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Reliability-Based Optimization for Maintenance Management in Bridge Networks

By
Xiaofei Hu

A dissertation submitted in partial satisfaction of the
requirements for the degree of
Doctor of Philosophy
in
Engineering – Civil and Environmental Engineering
in the
Graduate Division
of the
University of California, Berkeley

Committee in charge:

Professor Samer Madanat, Co-Chair
Professor Carlos Daganzo, Co-Chair
Professor Rhonda Righter

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Abstract

Reliability-Based Optimization for Maintenance Management in Bridge Networks

by

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Doctor of Philosophy in Engineering – Civil and Environmental Engineering
University of California, Berkeley

Professor Samer Madanat, Co-Chair

Professor Carlos Daganzo, Co-Chair

This dissertation addresses the problem of optimizing maintenance, repair and reconstruction decisions for bridge networks. Incorporating network topologies into bridge management problems is computationally difficult. Because of the interdependencies among networked bridges, they have to be analyzed together. Simulation-based numerical optimization techniques adopted in past research are limited to networks of moderate sizes.

In this dissertation, novel approaches are developed to determine the maintenance policies that best balance network performance and agency cost. For two different types of networks, two performance metrics are adopted, and the research is divided into two parts accordingly.

The first part focuses on moderate-size networks with limited redundancy. The network performance is quantified by a graph-theoretic indicator of network connectivity, since connectivity is the fundamental service function of a network. The objective is to ensure an adequate level of network connectivity at the lowest possible life-cycle maintenance cost. A novel two-stage approach is developed, which makes it possible to solve the problem by using standard optimization tools (with guaranteed convergence to optimality), as opposed to the heuristic algorithms used in related literature.

The second part studies large and redundant networks, and the network performance is quantified by the total user costs due to potential bridge failures. The objective is to minimize the total user costs, specifically, the extra travel distance over a planning horizon and under a budget constraint. It is conjectured and then verified that the expected increase in vehicle-miles travelled due to failures can be approximated by the sum of expected increases due to individual failures. This allows the network-level problem to be decomposed into single-bridge problems and tackled efficiently. The computational effort increases linearly with the number of bridges.

To my parents.

Contents

Contents

List of Figures	iv
List of Tables	v
Acknowledgements.....	vi
Chapter 1.....	1
1.1 Research Questions	2
1.2 Dissertation Contributions.....	3
1.3 Dissertation Outline.....	4
Chapter 2.....	6
2.1 Infrastructure Management	6
2.2 Reliability-Based Methods.....	7
2.3 Incorporating Network Topology.....	8
2.4 Summary	9
Chapter 3.....	10
3.1 Definitions and Assumptions	10
3.2 Connectivity Indicator.....	11
3.3 Formulation and Approach.....	14
3.3.1 Stage 1: Facility-Level Optimization	15
3.3.2 Stage 2: Network-Level Optimization.....	17
3.4 Numerical Example – Decomposable Network.....	18
3.4.1 Results and Analysis.....	20
3.4.2 Parametric Study.....	21
3.5 Numerical Example – Non-Decomposable Network.....	24
3.6 Extension to Networks with Multiple O/D pairs.....	26
3.7 Summary	28
Chapter 4.....	29
4.1 Problem Definition.....	29

4.2 Conjecture and Verification	30
4.2.1 Analytic proof for homogeneous grid networks	31
4.2.2 Verification in real world network.....	33
4.3 Network-Level Optimization	34
4.3.1 Formulation and Methodology	34
4.3.2 Numerical example.....	36
4.4 Summary	38
Chapter 5.....	40
5.1 Contributions.....	40
5.2 Future Work	41
Appendix A. Pseudocode for the network-level optimization.....	43
Appendix B. Upper bounds for the approximation errors	44
(1) <i>Approximation errors with two close failures</i>	44
(2) <i>Approximation error with $r \geq 3$ close failures</i>	47
(3) <i>Total approximation error</i>	49
Bibliography	50

List of Figures

Figure 1.1 Examples of moderate-size and large-size networks.....	3
Figure 2.1 Deterioration patterns in the absence of maintenance (left), and influence of maintenance and rehabilitation actions (right) (Source: Frangopol et al., 2001).....	7
Figure 3.1 Networks with bridges in series (a) and that with bridges in parallel (b).....	12
Figure 3.2 The shape of the function $f_i(\mathbf{p}_i^{ub})$	15
Figure 3.3 Decomposable network	19
Figure 3.4 Network reliability levels achieved with different agency costs	20
Figure 3.5 Optimal results from different step sizes of threshold discretization	21
Figure 3.6 Impact of bridge location on the optimal resource allocation	22
Figure 3.7 Impact of deterioration rate on the optimal resource allocation on bridges in parallel	23
Figure 3.8 Impact of MR&R cost on the optimal resource allocation on bridges in parallel	24
Figure 3.9 (a) Bridge network topology and (b) schematic bridge network (Reprinted from Liu and Frangopol, 2005)	25
Figure 3.10 Optimal results for different network reliability levels	26
Figure 4.1 The effect of bridge failures on VMT	30
Figure 4.2 Illustration of an individual failure.....	32
Figure 4.3 Numerical implementation for 6,000 bridges.....	37
Figure 4.4 Lifecycle budget allocated on different maintenance actions	38

List of Tables

Table 3.1 Unit costs of MR&R actions.....	19
Table 4.1 Comparison of the mean errors and the error bound	34

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Chapter 1

Introduction

It is widely acknowledged that the social and economic development of any region relies on satisfactory performance of its transportation network. In particular, highway bridges represent the most crucial and most vulnerable components of highway networks. Most highway bridges are built at the connections to on/off ramps, the intersections of highways, and the crossings over rivers or railways. The malfunction of bridges may severely degrade the overall performance of a network. In networks with limited redundancy, bridge closures can lead to disconnection of nodes. In redundant networks, bridge closures can sever effective links connecting origins and destinations. Vehicles need to bypass the unserviceable links, which can increase total travel time and distance.

Bridges deteriorate progressively over their lifetime due to environmental effects and traffic loads. This dissertation's scope is bridge failures caused by gradual deterioration. Taking bridge decks as an example, the major mechanism of concrete deck failure is spalling, or breaking off in fragments, which is caused by the corrosion of the steel rebars and delamination cracking in the concrete. Corrosion starts when chlorides reach the level of steel. Rust builds up around the rebars; the volume of steel expands. The expansion causes concrete fractures and the cohesion between concrete and steel gradually vanishes. The structural resistance of the bridge deck decreases, and it may eventually result in the deck's failure (i.e., the occurrence of spalling).

The problem is important because the current conditions of federal bridges are unsatisfactory, and maintenance and repairs budgets are limited. According to ASCE (2013), the average age of bridges in the U.S. is 42 years old; more than 30% of existing bridges have exceeded their 50-year design life. Timely and adequate maintenance activities must be carried out to ensure safety and satisfactory service levels. Faced with an \$8 billion annual investment shortfall for federal bridges, it is necessary to allocate and distribute the limited maintenance resources in an optimal manner.

The maintenance plans cannot be made arbitrarily, but should be supported by numerical analysis. Many studies have dealt with the optimal maintenance planning for individual bridges or systems of bridges (Markow et al., 1993; Frangopol et al., 2001; Kong and Frangopol, 2003; Robelin and Madanat, 2007; Robelin and Madanat, 2008). Since maintenance policies are usually made by agencies responsible for an entire highway network, research has expanded to networks with multiple bridges in recent years. Network-level problem formulations usually seek maintenance policies that best balance network performance and agency cost. In the literature, different criteria are chosen to quantify the network performance. Some studies focus on connectivity; i.e., the possibility to reach every node from every other node. Liu and Frangopol (2005, 2006) used a graph-theoretic indicator of network reliability or connectivity for networks with single O/D (origin/destination) pairs. Bocchini and Frangopol (2013) expanded the work to multiple O/D pairs. Other studies have adopted travel time and travel distance related indicators

for network performance. Bocchini and Frangopol (2010) introduced an indicator associated with the total travel time and total travel distance of the network.

The analysis of network-level management increases the computational complexity. The bridges in a network are functionally correlated; the impact of a bridge failure on the overall network performance depends on whether other bridges have failed. Because of the interdependencies among bridges, they have to be analyzed together. When uncertainties are involved, most previous research resorts to simulation-based numerical optimization algorithms, such as genetic algorithms, which do not guarantee convergence to optimality. As a result, only networks of moderate sizes (10-30 bridges) can be tackled. However, most real networks in urban areas include hundreds, if not thousands of bridges; for example, there are about 6,000 bridges in the San Francisco Bay Area.

This dissertation aims to develop comprehensive frameworks for bridge management problems of realistic sizes, using realistic metrics. It will provide agencies with decision-making tools to allocate the limited maintenance resources over the planning horizon as well as over networked bridges in a cost-effective manner. Before delving into the research, this chapter presents some preliminary ideas. Section 1.1 describes the scope of the research and the questions that will be answered in this dissertation. Section 1.2 presents the main contributions of this work. Finally, Section 1.3 provides the organization for the rest of this dissertation.

1.1 Research Questions

The bridge networks considered in this dissertation consist of bridges and connecting roads. Bridges are the only vulnerable components of the network, and they are managed by a single agency such as a state department of transportation. The research problem is to determine the optimal maintenance policies for bridge networks that best balance the overall network performance and the total maintenance cost.

To formulate the bridge maintenance management problem mathematically, the first question to be answered is how to measure network performance quantitatively. Some networks are of moderate size and limited redundancy, such as the example shown in Figure 1.1(a). The closure of bridges can sever paths connecting nodes. For this type of networks, it is necessary to ensure that there is at least one path for a vehicle to reach its destination, as connectivity is the fundamental service function of a transportation network. Therefore, a graph-theoretic indicator of network connectivity is chosen to measure the network reliability or connectivity. In large and redundant networks, such as the grid network shown in Figure 1.1(b), connectivity is irrelevant since there are many alternative paths for vehicles to reach their destinations. In the case of bridge closures, network users have to reroute to bypass the unserviceable links, which can result in extra travel time and travel distance. Therefore, the network performance is quantified by the increase in user costs, specifically, the extra travel distance due to potential bridge failures.

Using the performance metrics, the maintenance management problem can be formulated mathematically as an optimization problem with two conflicting objectives: maximizing the network performance and minimizing the total agency cost. The bridge network is a time variant system; the network degrades as the bridges deteriorate. Because the future information of the bridge conditions is not available, deterioration models must be developed to predict future conditions. Based on the deterioration models, maintenance resources need to be optimally allocated over the planning horizon and the networked bridges. Since the bridges are functionally

correlated, they have to be analyzed together. The computational complexity becomes tremendous in large networks. This so-called “curse of dimensionality” is a frequently encountered challenge in network-level management problems. Therefore, to develop efficient and accurate solution methods is the most difficult part of the research.

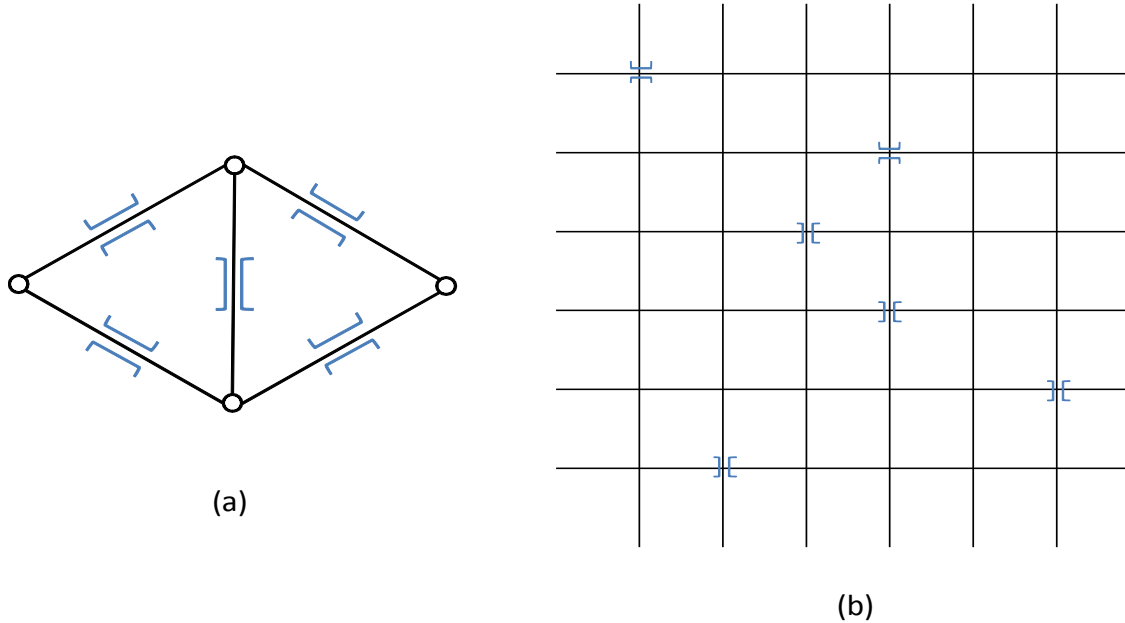


Figure 1.1 Examples of moderate-size and large-size networks

Last but not least, to help agencies make better maintenance decisions for bridge networks, it is necessary to translate the scientific analysis and results into practical use. This will also be discussed in the dissertation.

1.2 Dissertation Contributions

This dissertation produces both scientific and practical benefits.

The first contribution of the research is in identifying appropriate network performance criteria. Instead of using a uniform criterion, suitable metrics are chosen for different types of networks. For moderate size networks with limited redundancy, the network performance is quantified by a graph-theoretic indicator of network connectivity. For large and redundant networks, the network performance is measured directly by the total user costs associated with potential bridge failures.

Accordingly, the research is divided into two parts: retaining network connectivity in moderate-size networks and minimizing total user costs in large and redundant networks. Contributions in each part are as follows.

Retaining network connectivity:

- An approximation of network reliability function for non-decomposable networks of moderate sizes.
- A novel two-stage approach to tackle the connectivity problem, which makes it possible to solve the problem by using standard optimization tools (with guaranteed convergence to optimality), as opposed to the heuristic algorithms used in the literature.
- A framework to study networks with multiple O/D pairs.

Minimizing total user costs:

- Verification that the expected increase in user costs due to bridge failures can be approximated by the sum of increases due to individual failures, if a network is uncongested.
- An efficient solution method, capable of solving MR&R (Maintenance, Repair and Reconstruction) optimization problems for large networks with thousands of deteriorating bridges over multiple periods.

1.3 Dissertation Outline

The remainder of this dissertation is organized as follows.

Chapter 2 furnishes the background of this research by reviewing the existing literature on bridge management, with emphasis on reliability-based methods and network considerations.

Chapter 3 solves for the optimal MR&R policies for retaining network connectivity. The objective is to ensure an adequate level of network reliability at the lowest possible life-cycle maintenance cost. Instead of considering the evolution of network reliability over the whole lifecycle, the optimization is formulated with a constraint on the lower bound of network reliability, which reduces the network-level problem to one of optimizing the set of reliability levels for individual bridges. To evaluate and illustrate the approach, it is applied in two numerical examples, a decomposable network and a non-decomposable network. Finally, the framework is extended to networks with multiple O/D pairs.

Chapter 4 develops a simple framework to determine optimal maintenance plans for large networks with many bridges. The objective is to minimize disruption, specifically, the extra travel distance caused by potential bridge failures over a planning horizon and under a budget constraint. We show, through exact analysis for networks with a grid structure, and through simulations for a real-world network, that the expected increase in vehicle-miles travelled due to failures can be approximated by the sum of expected increases due to individual failures. This allows the network-level problem to be decomposed into single-bridge problems and tackled efficiently. A numerical example is implemented as an illustration.

Chapter 5 concludes the dissertation by summarizing the high-level message of this dissertation, and outlining directions for future work.

Some of the more technical proofs will be deferred to the Appendices.

Chapter 2

Background

Because bridge management belongs to the broader area of infrastructure management, most methodologies in infrastructure management can be applied to bridge management as well. Section 2.1 briefly reviews different frameworks in infrastructure management, with emphasis on bridge management. Section 2.2 reviews past research on bridge management that has used reliability-based methods. Section 2.3 discusses recent research in infrastructure management that has taken into account the network topology.

2.1 Infrastructure Management

Infrastructure management proceeds from the idea that decisions in maintenance, repair and reconstruction (MR&R) can be based on an optimal use of resources. The major task of infrastructure management is to determine the optimal selection and time scheduling of maintenance activities. Since the future information of facilities is not available, deterioration models must be developed to predict the future conditions of facilities. Based on the deterioration models, different optimization problems are formulated to solve for the optimal maintenance plan.

The existing research can be roughly categorized into facility-level management and system-level management. The facility-level infrastructure management problem solves for optimal MR&R policies for an individual facility. The most widely used framework is the MDP (Markov decision process) (Madanat and Ben-Akiva, 1994), where the deterioration process is modeled as Markovian process and dynamic programming is applied to obtain optimal maintenance policies. The MDP approach utilizes standard and efficient optimization techniques, and it accounts for uncertainty in deterioration. To provide insights into the optimal solution, analytical approaches in continuous time and state have been formulated and solved using nonlinear-optimization (Li and Madanat, 2002) and calculus of variation (Ouyang and Madanat, 2006). The drawback of these analytical approaches is that they are based on deterministic deterioration models.

The system-level infrastructure management problem determines the optimal MR&R policies for multiple facilities under a total budget constraint. Existing literature on system-level optimization can be broadly categorized into top-down approaches and bottom-up approaches. Top-down approaches perform optimization at the system level, and actions are recommended for fractions of the facility population (Kuhn and Madanat, 2005). In bridge management, the Pontis system (Thompson et al. 1998) adopted a top-down approach. It is efficient in handling large number of facilities, but it cannot account for the heterogeneity of facilities. The bottom-up approach first performs optimization for each facility, and then selects optimal solutions to account for the total budget constraint. This approach has been adopted in BRIDGIT (Hawk and

Small 1998) and Indiana Bridge Management System (Sinha et al. 1988). Although the bottom-up approach accounts for the heterogeneity of facilities, it is computationally expensive when the number of facilities becomes large.

These bridge management systems are based on the MDP, using discrete condition states obtained from visual inspections. Such discrete states can only represent the observable deterioration of bridge components, but cannot account for the load carrying capacity of the structures (or structural reliability). Besides, the transition probabilities are assumed to be time invariant, which is unrealistic. These have been revealed as the main limitations of the existing bridge management systems (Frangopol and Das, 1999).

2.2 Reliability-Based Methods

It is generally recognized that the bridge management should be addressed from a reliability viewpoint. The determination of MR&R policies depends on the structural reliability, rather than the visual conditions of bridges. Starting in the 1990s, a major research thrust has focused on including reliability theory in bridge management.

The reliability level of bridge structure is represented by the reliability index β . It is a measure of structure safety. By definition, the reliability index is related to the structure's failure probability in a time interval between inspections by the probit transformation; so that $\Phi(-\beta)$ is said probability, where Φ is the standard normal cumulative distribution function. Based on performance limit state analysis, Thoft-Christensen (1998), Frangopol et al. (1997) and Estes (1997) have derived whole life profiles for the progression of the bridge component reliability index. Figure 1 shows the piecewise linear model that has been frequently used. The parameters of the curve are treated as random variables following the prescribed probability distribution.

The entire bridge system consists of superstructure (decks, girders) and substructure (pier, footing). The performance of the bridge system can be very different from the performance of its components. To evaluate the system reliability of an individual bridge, it can be idealized as a series-parallel system of its components (Estes and Frangopol, 1999).

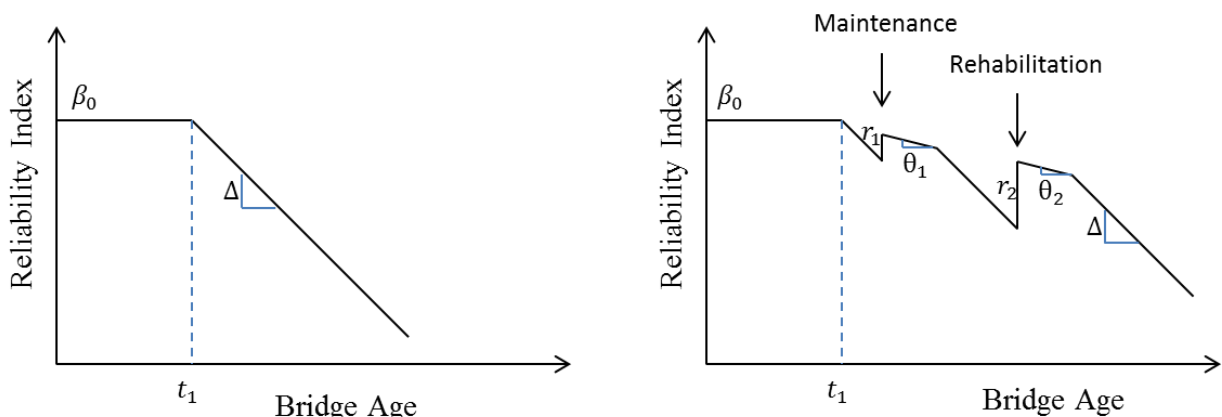


Figure 2.1 Deterioration patterns in the absence of maintenance (left), and influence of maintenance and rehabilitation actions (right) (Source: Frangopol et al., 2001)

Based on the reliability lifetime profiles, efforts have been made to develop optimization strategies that best balance bridge reliability and life-cycle cost. Examples include Frangopol et

al. (2001) and Kong and Frangopol (2003). Frangopol et al. (2000) studied a system of multiple bridges with the objective to minimize maintenance cost while maintaining the reliability of each bridge above an acceptable level. However, the limitation of their framework is that they only compare predetermined maintenance sequences, which is a static approach using an incomplete solution space. An optimal approach would compare maintenance policies dynamically through time, resulting in a wider solution space over the planning horizon.

To account for the history of deterioration and maintenance activities while including a more complete set of solutions, Robelin and Madanat (2007) used MDP with augmented states to determine the optimal maintenance actions for an individual bridge. Later, Robelin and Madanat (2008) proposed a computationally efficient bottom-up approach which provides optimal policies for realistic system sizes while accounting for the heterogeneity among individual facilities.

2.3 Incorporating Network Topology

Since maintenance policies are usually made by agencies responsible for the entire network, research on bridge management has expanded beyond bridge stock maintenance optimization to account for network considerations. Bridges in a network cooperate with one another to provide service to the network users. Therefore, the objective of maintenance management at network level is to achieve a satisfactory network performance, rather than to focus on the condition of individual bridges.

Incorporating bridge network topology in maintenance decision-making was first studied in the context of seismic risk reduction. Augusti et al. (1994) applied dynamic programming to determine the optimal structural upgrading interventions with the objective of maximizing the probability of network connection. Later, Augusti and Ciampoli (1998) extended their research by considering alternative objectives, including network capacity, out of service time and time efficiency of interventions. However, the bridge failures are assumed to be independent in these works, which is unrealistic. Recent studies have considered the statistical correlation among bridge serviceability under seismic hazards (Song and Ok, 2010; Bonstrom and Corotis, 2013; Ghosh et al., 2013).

In the context of network degradation due to gradual deterioration, as considered in our research, incorporating network topology is a more difficult problem. The bridge network is usually regarded as a spatially distributed and time invariant system. The limited financial resources need to be optimally allocated over a specified planning horizon, as well as over the networked bridges. Because of the interdependencies among the bridges, they have to be analyzed together. Computational issues arise frequently and are the main challenge in network-level management problems. Existing research invariably resort to simulation-based numerical optimization techniques, such as genetic algorithms, which automatically produce a Pareto frontier for two conflicting objectives, usually the minimization of agency cost and the maximization of a network performance index.

Different criteria have been adopted to measure the network performance. Some studies focus on the connection of the network, i.e., the possibility of reaching every node from every other node. For instance, Liu and Frangopol (2006a) used the probability that a specific destination can be reached from a fixed origin as the performance criterion. Bocchini and Frangopol (2011a) introduced the ‘Fully Connected Ratio’, the proportion of samples when all nodes are reachable. Some other studies have created performance indicators based on the total

travel time and total travel distance. Orcesi and Cremona (2010) built up a bridge network maintenance framework for the Pareto optimization of MR&R cost and user cost. Homogeneous Markov transition models were used to predict future condition and genetic algorithms were applied to obtain the optimal maintenance strategy. Bocchini and Frangopol (2010) introduced an indicator computed as a function of total travel time and total travel distance. The trips of the network users are distributed based on user equilibrium. A multi-objective genetic algorithm was implemented to obtain a Pareto frontier of the total maintenance cost and the performance indicator. One critical aspect of the methodology is that the use of Monte Carlo simulation nested in genetic algorithms is very time-consuming. The authors have to use lookup table and bookkeeping techniques to alleviate the computational burden.

2.4 Summary

Based on the literature, it is clear that the existing research on maintenance management for bridge networks is limited. Most bridge management problems are performed at component and system levels without accounting for the overall network performance.

In the research that has incorporated network topology, the criteria for evaluating the network performance have not been carefully selected for different networks. Although different performance indexes are developed, it is seldom discussed which is most relevant for different scenarios. Because bridge failure is very rare, retaining network connectivity is a problem of significance only for networks with limited redundancy. For large and redundant networks, it is more relevant to consider the increase in costs of network users due to bridge failures. Although some researchers have created travel time related indicators, the user costs are not measured by themselves.

Besides, the methodologies developed for the network-level management problems are mostly simulation-based numerical optimization techniques. These methods are computationally expansive, and therefore, limited to networks of moderate sizes (10-30 bridges). However, most networks in urban areas include hundreds, if not thousands of bridges; for example, there are about 6,000 bridges in the San Francisco Bay Area.

In order to contribute to the literature, this dissertation develops efficient solution methods for the network-level management problems, which are capable of handling large networks with many bridges. The maintenance management problems are twofold. For moderate size networks with limited redundancy, the objective is to minimize the agency cost while retaining a prescribed level of network reliability in terms of connectivity. For large and redundant networks, the objective is to minimize the traffic disruption caused by bridge failures under a budget constraint.

Chapter 3

Retaining Network Connectivity

Retaining network connectivity is important as it ensures that there is at least one path for vehicles to reach their destinations. In this chapter, we consider networks of moderate size and limited redundancy, which usually are the highway networks in rural areas, where bridge failures may lead to disconnection in these networks.

This chapter presents a framework to determine the optimal maintenance plan that retains a predefined network reliability level using minimum agency costs. Section 3.1 presents the definition and assumptions for the problem. Section 3.2 introduces a graph-theoretic indicator to quantitatively measure the network connectivity. Based on the connectivity indicator, Section 3.3 formulates the problem mathematically and proposes a novel two-stage approach to solve it. Section 3.4 uses a decomposable network as a numerical example to illustrate and evaluate the method. Section 3.5 applies the framework to a non-decomposable network, the highway network connecting Denver and Lafayette. Section 3.6 discusses a simple extension of the framework to networks with multiple O/D pairs. Finally, Section 3.7 summarizes the findings of this chapter.

3.1 Definitions and Assumptions

Network. The highway network considered in this paper consists of bridges and connecting roads. It is assumed that the network topology remains constant over the planning horizon, and that bridges are the only vulnerable components of the network. Bridges in the network are managed by a single agency such as a state department of transportation.

Bridge condition. Because the deck is the bridge component that deteriorates fastest and requires the largest maintenance budget, only deck condition is considered. This condition is represented by a deck's reliability index β . This index is related to the deck's failure probability p in a time interval between inspections by the probit transformation; so that $p = \Phi(-\beta)$ is said probability, where Φ is the standard normal cumulative distribution function.

Independent failure. Given a set of actions, $\vec{a}$, and a parameter vector π that characterizes the bridge itself (such as its design, material type, traffic, and environment), the deck reliability index $\beta(t|\pi, \vec{a})$ is a stochastic process over time. It is reasonably assumed that this deterioration process $\beta(t|\pi, \vec{a})$ is independent across bridges, because correlations in performance across bridges due to the environment and traffic are captured by the parameter vector π . In other words, conditioned on $(\pi, \vec{a})$, failures are independent across bridges.

Costs. Agencies incur costs when maintenance actions are performed on bridges. Because the maintenance management problem in this chapter seeks the trade-off between network connectivity and agency costs, user costs are not considered.

Maintenance and inspection. Although the deterioration of a bridge deck is a continuous process, the maintenance decisions are made at discrete points in time. Since agencies usually have yearly budgets, it is reasonable to assume that maintenance decisions are made every year for each bridge. The current condition of each bridge is assumed to be known perfectly, meaning that inspections are carried out every year and are error free.

3.2 Connectivity Indicator

For a network with single O/D pair, network failure is defined as the circumstance when the origin is disconnected with the destination. For a network with multiple O/D pairs, the network fails whenever any O/D pair is disconnected. The reliability level of a bridge network, in terms of connectivity, can be reversely reflected by the probability that the network fails. In the next few sections of this chapter, networks with single O/D pairs are considered. Extension to networks with multiple O/D pairs is discussed in Section 3.6.

Let N denote the number of bridges in the network. As the bridges deteriorate, the network degrades and its failure probability increases. Let p_{net} denote the network failure probability and p_i denote the failure probability of bridge i . Since bridges fail independently, p_{net} can be expressed as a function of p_i 's; i.e., $p_{net} = H(p_1, p_2, \dots, p_N)$. The network failure probability function H depends on the network topology.

A commonly used tool to obtain function H is the structure function. It is a binary function indicating the status of the network, with the status indicators for individual bridges as the independent variables (Hoyland and Rausand, 1994). The mathematical expression for the structure function is:

$$\Psi(I) = \Psi(I_1, I_2, \dots, I_N) = \begin{cases} 0, & \text{if network is connected} \\ 1, & \text{if network has failed} \end{cases} \quad (3.1)$$

where I_i is status indicator for bridge i , such that

$$I_i = \begin{cases} 0, & \text{bridge } i \text{ is in service} \\ 1, & \text{bridge } i \text{ has failed} \end{cases} \quad (3.2)$$

With these definitions, the network failure probability p_{net} is equal to the expected value of the structure function. Some networks are simple in structure and can be decomposed. Here, we define decomposable networks as those that satisfy the two conditions:

- 1) The links can be divided into sets that are either parallel or series.
- 2) After replacing each set by a simple link, you can do 1) again until you have only one link.

For the decomposable networks, the function H is relatively easy to obtain. Other networks are complex and non-decomposable, and a minimal cut method is used to obtain function H . The detailed analysis follows.

Decomposable Networks

Figure 3.1 shows two elementary kinds of networks; network (a) has bridges in series and network (b) has bridges in parallel. Because network (a) fails if and only if any bridge fails, the structure function is: $\Psi(\underline{I}) = 1 - (1 - I_1)(1 - I_2) \dots (1 - I_N)$. By taking the expectation of the structure function, the probability that network (a) fails is:

$$p_{a,net} = 1 - \prod_{i=1}^N (1 - p_i) \quad (3.3)$$

For network (b), the network fails if and only if all the bridges fail. The structure function is $\Psi(\underline{I}) = I_1 I_2 \dots I_N$, and the probability that network (b) fails is:

$$p_{b,net} = \prod_{i=1}^N p_i \quad (3.4)$$

The results for other decomposable networks can be obtained similarly. It should be noted that the failure probability function for network (b), as shown by Equation (3.4), is multiplicatively separable. The failure probability for network (a), shown by Equation (3.3), is also multiplicatively separable after removing the constant 1.

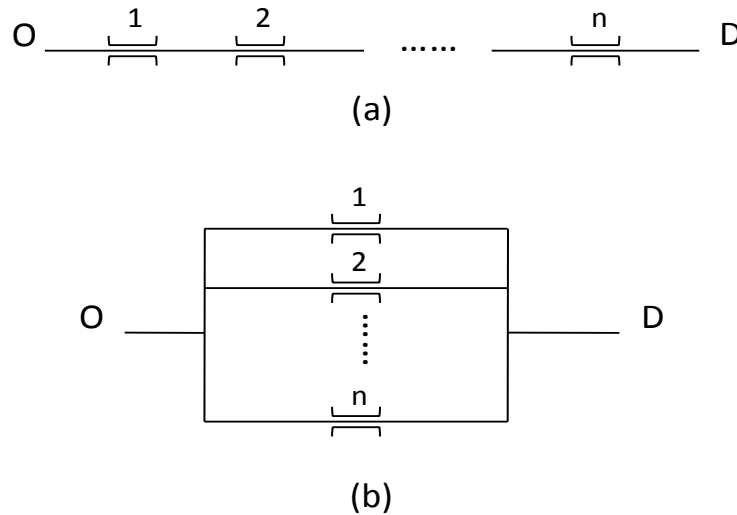


Figure 3.1 Networks with bridges in series (a) and that with bridges in parallel (b)

Non-decomposable Networks

For non-decomposable network, the structure function can be obtained by the minimal path or the minimal cut method (Hoyland and Rausand, 1994). The minimal cut method is applied here for convenience. A cut is a set of bridges such that if all the bridges in the cut fail, the network

will fail. A minimal cut is a set of bridges that comprise a cut, but the removal of any one bridge from the set causes the resulting set to not be a cut. So the network fails if and only if all bridges in the minimal cut fail.

Many algorithms have been proposed to enumerate the minimal cut set (Lin et al., 2003; Benaddy and Wakrim, 2012). If there are M_c minimal cut sets $S_1, S_2, \dots, S_{M_c}$ in a network, then the structure function of the network is

$$\Psi(\underline{I}) = 1 - \left(1 - \prod_{i \in S_1} I_i\right) \left(1 - \prod_{i \in S_2} I_i\right) \dots \left(1 - \prod_{i \in S_{M_c}} I_i\right) \quad (3.5)$$

The network failure probability is equal to the expected value of the structure function. Because two different minimal cuts may include the same bridge, polynomial expansion must be carried out before taking the expectation of $\Psi(\underline{I})$. The result is a long polynomial, as shown in Equation (3.6):

$$p_{net} = \sum_{\gamma} \left(\prod_{i \in S_{\gamma}} p_i \right) - \sum_{\gamma < \delta} \left(\prod_{i \in S_{\gamma} \cup S_{\delta}} p_i \right) + \sum_{\gamma < \delta < \rho} \left(\prod_{i \in S_{\gamma} \cup S_{\delta} \cup S_{\rho}} p_i \right) - \dots \quad (3.6)$$

where the indexes γ, δ , and $\rho \in \{1, 2, \dots, M_c\}$.

In Equation (3.6), $\prod_{i \in S_{\gamma}} p_i$ is the probability that cut S_{γ} is activated; $\prod_{i \in S_{\gamma} \cup S_{\delta}} p_i$ is the probability that cuts S_{γ} and S_{δ} are activated simultaneously; etc. As the failure probability of a bridge is very small, the probability that several cuts are activated simultaneously is the higher order term compared to the probability that a single cut is activated. Let p_{max} denote the largest values in p_i 's. We first compare the probability that S_{γ} is activated with the probability that two cuts (one of them is S_{γ}) are activated simultaneously:

$$\frac{\sum_{\delta > \gamma} \left(\prod_{i \in S_{\gamma} \cup S_{\delta}} p_i \right)}{\prod_{i \in S_{\gamma}} p_i} \leq (M_c - \gamma) \cdot p_{max} \leq M_c \cdot p_{max} \quad (3.7)$$

Then, the one-component terms in Equation (3.6) are compared with the two-component terms:

$$\sum_{\gamma < \delta} \left(\prod_{i \in S_{\gamma} \cup S_{\delta}} p_i \right) \leq \sum_{\gamma} \left(M_c \cdot p_{max} \cdot \prod_{i \in S_{\gamma}} p_i \right) = M_c \cdot p_{max} \cdot \sum_{\gamma} \left(\prod_{i \in S_{\gamma}} p_i \right) \quad (3.8)$$

If $M_c \cdot p_{max} \ll 1$, the probability that two cuts are activated simultaneously is negligibly small compared to the probability of single-cut activation. Similarly, the probability that more

than two cuts are activated has even smaller order of magnitude and can be neglected. Therefore, the network failure probability can be approximated by:

$$p_{net} \approx \prod_{i \in S_1} p_i + \prod_{i \in S_2} p_i + \dots + \prod_{i \in S_{M_c}} p_i \quad (3.9)$$

Each term in the right hand side of the equation represent the probability that the cut is activated. Because the failure rates are usually in the scale of 10^{-5} , the approximation is accurate for networks having hundreds of min-cuts.

It should be noted that two different minimal cuts may contain the same bridge, and thus the same p_i can exist in several terms. Therefore, the network failure probability function H is not multiplicatively separable for the non-decomposable network.

3.3 Formulation and Approach

Given the planning horizon is T years, the bridge management problem is formulated as the determination of the minimum-cost maintenance plan that achieves a certain network reliability level, or equivalently, that keeps the network failure probability below a certain threshold, p_c .

The mathematical formulation can be written as:

$$\begin{aligned} \min \quad & \sum_{i=1}^N C_i \\ \text{s.t.} \quad & p_{net}(t) = H(p_1(t), p_2(t), \dots, p_N(t)) \leq p_c, \text{ for } t = 1, 2, \dots, T \end{aligned} \quad (3.10)$$

where C_i is the agency maintenance cost for bridge i , $p_{net}(t)$ is the network failure probability at year t , and $p_i(t)$ is the failure probability of bridge i at year t .

The constraint is equivalent to keeping the maximum value of the network failure probability below the threshold; i.e., $\max_t p_{net}(t) \leq p_c$. Instead of considering the evolution of $p_{net}(t)$ over the planning horizon, an upper bound of network failure probability p_{net}^{ub} is used to substitute for $\max_t p_{net}(t)$ in the constraint. This upper bound is chosen to be $H(p_1^{ub}, p_2^{ub}, \dots, p_n^{ub})$, where p_i^{ub} is the maximum failure probability of bridge i .

Obviously, $p_{net}^{ub} \geq \max_t p_{net}(t)$, and they are equal when all bridges reach their worst conditions simultaneously. Therefore, this substitution may tighten the constraint, and it may result in a conservative solution, but this substitution is justified for two reasons. First, because the piecewise reliability profile adopted in bridge management is theoretical, it is associated with high uncertainty, which makes a conservative solution preferable. The second reason comes from the observation that bridges are very likely to reach their worst condition at the end of the planning horizon when there is no salvage value (Liu and Frangopol, 2005; Robelin and Madanat, 2007). In that case, $p_{net}^{ub} = \max_t p_{net}(t)$, and the two constraints thus become equivalent.

After the substitution, the formulation of the network-level problem is transformed to:

$$\begin{aligned}
& \min_{p_i^{ub}} \sum_{i=1}^N f_i(p_i^{ub}) \\
& s. t. \quad p_{net}^{ub} = H(p_1^{ub}, p_2^{ub}, \dots, p_N^{ub}) \leq p_c
\end{aligned} \tag{3.11}$$

where $f_i(p_i^{ub})$ is the minimum agency cost required to maintain the failure probability of bridge i below p_i^{ub} .

The optimization problem has been reduced to the determination of the optimal reliability levels, p_i^{ub} 's, for individual bridges, which is then tackled using a two-stage approach. In the first stage, the function $f_i(p_i^{ub})$ is solved for each bridge. Based on the results of the first stage, the second stage solves for the optimal set of reliability levels for individual bridges. After the maintenance policies are determined using the two-stage approach, the conservativeness due to the substitution can be evaluated quantitatively by comparing the actual reliability level to the prescribed value.

The details of each step are described in the following subsections. Subsections 3.3.1 formulates and solves the facility-level problem using MDP with augmented states. Subsection 3.3.2 develops algorithms to tackle the network-level optimization.

3.3.1 Stage 1: Facility-Level Optimization

For each individual bridge, the facility-level problem is solved for a range of thresholds of reliability index. This procedure produces a non-decreasing function $f_i(p_i^{ub})$ for each bridge i , which represents the present value of total MR&R cost that is required to maintain the failure probability below p_i^{ub} , as shown in Figure 3.2.

The function $f_i(p_i^{ub})$ cannot be determined for a continuous interval of p_i^{ub} , because this would require solving the facility-level optimization an infinite number of times. Therefore, the function $f_i(p_i^{ub})$ is defined only for a finite number of thresholds. It has been shown that the discrete implementation is a valid approximation to the optimal cost function (Robelin and Madanat, 2008).

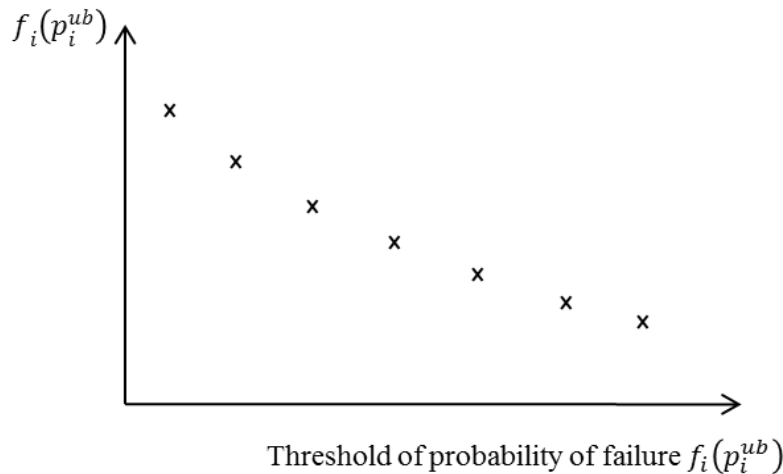


Figure 3.2 The shape of the function $f_i(p_i^{ub})$

The specific facility-level approach is not the focus of this dissertation as long as it yields the function f_i for each bridge. This section describes a dynamic programming method to solve facility-level problems. The bridge deterioration is formulated as Markov decision processes with augmented states. In traditional Markovian models with bridge component condition as the state, the history of the condition is not taken into account, which is seen as a limitation. According to Robelin and Madanat (2007), the effects of the history of deterioration and maintenance can be captured by a three-dimensional vector $x = (\beta, u, \tau)$, where β is the reliability index, u represents the type of the latest action performed on the bridge, and τ represents the time since the latest action. Therefore, the deterioration process can be modeled as Markovian, using x as the state. The advantage of this method is that it uses standard optimization techniques, while retaining the effects of deterioration and maintenance history as part of the model.

Although the reliability index β can take any value in a continuous interval, it is discretized for computational convenience. For a given threshold p_i^{ub} , the objective is to minimize the total discounted agency cost while keeping network failure probability below p_i^{ub} , or equivalently, the reliability index above β^c , and $\beta^c = -\Phi^{-1}(p_i^{ub})$. The Bellman's recursion is:

$$\begin{aligned} \forall x \in X, \quad V_t(x) &= \min_{a_{it} \in A} \{c_i(a_{it}) + q(x) + \alpha \sum_{y \in X} P_i(y|x, a_{it}) V_{t+1}(y)\}, \\ &\text{for } t \in \{0, 1, \dots, T-1\}. \\ V_T(x) &= 0 \end{aligned} \quad (3.12)$$

- X : state space of the Markov Chain;
- A : set of all possible MR&R actions, include all types of maintenance, repair, and reconstruction activities, and do-nothing;
- T : planning horizon;
- $c_i(a_{it})$: cost of action a_{it} on bridge deck i .
- $q(x)$: penalty cost, set to be an arbitrary large value when the reliability index of state x is below β^c , and zero otherwise;
- α : discount factor.
- $V_t(x)$: minimum cost-to-go for the agency to manage a bridge deck currently in state x from year t to the end of the planning horizon.
- $P_i(y|x, a_{it})$: transition probability for bridge deck i from state x to state y in the next year, given action a_{it} is applied.

In the above formulation, a large penalty cost is assigned to the cost-to-go function whenever the reliability index falls below the threshold. With this strategy, it is ensured that the reliability of the bridge is maintained above the threshold.

For each individual bridge, the facility-level problem is solved for a range of thresholds of reliability index. This procedure produces a non-decreasing function $g_i(\beta^c)$, $g_i(\beta^c) = f_i(\Phi(-\beta^c))$ for each bridge i , which represents the present value of total MR&R cost that is required to maintain the reliability index above β^c . Using the one-to-one relationship between the reliability index and the deck's failure probability, the function $f_i(p_i^{ub})$ can be easily obtained.

3.3.2 Stage 2: Network-Level Optimization

Based on the optimal cost function $f_i(p_i^{ub})$ obtained in the first stage, the second stage solves for the optimal reliability levels, p_i^{ub} 's, for individual bridges. In general, the network-level optimization is a nonlinear integer programming problem, because both functions H and f_i are nonlinear and the decision variables, p_i^{ub} 's are defined on a discrete domain.

In decomposable networks, the links can be divided into subsystems that are either in parallel or series. The failure probability function H for the subsystems is multiplicatively separable, as shown in Section 3.2. The optimization problem has the format of a one-dimensional resource allocation problem (Bellman and Dreyfus, 1962; Augusti, 1994). It can be easily solved in polynomial time using dynamic programming. After solving the problem for each subsystem, the subsystem is replaced by a single link. The whole process is repeated until we get the optimal cost function for the entire network.

For non-decomposable networks, the function H is not multiplicatively separable, which makes it more difficult to solve. To tackle this nonlinear integer optimization problem, an algorithm is developed to transform it into Binary Integer Linear Programming (BILP).

To keep the network failure probability below a certain threshold p_c , a necessary condition is that the activating probability of each cut is smaller than p_c . It is not a sufficient condition, because the network failure probability is the sum of the activating probabilities of all cuts. Here, the optimization is first solved under the necessary condition, which provides a lower bound to the optimal cost. Then an iterative solution method is introduced to gradually lead the solution into the feasible domain where the constraint on overall network failure probability is satisfied.

For an arbitrary minimal cut S_γ , it is required that $\prod_{i \in S_\gamma} p_i^{ub} \leq p_c$. This inequality constraint is equivalent to a set of binary linear inequality constraints by a change of variables. The optimal cost function $f_i(p_i^{ub})$ is only defined for a finite number of thresholds, which can be denoted as $q_1, q_2, \dots, q_R$, and the corresponding minimum agency cost for bridge i as $f_{i1}, f_{i2}, \dots, f_{iR}$. We use binary variables Z_{ik} 's to substitute for p_i^{ub} as the decision variables. Let

$$p_i^{ub} = \sum_{k=1}^R Z_{ik} q_k, \quad \text{for } i = 1, 2, \dots, N \quad (3.13)$$

where Z_{ik} 's are binary variables, and $\sum_{k=1}^R Z_{ik} = 1, \text{ for } i = 1, 2, \dots, N$.

After substituting Equation (3.13) into $\prod_{i \in S_\gamma} p_i^{ub}$, we can obtain a polynomial of Z_{ik} 's, where each of the coefficients is a product of a subset of $\{q_1, q_2, \dots, q_R\}$. To illustrate, an arbitrary term of the polynomial can be denoted as:

$$\left(\prod_{j=1}^{|S_\gamma|} q_{k_j} \right) \cdot Z_{i_1 k_1} \cdot Z_{i_2 k_2} \cdot \dots \cdot Z_{i_{|S_\gamma|} k_{|S_\gamma|}} \quad (3.14)$$

where $|S_\gamma|$ represents the number of bridges in the minimal cut S_γ . If the coefficient $\prod_{j=1}^{|S_\gamma|} q_{k_j}$ is greater than p_c , the term (3.14) must set to be zero; i.e., $Z_{i_1 k_1} \cdot Z_{i_2 k_2} \cdot \dots \cdot Z_{i_{|S_\gamma|} k_{|S_\gamma|}} = 0$. Because Z_{ik} 's are binary variables, this is equivalent to the linear inequality constraint:

$$Z_{i_1 k_1} + Z_{i_2 k_2} + \dots + Z_{i_{|S_\gamma|} k_{|S_\gamma|}} \leq |S_\gamma| - 1 \quad (3.15)$$

Using the strategy described above, the constraint on each cut activating probability produces a set of binary linear inequality constraints. After the variable substitution, the objective function is transformed to:

$$\min_{Z_{ik}} \sum_{i=1}^N \sum_{k=1}^R f_{ik} \cdot Z_{ik} \quad (3.16)$$

The optimization problem becomes a standard BILP, which can be tackled by commercial solvers such as AMPL and MATLAB.

As mentioned earlier, keeping the activating probability of any minimal cut below the threshold p_c is not sufficient to ensure the network failure probability is below p_c . If the solution results in a higher failure probability than p_c , more intense actions should be applied to maintain the network in a better condition than the current level. In other words, the total agency cost should be greater than the current value; i.e.,

$$\sum_{i=1}^N \sum_{k=1}^R f_{ik} \cdot Z_{ik} > \text{cost in current solution} \quad (3.17)$$

Therefore, we need to add in constraint (3.17), which requires using an iterative solution method. The pseudocode for the network-level optimization is provided in Appendix A.

Although most real highway networks cannot be decomposed, they may have subsystems that have elements in series or in parallel. To relieve the computational burden, it is always beneficial to run the optimization first for these subsystems, and then replace the subsystem with a single equivalent element when solving the entire network.

The main drawback of BILP is that it is NP complete and no solution in polynomial time has been found. The size of the BILP problem is usually determined by the number of decision variables and constraints. For our problem, the network size, after substituting the reducible parts, is constrained to hundreds of min-cuts. It can be handled by ordinary solvers on a PC.

3.4 Numerical Example – Decomposable Network

In this section, a seven-bridge decomposable network is used as a numerical example to illustrate and evaluate the proposed approach. The topology of this network is the same as the one

connecting San Francisco and Oakland, when only the seven major cross-bay bridges are included. The schematic network is shown in Figure 3.3.

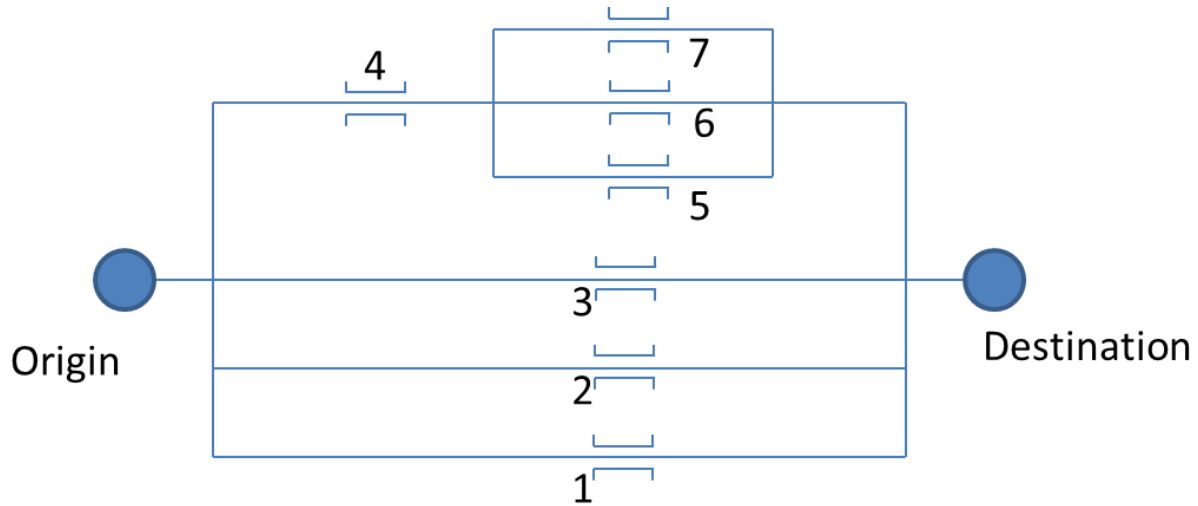


Figure 3.3 Decomposable network

Bridges 1, 2, and 3 are in parallel, and each of them independently provides a path from the origin to the destination. Bridge 4 is in series with the subsystem which has Bridges 5, 6, and 7 in parallel. The network failure probability can be written as:

$$p_{net} = p_1 p_2 p_3 (1 - (1 - p_4)(1 - p_5 p_6 p_7)) \quad (3.18)$$

Using the stochastic lifetime profile of the reliability index developed in Frangopol et al. (2001), the transition matrices are obtained by Monte Carlo simulation. The details of the simulations process are provided in Robelin and Madanat (2007). To capture the heterogeneity of different bridge decks, parameters are adjusted within realistic ranges. The reliability index is discretized to take integer values from 1 to 15.

Four different actions are considered, do-nothing, maintenance, repair, and reconstruction. The unit costs of maintenance activities are adopted from Kong and Frangopol (2003), as shown in Table 3.1.

No.	MR&R Type	Unit Cost (\$/m ²)
1	Do-nothing	0
2	Maintenance	200
3	Repair	300
4	Replacement	1000

Table 3.1 Unit costs of MR&R actions

The two-stage approach is implemented. The results and analysis are presented in Subsection 3.3.1. A parametric study is carried out to analyze the impacts of different factors on resource allocation, as presented in Subsection 3.3.2.

3.4.1 Results and Analysis

Figure 3.4 plots the network reliability levels against the minimum agency costs required to achieve these reliability levels. The horizontal axis represents the total discounted agency costs, or it can be interpreted as the total budget for the network maintenance. The vertical axis represents the network reliability index, which is related to the network failure probability by the probit transformation. The curve provides a Pareto frontier of two conflicting objectives, minimizing the agency costs and maximizing the network reliability. The non-decreasing trend of the curve is intuitive, since adding more money will never worsen the network condition. It can also be observed that the curve flattens as the budget increases, indicating a diminishing marginal effect of budget on reliability. When the budget is abundant to maintain all bridges in good condition, the effect of adding more money is small.

The curve shown in Figure 3.4 is not smooth, but has small steps and jumps. The small flat segments suggest that the network reliability level remains the same when the budget changes, as long as the budget falls in the range of the corresponding flat segment.

Since the facility optimal cost function is a step function, it is possible that part of the budget allocated on the facility has not been used, if the assigned budget is not equal to the starting point of a step. This unused part of budget is the budget residual. There are two causes for the budget residual. The first is that the cost of each MR&R action is fixed and indivisible, and thus the total discounted MR&R cost can only take a finite number of values, corresponding to different combinations of actions. The second cause is a consequence of the discretization of the reliability index. There is a minimum amount of budget required to increase the reliability level from a given state by one state. If the increment of budget does not reach the minimum amount, the reliability level remains unchanged and the increment of budget is unused.

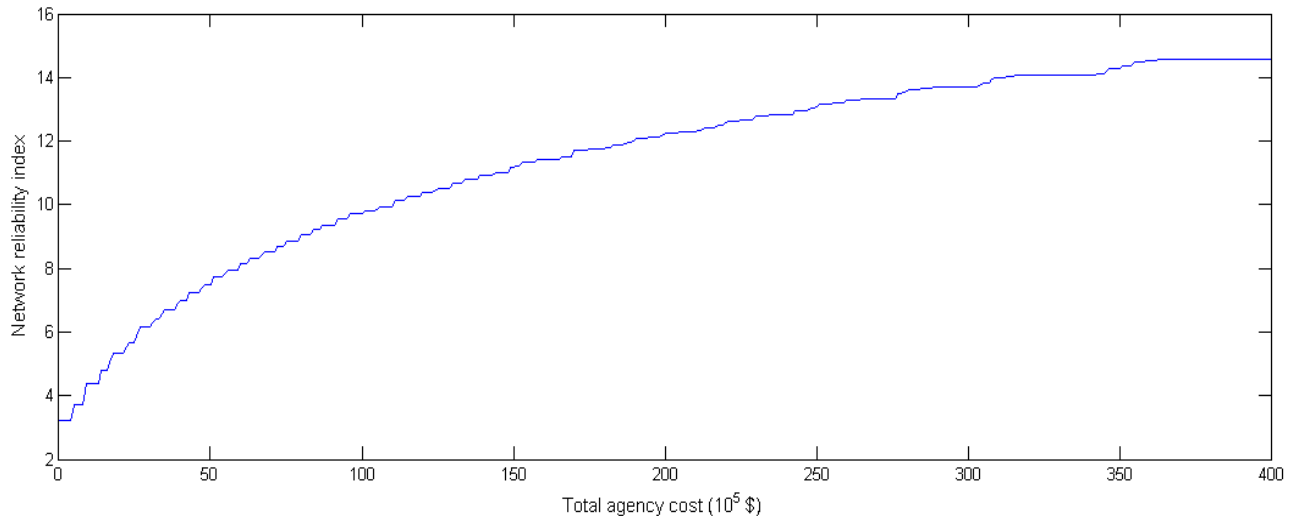


Figure 3.4 Network reliability levels achieved with different agency costs

In practice, the optimal cost value $f_i(\beta^c)$ is only determined for a finite number of thresholds of reliability index. This is because the reliability index has been discretized to take a finite number of values in the Markov decision process. Moreover, to determine $f_i(\beta^c)$ in a continuous interval requires solving an infinite number of facility-level problems, which is impossible.

Therefore, the reliability threshold is discretized and the results for different step size are shown in Figure 3.5. It can be observed that the curve becomes smoother as the step size decreases. Moreover, the curves with the smaller step sizes are above the curves with the larger step sizes, indicating that for the same budget, the model with smaller step size achieves higher network reliability. It is also observed that the distance between the step size 1 and step size 2 is much smaller than that between step size 2 and step size 4, which reveals a convergence pattern as the step size decreases. Since solving a model with smaller step size requires greater computational efforts, a suitable step size should be selected such that the results of the model satisfy the accuracy requirement, without an excessive increase in computational cost.

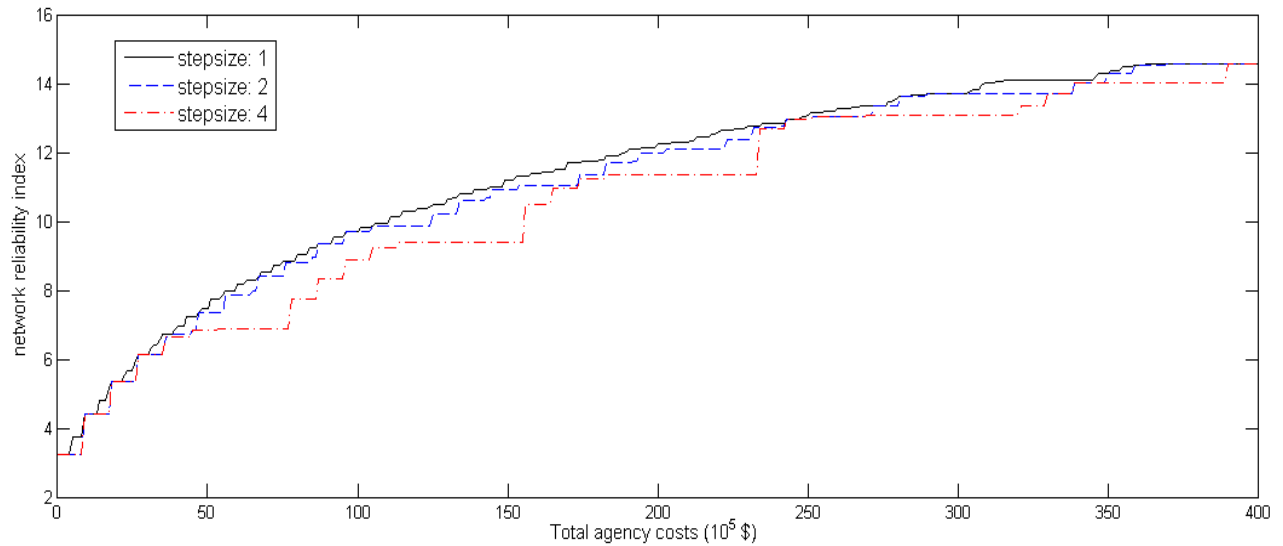


Figure 3.5 Optimal results from different step sizes of threshold discretization

3.4.2 Parametric Study

In this subsection, we study the impacts of three different factors on resource allocation over the network of bridges. These three factors are the bridge location in the network, the deterioration rate, and the unit cost of maintenance actions.

Location in Network

To isolate the impact of bridge location on the resource allocation, these bridges are set to be homogeneous.

Figure 3.6 shows the percentage of the agency costs consumed by each bridge in the optimal solution. It can be observed that more than 80% of the total budget is allocated to Bridges 1, 2, 3, and 4, while less than 20% is allocated to the other three bridges. Because Bridges 5, 6 and 7 cannot connect the origin to the destination without Bridge 4, their contribution to the network

connection is marginal compared to Bridges 1, 2 and 3. As expected, the budget is allocated to bridges that are more critical for network connectivity. In other words, the bridges in critical positions have priority in receiving maintenance resources.

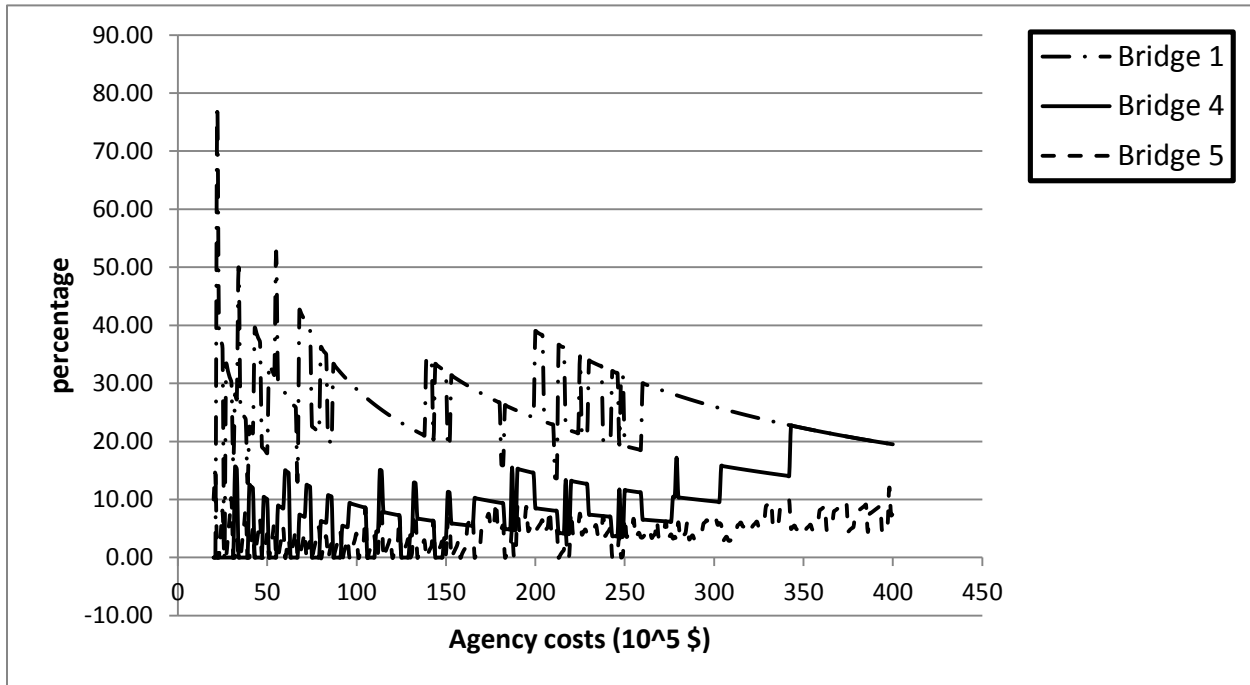


Figure 3.6 Impact of bridge location on the optimal resource allocation

Deterioration Rate

We focus on Bridges 1, 2, and 3 to study the impact of deterioration rate, since these three bridges are similar with regard to their positions in the network. Compared to Bridge 1, Bridge 2 has a relatively slower deterioration, with an expected deterioration rate of 0.075 per year, while Bridge 3 has a relatively faster deterioration, with an expected deterioration rate of 0.15 per year. Figure 3.7 shows the percentage of agency costs consumed by each bridge. If the budget is tight, only Bridge 1 and Bridge 2 are maintained, with no budget allocated to Bridge 3. When the budget is abundant, the bridges with higher deterioration rates receive more maintenance funds.

The phenomenon observed in Figure 3.7 can be explained as follows. The bridge with faster deterioration rate needs more resources to maintain it at the acceptable level. When the budget is tight, it is impossible to maintain all three bridges in satisfactory reliability states. Since the bridges are in parallel, the budget is preferably allocated to Bridges 1 and 2 in order to ensure the reliability of these two paths. When the budget is sufficient to maintain Bridges 1 and 2, resources are allocated on Bridge 3 as well to ensure the reliability of the third path. Finally, all three bridges receive adequate maintenance when the agency has a sufficient budget, and faster deteriorating bridges receive a larger proportion of the budget.

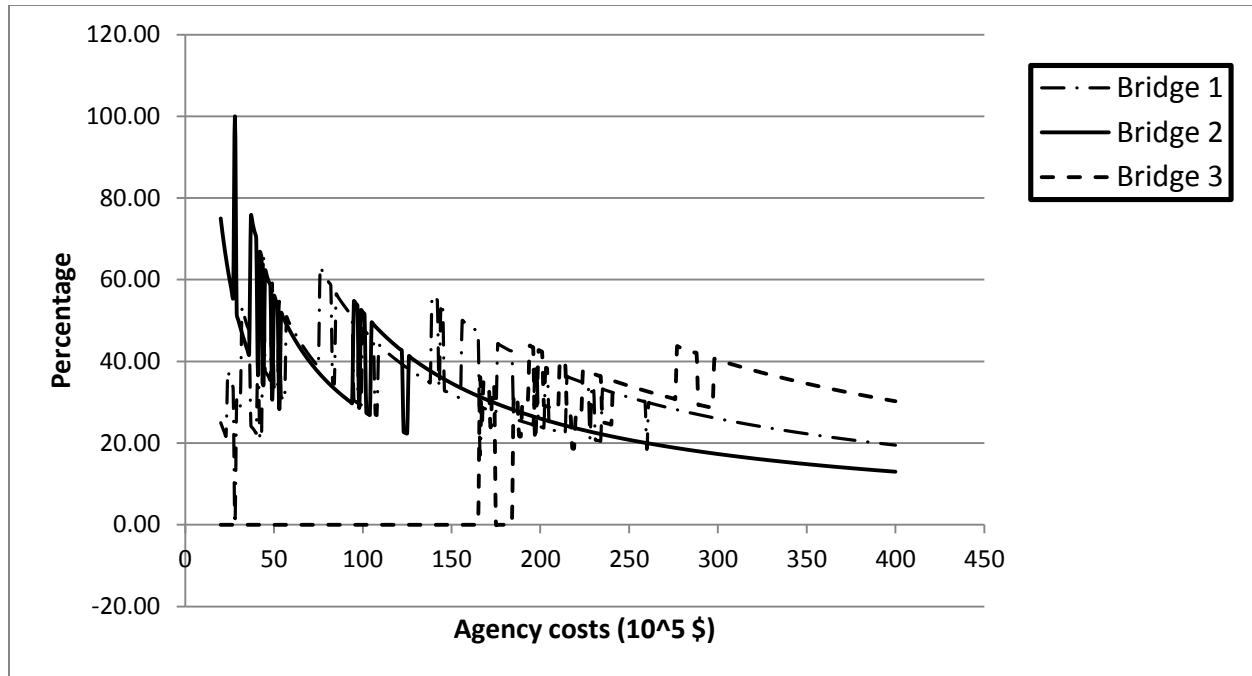


Figure 3.7 Impact of deterioration rate on the optimal resource allocation on bridges in parallel

Unit Costs of MR&R actions

To analyze the impact of costs of MR&R actions, the unit costs are adjusted, while other factors are unchanged. The unit costs of actions for Bridge 1 remain the same, while the unit costs of actions for Bridge 2 are reduced by 20% and the unit costs of actions for Bridge 3 are increased by 20%.

Figure 3.8 shows the percentage of budget allocated to each bridge. When the budget is tight (below $\$200 \times 10^5$), only Bridges 1 and 2 are maintained, with no budget allocated to Bridge 3. As the budget increases above $\$200 \times 10^5$, resources start to be allocated to Bridge 3. With an ample budget, Bridge 3 receives the largest fraction of the budget.

The phenomenon observed in this case can be explained similarly as in the previous section. When the budget is limited and only adequate to maintain one or two bridges in the network, Bridges 2 and 1 will be selected since the costs of maintenance activities for these two bridges are relatively small. This strategy efficiently uses the limited resources to upgrade the network reliability. When the total budget becomes large, there are available funds left after Bridges 1 and 2 receive necessary maintenance. At this point, maintenance is applied to Bridge 3, which has the highest cost.

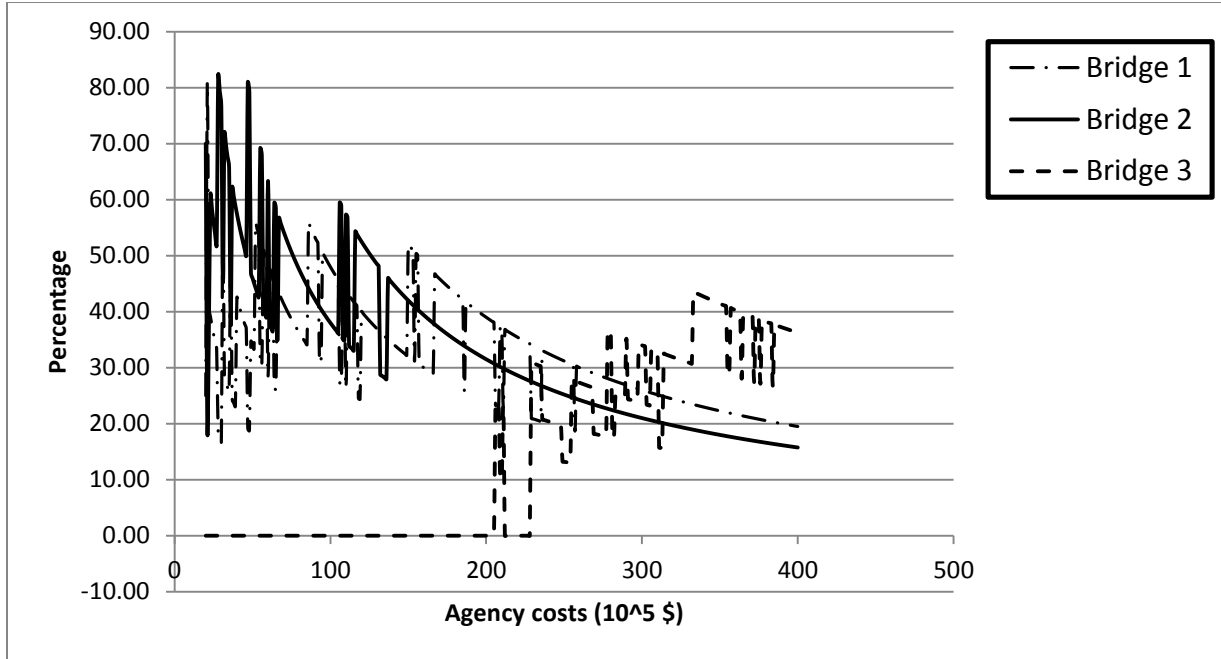


Figure 3.8 Impact of MR&R cost on the optimal resource allocation on bridges in parallel

3.5 Numerical Example – Non-Decomposable Network

In this section, a numerical example illustrates the proposed approach. The network under study is a 13-bridge network located in the northwest metropolitan area of Denver, Colorado (Liu and Frangopol, 2005). As shown in Figure 3.9(a), the bridge network is non-decomposable, but there are two subsystems, Group 1 and Group 2, which have bridges in series. Figure 3.9(b) shows the schematic layout of the network. There are four min-cut sets: {1, LE}, {1, HE}, {2, MU}, {HE, MU}.

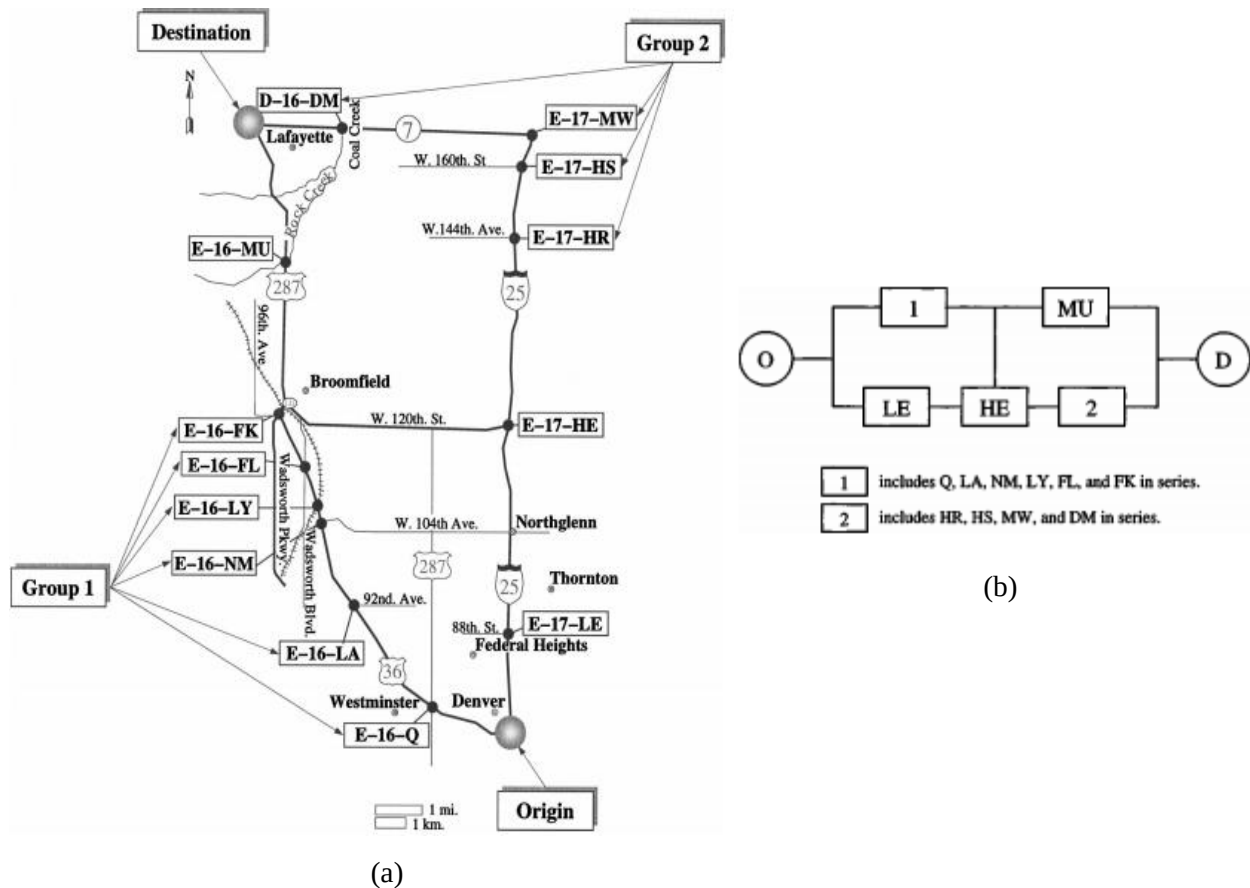


Figure 3.9 (a) Bridge network topology and (b) schematic bridge network (Reprinted from Liu and Frangopol, 2005)

The deterioration rates and initial states of bridges are adapted from Liu and Frangopol (2005). The planning horizon is set to be 30 years. The costs and the effect of maintenance activities are adopted from Kong and Frangopol (2003).

Figure 3.10 shows the results obtained after applying the two-stage approach on this network. The line plots the total maintenance cost required to achieve each reliability level (the allowed network failure probability), and the columns below the line show the allocation of the total budget. As expected, the cost increases for higher reliability level. Because Group 1 and Group 2 have bridges in series, they are more vulnerable and relatively expensive to maintain. MU is the sturdiest bridge, with the lowest deterioration rate. Therefore, when the network reliability level is low, most maintenance efforts are applied to LE, HE, and MU to ensure the connectivity of this path. When a higher network reliability level is required, maintenance resources are allocated to Group 1 to ensure the second path, through Group 1 and MU.

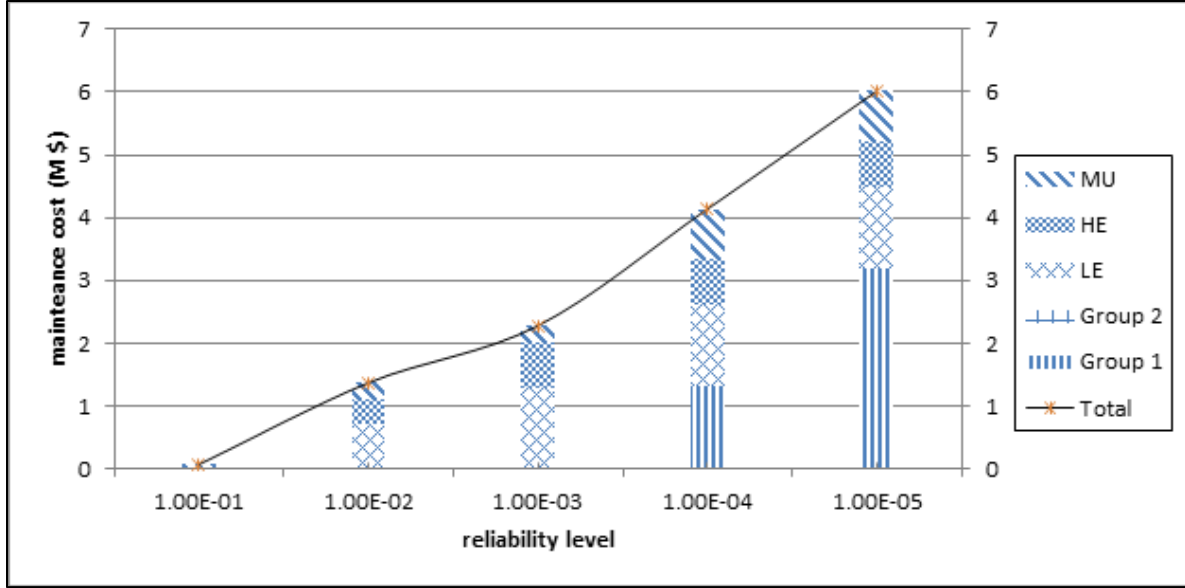


Figure 3.10 Optimal results for different network reliability levels

A simulation is carried out to evaluate the conservativeness of the solution. Using the optimal maintenance policy for the reliability level of 1.0×10^{-3} , 1000 deteriorating profiles of the network are generated. The result gives an expected network reliability index of 4.0, with a standard deviation of 0.37. The expected reliability is higher than the required reliability by 2.6 standard deviations, and therefore, the network reliability is kept above the threshold with 99.5% certainty.

3.6 Extension to Networks with Multiple O/D pairs

The methodology described above can be extended to the reliability of connectivity of a multiple-O/D-pair network. The minimal path sets and minimal cut sets in the multiple-O/D-pair network can be defined similarly as the single-O/D-pair network. A path for the network is a set of bridges, such that if all the bridges in the set have not failed, the network is connected. A minimal path is a set of components that comprise a path, but the removal of any one bridge will cause disconnection of the network.

Figure 3.11 is an example of the multiple-O/D-pair network, where each pair of the five nodes is an O/D pair. It can be observed that a minimal path corresponds to a spanning tree in the network. After all the minimal paths have been listed, the minimal cuts can be generated by subsequently selecting one bridge from any one of the minimal paths without repetition. However, because the number of spanning trees grows exponentially with the size of the graph, this approach may not be applicable to solve large scale networks.

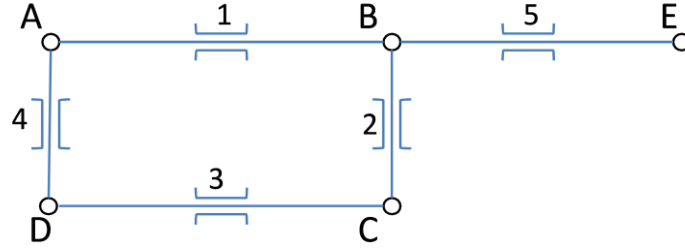


Figure 3.11 Multi-OD network example (Reprinted from Bocchini and Frangopol 2011)

Using the minimal cut sets, the algorithm developed in Section 3.3 for single-OD-pair networks can be used to solve for the optimal set of thresholds. For illustration, this approach is applied to the multiple-OD-pair network studied by Bocchini and Frangopol (2011), shown in Figure 3.11. There are seven minimal cut sets: $S_1 = \{5\}$, $S_2 = \{1,2\}$, $S_3 = \{1,3\}$, $S_4 = \{1,4\}$, $S_5 = \{2,3\}$, $S_6 = \{2,4\}$, $S_7 = \{3,4\}$. Therefore, the network failure probability can be written as:

$$p_{net}^f \approx p_5^f + p_1^f p_2^f + p_1^f p_3^f + p_1^f p_4^f + p_2^f p_3^f + p_2^f p_4^f + p_3^f p_4^f \quad (3.19)$$

Figure 3.12 displays the percentage of maintenance cost allocated to each bridge for different reliability levels. The five bridge decks' parameters are set to be equal in order to eliminate the effect of factors other than bridge location. Bridge 5 receives the largest amount of maintenance resources, about 30% ~ 40% of the total maintenance cost. The remaining budget is almost equally allocated to the other four bridges. The cause for this phenomenon is the special location of Bridge 5 in the network. The failure of Bridge 5 alone would lead to the disconnection of node E, while other disconnections require at least two bridges to fail. Thus, Bridge 5 is more important to network connectivity than other bridges.

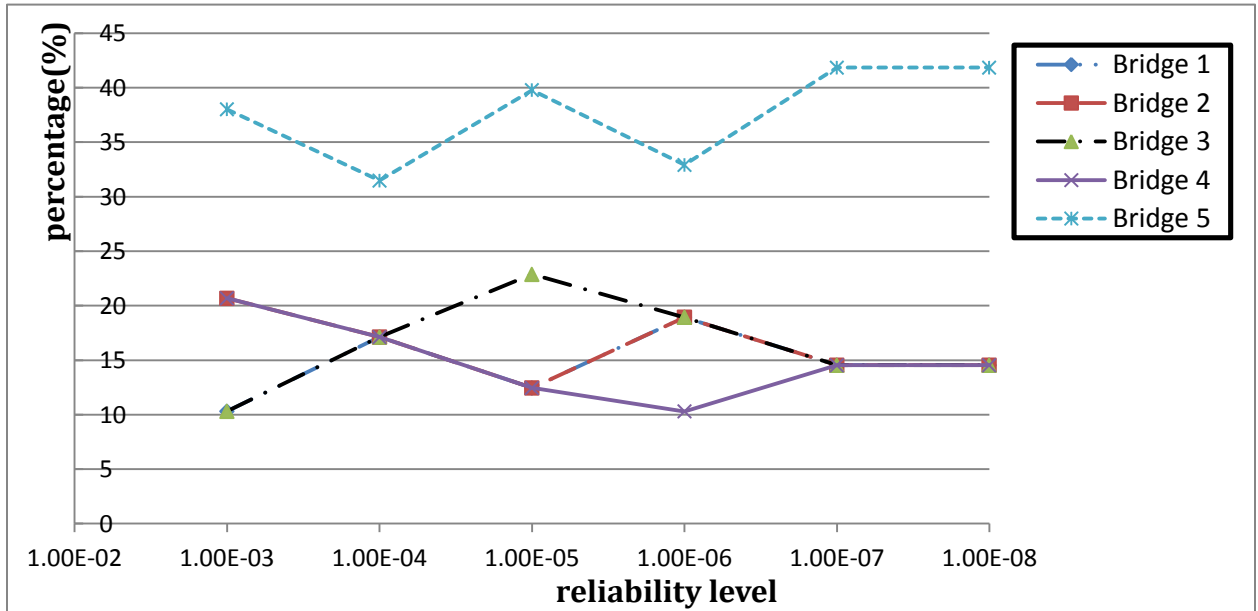


Figure 3.12 Proportion of maintenance cost over the five bridges

3.7 Summary

This chapter addresses the problem of optimizing maintenance decisions for deteriorating bridges at the network level. The objective is to ensure an adequate level of network reliability at the lowest possible life-cycle cost. A two-step framework has been proposed. The first step is the facility-level problem which solves for the optimal cost functions of individual bridges. Using the results of the facility-level optimization, the second step reduces the network-level problem to searching for the optimal reliability levels of individual bridges that minimize the total maintenance cost and retain a predefined network reliability level.

For simple networks which can be decomposed into subsystem in series or in parallel, the network-level optimization has the form of a one-dimensional resource allocation problem, and can be tackled using standard techniques such as dynamic programming. For non-decomposable networks, an approximation method is used to obtain an expression for the network reliability level directly from the minimal cut sets of the network. Then, an algorithm transforms the network-level problem into a BILP.

To illustrate and evaluate the methodology, we have applied it on three different network topologies: a decomposable network (composed of the 7 cross bay bridges in the San Francisco Bay Area), a non-decomposable network (the highway network from Denver to Lafayette), and a virtual network with multiple O/D's. The computational complexity increases linearly with the size of the decomposable network. For non-decomposable networks, the major drawback is that BILP is NP complete and no solution in polynomial time has been found. Problems with moderate size, as considered in this chapter, can be handled by ordinary solvers on a PC (guaranteed to optimality), as opposed to the heuristic algorithms used in related literature.

Chapter 4

Minimizing User Costs

Most real networks in urban areas are redundant and include hundreds, if not thousands of bridges. For these networks, connectivity is irrelevant as there are always alternative paths for vehicles to reach their destination. A more realistic metric is the user costs associated with bridge failures, i.e., the increase in travel time and travel distance caused by bridge failures. Using this performance metric, this chapter presents a solution method for management problems in large networks with many bridges. It is assumed that the network is not congested. Thus, user costs can be accurately reflected by the extra vehicle-miles travelled (VMT) caused by the closure of bridges.

The chapter is organized as follows. Section 4.1 defines the problem and introduces the user costs associated with bridge failures. Section 4.2 proposes and verifies a simplification conjecture. With this conjecture, Section 4.3 presents a Lagrangian decomposition method for the network optimal management. Finally, Section 4.4 summarizes the major findings of this chapter.

4.1 Problem Definition

As in the previous chapter, only bridge deck condition is considered. The deck reliability index $\beta(t|\pi, \vec{a})$ is a stochastic process over time. This deterioration process $\beta(t|\pi, \vec{a})$ is independent across bridges, and correlations in performance across bridges due to the environment and traffic are captured by the parameter vector π .

The bridges in a network are managed by a single agency such as a state department of transportation. The planning horizon is broken into discrete periods of one year, and the maintenance decisions are made every year.

Although bridge failures will not lead to disconnections of nodes in large and redundant networks, they can sever efficient paths connecting origins and destinations, increasing travel cost. Under a budget constraint, the objective of maintenance management is to minimize total increase in travel costs caused by potential bridge failures.

In the absence of congestion, the link user costs and the shortest paths connecting origins and destinations are fixed. The link cost for one trip can be reasonably expressed as a linear combination of the fixed link distance and link time. The cost of a set of failures can be measured by the difference in the travel costs of the users, assuming that they always choose the cheapest paths. This can be measured by calculating the difference in the costs before and after bridge failures for each O/D pair, multiplying each difference by the O/D flow and adding the results for all O/D pairs. This process is necessary because when several bridges fail simultaneously, the cost is not necessarily the sum of costs associated with individual failures.

Figure 4.1 illustrates the point. The alternative routes in both (a) and (b) represent costly paths. For case (a), the cost of a joint failure is much greater than that of a single failure, since vehicles have to take the costly alternative route when both bridges fail. On the other hand, for case (b), the joint failure cost is the same as the cost of a single failure. However, if the two failing bridges are far apart compared to a trip's length, then every O/D pair is affected by at most one bridge. Obviously, in this case, the added cost for two failures is the sum of the added costs for the single failures.

It is shown in the next section that if the failure probabilities are sufficiently low, these close failures are so unlikely that the added total cost can be accurately approximated by the sum of the costs for individual failures.

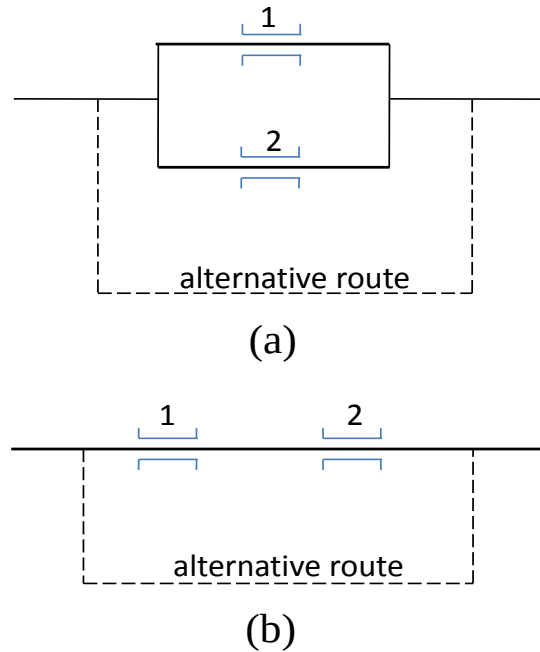


Figure 4.1 The effect of bridge failures on VMT

4.2 Conjecture and Verification

In this section, we show when the increase in cost for an entire network is approximately equal to the sum of the increases caused by individual failures. There are two conditions for this to happen. The first is that the failure rate of bridges is very low, and thus the circumstance that close bridges fail simultaneously is rare. The second condition is that the networks should be redundant; there must be alternative routes available for vehicles to reach their destination even if one or more links are out of service. These two conditions hold for developed regions, where highway facilities are dense and maintained regularly.

The conjecture is now verified. Subsection 4.2.1 proves it analytically for a class of homogeneous networks, and Subsection 4.2.2 verifies it in a real network.

4.2.1 Analytic proof for homogeneous grid networks

We choose idealized grids to prove the conjecture because they can represent very different scenarios by adjusting just a few parameters and can be modeled analytically. First, the features and parameters of the network are described. Then, a formula for the error bounds is presented. It is shown that the approximation error is negligibly small.

Network description and notations

The region under analysis is a square covered by a grid network with M arterials in each direction (north-south and east-west), as shown in Figure 4.2. This grid is overlaid over an infinitely dense grid of slower streets, which are used to access the arterials. The length of an arterial block is L . Bridges are located at intersections; there are m blocks between two neighboring bridges. The bridge failure probability is p , and the failures happen independently. All bridges are in the north-south direction. Thus, when a bridge fails, vehicles on the corresponding north-south arterial cannot go through the bridge, and afterwards vehicles cannot make turning movements at the intersection either, they can travel east-west.

It is assumed that trip origins are uniformly distributed, that all trips have a maximum length of nL , and that for each origin, destinations are uniformly distributed within the origin's reachable area. The demand density is q (veh/m⁴), representing the number of vehicles generated from a unit area and attracted to a unit area in the reachable area.

Vehicles access and egress the network at the arterial intersections closest to their origins and destinations. Because trips normally span many blocks (i.e. n is typically large), only vehicle-miles travelled on the network are considered; the access/egress distances are neglected.

Approximation formula and error bounds

As shown in Figure 4.2, when a bridge fails, vehicles going from area A to area B (or vice versa) need to take neighboring arterials to reach their destinations. These vehicles have to travel to the neighboring streets and then travel back, resulting in an additional distance of $2L$ per vehicle. All other O/D pairs remain unaffected. Thus, the increase in VMT due to the failure, denoted ω , is the product of $2L$ and the total number of vehicles affected by the failures:

$$\text{Number of affected vehicles} = 2 \cdot q \cdot \int_0^{nL} (nL - x)L \cdot Ldx = n^2 L^4 q. \quad (4.1)$$

Thus,

$$\omega = n^2 L^4 q \times 2L = 2n^2 L^5 q. \quad (4.2)$$

Now, use Ω to represent the expected increase in VMT of the entire network and Ω' the expected increase under the conjecture. Since there are M^2/m^2 bridges in the network, we have:

$$\Omega' \approx p \left(\frac{M^2}{m^2} \right) \omega = \frac{2pn^2 L^5 q M^2}{m^2}. \quad (4.3)$$

This formula is approximate because it uses (4.1) and (4.2) for failures close to the boundary, even though the boundary then reduces the number of affected vehicles. The formula is asymptotically exact as $M \rightarrow \infty$ and should be quite accurate for large networks; i.e. when $M/n \gg 1$.

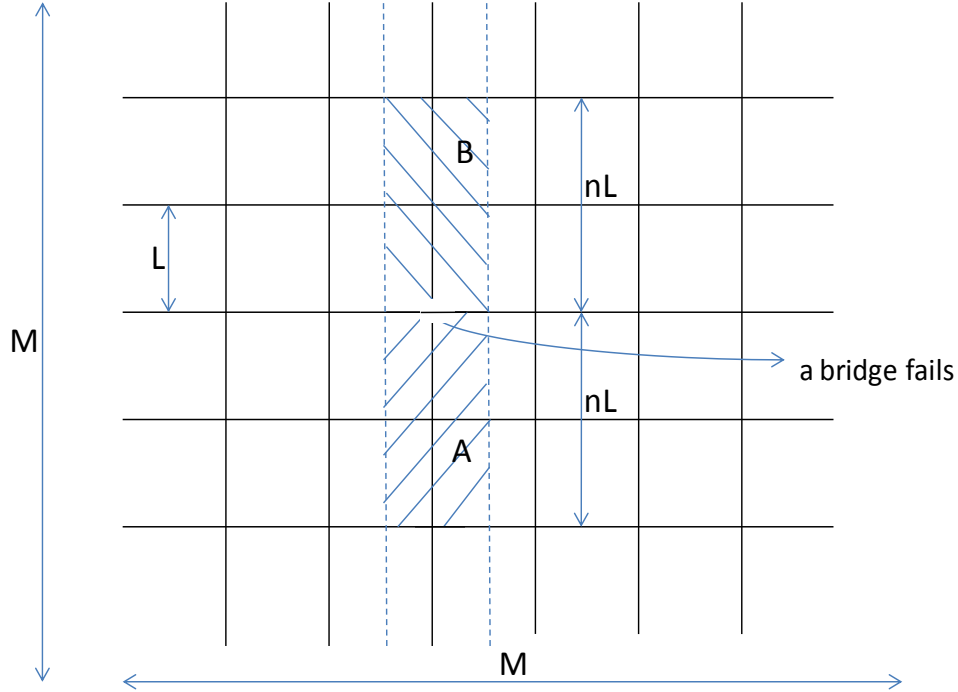


Figure 4.2 Illustration of an individual failure

The approximation error E is defined as the expected value of the absolute difference between the exact increase in vehicle-miles travelled due to simulated failures and the approximate increase derived by summing the increases obtained for each failed bridge in isolation from the others. To assess the error, Appendix B investigates the interaction effect between two close failures, and then derives error bounds for cases involving more failures. Finally, these results are synthesized to obtain an overall error bound.

It turns out that E is negligibly small. To quantify it, let $E' = E/\Omega$ be the relative error. Appendix A shows that $E' \leq B$, where

$$B = np + 13(6np)^2, \text{ if } m = 1 \quad (4.4a)$$

$$B = \frac{np}{3m} + 8\left(\frac{np}{m}\right)^2, \text{ if } m \geq 2 \quad (4.4b)$$

Since the right sides of these equations tend to 0 as $n/m \rightarrow 0$, this establishes that (4.3) is a very good approximation in practical cases, where typically $p \sim 10^{-5}$ and $n/m \sim 10^1$.

Equations (4.4) can be explained qualitatively as follows. For a given bridge failure, there are $O(n/m)$ bridges nearby that can interact with it. If $np/m \ll 1$, it is very unlikely that close failures occur, and therefore, their interactions are negligible. This should also be true if the

network and the demand are inhomogeneous. Thus, the conjecture should also hold for real networks.

4.2.2 Verification in real world network

Although many networks do not have exact grid layouts, they can be obtained by deforming a grid. Therefore, equations (4.4) are now re-expressed in terms of variables that can be measured for real networks. The accuracy of this approximation is then verified using the highway network of the San Francisco Bay Area.

To re-express equations (4.4), we need to replace n and m by equivalent parameters that would be meaningful for an arbitrary network. Since n and m are proxies for “trip length” and “distance between bridges”, let us introduce L_a as the average trip length, and D_a as the average distance between neighboring bridges. In a grid, $n = 3L_a/2L$, and $m = D_a/L$. Thus, (4b) can be rewritten in terms of the more generic parameters L_a and D_a as:

$$B = \frac{L_a p}{2D_a} + 18 \left(\frac{L_a p}{D_a} \right)^2, \text{ if } \frac{D_a}{L} \geq 2 \quad (4.5)$$

Equation (4.4a) is not included because in most cases $D_a \gg L$.

According to the National Bridge Inventory, there are 6,107 bridges in the Bay Area, 549 of which are structurally deficient. Given the total area of the Bay Area, D_a is estimated to be 1.07 miles. The original highway network is adopted from the Metropolitan Transportation Commission (MTC) travel model, and the traffic demand comes from MTC is demand forecasts for year 2010, from 6 a.m. to 10 a.m. on a weekday. Dividing total vehicle-miles travelled by the number of trips, L_a is estimated to be 5.72 miles.

Simulations were carried out to evaluate approximation errors in the real network and in an equivalent grid. The failure probability of deficient bridges was set to be 1.0×10^{-3} , corresponding to a low reliability index of 3. Other bridges have a smaller failure rate of 3.0×10^{-4} , although in reality $p \sim 10^{-5}$. For each of the ten instances generated, the failures were recorded, as was the added user cost. The increase in VMT due to a set of failures was then compared with the sum of the increases in VMT due to single failures to determine the absolute error. These absolute errors were then averaged across the 10 instances to estimate the absolute error, E_s , and the relative error, E_s' .

For the equivalent grid network, $n/m = 3L_a/2D_a \approx 8$. The absolute error, E_g , and the relative error, E_g' , were estimated by averaging 200 instances on a grid with $n = 16$, $m = 2$. The estimated relative errors from simulation, $\widehat{E_s'}$ and $\widehat{E_g'}$, and the error bound from formula (4.5), B , are shown in Table 4.1. The table shows that the analytic bound also holds for the real networks, and more importantly that the results for the real and idealized networks are very close. This supports our claim that the approximation can be used with confidence in real networks.

Mean Relative Error in the Real Network $\widehat{E}_s'$ (%)	Mean Relative Error in the Equivalent Grid $\widehat{E}_g'$ (%)	Error bound B (%)
6.42×10^{-2}	4.96×10^{-2}	1.04×10^{-1}

Table 4.1 Comparison of the mean errors and the error bound

Although the example used a distance metric, similar results would have been obtained with any reasonable linear combination of link distance, time and toll. After all, these changes could be calculated into equivalent distances, which may change the shape of the network; and we have seen from the first two columns of Table 4.1 that these changes have insignificant effects on accuracy.

4.3 Network-Level Optimization

Under the conjecture, the total user cost of the network can be explicitly expressed as the sum of user costs induced by individual bridges. The network problem can then be broken down into smaller subproblems using Lagrangian decomposition. The subproblems can be tackled with dynamic programming. In Subsection 4.3.1, the formulation and methodology are presented. In Subsection 4.3.2, the method is applied to a virtual network of 6,000 bridges as an illustration.

4.3.1 Formulation and Methodology

Lagrangian decomposition

The network-level maintenance management problem solves for the optimal maintenance policy that minimizes the expected increase in user cost due to possible bridge deck failures, subject to a multiyear budget constraint. Many agencies have a yearly budget constraint, but it is reasonable to assume that they are allowed to carry over unused funds in a given year to subsequent years. There are a range of maintenance actions that can be taken, and maintenance decisions are made every year for each bridge deck. The current condition of each deck is assumed to be known perfectly, meaning that inspections are carried out every year and are error free. It is also assumed that agencies are allowed to carry over unused funds in a given year to subsequent years.

There are N bridges in the network, and the planning horizon is T years. The set of decision variables is $\{a_{it}\}$, ($i = 1, 2, \dots, N$; $t = 1, 2, \dots, T$), whose elements are the actions applied on the bridges in each year. The failure probability of bridge i in year t , p_{it} , depends on the initial conditions and the history of maintenance actions. Therefore, p_{it} is a function of time t , initial condition p_{i0} , and the history of maintenance actions $\overrightarrow{a_{it}} = \{a_{i1}, a_{i2}, \dots, a_{it}\}$. Under the conjecture, the increase in user cost in the network is approximately equal to the sum of the increases in costs due to individual failures. The formulation of the optimization problem is then:

$$\begin{aligned}
& \min_{a_{it} \in A} \sum_{i=1}^N \sum_{t=1}^T \Delta_i(t) \cdot p_{it}(p_{i0}, \overrightarrow{a_{it}}) \\
& s. t. \quad \sum_{i=1}^N \sum_{t=1}^T c_i(a_{it}) \leq b
\end{aligned} \tag{4.6}$$

A – set of all possible MR&R actions, including maintenance, repair, and reconstruction activities, and do-nothing;

$\Delta_i(t)$ – increase in user cost due to the failure of bridge deck i in year t ;

$c_i(a_{it})$ – the maintenance cost of action a_{it} on bridge deck i ;

b – total budget.

The Lagrangian of the constrained optimization is:

$$\mathcal{L}(a_{it}, \lambda) = \sum_{i=1}^n \left(\sum_{t=1}^T (\Delta_i(t) \cdot p_i(t, p_{i0}, \overrightarrow{a_{it}}) + \lambda \cdot c_i(a_{it})) \right) - \lambda b \tag{4.7}$$

which is separable, so we can minimize over individual bridge decks separately, given the Lagrangian multiplier λ . For bridge i , the subproblem can be denoted as

$$g_i(\lambda) = \min_{a_{it}} \sum_{t=1}^T (\Delta_i(t) \cdot p_i(t, p_{i0}, \overrightarrow{a_{it}}) + \lambda \cdot c_i(a_{it})) \tag{4.8}$$

The dual problem to the original optimization is: $\max_{\lambda} \sum_{i=1}^n g_i(\lambda) - \lambda b$. The first order condition for maximum is $\sum_{i=1}^n \sum_{t=1}^T c_i(a_{it}) = b$. If we use the bisection method to update λ , a simple algorithm can be obtained:

```

initiate  $\lambda_{min}$  and  $\lambda_{max}$ ;
repeat
   $\lambda = (\lambda_{min} + \lambda_{max})/2$ 
  solve the subproblems (possibly in parallel);
  if  $\sum_{i=1}^n \sum_{t=1}^T c_i(a_{it}) < b$ , update  $\lambda_{max} \leftarrow \lambda$ ;
  if  $\sum_{i=1}^n \sum_{t=1}^T c_i(a_{it}) > b$ , update  $\lambda_{min} \leftarrow \lambda$ .

```

For a given λ , a set of subproblems needs to be solved to obtain $g_i(\lambda)$. Computing time can be shortened by solving subproblems in parallel and updating λ with other efficient algorithms for quicker convergence of the dual problem.

The following section describes a dynamic programming method for the subproblems. The framework is as in Section 3.3.1. The main difference is that user costs are now considered as part of the objective function.

Dynamic programming for subproblem

The subproblem is:

$$\min_{a_{it}} \lambda \cdot \sum_{t=1}^T \left(\frac{1}{\lambda} \Delta_i(t) \cdot p_i(t, p_{i0}, \overrightarrow{a_{it}}) + c_i(a_{it}) \right) \quad (4.9)$$

The first term in the brackets can be interpreted as the user cost for a given deck condition, and then the problem becomes a typical facility-level optimization. There are different approaches for solving the maintenance management problem for a single bridge. Here, we solve it as a Markov decision process with augmented states (Robelin and Madanat, 2007), as it uses standard and efficient optimization techniques and retains the effects of the history of deterioration and maintenance as part of the model. The state of a bridge deck is represented by a three-dimensional vector $x = (\beta, u, \tau)$, where β is the reliability index, u represents the type of the latest action performed on the bridge, and τ represents the time since the latest action. Given the state of the bridge deck, the failure probability p_i is equal to $\Phi(-\beta)$. The Bellman recursion is:

$$\forall x \in X, \quad V_t(x) = \min_{a_{it} \in A} \left\{ \frac{1}{\lambda} \cdot \Delta_i(t) \cdot p_i(x) + c_i(a_{it}) + \sum_{y \in X} P_i(y|x, a_{it}) V_{t+1}(y) \right\}, \quad (4.10)$$

for $t \in \{0, 1, \dots, T\}$.

$$V_{T+1}(x) = 0$$

- X : state space of the Markov Chain;
- $p_i(x)$: the probability of failure given that the current state is x .
- $V_t(x)$: minimum cost-to-go for the agency to manage a bridge deck currently in state x from year t to the end of the planning horizon.
- $P_i(y|x, a_{it})$: transition probability from state x to state y in the next year, given action a_{it} is applied.

4.3.2 Numerical example

The numerical study is implemented on a network with 6,000 bridges. The planning horizon is 60 years. To represent heterogeneity in the network, we randomly divide the bridges into 10 homogeneous groups. The transition probabilities matrices are obtained by Monte Carlo simulation, based on the stochastic lifetime profile of the reliability index from Frangopol et al. (2001). Details of this Monte Carlo simulation process used are provided in Robelin and Madanat (2007). The costs of the maintenance actions are adopted from Kong and Frangopol (2003). Our example uses a distance-based metric, and the increase in VMT caused by a bridge failure is estimated based on the traffic demand and the transportation network of the San Francisco Bay Area. To capture the heterogeneity across different groups, the parameters provided in these papers are adjusted within realistic ranges.

The algorithm is implemented to solve the optimization for different budgets. Figure 3 plots for different budget levels, the inverse of λ , which represents the extra budgetary dollars required to save one additional vehicle-mile of user “cost” (\$/veh-mile). As expected, as the budget increases, the cost of saving an additional vehicle-mile increases, meaning that the better the system, the more costly it is to find extra savings.

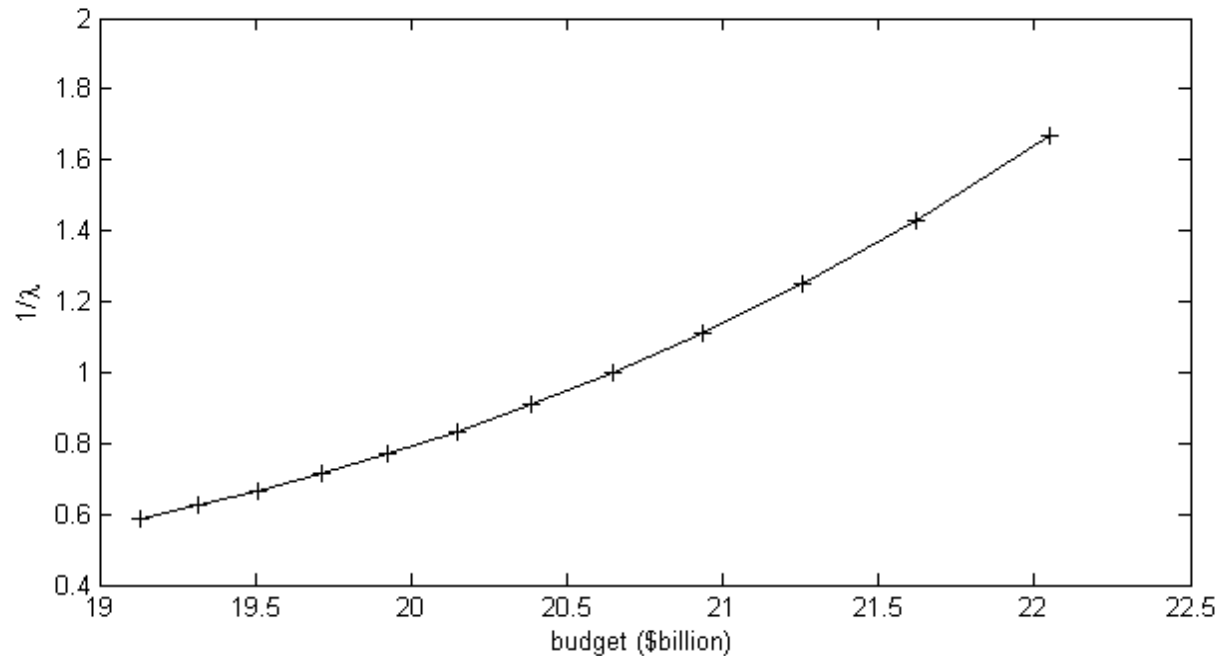


Figure 4.3 Numerical implementation for 6,000 bridges

Figure 4.3 also shows that the budget required to achieve optimality depends on how much the highway agency values one vehicle-mile of user cost. In our example, if this monetary value is \$1, then the budget required for the network’s lifecycle maintenance is around \$20.6 billion.

For a prescribed value of user costs, the method can yield the optimal maintenance policies for individual bridges. With $1/\lambda$ set to be \$1/veh-mile, Figure 4.4 plots the expected amount of budget consumed by the three different maintenance actions at each year during the lifecycle. In our example, most of the construction actions are conducted between year 15 to year 40, and most of the repairs are conducted in the later period. This type of analysis can help agencies plan the maintenance budget over the lifecycle of bridge networks.

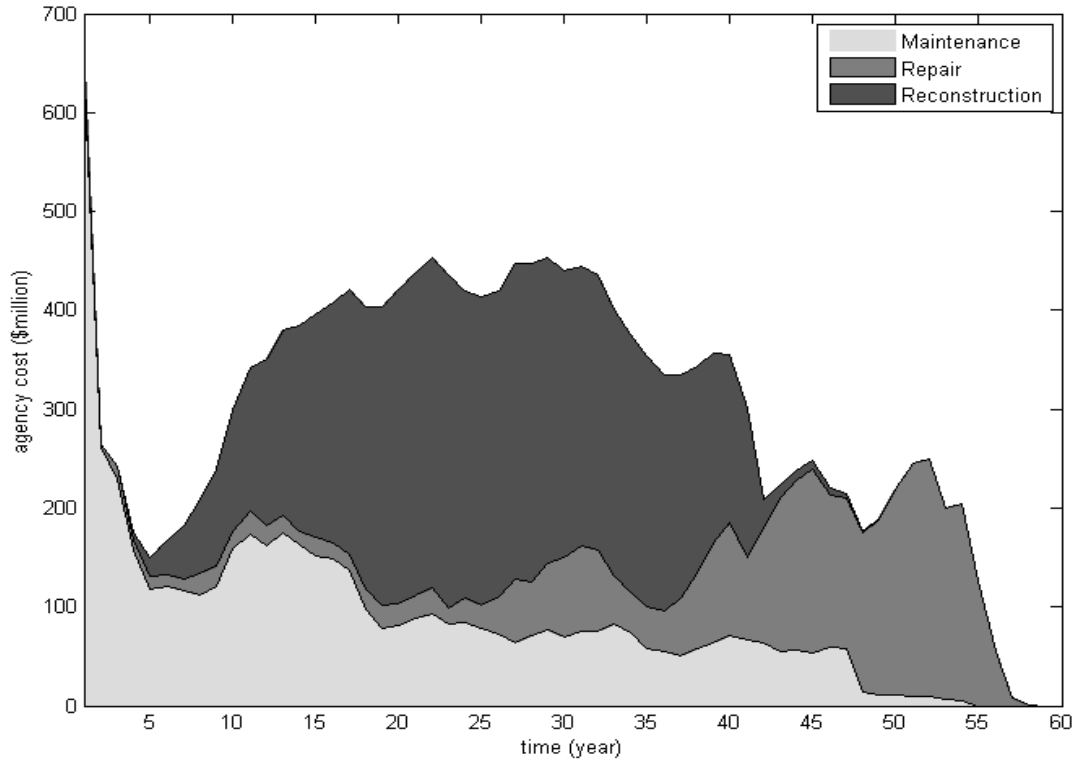


Figure 4.4 Lifecycle budget allocated on different maintenance actions

The solution time increases linearly with the size of the network. Assume it takes τ seconds to solve one subproblem. This is independent of N . To update λ , N subproblems should be solved, which takes $N\tau$ seconds, where N is the number of bridges in the network. If the error allowed for λ is 10^{-3} , the number of iterations needed is $\log_2 10^3(\lambda_{max} - \lambda_{min})$. Therefore, the solution time is $N\tau \log_2 10^3(\lambda_{max} - \lambda_{min})$, which is proportional to N as claimed.

4.4 Summary

This chapter shows how to find the optimal maintenance plans for deteriorating bridges in large-scale networks. The objective is to minimize the expected increase in user costs caused by potential bridge failures over a planning horizon, under a budget constraint. The main contributions are:

- The overall user cost is measured directly by the increase in travel time and distance, which, for large networks, are more relevant than connectivity or other metrics used in past research.
- It is shown that for uncongested networks our approximation is valid. As a result, the network-level optimization problem can be decomposed.

- An efficient approach is developed, capable of optimizing the maintenance activities of large networks with thousands of deteriorating bridges over multiple periods.

These contributions pertain to uncongested networks. They apply because bridges separated by large distance do not interact. In the real world, there can be congested links, especially in urban areas during peak hours. Although congestion can enlarge the critical distance above which two bridges do not interact, the enlargement should be small for moderately congested networks. (Unless a grid-like network is severely congested everywhere, the elimination of a link in this kind of a network does not appreciably change the congestion level of far-away links; see Newell (1993) for an analysis.) Thus, bound (4.4) should still hold in moderately congested networks, if one uses the congested link costs of the basic network (without failures) as the basis for evaluating the $\Delta_i(t)$ of the proposed methodology. If congestion is more severe, (4.4) can be violated, but it should be remembered that the bound is very tight. Thus, the method proposed in this paper should yield reasonable results even under typical congestion levels found in real cities. Since, to date, there are no alternative solution methods for large networks, the proposed methodology can serve as a benchmark against which improved methods can be compared.

Chapter 5

Conclusions

This chapter concludes the dissertation. Section 5.1 summarizes the contributions of this research. Section 5.2 points out several possible directions for future work.

5.1 Contributions

This dissertation addresses the problem of optimizing maintenance, repair and reconstruction decisions for deteriorating bridges in highway networks. The bridge deck condition is measured by its reliability index, and its deterioration process is modeled as Markovian process with augmented states. A comprehensive framework is developed to help agencies make optimal maintenance plans that best balance the overall network performance and the total maintenance cost.

Instead of using a uniform criterion, suitable metrics are chosen for different types of networks. For moderate size networks with limited redundancy, the network performance is quantified by a graph-theoretic indicator of network connectivity. For large and redundant networks, the network performance is measured directly by the total user costs associated with potential bridge. Therefore, the research problems are twofold: retaining network connectivity in moderate-size networks and minimizing total user costs in large networks.

The first part of the research focuses on moderate-size networks with limited redundancy. The problem is formulated as determining the optimal maintenance policies that minimize the total agency cost while retaining a prescribed network reliability level. The major contributions in this part of research include:

- An approximation of network reliability is obtained for non-decomposable networks of moderate sizes that is a function of individual bridge reliabilities.
- A novel two-stage approach is developed to tackle the connectivity problem, which makes it possible to solve the problem by using standard optimization tools (with guaranteed convergence to optimality), as opposed to the heuristic algorithms used in related literature.
- Different types of network topologies are considered, including decomposable and non-decomposable networks, single-O/D-pair and multiple-O/D-pair networks.

The second part of the research focuses on large and redundant networks. A simple method is developed to solve for the optimal policies that minimize the total user costs of the network under a budget constraint. The major contributions in this part of research are listed as follows:

- The overall user cost is measured directly by the increase in travel time and distance, which are more relevant for these networks than connectivity or other metrics used in past research.
- It is verified that the expected increase in user costs due to bridge failures can be approximated by the sum of increases due to individual failures, if a network is uncongested.
- An efficient solution method is developed, capable of solving MR&R optimization problems for large networks with thousands of deteriorating bridges over multiple periods.

Aside from these contributions in the methodology aspects, this research also produces practical benefits. Using the framework presented in this dissertation, analysis can be carried out on the resource allocation in the optimal solution, which can help the agencies plan the maintenance budget over the lifecycle and over the bridges in the network.

5.2 Future Work

The directions for future research include the following:

- For networks with multiple O/D pairs, the connectivity is inversely proportional to the network failure probability, i.e., the probability that any O/D pair is disconnected. Using this indicator, all the O/D pairs are equivalent in the formulation. However, some O/D pairs may have larger social or economic impact than others, and their connection may be more important. This consideration should be included in the future research.
- Our assumption that the expected increase in user costs due to bridge failures can be approximated by the sum of increases due to individual failures is validated for uncongested networks. It holds because bridges separated by large distance do not interact. In the real world, there may be congested links, especially in urban areas during peak hours. Congestion may enlarge the critical distance above which two bridges do not interact, and the error bound may not hold for severely congested networks. Although the method is expected to yield reasonable results even under typical congestion levels found in real cities, future research should investigate the impact of congestion in more detail and develop improved methods.
- The agency costs are computed as the total discounted maintenance cost of the network over multiple periods. In the numerical examples, the planning horizons are set to be long. The optimization problems produce Pareto frontiers of performance indicators and total agency cost. This type of analysis can help agency plan the budget for the lifecycle maintenance of networks. In practice, agencies usually have yearly budget constraint. To find the optimal policies that fit the yearly budget constraint can be a direction for future research.

- The network-level management problem seeks to achieve satisfactory overall network performance. The conditions of individual bridges are not considered separately. In practice, because bridge collapse can cause great economic loss and threaten the safety of its users, the bridge management should constrain the condition of individual bridges.
- Aside from bridge failures, maintenance actions may result in partial or fully closure of bridge lanes, and cause delays and detours to the network users. Future research should include user costs due to maintenance actions.

Appendix A. Pseudocode for the network-level optimization

```
1  Define NetProbability (predefined threshold of network failure probability).
2  Define Z [bridgeID, thresholdID] (decision variable).
3  Define CutSize (set of cut sizes, initially empty).
4  Input OptCost [bridgeID, thresholdID] (the optimal cost from facility-level optimization).
5  Input ThresholdValue [thresholdID] (threshold of probability of failure in facility-level problem).
6  Objective function:
   sum Z [bridgeID, thresholdID]* OptCost [bridgeID, thresholdID] over bridgeID and thresholdID
7  Equality Constraint:
8  FOR each bridgeID
9      Sum Z [bridgeID, thresholdID] over thresholdID = 1
10 END FOR
11 FOR each minimal cut set
12     IF the size of minimal cut  $\notin$  CutSize THEN
13         Add the size of minimal cut to CutSize.
14     END IF
15 END FOR
16 FOR each cutsize in CutSize
17     Build up a cutsize-dimensional matrix ActiveProb.cutsize:
       ActiveProb.cutsize [i1,i2,...,icutsz] = ThresholdValue [i1]* ThresholdValue [i1]*...
       *ThresholdValue[icutsz].
18     Index.cutsize = Find (ActiveProb.cutsize > NetProbability)
       (return the coordinates of the elements in ActiveProb.cutsize with values greater than NetProbability)
19 END FOR
20 Inequality constraints
21 FOR each minimal cut set
22     currentSize = size of the minimal cut set;
23     FOR each row of Index.currentSize
24         Add constraint:
           sum Z [currentcut[i], Index.currentSize[row,i]] over i (from 1 to currentSize)  $\leq$  currentSize - 1.
25     END FOR
26 END FOR
27 CALL BILP solver; return solution Z and currentCost.
28 current network failure probability = function of Z.
29 WHILE current failure probability > NetProbability
30     Add constraint:
       sum Z [bridgeID, thresholdID]* OptCost [bridgeID, thresholdID] over bridgeID and thresholdID > =
       currentCost + unit cost.
31     CALL BILP solver; return solution vector Z and currentCost.
32     current network failure probability = function of Z.
33 END WHILE
```

Appendix B. Upper bounds for the approximation errors

(1) Approximation errors with two close failures

Bridges are labeled by their coordinates in the network; i.e., bridge ij represents the bridge on the i th row and j th column. Then, the event of a specific pair of failures can be denoted as (i_1j_1, i_2j_2) , representing the failures of bridges i_1j_1, i_2j_2 and no other failures interacting with them. Therefore, the probability of the occurrence of event (i_1j_1, i_2j_2) , denoted as $p_{i_1j_1, i_2j_2}$, should be smaller than p^2 , where p is the failure probability of a single bridge.

Let $\delta_{i_1j_1, i_2j_2}$ denote the interacting effect of the two failures, and it is equal to the difference between the actual increase in VMT when both bridges fail and the sum of the increments due to two single failures. Use E_2 to represent the mean absolute error caused by two close failures, then $E_2 = \sum_{(i_1j_1, i_2j_2)} (p_{i_1j_1, i_2j_2} \cdot |\delta_{i_1j_1, i_2j_2}|)$. To obtain an analytical expression of E_2 , we first analyze the interacting effect between any two specific failures, $\delta_{i_1j_1, i_2j_2}$. Interaction exists if and only if these two failures are on the same or neighboring columns; otherwise, their impacts on VMT are independent.

Case 1: the two failures are on the same column j

Result 1. $\delta_{i_1j, i_2j} = -2(n - |i_1 - i_2|)^2 L^5 q$, $|i_1 - i_2| < n$

Proof. Let $k = |i_1 - i_2|$, which is the number of blocks between the two failures. As shown in Figure A1, the vehicles generated in area A/B with destinations in B/A are only interrupted once, when both bridges fail. Thus, the actual increase in VMT should be smaller than the sum increments of VMT caused by individual failures. The number of vehicles generated in A/B and with destinations in B/A can be calculated as:

$$q \cdot \int_0^{(n-k)L} Lx \cdot Ldx = \frac{1}{2}(n-k)^2 L^4 q$$

To bypass the unserviceable joints, affected vehicles need to travel to the neighboring corridors and then travel back, which results in an additional length of $2L$ per vehicle. Therefore, we have $\delta_{i_1j, i_2j} = -2 \cdot \frac{1}{2}(n-k)^2 L^4 q \cdot 2L = -2(n-k)^2 L^5 q$. $\square$

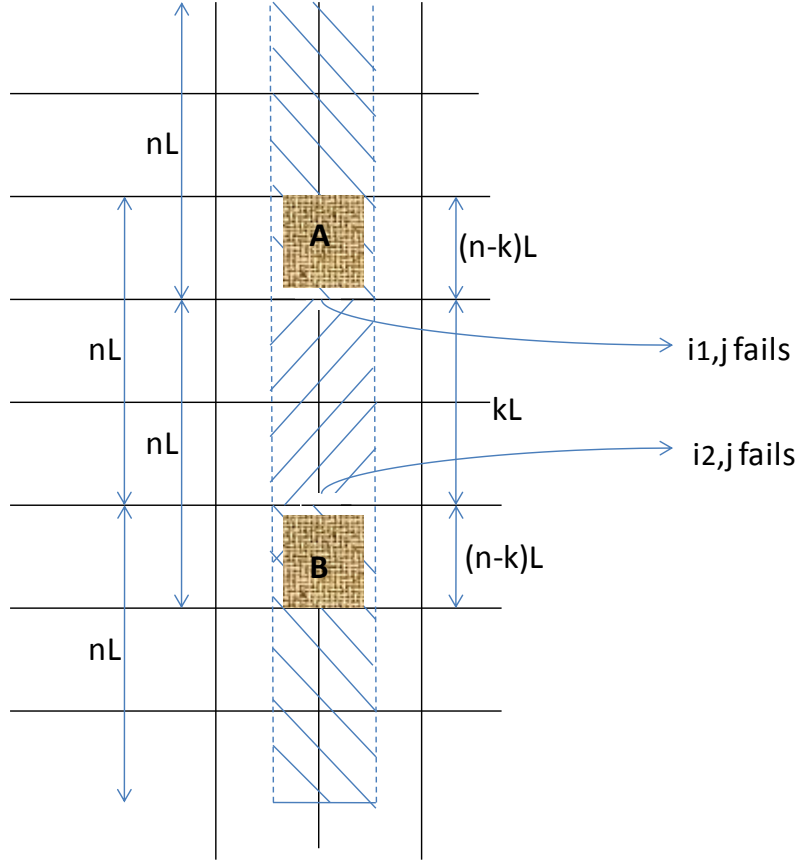


Figure B1 – Interaction between failures on the same column

Case 2: the two failures are on two neighboring columns

Result 2. $\delta_{i_1 j_1, i_2 j_2} = \begin{cases} 4(n-1)^2 L^5 q, & \text{if } |i_1 - i_2| \leq 1 \\ 2(n - |i_1 - i_2| - 1)^2 L^5 q, & \text{if } |i_1 - i_2| > 1 \end{cases}$

Proof. The actual increase VMT is greater than the sum of increments due to individual failures, since the crossing vehicles are also impacted. If the two failures are also on the same or the neighboring rows, vehicles with OD pairs of A1-B2 and A2-B1 need to reroute to reach their destinations, as shown in Figure A2. The number of vehicles generated in A1 with destinations in B2 can be calculated as:

$$q \cdot \int_0^{(n-1)L} L \cdot ((n-1)L - x) L \cdot dx = \frac{1}{2} (n-1)^2 L^4 q$$

Each vehicles need to travel 2L more distance. Thus,

$$\delta_{i_1 j_1, i_2 j_2} = 4 \cdot \frac{1}{2} (n-1)^2 L^4 q \cdot 2L = 4(n-1)^2 L^5 q$$

Note that although the above formula is derived for cases when $i_1 = i_2$, we use the same formula for cases when $|i_1 - i_2| = 1$. There is a small change in the number of affected vehicles in the latter case, but it is neglected for simplicity.

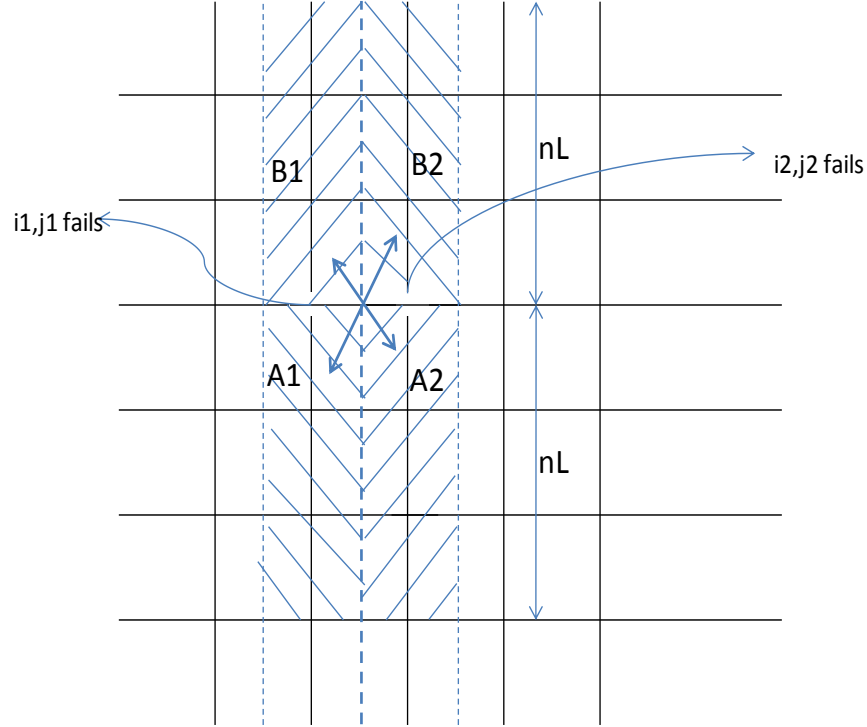


Figure B2 – Interaction between failures on neighboring columns

If there is at least one row between the two failures, the vehicles with OD pair of A2-B1 can take the row between the failures, and they do not need to travel additional miles. The number of vehicles generated in A1 with destinations in B2 can be calculated as:

$$q \cdot \int_0^{(n-1-k)L} L \cdot ((n-1-k)L - x) L \cdot dx = \frac{1}{2} (n-k-1)^2 L^4 q$$

where $k = |i_1 - i_2|$. Thus the excess due to interaction is:

$$\delta_{i_1 j_1, i_2 j_2} = 2 \cdot \frac{1}{2} (n-k-1)^2 L^4 q \cdot 2L = 2(n-k-1)^2 L^5 q \quad \square$$

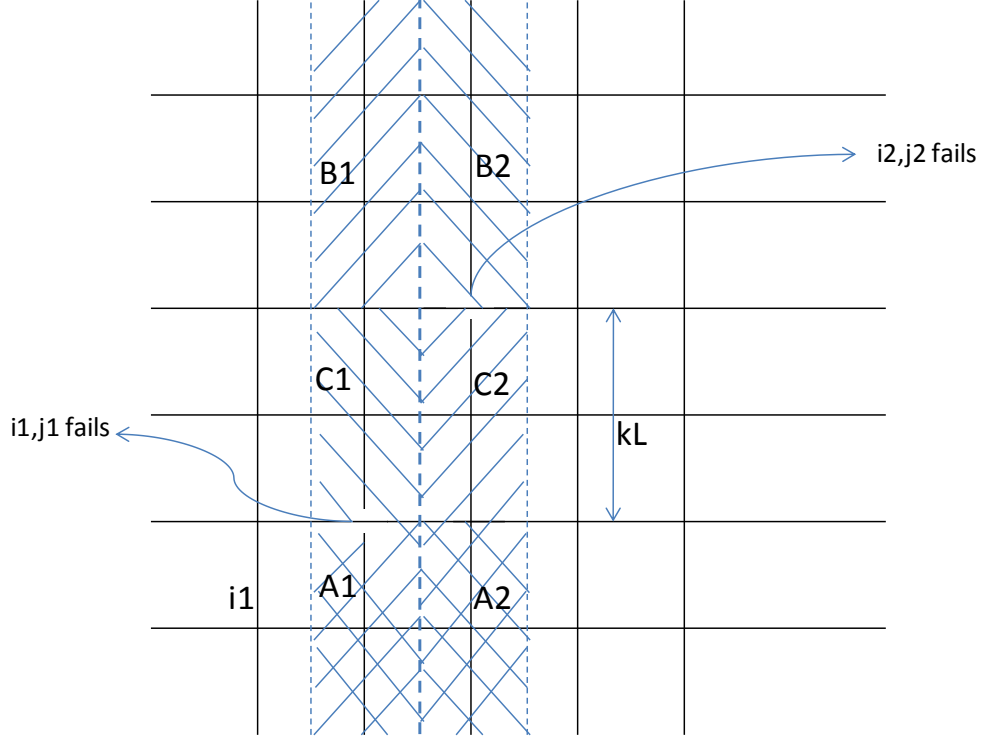


Figure B3 – Interaction between failures on neighboring columns

If $m = 1$, indicating that there is a bridge at each intersection, both of the two cases described above need to be accounted for. If $m \geq 2$, only the interaction in case 1 need to be considered, since bridges cannot be on the neighboring corridors. The relative error $E'_r = E_r/\Omega$, where Ω is the expected increase in VMT of the entire network under the conjecture.

Result 3a. $E'_2 \leq np$, when $m = 1$

Result 3b. $E'_2 \leq np/3m$, when $m \geq 2$

Proof. Recall that $E_2 = \sum_{(i_1j_1, i_2j_2)} (p_{i_1j_1, i_2j_2} \cdot \delta_{i_1j_1, i_2j_2})$ and $p_{i_1j_1, i_2j_2} \leq p^2$. Therefore, we have $E_2 \leq p^2 (\sum_{(i_1j_1, i_2j_2)} \delta_{i_1j_1, i_2j_2})$. Based on the expressions for the approximation error δ provided in **Result 1** and **Result 2**, the expression of the sum can be derived with algebra.

When $m = 1$, $E_2 \leq (2n^3 + n^2 + n - 8)p^2L^5qM^2$. Then, the relative error $E'_2 = E_2/\Omega \leq p(n + \frac{1}{2} + \frac{1}{2n} - \frac{4}{n^2}) \approx np$. When $m \geq 2$, $E_2 \leq \frac{1}{3}(\frac{2n^3}{m^3} + \frac{n}{m} - 3\frac{n^2}{m^2})M^2L^5qp^2$. The relative error $E'_2 = \frac{E_r}{\Omega} \leq \left(\left(\frac{m}{n} - \frac{3}{2}\right)^2 - \frac{1}{4}\right)\frac{np}{6m} \leq \frac{np}{3m}$. $\square$

(2) Approximation error with r ($r \geq 3$) close failures

Let E_r ($r \geq 3$) denote the mean absolute error due to r close failures, and $E'_r, E'_r = E_r/\Omega$, denote the relative error.

Based on the analysis in Appendix A, two failures should be geographically close for them to interact mutually; they should be on the same or neighboring columns and within a distance of nL . In other words, when a bridge ij fails, it only interacts with failures within a certain area, which is defined as the *interacting area* of bridge ij . Similarly, when r close failures happen, there is an interacting area such that the failures outside this area have no interaction with the r failures. We use N_r to denote the number of bridges in the interacting area of r close failures. The number of bridges in the interacting area of a single failure can be counted: when $m = 1$, $N_1 = 6n - 2$; when $m = 2$, $N_1 = 2n/m$. For $r \geq 2$, the interacting area of the r close failures is the union of that of individual failures. The number of bridges in the interacting area is not a constant; it depends on the specific combinations of the r close bridges that fail. However, the bounds for N_r can be obtained. First, the interacting area of r failures should be larger than that of a single failure, and therefore, $N_r \geq N_1$. Besides, because the interacting areas of the individual failures must have some overlaps for them to be “close”, an upper bound N_r^u for N_r is rN_1 .

Use b_r to denote a specific combination of r close failures. The set $B_r = \{b_r\}$, is the set of all specific combinations of r close failures. The occurrence of b_r requires that the r specific bridges fail and that the bridges in their interacting area are in service. Therefore, the probability that b_r occurs is equal to $p^r(1-p)^{N_r}$. Use $\delta(b_r)$ to denote the error encountered given that b_r happens. The formula for E_r is:

$$E_r = \sum_{b_r \in B_r} p^r(1-p)^{N_r} \cdot |\delta(b_r)| \leq \sum_{b_r \in B_r} p^r |\delta(b_r)|$$

Result 4a. $E'_r \leq (6np)^{r-1}r^2$, when $m = 1, r \geq 3$.

Result 4b. $E'_r \leq 2(2np/m)^{r-1}$, when $m \geq 2, r \geq 3$.

Proof. C_r is the number of all specific combinations of r close failures, and ξ_r is the upper bound of $|\delta(b_r)|$. Then, an upper bound for the mean absolute error caused by r close failures is $C_r p^r \xi_r$.

There are M^2/m^2 bridges in the network. We can think it as a selection process to find the number of different combinations of r close failures. For the first failure, we arbitrarily choose one from the M^2/m^2 bridges. The second failure should be within the interacting area of the first failure, and there are N_1 options. Similarly, there are N_2 options for the third failure, N_3 options for the fourth, etc. Thus, the number of different selections is $M^2 N_1 N_2 \dots N_{r-1}/m^2$. Because the order of the failures should not be considered in the combination, $C_r = M^2 N_1 N_2 \dots N_{r-1}/(m^2 \cdot r!)$. Recall that the upper bound of N_r is rN_1 . Then, $C_r \leq \frac{M^2 \cdot (N_1)^{r-1}}{rm^2}$.

Next, we analyze the bound of $\delta(b_r)$. When $m = 1$, an upper bound for the number of affected vehicles is $2 \times rnL^2 \times rnL^2 \times q = 2qr^2n^2L^4$. To bypass the r failures, vehicles need to travel rL miles at most. Therefore, $\xi_r = 2qr^2n^2L^4 \times rL = 2qr^3n^2L^5$. If $m \geq 2$, the interaction is among failures on the same column. The number of affected vehicles is no greater than $2 \times nL^2 \times nL^2 \times q \times r = 2qrn^2L^4$, and $\xi_r = 4qrn^2L^5$.

Recall that $N_1 = 6n - 2$ when $m = 1$, and $N_1 = 2n/m$ when $m \geq 2$. By the formula $E_r \leq C_r p^r \xi_r$, the approximation error $E_r \leq 2 \cdot 6^{r-1} \cdot r^2 (np)^r n M^2 q L^5$, and the relative error $E'_r = E_r / \Omega \leq (6np)^{r-1} r^2$, if $m = 1$. The approximation error $E_r \leq 4M^2 (pN_1)^{r-1} p n^2 L^5 q / m^2$. The relative error $E'_r \leq 2(2np/m)^{r-1}$, if $m \geq 2$. $\square$

(3) Total approximation error

Result 5. An upper bound for the total relative error is:

$$B = \begin{cases} np + 13(6np)^2, & \text{when } m = 1 ; \\ np/3m + 8(np/m)^2, & \text{when } m \geq 2. \end{cases}$$

Proof. The approximation error should be smaller than $E'_2 + \sum_{r=3}^{\infty} E'_r$. Based on **Result 3a, 3b** and **Result 4a, 4b**, when $m = 1$, the upper bound is:

$$np + \sum_{r=3}^{\infty} (6np)^{r-1} r^2 = np + 13(6np)^2$$

When $m \geq 2$, the upper bound is:

$$np/3m + \sum_{r=3}^{\infty} 2(2np/m)^{r-1} = np/3m + 8(np/m)^2 . \square$$

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