new technology and new management practices that encourage them to compete on agility and positioning. Product cycles (and rebranding) seem to be accelerating in consumer electronics and lifestyle goods, while in other industries like enterprise software – for example in web-base applications like those for analytics and customer relationship management – permit their makers to continually tweak their product and respond to competitor repositioning.

Outside of voter analysis, empirical tests of the Hotelling model have been sparse. For differentiated products, for example, there is often no consensus on how best to define the attribute space, and most firms have been understandably reluctant to allow access to their data.

Experimental methods offer an empirical approach that avoids these problems, but the results so far have been mixed at best. Brown-Kruse et al. [8] and Brown-Kruse and Schenk [9] investigate a duopoly model with varying customer densities over a finite one-dimensional action space. Collins and Sherstyuk [17] look into a three-agent model with inelastic demand and uniform prices. There exists no pure-strategy equilibrium in this set-up [26]. Shaked [64] finds a unique mixed strategy equilibrium in which players

randomize uniformly over the second and third quartiles. Collins and Sherstyuk [17] find little support for Shaked’s equilibrium hypothesis.

In work most closely related to this paper, Huck et al. [39] investigate the four-player implementation of the location-only model. Eaton and Lipsey show that all Nash equilibria are in pure strategies with two players located back-to-back at the first quartile, and the remaining two players similarly located at the third quartile. Like Collins and Sherstryuk, Huck et al. find that the empirical distribution of locations is quite different than this NE distribution — subjects exhibit a “W-shaped” distribution of locations, with significant clustering near the second quartile (the median) as well as near the first and third quartile.

Using novel software for conducting economic experiments, we investigate the four-player location-only model with no prices, uniform customer density, inelastic demand and a bounded, finite action space. We compare a discrete time treatment with two continuous time treatments in which subjects are able to adjust their locations either with (or else without) a speed limit. We replicate earlier results that human subjects generally fail to converge to the distinctive equilibrium in discrete time, but establish for the first time that they do converge fairly reliably to the equilibrium in both continuous time treatments.

## 1.2 Hotelling Location Model

Each firm  $i = 1, 2, \dots, n$  chooses a location  $s_i \in [0, 1]$ . Firms produce homogenous goods with identical mill prices and linear transport costs. Each of a uniform continuum of consumers inelastically purchase a single unit at the lowest delivered price, i.e., from the closest firm.

Payoffs are determined as follows. Sort the strategy profile  $(s_1, s_2, \dots, s_n)$ , so  $S_{[1]} = \min\{s_1, s_2, \dots, s_n\}$ ,  $S_{[2]}$  is the second lowest location, ..., and  $S_{[n]} = \max\{s_1, s_2, \dots, s_n\}$ , so  $S_{[1]} \leq S_{[2]} \leq \dots \leq S_{[n]}$ . If there are exact ties ( $s_i = s_j$ ), then average the payoffs defined below over all feasible assignments of the tied players.

Normalizing unit profit to 1.0, firm  $i$ 's payoff is the length of its territory. As illustrated in Figure 1.1, that territory (except for the 'edge' players [1] and [n]) extends from the midpoint of the interval  $[S_{[i-1]}, S_{[i]}]$  with the firm just below to the midpoint of the interval  $[S_{[i]}, S_{[i+1]}]$  with the player just above. Thus the payoff function is

$$\Pi_i = \frac{1}{2}(S_{[i+1]} - S_{[i-1]}), i = 2, \dots, n - 1 \quad (1.1)$$

with

$$\begin{aligned} \Pi_1 &= S_{[1]} + \frac{1}{2}(S_{[2]} - S_{[1]}) \\ \Pi_n &= (1 - S_{[n]}) + \frac{1}{2}(S_{[n]} - S_{[n-1]}). \end{aligned} \quad (1.2)$$

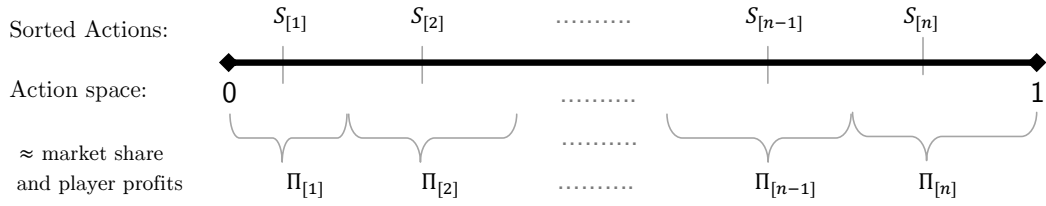


Figure 1.1: Hotelling Linear City action space

Equilibria in this game are sensitive to the number  $n$  of competing firms. It is well known that for  $n = 4$  there is a unique pure NE.

**Proposition.** The Hotelling location game for  $n = 4$  players has a unique pure Nash equilibrium, up to relabeling of players. The unique sorted equilibrium profile is  $S_{[1]} = S_{[2]} = \frac{1}{4}$  and  $S_{[3]} = S_{[4]} = \frac{3}{4}$ .

That is, in NE players are paired “back-to-back” at the first and third quartiles. It is easy to check that this is indeed a NE: by equation (1.2) a deviation to  $[0, .25)$  or to  $(.75, 1]$  clearly shrinks the deviator’s territory and payoff, while by (1.1) a deviation to  $(.25, .75)$  shifts the deviator’s territory but does not increase payoff. For a complete formal proof, see appendix B of Huck et al. [39], and for a proof of the uniqueness of pure NE, see Eaton and Lipsey [26].

### 1.2.1 Dynamic Considerations

In previous laboratory examinations of the Hotelling location-only game Collins and Sherstyuk, Huck et al., subjects were given random initial positions and allowed to select new actions simultaneously in discrete time, i.e., in a finitely repeated game. Although

the stage game is symmetric and constant sum (the total payoff is always 1.0), the players face considerable strategic uncertainty — to choose well, they must accurately predict their opponents' next location choices. The difficulty of predicting increases considerably as the number of players increases beyond  $n = 2$ . Players may also face a coordination issue; namely, where should each player go in the NE configuration? Thus it is easy to question the relevance of the static pure Nash equilibrium to a game with this type of dynamic structure.

To help understand what we might see in the dynamic game we ran computer simulations. In a discrete location space with 101 grid points and random initial locations, our automated agents played their myopic best response: in period  $t + 1$  each player allowed to move chooses a location that would maximize payoff given the period  $t$  location profile of the other players. Of more than 1000 simulations each of 600 periods in which agents moved simultaneously (everyone allowed to move every period), none (0%) converged to the NE and most locations were in the vicinity of the midpoint. (We tried several variants, but achieved convergence to NE in simultaneous move simulations only when initial locations were quite close to the NE and moves were limited to small increments.) By contrast, in more than 1000 asynchronous simulations in which players move one at a time in fixed order, all (100%) converged to NE, most of them within 200 periods in fixed order, all (100%) converged to NE, most of them within 200 periods and from a diverse set of initial locations. See the Online Supplementary Materials for more details.

These simulations suggest that the static model's predictive power may hinge on

the dynamic specifications. Moves are almost always asynchronous in continuous time, so in that respect it is similar to our turn-based simulations.

### 1.2.2 Testable Predictions

Our human subject experiment is designed to test two hypotheses suggested by the preceding theoretical discussion.

**Hypothesis 1:** Observed average deviations from the static Nash equilibrium (NE) will decrease over time in all treatments.

**Hypothesis 2:** Smaller average deviations from NE will be observed in continuous time (asynchronous) treatments than in the discrete time (synchronous) treatment.

## 1.3 Experiment

The experiment was programmed in ConG, software designed to implement continuous time economics experiments Pettit et al [57]. Subjects choose their target location using their mouse to click or to drag a slider, the black rectangle seen at the bottom of the screen in Figure 1.2. Subjects may also use the left and right arrow keys to shift locations incrementally. The horizontal position of the large green dot (currently with a score of 63.6) and black rectangle indicates the subject's current location, and other players' current locations are indicated with smaller dots of different colors. The vertical height of each dot indicates the players' current flow payoffs, also indicated by the number displayed by each players' dot. Accumulated flow payoffs are shown in the "Current Points" field, while points earned in all previous paid periods are indicated in

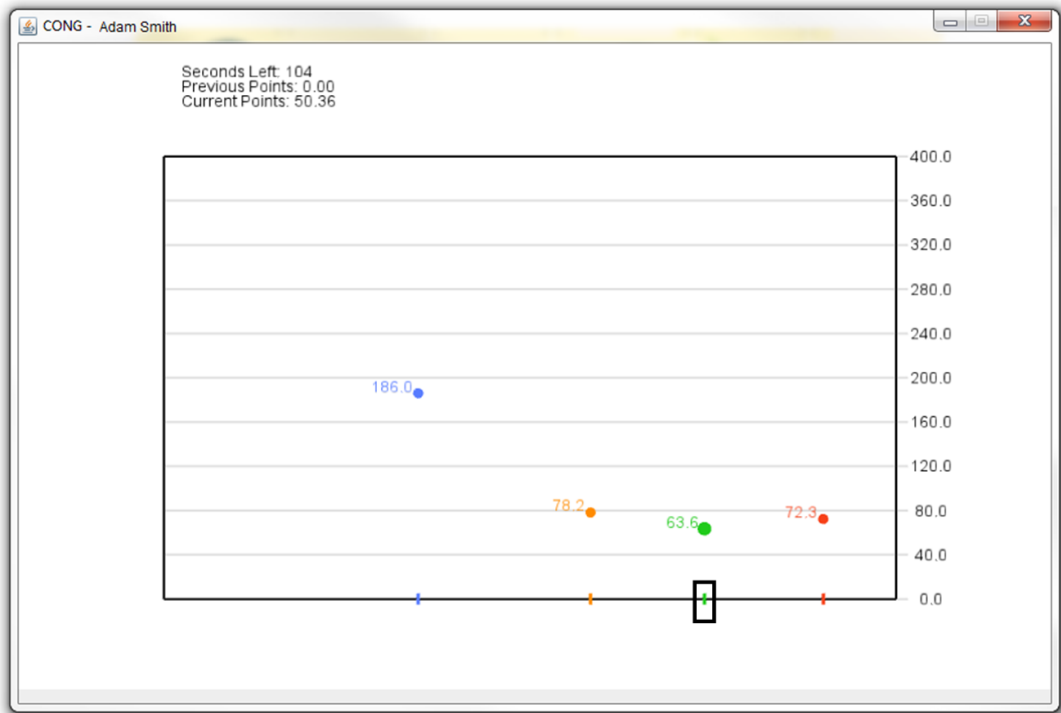


Figure 1.2: Hotelling game interface for Continuous Treatments.

the “Previous Points” field.

### 1.3.1 Treatments

We study three main treatments. The first is discrete time (“Discrete”). Periods are divided into  $n$  equal-length subperiods. Within each subperiod, subjects are freely able to move their location target using the mouse or arrow keys, but subperiod payoffs are determined solely by the target location profile chosen at the very end of the subperiod. Only at that point do subjects see the other players’ chosen locations. Subjects see a progress bar filling smoothly to indicate when the subperiod will end, and a “Subperiods Left” field counts down until the end of the period (neither are visible in the continuous

time UI shown in Figure 1.2). Payoffs for the entire period are the integral across subperiods of the piecewise constant flow payoffs, or equivalently, the average of the lump sum subperiod payoffs. A video of this and the other two treatments may be seen online at <http://youtu.be/NX6L1mV9iII>.

The other two treatments are continuous time. In continuous-time slow (“Slow”) when subjects select a new target location their current position moves toward the target at a constant rate (the “speed limit”). The chosen speed limit is such that it would take 30 seconds to traverse the entire interval.

In continuous-time instant (“Instant”), the subject’s current position moves immediately to the chosen target, with no perceptible delay. Actual latencies in our lab are less than 50 milliseconds; that is, no more than 50 ms elapse from the time a subject clicks a new location until the time when the effect of that click is revealed on all subjects’ displays. Of course, human reaction times are considerably longer than that, and subjects perceive the action as continuous in this treatment.

### 1.3.2 Procedures

Treatments are varied across sessions. Subjects are matched and rematched within “silos” of six subjects, and most sessions involve two silos. Sessions run for twelve periods, each period lasting three minutes. Discrete time periods are broken into 60 subperiods of 3 seconds each.



Table 1.1: Matching protocol

Subject	Period											
	1	2	3	4	5	6	7	8	9	10	11	12
1	R1	4p	4p	4p	4p	R2	4p	4p	R3	4p	4p	R4
2	R1	4p	4p	R2	4p	4p	4p	R3	4p	4p	R4	4p
3	4p	R1	4p	4p	R2	4p	4p	4p	R3	4p	R4	4p
4	4p	R1	4p	R2	4p	4p	R3	4p	4p	R4	4p	4p
5	4p	4p	R1	4p	R2	4p	R3	4p	4p	4p	4p	R4
6	4p	4p	R1	4p	4p	R2	4p	R3	4p	R4	4p	4p

*Note:* Player assignments are shown for each 6-player silo. The four subjects labelled “4p” in each column play the Hotelling location game that period; the other two subjects each play against three automated agents (“robots”) using one of four algorithms (R1 to R4) described in the text.

As shown in Table 1.1, each period a different subset of six subjects in the silo play the  $n = 4$  location game, while the two excluded subjects each play in a separate four-player game against three automated agents (“robots”). Robots reset their target position every 30 seconds following a specified sequence. For example subjects matched with robots in periods 1 through 3 face algorithm R1. The robot algorithms are held constant within each three-period set (periods 1-3, 4-6, 7-9 and 10-12). Subjects were told ahead of time that in one-third of periods their counterpart players would be players controlled by an automated computer program, and each subject faced each sequence once.<sup>1</sup>

<sup>1</sup>The location sequences for each robot group are as follows, **R1**: [0.25,0.5,0.75], [0.12,0.33,0.55], [0.23,0.76,0.85], [0.35,0.59,0.64], [0.25,0.26,0.75], [0.23,0.75,0.76], [0.45,0.5,0.8]. **R2**: [0.2,0.3,0.58], [0.29,0.59,0.6], [0.55,0.75,0.85], [0.25,0.26,0.74], [0.29,0.35,0.75], [0.23,0.75,0.76], [0.33,0.5,0.66]. **R3**: [0.75,0.5,0.25], [0.88,0.67,0.45], [0.77,0.24,0.15], [0.65,0.41,0.36],

Within each time treatment, each subject has equivalent opportunities to profit in robot periods, and so profit differences measure individual subjects’ relative ability to best respond. However, opportunities differ across time treatments — e.g., robots in Instant are sitting ducks for 30 seconds at a time while in Slow they often are moving steadily and the human subject is also constrained by the speed limit — so profit comparisons are less meaningful across treatments.

Our matching procedure balances several considerations. By the “Folk theorem,” a vast number of location configurations are supported in equilibrium with infinite horizon fixed matchings, and even with a finite horizon one might see fairly arbitrary conventions emerge as epsilon equilibria. Random matchings seem better suited to test static non-cooperative game predictions, but randomizing over large subject pools over many periods has two drawbacks. It produces relatively few “independent” observations, and enables a few confused subjects to “contaminate” a large fraction of the data.<sup>2</sup> Our solution was to match a different set of four subjects each period within a relatively small silo. Each silo gives us an independent observation, and confines possible contamination. As a bonus, the extra two subjects in each silo each period enabled individual performance comparisons against robots.

Sessions were conducted in the LEEPS laboratory at the University of California, Santa Cruz in February and March 2013. A total of 54 human subjects (randomly

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[0.75,0.74,0.25], [0.77,0.25,0.24], [0.55,0.5,0.2]. **R4**: [0.8,0.7,0.42], [0.71,0.41,0.4], [0.45,0.25,0.15], [0.75,0.74,0.26], [0.71,0.65,0.25], [0.77,0.25,0.24], [0.67,0.5,0.34]

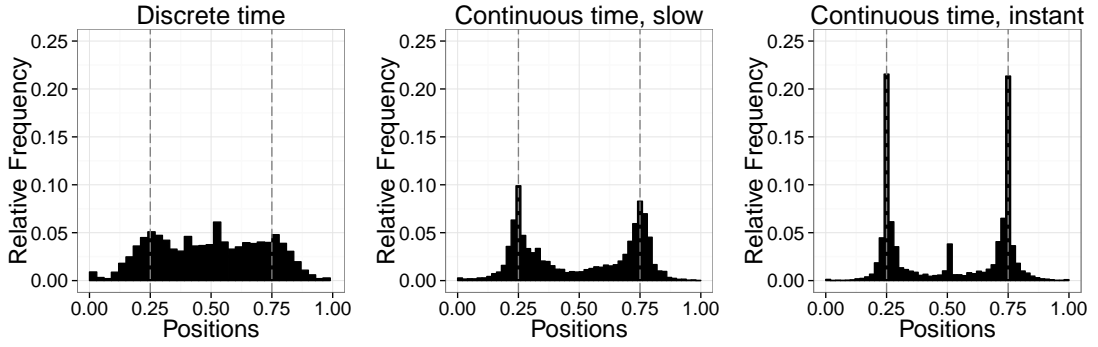
<sup>2</sup>In pilot sessions, post-experiment surveys and relatively low earnings for a few subjects indicated persistent confusion. Several other players exposed to one of these confused individuals exhibited less systematic behavior later in the session, even when no longer playing against that individual.

assigned into 9 silos) were drawn from the LEEPS Lab subject pool using recruitment package ORSEE [35]. Sessions lasted about 90 minutes, starting with instructions (reproduced in the Online Supplementary Materials), two practice periods and a quiz, followed by the 12 periods specified in Table 1.1, and finally cash payment. Subjects were told that they would be matched with three robots (algorithms unspecified, but subjects found it obvious that they were not human) in one-third of the periods, and with three other human participants in the room (silos unspecified, and their existence was not obvious to subjects) in the other two-thirds. Sessions also included a standard Holt-Laury risk preference test, but the elicited values turned out to provide negligible explanatory power and, to conserve space, are not discussed further.

Our location game is constant sum, and points were scaled so that each period the total score of all four players summed to 400 – or 100 points per-player per-period on average. Subjects received a \$5 show-up fee and between \$0.025 and \$0.034 for each point earned over the entirety over the session in excess of 1000 points, including both all-human periods and robot periods. Participating subjects received an average total payment of \$14.22.

## 1.4 Results

The three panels of figure 1.3 show the overall distribution of locations by treatment in the last half of each session (periods 7-12). In the first panel for the Discrete treatment, there is little evidence of the NE configuration. There are modest modes



*Note:* Location distributions are from the later half of sessions, excluding any period involving robots.

Figure 1.3: Relative frequency distribution of player locations, by treatment.

near the NE quartile points 0.25 and 0.75, but also a mode near the non-NE quartile (median) point 0.50, and a noticeable aversion to edge locations. The data of Huck et al. [39] has similar properties, but all three modes are sharper and location choices between the modes are less common than in our data. The second panel from the Slow treatment shows that we have strong modes in the vicinity of both NE quartile points, and no mode at 0.5; indeed, choices distant from .25 and .75 are rare and transient. The third Instant treatment panel shows sharp modes at the NE quartile points, and rather little activity elsewhere. (The small mode at 0.5 mainly reflects the behavior of a single player who stubbornly occupied that position for a number of periods in one session, despite unimpressive payoffs.)

The histograms in Figure 1.3 suggest better convergence to NE in Instant than in Slow, and little or no convergence in Discrete. To test our hypotheses more formally, we define a metric for the average absolute distance.

from NE, as follows. Sort the players' time- $t$  locations as usual:  $S_{[1]t}$  is the location

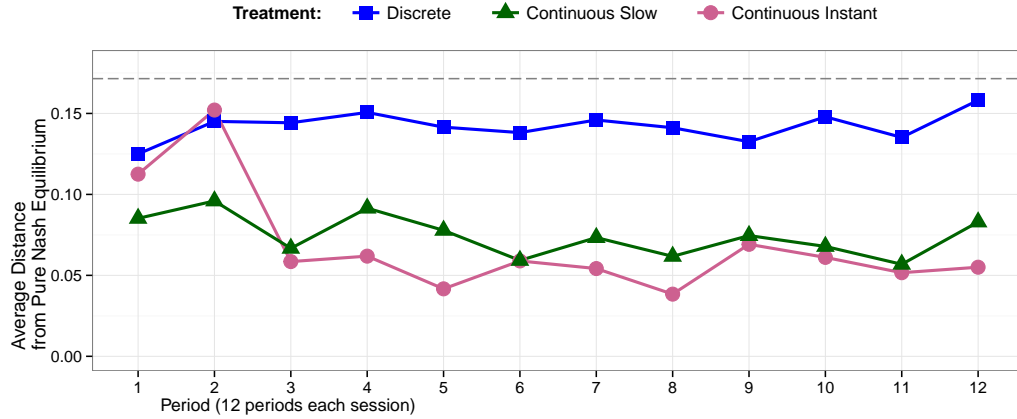


Figure 1.4: AvgAbsDist by period and treatment

of the “left most” player at time  $t$ ,  $S_{[2]t}$  is second from the left, etc., so that  $S_{[1]t} \leq S_{[2]t} \leq S_{[3]t} \leq S_{[4]t}$ . Then  $AvgAbsDist$  at time or subperiod  $t$  is:

$$AvgAbsDist_t = \frac{1}{4} (|S_{[1]t} - 0.25| + |S_{[2]t} - 0.25| + |S_{[3]t} - 0.75| + |S_{[4]t} - 0.75|) \quad (1.3)$$

Clearly  $AvgAbsDist = 0$  at a NE profile, and is bounded above by 0.5 (achieved when all  $S_{[i]t} = 0$  or all = 1). Ten million Monte Carlo simulations indicated that its expected value for a sample drawn from a uniform random distribution is 0.1715, with a median of 0.1646.

The overall median value of  $AvgAbsDist$  in our Discrete data is 0.1417, not much different from the random benchmark of 0.1715, while the median values in Slow and Instant, 0.0639 and 0.0461 respectively, are substantially lower. However, looking at the overall average  $AvgAbsDist$  of each session period we can reject the null hypothesis that the median is 0 in each treatment.

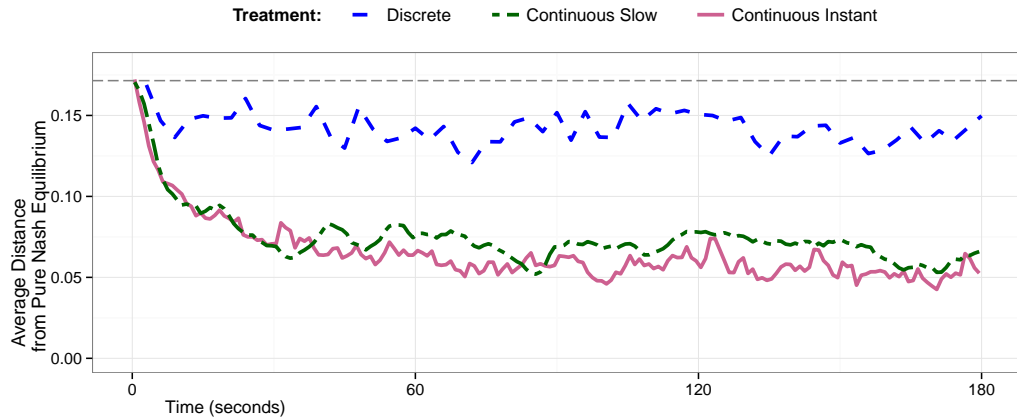


Figure 1.5: AvgAbsDist by second, by treatment over the period.

Is there a trend towards NE across periods? Figure 1.4 plots *AvgAbsDist* in human-only location games averaged period-by-period over all silos in each treatment. In the two Continuous treatments we do see a trend towards 0 (i.e., towards NE) after the first few periods, but it seems to level off thereafter. In Discrete periods, *AvgAbsDist* is not much below the random benchmark (the gray dashed line) and has no noticeable downward trend.

Are there trends within a typical period? Figure 1.5 plots *AvgAbsDist* in human-only location games averaged second-by-second (or, for Discrete, subperiod-by-subperiod) over all periods in each treatment. The software initializes subjects at uniform random locations, thus at time zero *AvgAbsDist* is about 0.17, as expected. Again, in the Discrete treatment, there is little trend towards NE within the period, while in the continuous treatments there is a clear trend at first but it stagnates after about 40 seconds.

Our main hypothesis tests are based on the following variant of the regression model

of Noussair et al. [53], applied to human-only data from latter half of all sessions (periods 7 through 12):

$$AvgAbsDist_{jt} = \beta_1 \left( \frac{1}{t} \right) + \beta_{21} \cdot D_1 \left( \frac{t-1}{t} \right) + \beta_{22} \cdot D_2 \left( \frac{t-1}{t} \right) + \beta_{23} \cdot D_3 \left( \frac{t-1}{t} \right) + u_{jt} \quad (1.4)$$

where  $AvgAbsDist_{jt}$  is the observed average absolute distance from the pure Nash equilibrium configuration, in treatment  $j$  at time  $t$  – each 100 milliseconds in continuous time or subperiod  $t$  in the case of discrete time.  $D_1$  is a dummy variable that is equal to 1 if the observation is from treatment  $j = 1$ , Discrete time. Likewise the dummies  $D_2$  and  $D_3$  are for observations from the Slow and Instant time treatments respectively. Finally,  $u$  is the random error that is normally distributed with mean zero.

The model offers an estimate of the direction of convergence. The  $\beta_1$  term has the interpretation as the y-axis intercept, i.e. where the time series starts at time  $t = 1$ . The coefficient  $\beta_{2j}$  can be interpreted as the value  $AvgAbsDist_{jt}$  converges toward as  $t \rightarrow \infty$  in treatment  $j$ , since  $\frac{t-1}{t} \rightarrow 1$  as  $t \rightarrow \infty$ . Equation (1.4) imposes a single origin coefficient, and imposes equal convergence target coefficients  $\beta_{2j}$  across silos and periods, but allows those targets to vary across treatments.

Estimates are shown in Table 1.2. Since we observe repeated interactions with individuals in the same silo, we cluster errors at the silo level and report standard errors corrected first-order autocorrelation using the Newey-West method for panel data sets, (see Newey and West [52]).

**Result 1:** Locations in the Discrete time treatment remain far away from the Nash

equilibrium profile.

Support for Result 1: Coefficient estimate  $\hat{\beta}_{21} = 0.140$  in Table 1.2 is statistically and “economically” very different from zero. There is a weak tendency for the distance from NE to decrease over time, inasmuch as  $\hat{\beta}_{21}$  is less than the estimate  $\hat{\beta}_1 = 0.264$  of the origin parameter, but the main point is that the estimated asymptotic distance  $\hat{\beta}_{21}$  is not much below the random benchmark of 0.17.

**Result 2:** Continuous time treatments exhibit convergence toward the pure Nash equilibrium.

Support for Result 2: The estimated asymptotic distances from NE in the Slow and Instant treatments are  $\hat{\beta}_{22} = 0.063$  and  $\hat{\beta}_{23} = 0.048$  respectively. These are statistically different from zero, but “economically” speaking they are much closer to zero than to the random benchmark 0.17. We find significant differences between the discrete and each continuous time treatment convergence targets estimates ( $H_0: \beta_{21} \neq \beta_{22}, \beta_{21} \neq \beta_{23}$  at  $p < 0.001$  in pairwise test correcting first order autocorrelation, and clustering errors at the silo level). We conclude that behavior moves decisively towards (but not all the way to) the Nash equilibrium in these treatments.

**Result 3:** Convergence toward NE is better in the continuous Instant treatment than in the continuous Slow treatment.

Support for Result 3: The coefficient estimate  $\hat{\beta}_{23} = 0.048$  for Instant is less than the estimate  $\hat{\beta}_{22} = 0.063$  for Slow, though these estimates are not statistically different. A more nuanced picture can be obtained by comparing the empirical distributions of average absolute distance from NE across treatments, as in Figure 1.6. One can see



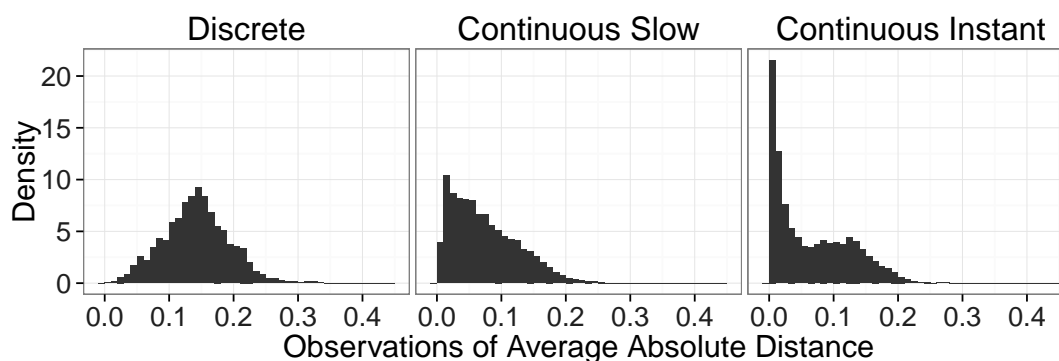


Figure 1.6: Observations of Average Absolute Distance

that the Discrete treatment has a vastly different distribution compared to the two continuous treatments, and that Instant has a much larger fraction of tiny distances (on or very near NE) than Slow. The Figure also shows that the distributions for the continuous time treatments are close for distances exceeding about 0.1, i.e., the upper tails are similar.

Table 1.3 lists the percent of AvgAbsDist observations for which it is implied that subjects are 1 percent and 5 percent away from the pure Nash equilibrium, i.e. the percent of AvgAbsDist observations less than 0.0025 and 0.0125 respectively. Regarding “within 5 percent” as close, we see that in 26 percent of observations players in continuous Instant treatments are close to the equilibrium configuration, compared to just 6 percent of observations in the continuous Slow treatment and much less than 1 percent of Discrete time observations. Regarding “within 1 percent” as close, the ratios of the percent of close-to-NE observations swings even more sharply towards the Instant treatment although, of course, the percentages are smaller than for the less stringent criterion for closeness.

**Result 4:** In the Instant treatment, groups of players that perform well against robot agents tend to converge more closely towards the pure NE.

Support for Result 4: We compute the average group normalized robot score,  $\rho$ , for each four-human-players group.<sup>3</sup> Column (2) of Table 1.2 shows that including this explanatory variable in equation 1.4 yields a very significant coefficient estimate of -0.035. That is, a standard deviation increase in a group’s average ability against robots is associated with a 3.5% smaller deviation from NE, holding treatment effects constant.

Column (3) of Table 1.2 shows that the strength of this effect varies by treatment. The interaction  $\rho_{DS} = -0.005$  is statistically but not economically significant, while the interaction  $\rho_{CS} = -0.018$  is three times larger but only marginally significant. The very significant Instant treatment interaction,  $\rho_{CI} = -0.045$  indicates that how well the four-player group did in our automated agent periods is particularly important in achieving convergence to the static NE in the continuous Instant setting.

## 1.5 Discussion

Our interest in Hotelling location games led us to focus on the  $n = 4$  player case. Our laboratory data replicate the negative result of Huck et al. [39] that behavior fails to converge in the Discrete time treatment. Thus we reject the Testable Hypothesis 1 for this treatment. More importantly, we have identified (as far as we know, for

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<sup>3</sup>Details are as follows. For each player  $i$ , we compute the total payoffs  $R_i$  against robots, and find the average  $\bar{R}_\tau$  and standard deviation  $\sigma_\tau$  across all players in each treatment  $\tau$ . The normalized individual score  $\rho_i = \frac{R_i - \bar{R}_\tau}{\sigma_\tau}$  is comparable across individuals and treatments. These individual level scores are averaged across the four players matched in a given group, providing an indication of group’s overall ability. Group-level  $\rho$  is used to match with group-period level *AvgAbsDist* time series.

the first time) conditions that lead to good NE convergence. In both continuous time treatments, Slow and Instant, we get convergence to a neighborhood of (but not precisely to) NE both within a typical period and across periods. The neighborhoods are tighter under Instant, as confirmed by parametric tests in the tradition of Noussair et al. [53]. This supports Testable Hypothesis 1 for these treatments, and also supports Testable Hypothesis 2.

We believe that our results have a practical implication: Hotelling location models now seem more relevant to understanding behavior in the wider world than might previously have been supposed. In particular, in terms of product positioning in a space of features or perceived characteristics, our Instant and Slow treatments capture important aspects of competition in the 21<sup>st</sup> century. In some situations (e.g., introducing new products) firms are able to place their product anywhere in the spectrum, while in other situations (e.g., repositioning) product development and marketing can only gradually adjust the way consumers perceive their product. Our results suggest that static Nash equilibrium may have predictive power in both situations.

There also may be theoretical implications, regarding the dynamic foundations of static Nash equilibrium in general and not just for the particular spatial model we investigate. Many early game theorists, including John Nash, offered intuitive dynamic arguments that later generations of theorists formalized as learning in games or evolutionary games; for a now-classic summary see Fudenberg and Levine [30]. Our work speaks to a followup question: when (and how) do continuous time dynamics alter convergence to static NE relative to discrete time dynamics? Friedman and Oprea [29]

and Bigoni et al. [5] find that continuous time dynamics dramatically delay convergence to the inefficient stage game NE in prisoner's dilemma games, i.e., they support cooperation in the finite horizon repeated game. Dorsey [23] and Oprea et al. [54] find that, by itself, continuous time is insufficient to reliably delay convergence to the inefficient NE in four player voluntary contribution public goods games, perhaps due to coordination issues. Deck and Nikifarakis [21] study a challenging pure coordination game: minimum effort or weakest link played on a circle network. Their results suggest that strategic uncertainty impedes convergence to the more efficient static NE. Our own results, including the simulation exercises, provide additional evidence.

Definitive answers to the followup question await further work, but theorists pondering that question may wish to distinguish the impact of asynchronous responses from rapid responses. Coordinating on a static NE (which emphasizes *unilateral* best response) may be aided mainly by asynchronous response opportunities, while coordinating on an efficient profile (not an issue in a Hotelling location model) may be aided mainly by rapid responses.

Several new avenues of laboratory research now come into focus. First, we note that unexplained discrepancies remain between our Discrete time results and those of Huck et al. [39]. They obtain sharper modes than we do, including the contra-NE mode at the center location.

A broader avenue is to apply the continuous time treatments to more general Hotelling models, including the no-edge case (the circle), joint decisions of price and location, and different numbers of players. Another broad avenue is to separate the

impact of continuous time per se from the impact of asynchronous choice. Our agent-based simulations suggest that taking turns in discrete time suffices to achieve Nash (or near-Nash) equilibrium behavior, but we do not yet know whether humans will agree.

	<i>Dependent variable:</i>		
	Average Absolute Distance		
	(1)	(2)	(3)
$\hat{\beta}_1$ : Intercept	0.264*** (0.0160)	0.264*** (0.0152)	0.264*** (0.0144)
<i>Convergence Targets by Treatment</i>			
$\hat{\beta}_{21}$ : Discrete Time	0.140*** (0.0004)	0.140*** (0.0022)	0.140*** (0.0003)
$\hat{\beta}_{22}$ : Continuous Slow	0.063*** (0.0068)	0.063*** (0.0047)	0.063*** (0.0052)
$\hat{\beta}_{23}$ : Continuous Instant	0.048*** (0.0080)	0.048*** (0.0036)	0.048*** (0.0042)
<i>Average Group Normalized Robot Score</i>			
$\hat{\rho}$		-0.035*** (0.0053)	
$\hat{\rho}_{DS}$ : Discrete Time			-0.005*** (0.0005)
$\hat{\rho}_{CS}$ : Continuous Slow			-0.018* (0.0091)
$\hat{\rho}_{CI}$ : Continuous Instant			-0.045*** (0.0058)
Adjusted $R^2$	0.616	0.6742	0.6835
Residual Std. Error	0.052	0.04789	0.0472
F Statistic	26,297*** (df = 4; 65488)	27,110*** (df = 5; 65487)	20,210*** (df = 7; 65485)
Number of Observations:	65,492	65,492	65,492

*Note:* Standard errors in parentheses; \*\*\* indicates  $p < 0.01$  and \* indicates  $p < 0.1$ . Data are from periods 7 through 12 for all games involving four human players.

Table 1.2: Estimates of Equation 1.4

	<b>Discrete</b>	<b>Continuous Slow</b>	<b>Continuous Instant</b>
<b>Periods 1-3</b>	0.0% (0.0%)	0.01% (2.53%)	0.3% (8.76%)
<b>Periods 4-6</b>	0.0% (0.0%)	0.02% (4.33%)	0.41% (22.08%)
<b>Periods 5-9</b>	0.0% (0.19%)	0.01% (6.59%)	8.5% (33.51%)
<b>Periods 10-12</b>	0.0% (0.0%)	0.74% (12.05%)	6.28% (39.06%)
<b>Overall</b>	0.0% (0.05%)	0.20% (6.38%)	3.87% (25.86%)

*Note:* Entries are the fraction of observations within 1 percent of Pure NE ( $\text{AvgAbsDist} \leq 0.0025$ ), and within within 5 percent of Pure NE in parentheses ( $\text{AvgAbsDist} \leq 0.0125$ ). Observations consist of subject location configurations sampled ten times a second in continuous time treatments, and each subperiod in the Discrete time treatment.

Table 1.3: Attainment of Near-Equilibrium Location Formations by Treatment

## Chapter 2

# Stability in Competition? Hotelling in Continuous Time

A version of this paper is coauthors with Liam Rose, University of California, Santa Cruz, [lrose1@ucsc.edu](mailto:lrose1@ucsc.edu).

### 2.1 Introduction

Hotelling's seminal paper *Stability in Competition* [38] characterized the stylized fact that individuals buy commodities from different sellers despite modest differences in price, and the work continues to garner citations at an impressive pace even 85 years after its publication. The model is often taught and discussed as a simple location model in which firms decide how to position their product in a linear product space. This space is generally taken to be location, although the model has been adapted extensively to numerous phenomena ranging from industrial organization to politics.



In the classic setup, firms face a constant price and a uniformly distributed mass of potential consumers that will buy at most one unit from one of the firms as determined by a specified utility function. Consumer utility is decreasing linearly in distance, and as such consumers will purchase the homogenous good from the closest vendor. With two firms in the market, this results in the firms locating adjacent to each other at the midpoint; this is to say, the firms produce identical products. This is known as the *principle of minimum differentiation*.

However, Hotelling suggested but did not prove that this spatial competition would lead to a price equilibrium between firms. This has since been proven to be incorrect for his specification of the model with linear transportation costs, as firms can always improve profits by moving to a new position after a competitor's move, be it a new location or a new price point. The absence of a pure strategy Nash equilibrium contrasts starkly with the “folk wisdom” that firms minimize the distance between one another and thereby maximize payoffs.

This contrast between formal theory and folk wisdom motivates our empirical work. With theoretical work unable to provide a clear equilibrium on such an entrenched model, we turn to the lab to test the predictions that Hotelling attempted to elucidate. We adhere closely to the original model and question whether Hotelling's Law — another name for the principle of minimum differentiation — holds. We also test whether the ability to rapidly adjust location and price can induce firms to cooperate to achieve higher profits. In a continuous time setting in the laboratory, we examine variants of the Hotelling model in which pairs of anonymously matched subjects can adjust their

price and location during four-minute periods. Treatments vary in how often they are allowed to adjust their position, ranging from free adjustment on either dimension to being limited to adjustment on only one dimension during set blocks of time. Subjects accrue flow payoffs throughout the period that depend simply on their location and price positions relative to their counterparts in that moment in time. Subjects are randomly rematched after each period, with sessions lasting 12 periods.

Our results indicate that despite theoretical ambiguity suggesting otherwise, Hotelling's principle of minimum differentiation largely holds. We also demonstrate that free and unlimited adjustment leads to higher payoffs for subjects, while limiting adjustment lowers prices and payoffs and increases competitive behavior. We provide circumstantial evidence that collusion arises from signaling from one of the subject pairs, either through momentary jumps to desired positions or willful loss in payoffs while waiting for a counterpart to fall in line. However, non-competitive behavior is not widely observed, and it is clear that the Hotelling model is not one that leads easily to a settled state. As such it should only be used very cautiously when explaining duopolies, whether they are political parties competing for voters or ice cream push-carts on a beach.

Section 1 provides background on Hotelling's seminal model and its theoretical development, and Section 2 recalls his original notation. Section 3 details the experimental design, Section 4 gives the results of the experiment, and Section 5 concludes.

## 2.2 Background

It is appropriate to distinguish between horizontal and vertical differentiation. With horizontal differentiation, not all consumers will agree on which firm or product is preferred. With fixed prices, Hotelling's model only discusses horizontal differentiation in the form of a location choice by the firm. Despite most colloquial discussion leaving the analysis there, Hotelling actually included prices and devoted a significant portion of his paper to discussing vertical differentiation. With vertical differentiation, all consumers will agree which product is preferred, all else held constant. The dimension is taken to be price for the purposes of this discussion, although it could also be product quality with prices held constant.

### 2.2.1 Evolution of Hotelling's Original Model

The original model has been appropriated to attempt to explain a wide range of phenomena, concentrated most densely in the political science and industrial organization literatures. These applications range from voting habits (Downs [25]) to entry deterrence (Schmalensee [63]) to competition in specific industries (Baum and Mezias [4] for hotels, Calem and Rizzo [13] for hospitals, and Iyer et al. [42] for religions, as examples).

Unfortunately, however, many of these applications do not take into account how sensitive the Hotelling location model is to small changes to the setup and set of assumptions. Eaton and Lipsey [26] detail equilibria for more than two players, and show that minimum differentiation does not generalize easily even if local clustering tends

to emerge. With four players, for example, the equilibrium has two players on each of the first and third quartiles. With three players, however, there is no way to satisfy the pure equilibrium conditions. Salop [62] changed the linear city to a circle — among other alterations — which results in maximum differentiation in product space, such that firms are evenly distributed around the circle.

Perhaps most significantly, D’Aspremont et al. [20] show that the principle of minimum differentiation does not hold due to the non-existence of a price equilibrium when firms are not sufficiently far from each other. This is because demand is discontinuous when firms are located close together. They propose a simple modification to the consumer utility function — quadratic instead of linear transport costs — that restores the continuity of demand and allows for a price equilibrium anywhere. However, this changes the sign of the derivative of the firms’ profit functions with respect to location from positive to negative; this is to say, firms then locate as far from each other as possible.

This prompted a number of authors to implement alterations to the model to remedy this equilibrium non-existence problem. Graitson [33] assumes “maximin” behavior, in which a firm that is too close to an opponent sets a price that maximizes its profit function taking the other firm’s price to be zero. This leads to a “maximin equilibrium” which gives an equilibrium with firms located at the first and third quartiles charging Nash-Cournot prices. Neven [51] integrated the D’Aspremont et al. suggestion of quadratic transport costs and gave the model two stages, where firms first engage

in horizontal differentiation before choosing a price.<sup>1</sup> He shows that a pure strategy price equilibrium exists for every pair of products, and confirms that firms maximize horizontal differentiation in equilibrium.

Generally, subsequent work — Cremer et al. [18] and Irmen and Thisse [41], for example — followed the quadratic transport costs approach, although [28] examined a range of utility costs of transportation and showed that not all specifications resulted in maximal differentiation. However, the problem of equilibrium existence within Hotelling’s original setup lingered, and increasingly nuanced approaches attempted to tackle the problem.<sup>2</sup> Close relatives of the Hotelling model — such as Shaked [64] — give the model an entry decision stage or alter the dimensions on which firms compete. More modern work has seemingly strayed even further from the original setup to attempt to capture real world phenomena, particularly in the IO literature. We cannot even begin on an exhaustive list here, as the original paper had over 8000 listed citations at the time of writing. Two things are clear from the existing literature, however. First, the predictions of the model are sensitive to small changes to the setup. Second, the model has defied many efforts to cleanly characterize its equilibrium, with the possibility of a pure-strategies equilibrium having been definitively eliminated.

But despite the result-altering breakthroughs, Hotelling’s conclusions are still widely cited — if only casually — with the equilibrium difficulties frequently ignored. This

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<sup>1</sup>Hotelling himself hinted at a two stage approach when prices were decision variables, for the reason that prices are easier to adjust than locations or product variety. Other papers — such as Graitson [33] and Economides [27] — tacitly followed this approach.

<sup>2</sup>See Gabszewicz and Thisse [32] for an early overview and Caplin and Nalebuff [14] for an approach that gives the additional assumptions and conditions needed for a pure strategy price equilibrium.

paper explores the applicability of his original result as well as the robustness of the more nuanced conclusions of subsequent models in an experimental setting.

### 2.2.2 Previous Experimental Work

There have been a number of attempts to test the Hotelling model in an experimental setting. Brown-Kruse et al. [9] — along with its predecessor, Brown-Kruse et al. [8] — investigate a two-player uncertain endpoint model, but focus on the effect of communication on collusion. They find that communication led participants to locate near the quartiles to maximize joint profit, but the principle of minimum differentiation did seem to hold in their results when communication was limited. Huck et al. [39] were the first to test a four-person Hotelling game, but found little support for the equilibrium hypothesis. Kephart and Friedman [44] showed that the four-player Nash equilibrium emerged more quickly with the ability to adjust location instantly.<sup>3</sup>

Very few authors have included vertical differentiation in an experimental setting. This could be due to the equilibrium existence problems discussed above, or due to a lack of technological capability. To our knowledge, only three works have tested a game with price as a choice variable in addition to location. The first attempt was by Mangani and Patelli [48], who specified their model with quadratic transport costs such that theory would suggest subjects should maximally differentiate in the location dimension to relax price competition. The authors tested this with three treatments: a two-stage location then price game mirroring Neven’s setup, a treatment with only

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<sup>3</sup>The four-player location-only game has its own form of the principle minimum differentiation as the equilibrium, with players located back-to-back on the first and third quartiles.

periodic location adjustments, and a treatment in which price and location were chosen in the same period. The one-shot game in the last of these treatments does not have a theoretical benchmark. Subjects, however, tended toward the center of the location space, although still 20-30 percent of the action space away from their counterparts, on average. The authors suggested risk aversion as an explanation, but not direct tests this.

Kusztelak [45] allowed limited communication between subjects as well as also including quadratic transport costs. In his first treatment, prices were automatically computed, reducing the game to a location-only decision with 101 discrete location fields (between 0 and 100). Subjects behaved as expected with maximal differentiation in this treatment. But when a price decision was added in the second treatment, the differentiation decreased significantly, with over 40 percent of location in the center of the action space. He hypothesizes that increased model complexity reduces differentiation, and also tests a market with two horizontal decision variables along with the price decision variable in the final treatment.

Finally, Barreda et al. [3] attempted to test the hypothesis that firms use product differentiation to relax price competition by focusing on a limited, discrete location decision. Specifically, subjects could only choose among either seven or eight location slots, depending on the treatment. In their most relevant treatments, the authors found less product differentiation than theory would predict, and relatively few high prices.

### 2.2.3 Our Contribution

Our work contributes to the literature in several important ways. First, this experiment is the cleanest test to date of a simple two-player Hotelling model with horizontal and vertical decision variables and linear transport costs, thus giving a better view into how Hotelling’s seminal result fairs in an ideal setting. Second, this experiment is the first to test the model in continuous time. Our work also takes a novel approach by blurring the sharp distinction between the continuous choice model and sequential models, providing insight on how the model should be applied and the effect the ability to adjust quickly affects firms behavior. Finally, superior lab software and programming gives participants a more intuitive interface and faster learning experience to allay concerns of participant apprehension skewing results in the competitive setting.

## 2.3 Model and Predictions

First, we begin by recalling Hotelling’s assumptions and notation. There are customers evenly distributed on a line of length  $l$ , with firms  $A$  and  $B$  selling a homogenous product with zero production cost.<sup>4</sup> Each customer consumes one unit of the good, and will buy from the seller who gives the least delivered price. Firms locate at points  $a$  and  $b$  respectively, such that  $a$  is the distance from 0,  $b$  is the distance from  $l$ ,  $a + b \leq l$ , and  $a \geq 0, b \geq 0$ . For simplicity, we normalize  $l$  to be 1 in our experiment. Firms also set prices  $p_A$  and  $p_B$ , respectively. Transport costs are linear and are denoted by  $c$ .

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<sup>4</sup>This experiment only examines the two-player game, but can be generalized to  $n$  sellers. See Brenner [6] for a derivation of the model with more than two players.



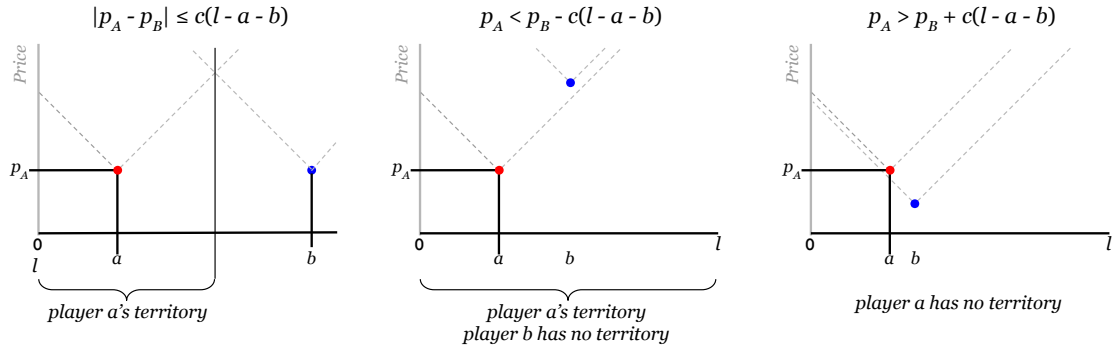


Figure 2.1: Player Configurations Under Three Cases

First, consider the case that price and location are chosen simultaneously. Payoff functions for  $A$  and  $B$  are given by:

$$\pi_A(p_A, p_B, a, b) = \begin{cases} ap_A + \frac{1}{2}(l - a - b)p_A + \frac{1}{2c}p_A p_B - \frac{1}{2}p_A^2 & \text{if } |p_A - p_B| \leq c(l - a - b) \\ lp_A & \text{if } p_A < p_B - c(l - a - b) \\ 0 & \text{if } p_A > p_B + c(l - a - b) \end{cases}$$

$$\pi_B(p_A, p_B, a, b) = \begin{cases} bp_B + \frac{1}{2}(l - a - b)p_B + \frac{1}{2c}p_A p_B - \frac{1}{2}p_B^2 & \text{if } |p_A - p_B| \leq c(l - a - b) \\ lp_B & \text{if } p_B < p_A - c(l - a - b) \\ 0 & \text{if } p_B > p_A + c(l - a - b) \end{cases}$$

Figure 2.1 diagrams three player configurations corresponding to the three cases in player  $a$ 's payoff function.

These profit functions are clearly discontinuous at the points where the delivered price of one firm is equal to the price of a rival at the rival's location. At these points,

a whole group of consumers will be indifferent between the two firms. D’Aspremont et al. showed that there is a Nash-Cournot equilibrium point only if sellers are sufficiently far from each other, or such that:

$$\left(l + \frac{a-b}{3}\right)^2 \geq \frac{4}{3}(a+2b)l \quad (2.1)$$

$$\left(l + \frac{b-a}{3}\right)^2 \geq \frac{4}{3}(b+2a)l \quad (2.2)$$

When sellers locate close to one another, it is optimal for them to undercut each other and capture the entire market. But if 2.1 and 2.2 hold, then both  $\partial\pi_A/\partial a$  and  $\partial\pi_B/\partial b$  are strictly positive, implying each firm should move closer to her rival. Once the firms are relatively close to one another, 2.1 and 2.2 are violated, implying a Nash equilibrium does not exist.<sup>5</sup> Therefore, subjects in the experiment have an incentive to push toward the center, then try to undercut each other in the price dimension to grab the entire market. This prediction would see subjects follow each other closely in the action space, with frequent adjustment to price and location and large volatility in profits.

A possible evasion of this problem is to assume the maximin strategy introduced by Graitson. Here, the seller charges the profit maximizing price if she is likely to be undercut by her competitor when charging the Nash-Cournot price, and the Nash equilibrium price if not. Graitson proves that a socially optimal equilibrium — i.e. one that minimizes transport costs — exists with this strategy in which firms charge

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<sup>5</sup>It is worth noting that there is a trivial Nash equilibrium at  $p_A^* = p_B^* = 0$  if  $a = b$ . This follows from Bertrand competition, in which there always exists an equilibrium uniquely determined by zero prices.

Nash-Cournot prices and locate at the first and third quartiles.<sup>6</sup>

In the two-stage game, firms first simultaneously choose location, then simultaneously choose a price with full information from the first stage. Apart from this feature, the setup is the same as above. Dasgupta and Maskin [19] prove that each price-setting stage has an equilibrium in mixed strategies and Osborne and Pitchik [55] examine the equilibrium that results from firms using mixed strategies in this stage. They characterize — but are unable to prove the existence of — a unique perfect equilibrium in the first stage in which each firm locates 0.27 from the endpoints of the unit interval, which is clearly quite close to the equilibrium that arises from minimax behavior. In the price-setting stage, for a symmetric location pair, the equilibrium price strategy is a union of two intervals — such that the CDF will be kinked. Prices then fall between .5 and 1, with most of the probability weight falling on price of 1. This prediction would see participants in the experiment at or near the profit maximizing positions.

In both versions, players can gain higher profits from collusion, but have incentive to cheat. This mirrors the classic Prisoner’s Dilemma, albeit with far more intermediate outcomes. Friedman and Oprea [29] showed that continuous time treatments greatly increase cooperation; as such, we would predict successful non-competitive behavior to be much more prevalent in continuous time treatments.

With the uncertainties in the equilibrium conditions in mind, we turn to the laboratory to answer lingering questions about the results and behavior of firms in Hotelling’s

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<sup>6</sup>Similar to Graitson, Takatoshi [66] purposes that firms will maximize on one dimension and minimize on the other in the two-stage game.

classic model of competition.

## 2.4 Experimental Design

The experiment was performed in sessions differing only in the timing of the game. We study three treatments: *Discrete*, *Continuous Instant*, and *Continuous Slow*. Sessions included of just one of the treatments, and consisted of two practice periods followed by 12 potentially paid periods. Subjects were randomly matched into two-person pairs, and rematched with a new counterpart each period. Periods lasted 4-5 minutes with random endings for subject pairs to avoid endgame effects, although practice periods lasted for 30 seconds. Sessions contained six participants, and subjects could be re-matched to any other subject at the start of another period.<sup>7</sup> Note that for all treatments, there was no difference between counterparts in any of the game’s parameters. In all treatments, participants choose their location and price by clicking in the x-y action space.

In all treatments participants choose their location and price by clicking in the x-y action space. Action selections could be made with pixel precision. In some previous laboratory investigations action selection grids have been limited from the single digits to several dozen discrete actions available. Our implementation - with several hundred thousand available x-y coordinates available to participants<sup>8</sup> - approximates continuous

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<sup>7</sup>In some pilot sessions, we had eight-subject sessions that were divided into four-subject silos. While subjective, we felt that groups of this size could be risky in terms of subjects being able to identify a counterpart they had previously been paired with, potentially altering behavior.

<sup>8</sup>We implement an action space that is 425 pixels square, resulting in about 180,000 potential location and price combinations available to subjects

action selection far more closely.

In the *Discrete* treatment, subjects played an n-stage game in which location is selected first, followed by price with full information about location decisions. Subjects were given 3 seconds to choose their location, indicated by a progress bar on the top of their computer screen. The screen then adjusts to reflect the location the subject and her counterpart have chosen, and subjects were given 3 seconds to choose price, again indicated by a progress bar. We define these 3 second intervals as subperiods. Subjects had four subperiods of price decisions before they were allowed to readjust location.<sup>9</sup> Figure 2.3b gives a screenshot of the user interface for this treatment. “Flow” payoffs are shown as bars in the graph on the right, and are updated after every subperiod. The blue dot indicates the subject’s position in the last subperiod, while the green dot indicates her counterpart’s position. The black line shows the subject’s current choice for that subperiod, while the grey line simply follows the mouse.

In the *Continuous Instant* treatment, subjects chose both location and price freely and instantaneously.<sup>10</sup> A screenshot of this treatment is shown in Figure 2.3a. Flow payoffs are shown in the graph on the right, and are updated continuously. The blue dot indicates the subject’s current position, and the pink dot shows her counterpart’s current position. The grey crosshairs simply follow the mouse.

The *Continuous Slow* treatment is identical to the previous treatment except for

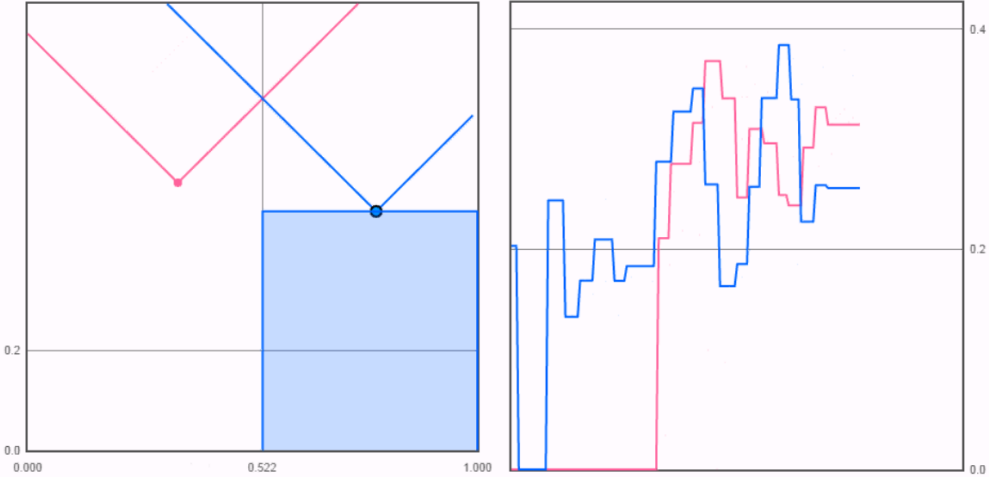
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<sup>9</sup>In pilot sessions, we also ran treatments in which location- and price-setting subperiods alternated, with no discernible difference in subject behavior.

<sup>10</sup>The latency between a subject’s click and seeing the action on the computer screen is around 50 milliseconds, or far faster than human reaction time. This latency did increase slightly during periods of very frequent position adjustment by subjects, but not above tolerable levels that would disrupt subject behavior.

Economics Experiment      Subject: Subject-1   Period: 4   Scoring Period      Current score: 0.019

Your color

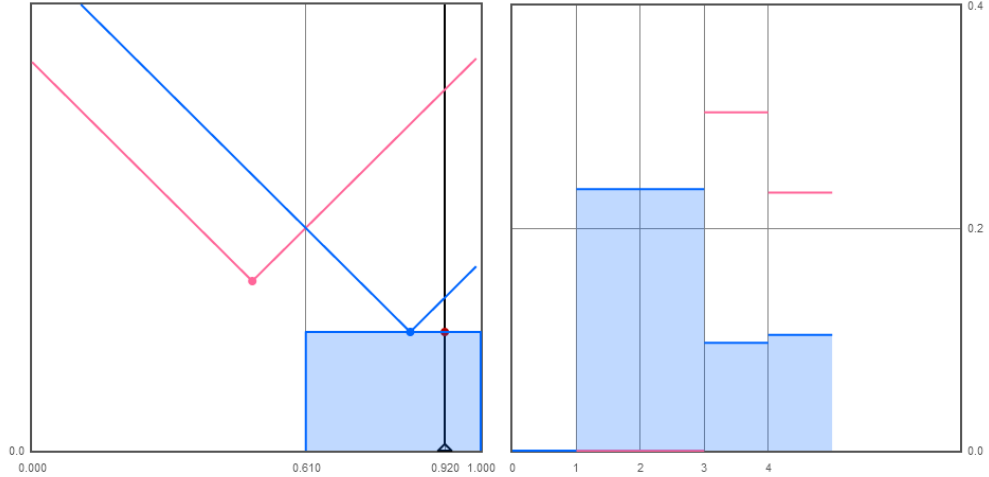


(a) Continuous Time

Economics Experiment      Subject: Subject-1   Period: 3   Scoring Period      Total Score: 0.000  
 Current score: 0.567

Your color

Choose x



(b) Discrete Time

Figure 2.2: User Interfaces

a “speed limit” on subject movement in action space. When a subject chooses a new location and price coordinate, a grey dot appears at that location while her actual position adjusts slowly to that point. If a subject wants to change direction while her position is adjusting, a new grey dot appears and her position immediately begins to adjust to the new target. As an analogue to the discrete time treatment, subject position could be adjusted four times quicker on the price dimension than on the location dimension.

In all treatments, subjects were given information about their current payoffs. The user interface included the linear transport costs running away from their position, the cutoff that determined the edge of the area they control, and a shaded region showing the area they control.

Subjects in all sessions were randomly selected using online recruiting software ORSEE, [35], at the University of California, Santa Cruz from our pool of volunteers, who are primarily undergraduates from all major disciplines. All were inexperienced, i.e., had never participated in a Hotelling experiment in our lab. Written instructions given for each treatment are included in the web appendix<sup>11</sup>, and these instructions were also read out loud. Following this, subjects saw a short, silent instructional video with on-screen text, which was read aloud as it appeared.<sup>12</sup> Pink noise — a full-frequency audio process that can be used to mask ambient noises — was played in the background to prevent subjects from hearing the mouse clicking of other subjects. Sessions lasted

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<sup>11</sup>The web appendix is at [www.cazaar.com/home/research](http://www.cazaar.com/home/research)

<sup>12</sup>Instruction videos are also available at the web appendix.

80-90 minutes each, and subjects were paid their point total multiplied by \$20 for two periods, which were decided by an overt dice roll by one of the participants. Following both pilot and paid sessions, we noticed only small differences in subject behavior between *Continuous Instant* and *Continuous Slow* treatments. Therefore, we follow the advice of List et al. [47] and substitute one discrete time session for one continuous slow session. Average earnings were \$16.21, and breakdown by treatment is available in Table 2.1.

Treatment	Number of Subjects	Average Payout
Continuous Instant	24	\$18.39
Continuous Slow	24	\$14.67
Discrete	24	\$16.71
Total	72	\$16.59

*Notes:* “Average Payout” includes the \$5 show-up fee.

Table 2.1: Subjects and Payouts by Treatment

## 2.5 Results

### 2.5.1 Subject Price and Location Decisions

To provide an overview of the results, Figure 2.3 gives heat maps of all players’ price and location decisions by treatment, respectively. In these figures, “hotter” colors mean players spent more time in these positions, while “cooler” colors indicate little time was spent in that area of the action space. The most striking feature of these figures is the heat distribution between continuous and discrete time treatments. Subject positions were clearly more concentrated in continuous time treatments, with discrete



	Discrete		Continuous Slow		Continuous Instant	
	Mean	Median	Mean	Median	Mean	Median
Price	0.5787 (0.0184)	0.606 (0.0247)	0.5117 (0.0070)	0.492 (0.0096)	0.5855 (0.0062)	0.597 (0.0090)
Profit	0.2534 (0.0096)	0.2325 (0.0141)	0.2380 (0.0041)	0.1980 (0.0058)	0.2729 (0.0037)	0.2550 (0.0059)

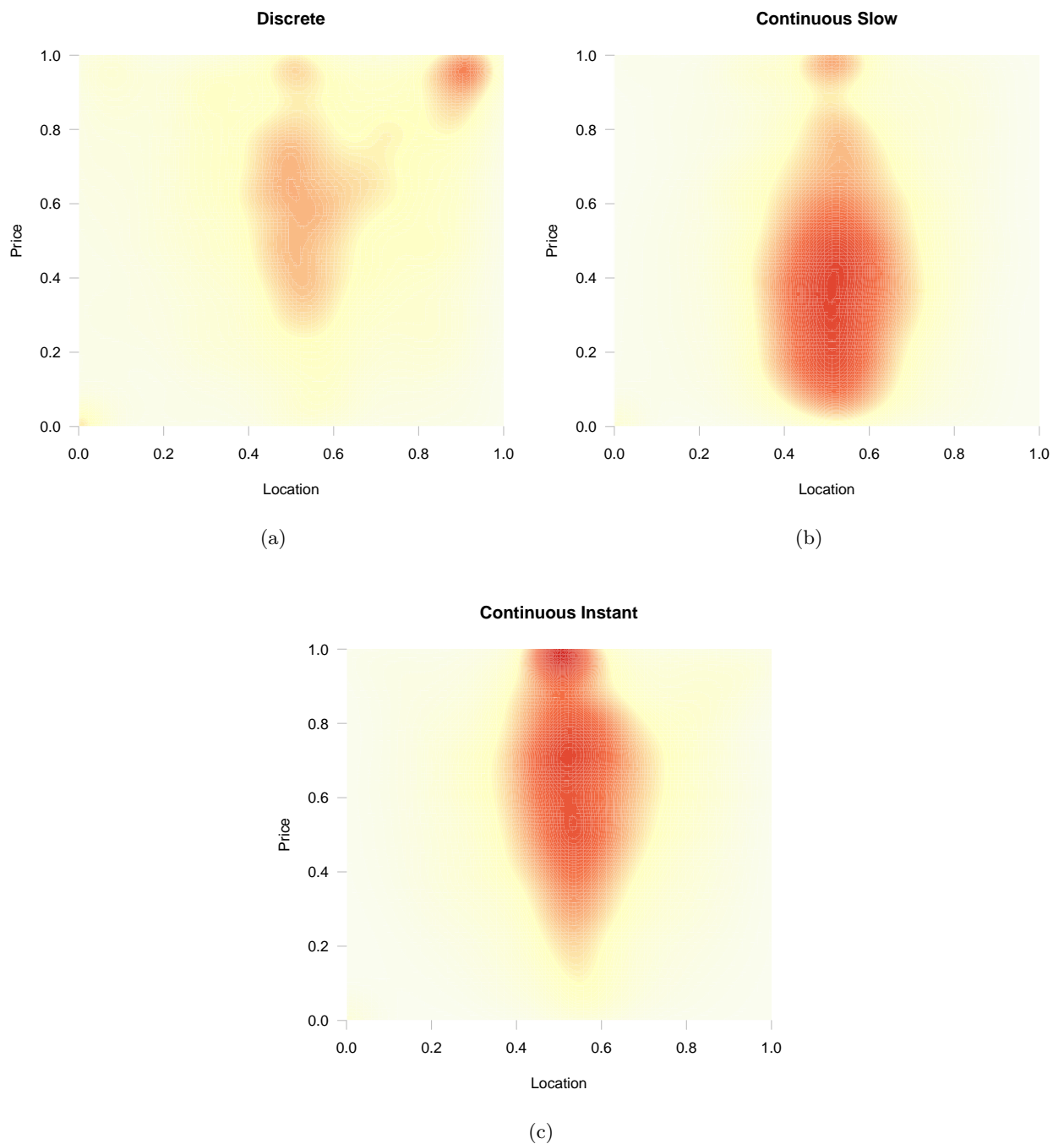
*Notes:* Means and block bootstrapped standard errors of prices and profits by treatment. Joint profit maximizing would lead to prices equal to 1 and profits equal to 0.5.

Table 2.2: Summary Statistics, by Treatment

time positions more evenly distributed in the action space. In continuous time, players tended to be centrally located on the x-dimension, while price positions varied more by treatment. Prices in *Continuous Instant* treatments tended to be the highest of any treatment, with a strong concentration around the highest possible price. Putting a speed limit on adjustment lowered prices and diminished the congregation around the highest prices.

Table 2.2 gives basic summary statistics by treatment. In the *Continuous Instant* treatment, subjects had the highest average prices and profits of any treatment. When subjects can adjust price quickly but not location, prices were lower than with instant adjustment, and median payoffs were the lowest of any treatment. For comparison, if both subjects exhibited joint profit maximizing behavior, prices would be one and profits would be equal to 0.5 for each subject in the pair.

While the straightforward summary statistics show relatively little difference between treatments, examining within subject pairs tells a different story. Table 2.3 gives



*Notes:* These figures show all players' price and location decisions by treatment. The heat maps run from cool to hot colors, with "hotter" colors indicating that players spent more time in those positions.

Figure 2.3: Heat Maps of Subject Price and Location Decisions by Treatment

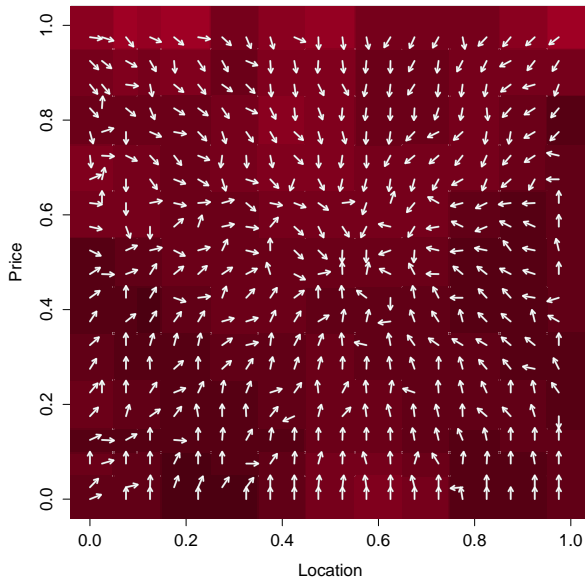
summary statistics on mean and median distance from a subject's counterpart, with "distance" specified as purely location, purely price, and euclidean distances. Note that both axes are scaled to one, so that a distance of .1 is very close while a distance of .5 is quite far from a counterpart. Subjects were much closer together on all measures of distance in the continuous treatments, such that the discrete stage game tended to push subjects apart in the action space. This can be seen easily in the heat maps discussed above. Price distance is consistently lower across treatments, even in the Continuous Time treatment that did not inhibit location adjustment in any way. Note that the median distances are consistently smaller than the mean distances. Observationally, this is due to some subjects consistently moving away from their counterpart to attempt to avoid the intense competition that characterized many subject pairs. This can be seen particularly clearly in the Continuous Instant treatment, where the median distance between counterparts in the price dimension is just six percent of the action space.

We have documented where subjects tended to locate in both dimensions, but we also wanted to characterize their movement when they did make adjustments. For this, we present in Figure 2.4 a form of an empirical vector field in which average subject movement from a given position is shown. Here, vectors show the average direction that subjects moved starting from that neighborhood. In the background, colors map to the percentage of observations in that neighborhood for which players changed their action set. Darker colors indicate that subjects tended to change their price/location decision in that area more often, with the direction of the change following the overlaid vector, on average.

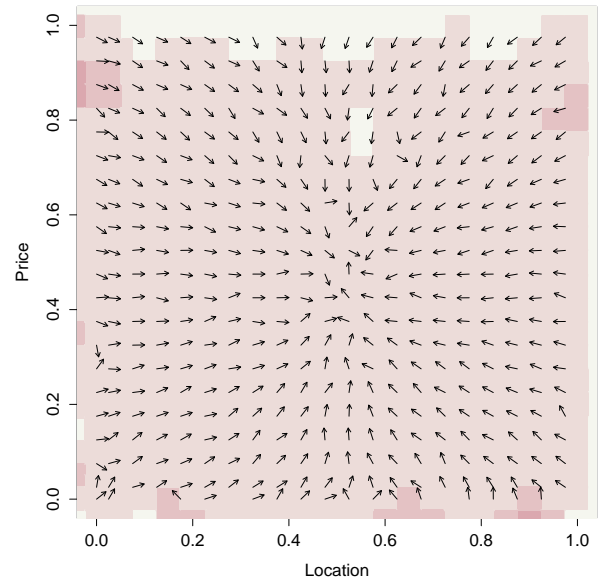
	Discrete	Continuous Slow	Continuous Instant
Location Distance (distance on x-axis from counterpart)			
Mean	0.2529 (0.0093)	0.1288 (0.0022)	0.1578 (0.0021)
Median	0.205 (0.0125)	0.0875 (0.0019)	0.1020 (0.0029)
Price Distance (distance on y-axis from counterpart)			
Mean	0.1714 (0.0046)	0.1140 (0.0016)	0.1097 (0.0016)
Median	0.109 (0.0052)	0.074 (0.0013)	0.062 (0.0015)
Euclidean Distance (from counterpart)			
Mean	0.3433 (0.0086)	0.1921 (0.0024)	0.2122 (0.0026)
Median	0.3097 (0.01)	0.1534 (0.0025)	0.1615 (0.0035)

*Notes:* Mean and median distances on specified dimension by treatment. Block bootstrapped standard errors in parentheses. Axes are scaled such that maximum differentiation on one dimension would give a distance of one.

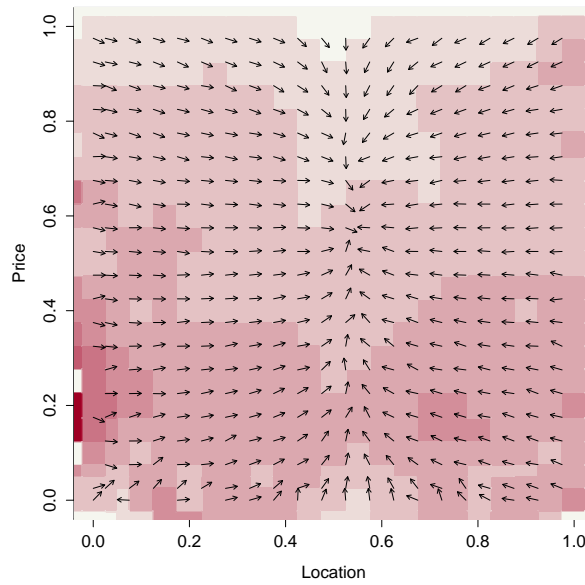
Table 2.3: Comparison to Counterpart Statistics by Treatment



(a) Discrete



(b) Continuous Slow



(c) Continuous Instant

*Notes:* These figures show all players' price and location decisions by treatment. Arrows indicate average direction of action set changes starting from the arrow's neighborhood. The heat maps run from cool to hot colors, with "hotter" colors indicating that players were more likely to change their action set while in that neighborhood.

Figure 2.4: Vector Fields of Subject Position Adjustments by Treatment

Subject adjustments vary greatly by treatment. In the *Discrete* treatment, subjects tend to decrease high prices and raise low prices, and tended to change their actions no matter where they were positioned. Beyond this, however, behavior is somewhat erratic. Movement in the *Continuous Slow* treatment is a bit more clear, with subjects tending to adjust towards the center. The heatmap for *Continuous Slow* is a bit deceptive, since action changes were rate limited by the “slow” speed limit. The heatmap for this treatment shows changes in players *target* location and prices, which were far less frequent than in either *Discrete* or *Continuous Instant* treatments. But the clearest story emerges from the *Continuous Instant* treatment. Here, the lower edges of the figure are darker as subjects made more frequent adjustments to avoid being “boxed in” by a counterpart. Prices tended to be adjusted upwards until about 0.6 — which was the median in this treatment — and downward above that. Central locations with medium to high prices tended to be the most stable action sets.

### 2.5.2 Non-competitive Behavior

As detailed previously, subjects have an ever-present incentive to undercut on either dimension. Thus, it may come as somewhat of a surprise that we see mean and median price decisions between 0.5 and 0.6. We compile non-competitive behavior rates for player pairs. Informally, non-competitive behavior is a situation where two players are able to settle into relatively stable and jointly profitable positions. To capture this sense, for our analysis we define  $\rho_{ik}$  as the fraction of time player pair  $i$  maintain jointly positive profits within 10 and 20 percent of each other — and by implication refrain

	10%		20%	
	Mean	Median	Mean	Median
Discrete	0.0846 (0.0146)	0.0607 (0.0069)	0.1685 (0.0202)	0.1379 (0.0178)
Continuous Slow	0.1131 (0.0169)	0.0892 (0.0088)	0.2185 (0.0225)	0.1894 (0.0192)
Continuous Instant	0.1425 (0.0278)	0.0985 (0.0077)	0.2535 (0.0298)	0.1980 (0.0092)

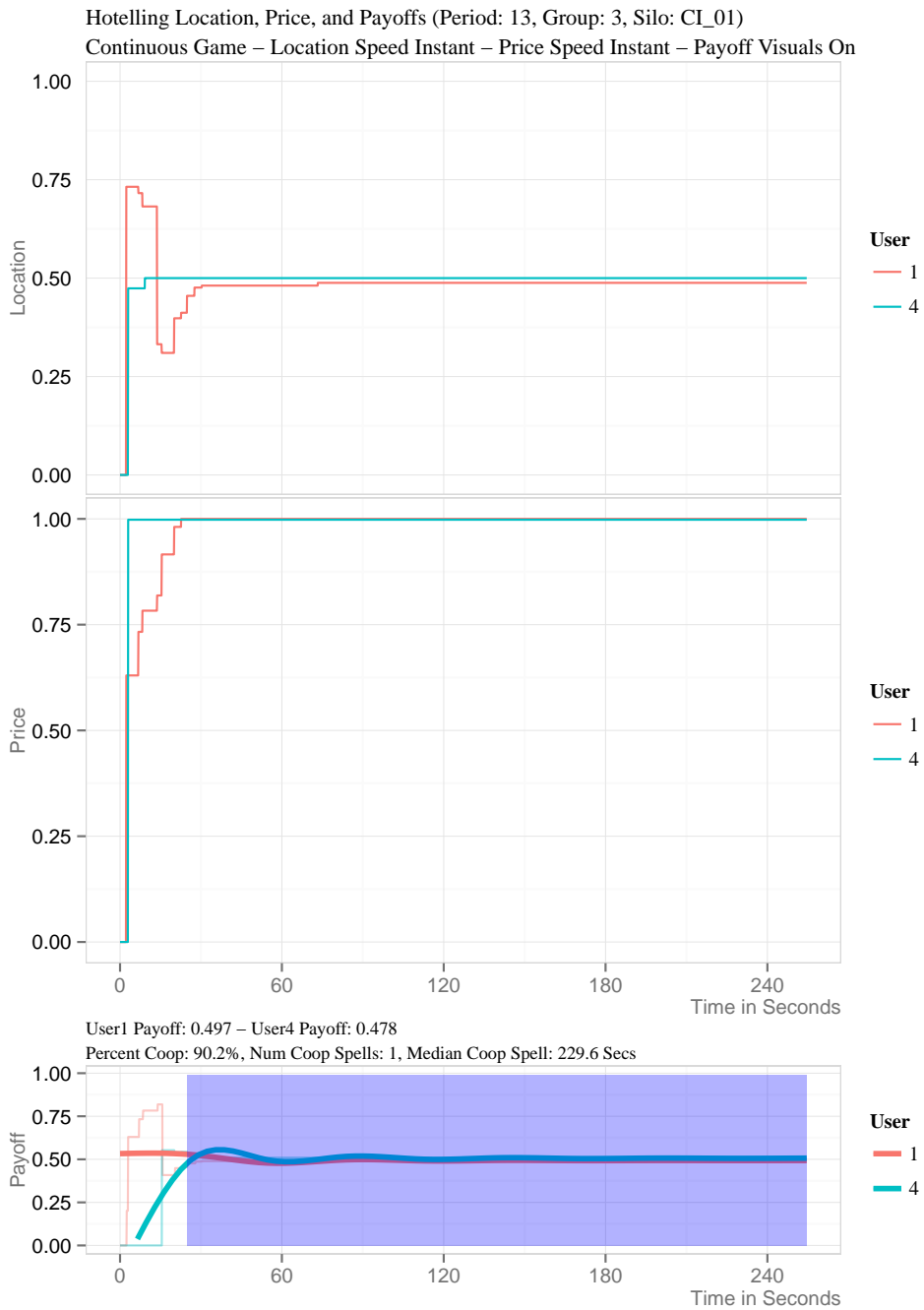
*Notes:* Mean and median non-competitive behavior rates with bootstrapped standard errors. The percentage refers to the threshold defining when subjects are not exhibiting competitive behavior.

Table 2.4: Non-competitive Behavior Rates by Treatment

from undercutting one another — in session  $k$ . Although these are conservative and arbitrary thresholds, our results are robust to a range of changes to this threshold.

Table 2.6 shows mean and median non-competitive behavior rates by treatment for the two thresholds for the last six periods of sessions.<sup>13</sup> The continuous time treatments were clearly more conducive to non-competitive behavior than the discrete time treatment, with rates exceeding a quarter of the period on average. Note again that median values were generally well below mean values. This is because some subject pairs were able to quickly come to an agreeable state — thus spending large portions of periods in cooperation — while others could only manage short-lived tacit agreements, or none at all. As one might expect, non-competitive behavior increases with fewer adjustment restrictions.

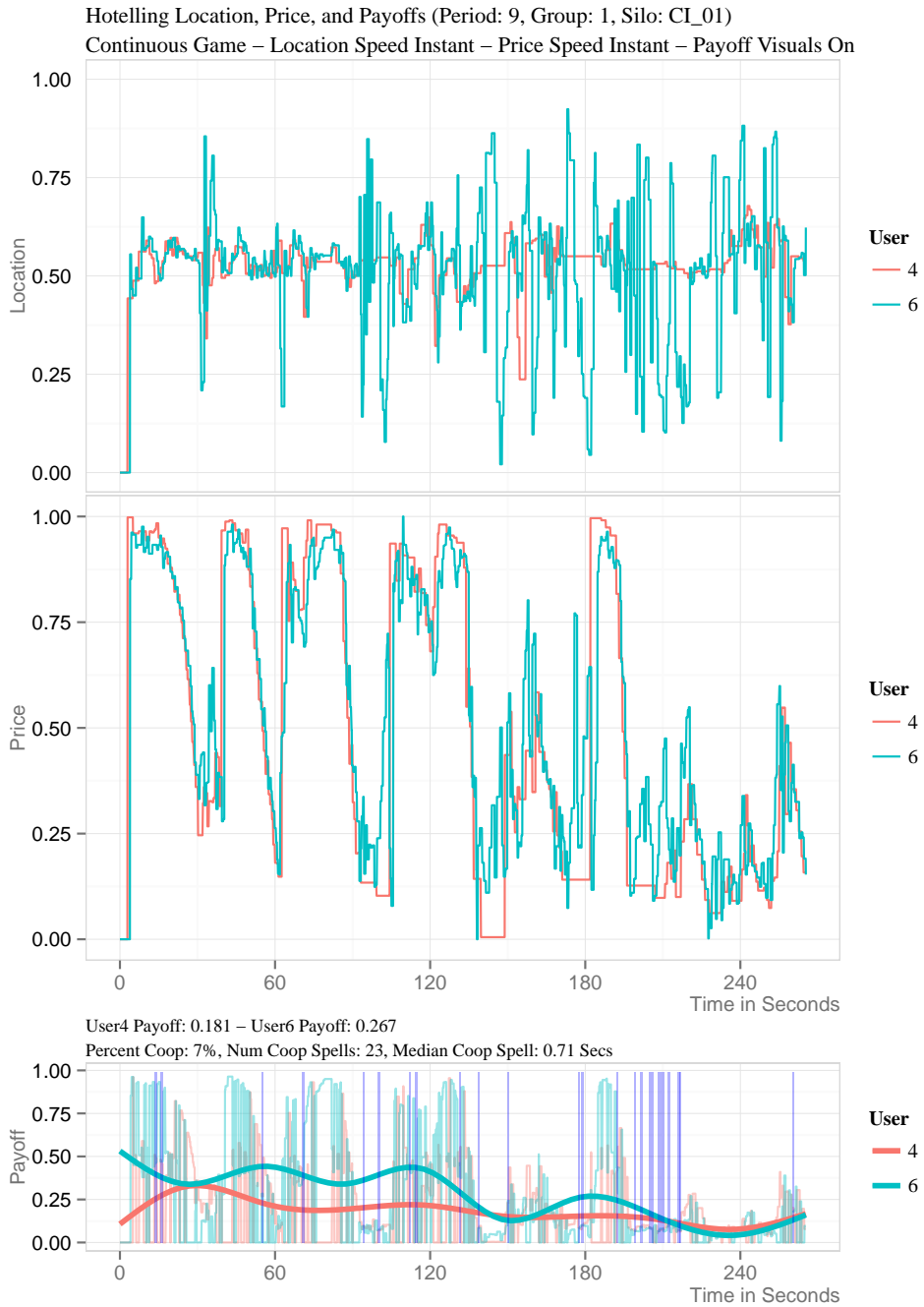
<sup>13</sup>Despite having two practice periods, we focus on settled behavior here to avoid any learning effects from the opening periods. The results from all periods are shown in Table 2.6 in the appendix.



*Notes:* This figure shows two subjects that successfully colluded for the majority of a period in the *Continuous Instant* treatment. The first panel is the subject-pairs location decisions over time, the second panel is their price decisions over time, and the final panel is their flow payoff. Note that Player 4 was an “aggressive colluder,” as she willingly took losses at the beginning of the period while waiting for her counterpart to conform.

Figure 2.5: Collusion Evidence, Successful Collusion





*Notes:* This figure shows two subjects that were unable to collude in a period in the *Continuous Instant* treatment. The first panel is the subject-pairs location decisions over time, the second panel is their price decisions over time, and the final panel is their flow payoff. Note that Player 4 is the same subject shown in Figure 2.5, but with a more aggressive counterpart.

Figure 2.6: Collusion Evidence, Unable to Collude

Note that we refrain from calling the behavior discussed above “collusion.” Aside from the issues of using a somewhat loaded term, it is true that subjects in this game can be both non-competitive and earning trivial payoffs. However, a player would be better off undercutting and taking the entire market for *any* non-zero counterpart price, and our definition above encapsulates this wider notion of eased competition.

Even though subjects clearly displayed more anti-competitive behavior in continuous time, it is somewhat puzzling that non-competitive rates were relatively low. Figure 2.5 gives circumstantial evidence of how players were able to coordinate. It shows a subject pair in the middle of the session playing a *Continuous Instant* treatment. Shaded regions indicate non-competitive behavior between subjects. In the bottom panel on each figure, the thick lines are smoothed flow payoffs for each subject, while the actual flow payoffs are shown in the background. In the very beginning of the period, player 4 — the orange player — immediately adjusts her price to the maximum allowed (normalized to one) and her location to the middle. Notice that this reduced her payoff to lower than her counterpart’s while she waited for her counterpart to fall in line with her strategy. The subject pair colluded for almost the entirety of the period, indicated by the blue bars in the payoff figure. The subject pair obtained much higher than average payoffs in this period as a result. Note that we are comfortable using the word “collude” here, as joint maximizing profits surely fits any definition of the term. On the other hand, Figure 2.6 shows a typical case of players following each other in the action space throughout the period. Player 4 is the same player that aggressively pushed for a collusive state in Figure 2.5, but is now matched with a more competitive

	Mean	Median
Continuous Instant	0.6235 (0.0161)	0.6162 (0.0149)
Continuous Slow	0.5402 (0.019)	0.5646 (0.0217)
Discrete	0.5992 (0.0255)	0.5914 (0.0311)

*Notes:* Mean and median non-competitive behavior rates with bootstrapped standard errors. Steady Positive Payoffs refers to spells in which both players in a subject pair have positive flow payoffs.

Table 2.5: Steady Positive Payoffs Non-Competitive Rates by Treatment

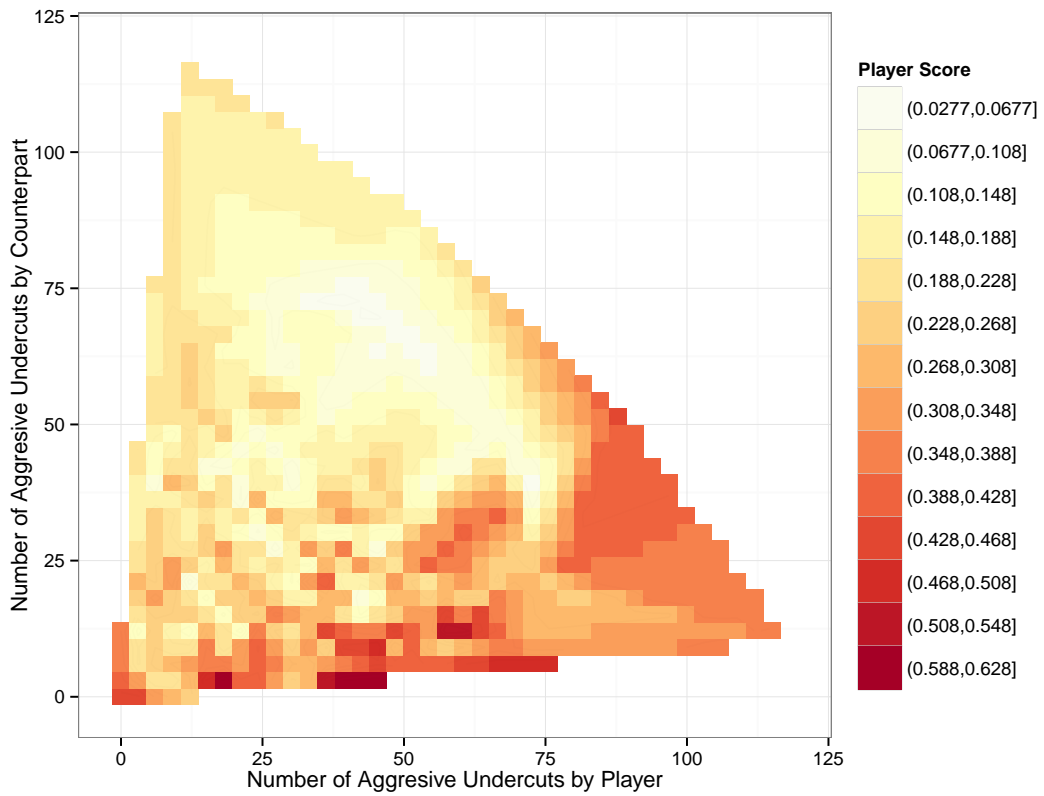
player. Notice that she repeatedly attempts to drive the prices is higher, thus taking a momentary loss. But Player 4’s counterpart immediately undercuts her, forcing her to be drawn into tight competition. At the end of the period, Player 4’s payoffs are much lower than her counterpart’s due to her attempts to ease competition. This kind of behavior was typical in the game, as “aggressive colluders” were only able to coax anti-competitive behavior out a relatively low number of counterparts.

We define an alternative state of non-competitive behavior that we call *Steady Positive Payoffs*. This concept abstracts away from a specific threshold, with a subject-pair in this state when both subjects have positive payoffs. The subject-pair’s spell in Steady Positive Payoffs is then broken if one of the players undercuts her counterpart causing their profits to fall to zero. The rates that come from this definition are reported in

Table 2.5. This shows that subjects were able to carve out some portion of the market well over 50 percent of the time. As expected, the *Continuous Instant* treatment had the least intense competition. However, *Continuous Slow* treatments had lower non-competitive rates by this measure, and were lower than even the *Discrete* treatment. We speculate that this is a consequence of the location adjustment speed limit, which seemed to drive subjects to compete more aggressively on price alone and undercut more frequently.

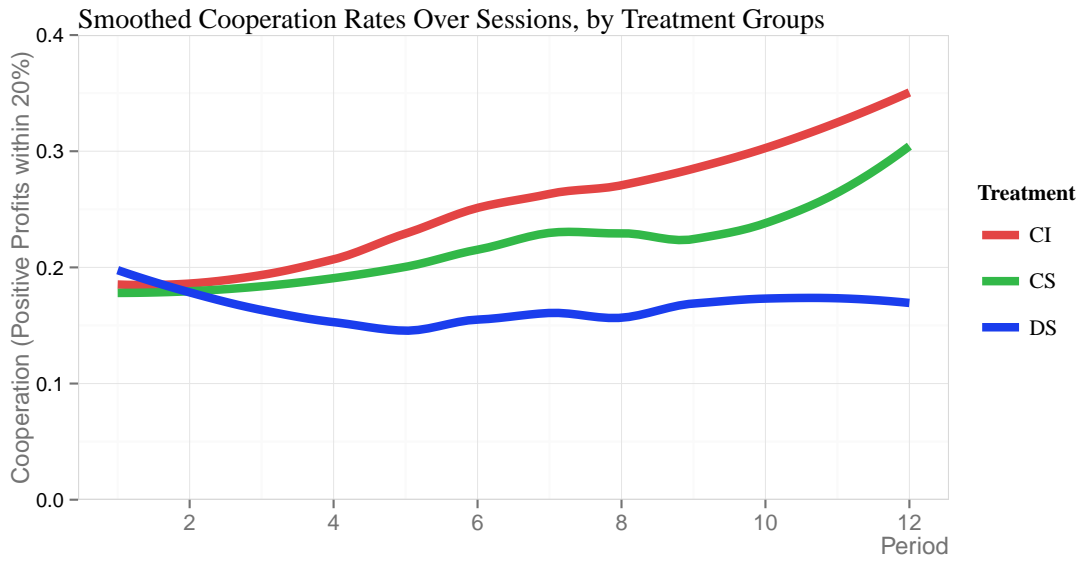
Similarly, we examine the connection between a subject’s tendency to undercut her counterpart, her counterpart’s tendency to undercut, and the subject’s payoff. This relationship is shown visually in Figure 2.7. The subject’s tendency to undercut is shown on the x-axis, given as a count of the number of times a subject undercuts her counterpart — such that the counterpart’s flow payoffs are reduced to zero by the move — in a specific period. This is plotted against the number of times that subject was undercut by her counterpart in the same period. As such, there is intentional “double counting” in the figure, with a subject being counted both as a subject and as another subject’s counterpart. The colors represent the subject’s payoff, with “hotter” colors indicating higher payoffs (note that the counterpart payoff is not shown in the figure). If multiple subject-pairs occupy the same cell, then the average payoff is taken.

We first note that abstaining from these aggressive undercuts are beneficial to the subject only if her counterpart exhibits similar restraint. This can be seen easily by the hot spot in the lower-right corner. It is clear that subjects that were more aggressive than their counterparts tended to have higher payoffs, as evidenced by the darker spots



*Notes:* An “aggressive undercut” is defined as a player movement that causes her counterpart’s flow payoffs to be reduced to zero. The x-axis shows the number of instances in a period that the player completed this action, while the y-axis is the number of times her counterpart performed an aggressive undercut. The colors indicate the player’s score at the end of the period, with darker colors indicating a higher score.

Figure 2.7: Subject-Pair Tendency to Undercut and Payoffs



*Notes:* This shows smoothed non-competitive behavior rates by period, separated by treatment. Rates are measured by the 20 percent threshold, with 90 percent confidence intervals shown in the background.

Figure 2.8: Learning Effects

on the lower right section of the figure. Conversely, the subject's payoffs suffered if she met a counterpart that was more aggressive than her, as seen by the light areas in the top-left. We find this revealing of the incentives subjects faced in the game, as this shows that they consistently did better by being more competitive than their counterparts. However, it is also clear that successful collusion leads to higher payoffs, with high payoffs for the subject when both she and her counterpart did little to no undercutting. Finally, if both players were particularly aggressive and in relatively equal proportions, both subjects saw their payoffs suffer, as evidenced by the lightest region in the middle close to the 45 degree line.

Exit surveys from our experiments indicated that subjects did not initially attempt to come to tacit agreements with their counterparts, even among subjects with high non-

competitive behavior rates. As such, we examine learning effects by treatment in Figure 2.8. It shows non-competitive behavior rates by period with the 20 percent threshold. Early in the sessions, rates were not statistically different between treatments. These differentiate relatively quickly, however, with continuous time treatments increasing and the discrete time treatment decreasing in non-competitive behavior. Our data from the *Continuous Slow* treatment are noisier, and hence is not as stable as the other two treatments and is not distinguishable from the *Continuous Instant* treatment.

## 2.6 Discussion & Concluding Remarks

Our principal findings can be summarized briefly. First, subjects tended to locate close together in the middle of the action space, especially in continuous time treatments. In the *Continuous Instant* treatment with instantaneous price and location adjustments as well in the *Continuous Slow* treatment with delayed price adjustment, subjects were heavily congregated in the center, and were only 10 percent of the action space away from their counterpart on average. This lends strong support to Hotelling's principle of minimum differentiation.

Second, non-competitive behavior was higher in continuous time treatments. Our results are consistent with previous laboratory experiments that showed the ability to respond quickly can increase cooperation, though in this case it is not as nearly dramatic as in a simpler game such as a Prisoner's Dilemma. Our results show that the free and instantaneous adjustment gives the least intense competition, and we give

circumstantial evidence of subjects aggressively pushing for a collusive states at the expensive of short-term payoffs.

However, this eased competition was far from ubiquitous, averaging about 25 percent of the time in settled behavior in the *Continuous Instant* treatment compared to roughly 17 percent in the *Discrete* treatment. Many subject-pairs failed to settle on a price equilibrium, even if one of the subjects in the pair was a willing collaborator. This, combined with our previous findings, can only lead us to conclude that Hotelling was largely correct in the principles and predictions of *Stability in Competition*, even if the model outlined in the seminal paper refused to yield an identifiable equilibrium. A passage from the original paper itself is particularly salient:

*“For two independent merchants to come to an agreement of any sort is notoriously difficult, but when the agreement must be made all over again at frequent intervals, when each has an incentive for breaking it, and when it is frowned upon by public opinion and must be secret and perhaps illegal, then the pact is not likely to be very durable.”*

With this in mind, we make a quiet appeal to economists, political scientists, and teachers of economic theory to use caution with their use of the “folk wisdom” version of the Hotelling model. Many applications of Hotelling’s model — from gas station placement and even to homogeneous price voting theory — should be viewed with extreme caution in light of the instability — especially on the price dimension — shown in our experiment. For instance, according to the median voter theorem in a two-party system, political parties should track to the center to capture the maximum amount of votes. But it is clear that this works only if party differences can be characterized only by horizontal differentiation, as introducing vertical differentiation — as we followed



	10%		20%	
	Mean	Median	Mean	Median
Continuous Instant	0.1605 (0.0313)	0.1072 (0.0076)	0.2669 (0.0334)	0.2053 (0.0143)
Continuous Slow	0.1413 (0.0191)	0.1006 (0.0126)	0.2623 (0.0201)	0.2311 (0.0184)
Discrete	0.097 (0.0137)	0.0712 (0.0088)	0.1734 (0.02)	0.1489 (0.0169)

*Notes:* Mean and median non-competitive rates with bootstrapped standard errors. The percentage refers to the threshold defining when subjects are not competing.

Table 2.6: Non-competitive Rates by Treatment, All Periods

Hotelling in doing so here — will obscure the clean result and require robust cooperation to achieve some degree of stability.

## Part II

# Economics of a Virtual Economy

## Chapter 3

# Aggregate Dynamics in a Large Virtual Economy: Prices and Real Activity in

## Team Fortress 2

[A] synthesis of macro and micro approaches is required to analyze policies instituted at the national level with general equilibrium impacts  
~ James Heckman [36]

### 3.1 Introduction

Heckman's Nobel-lecture proposal is to estimate all macro-level indicators directly from micro-level data. That proposal is difficult to implement in major national economies, but it is entirely feasible in some large virtual economies such as the one we study here.

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A version of this paper is a working paper with coauthor Matthew Baumer, mbaumer@ucsc.edu

The object of our study is the virtual economy of Team Fortress 2 (TF2) developed and overseen by Valve Software. This economy and others like it hold great potential for researchers: millions of users engaging in billions of economic transactions involving thousands of different types of goods; the game designers are near-omnipotent social planners able to create and destroy goods and implement policy at will; and they gather essentially complete micro data that enables precise construction of macro variables.

The TF2 economy has some features that are unusual, even for a virtual economy. There is no explicit currency good, and trading occurs exclusively through decentralized barter. Goods are homogeneous and of known quality (i.e. there are no “lemons” as in Akerlof [1]). Items are also durable and do not depreciate due to “wear and tear” in the way that a physical item would. Another issue likely important is that items are indivisible and can only be exchanged in discrete quantities (e.g. it is impossible to trade half of a common currency item, the treasure key, as keys are not capable of being split). There is also a significant amount of activity that is due to a very small number of very active individuals, which we will refer to as “high net worth individuals” (HNWIs). These quirks will be leveraged in future papers to discuss the issue of the spontaneous emergence of money, the emergence of trade intermediaries, and information brokerage services by applying concepts from network theory to map the interactions between different types of user.

Our approach advances ideas presented in Castronova et al. [16] and Castronova [15] by implementing more rigorous economic indicators of aggregate economic behavior in a large virtual economy. But there are also some crucial differences in our work:

Castronova studies Everquest II, a economy with explicit currency (gold pieces) and in-game posted-price markets available to the users, thus trade in that environment would not be considered barter or decentralized in any sense. Our work also more directly adheres to methodology commonly used in modern empirical economic techniques.

Everquest II and TF2 are far from the only such examples of large virtual worlds with economic activity: “Second Life” is an entire virtual world, complete with in-game real estate, stores, jobs, and of course other people. “World of Warcraft” has players fight monsters and each other with the hope of saving the realm from the great evil that threatens it and has players engaging in money-mediated trade with each other to facilitate this end.

Even the NYSE has made its operations completely digital. Traders physically standing on the trading floor on Wall Street in fact have their business conducted through computer servers located in Mahwah, New Jersey. The NYSE and its affiliated traders have had almost 200 years to develop their institutions; what commerce in virtual economies will look like once it matures is an open question.

## **3.2 Research Questions**

**Q1:** What is the trend in real growth per-capita and how can we explain this trend?

Our primary goal is a basic macroeconomic characterization of this large virtual economy. We will examine the dynamics of real growth and explain what are the

economic causes of dynamics. We also will perform a decomposition of nominal growth into its constituent components: growth of the price level, real growth, and population growth.

**Q2:** How do macroeconomic aggregates (e.g. the price level) respond to macro-level shocks?

An appealing consequence of the complete nature of our dataset is the ability for us to pinpoint precisely what might be causing, for example, bouts of inflation or deflation. There are also numerous exogenous policy changes and events that appear to have influenced this economy and can be detected in our indicators.

**Q3:** How do markets for individual items respond to micro-level shocks?

A quirk of this environment is that there are numerous unexpected events that can be taken as exogenous by market participants. For example, a number of cosmetic items were suddenly “retired”, meaning they were removed from the store and new items of these types could no longer be acquired, fixing their number in the economy. We might expect this intervention to increase prices – essentially a negative shock to supply – but it is also possible that market participants’ speculation “overshoots” the new (post-announcement) fundamental value.

### **3.3 Environment and Data**

Team Fortress 2 is a competitive multiplayer first-person-shooter game which has two teams of typically 6 to 10 combatants vying for supremacy. Winning could result

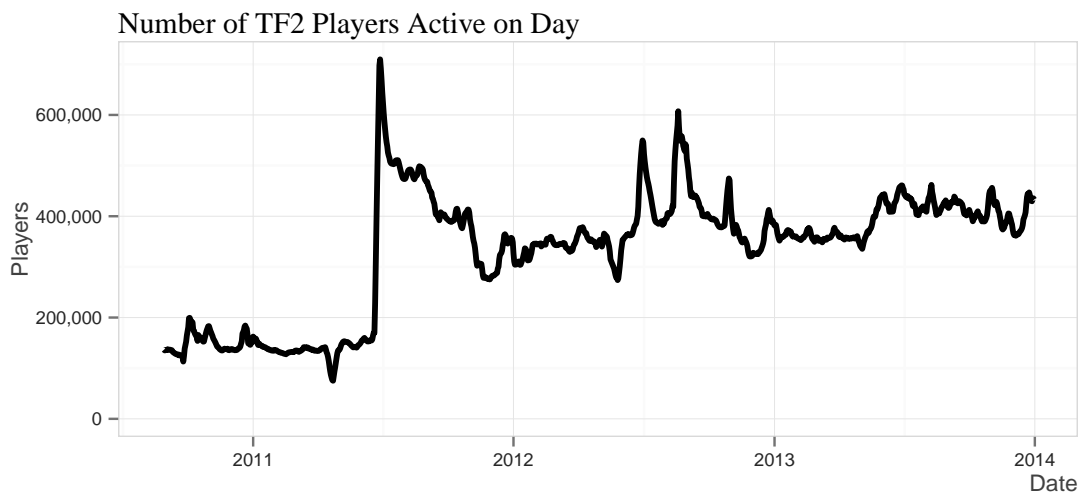


Figure 3.1: Daily TF2 Players

*Note:* Count of unique players logged into TF2 over day. Seven day moving average applied.

from (depending on the game mode) killing enough of your opponents (but don't worry, death is only temporary!), capturing a briefcase full of valuable intelligence from the heart of your opponent's base and sneaking it back to your home base, or successfully pushing a cart full of explosives to your opponents base to blow them up. One round of the game typically takes ten to twenty minutes. Each player in a game chooses their character class from nine different options such as quick and agile Scout, the pro-social Medic, or the deceptive Spy and try to do their best to help their team achieve glorious victory.

### 3.3.1 Economic Environment

TF2 debuted in 2007 and initially followed the standard video game business model: players pay for a copy and can play to their hearts' content. Then, in 2008, a new dimension was added to the game: an item system which allowed users to collect virtual

goods which would customize the look and play style of their characters. As people played, they would randomly receive item drops (and some special items could result from completing a list of in-game achievements), but there was no way to exchange items with each other. A September 2010 update introduced two institutions which continue on until today: a barter platform to exchange items with other players, and a virtual store where items could be directly purchased from Valve using a credit card. Figure 3.1 displays the number of active players on a daily basis. In November 2013 alone, there were more than 2.1 million different users that spent some amount of time playing TF2. At the end of July 2011, TF2 went “free-to-play” (F2P), removing the requirement to purchase a game license before people are allowed to play, at which point the game generated revenue only by selling in-game items on the official store.

An item in the context of the Valve marketplace is any virtual good that can be stored in a player’s inventory (henceforth referred to as a “backpack”) and be traded. These may include TF2 items, installation licenses for other games on Valve’s digital distribution platform called Steam, and items from games other than TF2 on the Steam platform. Backpacks have finite space, but the capacity is large enough (300 item slots) that most users are unlikely feel this constraint. As well, there are “backpack expanders” that can be purchased from Valve for \$.99 which loosen this constraint by granting an additional 100 item slots.

The process of successfully completing a trade is as follows: Find a trading partner through communication channels that can be internal or external to TF2, add them to your contact list, request a trade session, arrange an exchange in that session which



makes both parties happy, and then execute the trade after multiple layers of confirmation. This is a quite inconvenient system for the market participants, but it represents an opportunity for inquiring economists to study actual human behavior in an environment in which we are theoretically well versed. It is important to point out that the economy by construction was designed to support only barter.

Our sample consists of a full log of all transactions occurring between 9 August 2011 and 31 May 2013, a 661 day interval. There were more than 70 million barter transactions, which averages out to more than 100,000 trades per day or over one trade per second. This is the primary source of the data set which we will use to do the following analysis. Across these 70 million individual transactions, over 300 million virtual items changed hands. There were 4,267,832 unique traders participating in the barter market, with the median trader conducting 4 exchanges, and with approximately one third of traders exchanging ten or more items over the sample period. Some traders participated in a large number of trades; the top ten accounts by trade count each conducted over 150,000 barter transactions.

The Team Fortress 2 trading environment represents the largest dataset of a barter exchange market that we are aware of. This is all the more remarkable since barter markets today tend to emerge in environments which feature weak institutions and consequently have meager record-keeping.

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User Privacy: In order to protect the privacy of individuals involved in the TF2 Economy, user identities were anonymized, timestamps masked, and any data containing unique user identifiers was held on Valve Corporation machines. Though the researchers were given access to the full log of market transactions, all other company supplied metrics removed users who marked their Steam backpacks to private.

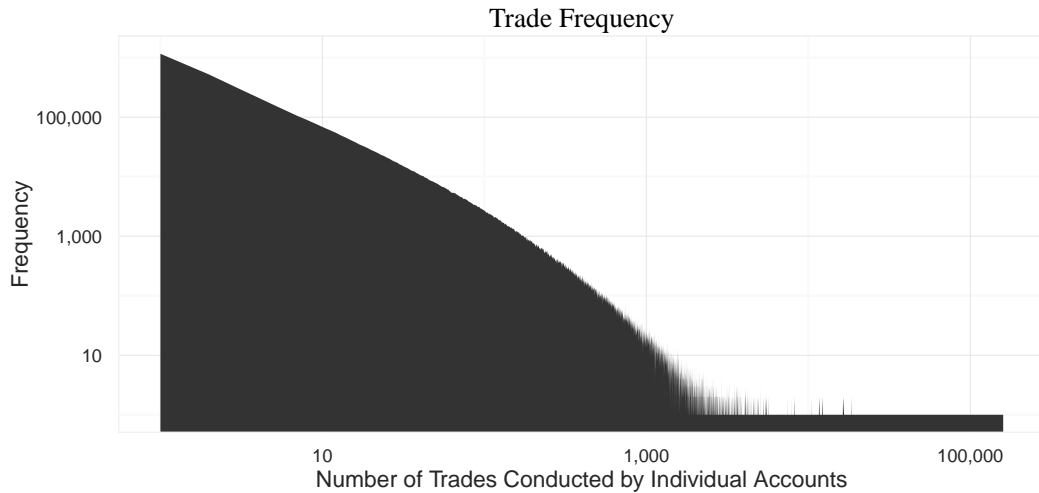


Figure 3.2: Histogram of User Trade Frequency

*Note:* Due to length of tail, top 10% of traders are not visible. Includes trading of non-TF2 game items and “Steam” game licenses. Does not include players who did not trade at least once.

Items in TF2 have various types. There are consumables that are used in conjunction with other items (e.g. a can of paint that can be used on a cosmetic that changes the item’s color palette, or a name tag that lets the player choose a custom name for their item) and durables which can be used for as long as the owner wishes and do not undergo any sort of depreciation as a result of use. All durables have associated class restrictions; some durables can be equipped by any class and others can only be equipped by one or a few classes.

In addition, each individual item is designated one of a number of different “qualities”, which serve primarily to signal scarcity and characteristics of the item. These include:

- Unique: counterintuitively, the most common item quality

- Unusual: adds a custom effect to the item such as flames erupting from the item's surface and are overall the rarest and most sought after quality
- Strange: track various statistics for the player when worn

There are a few other qualities of items, but they are generally simple variations of unique.

Players can gain items from a number of different sources: random drops from playing (although there is a cap of how many items can be received per time period from this source), direct purchase from the “Valve store” using real cash, special promotions (e.g. holidays, as a reward for completing some achievement, or as an incentive for buying another game), trading with other players, by opening crates which require a key which is then consumed along with the crate, and through a crafting system introduced in December of 2009.

From observation of the set of items most commonly used as a unit of account on independent community-created trading posts, there is evidence that the widely accepted commodity currencies include three denominations of “metals”, as well as “keys”, “Bill’s Hats”, and “Earbuds”. The three different types of metals in order of increasing value are scrap, reclaimed, and refined metal. There exists an in-game system that allows conversion of one denomination into another in either direction at the rate of 3 lower valued to 1 of the next higher valued. For example, anyone can convert 3 scrap metals into 1 reclaimed, then combine that reclaimed with 2 more reclaimed to create a refined, then break that refined back into 3 reclaimed. There is no cost associated with

these conversions beyond the time it takes to perform the necessary mouse clicks.

Metals result from scrapping (deleting) weapons from your backpack and are used in combination with other metals and items to create new items via predefined recipes. Keys only originate from store purchases and may be used to open crates that contain new items with various probabilities. Crates are analogous to raffle tickets; if you pay the cost of one key to open a crate, you will most likely get an item worth somewhat less than the key but there is a small chance to get a very valuable item worth much more.

Metals and keys are created and consumed regularly. Bill's Hats and Earbuds, in contrast, entered the market as promotional items given away in the past and can no longer be found or purchased directly from Valve. Their supply is bounded by the current number in existence and slowly shrinks due to people quitting the game or deleting them.

Once a player is in possession of an item, they will not lose it unless they either trade, delete, or consume the item in the case of consumables. At the end of 2012, the ability for players to sell items directly to other players for Valve store credit in an official centralized posted-price marketplace was added. This store credit is denominated in the player's local currency and is redeemable for TF2 items purchased from the Valve store as well as the purchase of licenses for other games from Valve's digital distribution platform called "Steam".

This demonstrates an important distinction between this economy and the physical world; in order to produce a good there are raw materials that necessarily must be

consumed due to conservation of mass. But the production of an additional good in this virtual economy requires no more than a additional line saved to a database. There is still technically an upper bound on how many items can exist, but for practical purposes this horizon is infinite and the marginal cost of production of these goods is zero for Valve.

Another distinction between this environment and physical economies comes from the nature of consumption. Most real goods are actually consumed at some rate and once they are used up, are no longer usable again. This does not happen in TF2. Most consumption is of goods which are perfectly durable (with the exception of tools, but tools either result in or modify durables). We can then think of the size of this economy as being the aggregate value of the stock of durables and tools.

### **3.3.2 Data**

Much of our data takes the form of logs documenting barter transactions of virtual items between two users. These are lists of transactions linked to users and the individual items associated with the trade. These data were supplied to us via a half terabyte sized relational database from which we generated observations in the form of Table A.1. Each row in the transactions log represents the movement of a single item and is associated with a unique trade identifier, two unique player identifiers (one for the sender of the item and one for the recipient), a unique item-level identifier which no two items share (AssetID), and an identifier for the specific item type which identical items would share with each other (EconAssetClass). For example, if a player possesses two unique quality

“Bill’s Hats” that are otherwise identical, they would share an EconAssetClass but also each will be associated with unique AssetID that represents the specific individual item. When an item is traded its old AssetID is removed from the originating user’s inventory and a new one is created for the user receiving it. Thus, we can track both individual items as well as individual classes of items, defined as items which share a type and quality which makes them functionally identical.

TradeID	PartyA	PartyB	Time	AppID	AssetID	NewAssetID	Origin	EconAssetClass
1	1203	1876	1351926000	440	38818	41361	1	100
2	4256	172	1351927010	440	39425	41362	0	194921
2	4256	172	1351927010	440	41359	41363	1	158535
3	993	8384	1351928320	440	41339	41364	0	207

Table 3.1: Example data snippet

We present as an example Table 3.1. By looking at trade IDs, we can classify each individual trade into categories such as simple monetary trades or simple barter trades, as will be discussed in detail later. Party A and B allow us to track the trading behavior of individual traders and the AssetID and NewAssetID let us track the movements of individual items as they pass from user to user. Origin indicates which user is the recipient of the item transfer and EconAssetClass is the identifier which lets us determine the specific item type that was traded. In this fabricated example, the first trade was a one-way exchange where a player with ID number 1876 gave an item to another player with ID number 1203 and received nothing in return. The item that was given away was of type 100. The next trade involved the player 4256 giving an item of type 194921 to player 172 and receiving an item of type 158535 in exchange.

### 3.4 Estimating Prices from Barter Data

Our approach to generating prices for individual items is to define one good among the emergent currencies to be our numeraire, calculate spot exchange rates between the other currencies and our numeraire, and convert goods exchanged for those alternative currencies into the corresponding value in terms of the numeraire. This approach gives us price estimates which allow for direct value comparisons between all items. We also generate statistics for each item including daily turnover, number of trades, and stocks.

The question of how to define which goods are used as “currencies” and which are not is not a trivial one, but this discussion is not something we shall delve into in this paper. Since the different metals can be converted costlessly into each other in either direction at the rates mentioned previously, we convert all price observations involving metal into the equivalent value in terms of refined metal.

From all of the goods used as commodity currencies, we choose keys to be the numeraire. Keys were selected because they appear to have the most stable value, likely due to the fact that their supply is allowed to expand as well as contract and the price is anchored to the dollar since keys can only be produced in the economy through direct purchases from the Valve store at a price of \$2.49 per key. The other potential currency goods either were introduced later on (Bill’s Hats and Earbuds) or displayed rapid expansion of supply (faster than the growth of population) causing instability in

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In upcoming work we will rigorously identify goods that appear to be the most “money-like” based on their characteristics in the data, but for this paper we will simply take money goods for granted and assign currency status to those items which are used as a unit of account in the major community-run pricing resources.

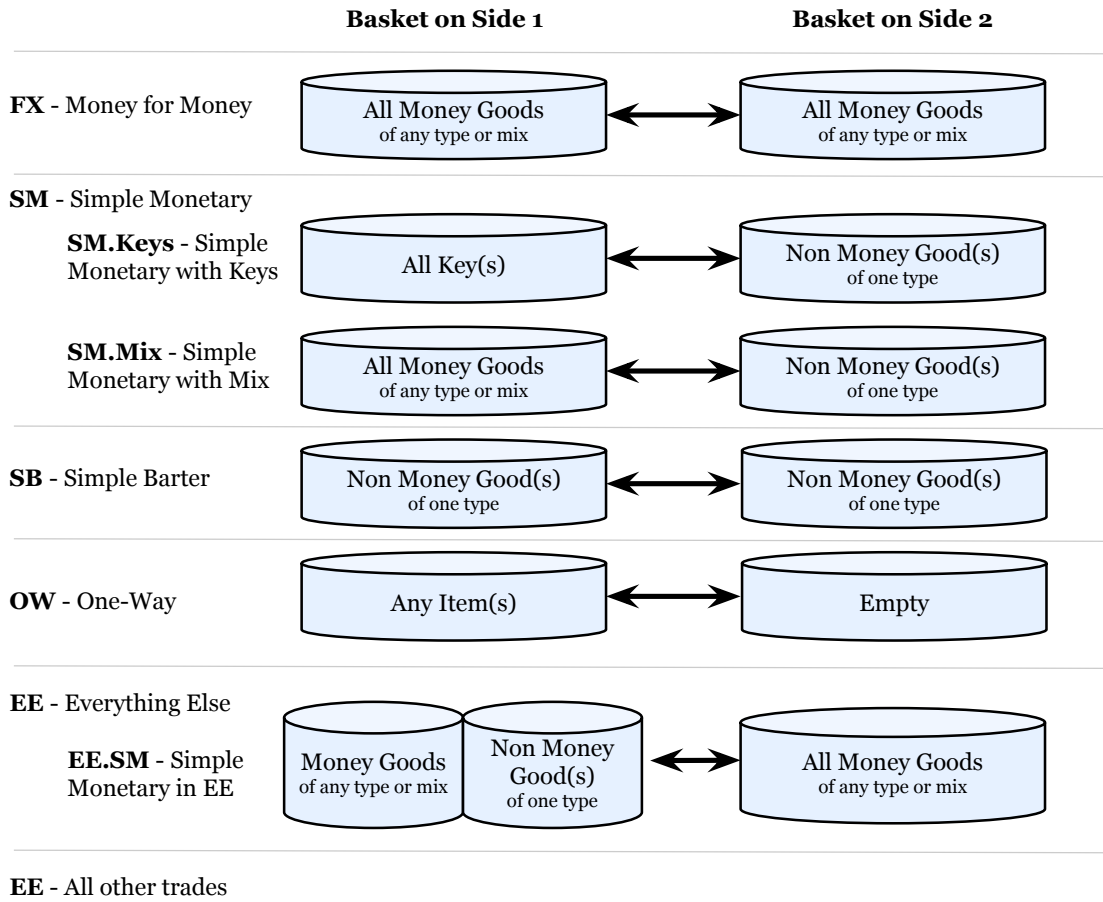


Figure 3.3: List of Trade Classification Types.

estimated prices.

We define a **simple monetary** (SM) trade observation as an exchange involving a single non-currency item type and any basket of commodity currency items. In order to use SM trades to estimate prices that are comparable to each other, prices need to be measured in a common unit, which we refer to as “synthetic keys”. A synthetic key price is the equivalent key-value of a good perhaps exchanged for non-key money(s). We calculate daily exchange rates between different types of money items by looking at the



subset of trades that are **money for money** (FX), which are defined as trades which have only money goods on both sides. See Figure 3.3 for a complete classification of all possible trade types.

By looking at these FX trades, we generate daily inter-money exchange rates as follows. Define  $Q_{it}^{KM}$  as the quantity of *keys* traded for *metals* on date  $t$  in transaction  $i$ , where transaction  $i$  is among the subset of trades involving only metal on one side and only keys on the other.  $Q_{it}^{MK}$  is likewise the quantity of *metals* (expressed in terms of refined) traded for *keys* in the same exchange. A single metal/key exchange rate observation is thus,

$$R_{i,t}^{KM} = \frac{Q_{i,t}^{KM}}{Q_{i,t}^{MK}}$$

The daily spot metal/key exchange rate is then the median of all  $i$  exchange rate observations on date  $t$  (weighted by the number of keys in each observation), allowing us to value any quantity of metal in terms of the going rate for keys at that moment. By a similar process we derive daily synthetic key values of Bill’s Hats and Earbuds. Over 910,000 transactions inform our FX sample (approximately 1,300 a day), of which 700,468 are metal-for-keys exchanges, 107,651 are Bill’s Hat for some combination of metals and keys, and 104,566 involve Earbuds for some metal-key combination. Spot Bill’s Hat and Earbuds exchange rates are based on trades involving keys and/or metals, converting metals into synthetic keys at the day’s metal/key exchange rate.

Using these spot exchange rates to express all SM trades in terms of keys, a simple

monetary price observation is as follows:

$$P_{it}^{SM} = \frac{V_{2,it}^{S.Key} - V_{1,it}^{S.Key}}{Q_{1,it}^{SM}}$$

Where  $V_{2,it}^{S.Key}$  is the value, in terms of synthetic keys, of the all-money side of a SM trade,  $V_{1,it}^{S.Key}$  is the synthetic key value of any *currency goods* on the side of the trade that involves a non-money item, and  $Q_{1,it}^{SM}$  is the quantity of the non-money good involved in the SM trade.  $V_{1,it}^{S.Key}$  can be thought of as a cashier making change when a larger than necessary denomination of currency is used to make a purchase.

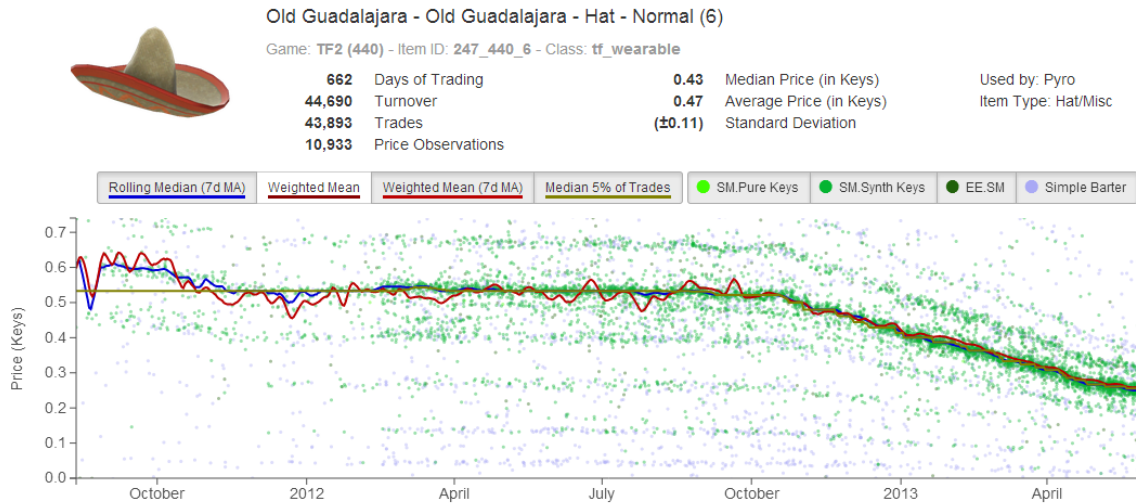


Figure 3.4: Price time series and meta data.

*Note:* A typical individual item price time series. Scatter points reflect individual transactions and their implied valuation. Multicolored lines reflect various temporal aggregate methods deriving daily prices.

Over 9 million trades provide SM price observations, or an average of approximately 14,000 per day. We aggregate our sample of asynchronous price observations on time period and EconAssetClass (item type) to generate price time series for each individual

variety of item. Figure 3.4 demonstrates our price time series for an arbitrarily chosen item, namely a stylish sombrero hat called “Old Guadalajara” which is wearable only by the Pyro character class. Notice that there are discrete bands above and below the price trend line; this is a consequence of the indivisibility of the currency goods. Prior to October 2012, the first of these bands are .1-.15 keys away from each other, which would correspond to the value of one reclaimed metal at contemporary market exchange rates.

An additional 8.5 million trades offer Simple Barter (SB) item value observations as well – trades that involve only two non-money items. However, we only use SM price observations and did not incorporate SB prices because they appear to have a more complicated valuation method than SM trades. It appears that when traders meet, if the buyer of the specific item does not or can not pay in currency items, they must pay a premium with their non-money items, meaning the trade won’t be balanced in terms of value. This would simply introduce mean-zero noise to valuations if we assume that all items are equally sought after by barter traders. But if some items were relatively more sought after than other for barter exchange, there would be some item-specific fixed term that would need to be controlled for. We therefore choose to exclude SB observations from our price estimations as we determined that the number of SM trades is sufficiently large that our estimation process will be precise.

Our temporal aggregation approach assumes that each item at every moment possesses an underlying “fundamental market valuation” based on its characteristics and relevant market conditions. We then take each individual price observation as a noisy

signal for that item's contemporary fundamental value. That is, we assume SM price observations are drawn from their true values, plus some error process. It is worth mentioning that some items appear to have reasonably complex profiles, such as bimodality in price, which we take as further evidence of the economic significance of currency indivisibility.

To estimate the price of a given item on a given day, we start with a seven day window centered on that day and collect all observed SM transactions involving that item. We then clean out observations beyond the 1st and 9th price deciles as there are outliers which, for thinly traded items, can lead to a large amount of volatility. To estimate prices using a rolling average, we then apply a weighting function to these price observations based on temporal distance from the day in question and widen the time window beyond one day if necessary.

A distinguishing characteristic of this environment is the constant addition of new types of items that players can buy or find. This methodology involves taking observed transactions around a given day and using those to estimate spot prices. This approach is not ideal for pricing items soon after their introduction because there will be relatively few observations. To mitigate this issue, we also develop a hedonic pricing model that imputes prices of items based on observable characteristics and supplement price estimates directly as above with estimates from this hedonic model for use in our price index. This hedonic model will be discussed further in the next section.

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See Appendix for more details regarding determination of appropriately wide time windows.

## 3.5 Methods

### 3.5.1 Market Capitalization

We now turn to characterizing the size and growth rate of the TF2 virtual economy. Due to the relative lack of production, GDP is not an appropriate measure for this. We instead calculate the “market capitalization” which we are defining as the total key-value of aggregate item stocks held by active players, where a player is designated “active” if they have played within 90 days. To calculate this, we take the level of existing stocks of each item in each time period and multiply them by the prevailing price in that time period, then sum over all items. We will denote aggregate nominal wealth in period  $t$  as  $W_t$  and is defined as

$$W_t = \sum_{i=1}^{N_t} p_{i,t} S_{i,t}$$

where at time  $t$  there are  $N_t$  total different goods,  $p_{i,t}$  is the price of good  $i$  and  $S_{i,t}$  is its outstanding stock. One quirk of this economy is that a large majority of existing goods do not undergo any sort of depreciation. This means that value is constantly being created but relatively rarely being destroyed; compare this to, for example, the value created by a pig farmer. He creates value by raising pigs and selling the pork, but this value ceases to exist once the pork is eaten. Keys and metals are consumed in a similar fashion to this pork, but it is rare for other economically significant items to disappear. But this begs the question: even though item stocks should be increasing

over time, is the real wealth of the average individual agent increasing along with it?

The nominal growth of all active players' inventory holdings can be written

$$W_{t+1} = G_t W_t \tag{3.1}$$

where  $W_t$  represents the nominal wealth and  $G_t$  represents the growth rate of nominal wealth in period  $t$ .  $G_t$  is the product of three components, population growth  $G_t^P$ , per-capita real growth  $G_t^R$ , and growth of prices (i.e. inflation)  $G_t^I$ . Thus, we can take logs of equation 3.1 to find (where lower cases denote log levels):

$$\Delta w_t = w_{t+1} - w_t = g_t^P + g_t^R + g_t^I \tag{3.2}$$

To better understand the causes of shifts in nominal aggregate wealth, we will take advantage of this decomposition but before we can do this we will need measures for each of these components.

### 3.5.2 Törnqvist Price Index

The simplest starting point for a basic price index is a Laspeyres index which uses a quantity basket fixed to a base year and estimates price increases by allowing prices to adjust in each time period:

$$P_t^{Laspeyres} = \frac{\sum_i^N p_{it} q_{i0}}{\sum_i^N p_{i0} q_{i0}}$$

However, there is a particular problem with direct implementation of a basic Laspeyres index: New items are constantly being introduced. If we choose a base period early in our timeline, we will leave out all of the items which were introduced later on which are likely to be economically important. But if we choose a base period late in our timeline, since there are some items which did not exist early in the sample, we can have no prices for items in early periods. And, indeed, this is a significant issue for our environment. At the beginning of our data set, there are about 630 different item types traded, and at the end there are over 1600. The common alternative to a basic Laspeyres index is a Paasche index. Paasche indices suffer from a closely related issue; they take the quantity index from the current year in the denominator rather than quantities from the base year. But we can have no prices in the base time period for items which were introduced later on since we have no observed trades of goods that did not exist. Our strategy for solving this problem is twofold. First, we use a modified Törnqvist index rather than Laspeyres or Paasche. Second we use a hedonic model to estimate what prices for goods would have been just before their introduction.

Our modified Törnqvist [67] index modeled after the way the US C-CPI-U handles its upper level price indices. The Törnqvist index is superlative and built from Translog preference functions. A Törnqvist price relative is as follows:

$$P_{t,t-1}^{\mathbb{T}} = \frac{P_t}{P_{t-1}} = \prod_{i=1}^n \left( \frac{p_{i,t}}{p_{i,t-1}} \right)^{\frac{1}{2} \left( \frac{p_{i,t-1} q_{i,t-1}}{V_{t-1}} + \frac{p_{i,t} q_{i,t}}{V_t} \right)}$$

---

For more details, see Cage et al. [12] and Bureau of Labor Statistics [11] and ILO-IWGPS [40].

See chapter 18 of the *Export and Import Price Index Manual* [31] released by the IMF for a detailed discussion of superlative indices

where  $V_t$  is the total nominal value of all goods in the quantity basket in period  $t$ , thus  $\frac{p_{i,t}q_{i,t}}{V_t}$  is the expenditure share of good  $i$  in period  $t$ . The quantity index we use to calculate was built by drawing a weekly sample of active players from the population and observing what those players were holding in their backpack. For a detailed description of our sampling methodology, please see Appendix [A.7.2.1](#).

The Törnqvist index helps to avoid the problem discussed above with the simple Laspeyres: since the base period for each calculation is the previous period, the number of new items introduced between base and current periods are minimized. As well, since the weights are value shares, new items being introduced simply decreases the weights of already existing items so the index does not increase due to increasing quantities of items. The chain Törnqvist price index from base period  $t = 0$  to period  $T$  is thus:

$$\text{Chain } P_T^{\mathbb{T}} = \prod_{t=1}^T \left( P_{t,t-1}^{\mathbb{T}} \right)$$

One issue with our approach is due to the existence of items which are untradable - that is we observe no prices - but which appear in our representative bundle. These items certainly have a non-zero value and they do enter and leave people's inventories, but we have no choice to exclude these from our index. This is the same way that national statistical offices handle non-priced services like family household services.



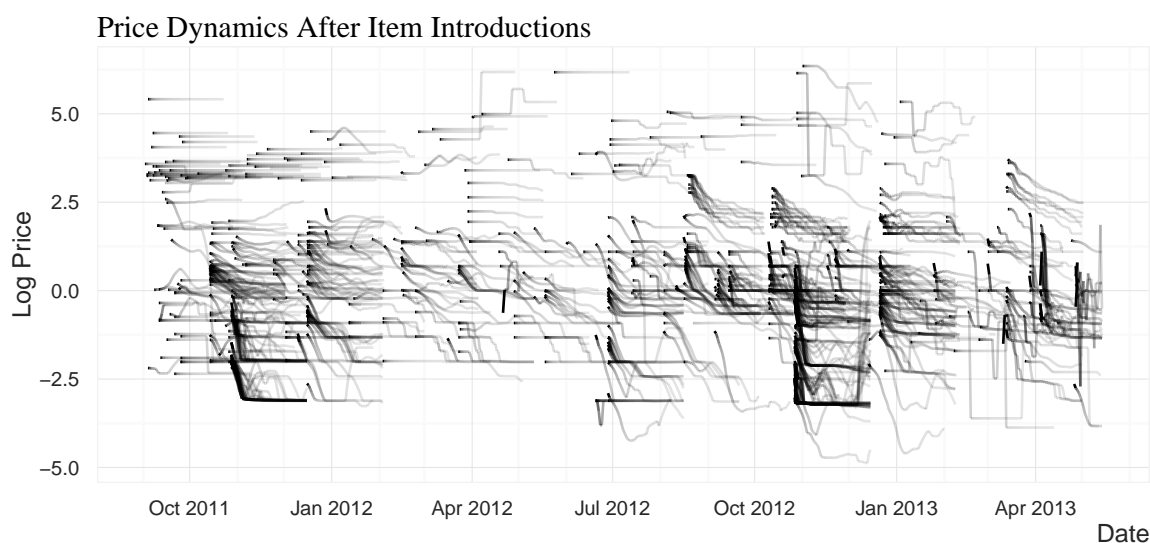


Figure 3.5: New item price time series.

*Note:* Each sparkline represents an item's price over its first fifty days.

### 3.5.3 Hedonic Pricing Model

Another potential issue is that newly introduced items generally exhibit a commonality in price trajectories. Most new items start at a premium relative to similar items, and then steadily trade lower in price. Figure 3.5 displays the price dynamics of items starting with their introduction and tracing the time path of their log prices for the first fifty days thereafter. Log prices are used to shrink the visual distance between item time series, hopefully helping to focus on general price dynamics. Note that there are clusters of new items around Halloween and the December holidays. Items with high starting prices (log price greater than 2.5, about 12 keys or more) appear to hold their value in most cases, but items with lower initial values nearly always trend downward.

The Törnqvist price relative discussed above ignores items for which price informa-

tion is not present in adjacent periods, and thus the initial premium price on most new items is not captured by the existing methodology. Though this issue is likely mitigated by the fact that new items are infrequently traded and seen in relatively few inventories when first introduced—and so their weights would be quite low—the omission of item introductions likely biases our price index downwards.

We deal with the problem of new item introductions by implementing a hedonic pricing model (Rosen [59] and Diewert [22]) which estimates the prices of items based on that item’s characteristics compared to the characteristics of other items with known prices. A similar hedonic price imputation approach is used by national statistical bureaus to estimate prices in conditions of changes in quality. We use the hedonic method as a best estimate of the initial values of each item based on the item’s observable characteristics. This is accomplished by regressing dummies for each of these characteristics interacted with time dummies on each item’s prices over time. For a given time period, this gives an estimated value for each characteristic an item can have. If we apply the assumption that an item’s value is approximated by the sum of values of its parts, we can estimate the price of an arbitrary item given only its vector of characteristics. We then use these imputed prices as our best estimates for the value of items the day before they are introduced.

We impute unobserved prices via the following hedonic price model:

$$\ln(p_{it}) = \alpha + \delta_t D_t + \sum_{k=1}^K (\beta_{kt} \cdot x_{ik}) + \varepsilon_{it} \quad \text{for } t = 0, 1, \dots, T \quad (3.3)$$

For the price  $p_{it}$  of item  $i$  in period  $t$ .  $D_t$  are fixed-effect time dummies (by week),  $x_{ik}$  is a dummy indicating whether or not item  $i$  possesses item characteristic  $k$  (such characteristics are time invariant), with error epsilon which has the standard assumption of being equal to zero in expectation. Thus  $\delta_t$  is the parameter for the fixed effects of week  $t$  and  $\beta_{kt}$  is the parameter on characteristic  $k$  in week  $t$ .

The different characteristics  $x_{ik}$  we include in this model are item quality, class equipability as some items can be used only by certain classes and others can be used by any class, item equip slot such as weapon or hat, and finally a dummy indicating items held by a large proportion of active players which took a value of 1 if 3% or more of users held the item and applied to less than 25% of items. We believe that these characteristics sufficiently describe different items. We are limited by the fact that a certain degree of the differentiation between items is due to non-quantifiable aesthetics (e.g., two items can be identical with respect to the observables mentioned above, but one of them might have art design that is in some sense “more attractive” and thus would command a premium), but we believe that the number of different items is large enough that these will be sufficiently averaged out when we conduct our regression.

## 3.6 Results

Our primary goal is the characterization of macroeconomic growth of this virtual barter environment. This requires the development of an aggregate price index and hedonic pricing models. Next, we present possible explanations for some of the ob-

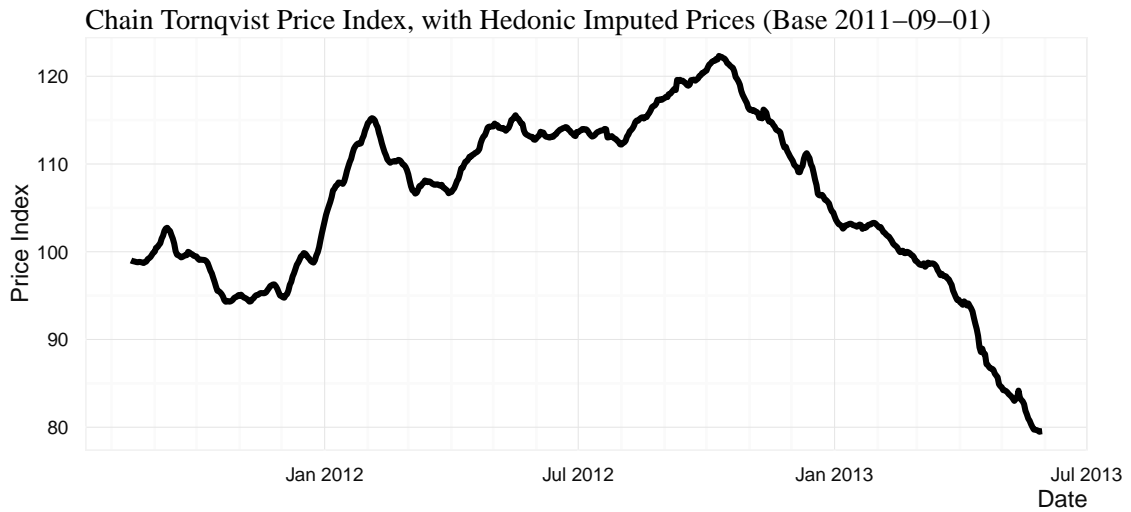


Figure 3.6: Aggregate PI using Chain Törnqvist with Hedonically Imputed Prices

served macro-level behavior. We conclude with our analysis of the impact of micro-level shocks on individual items with evidence of an asset price bubble, the first bubble to be documented in a barter market as far as we are aware.

### 3.6.1 Aggregate Price Level

In Figure 3.6, we present the calculated chain Törnqvist price index. Overall, the price level based on representative backpack contents is relatively stable with slight deflation until approximately mid-December of 2011, when there is a surge of inflation that is possibly related to a Christmas event which brought an influx of new users into the game and introduced holiday-themed items from new crates. This is followed by a dip towards the end of the first quarter of 2012 which proved to be temporary as prices return to their initial level and remain there for several months before seeing steady inflation until October 2012, where we see the most striking feature of our price index.

Starting with the Halloween event of 2012, we see a sustained deflationary period. Our index returns to its initial level around March 2012 and keeps falling until the end of our sample.

### 3.6.2 Insights from the Hedonic Model

The hedonic hypothesis postulates that for any given period, a good is a bundling of potentially time-varying price determining characteristics along with some possible aggregate price level effects that change from period to period. Plotted in Figure 3.7 are the coefficients on the weekly fixed effect dummies  $D_t$  along with their first and second standard errors bands. These can be interpreted as an estimate for changes in the overall price level in a given week relative to the first week. Compare Figure 3.6 to Figure 3.7; with the exception of a peak in the first quarter of 2012 which does not appear in Figure 3.7, the dynamics are remarkably similar. These are both estimating the same thing using entirely different methodology but both tell generally the same story.

Figure 3.8 plots how item characteristics have evolved over the sample using the hedonic model from equation A.7.2.7. In Figure 3.8 we see the evolution of value premiums based on item quality. For example, haunted items tend to have their highest premiums around Halloween (technically, we observe haunted items' least discounts

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Since item-level characteristics are fairly well defined in this context – item quality, character class equipability, and broad item type – it may be informative to run a simplified hedonic regression which eliminates time-variation in the  $\beta$  coefficients. Results from such a model could be interpreted as the average value placed on each observable characteristic for items in our sample and are presented in Appendix ??.

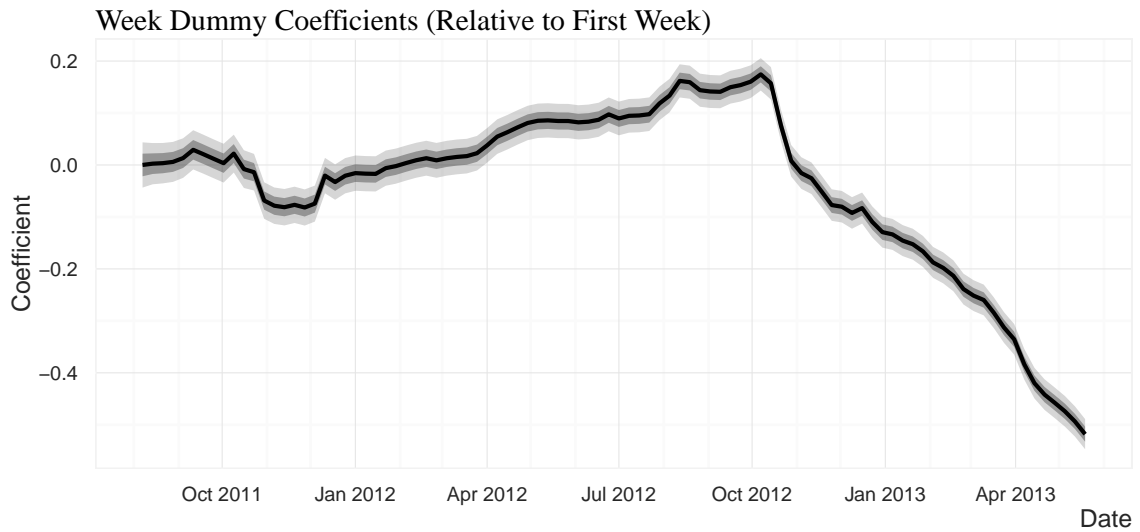


Figure 3.7: Time dummy estimate from simple hedonic price model.

*Note:* Dark and light gray ribbons reflect first and second standard error bands respectively. Coefficient estimates on time dummies from model Equation ??.

around Halloween – haunted items are essentially identical to unique items, except for their text color and quality designation), but haunted quality items otherwise tend to trade at a discount relative to unique items. Unusuals clearly trade at a consistent and increasing premium relative to uniques and other qualities. Interestingly, in the weeks preceding Halloweens, unusuals exhibit an increase in their value premium. This is possibly due to the introduction of a number of highly coveted Halloween themed visual effects (e.g. circling ghosts, cauldron bubbles, and “Demonflame”) at this time. Vintage items exhibit a consistently increasing premium relative to uniques. Vintages are defined by having been in existence prior to the introduction of item trading. These likely show steady increase due to the fixed nature of their supply.

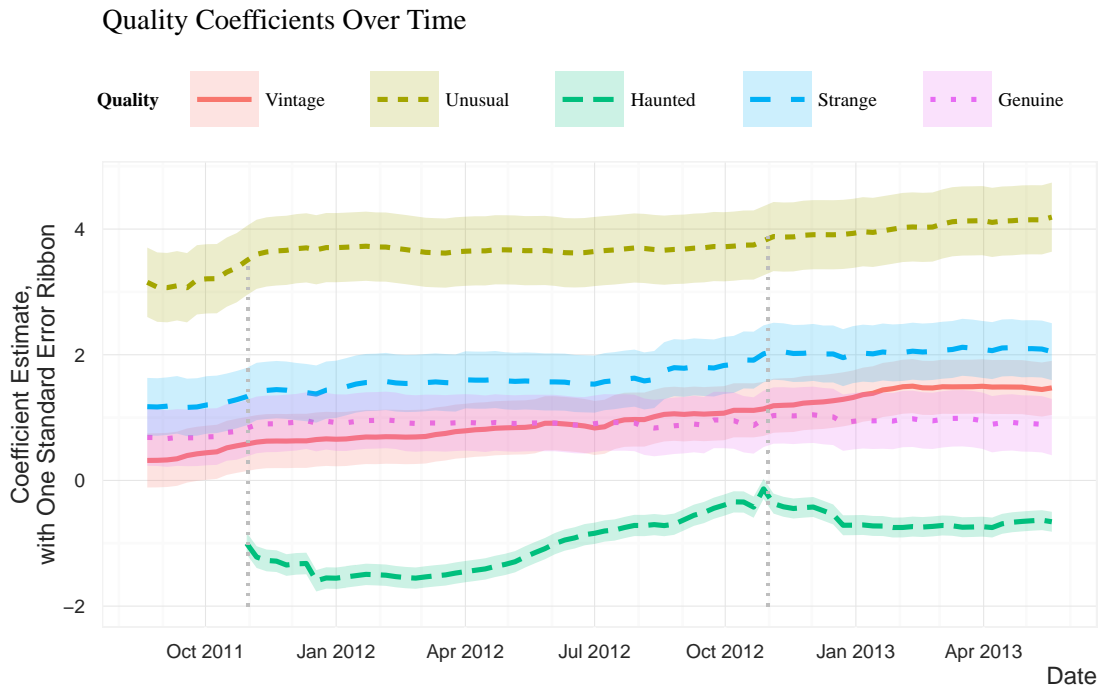


Figure 3.8: Coefficient estimates on time dummies interacted with item quality.

*Note:* Showing how premiums relative to unique have evolved over the trading sample. Standard error bands shows in transparent ribbons. Halloween 2011 and 2012 are indicated by vertical grey dashed lines.

### 3.6.3 Aggregate Value and Growth

Figure 3.9 shows the total nominal value of all items in active players’ inventories (what we call “market capitalization”) on a daily basis. This is calculated by taking the daily price of each item multiplied by the outstanding quantity in active players’ inventories, and summed over all items. We estimate that on the last day of our sample the total value of the economy was approximately 10 million keys – or using a very conservative US Dollar value exchange rate of \$2 per key (keys are available on the store at a price of \$2.49, which acts as a price ceiling) – \$20 million. Expanding stocks to include all TF2 items from *all users’* inventories, not just active players, market

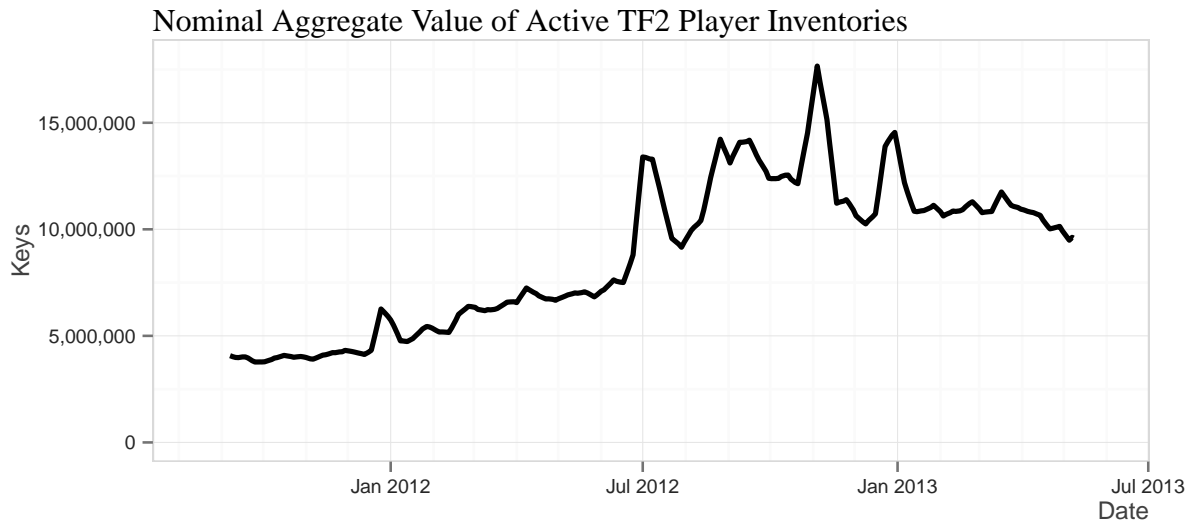


Figure 3.9: Nominal aggregate value of active TF2 player inventories.

*Note:* Keys are sold on the store for \$2.49 each

capitalization on the last day is over 50 million keys, or over \$100 million. Note that towards the end of our two year sample there appears to be a decline in aggregate value. This is explained by the decline in price level causing the bulk of commonly-held items (usually traded for metals) to drop in value with respect to our numeraire.

In previous sections, we elucidated the trends of the price level and per-capita real wealth. Applying those along with data regarding changes in active population to the decomposition presented in Equation 3.2 results in Figure 3.10. The levels displayed are all in percentage terms with respect to the levels in period 0. E.g. at the beginning of July 2012, the nominal economy is approximately 120% larger than it was at the beginning, of which approximately 10% can be attributed to growth in the price level, 35% of which can be attributed to growth in real per-capita wealth, and the remainder attributed to growth in the number of active players.



## Nominal Growth

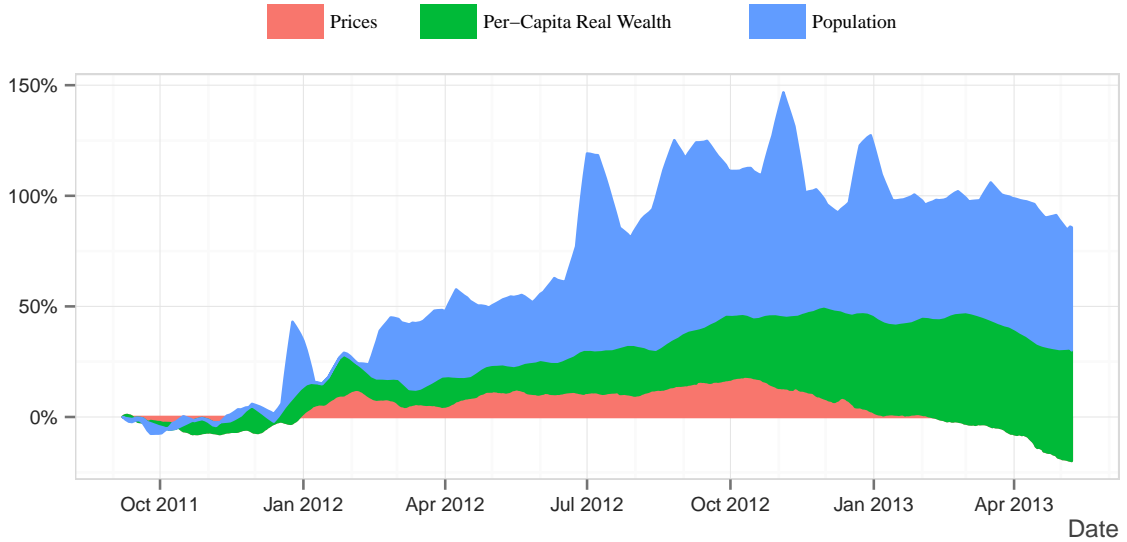


Figure 3.10: Growth of nominal active player wealth

*Note:* Nominal growth since August 2011. Aggregate nominal value of active player wealth is the product of prices, population, and per-capita real inventory values. The natural log of nominal wealth is thus plotted as the stack of these logged components.

We see that real per-capita inventories generally displayed a slowly increasing contribution to the total growth for the duration of our sample. It also shows that practically all of the volatility displayed in Figure 3.9 can be explained by volatility in the population of active players and that there is actually a steady and increasing contribution to economic growth from the real per-capita component. This signifies a healthy and growing economy, even during periods which players are rapidly switching between being active and inactive.

Prices consistently increase after January 2012 until a peak in October 2012, thereafter steadily pulling down net growth until the end of the sample. It can be seen that the contribution from prices disappears (and in fact becomes negative) on precisely the

date just after January 2013 at which the price index in Figure 3.6 shows that the price level dips below its starting point of 100.

#### 3.6.4 Nominal Rigidities and the Decline of the Price Level

Here, we present a plausible case in which this decreasing value of metals can translate to a decreasing aggregate price level. We observe that items tend to be primarily traded for a single currency. Low value items tend to trade for metals, mid value items tend to trade for keys, high value items tend to trade for Bill's Hats, and very high value items tend to trade for Earbuds as a result of the indivisibility of these currencies. It is therefore difficult to profit from currency arbitrage across "value-tiers" of items. It is this combination of price rigidities across currency denominations along with depreciation of metals that may have led to the sustained deflation we observe.

Our best explanation for the deflation towards the end of the sample is monetary and due to the quirks of a barter system with multiple de facto commodity currency goods. See Figure 3.11 for the daily spot exchange rates between keys and each alternative currency. Notice that decline in the price level starts at the end of 2013 – as seen in the price index in Figure 3.6 – syncing up with a sustained appreciation of keys against metals in Figure 3.11. This appreciation is quite significant: at the beginning of our sample it took a little more than two refined metals to purchase a key, but towards the end it took nearly six metals. Thus, the metal-price of keys more than doubled over this period. Also interesting to note is that the path of Bill's Hats/Key and Earbuds/Key exchange rates track each other closely (with a few exceptions near the end of the

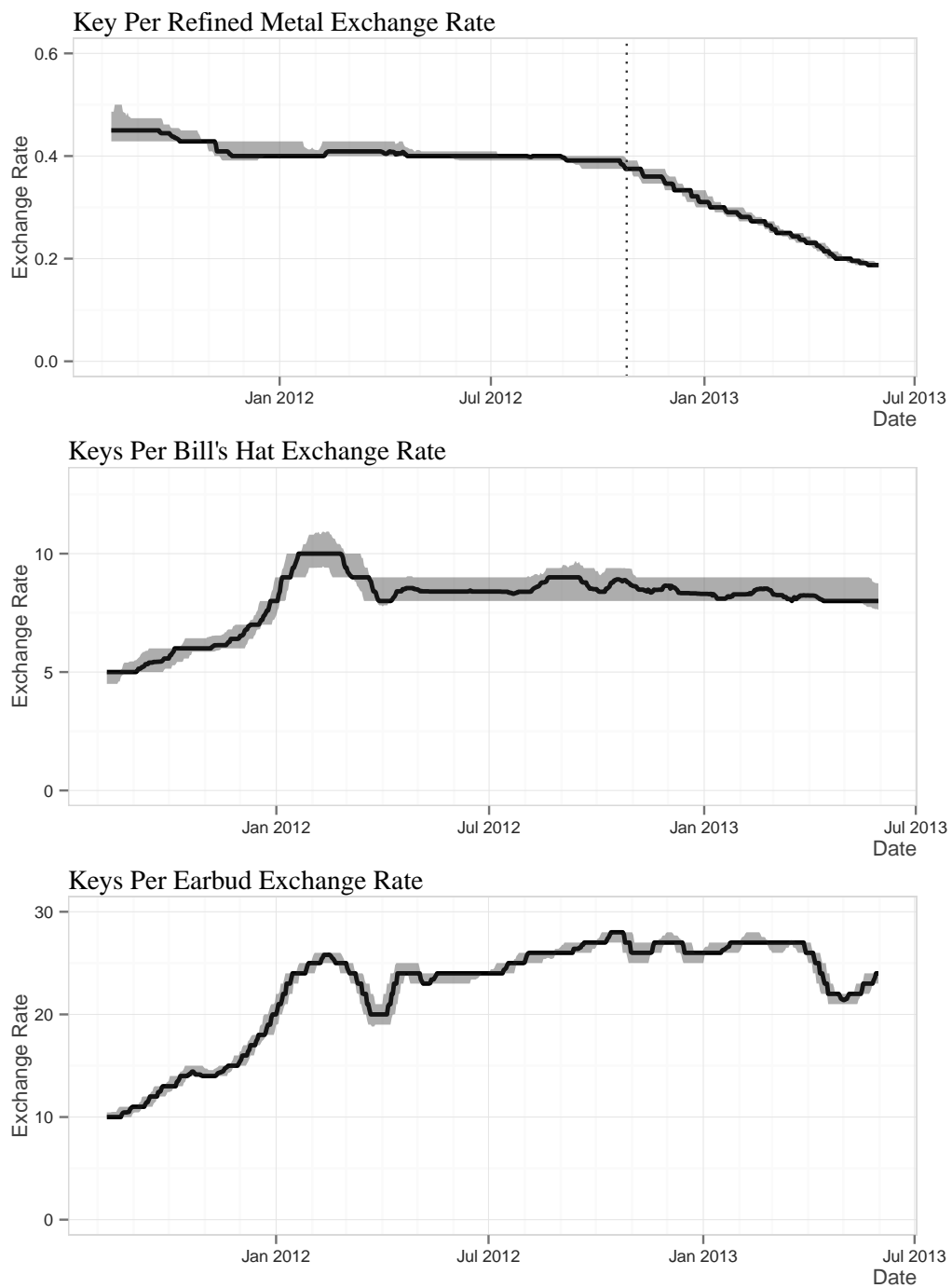


Figure 3.11: Inter-money exchange rates

*Note:* Daily median exchange rate with three-week smoothing. Grey ribbons reflect first and third quartiles of observed daily exchanges, meaning 50% of trades occurred within gray ribbon. 31 Oct 2012 indicated by a black dotted line in the top figure.

sample). This may imply that the higher-value currency goods are better substitutes for each other than the low-value metals, and is also likely due to the fixed nature of supply of these goods compared to the increasing supply of metals and keys.

To illustrate this point, consider how profitable arbitrage would occur if one currency (metal) is becoming devalued relative to the other currencies but metal prices remained fixed. One would trade metals for goods, then trade those goods for non-metal currencies, then trade the non-metal currencies back for more metal than they started with. This is only worth it if costs associated with trading the goods for non-metal currency is lower than the surplus from completing the cycle.

If these search and transactions costs are large enough, it is not worth it to engage in the arbitrage that would keep prices constant across all currencies. We see that as metal-key exchange rates decline and the value of metal to decreases, this does not appear to fully translate to the metal-price of metal-denominated items. Indeed, we see that for most metal-denominated items, their key-prices fall as metal depreciates. Thus, as the key-price of metals drops, the key-prices of metal-denominated items tend to drop with it. This leads to the component of our quantity bundle which consists of items that are primarily traded for metals to drop in lock-step with the metal depreciation. If this component of the aggregate quantity index is “large”, it alone can drive large movements in our aggregate prices.

We argue that this is due to frictions imposed by a barter market. If buyers were equally willing to pay with keys as metals for the purchase any good, it is likely that the prices of goods as denominated in the more consistently valued currency would be

constant and there would be an increase in the price in terms of the currency which sees a declining value. But, if most traders will only offer metals for some subset of goods because it is impractical to trade for goods which are worth a tenth of a key or less using keys or higher value currencies, such a scenario is plausible.

We now present evidence for the presence of nominal rigidities discussed above, which would imply that items which happen to be priced in terms of metals – likely due to their low value and therefore difficulty in trading with indivisible higher value commodity currencies – have their value linked to the value of metals.

We investigate this by linking the frequency that metal is used to pay for items to the price change from Oct 2012 to the May 2013. We estimate the following weighted OLS model:

$$\rho_i = \beta_0 + \beta_1 \cdot m_i + \varepsilon_i$$

In this regression,  $m_i$  represents the value proportion of SM trades for item  $i$  in which the item trades for metal and thus  $1 - m_i$  is the value proportion of trades which the item was exchanged for non-metal currencies. For example, an item that *always* traded for metals would have an  $m_i$  of 1 and an item for which half of the value of trades was from metals and half was from keys,  $m_i$  would be .5. The regression relies on value share percentages derived from October 2012 observations and these value share percentages hold a 0.95 correlation with observations in May 2013, implying that these value shares seem relatively stable over our time horizon. The dependent variable  $\rho_i$  represents the percent change of the price of item  $i$  with respect to this item's price in

October 12, 2012, just before the start of the deflationary period.

The model is weighted by the total value of each item  $i$  in the month of October 2012, thus more economically significant items were given heavier weights. We only looked at items for which prices were observed in both Oct 2012 and May 2013, there were 1,288 such items. We remove observations for which percent price changes were above the 99th or below the 1st percentile, leaving 1256 items with prices in both periods.

The interpretation of this regression is straight forward: the sign of the coefficient on  $m_i$  tells us if items which were primarily traded for metals tended to undergo price increases (positive  $\beta_1$ ) or price decreases (negative  $\beta_1$ ) over the period of deflation which started in October 2012.

<i>Dependent variable:</i>	
Percent Change in Price	
Metal Value Share	-0.1867*** (0.0406)
Constant	-26.0422*** (1.4763)
Observations	1,256
R <sup>2</sup>	0.0166

Table 3.2: Regression Estimates from WOLS of Price Change on the Trading Value Share of Metal.

*Note:* \*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Our regression coefficients are reported in Table 3.2. It shows that on average, items that traded 100% with metals tended to experience an 18% decrease in price compared to items which never traded for metals. This is evidence that items which

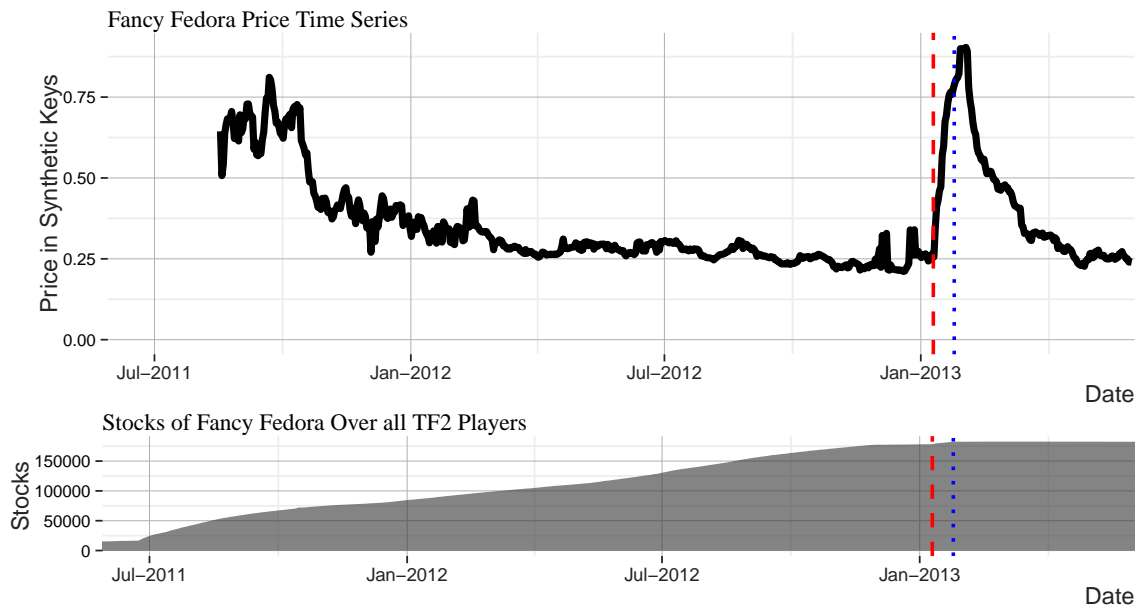


Figure 3.12: Price time series of the unique (normal) quality Fancy Fedora.

*Notes:* The date of the store retirement announcement is indicated by a dashed red line, and the actual retirement date is indicated by the second dotted line, in blue.

trade primarily for metals tend to have a corresponding decline in price. But the decline in price is also less than the decline in the exchange rate of metals (approximately 50% from October 2012 to May 2013, as can be seen in Figure 3.11) which means that this is likely only a part of the whole story.

### 3.6.5 Response of Individual Items to Micro-level Shocks

We conclude our results with a discussion of the impact on individual items of micro-level shocks. Notice in Figure 3.12, the price of the Fancy Fedora starts high and over a few months drops down and stabilizes, as is typical for newly introduced items. But

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The price time series was generated using trailing price estimates rather than the centered prices discussed above. This was because centered prices cause price estimates to increase before the announcement which is not representative of what was happening in the market on this day.

at the beginning of 2013, there is a sudden spike in interest. This is driven by a 10 Jan 2013 announcement, as indicated by a red dashed line, that this hat and 8 others would be “retired” on 25 Jan 2013 as indicated by a blue dotted line. Retirement of these items means that they are no longer acquirable except by trading with other players and thus the total supply would be capped at the current level on 25 Jan.

This announcement led to rampant speculation on these items which drove up the price by approximately 120% over the two week time period between announcement and retirement. But this price boost ultimately proved to be temporary as the price falls almost as rapidly as it surged in the first place. This represents the first evidence of a possible speculative bubble in a barter market that we are aware of.

Figure 3.12 also shows the stocks of Fancy Fedoras. On January 10, 2013 there were 178,400, which increased by 2.26% to 182,440 by January 25th. Our best explanation is that there was a sufficient quantity of these hats in existence to satisfy the demand for them for the purpose of durable consumption at the price of approximately 0.2 keys, but the retirement announcement caused a positive demand shock as market participants anticipated a negative future supply shock, driving up current prices (red dashed line). Soon after this negative supply shock took place (blue dotted line), it became clear that the act of fixing supply did not actually do much to shrink quantity available and – as well as the fact that there are likely a large number of close substitutes and the influx of supply by speculators after the January 10 announcement – meant that people interested in durable consumption of the item could simply buy a different hat that didn’t see the price more than double. Thus, the announcement and subsequent



retirement did not effectively change long run demand and had a small but positive effect on long run supply, so the price returned to its initial level and the speculators that went long on them figuratively lost their shirts.

Another item example demonstrating clear market responses to micro-shocks is the strange Scattergun, a strange-quality version of the default class weapon of the Scout. Strange quality items are notable because they record some sort of player statistic while the player uses the item (e.g. a counter that tracks total number of other players killed with the gun).

On 9 October 2012, as shown in Figure 3.13 with a red dotted line, these stranges were suddenly made available from a newly introduced and particularly ubiquitous series of locked crates and found inside these crates with a probability of approximately 20%. The effect of this policy shock on supply can be seen in Figure 3.13. The total stock on 9 Oct 2012 was approximate 71,000 and had been increasing at the rate of approximately 50 per day for months. After this new crate was introduced the rate of increase of the inventory stock suddenly exploded: after one month there were more than 101,000 strange Scatterguns in existence. And after three months, the stock had doubled.

The impact of this sudden large positive supply shock can clearly be seen in the price of the gun, depicted in Figure 3.13. In contrast to the story of the Fancy Fedora, in which the retirement of an item did not appear to have a long lasting impact on the market supply or demand leading to long run prices being the same as before the retirement, this event obviously actually impacts the long-run supply which causes an unambiguous decrease in long-run market price. Thus, individual prices in the economy

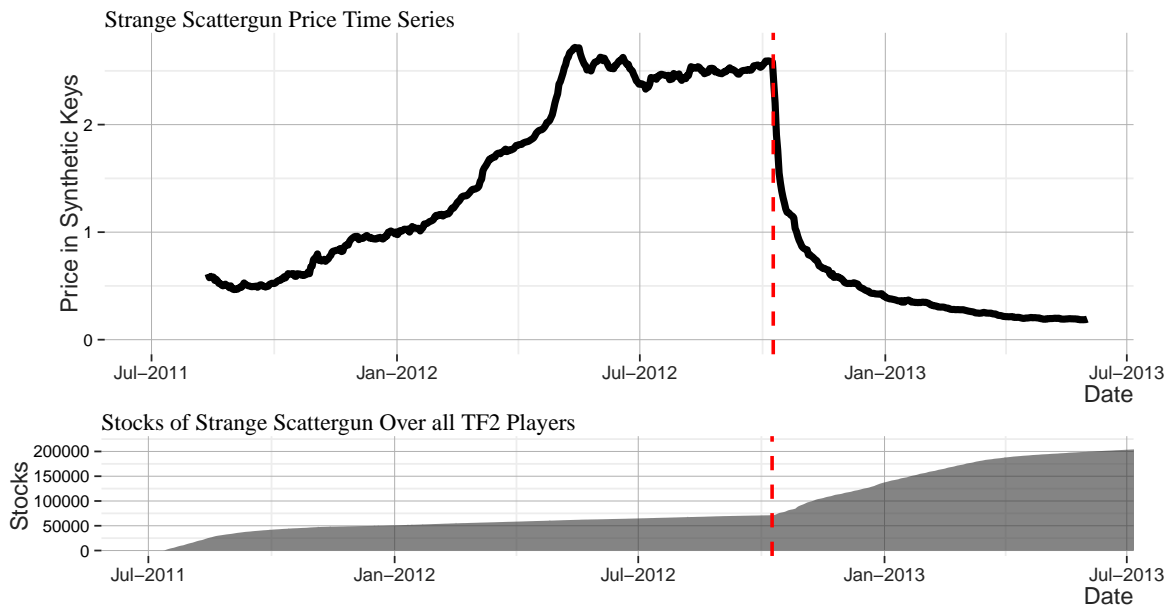


Figure 3.13: Price time series of the strange quality Scattergun.

*Note:* The red dotted line is at October 9th, the date which the item became more widely available.

do indeed appear to respond to specific micro-level shocks in the ways consistent with basic microeconomic intuition given the direction of the shocks to supply and demand.

### 3.7 Conclusion

With this work, we present an examination of an economy which is interesting for at least two reasons. First, it is a remarkably rich dataset which documents a true barter market, the likes of which have been pondered by economists for centuries. Second, it's a virtual economy consisting entirely of non-tangible goods which people nonetheless assign value to.

Our primary goal was to calculate macroeconomic growth in this novel environment and concluded that an increasing component of nominal growth was due to increases in

real per-capita holdings. Per-capita real wealth displays a slow and steady growth for the duration of our sample and most of the volatility in aggregate economic value can be explained by volatility in the active player population.

We presented a hedonic pricing model which we used to impute prices for a Törnqvist price index. We show that not all classes are created equal when it comes to item values. The index indicates that the price level tended to rise until October of 2012, at which point the price level starts declining due at least in part to the declining value of metals. We then traced the source of this depreciation of metals to a shock to both the stocks of metals and keys as well as the rate of increase of their respective supplies. We then demonstrated that items which trade for metals tended to have prices that decreased as the value of metals declined, indicating possible nominal rigidities. But the price decline was less than the decline of the value of metals, so this is likely not the only thing affecting these items. Thus, we did find evidence that macro-indicators responded to macro-level shocks.

Finally, we find in these virtual economies evidence of the same sorts of forces which evidently influence “real world” markets in our micro-level case studies. If a credible central authority makes some decree that could increase expectations of future prices, prices move in that direction. If there is a sudden exogenous positive supply shock in the market for a specific good, the price of that good falls. These goods are but two of many items that have been impacted by idiosyncratic shocks, and their behavior is mirrored in similar goods which were subjected to similar shocks. None of this news should be surprising, but it supports our position that other such virtual economies

(which are certainly only going to become more common in the coming years) are fertile ground for further research and the fact that these virtual economies will typically have impeccable record keeping should be enough to get researchers excited.

Future work will investigate the emergence and evolution of number of fundamental market institutions in the tradition of Radford [58], Burdett et al. [10], and Lankenau [46] and we will search for the origin of media of exchange and the development of trade intermediaries by mapping trade networks and behaviors of these intermediaries. In doing so, we hope to answer questions related to how much surplus such intermediary activity brings to the economy as a whole, and how is that surplus is distributed amongst various types of users, deep questions that go to the heart of classic economic inquiry and are issues which many modern economists have struggled to answer empirically.

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See Smith (1776) [65], Jevons (1885) [43], and Menger (1892) [49]

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# Appendix A

## Valve Economy - Macroeconomic Analysis Methodological Appendix

### A.1 Introduction

In the paper “Aggregate Dynamics in a Large Virtual Economy: Prices and Real Activity in Team Fortress” authors Matthew Baumer and Curtis Kephart examine economic activity in a large virtual economy designed to allow decentralized barter as the sole exchange institution. This paper explores in detail the technical considerations and methodological steps taken to map these barter transactions logs to standard economic metrics.

## A.2 Overview of the trading platform

This section briefly introduces background information about Valve, the Steam videogame platform, games on the platform, and the Steam Trading marketplace.

Valve Corporation maintains Steam, an internet based platform for the distribution and administration of digital rights related to PC games. On Steam, users may purchase and sell videogames, and the platform offers tools for social networking and connecting to multiplayer games. As of February 23, 2015 Valve states Steam supports over 125 million active users and over [4500 games](#). On a single day Steam has served nearly [10 million concurrent users](#).

In addition to distributing other makers games, Valve also creates and distributes its own licenses. Prominently in the Steam virtual economy are Team Fortress 2 (TF2) and DOTA2 (D.O.T.A. once stood for Defense of the Ancients).

TF2 is a lighthearted online multiplayer first-person-shooter. Two teams compete on maps for various objectives (capture the flag, deathmatch, king of the hill, etc.). Players choose from [nine character classes](#). Each character class has its own particular strengths, weaknesses weapons and other items (some weapons and other items may only be used by certain classes, other items may be used by more than one, or all classes). Many items are purely cosmetic (e.g. hats, sunglasses, or paint that changes the color of a weapon, hat, or sunglasses).

TF2 was originally released for sale in October 2007. On June 23 2011 the game was made free-to-play. Any person could play the game at no costs. To help support

the game Valve Software introduced a game-linked store in which users could purchase virtual items (weapons, apparel, etc.), complimenting the items players receive while playing the game.

Up to this point players had no way to exchange or transfer the items they possessed. If a player purchased an item in the store, were awarded it for achieving some milestone, or were given it by the game randomly they could not gift or trade the item with others. On August 9 2011 Valve Software offered the ability to exchange or gift items, effectively creating a secondary market for its digital items ([August 9 2011 marks the beta release of Steam Trading](#), it became available to [all users September 6 2011](#), and technically some form of trading was permitted as of [Sept 30 2011 on a very limited basis](#)).

Our dataset starts on August 9 2011 and ends May 31 2013. Trades in the first half of our log mostly involve TF2 items. In June 2012 Valve introduced their game DOTA2. The game also incorporated virtual goods that could be traded and quickly became popular. By the end of our sample about half of all items traded on the platform were DOTA2 items, with the remaining mostly TF2 items, in addition to some game licenses and items from other games.

An item in the context of the Valve marketplace is any virtual good that can be stored in a player's inventory and be traded. These may include TF2 items or items from other games, or installation licenses for other games on Steam. User inventories have finite space, but the capacity is large enough (300 item slots) that most users are unlikely feel this constraint. As well, there are "backpack expanders" that can be purchased from Valve for \$.99 which loosen this constraint by granting an additional



100 item slots.

A trade is completed by the following process: find a trading partner through communication channels that can be internal or external to your game, add the counterparty to your contact list, request a trade session, arrange an exchange in that session which makes both parties happy (which may be a one-way transfer), and then execute the trade after multiple layers of confirmation.

One important note is that Steam Trading was designed to support only barter. According to sources in Valve trading was simply a feature implemented to please a user base demanding the ability to gift and trade their virtual items and game licences. Goods could not be explicitly traded for any close proxy to money on Valve's platform until over a year later, [December 12th 2012](#). Though, many third party agents emerged to offer the ability for players to sell or purchase items for money.

Secondly, since our dataset is limited to transactions executed on the Steam Trading platform, we may miss other transfers of value in the economy. For example, a large number of trades are one-way trades, where parties to the trade appear to be transferring items from one person to another with nothing in exchange. Many of these transfers are likely to in fact be third party trades, where a user is buying or selling an item with a marketmaker, escrow service, or speculator for cash or other value. Though we observe all transfers of items, we would not observe transfers of value outside the Steam Trading platform, whether that be cash, credit, or favors.

The trading dataset constitutes one of the largest and best documented datasets of a barter market. This is all the more remarkable since barter markets today tend to

emerge where institutions are weak or where market transactions are prohibited, thus recordkeeping is consequentially poor. This dataset bears witness with a remarkable degree of richness to a plethora of economics institutions and phenomena.

### A.2.1 Dataset

This section outlines the structure of our dataset. It borrows heavily from Baumer and Kephart (2015).

Much of our data takes the form of logs documenting barter transactions of virtual items between two users. These are lists of transactions linked to users and the individual items associated with the trade. These data were supplied to us via a half terabyte sized relational database. The Table A.1 below gives of the form of the transaction log.

Each row in the transactions log represents the movement of a single item and is associated with a unique trade identifier, two unique player identifiers (one for the sender of the item and one for the recipient), a unique item-level identifier which no two items share (AssetID), and an identifier for the specific item type which identical items would share with each other (EconAssetClass). For example, if a player possesses two unique quality Bill's Hats" that are otherwise identical, they would share an EconAssetClass but also each will be associated with unique AssetID that represents the specific individual item.

Technically, when an item is traded its old AssetID is removed from the originating user's inventory and a new one is created for the user receiving it. Thus, we can track both individual items as well as individual classes of items, defined as items which share

a type and quality which makes them functionally identical.

TradeID	PartyA	PartyB	Time	AppID	AssetID	NewAssetID	Origin	EconAssetClass
1	1203	1876	1351926000	440	38818	41361	1	100
2	4256	172	1351927010	440	39425	41362	0	194921
2	4256	172	1351927010	440	41359	41363	1	158535
3	993	8384	1351928320	440	41339	41364	0	207

Table A.1: Example data snippet

By looking at trade IDs, we can classify each individual trade into categories such as simple monetary trades or simple barter trades, as will be discussed in detail later. Party A and B allow us to track the trading behavior of individual traders and the AssetID and NewAssetID let us track the movements of individual items as they pass from user to user. Origin indicates which user is the recipient of the item transfer and EconAssetClass is the identifier which lets us determine the specific item type that was traded.

In this fabricated example, the first trade was a one-way exchange where a player with ID number 1876 gave an item to another player with ID number 1203 and received nothing in return. The item that was given away was of type 100. The next trade involved the player 4256 giving an item of type 194921 to player 172 and receiving an item of type 158535 in exchange.

The database has tables listing characteristics of each EconAssetClass, and logs on each AssetID. Due to user privacy concerns the company declined to offer any information on Party IDs other than what we are able to glean via transaction logs. Timestamps were slightly masked, Unix time (seconds since the epoch) being offset by some unknown

amount that allowed us to know the transactions date but not the precise time of day.

### **A.2.2 Economics of videogame players**

One may wonder, are the actions of users of a videogame close to those of economies we care about? Items are not produced through capital and labor in the sense we are used to. They arrive randomly while playing the game, or are purchased at fixed rates via a store with zero marginal costs to the retailer. Players play for fun, not for survival. Very few of the items available aid much with gameplay, and many of the most valuation reduce a player's effectiveness. Though some free-to-play videogames sell virtual items to improve game performance ("pay-to-win" or "freemium" games), in this marketplace nearly all items serve a purely cosmetic role (e.g. hats, sunglasses) or cosmetic variation of functional items (e.g. the festive or haunted variety of most weapons).

It is true, a videogame is not a national or a city economy, with economic decisions directly affecting one's well-being. Institutional and policy choices do not influence the physical health and welfare of lives. A video game's equivalent to a recession is not likely to result in anyone's unemployment. No one pretends that matters of the game are more than of local and passing significance. But in the context of the game - for players that play it - these matters can be of the highest order of importance. This is the case even in games in which acquirable items are not vital to player performance. Items signal fashion and accomplishment. One's social standing may be assessed merely by the presence of a subtle cosmetic feature. The exchange of virtual goods may be of supreme importance for some, more important than the gameplay for which the items

were originally intended to supplement.

In many ways this study shows the pervasiveness of economic concepts considered almost trivial in their universality — economic concepts which are now practically taken for granted — but for which we can show in with non-trivial amount of observational richness. Further, the details of the dataset allows us to understand some poorly observed economic phenomena, such as the spontaneous emergence of commodity currencies, the formation of a basket of commodities used as media of exchange, the development of middlemen services, and the effect of market design features on the functioning of this barter economy.

Where the macroeconomy will be subject to the random whims of market forces, the supply of natural resource and development of technology, the vagaries of demand, imperfect recordkeeping, and the choices of government institutions, many aspects of a video game economy are in complete control by a central authority, with near omnipresent documentation.

### A.2.3 Market summary statistics

<b>Date</b>	<b>Event</b>
October 2007	TF2 Released
June 23 2011	TF2 goes free-to-play
August 9 2011	Steam Trading introduced
June 1 2012	DOTA2 free-to-play with trading
May 31 2013	End of our sample

Our sample constitutes 661 days of trading data, running from August 9 2011 when Steam Trading was introduced until May 31 2013. In this sample there are 70,118,655

trades, involving 4,171,719 unique trader IDs.

On what constitutes a trader, it should be noted that an individual may possess more than one trading account, and more than one individual may possibly control a single trader identifier. Going forward we will generally work with a unique trader ID as if they are a unique trader (the identifier PartyA and PartyB in the trade sample above). However, any analysis should be aware of this issue.

On the first day of the sample there were 3,551 trades involving 4,667 traders. On the final day of the sample there were 304,995 trades involving 181,094 traders. Clearly trading activity grew greatly over this time, and in fact was continuing to grow into the end of our nearly two year sample. The highest number of daily trades was on May 24 2013 with 447,258 trades involving 220,502 traders.

Figure [A.1](#) plots the growth in daily trading volumes. Spikes in trading activity around the end of December 2011, July 1 2012, October 20 through mid November 2012 and the end of December 2012 correspond with special events and sales on the Steam platform or in TF2. These times often brought an influx of newly active players to the game and to trading.

#### **A.2.3.1 Trading by Game**

All goods traded on Steam are either associated with a game (all items are associated with only one game) or with a license to play a specific game. Figure [A.2](#) below breaks down items traded each day by the game the item is linked to. Percentages are based on nominal counts of virtual goods traded on that day and do not weight by price.

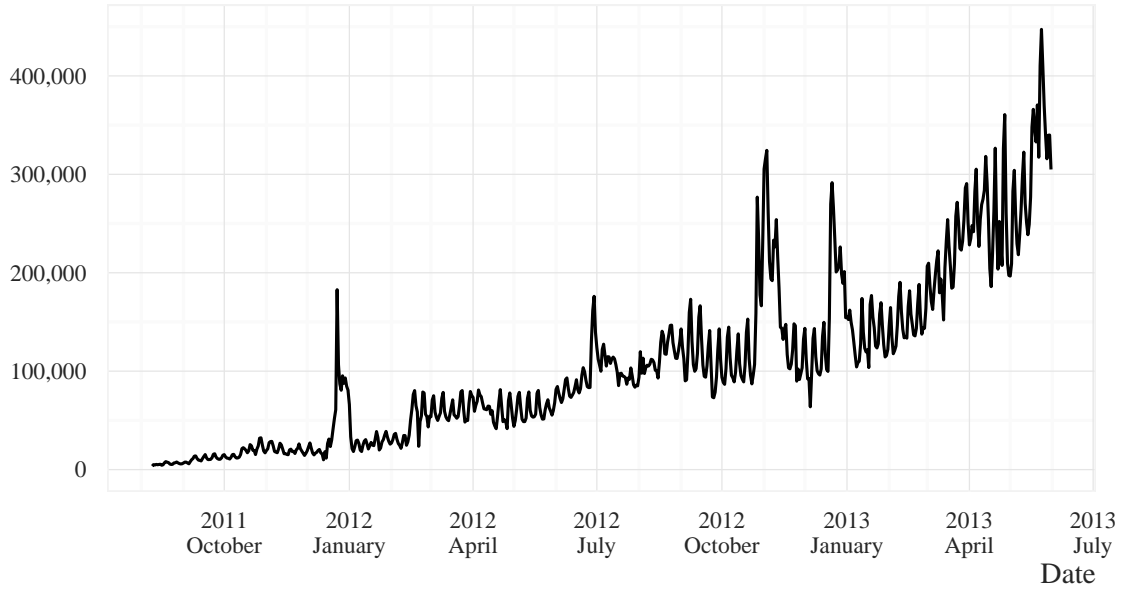


Figure A.1: Number of Trades, Daily

For the first half of the sample the vast majority of items exchanged were from the game TF2, with some trading of Steam licences (which gives digital rights to download games from Steam’s platform). June 1 2012 also marks the release of a new game DOTA2. After the release of DOTA2 trading in its items quickly take off. By the end of our sample a full half of items traded (based on nominal counts) are DOTA2 items.

#### A.2.4 What is an item?

In a game like Team Fortress 2 there are many types of items, from weapons (items usable by players in combat) to purely cosmetic items such as hats (items in which player characters wear on their heads). Some items transform others, for example in TF2 a [Paint Can](#) will change the color of certain items, a Name Tag will attached a

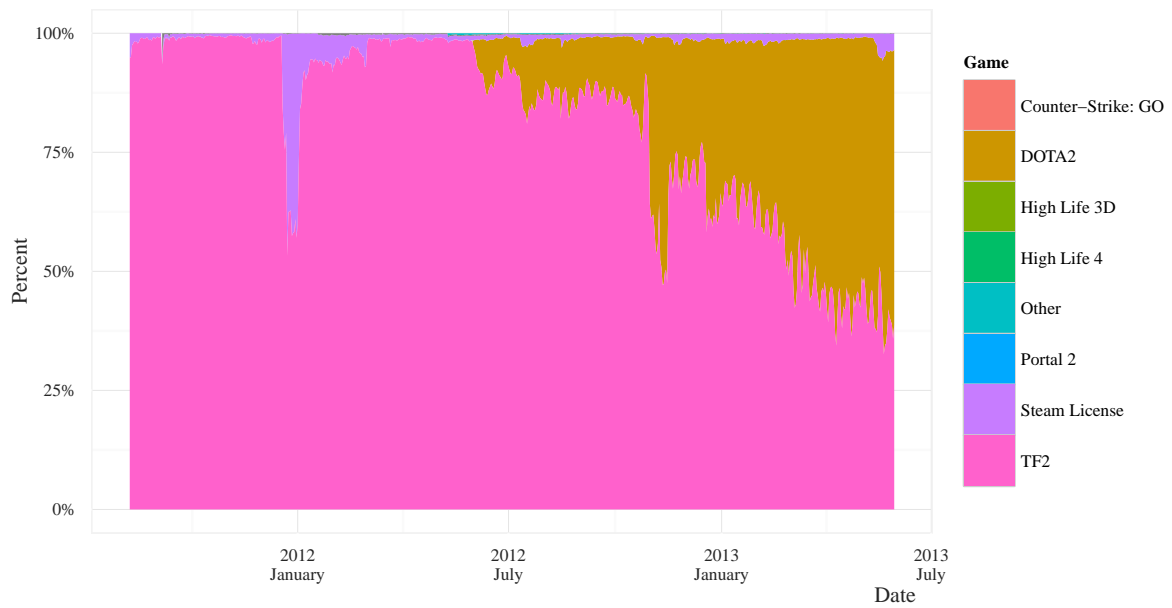


Figure A.2: Percent of Items Traded, by Game

name to an item, a key will unlock a Crate revealing a hidden new item, each of these items are destroyed in the process.

But the objects users may trade are not exclusive to virtual items linked to a game, they may also include games themselves. Players may trade unclaimed game licenses which they possess, essentially the right to access and play a particular game on Valve’s Steam gaming platform.

We define an “item” in the context of the Steam Trading marketplace as any virtual good that can be stored in a player’s backpack (a player’s personal inventory) and traded. These may include in-game items (from TF2 or Dota 2) or installation licenses for games.



Many goods fit most of this description but are untradable. From a player’s point of view these non-traded goods may be otherwise identical to traded goods. However, since we are unable to observe transactional information we exclude them from our analysis.

#### A.2.4.1 Item Granularity

In thinking about what an item is there is also a question; at what level of item should we focus our analysis? There are several levels to consider:

- Application ID (or AppID for short) Each game has an AppID. Items originate and are only useful in the game linked to its AppID. For example TF2’s AppID is 440, the AppID used to indicate a game license is 753.
- Definition Index (DefID) This might be thought of as the highest level of an item. All items with the same DefID have the same 3D design and headline name — but they may vary by color, particle or other special effects, age in the game, quality, and going price on the community market.
- “Quality” is a simple classification to help differentiate items with the same DefID (see TF2wiki at [wiki.teamfortress.com](http://wiki.teamfortress.com) on Quality for more information). Table A.2 lists all item TF2 qualities and their description. Other game like Portal and Dota2 items may also have qualities, but these will usually differ from TF2’s, e.g. DOTA2’s items qualities possess markedly different meanings.

- Economic Asset Class (EAC). This is the lowest level of what constitutes an item, without looking at individual items owned by individual players. Items with the same EAC will share DefID, quality, and all other distinguishing characteristics like color and particle effects.
- AssetID tracks each individual item. Each virtual item in existence is given a unique AssetID when a player receives it. Even if an item changes hands via a trade, the item will lose its old AssetID and receive a new one.

Consistent with convention in the Valve marketplace, the combination of DefID, AppID, and Quality is the primary item level on which we conduct our analysis. That is, for the sake of prices, turnover and other metrics, we define a unique “item” as the combination of DefID, AppID and Quality.

Items sharing the same DefID and game may vary widely in going price. Variation in the going price of same-DefID items depends on how these items are differentiated. Differentiating characteristics such as color, visual effects, the item’s history and others are fully encapsulated by Economic Asset Class. However EAC has proven too granular a view of items, with pricing data often too shallow (some DefID\_AppID\_Qs share hundreds of EACs), and many items that are otherwise similar or the same may vary by EAC. Item quality appears to encapsulates most of the ways same-DefID-AppID items are differentiated in the marketplace.

Two items that differ by EAC but share the same DefID\_AppID\_Q will usually hold the same characteristics that anecdotally define the good’s economic value, these EACs

will usually trade at the same value in the marketplace at any given moment.

#### A.2.4.2 Additional Ad-Hoc Item Specifications

We do occasionally deviate from the DefID-AppID-Quality level when calculating certain item prices.

- Some items should be even further distinguished from the DefID-AppID-Quality level. For example the “Rare Supply Crate” (DefID: 5068, AppID: 440 and Quality: 6, Normal) has three different types. These different crates differ in value based on the expected value of their contents (Thus these items include a “set.supply.crate.series” field, indicating the crate type. E.g. 5068\_440\_6\_ ...)
- Some items may be “gift wrapped” (in fact, some items are untradeable unless gift wrapped). The DefID-AppID-Q of that item is trade is that of the item “A Carefully Wrapped Gift”. Thus to discover the actual item traded, we must look into the gift to discover the enclosed DefID, game and quality.

#### A.2.4.3 Where Do Items Come From?

Items in TF2 may come from one of the following sources (see [TF2 wiki online documentation on Obtaining Items for full details](#)):

- **Item drop system.** Items randomly given to users after playing a certain amount of time. “Like mana from Heaven.”

- **Achievements.** As users meet certain character class specific milestones (e.g. achieving a certain number of types of kills) they may be awarded an item.
- **Store Purchases.** Some items may be bought directly from Valve (Mann Co. Store for TF2 items and the or DOTA2 store for DOTA2 items) at a generally fixed posted price. However, not all items have been for sale at the store, some items have been permanently removed from the store, and some prices have changed. Some items were created by non-Valve-employees, these people receive a percentage of their item's sales.
- **Crates.** An item acquired by a crate opening. Crates may only be opened by keys, in the process destroying both the key and the crate.
- **Crafting.** An item created via crafting (combining items and metals to form new items). Only certain items can be created by crafting, and only certain items can be used to craft new ones.
- **Gifts.**
- **Trading.** Of keen interest of this study.
- **Promotions.**
- **Events.**
- **Dueling.**

- **Community contribution.** Some items that are added to the store are the result of development by community designers whom then share in the profits resulting from sales of the items which they created. People that submit an item which is accepted get a copy of the item in a special quality (typically one-of-a-kind)
- **Steam Community market.** Starting December 12 2012 (announcement) users could sell certain items to other users in exchange for “Steam Wallet” money (a balance good for purchases on Valve’s gaming platform).

### A.3 Methodology

#### A.3.1 Methodology motivation and outline

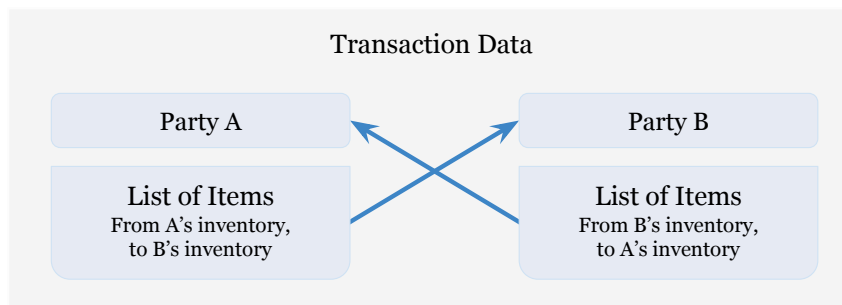


Figure A.3: A Valve Trade

As briefly introduced, data are logs documenting barter transactions. For our date range these are the complete list of individual items traded, linked to counterparties to the trade and timestamps. Standard macroeconomic metrics like inflation, Real and

Quality	English name — description
NULL	<b>Normal/Unique</b> – Null maps to unique.
0	<b>Normal</b> – Stock weapons. People familiar with the game inform us that normal and unique goods are essentially of the same quality.
1	<b>Genuine</b> – Item was obtained during a pre-order special.
2	<b>NA</b> – empty
3	<b>Vintage</b> – Either the item was obtained (1) before Valve went free to play (June 23 2011), or (2) before the item system started (not clear on the date, but before Aug. 2011).
4	<b>NA</b> – empty
5	<b>Unusual</b> – Item come with effects that visually separate it from others. Generally high value stores
6	<b>Normal/Unique</b> – Essentially a “normal” good. (Our analysis groups Null, 0 and 1 qualities into the same “normal” group)
7	<b>Community</b> – If a person is recognized by Valve for making a large contributions to the community, they get an item with a different color of text, (e.g. people who work on the wiki, server operators, people who do something for which Valve chooses recognize them). Very rare items
8	<b>Developer</b> – An item given to developers. (Very rare)
9	<b>Self-made</b> – Given to creator of item. Described as “the most original item” and “the die that casts the first item of this type”. If multiple people made the item, then multiple people will get a self-made. Items of this quality are usually quite valuable.
10	<b>Customized</b> – unclear description. Very rare and essentially unused.
11	<b>Strange</b> – An item that originated from a crate. (Players cannot buy a strange item from the store, the only way to acquire a strange items is via the secondary market or via crates)
12	<b>Completed</b> – unused.
13	<b>Haunted</b> – Seasonal for Halloween (limited time, seasonal item). E.g. spells, full moons. Kind of like unusual.,All turn on and off at same time (2-3 week window around Halloween
14	<b>tobor_a</b> – unknown

Table A.2: List of TF2 Item Qualities

Nominal Gross Domestic Product require prices, we thus first derive detailed price time series for all relevant items, in addition to item stocks and flow data.

One issue with a barter economy is we do not observe any explicit price information since there is no officially designated currency. No money or close money-proxy was incorporated into the online trading system until the introduction of the Marketplace, well into our sample. Our methodology estimates prices by aggregating valuations implied by “going rates” in the transactions we observe. It should be noted that there have emerged third party websites offering going rates for many items in TF2 and DOTA2, however the first of these websites arrived months into our sample. Additionally, many websites vary their pricing methodology, with some set by editorial board, some by voting, others sampling trading logs of voluntarily submitted trades, and we do not have complete logs of how these prices have changed over time.

Since we cannot observe how much money (whether Dollars, Euros, Yen or something else) were traded for items, it begs the question: How should we define a price in a barter economy? And how one might observe sufficient prices to aggregate into a time series?

Based on discussions with economic minded individuals working at Valve as well as affiliated with various Steam trading community resources, we were informed that the Valve Steam economy uses several items as media of exchange. These “Money Items” are listed in the Table below. There are four primary types of money goods: metals, keys, Bill’s Hats, and Earbuds.

Here we briefly outline the methodology organizing our analysis to come:

## 1. Item Price Time Series

- (a) Money Items. Building A Synthetic Key. Our first goal is to calculate daily exchange rates between money items (e.g. given an Earbud for sale, how many keys or how much metal can we expect in return?). With a time series of such inter-money rates we are able to convert any basket of money items into a value of synthetic keys (keys are chosen as numeraire because they are anecdotally known as the dominate money item, however many disagree, suggesting refined metals should be our unit of account). We calculate these exchange rates by looking at the subset of trades between only money goods.
- (b) Collect Prices Observations. We next calculate prices of all non-money items in terms of synthetic keys. We do this by looking at the subset of trades which are have non-money items on one side and some combination of only money items on the other side (e.g. if an item is usually exchanged for one key it is given the price of 1 key). This process generates millions of price observations.

## 2. Item Price Panel. Temporal aggregation of asynchronous price observations.

Given sufficient price observations, we calculate daily price time series — prices in terms of synthetic keys — for each item on each day.

## 3. Alternative Pricing Method, Item Prices via Transaction Regression Loop: We

also consider an alternate pricing method, using regressions to estimate prices that minimize the disparity in values on both sides of each trade.



4. **Market Summary Statistics.** Throughout this process we also collect item level statistics about the barter market, such as daily item turnover, number of traders and number of trades, daily stocks of each item, percent of trades which rely on the commodity currencies.
5. **Macroeconomic Metrics.** We calculate inflation via a chain Tornqvist price index weighting price observations by player item inventories. Items regularly come into existence throughout the sample, we thus impute unknown or imprecisely know prices via a hedonic pricing model. While an aggregate metric like GDP does not make sense in this transactional barter economy, we do calculate nominal market value of all items in active traders inventories. Using our price index we then decompose aggregate wealth into real per-capita wealth, population, and price time series.

### **A.3.2 Anecdotal Money Items for TF2**

Conversations with Valve economists and numerous player forums anecdotally indicate a few items act as commodity currencies in the Steam marketplace. Figure [A.4](#) lists all anecdotal money items in the TF2 marketplace, along with DefID and additional information. Figure [A.5](#) shows images of the top six money items and Figure [A.6](#) shows images of additional items that some in the marketplace consider money.

- **Metals:** There are three types of metals; scrap, reclaimed and refined metal.

These metals has fixed exchange rates between one another, with 3 scrap metals

creating one reclaimed metal, and 3 reclaimed metal to one refined metal. A single scrap metal is created when two weapon-class items are “smelted” down to scrap. Metal may be combined with other game items to create entirely new items via a process call “crafting”. There are numerous and continuously evolving crafting recipes.

- **Keys:** Keys are purchasable from the game store. A single key has nearly always cost \$2.50 USD or similar prices when denominated into other currencies. A key is able to open up a crate, revealing a new item and destroying the key and crate in the process. Crates reveal new items with varying probabilities. Precise probabilities that specific new items appear are not known by players, but may be estimated by examining the distribution of items unlocked by crates. There is a 1 percent chance of a very rare items being created, many of which are worth more than \$100. Crates appear in user inventories via the item drop system, and are the most abundant item in the economy.
- **Special Event Keys:** During certain times of the year like Christmas, Halloween, or the Summer, special crates with special keys have been released. These special crates will have items linked to the event, e.g. haunted items for a Halloween event or festive items around the holidays. Only event keys are able to open event crates. These special events usually expire after a couple of weeks, after which the event crates disappear and the event special keys are only able to open regular crates.

- Bill's Hats: Bill's Hats were items given out to people that pre-purchased a certain game in November 2009. There is now largely a fixed supply of these items. Bill's hats have tended to be worth eight to nine keys, but varied in value over the sample.
- Earbuds: Earbuds were awarded to players who pre-purchased Team Fortress 2 using Apple's OS X between June 10, 2010 and June 14, 2010. There is now mostly a fixed supply of these items. They tend to trade around 25 keys throughout our sample.
- Other potential money items
  - Tour of Duty Ticket: Some market participants suggest the tour of duty ticket should be considered a money item. It was introduced late into our sample and has tended to be worth about 0.5-key. Although it did participate in a large number of trades (677,551 trades with a turnover of 1.2 million) and tends to trade at a fairly consistent value relative to keys, it is not treated as money in our analysis.
  - High value items: In addition to all the items we treat as money, a number of other items are argued by some to serve as money. Max's Severed Head worth about 45 keys, and the Hat of Undeniable Wealth and Respect worth about 200 keys are suggested to serve as media of exchange for high value goods. However, our analysis will not treat them as such.

Def. Index	Def. Name	ItemNameEN	ItemType NameEN	Notes	Name of Money Type
126	L4D Hat	Bill's Hat	Veteran's Beret		Bill's Hat
143	OSX Item	Earbuds	Hat		Earbuds
162*	TTG Max Hat	Max's Severed Head	Hat	Removed from com money list.	---
334*	Treasure Hat 3	Hat of Undeniable Wealth And Respect	Hat	Removed from Com money list	---
5000	Craft Bar Level 1	Scrap Metal	Craft Item		1/9 metal
5001	Craft Bar Level 2	Reclaimed Metal	Craft Item		1/3 metal
5002	Craft Bar Level 3	Refined Metal	Craft Item		metal
5021	Decoder Ring	Mann Co. Supply Crate Key	Tool		key
5049	Winter Key	Mann Co. Supply Crate Key	Tool	Became normal key on 1/3/2011	key
5057	Christmas Key 2010	Stocking Stuffer Key (non tradable key)	Tool	Became normal key on 12/31/2010	key
5067	Summer Key	Mann Co. Supply Crate Key	Tool	Became normal key on 7/12/2011	key
5072	Naughty Winter Key 2011	Mann Co. Supply Crate Key	Tool	Became normal key on 1/4/2012	key
5073	Nice Winter Key 2011	Mann Co. Supply Crate Key	Tool	Became normal key on 1/4/2012	key
5079	Scorched Key	Mann Co. Supply Crate Key	Tool	Became normal key on 7/11/2012	key
5081	Fall Key 2012	Mann Co. Supply Crate Key	Tool	Became normal key on 9/21/2012	key
5628	Eerie Key	Mann Co. Supply Crate Key	Tool	Became normal key on 11/8/2012	key
5631	Naughty Winter Key 2012	Naughty Winter Crate Key 2012	Tool	Became normal key on 1/3/2013	key
5632	Nice Winter Key 2012	Nice Winter Crate Key 2012	Tool	Became normal key on 1/3/2013	key

Figure A.4: List of Money Goods

*Note:* Only items of normal quality are treated as commodity currencies - a discussion of quality follows shortly. Event keys are only treated as a money item worth one-key starting the date listed in “notes”. Before this date, event keys are treated as non-money items (event keys typically trade at a premium during their event period, after which their practical value is that of a standard key).

\* Items we do not trade as money when calculating prices.

- Dota2 money items: Gameplay and the role of items work slightly differently in Dota2 compared with TF2. The game itself and trading in its items became popular over our sample and a number of its items are anecdotally thought to play a money role in the marketplace. One in particular is the Dota2 Treasure Key. However this analysis, perhaps mistakenly, ignores these items when pricing others.



Figure A.5: Images of Anecdotal Money in TF2



Figure A.6: Images of Other Possible Money Items in TF2

### A.3.3 Categorizing transactions types

For the purpose of discovering prices, we will categorize trades by how money-items are involved. Different types of trades will offer different types and quality of item-level value observations.

All trades involve two baskets of items. Depending on the contents of these baskets (or lack of contents) we classify each trade as one of the following:

- **Money for Money (FX):** A trade in which one basket of money goods exchanges for another basket of money goods. “FX”, as in foreign exchange for short. Examples of money-for-money trades include transactions with keys on one side and metals on the other (of which there are over 700,000 such trades in our sample), or a trade with keys and metal on one side and a Bill’s Hat on

the other. This subset of trading may offer clear exchange rates between the various money items. If confident about such inter-money exchange rates, then given any combination of money items we can value these in terms of keys (or metals or Earbuds for that matter), allowing for the creation of a “synthetic key” numeraire.

- **Simple Monetary (SM):** One non-money good in exchange for a basket of money goods. On the non-money good side there may be any number of items, so long as they all share the same DefIDQ.
- **Simple Barter (SB):** An exchange of one non-money good of one type (the basket must have only one DefIDQ) in exchange for another non-money good of one type. If one side of an SB trade involves an item for which we have high quality prices (i.e. many consistent price observations on the current date) we may use that item to value the other side of the transaction.
- **One-Way (OW):** A transaction between one or more items on one side and an empty basket on the other. Many OW trades likely involve middlemen services, third party website that allow users to sell their virtual goods for cash. We will discuss this topic later.
- **Everything Else (EE):** exchange of one bundle (possibly including money) for another bundle (one of the bundles possibly trivial).

– Subclasses under EE (everything else) trade class. SM and SB above are

able to estimate prices of goods. To double check that these generally hold, we can compare them to the following in the Everything Else group:

- **EE\_SM**: All money items on one side, and a bundle of one-type of non-money item and some money on the other side.
- **EE\_SB**: Simple Barter. On one side, one type of non-money, and on the other side multiple types of non-money items.
- **EE\_E**: Everything everything else.

Figure A.7 breaks down trading in each type over time. Note that Money-for-Money trades are barely visible (the light green at the bottom of the chart) but still comprise tens of thousands of transactions over the two year sample. One-way trades constitute about half of all trades.

## A.4 Item Pricing and Commodity Currency Exchange Rates

In this section we discuss inter-commodity currency exchange rates, and how these are used to compute implicit prices for all other virtual goods on the Valve market.

### A.4.1 Simple monetary trades and item price observations

Simple monetary transactions will offer the clearest indication of going prices of items. A SM transaction involves just one type of non-money good on one side of the trade and money goods on the other side. As an example of how this offers a clear price signal, consider that we witness trades at which the going rate for one “Gabe’s Hat” is



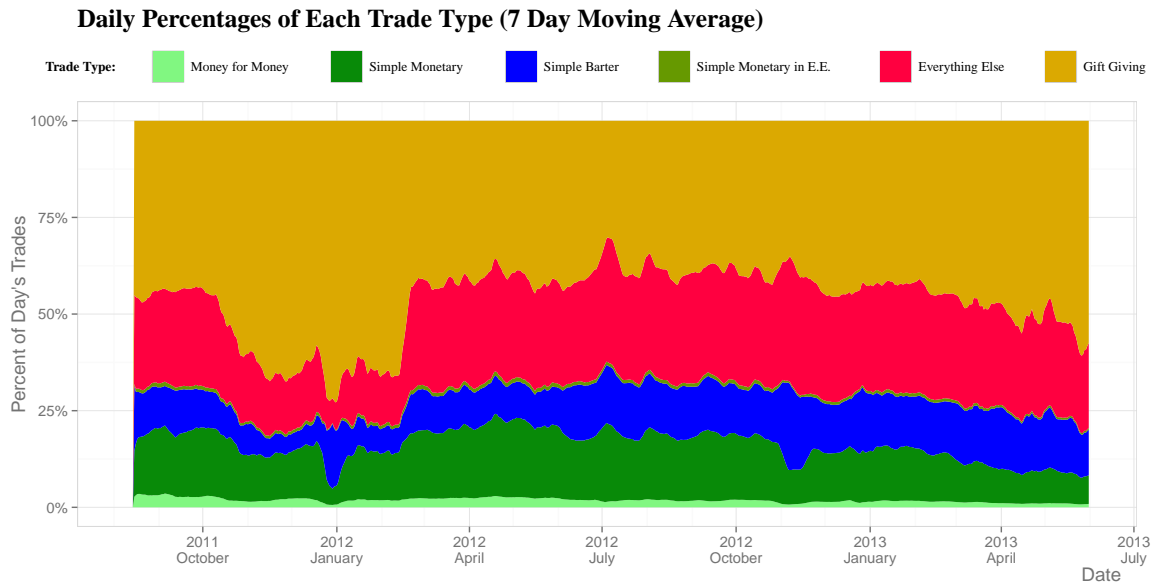


Figure A.7: Trades by Type

two keys, we'd obviously compute the price observations of Gabe's Hat as 2 keys (you may see "2k" or in future).

Given a simple monetary transaction involving only keys (this type of trade is referred to as an SM.Keys trade) involving item  $i$  at time  $t$ , we derive implicit key-valuation by the following:

$$Price.Observation.SM.Keys_{i,t} = \frac{\text{Number of Keys}}{\text{Number of Non-Money Goods}}$$

But not all trades have only keys on one side and the item on the other. Items worth less than a key are often traded for refined metal (e.g. Master's Yellow Belt). Items for which going prices are not perfectly divisible by keys also often include metal along with keys (e.g. Brigade Helm). If we can value other money items - metals, Bill's

Hats, and Earbuds - in terms of keys, it is straightforward to find these additional price observations. But this begs the question: given some basket of money goods (some metals, keys, Bill's Hats, and/or Earbuds), how do we value them in terms of keys?

#### A.4.2 FX trades and building a synthetic key

We first turn to the subset of trading that involves only anecdotal money items, (money-for-money trades, or FX trades for short). Generating daily intermoney exchange rates we can be confident in will allow us to price any money items into another money item. With clear money-item exchange rates (i.e. consistent daily values of metal, Earbuds and Bill's Hats in terms of their going rate for keys), we can then value mixed combinations of these items in terms of the numeraire keys. Whenever we do this we will refer to the value of the basket of mixed money items as denominated by "Synthetic Keys".

**Special event keys** – We should note that special event keys (listed in Figure A.4), enter the world as a special key that can open special crates in addition to normal crates. Examples of special event crates have been Christmas crates, where items inside the crate might possess a particularly festive decoration (e.g. Festive Bat). But these special event crates eventually disappear from player's inventories at the end of the special event period (at the date also listed in Figure A.4), usually two weeks after introduction. During the event time the special keys tend to trade at a premium above normal keys, and thus are not treated as a "Key" for the purposes of calculating prices of other items. However, after the event period ends and the special event crates disappear

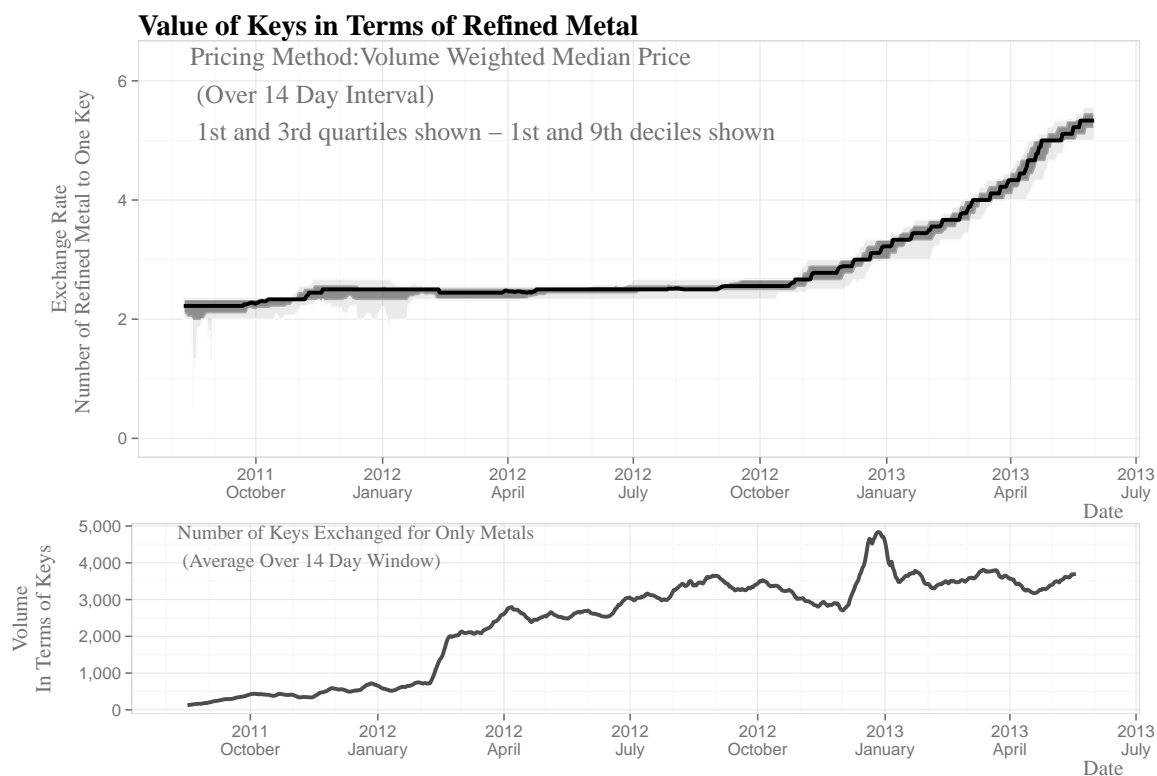


Figure A.8: Metals to Keys Exchange Rate

from the game, we treat them as normal keys when we observe one in a transaction. Though these event keys are sometimes visually different from normal keys (e.g. the mechanical looking Robo Key), after the special event period is over they are only able to open regular crates.

Now we shall look at each of these exchange rates.

In Figure A.8 we show the daily median exchange rate between refined metals and keys, (i.e. the number of refined metals used to purchase a single key). To derive this plot we convert all metals into refined metals according to their fixed exchange rate.

Bilateral trades between keys and metals are by far the most numerous type of

trade. At the end of 2012 some days had over 6,000 keys traded for metal. These trades account for about 3.6 percent of all trades in our record.

For much of the time series the exchange rate between the two was steady at slightly less than 2.5 refined metals to a key. In the fall of 2012 the value of metal relative to keys starts a precipitous decline, ending at 5.5 refined metals to a single key by the end of May 2013.

Also shown with dark gray bands are the evolving interquartile range and in light bands the interdecile range (i.e. 50 percent of all key-metal trades occur within the dark gray band, 90 percent within the light gray band). These give a sense of the distribution of prices around the median. With few exceptions it reveals remarkable consistency as to the going rate in the market. Now, we can value any basket containing some keys and some metal in terms of synthetic keys.

Figure A.9 shows the exchange rate between Bill's Hats and Key-Metal baskets. The scatter plot, time series and rolling quartiles and deciles reflect all trades between Bill's Hats and either keys, metal, or some combination of keys and metal (with metal converted into synthetic keys at the day's key-metal exchange rate).

At the start of this market Bill's Hats typically exchange for just under 5 keys. After about April 1st 2012, Bill's Hat trades for between 8 and 9 keys until the end of our sample.

In comparison to key metal exchange rates we see a noticeably wider dispersion of exchange rates. However the overall value of Bill's Hats relative to synthetic keys remains more consistent.

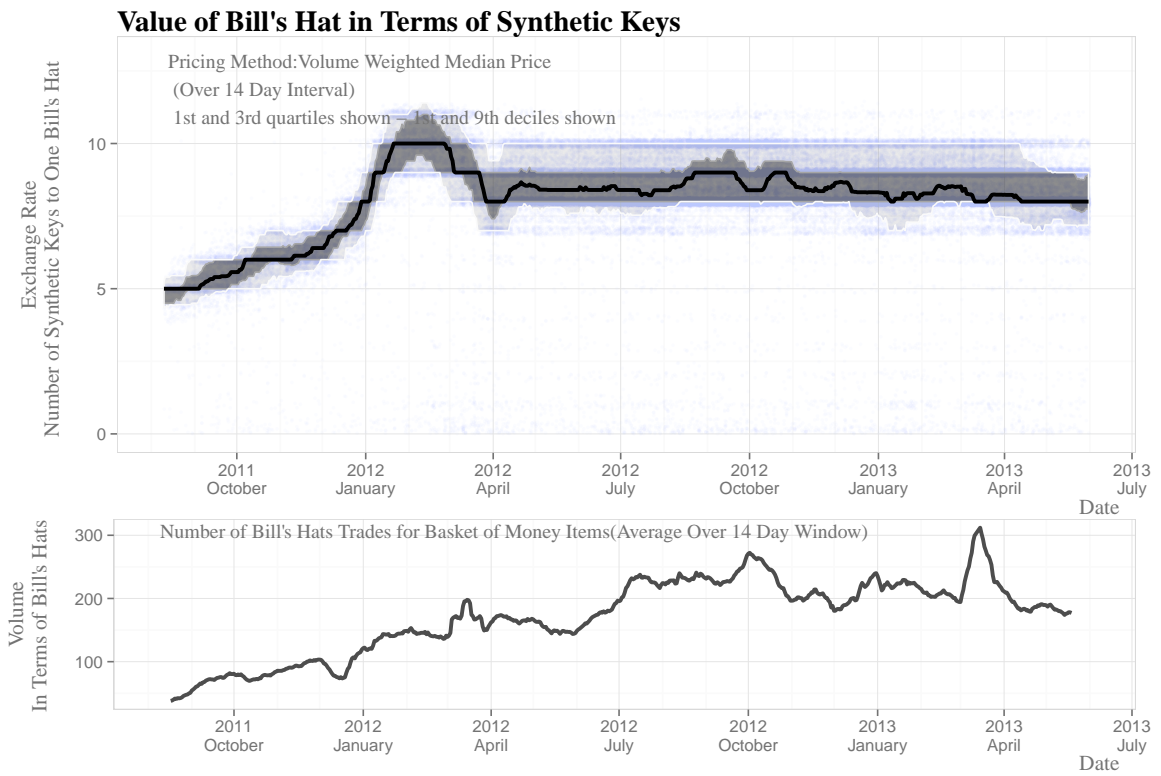


Figure A.9: Keys to Bill's Hat Exchange Rate

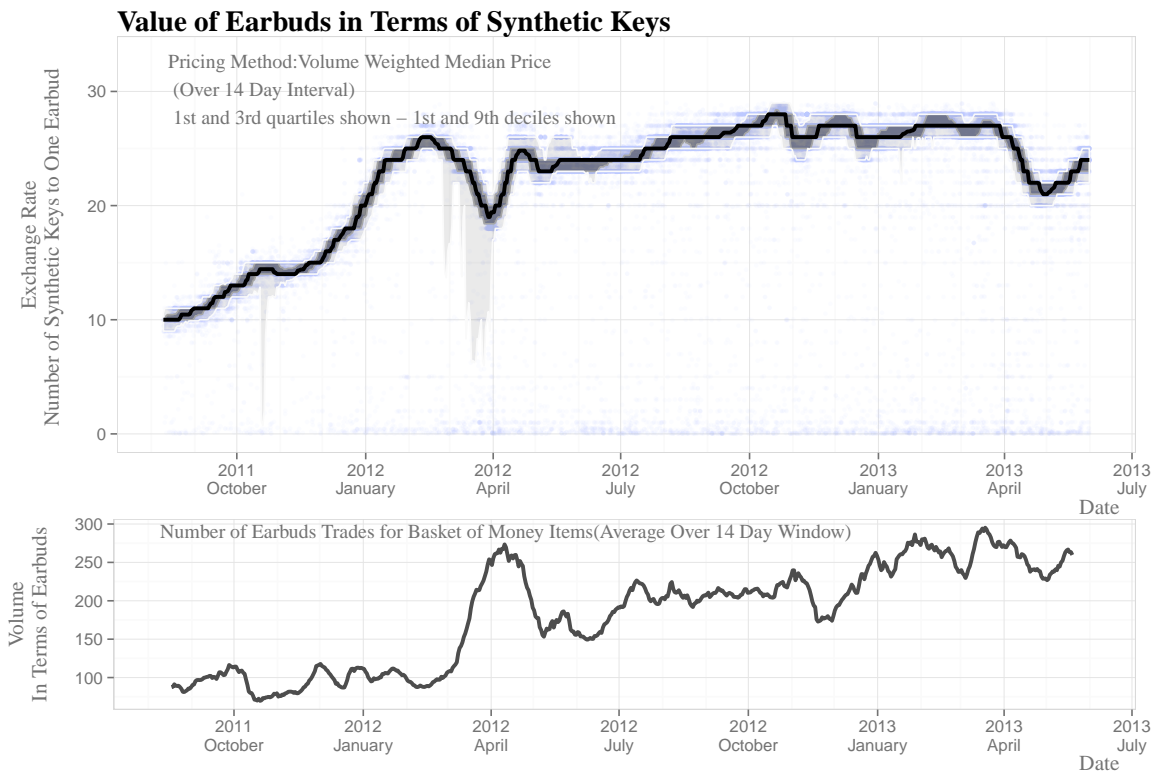


Figure A.10: Keys to Earbud Exchange Rate

Figure A.10 shows the exchange rate between Earbuds and key-metal baskets. Again, the scatter plot, time series, and rolling quartiles and deciles reflect all trades between Earbuds and either keys, metal, or some combination of keys and metal (with metal converted into synthetic keys at the day's key-metal exchange rate).

Earbuds have a more dynamic history of price changes in comparison to Bill's Hat. Earbuds traded for 10 keys at the start of the sample and end close to 24 keys.

Curiously, there are also a large number of trades between Earbuds and metal that value Earbuds close to zero (trades between an Earbud and a few small pieces of metal). Note the decile band just before April 2012, and the scatter points near the bottom of the vertical axis. This observation does not affect the rolling value of Earbuds, and is the case with Bill's Hats too, but to a much lesser extent.

#### **A.4.2.1 Discussion of Sufficient Sample Size to Be Confident in Exchange Rates**

Figure A.11 shows the number of money for money trades. We see that on any given day there are hundreds of FX trades and after February 2012 there are thousands of such trades. With the exception of some outliers like the single metal for Earbuds mentioned above, for a given short period of time the vast majority of exchanges between these money items occur at or very near the median rate. These exchange rates match closely with a number of trader price spreadsheet websites that have emerged to support Steam Trading, querying item exchange rates by sampling and surveys.

We have collected all these intermoney exchange rates into an interactive plot

## Money for Money Trades, Daily Count

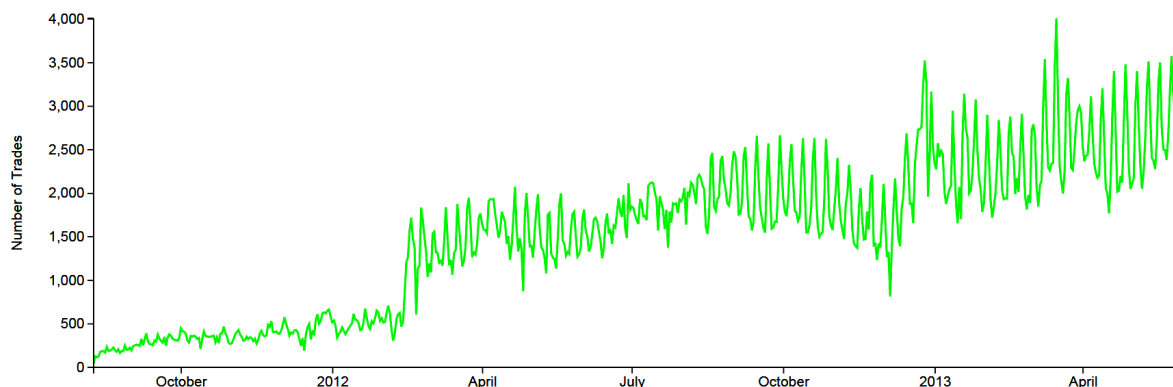


Figure A.11: Number of Money for Money Trades

posted to a University of California Santa Cruz Economics Department website, available at [econ-valve.ucsc.edu/DatApp/fx\\_rates](http://econ-valve.ucsc.edu/DatApp/fx_rates).

### A.4.2.2 Synthetic Key Calculation

We now have daily intra-money exchange rates. Given a basket composed of any combination of metal, keys, Bill's Hats and Earbuds we can value that basket in terms of synthetic keys.

Calculating the value of money items in terms of keys (in our price.Observation.SM.Mix equation above), we take the

$$V_{it} = \sum_{i=1}^N \{p_{it} \cdot q_{it}\}$$

Where  $p_{it}$  is the price of money-item  $i$  at date  $t$ , and  $q_{it}$  is the volume of that good.



### A.4.3 Item Price Observations

Up to this point we have grouped items up from the AssetID (unique, individual items) and EconAssetClass (where one item is essentially indistinguishable from another with the same EAC) up to the DefID\_AppID\_Quality level. We then selected a few items as money items, based on our anecdotal understanding of the economy. Looking at just inter-commodity-currency exchange rates we find that it is fairly clear that at a given time there is a well understood exchange rate between these money items. This allows us to create a synthetic key, that is, we can value any combination of money items in terms of keys (or in terms of metal or Earbuds for that matter).

We now detail how we use this information to generate item level price observations.

#### A.4.3.1 Different Types of Price Observations

1. Simple monetary (SM) is calculated as follows:

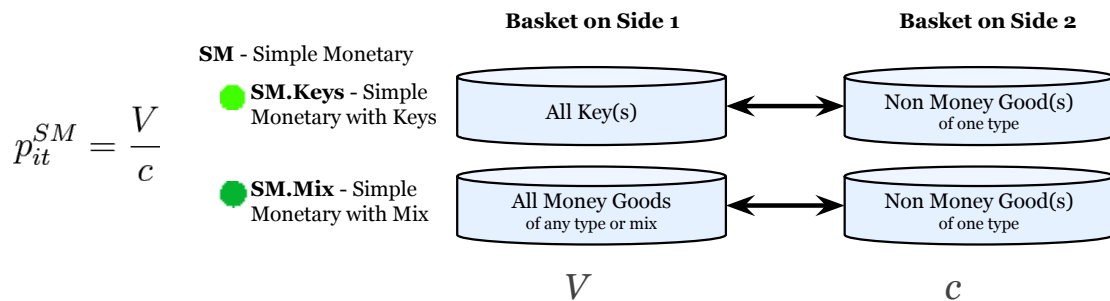


Figure A.12: Simple Monetary Trades

Given a simple monetary transaction, the price observation on that transaction is  $V$  (is the value the money-side of that trade in terms of synthetic keys) over  $c$  (the number

of non-money goods in this exchange.) Where  $i$  is the non-money item being priced and  $t$  is the timestamp of this price observation.

We've also differentiated between *SM.PureKeys* and *SM.SynthKeys*. A trade that only involves the non-money item on one side and key(s) on the other are designated *SM.PureKeys*. Trades that have a mix of metals, keys, Bill's Hats and or Earbuds (just not all keys) are labeled *SM.SynthKeys*. In practice both types of simple monetary trades give price observations of the same high quality.

2. A simple barter (SB) trade is an exchange of one non-money good of one type (the basket must have only one DefID, AppID & Quality combination) in exchange for another non-money good of one type.

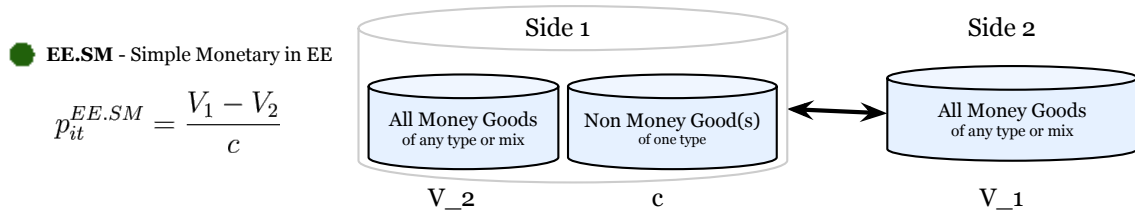


Figure A.13: Simple Barter Trades

Given a trade fits this descriptions, the implicit price is equal to the difference between the all-money side of the trade and the value of the money-items on the other side of the trade (that is  $V_1 - V_2$ ), divided by the number of non-money goods (the term  $c$ ). Where  $i$  is the non-money item being priced and  $t$  is the timestamp of this price observation.

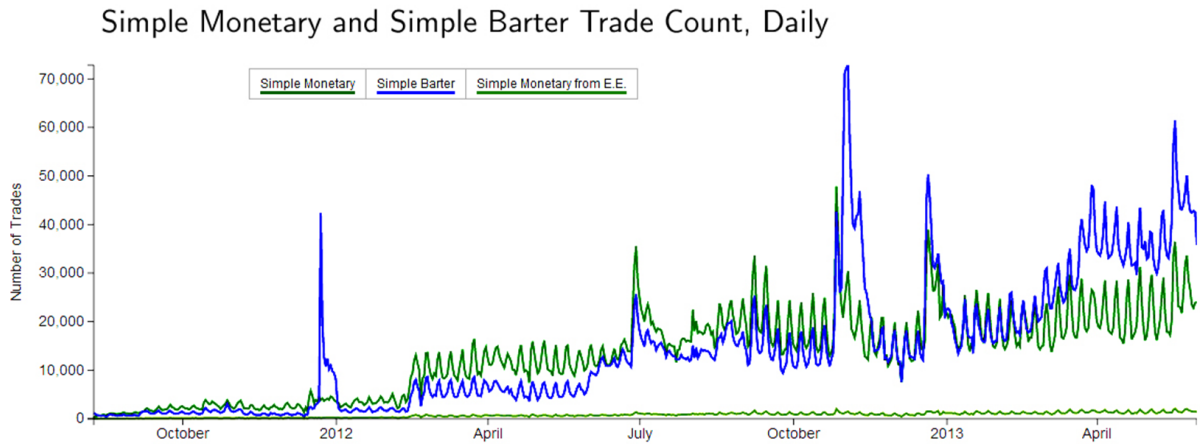


Figure A.14: Number of Simple Monetary and Simple Barter Trades

#### A.4.3.2 Price Observation Summary Statistics

**Number of Price Observations** – Implementing this methodology we find millions of price observations for thousands of virtual goods. Figure A.14 shows the daily count of trades by each type. Over the full sample there are 1,158,455 simple monetary trades involving only keys as money good, there are 7,674,123 simple monetary with synthetic keys (mix of keys, metal, Bill’s Hats and/or Earbuds), 504,944 simple monetary from everything else trades, and 8,581,937 simple barter trades. In total 18 million trades, or about 25 percent of all trading, offer a price observation in some form. Simple monetary trades in some form account for 13 percent of all trading.

**Number of Virtual Items** – At the DefID\_AppID\_Quality level we find 9,511 distinct items, 1,708 are from TF2, 2,156 are DOTA2, 5,639 are listed at Steam game licenses, and a remaining 8 are catchalls for all items from other games (of which there is very light trading).

**Variability of SB Prices** – Comparing simple barter to the various classes of simple monetary price observations, we also find SB observations to have much higher variance. Simple barter trades seem to offer relatively poor price observations. We can trace this to two reasons. First, since the value of one item is determined by the day’s value of the item on the other side of the trade, if the valuation we observe for the second side substantially deviates from the correct value (perhaps because of thin trading or noise), that will feed into a poor price observation for the main item, amplifying variability in price observations. Secondly, TF2 traders have suggested that it is convention among them to charge a premium when a potential counterparty to a trade wishes a barter trade instead of using one of the established commodity currencies. For example, if one trader wishes to buy a Bill’s Hat that has a price of eight keys, they can pay the eight keys, or alternatively pay with another item worth a bit more than eight keys, which may lead to a simple barter price observation that deviates from a more accurate sense of the true accepted value of an item.

## **A.5 Temporal Aggregation of Asynchronous Price Observations**

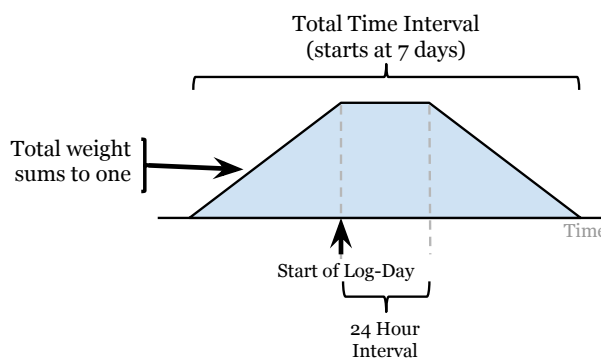
Our temporal aggregation approach assumes that each item at every moment possesses an underlying “fundamental market valuation” based on its characteristics and

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On a UCSC Economics Department website, we have created an interface to search and explore all items. On an item’s price page visitors can see all price observations broken down by observation type, SM, SB, etc. The URL is [econ-valve.ucsc.edu/DatApp/](http://econ-valve.ucsc.edu/DatApp/).

relevant market conditions. We then take each individual price observation as a noisy signal for that item’s contemporary fundamental value. That is, we assume SM price observations are drawn from their true values, plus some error process.

To estimate the price of a given item on a given day, our temporal aggregation process starts by collecting all SM price observations from a seven day window centered on a log-day. Since there are a large number of outliers which, for thinly traded items, can lead to a large amount of volatility from one period to the next, we then trim prices beyond the 1st and 9th price deciles. Then prices are estimated using a rolling average. First weighting for item quantities in each trade, and then a weighting function is applied based on temporal distance of the price observation from the log-day. More precisely, we apply a trapezoidal weighting function as illustrated below.



**Window Widening** – There are initially three days on either side of the log-day for which item-price is estimated. Many items have very high volumes with many price observations over which we can be fairly confident in their estimated prices. For some items, however, there is relatively low volume such that a week does not give us sufficient

observations to be confident in estimated prices reflected by fairly high coefficients of variation.

To account for this issue we define a control system that utilizes the coefficient of variation:  $c_v = \frac{\sigma}{\mu}$ , where  $\mu$  and  $\sigma$  are the mean and standard deviation of our sample. Our control system sets a cutoff value for coefficient of variation  $c_v^*$  and we calculate the coefficient for each item in a given time period  $c_v^{it}$  and if it is true that  $c_v^{it} > c_v^*$ , we increase the window for that item on that day by one day and recalculate. This process is repeated until the window includes sufficient observations such that  $c_v^{it} \leq c_v^*$ . The cutoff we use for this process is  $c_v^* = 0.6$  as this number appears to consistently select an appropriate window width.

During this process we also calculate price time series by a number of other temporal aggregation methods, including median with the above weighted and windowing methodology, median price over the 5 percent of prices nearest the log-day, and others. We also apply the above methodology with a trailing weighting, see figure below. Many of these item-level price time series are visible on the Valve data explorer at [econ-valve.ucsc.edu/DatApp/](http://econ-valve.ucsc.edu/DatApp/).

### A.5.1 Example of Item Prices

Figures A.15 and ?? show two example items from TF2. In each figure, each colored dot reflects an individual trade that implied that valuation — y-axis location — at that date. Different colors indicate which type of trade indicated the valuation (simple monetary with pure keys, simple barter, etc.). Also included are seven-day moving

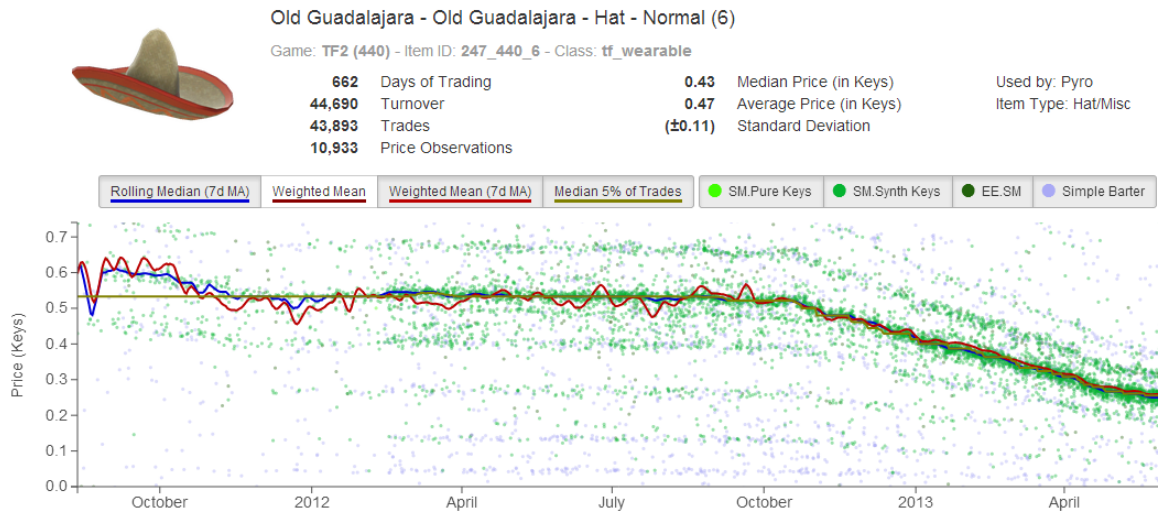


Figure A.15: Example Price History, Old Guadalajara

averages of the temporal aggregation methodology discussed. Additionally included is a line indicating the median price over the 5 percent of trades nearest the date.

The Old Gaudalajara of Figure A.15 is a hat item (worn on a video game characters head), and serves no functional value in the game. (In fact, if anything the large hat calls attention to the wearer, increasing risk of being seen and shot at.) At the start of the sample the hat is worth a little above half a key. The item maintains a fairly constant price until October 2012 after which it starts a steady decline.

Figure A.16 shows the item price history for the Strange Scattergun. Although the item is only a bit more powerful than standard issue weapons, its “Strange” quality means it tracks player gameplay statistics. It was a relatively rare and popular for much

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At the Valve data explorer website ([econ-valve.ucsc.edu/DatApp/item?id=247.440.6](http://econ-valve.ucsc.edu/DatApp/item?id=247.440.6)) you may change the y-axis price denomination to Refined Metal or Earbuds. When set to Refined Metal, we see the item maintains a fairly steady price of 1.33 refined metals throughout the entire sample. Making it worth about one Refined and one Reclaimed Metal.

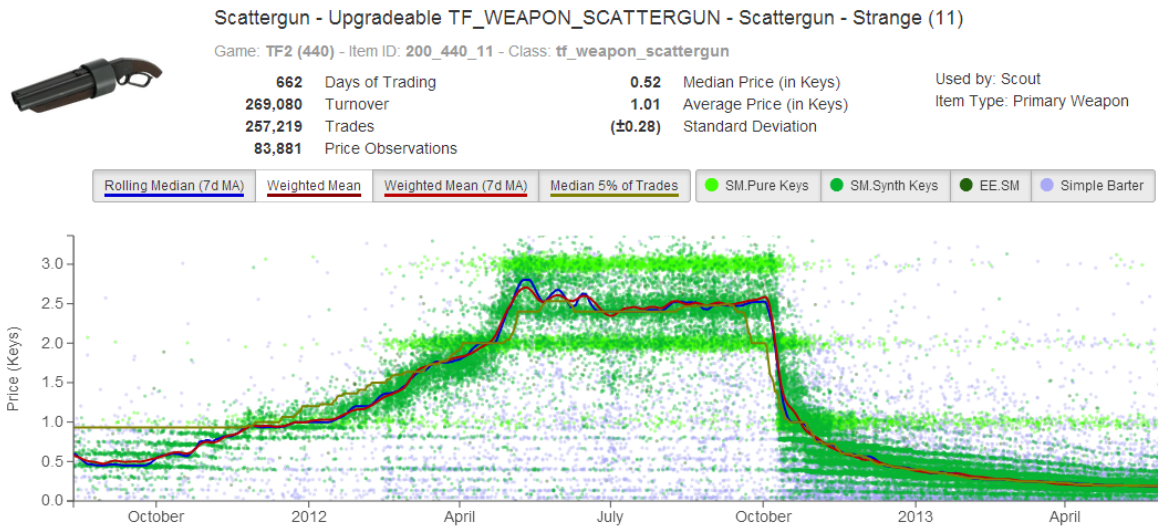


Figure A.16: Example Price History, Scattergun Strange

of the sample, starting off at a value of about half a key and increasing to about 2.5 keys. After this, the item was added to a new crate and would appear in uncratings with probability 20 percent. The Valve data explorer item page for the Strange Scattergun shows the dramatic increase in item stocks starting mid October 2012. This precipitated a dramatic decline in the price of the Strange Scattergun. By the end of the sample it was value at 0.2 keys.

### A.5.2 Additional Pricing Notes

When determining item level prices, some documentation bugs meant that we had to adjust individual item prices by hand. There are some items that are held by most players, and appear to have high and-or volatile price histories. It turns out that some of these items are largely untradable, except for a few that have a bug that permits



tradability. Perhaps because of the novelty of this tradability bug, the few instances of these items that are tradable sometimes go for the high or usually erratic values. However, the items of these types in most players inventories are not in fact tradable and therefore should be really be valued when determining  $W_t$ . Unfortunately, at the DefID, AppID, quality level we are unable to distinguish between the essentially worthless but well represented untradable verity from the erratically price tradable versions. If we do not correct for these biases the value of representative user baskets can appear higher and more volatile they it is in actuality.

Some examples, the “[MONOCULUS!](#)”, values range between 0.04 and 27 keys and appear in most users’ inventories. The “[RIFT Well Spun Hat Claim Cod](#)”, with a single price observation of 62 keys, and held by many players. The “[Horseless Headless Horsemann’s Head](#)”, valued imprecisely between 6 and 24 keys, is mostly untradable, and held by a large number players.

## A.6 Nominal Aggregate Values

We now turn to characterizing the size and growth rate of the TF2 virtual economy. Due to the relative lack of production, GDP is not an appropriate measure for this. Our dataset does contain daily stocks and turnover of each TF2 item, with our price panel dataset this permits calculation of nominal market capitalization and trading volume. Regarding DOTA2 and other items, although we observe trading we do not observe item stocks, and therefore cannot value that portion of the Steam Trading marketplace.

We instead calculate market capitalization as the total key-value of aggregate item stocks held by active players, where a player is designated active if they have played within 90 days. To calculate this, we take the level of existing stocks of each item in each time period and multiply them by the prevailing price in that time period, then sum over all items. We will denote this measure of aggregate nominal wealth in period  $t$  as  $W_t$  as

$$W_t = \sum_{i=1}^{N_t} \{p_{i,t} \cdot S_{i,t}\}$$

Where at time  $t$  there are  $N_t$  total different goods,  $p_{i,t}$  is the price of good  $i$  and  $S_{i,t}$  is its outstanding stock.

Figure A.17 shows  $W_t$  plotted weekly over the sample.

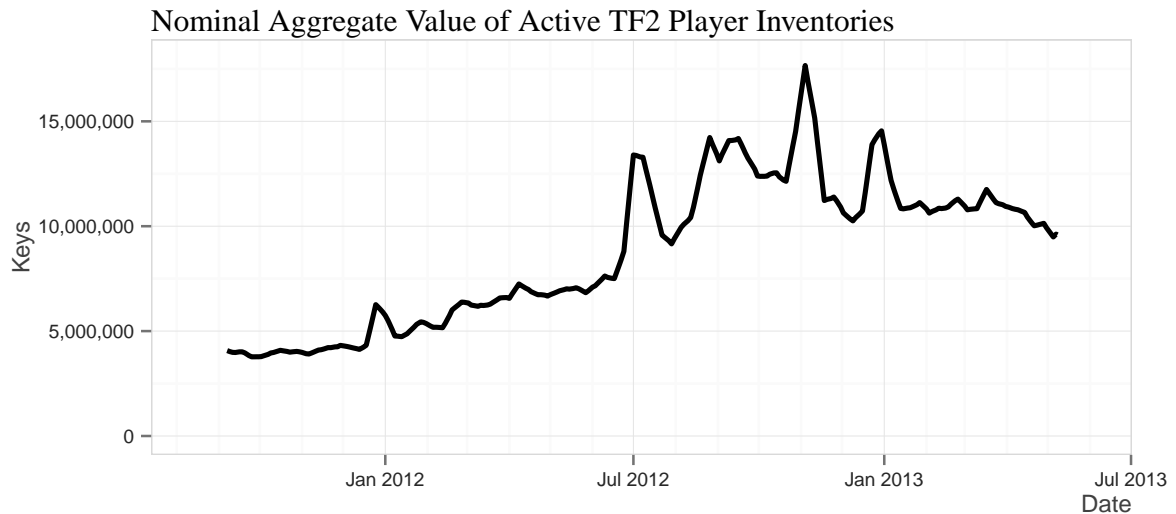


Figure A.17: Nominal Value of Active TF2 Player Inventories, in Keys, Daily

Figure A.18 shows the nominal value of trading in TF2 items over the full sample.

Note that this value values both sides of trades. That is, if a key is traded for a key, the nominal value of that trade would be logged at 2.

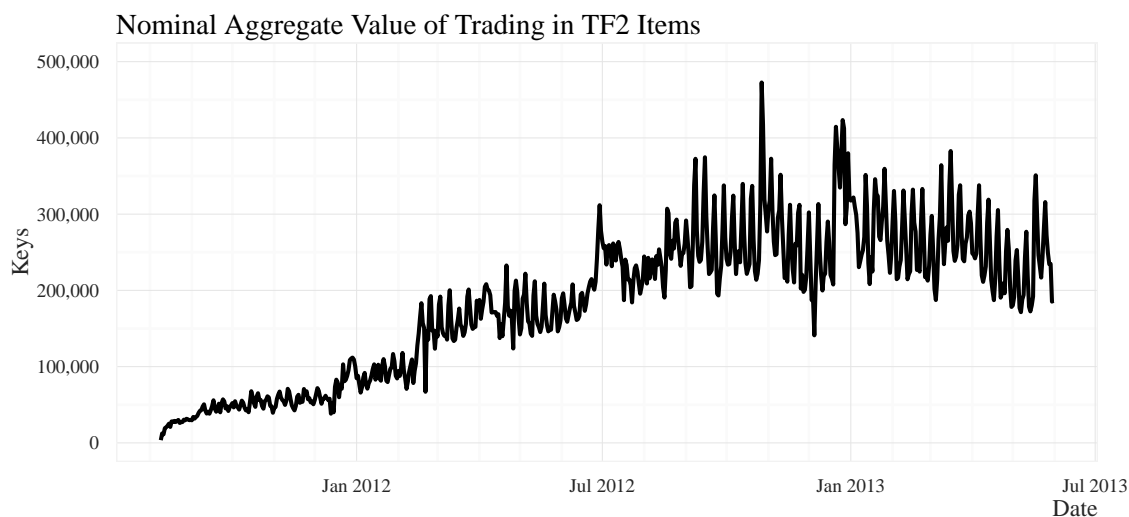


Figure A.18: Nominal Value of TF2 Trading, in Keys, Daily

We see a similar pattern in both nominal wealth and nominal value of TF2 trading. Values start quite low, gradually working up to a peak around Halloween of 2012 and then leveling off or declining, with a number of spikes around holidays and major events. For the first few trading the value of trading volume is close to zero. The first day of trading is valued at about 4000 keys. As we shall see shortly, much of the spikes in nominal wealth are due to volatility in the number of “active” players in TF2.

## A.7 Nominal Aggregate Decomposition

In this section we decompose nominal aggregate wealth in Figure A.17 into three constituent parts; the population of active TF2 players, the price level, per-capita real

wealth.

There are a number of approaches to this decomposition. Here, we will separate nominal wealth from the number of active players on TF2 and our estimate of the price level, with the remaining residual delivering per-capita real wealth.

### A.7.1 Population

Figure A.19 plots the number of active TF2 players, where an active player is defined as someone who logged into TF2 within the past month. We see a number of spikes around holidays and sales events seen in figure A.1 plotting the total number of trades in the Steam Trading economy (which include the trading of other game items and licenses).

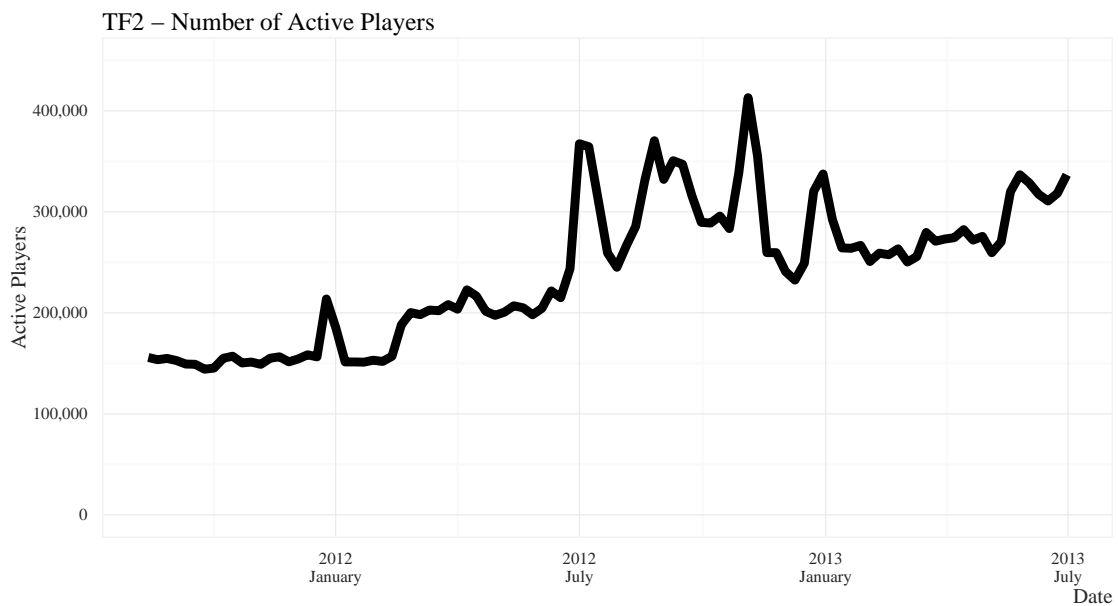


Figure A.19: TF2 Active Player Population

## A.7.2 Price index

A price index weights prices for a certain class of goods into a normalized average. We have already discussed how prices have been estimated for this virtual economy. In terms of weighting these prices there are a number of options. In consumer inflation indexes like CPI, quantities strive to reflect typical consumption baskets. In contrast, quantities reflect producer purchases in input producer price indexes, and production quantities in the Gross Domestic Product deflator. Our quantity index reflects the bundle of goods held by a “representative player”, i.e. a metric for player wealth.

### A.7.2.1 Representative item holdings

These representative player inventories are generated by drawing random samples of users from the active player population, where an active player is defined as one who logged into Team Fortress 2 within 30 days of the sample date. We identify the average quantity of each TF2 item held in the sampled inventories.

$$q_{it} = \frac{1}{N} \sum_{j=0}^N q_{jit}$$

There are some unique issues with our sampling in this environment due to the presence of an upper tail of inventory value distributions composed of people with very large inventory values. These high net worth individuals (HNWIs) are rare enough that we are unlikely to have a good balance of them represented in each sample, and large enough outliers to move summary statistics greatly from one sample to the next.

Increasing our sample size sufficiently beyond 1 percent of the population is also technically infeasible given the number of active players (typically more than 250,000 each week) many of whom possess scores of items. Therefore without adjustment the price index could exhibit big movements from one period to the next due more to sudden shifts in the quantity index than shifts in price.

Our approach to dealing with these HNWI is first to tag the top proportion of wealth-holding individual users as HNWI, where we define the inventory value cutoff as a nominal inventory value above 800 keys, or approximately \$1600. If an active player is classified as a HNWI in one of these censuses, their inventories are logged each week for the entire year and they are excluded from the non-HNWI sample for that year. These HNWI players account for approximately 0.3 to 0.4 percent of the active player population.

We then track inventories of all HNWI each period along with the random 1% sample of *non-HNWI*s, and derive average item inventories for each group. The composition of the basket derived from these 1% samples does not fluctuate greatly from time period to time period. Finally, the HNWI and non-HNWI representative inventories are combined weighting item quantities based on each groups' relative proportion of the overall active player population at each period.

All inventory data excludes individuals who have marked their "Steam Profile" as private. Of the approximately 1,500 unique active players classified as HNWI, 255 have been excluded this way. A significantly larger number but roughly a similar proportion of non-HNWI were similar excluded from our representative item holdings sampling.

Our methodology thus assumes the omission of these privacy preferring players does not significantly bias the representativeness of our HNWI and non-HNWI sample.

Once representative baskets are found for each tier, they are average together weighted by the relative proportion of each group to the overall population. Thus,  $q_{it}$  indicates our metric for the typical holdings of item  $i$  at date  $t$  among active TF2 players.

#### **A.7.2.2 Non-Priced items**

There exist numerous untradable and non-giftable items, many of which appear in many players' inventories and many that could be quite valuable if traded. These include certain items all players possess, contest awards, self-made items (given to the original creator of an item), and community items, (a special quality of item awarded to players recognized by the the video game company as having made some special contribution). We handle such non-priced goods similarly to how most national statistical office handle non-priced services like family household services, by excluding them from our price index. Although some measure of the value of these items might evolve through time, and ideally our price index would capture that, since we have no clear method to measure their value we must ignore them.

#### **A.7.2.3 Initial price indexes**

We first consider two canonical fixed weight price indices. The fixed-weight Laspeyres price index takes base quantities from the first time period, here the first day of trading,

August 8, 2011.

$$P_{0,t}^L = \frac{\sum_{i=1}^N p_{it}q_{i0}}{\sum_{i=1}^N p_{i0}q_{i0}}$$

Where  $P_{0,t}^L$  is the fixed-weight Laspeyres price index,  $p_{it}$  is the price of item  $i$  at time  $t$ , and  $p_{i0}$  and  $q_{i0}$  is the price and quantity of item  $i$  at the base date.

The Paasche price index sets the current date as quantity base:

$$P_{0,t}^P = \frac{\sum_{i=1}^N p_{it}q_{it}}{\sum_{i=1}^N p_{i0}q_{it}}$$

Where  $P_{0,t}^P$  is the fixed-weight Paasche price index,  $p_{it}$  is the price of item  $i$  at time  $t$ , and  $p_{i0}$  and  $q_{i0}$  is the price and quantity of item  $i$  at the base date.

Figure [A.20](#) shows these two price indices over the sample.

#### **A.7.2.4 Dealing with new items, part one**

One issue arises with the introduction of new item. Although no item disappeared from the marketplace during this sample, many new items were introduced. In period zero, August 8th 2011, we had prices on 628 TF2 items. By the end of the sample we had prices on 1,606 TF2 items. Since both Laspeyres and Paasche require item prices in the base period, the indices in figure [A.20](#) are actually only aggregating price changes of these 628 items. Thus in a real sense, these are subindices of old items.

With identical prices, the difference in trajectories of the higher Laspeyres and lower Paasche indices is a result of Paasche's changing quantities. For the Laspeyres Price



Laspeyres and Paasche Price Indexes (Base Period: 2011–08–14)

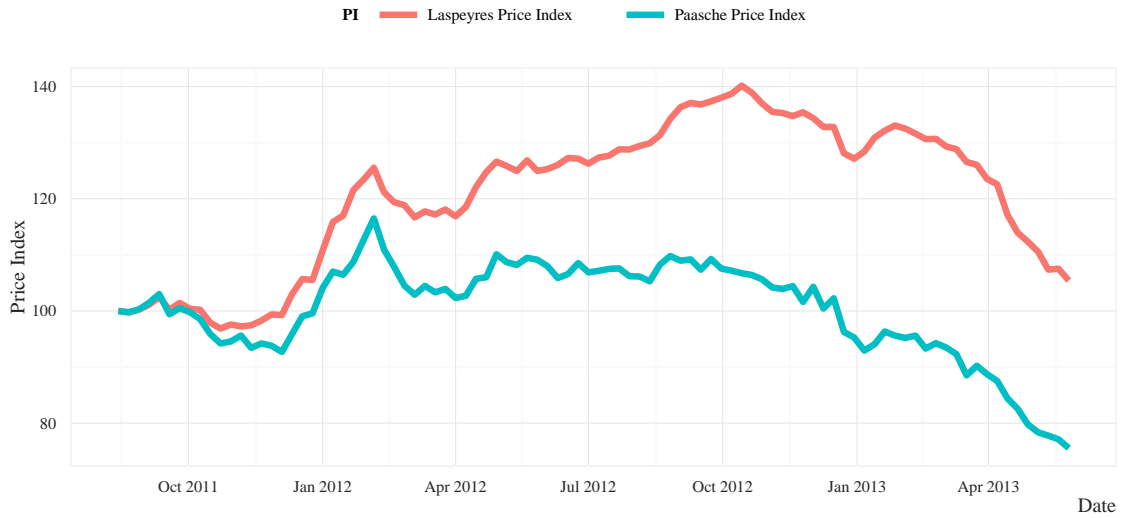


Figure A.20: Laspeyres Paasche Price Indexes

Index these quantities are fixed at the base year's quantities, the index thus tracks the general increase in prices up to a peak in mid-October 2012, and a general decline after. In the Paasche Price Index these quantities - the average holding of items in a representative active player's inventory - constantly change. As more and more items come into existence and as new players start playing the game with fresh, mostly empty inventories, the average quantities of items that have prices in the first period generally decreases through the sample.

As a representative example of this phenomena, the [Normal/Unique quality Earbuds](#) item increased in price by 101% over the sample. But the average holding of Earbuds declined by half, from 0.21 in August 2011 to 0.11 Earbuds by May 2013. While an item like [Normal/Unique Conjuror's Cowl](#) was worth only 40% of its initial value by

May 2013, while average holdings of it increased by over 300% over the same period.

One possible explanation for this pattern is that some items are no longer available for purchase for via uncrating after some initial introduction period, while other items have continued to be available throughout the sample. Supply constrained items have tended to hold or increase in value relative to the unconstrained. As more and more new players have entered the active player pool average holdings of the supply constrained items have declined shifting weights in the Paasche price index away from the higher priced goods.

Some evidence of this, regressing item-level percent changes in price from the first month to the last month over the percent change in average holdings from the first month to the last. Results are reported in table A.3. Though not a causal interpretation, we find that on average, if quantities did not change from the first to last month, prices reduced by 15%. Additionally, for each 100% increase in average holdings, prices decreased by 15.9%.

We deal with the steady introduction of new items in two ways, firstly via a chain index, and additionally by imputing the prices of not yet introduced items by a Hedonic price model.

#### **A.7.2.5 Chain price index**

Simple Laspeyres and Paasche price indices require a fixed basket of items. In these formulas there is no way to account for price and quantity information for items that come into existence or disappear after the base period. This usually becomes a bigger

	<i>Dependent variable:</i>
	Percent Change in Price From First to Last Week
Percent Chng. Avg Inventory Holding	-0.101*** (0.028)
Constant	-0.159*** (0.032)
Observations	615
R <sup>2</sup>	0.020
Adjusted R <sup>2</sup>	0.019
Residual Std. Error	0.794 (df = 613)
F Statistic	12.718*** (df = 1; 613)
<i>Note:</i>	*p<0.1; **p<0.05; ***p<0.01

Table A.3: Relationship between item price changes and average holdings

problem the greater the distance between the base period and time  $t$ , this distance increases the likelihood of new items being introduced into the Valve economy. In Figure A.20 we saw that Laspeyres and Paasche indices tracked each other closely for over a month after the base period.

To help address this issue we turn to chain linked price indices. Chain price indices use price comparisons in adjacent periods to derive an aggregate index. This maximizes the number of item price comparisons possible.

We first consider the chained Fisher Ideal price index. We first need to calculate Laspeyres and Paasche price relatives. “Price relative” here refers to aggregate measures of prices changes between two adjacent periods using only price and quantity data from those two periods.

Laspeyres price relative:

$$P_{t,t+1}^L = \frac{\sum_{i=1}^N p_{i,t+1} q_{i,t}}{\sum_{i=1}^N p_{i,t} q_{i,t}}$$

Paasche price relative:

$$P_{t,t+1}^P = \frac{\sum_{i=1}^N p_{i,t+1} q_{i,t+1}}{\sum_{i=1}^N p_{i,t} q_{i,t+1}}$$

Fisher Ideal price relative:

$$P_{t,t+1}^{FI} = \sqrt{P_{t,t+1}^L \cdot P_{t,t+1}^P}$$

It should be noted that the Fisher “ideal” is ideal in name only, and perhaps not more ideal than any reasonable price index formula. The chained Fisher Ideal price index is then:

$$\text{Chain } P_T^{FI} = \prod_{t=0}^T (P_{t,t-1}^{FI})$$

With chained Fisher Ideal price index originating at some base period. We use August 8, 2011, the first period as our base, setting the price index to 100.

As a popular alternative price index, a Törnqvist price relative takes the following form,

$$P_{t,t-1}^{\mathbb{T}} = \prod_{i=0}^n \left( \frac{p_{it}}{p_{i,t-1}} \right)^{\frac{1}{2} \left[ \frac{p_{i,t-1}q_{i,t-1}}{\sum_{j=0}^n (p_{j,t-1}q_{j,t-1})} + \frac{p_{i,t}q_{i,t}}{\sum_{j=0}^n (p_{j,t}q_{j,t})} \right]}$$

Törnqvist Price Index at time T is,

$$\text{Chain } P_T^{\mathbb{T}} = \prod_{t=1}^T \left( P_{t,t-1}^{\mathbb{T}} \right)$$

### TF2 Aggregate Price Indexes

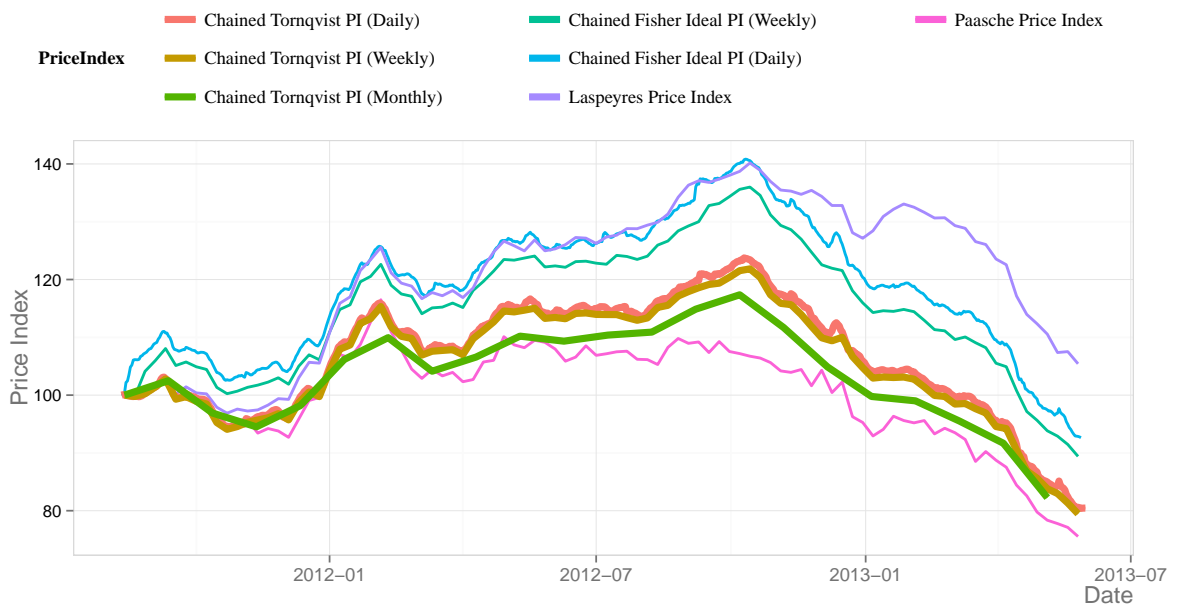


Figure A.21: Chain Price Indexes

Figure A.21 showing chain Törnqvist and Fisher Ideal with varying period frequencies, along with Laspeyres and Paasche price indexes.

### A.7.2.6 Chain drift

Note that chain Fisher Ideal daily price index does not intersect with its weekly price index. Instead, daily chain Fisher Ideal drifts somewhat about weekly chain Fisher Ideal. This is also true for chain Törnqvist weekly and daily, though to a lesser extent and not noticeably in this plot. These deviations between weekly and daily versions of the same index likely indicate chain drift. Chain drift occurs where the measure of long-term price changes via chaining does not match the measure of long-term price changes using prices and weights only from the start and end periods. Generally, price index that does not suffer from drift is desirable, making index levels between two periods directly comparable.

The more prices and quantities tend to oscillate, the more a chained price index will suffer from chain drift, and we do not have any reason to suspect Valve economy items to not oscillate in price and quantity.

One way to avoid chain drift is to stick with weights derived from the first and final periods, but we have already discussed why the use of a non-chained price index is not desirable in our situation; it forces us to omit price and weights of a majority of items in the TF2 economy. Although Chain Fisher Ideal is more susceptible than Törnqvist, Törnqvist indexes are still subject to chain drift. For example, in Figure A.21 note the drift in daily and weekly Törnqvist relative to monthly (which in fact is partly the result of new items introductions).

Going forward we will work with chain daily Törnqvist price index as it appears

to minimize these issues. However, it should be cautioned that because of chain drift, same index readings on two different periods may not imply identical prices levels in those periods.

#### **A.7.2.7 Dealing with new items, part two**

In the chained Törnqvist price index, we use price relatives to chain the price index. Although this maximizes the possible number of item price comparisons to aggregate into a price index, what if we have quantity or price information in one period but not its adjacent? This issue is largely avoided in national statistical office official calculations by first deriving numerous infinitely lived product-category price subindices that are then combined to the headline aggregate price index. These subindexes are made up of the prices of many individual goods within the category. Should a new item come into existence, it is incorporated into the subindex based on its popularity among consumers, characteristics that differentiate it among similar items, and other factors through a set methodology. Its price before introduction is often imputed based on some of these product characteristics (See BLS CPI “Cage, Greenlees, and Jackman 2002” and ILO et. al. PI Manual Chapter 17, more info below).

Until this point if any price or weight does not exist in a price relative (i.e. an NA for  $p_{i,t}$ ,  $p_{i,t-1}$ ,  $q_{i,t}$ , or  $q_{i,t-1}$ ), we ignore this item when calculating period  $t$ 's index. Since in our economy items have never been completely removed, this issue only occurs as items come into existence. That is when there is a price and quantity information in period  $t$ , but none in  $t - 1$ .

Since price indexes strive to capture movements in aggregate price behavior, throwing out any prices is potentially problematic, possibly biasing our indices. As items come into existences it usually takes some time before a new item is available to users by unlocking a new crate series and when we observe sufficient trading in the secondary market to infer prices via currently methods. Reviewing the price time series of numerous items, they usually trade for premiums upon release (likely due to their novelty and relative scarcity), and then quickly decline in value.

Figure A.22 shows the first 50-days of price dynamics for all tradable items introduced after Summer 2011. There are approximately 950 TF2 items introduced in this interval. The y-axis plots the natural log of Price, since items vary greatly in price, examining log price shrinks the visual distance between item prices, helping to focus on general price dynamics.

Firstly, note the large number of items appearing for the first time around Halloween and Christmas. These corresponds to many of the spikes in trading activity seen in previous figures. Also note that with few exceptions, price time series exhibit a downward trajectory after introduction. There are noticeable exceptions in this general trend. For example high value items tend to maintain their prices better than lower priced items. Also items introduced in large groups — noticeably around Halloween and the winter holidays — tend to see sharp declines in price after the first few days of trading. In fact the average price change from the first day of price observation to the second day, weighting for turnover, is  $-5.2\%$  After one week that average goes down to  $-21.8\%$ , and  $-50\%$  after 30 days.



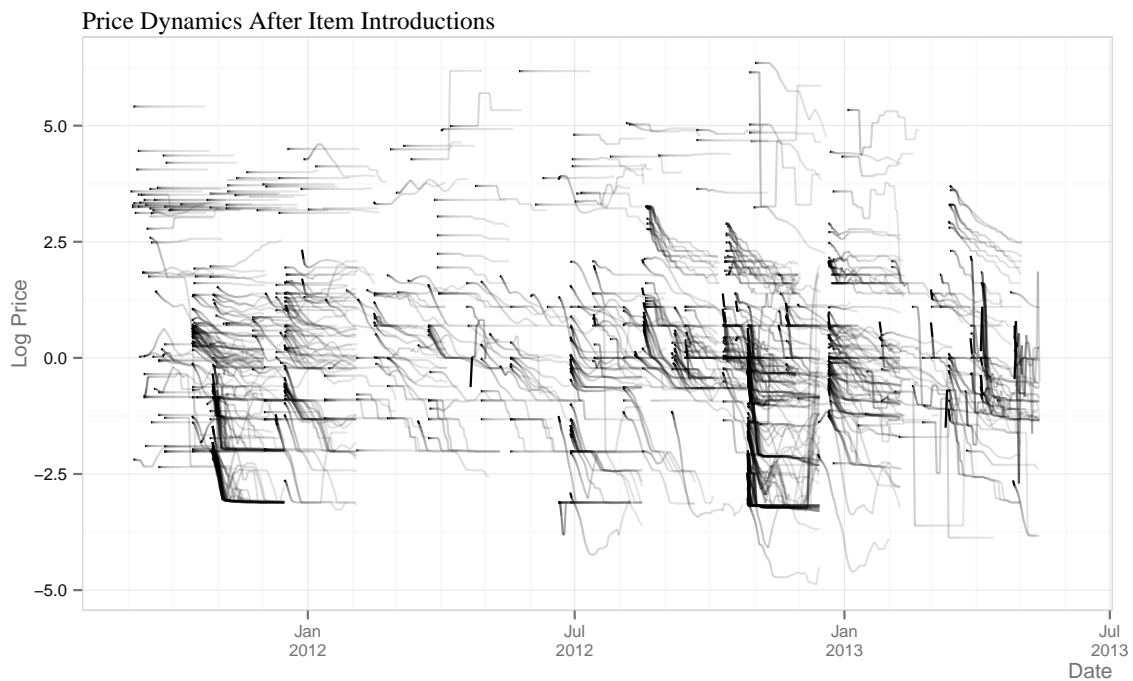


Figure A.22: New Item Price Dynamics

This observation suggests that dropping price relatives of newly introduced items may bias downward our aggregate price index, since our indices would then ignore the initial high price (One example is the [Strange Baby Face's Blaster, Scattergun](#)).

This downward bias in our price index estimate is mitigated by a few factors. Firstly, the scarcity of new items works in favor of a less biased price index. Since we weight price changes by the relative value in a representative active player's inventory - and since new items are usually quite rare - even if we somehow could incorporate the initial high prices in our price indices they likely account for a minuscule share of all price changes and so have a small effect on our price index.

Secondly, suppose a newly introduced item quickly entered numerous players' inven-

tories. This item would not be particularly scarce and thus unlikely to be valued at an initial premium. In fact, we could not find any instance of items that quickly accounted for a significant share of active player inventories after introduction.

To check whether or not this is a serious issue in our price indexes we closely examined the individual contributions of items to aggregate price level. In all cases, these new items account for a relatively tiny share of the value of a representative active player's inventory.

**Imputing prices of pre-traded items with a Hedonic price model.** National statistical bureaus have a standard methodology for dealing with new and discontinued items. They impute prices for these items, and map changes from imputed to observed prices to aggregate price indexes. Using a hedonic price model we regress item characteristics over observed prices and time dummies. For a given time period, this gives an estimated value of an item given its set of traits.

The Hedonic hypothesis postulates that an item is a bundle of  $K$  characteristics. By this reasoning, the price of an item is the sum of the premiums (or discounts) on the characteristics for which the item possesses. Of course, exactly how item characteristics map to prices may be very complex. We, however, impute unobserved prices via the relatively simple Hedonic price model below.

$$\ln(p_{it}) = \alpha + \delta_t D_t + \sum_{k=1}^K (\beta_{kt} \cdot x_{ik}) + \varepsilon_{it} \quad \text{for } t = 0, 1, \dots, T$$

For item  $i$  in period  $t$ , where  $p_{it}$  is the price of item  $i$  at period  $t$ .  $D_t$  are time dummies

(week),  $x_{it}$  is a dummy indicating whether or not item  $i$  possessed item characteristic  $k$  (note that item characteristics are time invariant), with error epsilon, with expected value zero.

The coefficient  $\beta_{kt}$  is the parameter of characteristic  $k$  at period  $t$ , with  $\delta_t$  is the parameter for period  $t$ .

In  $x_{it}$ , we include item Quality (again, restricted just to TF2 items), player class equipablility (which class of players are able to play with this type of item), item type (types include Hat or Misc, Non-Play Item, Primary Weapon, Secondary Weapon, Spy Sapper, Melee Weapon, EngineerPDA Invisibility Watch EngineerPDA or Spy-DisguiseKit.)

We also include a single quantity dummy variable, indicating the item is possessed by more than 3% of users, to capture the effect of an item being widely held (a sense of scarcity).

Figure [A.23](#) shows four daily price indexes. Two are the familiar daily chain Fisher Ideal and Törnqvist price indexes. We also show the two variants of the the Hedonic model above, one with a time interaction with item characteristics and one without.

As expected, the price indexes with Hedonically imputed prices bring the price index above the earlier chain Törnqvist price index without imputed prices. This provides further evidence that ignoring the introduction of items did bias down price level changes. Interestingly, we also see little difference between the two Hedonic price indexes (one with time varying characteristics and allowing for time invariant characteristics).

From this point, the discussion in Section [3.6](#) on the decomposition of aggregate

### Price Index Compared (Base 2011–09–01)

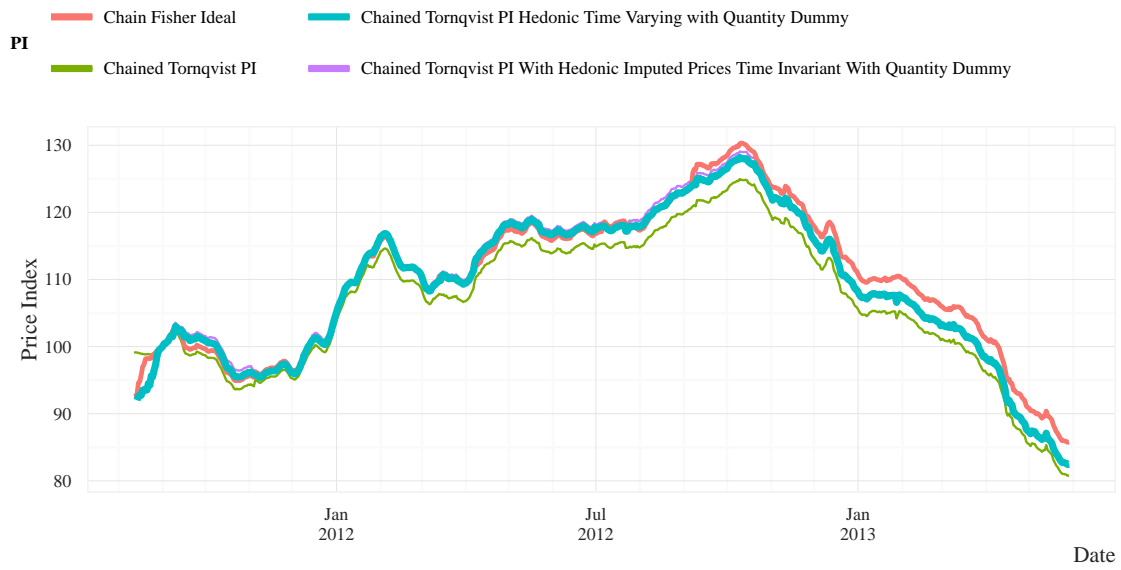


Figure A.23: Price Indexes with Hedonically Imputed Prices

nominal wealth into population, per-capita real wealth and the price index as well as insights from the hedonic pricing model is quite adequate.