Title
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Publication Date
1984

Peer reviewed

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Research Papers in Economics No. 84-4

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This paper has benefited from discussions with Avinash Dixit, Curtis Eaton, Drew Fudenberg, Steven Salant, and from comments by an anonymous referee. We gratefully acknowledge financial support from the National Science Foundation (SES-8207925) and the University of California at Berkeley.

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In markets with increasing returns to scale in investment, competition will occur over both the amount and the timing of new capital construction. This paper develops a theory of competition in markets with indivisible and irreversible investments. The consequences of competition depend on the strategies and information available to the competitors. If firms act as Nash competitors with binding contracts, revenues will exceed costs for any number of firms and otherwise identical firms will earn different profits. In the absence of binding contracts, competition over the timing of investment can completely dissipate profits in a sub-game perfect equilibrium with two or more firms.

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1. INTRODUCTION

Scale economies play an important role in the theory of industrial organization as a determinant of industry structure, conduct and performance. Joe Bain's pioneering study of the structure of U.S. industry identified scale economies at the plant level that are substantial in many industries, but rarely large enough to explain observed measures of firm concentration.[1] Since Bain published his findings, most of the theoretical and empirical literature has concentrated on a static concept of scale economies and ignored the crucial distinction between economies of scale in the short versus long run. This paper is concerned with competition in an industry in which short run scale economies exist, but in a long run sense there are constant returns to scale. Investment in capacity is indivisible or "lumpy", which implies that over the short run, defined as a period in which capacity is fixed, average costs fall as output increases up to a capacity limit; however, the "envelope" of all the short run cost curves is an approximately flat long run average cost curve. This type of cost structure is relevant for many industries, even some we might refer to as competitive.

The industry examined in this paper faces a growing demand for a homogeneous product. At any moment of time the next increment to industry capacity should be provided efficiently by only one firm, yet many firms are competing for the next investment. This leads to what Richardson (1960) has termed the "nomination problem"--how do firms resolve who is to carry out the investment? Another aspect of the investment process is that new capacity is long lived and firm specific.[2] The irreversibility of capital implies that the market is not "contestable" as defined by Baumol (1982) because there is no freedom of exit. Also, preemptive investment may be a credible threat, and this can have important consequences for the nature of competition.

No discussion of industrial evolution can be independent of behavioral assumptions that determine the nature of firm interactions. This paper considers two different equilibrium concepts. Both are Nash, but differ in the type of strategies used by firms. The first is an oligopoly where each firm chooses a sequence of investment dates, taking the sequence of competitors' investments as given and independent of its actions. An implicit assumption is that firms write binding contracts for the construction of new capacity. This equilibrium concept appears frequently in discussions of "dynamic" games where each agent takes the time paths of other agents' actions as given, and it is implicit in "open-loop" control-theoretic models of competition. Since agents do not choose actions that depend on the actual history of the game, this equilibrium concept is equivalent to a one-shot Cournot game with the actions defined over functions of time.

A characteristic of the Cournot-Nash game is that identical firms typically earn different profits. While the Cournot-Nash equilibrium concept is appropriate in games where agents can precommit to actions that do not change over time, the absence of rent equalization suggests difficulty with maintaining such an outcome. The second equilibrium concept deals directly with the rent equalization problem. In this game, agents form strategies that depend on the history of the game and provide for investment whenever a new plant can earn positive profits. Any firm can invest and upset the plans of any other firm, a tactic which avoids unequal profits by otherwise identical firms. We refer to this game as a preemptive equilibrium. Preemption in this paper refers to the use of anticipatory investment strategies by competing firms. Rivals are aware of opportunities for profitable investments and know that these opportunities can be lost to others. This leads to aggressive competition in new plant construction.[3]

The two equilibrium concepts illustrate the importance of behavior in industry dynamics. In the Cournot-Nash oligopoly, firms' profits are always positive given any number of competitors. Profits among firms differ, with higher profits going to the firms that invest earlier. The pattern of industry investment is unique and displays the characteristic that small firms grow faster. The welfare implications of the Cournot-Nash equilibria are unambiguous and lie between monopoly and a surplus-maximizing investment program.

The preemption equilibria are characterized by zero profits earned on every new investment. This outcome, which resembles Bertrand competition, can occur with only two firms. Formally, the equilibria are Perfect Nash equilibria in suitably defined history-contingent strategies. The welfare characteristics of these equilibria are ambiguous. It is possible, for example, that investment may occur too soon or too late relative to surplus-maximizing investment sequences. The divergent outcomes of the two models underscore the importance of a behavioral and dynamic theory of industrial organization.[4]

2. COMPETITION IN INVESTMENT PLANS

The basic model used throughout the paper is a stylized representation of an industry facing growing demand with indivisibilities in installing new capacity. The technological side of the model is similar to that developed by Manne et al (1967) in the planning literature.

Time is treated as continuous and indexed with t; $t=0$ will denote the initial date. The (inverse) demand curve facing the industry is $P(Q,t)$, a continuously differentiable function with Q denoting total output. Assume

$$\text{(A.1)} \qquad \frac{\partial P(Q,t)}{\partial t} > 0, \ \frac{\partial P(Q,t)}{\partial Q} < 0, \text{ and } P(Q,t) > 0 \text{ for all } (Q,t).$$

Any firm in the industry has access to the same technology which consists simply of a fixed cost C, (with the dimension of a stock), for installing one "plant". A plant is infinitely durable, is operated with no variable costs, and has a capacity of one unit of output.[5] While these assumptions are extreme, they define a basic model that is simple, yet rich enough to treat most of the problems that interest us.

The dynamic nature of investment competition allows firms to signal threats and make commitments that influence the behavior of their competitors. These actions permit a range of equilibrium outcomes. Nonetheless, a logical beginning for the study of investment competition is the dynamic analogy to Cournot competition, in which each firm takes the investment plans as well as the outputs of competitors as given.

The game assumed in this section is defined as follows. There are a finite number of firms, $j=1,...,J$. At date $t = 0$, each firm announces a sequence $s^j = \langle t_n^j \rangle$, which specifies the dates at which firm j will build its plants. The profit functions are defined in the usual way given the J-tuple of strategies $(s^1,\ldots,s^J)$ and the outputs of all other firms. Each firm chooses an investment strategy and outputs that are profit-maximizing taking as given the investments and outputs of all competitors. The competition as defined, although it includes time in the strategy choices, is actually a single play Cournot-Nash (C-N) game. At the initial date, firms announce their investment and output sequences and a C-N equilibrium exists if each sequence is profit-maximizing given the sequences determined by competitors. After firms announce their investments and outputs, there are no opportunities to revise their plans as time unfolds.

This formulation is restrictive in that firms' decisions do not depend on the history of the industry. History-dependent strategies are introduced in the next section. The game described in this section would be an accurate description of competition if law or technology requires that firms make binding contracts for investment and production plans. Many "dynamic" models in the literature are in fact one-shot Cournot-Nash games. These include control-theoretic models where agents determine the time paths of their actions at a single date, with no opportunity for revision (the "open-loop" assumption in control theory).

Consider first a duopoly where the firms have perfect foresight and full knowledge of their opponents' pay-offs and assume

$$\text{(A.2)} \qquad \frac{\partial}{\partial Q}\,[QP(Q,t)] > 0 \qquad \text{for all } (Q,t)\ .$$

This assumption of everywhere positive marginal revenue excludes excess capacity from being optimal given Cournot behavior. Then firm 1's best response to s^2 is given by the solution to

$$(1) \qquad \max_{s^1} \Pi^1(s^1,s^2) = \sum_{n=0}^{\infty} \int_{t_n}^{t_{n+1}} nP(n+m(\tau),\tau)e^{-r\tau} d\tau - Ce^{-rt_n}$$

where $m(\tau)$ is the capacity of firm 2, equal to its output by the no-excess capacity assumption, (A.2). Here t_n is the date at which firm 1 builds its n^{th} plant. Although the summation in (1) extends to infinity, no firm is forced to build an arbitrarily large number of plants. The last plant built is the largest n for which t_n is finite.

If firm 1 does not invest at the same date as firm 2 and if t_n is finite, a necessary condition for firm 1's best response to s^2 is that, for firm 1's n^{th} plant,

$$nP(n+m,t_n) - (n-1)P(n-1+m,t_n) = rC.$$

<u>Lemma 1</u>: Except for the initial date $t = 0$, firm i's best response to the investment sequence $\langle t_m \rangle$ of firm $j \neq i$ will never involve a sequence $\langle t_n \rangle$, where $t_n = t_m$ for any (n,m).

The marginal revenue earned by firm i would increase discontinuously by advancing the construction date before any date at which firm j invests. Thus for $t > 0$, along an equilibrium path $t_n = t_m$ could not be optimal. Firms may invest simultaneously at the initial date if the initial industry capacity is more than one plant short of the equilibrium level.

The same argument implies that firms will never build more than one plant at a time, with the possible exception of the initial date. If an equilibrium exists in the C-N game, firms necessarily will "space" their investments.

Define the incremental revenue flow to a firm from plant n when industry capacity immediately before the investment is $n + m - 1$ as

$$\Delta_N(n,k,t) = nP(k,t) - (n-1)P(k-1,t)$$

where $k = n + m$. The subscript N represents Nash behavior. A necessary condition for a C-N equilibrium is that at the investment dates $\langle t_n \rangle$ with $t_n > 0$,

$$\Delta_N(n,k,t_n) = rC. \tag{2}$$

Incremental revenue is the revenue flow with the new plant less the revenue lost on all plants built previously as a result of the lower price. Since revenue earned on the new plant is the same for all firms and the revenue loss on old plants increases with the size of the firm, incremental revenue is larger for smaller firms.

Market forces in a C-N oligopoly tend to push the industry toward equal market shares as smaller firms invest to catch up with larger firms. In a symmetric equilibrium, firm sizes along an equilibrium expansion path differ by no more than one unit, the minimum indivisible unit of investment. While asymmetric equilibria may exist in the C-N game, we limit attention in what follows to symmetric equilibria in pure (non-stochastic) strategies. One more assumption is needed.

(A.3) $\Delta_N(n,k,t)$ is a strictly increasing function of t and a strictly decreasing function of both n and k.

Assumption (A.3) implies that each firm's revenue is globally concave in its own output (capacity), taking the outputs of others as fixed. The assumption that incremental revenue increases with time implies that it never pays to delay investment after the date when incremental revenue equals the interest cost on a new plant (the cost of advancing construction).

Proposition 1: Let $n(t)$ and $m(t)$ be the capacities of firms 1 and 2. Given assumptions A.1–A.3, there exists a C-N equilibrium with $|n(t) - m(t)| \leq 1$ (a symmetric C-N equilibrium).

Proof: Let $\langle t_k \rangle$ be the sequence of industry investments with $k = n + m$. Suppose firm 1 builds the industry's k^{th} plant at t_k and fix the dates of all other plants. Let this be firm 1's n^{th} plant. Given A.1–A.3, firm 1's objective function is concave in t_k over the interval (t_{k-1}, t_{k+1}). Thus, firm 1 attains a unique maximum if it invests in this interval and the profit-maximizing date is determined by

$$\Delta_N(n,k,t_k) = rC \quad \text{for} \quad t_k \in (t_{k-1}, t_{k+1}).$$

We need only show that firm 1 will not choose to invest at any date after t_{k+1} or before t_{k-1} (Lemma 1 rules out investing at t_j for $j \neq k$).

Since firms' capacities at any date differ by no more than one unit, at t_{k+1} either firm 2 or firm 1 builds either its n^{th} or

$(n+1)^{st}$ plant. In a C-N equilibrium, one of the following necessary conditions must hold at t_{k+1} (recall that both firms have the same incremental revenue functions):

(3a) $$\Delta_N(n,k+1,t_{k+1}) = rC$$

or

(3b) $$\Delta_N(n+1,k+1,t_{k+1}) = rC.$$

Suppose the first condition (3a) holds, which corresponds to firm 2 building its n^{th} plant at t_{k+1}. Now suppose firm 1 postponed its n^{th} plant until some $t \in (t_{k+1},t_{k+2})$. Firm 1's incremental revenue is $\Delta_N(n,k+1,t)$. Since incremental revenue increases with time and $\Delta_N(n,k+1,t_{k+1}) = rC$, it follows that $\Delta_N(n,k+1,t) > rC$ for $t \in (t_{k+1},t_{k+2})$. Thus firm 1 increases the profit from the n^{th} plant by investing closer to t_{k+1}, but the profit function is discontinuous at t_{k+1}. (See Figure 1, which shows firm 1's profit as a function of the timing of investment in the n^{th} plant, holding fixed the investment dates for all other plants. The investment date for the n^{th} plant is t_n and $\pi^1(t_n)$ is firm 1's profit with all other investments fixed.)

Figure 1 here

By investing before t_{k+1} there is one less unit of industry capacity and firm 1's incremental revenue increases abruptly to $\Delta_N(n,k,t)$. Profit from the n^{th} plant is higher when firm 1 invests before t_{k+1}, and for $t \in (t_{k-1},t_{k+1})$ profit is a maximum at t_k.

The second condition (3b) corresponds to either firm 1 or firm 2 building its $(n+1)^{st}$ plant at t_{k+1}. In either case, if firm 1 delays investment of the k^{th} plant until $t \in (t_{k+1}, t_{k+2})$, then $\Delta_N(n+1, k+1, t) > rC$ and it pays to move the investment back to the original interval (t_{k-1}, t_{k+1}).

Similar arguments apply if firm 1 chooses to advance the investment date for the n^{th} plant to some $t \in (t_{k-2}, t_{k-1})$, and if firm 1 changes the investment date for the n^{th} plant to any $t > t_{k+2}$ or $t < t_{k-2}$. Hence t_k is a global maximum for the n^{th} investment by firm 1. The results are summarized in Figure 1. By symmetry, the same arguments apply to all other investment dates by either firm. Thus the sequence $\langle t_k \rangle$ defined by

$$\Delta_N(n, k, t_k) = rC$$

with $|n(t) - m(t)| \leq 1$ is a C-N equilibrium.

The time path of investment in a (symmetric) C-N equilibrium is as follows. Suppose $j=1,\ldots,J$ firms begin with initial capacities $(n_1, \ldots, n_J)$, ordered so that $n_i \leq n_j$ for $i < j$. To avoid a jump to the equilibrium path at the initial date, assume that no firm has an incentive to invest until some $t > 0$. If firm 1 is strictly smaller than any other firm, it has the largest incremental revenue and will build first when

$$\Delta_N(n_1+1, k, t) = rC,$$

where k is industry capacity after the firm invests.

Firm 1 will continue to build (at discrete intervals) until it catches up in size to firm 2. Either firm 1 or firm 2 will build the next plant, but not both. (If firms 1 and 2 were the same initial size, the identity of the firm that build the first plant would not be determinate.) Firms 1 and 2 will take turns building until they catch up in size to firm 3. Then firms 1, 2, and 3 will take turns in an investment round robin until they catch up in size to firm 4, and so forth.

Define an investment "round" as a sequence in which all firms augment their capacity by one unit. The ordering of firms in an equilibrium sequence is arbitrary. What is determinate is the date at which each plant is built and that at the end of each investment round firms must be of equal size. There are multiple C-N equilibria corresponding to the ordering of firms in each investment round. Indeed if it always pays to build another plant at some date, there is a countable infinity of C-N equilibria.

Proposition 2: The C-N sequence of industry investment dates, $\langle t_k^N \rangle$, is uniquely determined in the following manner:

For $k \leq J$, t_k^N is the solution to

$$\Delta_N(1,k,t_k^N) = rC. \tag{4a}$$

For $k = nJ+m$, $0 < m \leq J$, where n denotes the largest number of times that all firms have invested, t_k^N is the solution to

$$\Delta_N(n+1,k,t_k^N) = rC. \tag{4b}$$

The proof follows directly from the above discussion.

Will a C-N oligopoly build plants sooner or later than a monopolist? Let $s^M = \langle t_k^M \rangle$ be the monopoly investment sequence, which solves

$$(5) \qquad \max_s \Pi(s) = \sum_{n=0}^{\infty} \int_{t_n}^{t_{n+1}} R(n,t)e^{-r\tau}d\tau - Ce^{-rt_n}$$

where $R(n,t) = nP(n,t)$.

Define the monopoly incremental revenue from the k^{th} plant, $\Delta_M(k,t) = kP(k,t) - (k-1)P(k-1,t)$. Given the assumed behavior of demand, a necessary and sufficient condition for optimality of the monopoly investment sequence is $\Delta_M(k,t_k) = rC$ for $k = 1,2,\ldots$.

<u>Proposition 3</u>: $t_k^N < t_k^M$ for all $k \in K$, where K is the largest number of plants built in the C-N oligopoly.

A monopolist's expansion path lies below that of a C-N oligopoly, in that at any date a monopolist's capacity is no more, and may be less, than total capacity in a C-N oligopoly. The proof follows directly from the observation that incremental revenue for a C-N firm, $\Delta_N(n,k,t)$ exceeds the monopoly incremental revenue, $\Delta_M(k,t)$ for all n,k, and t.

Let $\langle t_k^* \rangle$ be the sequence of surplus-maximizing investment dates. The sequence $s^* = \langle t_k^* \rangle$ solves (5) with revenues $R(n,t)$ replaced by total surplus

$$S(n,t) = \int_0^n P(x,t)dx.$$

Define the incremental surplus contributed by the k^{th} plant,

$$\Delta_W(k,t_k) = S(k,t_k) - S(k-1,t_k).$$

A necessary and sufficient condition for the surplus-maximizing investment sequence is $\Delta_W(k,t_k) = rC$. It is straightforward to show that the incremental surplus from a k^{th} plant exceeds the incremental revenue from a k^{th} plant in a C-N oligopoly (and a fortiori exceeds the monopoly revenue). Hence

Proposition 4: $t_k^N > t_k^*$ for all $k \in K$. A C-N oligopoly builds plants uniformly later than the surplus-maximizing dates.

Price in a C-N equilibrium exceeds the flow average cost rC at every date, and therefore all firms make strictly positive profits. This follows from the investment condition for the k^{th} plant,

$$\Delta_N(n,\ k, t_k^N) = rC$$

which implies

$$P(k,t_k^N) = rC + (n-1)[P(k-1,t_k^N) - P(k,t_k^N)]. \tag{6}$$

The right-hand side of equation (6) equals rC only if the firm is building its first plant so that $n = 1$, and it exceeds rC if $n > 1$. Since price increases with time for a given capacity, the market price will exceed rC with the exception of those dates at which firms build their first plants. Thus, even as the number of firms gets arbitrarily large, the price will strictly exceed flow average cost except at investment dates for a firm's first plant and all firms will earn strictly positive profits.

Adding more firms in a C-N equilibrium simply stretches the period of time during which firms build their first plants. Increasing the number of firms from J to $J + 1$ has no effect on the dates at which the first J firms make their initial capacity investments. For any finite date T, if J is sufficiently large, no firm will build more than one plant in the interval (0,T). During the interval (0,T), the industry consists of a succession of new entrants, each of which builds one plant.

A C-N model where the number of firms is arbitrarily large has, in the limit, the characteristic of free entry into the industry. Every industry investment is a firm's premiere plant, and the investment date for the industry's k^{th} plant is determined by

$$\Delta_N(1,k,t_k) = P(k,t_k) = rC.$$

Price equals flow average cost when a plant is built, and exceeds flow average cost at every other date (given the assumption that demand grows with time). Thus profits are positive for any number of competitors. Also, the C-N equilibrium does not converge to an efficient allocation as the number of competitors increases. This accords with Proposition 4, which states that the C-N and surplus-maximizing investment dates always diverge. It is easily demonstrated by noting that the revenue flow from a firm's premiere plant, $P(k,t_k)$, falls short of the incremental surplus from that plant.

Precise bounds can be placed on the extent of suboptimal capacity. The next proposition shows that the C-N capacity with arbitrarily many firms is always within one plant of the surplus-maximizing capacity.[6]

Proposition 5: Let $k(t)$ denote the C-N capacity at date t, and $k^*(t)$ the surplus-maximizing capacity. Then in the limit as $J \to \infty$

$$k^*(t) - 1 \leq k(t) \leq k^*(t).$$

Proof: If J is sufficiently large, no firm will build more than one plant and the C-N investment rule is $P(k,t_k) = rC$, while the optimal investment rule is $\Delta_W(k,t_k^*) = rC$. Let $k(t)$ and $k^*(t)$ denote the C-N and the optimal capacity levels at date t. From Proposition 4, we know that $k(t) \leq k^*(t)$.

Consider the interval $[t_k^*, t_{k+1}^*)$, beginning with the k^{th} plant along the optimal path and ending just before the $(k+1)^{st}$ plant. At t_k^*,

$$P(k^*-1, t_k^*) > \Delta_W(k^*, t_k^*) = rC.$$

Suppose $k(t) \leq k^* - 2$. Then

$$P(k(t)+1, t_k^*) \geq P(k^*-1, t_k^*) > rC.$$

Thus $k(t) \leq k^* - 2$ cannot hold at t_k^* along a C-N equilibrium path. (It would pay for someone to build another plant.) But $k^*(t)$ is constant for $t \in [t_k^*, t_{k+1}^*)$ while $k(t)$ is nondecreasing, and the argument holds for any k^*. Thus

$$k^*(t) - 1 \leq k(t) \leq k^*(t)$$

Merger activity has predictable consequences in the Cournot-Nash model of industry competition. A merger changes the number of active firms in the industry and the distribution of firm capacities. Prices depend only on total industry capacity because by assumption all capacity is fully utilized. Thus, the impact of any merger is on the timing of capacity investment undertaken by the industry. Note that since smaller firms invest before larger firms in a C-N equilibrium (when all firms have access to the same technology), a merger of small firms has a greater negative impact on capital formation in the short run than does a merger involving relatively large firms.[7]

Consider as an example three firms, $j=1,2,3$, with initial capacities (1,2,3). Firm 1 would build first, followed by either firm 2 or by another investment by firm 1. Suppose firms 2 and 3 merge, so that initial capacities are (1,5). The merger would have no effect in the short run. Investment incentives (incremental revenues) depend only on a firm's own capacity and on total industry capacity. Hence, firm 1 would have the same incentive to build a second (or a third) plant, although subsequent investments would be delayed.

Now suppose firms 1 and 2 merge, so that the new initial market structure is (3,3). The merger eliminates firm 1, which would have had the greatest incentive to invest. The smallest initial capacity is now 3 rather than 1, and the incremental revenue is correspondingly reduced. Hence, investment is delayed and price is higher relative to the price and investment paths for the original industry structure.

The merger involving the two smaller firms is more detrimental to economic performance than the merger involving the two larger firms. Price is higher and investment is slowed. The negative impact from the merger of firms 1 and 2 is more substantial not because it creates a larger firm, but because it eliminates a smaller firm that had a greater incentive to expand.

Although profits are positive in a C-N equilibrium with any number of firms, the profits earned by any particular firm depend on its position in the equilibrium sequence of investments. There is an infinity of C-N equilibria, each with the same industry investment dates, but a different order of investments by firms. Since total investment and hence prices are identical in each of the equilibria, a firm earns a higher profit if it occupies an earlier position in any investment round. This suggests a conceptual difficulty with the behavioral assumptions underlying the C-N oligopoly. Since each firm prefers the equilibrium in which it invests first, we can expect that firms will compete for the opportunity to build the first plant. Yet this competition to be first is inconsistent with the assumptions of a Cournot-Nash game with pure strategies, where each firm takes the actions of others as fixed. The next section considers an oligopoly where firms make investment decisions based on the history of the industry, and each new investment can be made by any firm.[8]

3. PREEMPTIVE COMPETITION

A failure of the Cournot-Nash equilibrium is its inability to capture the element of "animal spirits" that leads firms to compete for the right to make each new investment. In the C-N model, each firm assumes it cannot upset the investment plans of competitors. Every investment earns profits in a C-N equilibrium, yet firms do not compete for these profits, even when some firms must wait their turn and settle for profits that are strictly lower than the profits of firms that invest at earlier dates. This section considers a market where firms actively compete for each new investment. The model is dynamic and employs firm strategies that depend on the history of the industry.

In a continuous time model a difficulty arises in distinguishing history from current actions and in dealing with the possibility of simultaneous moves. The problem is compounded because decision variables of the firms induce "jumps" in the state variables making the stock-flow distinction difficult. With simultaneous moves, competitors could make decisions they would regret after they learn the actions of their rivals. The possibility of such outcomes depends on the information structure of the game and on the ability to reverse undesired moves.

The competitive situation described in this section assumes an information and decision structure where redundant investment can be avoided. Agents are subject to decision lags between observations and actions. An action if planned, but not undertaken, can be cancelled. Moreover for one reason or another different firms are subject to different decision lags. These reasons can be systematic, reflecting organization, or stochastic. In the duopoly model, the firm with the shorter decision lag will, upon receipt of the same information, be able to act before the firm with the longer decision lag. Let $I(t)$ denote the common information on the state of the game at time t. Firm i with decision lag h_i can take an action at time t, $a_i(t)$, depending only upon the history prior to $t - h_i$; we summarize this history as $I(t-h_i)$. Suppose firms 1 and 2 have decision lags h_1 and h_2 with $h_1 < h_2$. Firm 1 has the advantage at any t of being able to act upon more recent information. At $t = 0$ both firms make an observation on the state and firm 1 takes an action after a time interval of h_1. Firm 2 observes Firm 1's actions and takes an action after a further $h_2 - h_1$ units of time pass. Firm 1 cannot act in the interval immediately after its action.

We now let the decision lags approach arbitrarily close to zero, but maintain $h_2 > h_1$. The delay between information and action becomes negligible but firm 1 retains the advantage of being able to act sooner. In this limiting situation at time t both firms observe $I(t)$, but $a_1(t)$ is realized instantaneously before $a_2(t)$. Firm 2's limiting information set at t is $I(t)$, augmented by whatever action firm 1 initiates. Hence, in the limiting continuous time formulation, the firms have different information sets, which induce a natural momentary first mover advantage on firm 1. In order to incoporate this assumption into our analysis we denote by $x^+(t)$ the total industry capacity after firm 1 has moved, but before firm 2 has moved. Let $x^-(t)$ denote total industry capacity before firm 1 has moved. By "move" we mean that sufficient time has elapsed for the firm to be able to take an action, but it need not choose to invest. The terms $x^-(t)$ and $x^+(t)$ are the state variables which represent the information sets of firm 1 and firm 2 at time t.

We will show that the first-mover advantage implied by the decision lags is trivial in that both firms will earn zero profits on new investments in a preemption equilibrium. The importance of the decision lag structure is to rule out instances of simultaneous capacity expansion. Firm 2, with the longer lag, would not invest immediately after firm 1 invests. Firm 2 acts to "police" the actions of firm 1 by offering a credible investment threat if firm 1 chooses not to invest. As both h_1 and h_2 approach zero, this policing function becomes increasingly effective and forces firm 1 to invest and earn only normal profits. Note that the assignment of the decision lags to the firms is arbitrary, but in the absence of differential lags we could not rule out the possibility of simultaneous and redundant investment activity.[9]

In the development of the preemptive equilibrium, we assume as in the Cournot-Nash case that firms play Cournot conditional on installed capacity and that all capacity is fully utilized (marginal revenue is positive). The crucial difference is in the timing of capacity expansion. A preemption equilibrium will be described for a duopoly, although the arguments extend to a J-firm oligopoly with appropriate expansion of the strategy spaces. Let the two firms be denoted by 1 and 2, and let $k=n+m$ represent total industry capacity at date τ.

A particular sequence of dates plays a central role in the preemptive equilibrium. Let (k,τ) summarize the state of the industry and define a <u>threat sequence</u> $s(k,\tau) = \langle \hat{t}_j \rangle_{j=k+1}^{\infty}$ associated with the state (k,τ) by the following infinite set of equations:

$$(7) \qquad \sum_{i=j}^{\infty} \int_{\hat{t}_i}^{\hat{t}_{i+1}} P(i,t)e^{-rt}dt - Ce^{-rt_j} = 0, \quad j = k+1, k+2, \ldots, \infty$$

with $\hat{t}_{k+1} \geq \tau$.

A solution to this set of equations can be shown to exist, given a mild growth condition on $P(j,t)$. In general the solution is not unique; there is a continuum of solutions indexed by the value of $\hat{t}_{k+1}$. It will be convenient (although arbitrary) to use a particular threat sequence conditioned on the state (k,τ). First, note that if the sequence $\langle \hat{t}_j \rangle$ is "separated", with $\hat{t}_j > \hat{t}_{j-1}$, for all j, then (7) is equivalent to the set of equations

$$(8) \qquad \int_{\hat{t}_j}^{\hat{t}_{j+1}} [P(j,t)-rC]e^{-r(t-\hat{t}_j)} dt = 0 , \qquad j = k+1,k+2,\ldots,\infty$$

A separated sequence can exist only if $P(k+1,\hat{t}_{k+1}) < rC$. If $k+1$ is too small at date τ so that the price exceeds rC, equation (8) will not be satisfied for any $\hat{t}_{k+1} \geq \tau$. Define $N(k,\tau)$ as the smallest integer $\geq k+1$ such that $P(N(k,\tau),\tau) < rC$. The <u>threat sequence</u> $s(k,\tau)$ will be defined as the sequence $\langle\hat{t}_j\rangle$ satisfying (8), with $j = N(k,\tau)$, $N(k,\tau)+1, N(k,\tau)+2,\ldots$, and $\hat{t}_{N(k,\tau)} = \tau$. The <u>threat capacity</u> at date t associated with the threat sequence $s(k,\tau)$ is defined as $N(t) = \max_j\{j:\hat{t}_j \leq t\}$. Note that of the continuum of possible threat sequences defined by equation (7), $s(k,\tau)$ is the threat sequence which begins at the earliest date.

All threat sequences, and the sequence $s(k,\tau)$ in particular, have the property that all plants built in the sequence earn exactly zero profits. Note that if $N(k,\tau) > k+1$, so that more than one plant must be built at date τ for (8) to hold, all plants built at τ (as well as those built at $\hat{t}_j > \tau$) will just break even, given that the threat sequence $s(k,\tau)$ of investment dates is realized. The task ahead is to show that the threat sequence $s(k,\tau)$ represents a perfect (as defined by Selten (1975)) Nash equilibrium in the investment competition.

The demonstration of an equilibrium begins with the definition of strategy functions. A strategy function value of 1 will indicate the firm in question invests; a value of 0 indicates the firm does not invest. In general, if a strategy function takes on a positive integer value, n, that means the firm in question will build n plants.

Consider the following strategies ϕ^1 and ϕ^2, given the states (x^-,t) and (x^+,t), with x^- and $x^+ \geq k$ and $t \geq \tau$.

(9a)
$$\phi^1(x^-,t) = \begin{cases} N(t) - x^- & \text{if} \quad x^- < N(t) \\ 0 & \text{if} \quad x^- \geq N(t) \end{cases}$$

(9b)
$$\phi^2(x^+,t) = \begin{cases} N(t) - x^+ & \text{if} \quad x^+ < N(t) \\ 0 & \text{if} \quad x^+ \geq N(t). \end{cases}$$

Strategy (9a) says that firm 1 will invest at the beginning of period t if the industry capacity level is below the threat level at date t. In this case firm 1 will build all the capacity necessary to bring industry capacity up to the appropriate threat level for that date. Strategy (9b) says that, after having observed the action of firm 1 (as represented by the state x^+), firm 2 will invest whatever is necessary in order to bring industry capacity up to the appropriate threat level.

We will now establish that ϕ^1 and ϕ^2 are equilibrium strategies and, indeed, are perfect equilibrium strategies.

Proposition 6: The market outcome starting from state (k,τ) given the strategy pair (ϕ^1,ϕ^2) is that firm 1 builds all plants $j = k+1, k+2, \ldots,$ at threat dates $\langle \hat{t}_j \rangle = s(k,\tau)$.

Proof: Follows trivially from the definitions in (9a) and (9b). The outcome where firm 1 builds all of the new plants follows from the assumption that firm 1 has the shorter decision lag. Firm 2 does not invest, but is "waiting in the wings" to invest if firm 1 chooses not to act. For the limiting case as the lags approach zero, the presence of firm 2 is sufficient to assure that firm 1 earns zero profits on all new plants. The assignment of decision lags determines industry structure in the preemption game, but profits earned on new plants are zero for any assignment of arbitrarily short lags.

The demonstration of the Nash and Perfect Nash properties of the strategy pair ϕ^1 and ϕ^2 makes use of the value to each firm of investing before or after the threat dates $\langle \hat{t}_j \rangle$. A useful construction is that of an "entry value function" for the j^{th} plant, which gives the value of a plant built between $\hat{t}_{j-1}$ and $\hat{t}_{j+1}$, given that all future plants are built at the threat dates $\hat{t}_k$ for $k \geq j+1$. Let s^{-j} represent the threat sequence $s(k,\tau)$ with $\hat{t}_j$ deleted. The entry value function for plant j built at date t is (discounted to the investment date, t)

$$V_j(t,s^{-j}) = \int_t^{\hat{t}_{j+1}} e^{-r(u-t)} P(j,u)du + \sum_{i=j+1}^{\infty} \int_{\hat{t}_i}^{\hat{t}_{i+1}} e^{-r(u-t)} P(i,u)du - C$$

It is easy to show that $V_j(t,s^{-j})$ is positive in the interval $(\hat{t}_j,\hat{t}_{j+1})$, negative in $(\hat{t}_{j-1},\hat{t}_j)$ and continuous over $(\hat{t}_{j-1},\hat{t}_{j+1})$. This is illustrated in Figure 2. Thus if any firm builds before $\hat{t}_j$, it would lose money on the j^{th} plant, and if the firm builds after $\hat{t}_j$, but before $\hat{t}_{j+1}$, it would profit on the j^{th} plant.

Figure 2 here

Proposition 7: (ϕ^1, ϕ^2) is a Nash equilibrium strategy pair from state (k, τ).

Proof: First we prove ϕ^1 is a best response to ϕ^2. Our point of reference in this proposition is only the initial state (k, τ). Given ϕ^2, if firm 1 does not invest at any date along the threat sequence $\langle \hat{t}_j \rangle$, when capacity is below the threat level, then firm 2 will invest. Investing at the threat dates earns zero profits on new plants and positive profits on existing capacity. If firm 1 invests in the j^{th} industry plant before the threat date $\hat{t}_j$, this lowers the price received on all existing plants and, given the properties of the entry value function V_j, yields a negative value for the new plant. If firm 1 attempts to invest in plant j after $\hat{t}_j$, firm 2 will invest first. This proves that ϕ^1 is a best response to ϕ^2 given the initial state (k, τ).

Now consider firm 2. Given ϕ^1 and the initial state (k, τ), firm 2's only option is to invest at dates before or after $\hat{t}_j$. By exactly the same argument as was used in firm 1's case, this is not preferred to ϕ^2. Hence ϕ^2 is a best response to ϕ^1. Q.E.D.

Note that since both firms have identical costs, the identities of firm 1 and firm 2 are irrelevant. Either firm could do the investing.

Perfectness of the strategy pair requires that ϕ^1 and ϕ^2 be Nash equilibrium strategy pairs for any possible disequilibrium and equilibrium histories leading up to a feasible state (x^+, t) or (x^-, t), for x^- and $x^+ \geq k$ and $t \geq \tau$. Essentially this involves checking whether the contingent part of each strategy is a best response given that the contingency occurs.

Proposition 8: (ϕ^1,ϕ^2) is a Perfect Nash equilibrium strategy pair.

Proof: There are a number of cases to consider.

(i) Consider any subsequent state (x,t) and (x^+,t) along the equilibrium outcome described in Proposition 4. The argument of Proposition 5 remains valid and ϕ^1 and ϕ^2 are equilibrium strategies for that state. If $x^- = N(t)$, but $\hat{t}_{N(t)} < t < \hat{t}_{N(t)+1}$, then by definition the state (x^-,t) is in the equilibrium sequence, independent of how it was reached, and the argument in (i) holds. If $x^- = N(t)-1$, and $t = \hat{t}_{N(t)}$, then this also is an equilibrium state in which firm 1 will immediately invest and the subsequent outcome will be the equilibrium sequence.

(ii) Suppose the actual capacity levels, x^+ and x^-, are greater than the threat level, $N(t)$. Given (ϕ^1,ϕ^2) neither firm will invest, and given the properties of the entry value function it is clearly optimal for both not to do so, as this would yield negative profits on the new plant and reduce revenues on the old plant.

(iii) The important case to consider is a disequilibrium state with x^+ or $x^- < N(t)$. The easiest case to consider is a state which is one short of the appropriate threat level. So either (a) $x^+ = N(t)-1$ and $t \geq \hat{t}_{N(t)}$, or (b) $x^- = N(t)-1$, and $t > \hat{t}_{N(t)}$. In case (a) firm 1 should have invested at t but did not. Given ϕ^1, if firm 2 does not invest, then firm 1 will invest the next instant and at all subsequent dates given by $s(k,\tau)$, yielding non-negative profits on the next investment. If firm 2 invests at $\hat{t}_{N(t)}$, this is a best response which also yields non-negative profits. In (b) with $x^- = N(t)-1$ the argument is similar to (a), although in this case firm 1 makes positive profits on the new plant when future investments are made at the threat dates. When $t > \hat{t}_N$ and $x^- = N(t)-1$, then firm 1, given ϕ^2, can preempt firm 2 and earn positive profits. If firm 1 fails to invest at t, then given ϕ^2, firm 2 will invest at t, causing firm 1 to forego a profitable opportunity on a new plant, and giving rise to the same losses on old plants.

(iv) If current industry capacity falls more than one plant short of the appropriate threat level, then given ϕ^1, firm 1 will build all of the necessary capacity. The argument here is an obvious extension of the argument in part (iii). Q.E.D.

The implication of Proposition 8 is that the threats implicit in the strategy pair (ϕ^1,ϕ^2) are credible. For example, firm 2 always threatens to invest if firm 1 does not at the appropriate dates. It turns out that if firm 1 does not invest, then given the future course of the game and firm 1's strategy, it is in firm 2's best interest to invest.

The preemptive equilibrium captures what seems to be the essence of competition in an intertemporal model with lumpy additions to capacity: the use of anticipatory investment to preempt rivals and capture profitable opportunties. The equilibria corresponding to the strategy pair (ϕ^1,ϕ^2) have the property that the new plants built by either firm earn zero profits. Any profits arise as a consequence of sunk costs on existing capital. Appendix A shows that profits discounted to date τ for a firm with initial capacity n, when both firms follow strategies (ϕ^1,ϕ^2) for $t > \tau$ are

(10) $$\Pi^1 = n \int_{\tau}^{\hat{t}_{k+1}} P(k,t)e^{-r(t-\tau)}dt + n e^{-r(\hat{t}_{k+1}-\tau)} C$$

where k is the total industry capacity at date τ. Since the new plants break even, all profits appear as revenues earned on existing capacity.

The threat strategy $s(k,\tau)$ which forms the basis of the preemption strategy was determined by choosing the earliest investment sequence that yields zero profits on all new plants. For this case, $\hat{t}_{k+1} = \tau$ and the present-value profit for firm 1 is simply nC. In general it is possible to find perfect Nash preemptive equilibria based on threat strategies that begin at a later date, with $\hat{t}_{k+1} > \tau$. All new plants would continue to earn zero profits, but the market price for $\tau < t < \hat{t}_{k+1}$ would be higher than the price under the strategy $s(k,\tau)$. Thus revenues earned on existing capacity would be higher, as indicated by equation (10).

The infinite time horizon of the game permits a multiplicity of preemption equilibria. There is no "last date" which serves to fix the timing of investments that earn zero profits. Note that while all the preemption equilibria earn zero profits on newly constructed plants, the infinite time horizon suggests the possibility of supergame strategies that earn positive profits on new plants.

The ordering of investment by the two firms using the strategy pair (ϕ^1,ϕ^2) is determined by the decision lags. Profits depend only on sunk capacity and not on the identity of the firm that adds new plants. Both firms have equal incentives to invest at the threat dates, and the incentives are independent of existing capacity. The rule is invest whenever a new plant can earn profits, and this rule is independent of the firm's own existing capacity. Just as the Cournot-Nash oligopoly in Section 2 is the dynamic analog of static Cournot competition, the preemption equilibrium parallels the outcome of a static Bertrand game.

Comparing the preemption and Cournot-Nash equilibria, the preemption game results in a more rapid accumulation of industry capital (recall that profits are strictly positive in a C-N equilibrium). Relative to the surplus-maximizing investment dates, the preemption investment dates may be earlier or later. Appendix B shows that at the surplus-maximizing investment dates, profits earned on new plants may be positive or negative; hence an unambiguous comparison with the preemption dates is not possible.

Given the symmetry of the preemption game, extending the duopoly strategies (ϕ^1, ϕ^2) to an oligopoly and allowing for the possibility of mergers would have no effect on the timing of industry investment, short of merger for monopoly. The strategies call for investment whenever a new plant can break even at prices corresponding to the complete utilization of existing capacity. As long as there exists a viable competitor in the industry, the timing of new investment is determined by the breakeven rule. Merger activity in the preemption equilibrium, short of monopoly, would not affect the timing of new investment or total industry profits when all capacity is fully utilized.

4. CONCLUDING REMARKS

The Cournot-Nash game represents a basic element of interfirm rivalry and appears frequently as an equilibrium concept in dynamic games. Yet the results of Section 2 suggest that, at least with increasing returns to scale in new investment, the C-N model yields predictions that are often at odds with intuition. For example, increasing the number of competitors need not lead to an erosion of profits. Firms space their investments over time. Price is at least as large as flow marginal cost at every investment date, and with investment nonconvexities, the average price always exceeds average cost.

The peculiar results of the C-N competition stem from the absence of incentives for preemptive investment behavior. In a C-N game, each competitor expects rivals to maintain their investment sequences fixed over time. This conjecture is implicit in the "one-shot" game where each rival specifies an investment sequence at the start of the game and has no opportunity for change. The essence of preemptive behavior is the anticipation that rivals will adjust their actions conditional on the actual evolution of the industry. Thus a competitor may profitably invest just before a rival's planned investment, because the competitor anticipates that the rival will respond by delaying its plans. The C-N behavioral assumption presumes rivals will not change their plans, and hence leaves no room for preemptive competition.

The model of preemptive competition in Section 3 includes a large dose of "animal spirits," in that firms stand ready to invest whenever a new plant can earn positive profits. Firms invest in the preemption equilibrium even though the incremental revenue from the new plant may be negative. The new plant just breaks even, but the investment lowers revenues from existing plants.

Investment with zero profits on new plants is an equilibrium because either actual or potential competitors exercise a credible threat to invest whenever a firm delays a construction program with the purpose of raising prices and profits. Although there is a continuum of preemption equilibria, they all have the property of zero profits on new plants, even with only two firms. Free entry is not necessary to dissipate profits. The preemption equilibria are "perfect": the assumed strategies yield best responses not just along an equilibrium path, but also along any disequilibrium path leading to a feasible state.

Since there is no terminal time to restrict the possible credible equilibria in this game, it is not surprising that the preemption equilibria are not unique, and indeed credible equilibria with properties very different from those described in Section 3 could emerge. Perfect Nash equilibria may well exist that do not exhibit the property of rent dissipation that characterizes the preemption game. Nonetheless, the preemption equilibrium strategies described in this section demonstrate the properties of dynamic perfect equilibria that allow for neither explicit or implicit collusion, and serve as a contrast to the more familiar Cournot-Nash formulation.

FOOTNOTES

[1] See Bain (1954, 1956).

[2] Eaton and Lipsey (1981) lucidly discuss the importance of capital specificity as a determinant of industry structure.

[3] Preemption in this paper refers to anticipatory investment. A large literature deals with preemption for entry deterrence. Some of the references include Dixit (1980), Eaton and Lipsey (1979), Gilbert and Harris (1981a,b), Gilbert and Newbery (1982), Gilbert (1984), Harris and Lewis (1982), and Spence (1977).

[4] Baumol, Panzar and Willig (1982, Ch. 14) come to similar conclusions from a quite different perspective.

[5] The extension to constant marginal costs (the same for all firms) is straightforward and left to the reader.

[6] Note that the Cournot-Nash equilibrium is trivially cost efficient in the sense that the present value cost of delivering a given output stream is minimized. This follows from the assumptions that all firms have identical costs and capacity is fully utilized.

[7] The hypothetical merger is exogenous in that stockholders' or managers' incentives to effect or block the merger are ignored (see Salant et al. (1983)).

[8] The problem of unequal profits appears in a number of dynamic models which use the "open-loop" equilibrium concept of differential games (e.g. Reinganum (1981a,b)), but it can also occur with subgame perfect preemptive strategies (see Fudenberg and Tirole (1983)).

[9] What is crucial here is the assumed ordering in the delays associated with a commitment to build a plant. This ordering allows the firm that commits last to cancel rather than invest at the same time as its opponent. Hence "mistakes" where firms invest simultaneously (to their mutual disadvantage) are avoided. An example of an open-loop game where firms make simultaneous decisions is in Pitchik (1982). Examples of closed-loop games with simultaneous moves are in Fudenberg, et al. (1983) and Fudenberg and Tirole (1983).

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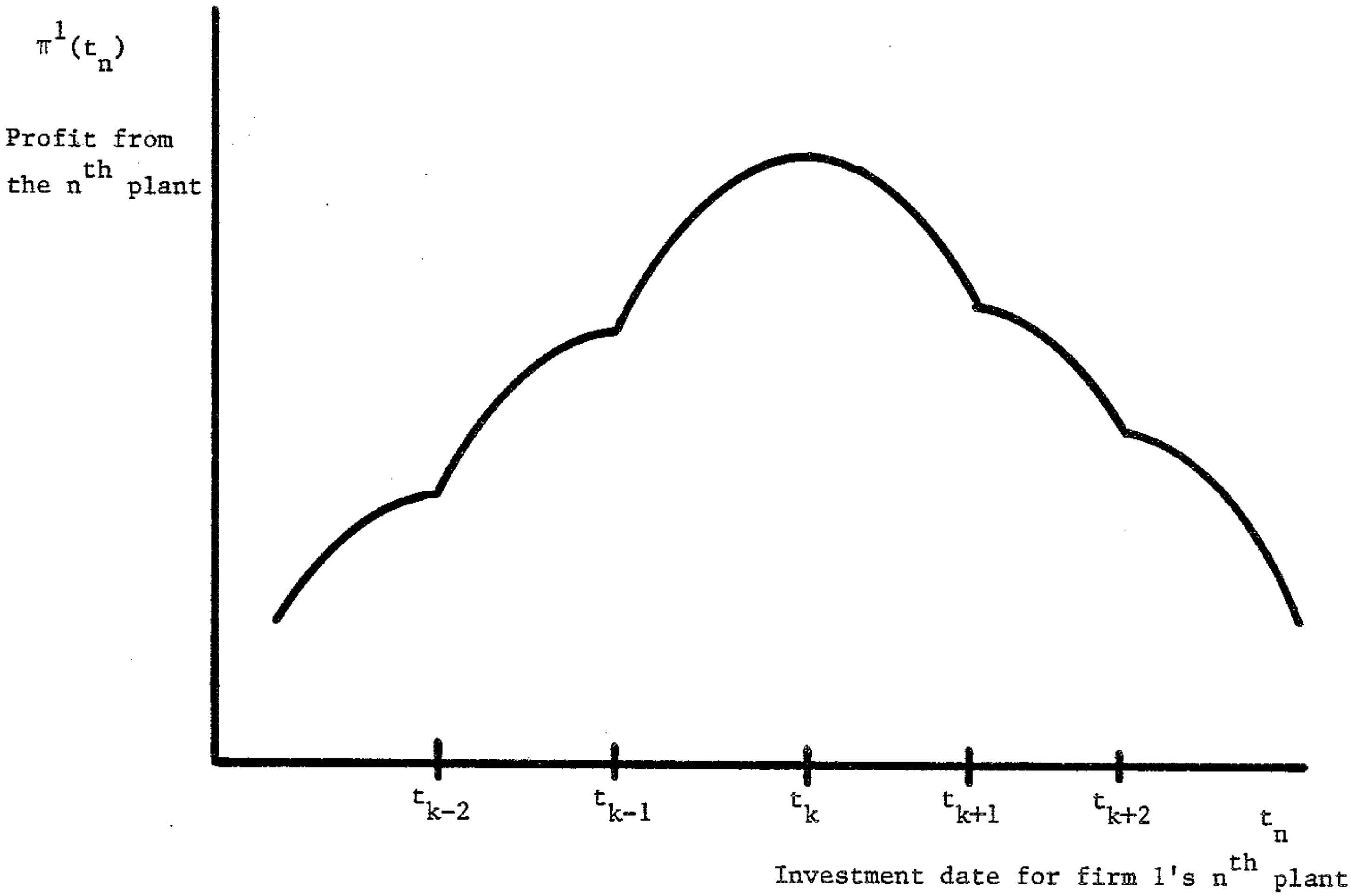

Figure 1. Firm 1's profit as a function of the investment date for the n^{th} plant, holding fixed the dates of all other investments.

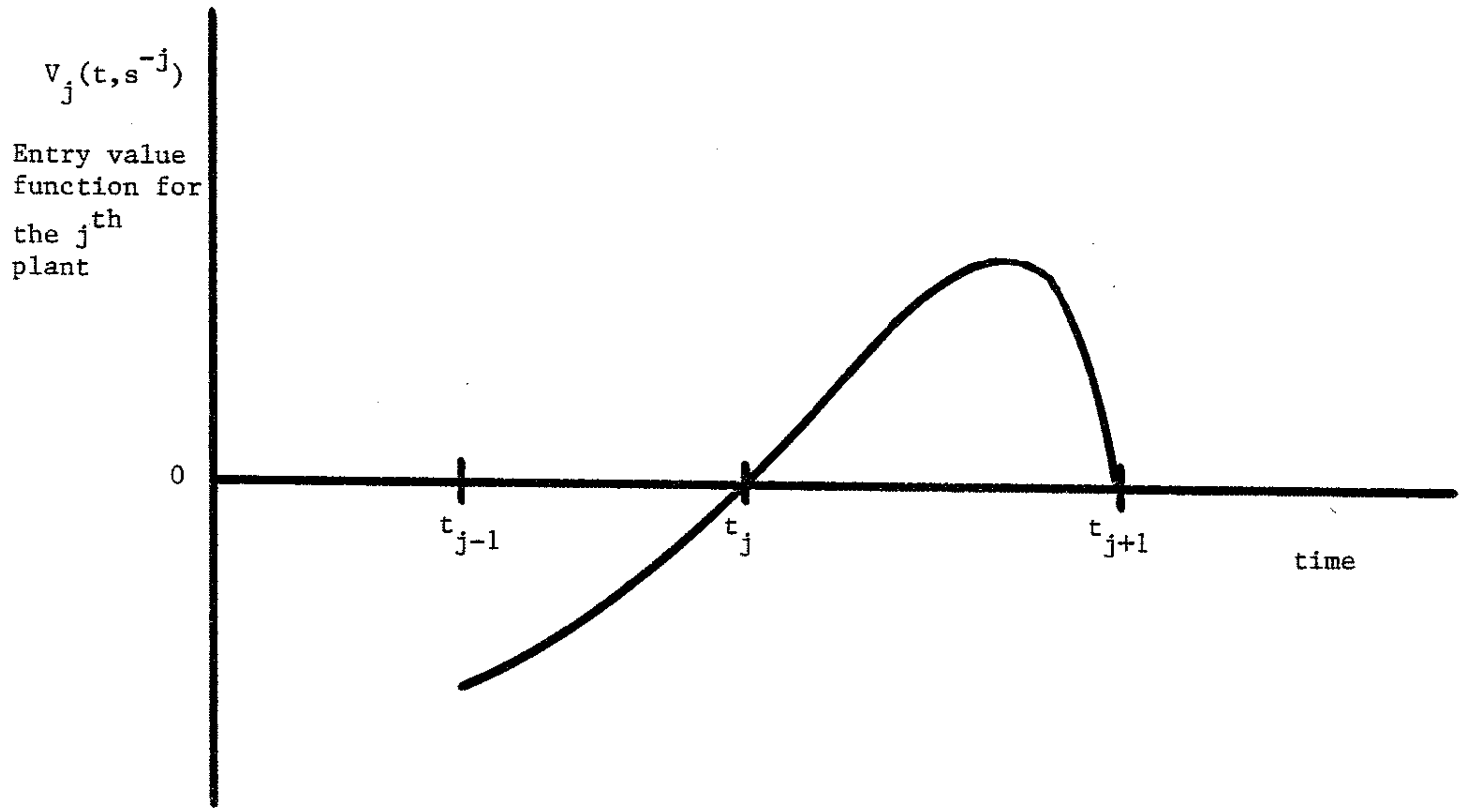

Figure 2. Behavior of the entry value function for investment in the j^{th} plant.

APPENDIX A

Calculation of profits earned using the preemption equilibrium strategy pair (ϕ^1, ϕ^2).

Given the initial state (k,τ) with initial firm capacities n and m, if firm 1 invests at each threat date $\langle \hat{t}_j \rangle$ for $j = k+1, \ldots,$ all new plants just break even and any profits appear only on existing firm capacity. The profit for firm 1 is discounted to date τ is

$$\text{(A1)} \qquad \Pi^1 = n \left[\int_{\tau}^{\hat{t}_{k+1}} P(k,t)e^{-r(t-\tau)}dt + \sum_{j=k+1}^{\infty} \int_{\hat{t}_j}^{\hat{t}_{j+1}} P(j,t)e^{-r(t-\tau)}dt\right]$$

Also, from the definition of the threat strategy (equation (7)),

$$\text{(A2)} \qquad \sum_{j=k+1}^{\infty} \int_{\hat{t}_j}^{\hat{t}_{j+1}} P(j,t)e^{-r(t-\tau)} dt = Ce^{-r(\hat{t}_{k+1}-\tau)} .$$

Thus,

$$\text{(A3)} \qquad \Pi^1 = n \left[\int_{\tau}^{\hat{t}_{k+1}} P(k,t)e^{-r(t-\tau)} dt + Ce^{-r(\hat{t}_{k+1}-\tau)}\right],$$

which is equation (10) in Section 3.

Since firm 1 earns zero profits on every new plant $j = k+1, \ldots$, if firm 2 had invested at dates $\hat{t}_j$, firm 1's profits also would be given by equation (A1). Firm 1's profits are the same whether or not it invests provided firm 2 uses the strategy ϕ^2. It is still optimal for firm 1 to invest at the threat dates because it earns positive profits that are strictly higher than the profits it would earn by investing at any other date. The equality of profits earned by investing at the threat dates versus not investing at all when firm 2 uses ϕ^2 merely shows that the strategies ϕ^1 and ϕ^2 are symmetric.

APPENDIX B

Proof that the value of a plant built in the surplus-maximizing investment sequence may be positive or negative. (See also Starrett (1978) for an alternative approach.)

Define $P^*(t) = P(Q^*(t),t)$, where $Q^*(t)$ is the maximizing output at t. The value of a plant built at t_k and discounted to t_k is

$$\text{(B1)} \qquad \Gamma(t_k) = \int_{t_k}^{\infty} e^{-r(\tau - t_k)} P^*(\tau)d\tau - C$$

$$= \sum_{j=k}^{\infty} \int_{t_j}^{t_{j+1}} e^{-r(\tau - t_k)} [P^*(\tau) - rC]d\tau$$

At the surplus-maximizing investment dates, $\Delta_W(j,t_j) = rC$ for $j = k,k+1,\ldots$ and for $t \in (t_j,t_{j+1})$

$$\lim_{t^+ \to t_j} P^*(t) < \Delta_W(j,t) < \lim_{t^- \to t_{j+1}} P^*(t)$$

The sign of $\Gamma(t_k)$ therefore depends on the time path of $P^*(t)$. However, the investment dates depend only on $P^*(t_j)$ for $j = k,k+1,\ldots$ and not on the time path of $P^*(t)$ for $t \in (t_j,t_{j+1})$. Thus there exist time paths of prices $P(Q,t)$ with the property that the surplus-maximizing investment sequence is the same for all time paths of prices, but $\Gamma(t_k) > 0$ for some paths and $\Gamma(t_k) < 0$ for others.