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ANALYSIS OF PLATES OF LINEAR STRAIN-HARDENING MATERIAL

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ANALYSIS OF PLATES OF LINEAR STRAIN HARDENING MATERIAL

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November 1960

ERRATA TO

"ANALYSIS OF STRESS CONCENTRATIONS IN THIN ROTATIONAL SHELLS
OF LINEAR STRAIN HARDENING MATERIAL"

By J. F. Brotchie, J. Penzien, and E. P. Popov

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Page	Eqn. No.	Read	For
13	19 b]	$-\frac{R}{Et} (N_{\phi} + N_{\theta})$	$+\frac{R}{Et} (N_{\phi} + N_{\theta})$
19	34]		
20	37]		
21	38]		
21	39]		
17	29 a	$-\frac{1}{x} \frac{dF}{dx}$	$\frac{1}{x} \frac{dF}{dx}$
17	29 b	$-\frac{d^2F}{dx^2}$	$\frac{d^2F}{dx^2}$
19	32 a	$-\frac{aD_1}{L_1^2} \quad \lceil \quad \rceil$	$\frac{aD_1}{L_1^2} \quad \lceil \quad \rceil$
19	32 b	$-\frac{D_1}{L_1^2} \quad \lceil \quad \rceil$	$\frac{D_1}{L_1^2} \quad \lceil \quad \rceil$
19	33 a]	$-\frac{aD}{L^2} \quad \lceil \quad \rceil$	$\frac{aD}{L^2} \quad \lceil \quad \rceil$
19	33 b]		
20	35]	$-\frac{R}{aEt} (N_{\phi} + N_{\theta})$	$+\frac{R}{aEt} (N_{\phi} + N_{\theta})$
20	36]		
41	95	$+\frac{aEt\omega}{R}$	$-\frac{aEt\omega}{R}$
41	Line 14	Eqn. 84	Eqn. 83
43	104]	$\frac{Et}{R}$	$\frac{R}{Et}$
46	112 a]		
46	112 b]		
47	112 c]		
43	106	$\frac{aEt}{R}$	$\frac{R}{aEt}$
43	107	$\frac{a^{1/2}Et}{R}$	$\frac{R}{a^{1/2}Et}$

FOREWARD

This report is submitted in fulfillment of Contract Number UCX 2210 with the Lawrence Radiation Laboratory, Livermore, California.

The investigation was conducted by J. F. Brotchie under the general supervision and technical responsibility of J. Penzien and E. P. Popov, Department of Civil Engineering, University of California, Berkeley, California.

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NOTATION

- M_r, M_ϕ = moments per unit width in radial and tangential directions respectively
 $M_{r\phi}$ = twisting moment per unit width of plate, normal to the radial direction
 Q_r, Q_ϕ = shearing forces per unit width in radial and tangential directions respectively
 w = deflection at any point
 M_u = ultimate moment capacity per unit width of plate (Fig. 3)
 M_p = plastic moment, at which yield is assumed to initiate (Fig. 3)
 M_c = equivalent moment ordinate, at which yield would initiate for rigid, strain - hardening behavior (Fig. 3)
 σ_u = ultimate stress (Fig. 1)
 σ_p = yield stress (Fig. 1)
 σ_c = equivalent yield stress for rigid, strain - hardening material (Fig. 1)
 r, ϕ = polar coordinates
 x = dimensionless radial coordinate = $\frac{r}{s}$
 s = parameter defined by $s = \left(\frac{D}{\gamma}\right)^{\frac{1}{4}}$, the radius of relative stiffness of an elastically supported plate
 γ = density or reaction modulus of the supporting medium
 D = flexural rigidity in the purely elastic range = $\frac{Et^3}{12(1-\nu^2)}$
 D_1 = flexural rigidity of a semi-elastic plate, normal to the direction of yielding, = $\frac{Et^3}{12(1-\alpha\nu^2)}$
 t = plate thickness
 ν = Poisson's ratio
 E = elastic modulus

E_p = inelastic modulus

a = E_p/E

q = loading intensity

∇^2 = Laplace operator

$J_n(\)$ = Bessel function of first kind and order n

$H_n^{(1)}(\)$ = Bessel function of the third kind (Hankel function) and order n

i = $\sqrt{-1}$

Re, Im = real and imaginary parts respectively, of complex functions

SYNOPSIS

The flexural behavior of plates in the post elastic range is described. The case of linear strain-hardening is considered, and the criterion of failure is defined by a maximum principal stress or maximum principal moment in the plate. General solutions are developed from small deflection theory, for axi-symmetrical and unsymmetrical bending in the plate. Where bending is axi-symmetrical, three separate cases are considered: strain hardening in the radial direction only, in the tangential direction only, and in both directions simultaneously. For unsymmetrical bending, only the case where strain hardening occurs in both directions of principal stress is considered.

Tables and formulas are presented for the analysis of edge supported plates, elastically supported plates, and spherically curved plates, or shallow shells.

THEORY

General

As a basis for analysis, the plate is considered to be uniformly thin and to be composed of a homogeneous, isotropic material exhibiting linear strain-hardening in the post-elastic range, (Fig. 1). The term linear strain-hardening in this sense refers to any linear relationship between stress and strain occurring at strains exceeding the limit of proportionality of the material.

Thus, the distribution of bending stress in the elastic range is assumed to be proportional to strain (Fig. 2a). In the inelastic range, a simplified stress block is introduced in which the inelastic relationship between stress and strain is considered to extend through to the middle surface of the plate (Fig. 2b). Consequently, the relationship between moment and curvature may be approximated as in Fig. 3.

Fig. 3 gives principal moments for pure, one-way bending in the plate. The moment ordinate M_u represents the ultimate moment capacity and, from Figs. 1 and 2b, is given by

$$M_u = \left(\sigma_u + \frac{1}{2} \sigma_c \right) \frac{t^2}{6} \quad (1)$$

where σ_u is the ultimate stress and σ_c is the assumed stress ordinate at the middle surface, i.e. the equivalent yield stress for a rigid--strain-hardening material. The yield moment ordinate M_p (Fig. 3) is also based on the stress block of Fig. 2b, viz.

$$M_p = \left(\sigma_p + \frac{1}{2} \sigma_c \right) \frac{t^2}{6} \quad ; \quad (2)$$

as is the ordinate M_c (Fig. 3), which is therefore given by

$$M_c = \frac{\sigma_c}{4} t^2 \quad (3)$$

The magnitudes of the moments M_u and M_c are assumed to remain unchanged for the case of two-way bending in the plate, and the condition $M = M_u$ where M is the maximum moment in the plate, is considered as a failure criterion.

The term yield will be used herein to designate curvature increments past the assumed proportional limit (at M_p) of the plate. The condition of purely plastic yielding, i.e. of zero rate of hardening, is included as a special case.

For bending past the elastic range, yielding occurs over areas or zones of the plate, and for the idealized relationship of moment and curvature assumed in Fig. 3, each zone may be assumed to have a definite boundary. Yielding may occur in the direction of only one principal moment, or in both

directions, and other areas of the plate may remain elastic. In the case of purely plastic yield, a zone of yield may, under certain conditions, be effectively reduced in width to a line and is here termed a yield line. When the rate of hardening is finite and positive however, the yield zones are always of finite width.

Symmetrical bending

For symmetrical bending in the inelastic range, the various zones of yield occur concentrically, and three different inelastic zones are possible. In one, yielding may occur in the tangential (or circumferential) direction; in another, in the radial direction; and in the third, yielding may occur in both directions simultaneously.

For the purpose of analysis, each zone may be considered separately and a separate differential equation obtained for the deflection in each.

The analysis of a plate on a liquid type elastic foundation will be considered initially as a general case. The solution for an edge supported plate may be obtained from this as the limiting case where the density or reaction modulus of the liquid approaches zero.

Hence for symmetrical bending in the elastic range, the classical differential equation for deflection is

$$\nabla^4 w = w^{iv} + \frac{2}{r} w^{iii} - \frac{1}{r^2} w^{ii} + \frac{1}{r^3} w^i = (q - \gamma w)/D \quad (4)$$

The corresponding equations for deflection in the inelastic zones are derived in the Appendix.

Hence for strain-hardening in the tangential direction only, with elastic bending in the radial direction, the differential equation is

$$w^{iv} + \frac{2}{r} w^{iii} - \frac{a}{r^2} w^{ii} + \frac{a}{r^3} w^i = (q - \gamma_w)/D_1 . \quad (5)$$

Where the plate is inelastic only in the radial direction,

$$w^{iv} + \frac{2}{r} w^{iii} - \frac{1}{ar^2} w^{ii} + \frac{1}{ar^3} w^i = (q - \gamma_w)/aD_1 ; \quad (6)$$

and when yielding occurs in each direction of principal stress,

$$w^{iv} + \frac{2}{r} w^{iii} - \frac{1}{r^2} w^{ii} + \frac{1}{r^3} w^i = (q - \gamma_w)/aD . \quad (7)$$

Primes in the above equations denote differentiation with respect to r ; a is the ratio E_p/E of the inelastic and elastic moduli; q is the intensity of lateral load; γ is the density of the supporting liquid; and D , D_1 , aD_1 , and aD are the effective flexural rigidities in the radial direction of the respective zones of the plate. From the linear relationships between stress and strain,

$$D = \frac{Et^3}{12(1-\nu^2)} \text{ and } D_1 = \frac{Et^3}{12(1-a\nu^2)} ,$$

where t is the thickness of the plate, and ν is Poisson's ratio.

For an edge supported plate, $\gamma = 0$, and Eqns. 4 to 7 reduce respectively to

$$w^{iv} + \frac{2}{r} w^{iii} - \frac{1}{r^2} w^{ii} + \frac{1}{r^3} w^i = \frac{q}{D} \quad (8)$$

$$w^{iv} + \frac{2}{r} w^{iii} - \frac{a}{r^2} w^{ii} + \frac{a}{r^3} w^i = \frac{q}{D_1} \quad (9)$$

$$w^{iv} + \frac{2}{r} w^{iii} - \frac{1}{ar^2} w^{ii} + \frac{1}{ar^3} w^i = \frac{q}{aD_1} \quad (10)$$

and

$$w^{iv} + \frac{2}{r} w^{iii} - \frac{1}{r^2} w^{ii} + \frac{1}{r^3} w^i = \frac{q}{aD} \quad (11)$$

Introducing the dimensionless variable $x_j = r/s_j$, ($j = 1, 2, 3, 4$), where $s_1 = \left(\frac{D}{\gamma}\right)^{\frac{1}{4}}$, $s_2 = \left(\frac{D_1}{\gamma}\right)^{\frac{1}{4}}$, $s_3 = \left(\frac{aD_1}{\gamma}\right)^{\frac{1}{4}}$, and $s_4 = \left(\frac{aD}{\gamma}\right)^{\frac{1}{4}}$ in the respective zones of the plate, and setting $q = 0$, Eqns. 4 through 7 simplify respectively to

$$w^{iv} + \frac{2}{x_1} w^{iii} - \frac{1}{x_1^2} w^{ii} + \frac{1}{x_1^3} w^i + w = 0 \quad (12)$$

$$w^{iv} + \frac{2}{x_2} w^{iii} - \frac{a}{x_2^2} w^{ii} + \frac{a}{x_2^3} w^i + w = 0 \quad (13)$$

$$w^{iv} + \frac{2}{x_3} w^{iii} - \frac{1}{ax_3^2} w^{ii} + \frac{1}{ax_3^3} w^i + w = 0 \quad (14)$$

and

$$w^{iv} + \frac{2}{x_4} w^{iii} - \frac{1}{x_4^2} w^{ii} + \frac{1}{x_4^3} w^i + w = 0 \quad (15)$$

where primes in this case, and from here on, denote differentiation with respect to x . For $q = 0$, Eqns. 8 through 11 may be correspondingly written in the form

$$w^{iv} + \frac{2}{x} w^{iii} - \frac{1}{x^2} w^{ii} + \frac{1}{x^3} w^i = 0 \quad (16)$$

$$w^{iv} + \frac{2}{x} w^{iii} - \frac{a}{x^2} w^{ii} + \frac{a}{x^3} w^i = 0 \quad (17)$$

$$w^{iv} + \frac{2}{x} w^{iii} - \frac{1}{ax^2} w^{ii} + \frac{1}{ax^3} w^i = 0 \quad (18)$$

$$w^{iv} + \frac{2}{x} w^{iii} - \frac{1}{x^2} w^{ii} + \frac{1}{x^3} w^i = 0 \quad (19)$$

where x in this case is introduced only for dimensional consistency with Eqns. 12 through 15 and may be replaced by r .

The solution of Eqn. 12, and hence, also, the solution of Eqn. 15, may be written

$$w = \text{Re} \left[A J_0(x_j \sqrt{i}) + B H_0^{(1)}(x_j \sqrt{i}) \right] \quad j = 1, 4 \quad (20)$$

where $J_0(\)$ and $H_0^{(1)}(\)$ are Bessel functions of the first and third kinds respectively, and of zero order. A and B are complex constants and are determined from the boundary conditions.

The solution of Eqn. 13 may be expressed in the form

$$w = C_1 R_1 + C_2 R_2 + C_3 R_3 + C_4 R_4 \equiv C_k R_k \quad (21)$$

in which R_k are four independent series solutions and C_k are real constants.

$$R_1 = 1 + \sum_{4,8}^{\infty} \frac{[-1]^{n/4} x_j^n}{n(n-2) \left[(n-1)^2 - a \right] \cdot (n-4)(n-6) \left[(n-5)^2 - a \right] \cdot \dots \cdot 4 \cdot 2 \cdot (3^2 - a)}$$

$$R_2 = x_j^2 + \sum_{6,10}^{\infty} \frac{[-1]^{(n-2)/4} x_j^n}{n(n-2) \left[(n-1)^2 - a \right] \cdot \dots \cdot 6 \cdot 4(5^2 - a)}$$

$$R_3 = x_j^{1+a^{1/2}} + \sum_{5+a^{1/2}, 9+a^{1/2}}^{\infty} \frac{[-1]^{(n-1-a^{1/2})/4} x_j^n}{n(n-2) \left[(n-1)^2 - a \right] \cdot \dots \cdot (5+a^{1/2})(3+a^{1/2})(4+2a^{1/2})4}$$

and

$$R_4 = x_j^{1-a^{1/2}} + \sum_{5-a^{1/2}, 9-a^{1/2}}^{\infty} \frac{[-1]^{(n-1+a^{1/2})/4} x_j^n}{n(n-2) \left[(n-1)^2 - a \right] \cdot \dots \cdot (5-a^{1/2})(3-a^{1/2})(4-2a^{1/2})4}$$

The series R_k are independent except at $a = 0, \pm 1, \pm 4, \dots$. At $a = 1$, Eqn. 13 reduces to Eqn. 12, whose solution is given by Eqn. 17; and R_j as defined above reduced to only two independent series, representing the Bessel functions $\text{Re} J_0(x \sqrt{i})$ and $\text{Im} J_0(x \sqrt{i})$. There is, in effect, no inelastic range in this case. On the other hand, when $a = 0$, the behavior is purely plastic, past the elastic range, and R_k are given by

$$R_1 = 1 + \sum_{4,8}^{\infty} \frac{[-1]^{n/4} x^n}{n(n-1)^2(n-2) \dots 4 \cdot 3^2 \cdot 2}$$

$$R_2 = x + \sum_{5,9}^{\infty} \frac{[-1]^{(n-1)/4} x^n}{n(n-1)^2(n-2) \dots 5 \cdot 4^2 \cdot 2}$$

$$R_3 = x^2 + \sum_{6,10}^{\infty} \frac{[-1]^{(n-2)/4} x^n}{n(n-1)^2(n-2) \dots 6 \cdot 5^2 \cdot 4}$$

$$R_4 = R_2 \log x + \sum_{n=5,9}^{\infty} \left[\frac{[-1]^{(n-5)/4} x^n}{n(n-1)^2(n-2) \dots 5 \cdot 4^2 \cdot 3} \sum_{r=5,9}^n \frac{2(2r^2 - 4r + 1)}{r(r-1)(r-2)} \right]$$

The solution for w from Eqn. 14 is determined similarly, as

$$w = C_k S_k \quad (22)$$

where S_k may be obtained by substituting $\frac{1}{a}$ for a in R_k , Eqn. 21.

For the case of an edge supported plate, the solution of Eqn. 16 for bending in the elastic range is

$$w = C_1 + C_2 x^2 + C_3 \log x + C_4 x^2 \log x \quad (23)$$

Likewise, the solution of Eqn. 17 for tangential strain-hardening is

$$w = C_1 + C_2 x^2 + C_3 x^{1+a^{\frac{1}{2}}} + C_4 x^{1-a^{\frac{1}{2}}} \quad (24)$$

For radial strain-hardening, from Eqn. 18,

$$w = C_1 + C_2 x^2 + C_3 x^{1+a^{-\frac{1}{2}}} + C_4 x^{1-a^{-\frac{1}{2}}} \quad (25)$$

and for strain-hardening in each direction, the solution for w is given by Eqn. 23, since Eqns. 16 and 19 are identical.

Eqns. 23 to 25 might alternatively have been obtained from the condition that in the limit $\delta \rightarrow 0$, $x_j (= r/s_j)$ also approaches zero and that only the first term in each series solution, for the

plate on a liquid support, is significant, e.g. in Eqn. 21 as $\bar{\nu} \rightarrow 0$, $R_1 \rightarrow 1$, $R_2 \rightarrow x^2$, $R_3 \rightarrow x^{1+a^2}$ and $R_4 \rightarrow x^{1-a^2}$, so that the solution for w in Eqn. 21 approaches the solution for w in Eqn. 24. In the case of symmetrical bending in the elastic range, moments and shears are given by

$$M_r = -D \left[\frac{d^2 w}{dr^2} + \nu \frac{1}{r} \frac{dw}{dr} \right] \quad (26)$$

$$M_\phi = -D \left[\frac{1}{r} \cdot \frac{dw}{dr} + \nu \frac{d^2 w}{dr^2} \right] \quad (27)$$

$$Q_r = -D \left[\frac{d}{dr} \nabla_r^2 w \right] \quad (28)$$

Where strain-hardening occurs in the tangential direction, the corresponding equations (from the appendix) are

$$M_r = -D_1 \left[\frac{d^2 w}{dr^2} + a \nu \frac{1}{r} \frac{dw}{dr} \right] + \nu M_c \quad (29)$$

$$M_\phi = M_c - a D_1 \left[\frac{1}{r} \frac{dw}{dr} + \nu \frac{d^2 w}{dr^2} \right] \quad (30)$$

$$Q_r = -D_1 \left[\frac{d^3 w}{dr^3} + \frac{1}{r} \frac{d^2 w}{dr^2} - a \frac{1}{r^2} \frac{dw}{dr} \right] - \frac{(1-\nu)}{r} M_c \quad (31)$$

and for radial strain-hardening,

$$M_r = M_c - a D_1 \left[\frac{d^2 w}{dr^2} + \nu \frac{1}{r} \frac{dw}{dr} \right] \quad (32)$$

$$M_\phi = -D_1 \left[\frac{1}{r} \frac{dw}{dr} + a \nu \frac{d^2 w}{dr^2} \right] + \nu M_c \quad (33)$$

$$Q_r = -D_1 \left[a \frac{d^3 w}{dr^3} + a \frac{1}{r} \frac{d^2 w}{dr^2} - \frac{1}{r^2} \frac{dw}{dr} \right] + \frac{1-\nu}{r} M_c \quad (34)$$

For strain-hardening in each direction

$$M_r = M_c - a D \left[\frac{d^2 w}{dr^2} + \nu \frac{1}{r} \frac{dw}{dr} \right] \quad (35)$$

$$M_\phi = M_c - a D \left[\frac{1}{r} \frac{dw}{dr} + \nu \frac{d^2 w}{dr^2} \right] \quad (36)$$

$$Q_r = -aD \left[\frac{d^3 w}{dr^3} + \frac{1}{r} \frac{d^2 w}{dr^2} - \frac{1}{r^2} \frac{dw}{dr} \right] \quad (37)$$

Hence, for a plate on an elastic foundation and symmetrical bending in the elastic range, setting

$$w = \frac{s^2}{D} \operatorname{Re} [Z] \quad (38)$$

where

$$Z = A J_0(x\sqrt{i}) + B H_0^{(1)}(x\sqrt{i})$$

gives

$$M_r = -\operatorname{Re} \left[Z^{ii} + \nu \frac{1}{x} Z^i \right] \quad (39)$$

$$M_\phi = -\operatorname{Re} \left[\frac{1}{x} Z^i + \nu Z^{ii} \right] \quad (40)$$

$$Q_r = + \frac{1}{s} \operatorname{Re} [i Z^i] \quad (41)$$

Primes again denote differentiation with respect to x , and each of these functions is available in tabulated form (References 1, 2, 3 and 4).

In the inelastic range, with tangential strain-hardening, setting

$$w = \frac{s^2}{D_1} [C_k R_k] \quad (42)$$

gives

$$M_r = -C_k \left[R_k^{ii} + a \nu \frac{1}{x} R_k^i \right] + \nu M_c \quad (43)$$

$$M_\phi = M_c - a C_k \left[\frac{1}{x} R_k^i + \nu R_k^{ii} \right] \quad (44)$$

$$Q_r = -\frac{1}{s} \left[C_k \left[R_k^{iii} + \frac{1}{x} R_k^{ii} - \frac{a}{x^2} R_k^i \right] + \frac{1-\nu}{x} M_c \right] \quad (45)$$

For radial strain-hardening, and

$$w = \frac{s^2}{aD_1} [C_k S_k] \quad , \quad (46)$$

$$M_r = M_c - C_k \left[S_k^{ii} + \nu \frac{1}{x} S_k^i \right] \quad (47)$$

$$M_t = - C_k \left[\frac{1}{ax} S_k^i + \nu S_k^{ii} \right] + \nu M_c \quad (48)$$

$$Q_r = - \frac{1}{s} \left[C_k \left[S_k^{iii} + \frac{1}{x} S_k^{ii} - \frac{1}{ax^2} S_k^i \right] + \frac{1-\nu}{x} M_c \right] \quad (49)$$

The functions R_j , and hence S_j , and their derivatives, are tabulated herein.

With strain-hardening in each direction, setting

$$w = \frac{s^2}{aD} \operatorname{Re} [Z] \quad , \quad (50)$$

gives
$$M_r = M_c - \operatorname{Re} \left[Z^{ii} + \nu \frac{1}{x} Z^i \right] \quad (51)$$

$$M_\phi = M_c - \operatorname{Re} \left[\frac{1}{x} Z^i + \nu Z^{ii} \right] \quad (52)$$

$$Q_r = + \frac{1}{s} \operatorname{Re} [i Z^i] \quad (53)$$

In the case of an edge supported plate under symmetrical bending in the elastic range, setting

$$w = \frac{s^2}{D} \left[C_1 + C_2 x^2 + C_3 \log x + C_4 x^2 \log x \right] \quad , \quad (54)$$

gives

$$M_r = - \left[2(1+\nu)C_2 - (1-\nu)C_3 \frac{1}{x^2} + C_4 \left[2 \log x + 3 + \nu(2 \log x + 1) \right] \right] \quad (55)$$

$$M_\phi = - \left[2(1+\nu)C_2 + (1-\nu)C_3 \frac{1}{x^2} + C_4 \left[2 \log x + 1 + \nu(2 \log x + 3) \right] \right] \quad (56)$$

$$Q_r = - \frac{1}{s} \left[4C_4 \frac{1}{x} \right] \quad (57)$$

For inelastic bending, with tangential strain-hardening,

$$w = \frac{s^2}{D_1} \left[C_1 + C_2 x^2 + C_3 x^{1+a^{1/2}} + C_4 x^{1-a^{1/2}} \right] \quad (58)$$

$$M_r = - \left[2C_2(1+\nu) + C_3(1+a^{1/2}) (a^{1/2} + \nu a) x^{-1+a^{1/2}} + C_4(1-a^{1/2}) (-a^{1/2} + \nu a) x^{-1-a^{1/2}} \right] + \nu M_c \quad (59)$$

$$M_\phi = M_c - a \left[2C_2(1+\nu) + C_3(1+a^{1/2}) (1+a^{1/2}\nu) x^{-1+a^{1/2}} + C_4(1-a^{1/2}) (1-a^{1/2}\nu) x^{-1-a^{1/2}} \right] \quad (60)$$

$$Q_r = -\frac{1}{s} \left[\frac{2C_2}{x} \left(1 + \frac{1}{a}\right) + \frac{1-\nu}{x} M_c \right] \quad (61)$$

and for radial strain-hardening,

$$w = \frac{s^2}{aD_1} \left[C_1 + C_2 x^2 + C_3 x^{1+a-\frac{1}{2}} + C_4 x^{1-a-\frac{1}{2}} \right] \quad (62)$$

$$M_r = M_c - \left[2C_2(1+\nu) + C_3(1+a^{-\frac{1}{2}})(a^{-\frac{1}{2}} + \nu) x^{-1+a-\frac{1}{2}} \right. \\ \left. + C_4(1-a^{-\frac{1}{2}})(-a^{-\frac{1}{2}} + \nu) x^{-1-a-\frac{1}{2}} \right] \quad (63)$$

$$M_\phi = - \left[2C_2\left(\frac{1}{a} + \nu\right) + C_3(1+a^{-\frac{1}{2}})\left(\frac{1}{a} + \nu a^{-\frac{1}{2}}\right) x^{-1+a-\frac{1}{2}} \right. \\ \left. + C_4(1-a^{-\frac{1}{2}})\left(\frac{1}{a} - \nu a^{-\frac{1}{2}}\right) x^{-1-a-\frac{1}{2}} \right] \quad (64)$$

$$Q_r = -\frac{1}{s} \left[\frac{2C_2}{x} \left(1 - \frac{1}{a}\right) - \frac{1-\nu}{x} M_c \right] \quad (65)$$

With strain-hardening in each direction,

$$w = \frac{s^2}{aD} \left[C_1 + C_2 x^2 + C_3 \log x + C_4 x^2 \log x \right] \quad (66)$$

$$M_r = M_c - \left[2(1+\nu)C_2 - (1-\nu)C_3 \frac{1}{x^2} + C_4 \left[2 \log x + 3 + \nu(2 \log x + 1) \right] \right] \quad (67)$$

$$M_\phi = M_c - \left[2(1+\nu)C_2 + (1-\nu)C_3 \frac{1}{x^2} + C_4 \left[2 \log x + 1 + \nu(2 \log x + 3) \right] \right] \quad (68)$$

$$Q_r = -\frac{1}{s} \left[4 C_4 \frac{1}{x} \right] \quad (69)$$

Unsymmetrical bending

For unsymmetrical bending in the elastic range, the differential equation to deflection for the plate on an elastic foundation is

$$\nabla_{r,\phi}^4 w = (q - \gamma w)/D \quad (70)$$

where ∇^2 in this case is:

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} ,$$

and r and ϕ are the radial and angular coordinates respectively.

Similarly, when strain-hardening has occurred in each direction of the plate, the differential equation for deflection is

$$\nabla_{r,\phi}^4 w = (q - \gamma w)/aD \quad (71)$$

Where yielding occurs in one direction only, and the bending is unsymmetrical, the directions of principal stress, and hence the direction of yield, will vary from point to point, and a general solution in a usable form appears to be impracticable.

For $q = 0$, Eqns. 70 and 71 may each be reduced to

$$\nabla_{x,\phi}^4 w + w = 0 \quad (72)$$

where $x = \frac{r}{s}$ and $s = \left[\frac{D}{\sigma}\right]^{\frac{1}{4}}$ or $\left[\frac{aD}{\sigma}\right]^{\frac{1}{4}}$ in the respective zones of the plate.

The solution for w of Eqn. 72 is

$$w = \sum_{n=0}^{\infty} \operatorname{Re} \left[A_n J_n(x\sqrt{i}) + B_n H_n^{(1)}(x\sqrt{i}) \right] \frac{\cos n\phi}{\sin n\phi} \quad (73)$$

where A_n and B_n are complex constants, and $J_n(\)$ and $H_n^{(1)}(\)$ are Bessel functions of the first and third kinds respectively, and of order n .

In the elastic range, moments and shears are given in terms of deflections by

$$M_r = -D \left[\frac{\partial^2 w}{\partial r^2} + \nu \left[\frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \phi^2} \right] \right] \quad (74)$$

$$M_\phi = -D \left[\frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \phi^2} + \nu \frac{\partial^2 w}{\partial r^2} \right] \quad (75)$$

$$M_{r\phi} = (1 - \nu) D \left[\frac{1}{r} \frac{\partial^2 w}{\partial r \partial \phi} - \frac{1}{r^2} \frac{\partial w}{\partial \phi} \right] \quad (76)$$

* In a particular case, choice of a suitable origin for ϕ usually enables the solution to be expressed in terms of the cosine series only.

$$Q_r = -D \frac{\partial}{\partial r} \left[\nabla_{r,\phi}^2 w \right] \quad (77)$$

$$Q_\phi = -D \frac{\partial}{r \partial \phi} \left[\nabla_{r,\phi}^2 w \right] \quad (78)$$

In the inelastic range the corresponding expressions are

$$M_r = M_c - aD \left[\frac{\partial^2 w}{\partial r^2} + \nu \left[\frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \phi^2} \right] \right] \quad (79)$$

$$M_\phi = M_c - aD \left[\frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \phi^2} + \nu \frac{\partial^2 w}{\partial r^2} \right] \quad (80)$$

$$M_{r\phi} = (1-\nu) aD \left[\frac{1}{r} \frac{\partial^2 w}{\partial r \partial \phi} - \frac{1}{r^2} \frac{\partial w}{\partial \phi} \right] \quad (81)$$

$$Q_r = -aD \frac{\partial}{\partial r} \left[\nabla_{r,\phi}^2 w \right] \quad (82)$$

$$Q_\phi = -aD \frac{\partial}{r \partial \phi} \left[\nabla_{r,\phi}^2 w \right] \quad (83)$$

Hence for a plate on an elastic foundation and unsymmetrical bending in the elastic range, setting

$$w = \frac{s^2}{D} \operatorname{Re} \left[Z_n \right] \cos n\phi^* \quad \text{where } n^{**} = 0, 1, 2, \dots, \infty$$

$$\text{and } Z_n = A_n J_n(x\sqrt{i}) + B_n H_n^{(1)}(x\sqrt{i}) \quad (84)$$

gives (for $\nu = 0$),^{***}

$$M_r = -\operatorname{Re} \left[\frac{\partial^2}{\partial x^2} Z_n \right] \cos n\phi \quad (85)$$

$$M_\phi = -\operatorname{Re} \left[\left(\frac{1}{x} \frac{\partial}{\partial x} - \frac{n^2}{x^2} \right) Z_n \right] \cos n\phi \quad (86)$$

$$M_{r\phi} = + (1-\nu) \operatorname{Re} \left[n \left(\frac{1}{x} - \frac{1}{x} - \frac{1}{x^2} \right) Z_n \right] \sin n\phi \quad (87)$$

* For greater generality, a sine series would again be included. It is omitted here for simplicity.

** A summation convention, i.e. a summation from $n = 0$ to $n = \infty$ is intended.

*** For other values of ν , moments are given by superposition.

$$Q_r = \frac{1}{s} \operatorname{Re} \left[i \frac{\partial}{\partial x} (Z_n) \right] \cos n \phi \quad (88)$$

$$Q_\phi = -\frac{1}{s} \operatorname{Re} \left[i n Z_n \right] \sin n \phi \quad (89)$$

In the inelastic range, setting

$$w = \frac{s^2}{aD} \operatorname{Re} \left[Z_n \right] \cos n \phi \quad n = 0, 1, 2, \dots, \infty \quad (90)$$

gives (for $\nu = 0$)

$$M_r = M_c - \operatorname{Re} \left[\frac{\partial^2}{\partial x^2} Z_n \right] \cos n \phi \quad (91)$$

$$M_\phi = M_c - \operatorname{Re} \left[\left(\frac{1}{x} \frac{\partial}{\partial x} - \frac{n^2}{x^2} \right) Z_n \right] \cos n \phi \quad (92)$$

$$M_{r\phi} = + (1-\nu) \operatorname{Re} \left[n \left(\frac{1}{x} \frac{\partial}{\partial x} - \frac{n^2}{x^2} \right) Z_n \right] \sin n \phi \quad (93)$$

$$Q_r = \frac{1}{s} \operatorname{Re} \left[i \frac{\partial}{\partial x} (Z_n) \right] \cos n \phi \quad (94)$$

$$Q_\phi = -\frac{1}{s} \operatorname{Re} \left[i n Z_n \right] \sin n \phi \quad (95)$$

For an edge supported plate the homogeneous differential equation for deflection reduces to

$$\nabla_{x,\phi}^4 w = 0 \quad (96)$$

and the solution for w in the elastic range may be written

$$w = \frac{s^2}{D} \left[\begin{array}{l} C_1 + C_2 x^2 + C_3 \log x + C_4 x^2 \log x \\ C_{11} x + C_{12} x^3 + C_{13} x^{-1} + C_{14} x \log x \end{array} \right] \frac{\cos \phi}{\sin \phi} \\ + \left[\begin{array}{l} C_{n1} x^n + C_{n2} x^{n+2} + C_{n3} x^{-n} + C_{n4} x^{-n+2} \end{array} \right] \frac{\cos n \phi}{\sin n \phi} \quad]_{n=2,3,\dots,\infty} \quad (97)$$

where C_n are real constants, and moments and shears are given in terms of deflection by Eqns. 74 through 78 above.

In the inelastic range, the differential equation for deflection is again Eqn. (96) and the solution for w may be similarly expressed, as

$$w = \frac{s^2}{aD} \left[C_1 + C_2 x^2 + C_3 \log x + C_4 x^2 \log x + \left[C_{11} x + C_{12} x^3 + C_{13} x^{-1} + C_{14} x \log x \right] \frac{\cos \phi}{\sin \phi} \right. \\ \left. + \left[C_{n1} x^n + C_{n2} x^{n+2} + C_{n3} x^{-n} + C_{n4} x^{-n+2} \right] \frac{\cos n \phi}{\sin n \phi} \right] \quad n=2,3,\dots,\infty \quad (98)$$

which differs from Eqn. 97 only by the arbitrary presence of the parameter 'a' in the coefficient. Moments and shears are obtained from Eqns. 79 through 83.

Boundary Conditions

The constants in every case are evaluated from the boundary conditions, which, except when yielding is confined to a line, are the same as for an entirely elastic plate. Hence at a clamped edge

$$w = 0 = \frac{dw}{dn} \quad , \quad (99)$$

at a simple support $w = 0 = M_n \quad , \quad (100)$

and at a free edge $M_n = 0 = V_n \left(\equiv Q_n - \frac{\partial M_{nt}}{\partial t} \right) \quad (101)$

where n is the normal to the edge. At boundaries between the different zones, deflections, slopes, moments, and shears are continuous.

For purely plastic yield ($a = 0$), and yielding is considered to be confined to a line, a discontinuity of slope results at the line, and the condition $M_n = M_p$ is substituted for the slope condition.

If the line occurs at a boundary, a specified deflection or shear provides the second determinative condition, and if the line is straight but is not at a boundary (of a load, reaction, or change in plate thickness), the required condition is $Q_n = 0$.

Typical applications are now considered.

APPLICATIONS

Simply supported circular plate, ring loading

Consider the case of a simply supported circular plate loaded with a ring load about the center, Fig. 4.

In the elastic range, maximum moment occurs within the circle of loading where a state of pure bending exists, i.e. $M_r = M_\phi = M_o$ (constant).

If the load is steadily increased, M_o will increase to M_p (Fig. 3) and yielding may occur in each direction throughout the loaded circle. The surrounding plate is still elastic at this stage.

Under increased loading, radial and tangential yielding each extend concentrically into the surrounding plate, the radius of tangential yield extending the farther of the two. Radial yield tends to be limited to a narrow zone surrounding the loaded circle, but tangential yield may extend to the edge of the plate before failure occurs, depending on the proportions involved.

Failure is produced when the maximum moment, the moment within the loaded circle, reaches the ultimate moment, M_u .

Hence for inelastic analysis, at a specific load, the plate might be divided into three concentric zones. In the central zone (extending outside the loaded circle) yielding occurs in each direction; in the intermediate zone only tangential yielding occurs; and in the outer portion, the plate remains elastic.

Moments, shears, and deflections in the outer zone are given by Eqns. 54 through 57, in the next, by Eqns. 58 through 61, and in the inner zone, by Eqns. 66 through 69. The radii of yield and the

constants involved are evaluated from the total load, the continuity conditions between the zones and the conditions of simple support at the edge.

Plate on elastic foundation, concentrically loaded circular insert

The case of a large or infinite plate on a liquid type elastic foundation and with a concentrically loaded rigid circular insert, is now considered (Fig. 5).

In the elastic range of stresses the deflection is given by Eqn. 38. Constants are evaluated from the condition of zero slope at the edge of the insert, and from the magnitude of the shear at this point. Hence from Eqn. 39 the maximum moment occurs at the edge of the insert, in the radial direction, and it is here that yielding commences. Yielding begins when $M_r = M_p$, say at a load P_1 , and under increased load a narrow zone of radial yield develops around the insert edge, forming in effect a plastic hinge, and allowing symmetrical rotation of the plate. The increased slope causes tangential moment to increase until it too reaches M_p , at load P_2 .

Under larger loads, yielding occurs in the tangential direction, commencing near to the edge of the insert and spreading concentrically outwards.

At loads equal to or less than P_2 , in which yielding occurs only in the radial direction, deflections, moments, and shears within the yielded zone are given by Eqns. 46 through 49. Deflections, moments, and shears in the surrounding plate are given by Eqns. 38 through 41.

At loads greater than P_2 two additional zones may be formed, since tangential yielding can occur both within the area of radial

yield, and outside it. Deflections, moments, and shears within these zones are given by Eqns. 50 through 53 and Eqns. 42 through 45 respectively.

Boundary conditions are that: slope equals zero at the insert edge, $M_r = M_p$ at the edge of radial yield, $M_\phi = M_p$ at the edge of tangential yield, there is continuity between the zones, and $w = 0 = \frac{dw}{dr}$ at infinity. Failure occurs when $M_r = M_u$ at the insert edge.

In the limit $a \rightarrow 0$, the radial yield zone is considered to be reduced in width to a line, a yield line. The above conditions at each edge of the zone (or line) still hold, necessitating a discontinuity of slope across the line, as previously noted, and infinite strains in the outer fibres of the plate. This, however, is merely a convenient approximation for analysis of the surrounding plate. In practice, yielding in the outer fibres again occurs over a finite width of plate so that strains remain finite and continuity of slope is preserved.

For the particular case where the radius of the insert is $0.4s$, and $a = 1/2$, moments at various stages of yield are shown in Fig. 5. For load P_1 , at which yield commences, moments are shown in Fig. 5b; and moments through the elastic range are proportional to these. At load P_2 , tangential yield commences and moments are as shown in Fig. 5c. A zone of tangential yield has developed ($M_\phi \geq M_p$) in Fig. 5d, and radial moment at the insert edge continues to increase. In Fig. 5e the radial moment at the edge increases to M_u and the failure load is reached.

Thin shallow spherical shell

The case of a plate on an elastic foundation is of additional interest in that it is approximately analogous, during limited yielding

(say $P \leq P_2$), to the case of a shallow segment, or zone, of a thin spherical (or paraboloidal) shell.

Consider a thin spherical shell with a concentrically loaded, rigid circular insert centered at the pole (i.e. origin). Since significant displacements are limited to a shallow segment or zone surrounding the insert, moments, shears, and deflections in the elastic range are given by Eqns. 38 through 41--from either shallow shell or shallow zone theory (depending on the expression used for x --see Reference 4).

The parameter s (or L in the present context) is given in this case by

$$s \text{ (or } L) = \frac{t^{1/2} R^{1/2}}{[12(1-\nu^2)]^{1/4}}$$

where R is the radius of the shell. The elastic foundation is replaced in effect by the elastic membrane forces (N_r and N_t) in the shell, and in the elastic range, the sum of the magnitudes of these forces is approximately proportional to the deflection w ,

$$\text{i.e. } N_r + N_t = \frac{Et w}{R} \equiv \gamma w \cdot R$$

In the inelastic range, provided the extent of yield is limited, its effect might be considered to be restricted to the moment-curvature relationship, the inplane forces remaining proportional to net inplane strains so that the above relationship would again apply. Hence in the range of yielding for which this assumption is sufficiently accurate, bending in the shell is again analogous to that in the plate on an elastic foundation, and the equations for this latter case are again applicable.

For the extreme case where $a = 0$ (i.e. pure plasticity), radial yielding is confined to a line and no approximation is here involved. In

the other extreme where $a = 1$, the shell remains elastic and the above assumption is again exact. Between these limits, it appears that at least sufficient accuracy for design calculations may be obtained.

Thus for loads less than or equal to P_2 , where only radial yielding occurs, and is confined to a narrow band, deflections, moments, and shears within the yielded area are given by Eqns. 46 through 49, and in the surrounding shell by Eqns. 38 through 41.

Similarly, the effect of a symmetrical moment concentration due to a fitting in the shell, and its relief by limited yielding around the fitting, may be readily analysed, allowing the ultimate load capacity to be determined.

DISCUSSION

Thus for strain hardening ($a > 0$) in the post elastic range, yielding occurs over finite areas of the plate. For the idealized moment-curvature relationship shown in Fig. 3, these areas may be considered to have definite boundaries.

Yielding may occur in one direction of principal moment, or in both. For yielding in one direction, only the symmetrical cases of radial yield alone and of tangential yield alone are readily handled, and solutions for these are presented. For yielding in two directions, solutions are developed for both symmetrical and unsymmetrical bending in the plate. The solutions in each case are applicable to bending in any quadrant* of the plate, that is, for moments of the same, or opposite, sign.

For design calculations, however, particularly where the ratio a , of $\frac{E_p}{E}$, is small compared with unity, the approximation might be introduced that narrow bands of yield be considered as lines--as in the purely plastic case. This would not only further simplify the analysis, but extend its application to many cases of unsymmetrical bending, with yielding in one direction only.

For structural materials the magnitude of 'a' will normally lie between 0 and 1 (Fig. 6a and b). Values outside this range, either greater than 1 or less than 0, are also admissible mathematically, and these may have certain applications, Fig. 6c and d. Multiple values of a , for which the moment curvature diagram may be approximated by three or more straight lines, can also be handled, Fig. 6e. Each

* In the case of purely plastic yield, $a = 0$, the Tresca yield criteria may be introduced, providing different behavior in each quadrant of bending, as described in Reference 7.

value of 'a' is considered to produce a separate zone of yield in the plate, each of which has a definite boundary as before. Multiple values of 'a' could be used to approximate more accurately a given moment-curvature relationship. If one of the values chosen were $a = 0$, the effect of strain-hardening in mild steel could be considered, Fig. 6f. In general, though, the use of multiple values of 'a' would be an unnecessary refinement.

Using superposition, the solutions presented may be readily applied to the analysis of limited yielding in continuous plates with intermittent supports, and this is outlined for the special case of $a = 0$, in Reference 7.

The application of the solutions to limited yielding in shells is of special interest and will be developed further in a later report.

Use of Tables

The functions R_j and their derivatives with respect to x are evaluated in the range $0 \leq x \leq 10$, for various values of a . It will be noted that the table R_j ($a=4$) is also the table for S_j ($a=1/4$) and more generally, R_j ($a=a_1$) \equiv S_j ($a=\frac{1}{a_1}$), allowing both R_j and S_j to be determined from one set of tables.

The functions are tabulated to eight significant figures. The last two digits in the number indicate the exponent power of 10, plus 50, and are used to locate the decimal point, vis.

$$\begin{aligned} 1111111150 & \text{ is } 0.11111111 \times 10^0 \\ 1111111152 & \text{ is } 0.11111111 \times 10^2 \\ 1111111148 & \text{ is } 0.11111111 \times 10^{-2} \\ 1111111149- & \text{ is } -0.11111111 \times 10^{-1} \end{aligned}$$

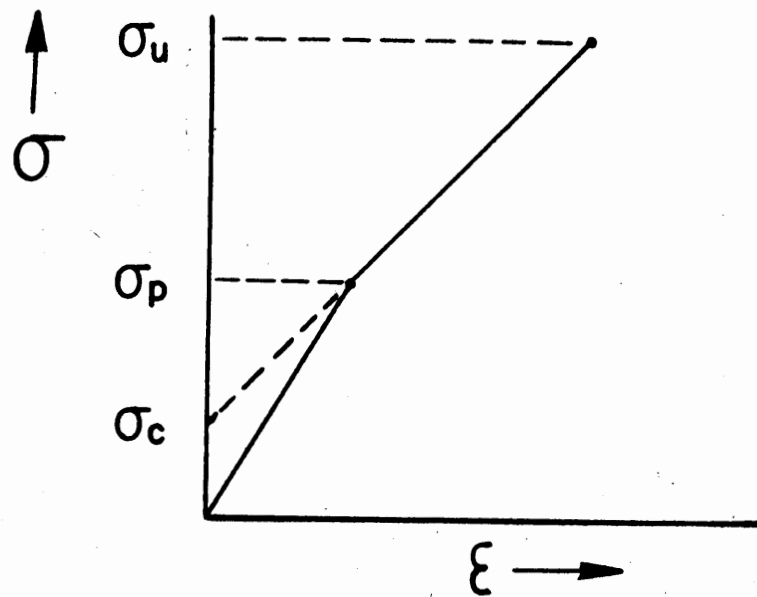


Fig.1 - Stress - strain relationship

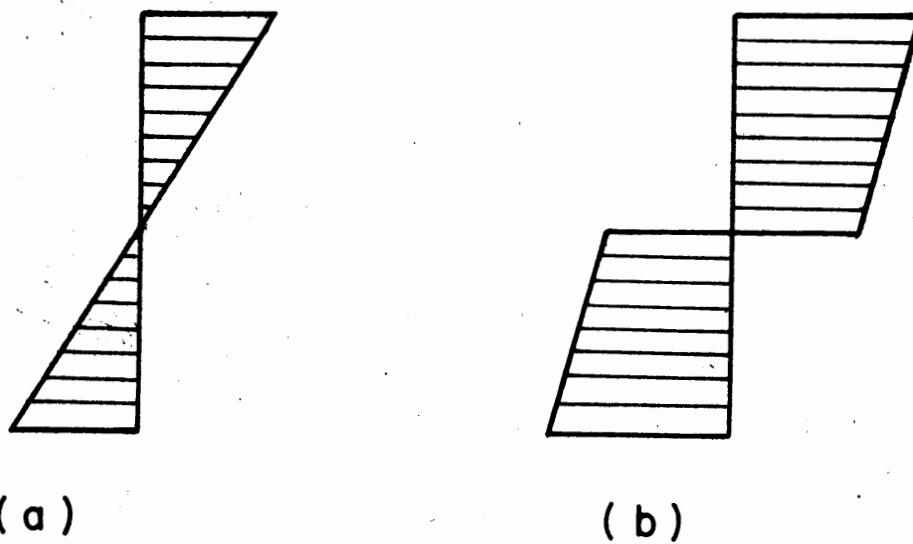


Fig.2 (a) elastic, (b) inelastic, stress blocks

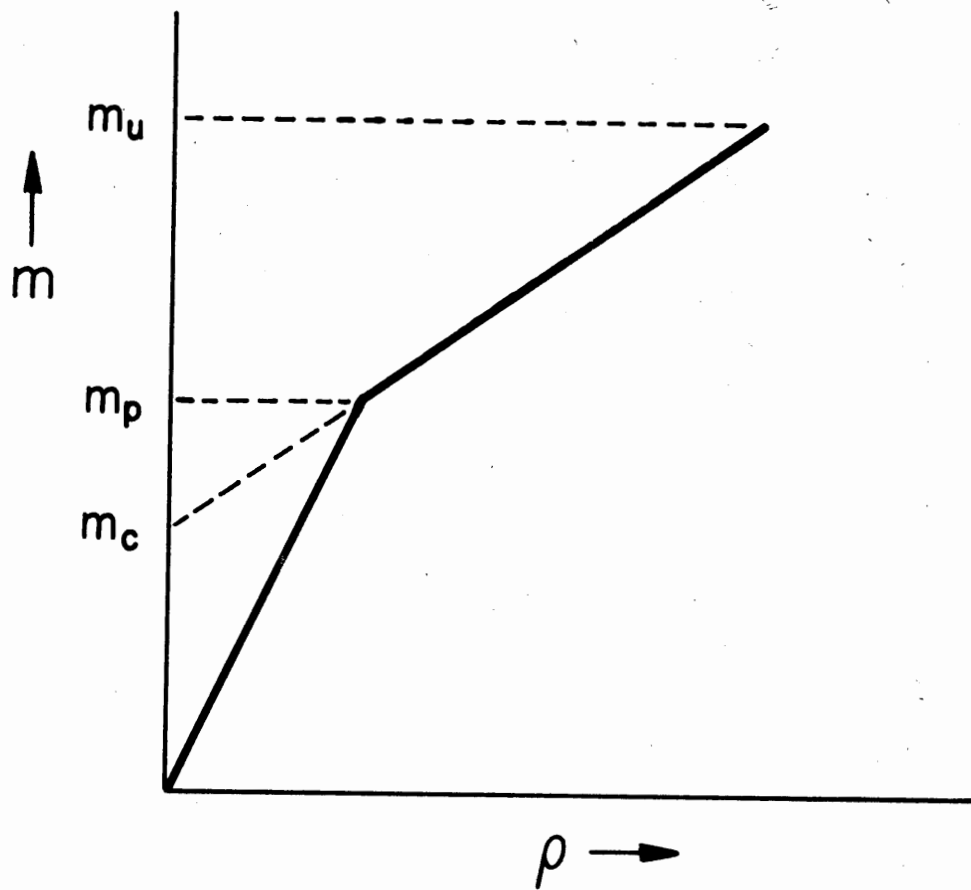


Fig. 3 Moment curvature relationship

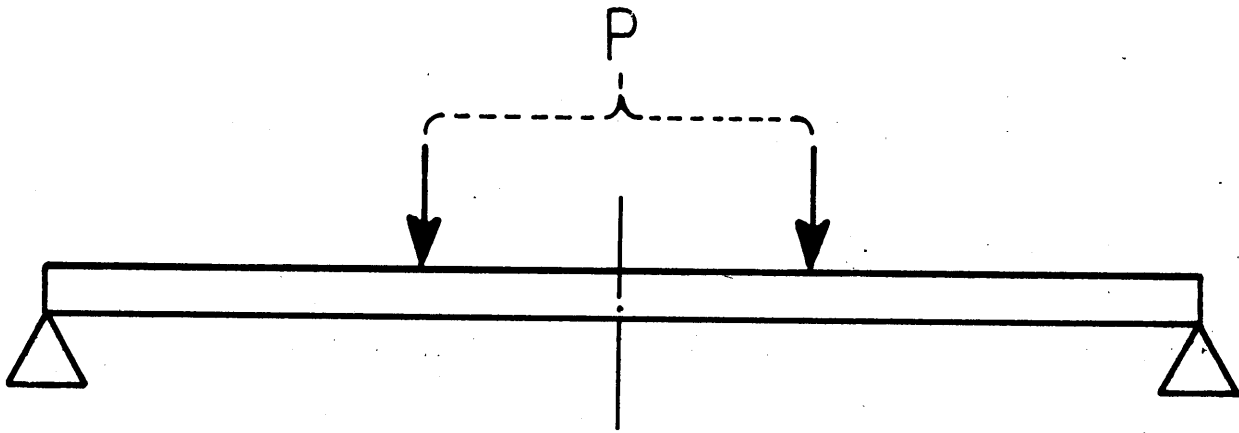


Fig. 4 Simply supported circular plate with ring load P

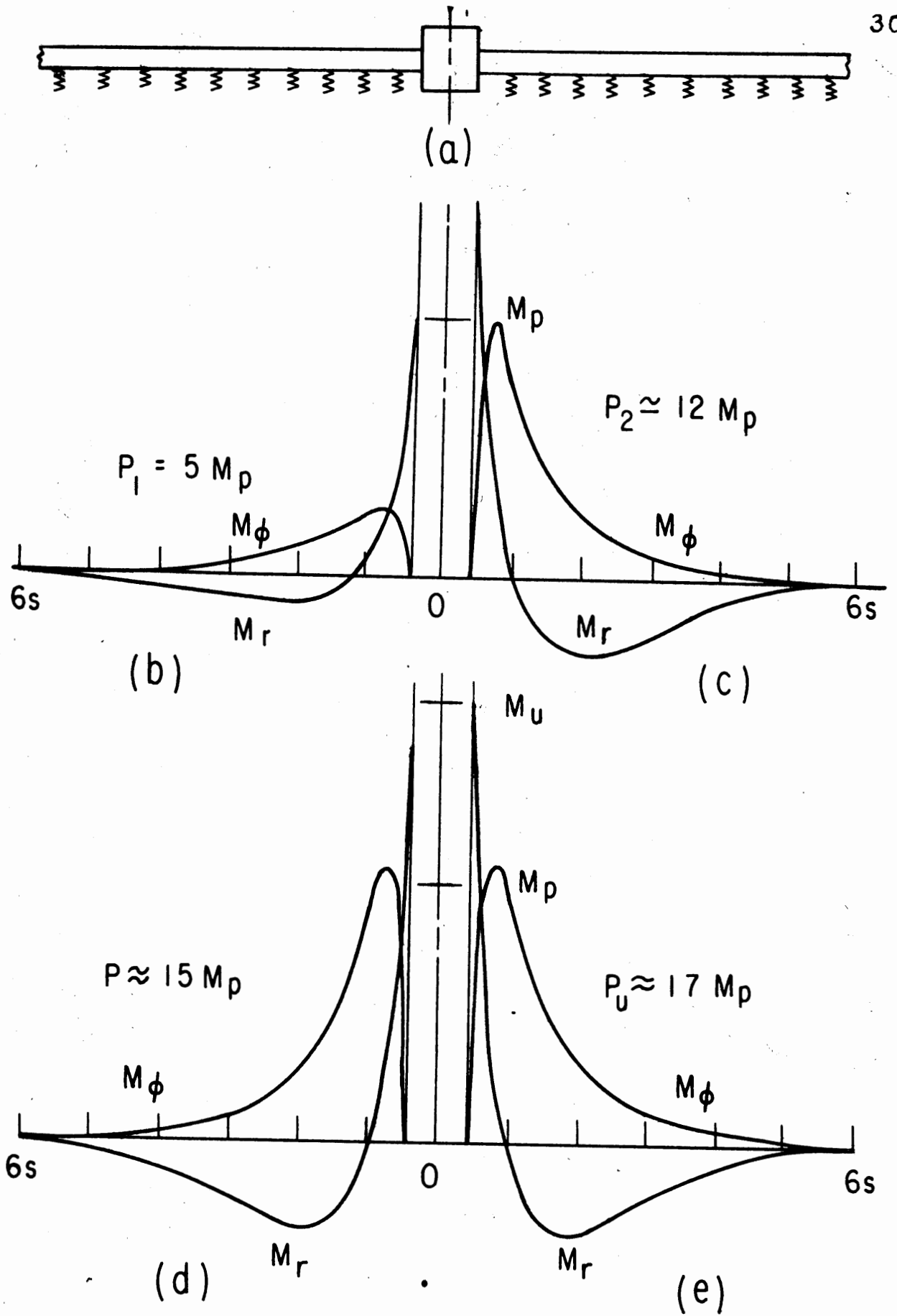


FIG.5 DEVELOPMENT OF MOMENTS AROUND A CONCENTRICALLY LOADED RIGID CIRCULAR INSERT IN A PLATE ON AN ELASTIC FOUNDATION .
(NOT TO SCALE)

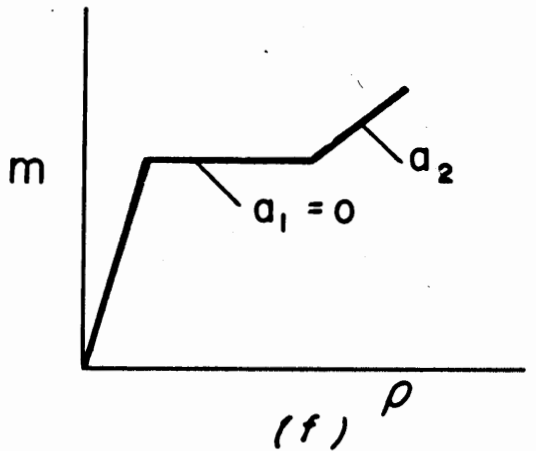
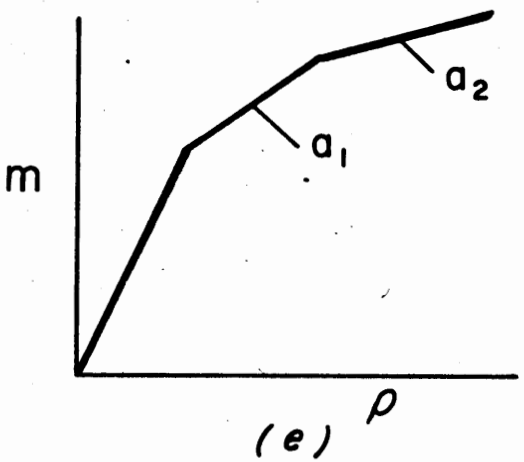
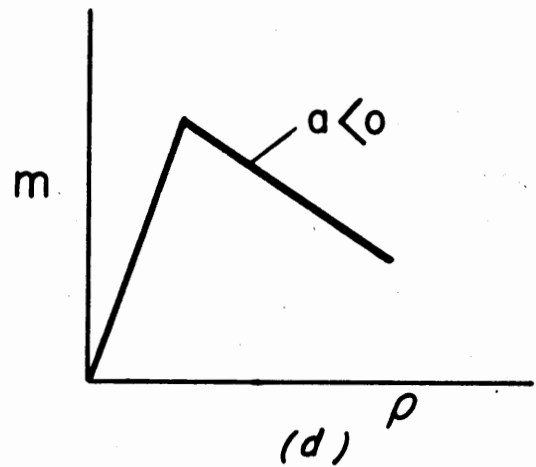
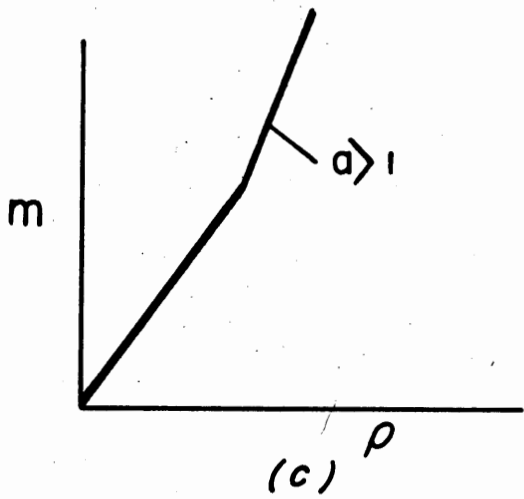
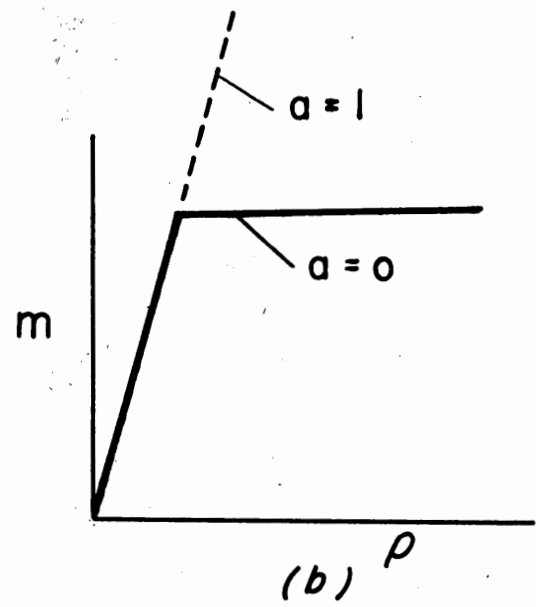
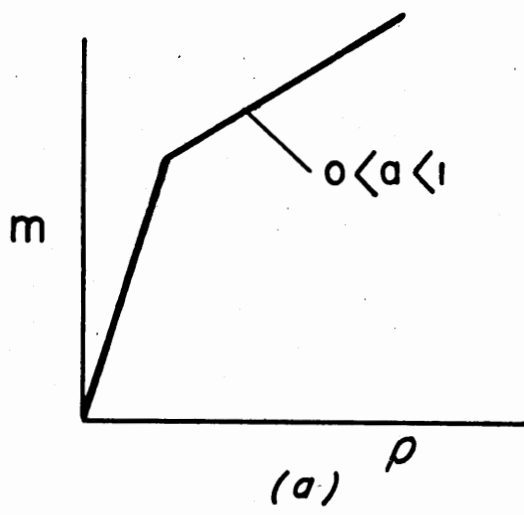


Fig. 6 Applications - Possible moment curvature relationships

TABLE I

$\alpha = 0$

X	R ₁	R ₁ '	R ₁ ''	R ₁ '''
0	1000000051			
0.1	9999986150	5555555146	-1666666348	-3333331349-
0.2	9999777850	4444438447	-6666645548	-6666603349-
0.3	9998875050	1499989748	-1499975949	-9999518049-
0.4	9996444550	3555478248	-2666531249	-1333130250-
0.5	9991319750	6944075648	-4166150049	-1666046650-
0.6	9982001050	1199867849	-5998457249	-1998457250-
0.7	9966656250	1905166549	-8162776349	-2329998850-
0.8	9943121050	2843453849	-1065799950	-2660165850-
0.9	9908900450	4047740649	-1348242850	-2988286050-
1.0	9861170250	5550831949	-1663360350	-3313497450-
1.1	9796779350	7385240049	-2010809750	-3634724750-
1.2	9712253950	9583076149	-2390129050	-3950656350-
1.3	9603801050	1217592150	-2800712350	-4259721050-
1.4	9467315750	1519466650	-3241783050	-4560066350-
1.5	9298387950	1866932650	-3712363550	-4849536750-
1.6	9092312950	2262882950	-4211245550	-5125651350-
1.7	8844103050	2710076350	-4736954850	-5385585350-
1.8	8548502750	3211109450	-5287717350	-5626150050-
1.9	8200006150	3768384950	-5861420750	-5843775050-
2.0	7792878350	4384075750	-6455575850	-6034492350-
2.1	7321180050	5060085450	-7067276750	-6193922350-
2.2	6778797250	5798004450	-7693157750	-6317260750-
2.3	6159474150	6599061150	-8329351750	-6399270050-
2.4	5456852650	7464072250	-8971445350	-6434272750-
2.5	4664515850	8393381150	-9614435750	-6416147350-
2.6	3776038750	9386801750	-1025268651	-6338330750-
2.7	2785043850	1044355151	-1087987951	-6193821350-
2.8	1685265250	1156217651	-1148898051	-5975190050-
2.9	4706183749	1274048451	-1207218651	-5674595350-
3.0	8647216249	-1397546351	-1262089351	-5283804050-
3.1	2326234050	-1526319951	-1312565451	-4794221350-
3.2	3918957650	-1659878651	-1357614751	-4196923350-
3.3	5647388250	-1797624651	-1396114151	-3482702750-
3.4	7515365650	-1938842751	-1426847551	-2642120550-
3.5	9525952150	-2082691651	-1448503351	-1665567350-
3.6	1168130251	-2228193951	-1459673351	-5433353049-
3.7	1398251851	-2374226751	-1458852051	-7342979749
3.8	1642950751	-2519511351	-1444437251	-2176975550
3.9	1902081351	-2662604451	-1414730851	-3794153350
4.0	2175344851	-2801888251	-1367941951	-5594979350
4.1	2462272151	-2935560851	-1302190051	-7588158750
4.2	2762203751	-3061628751	-1215510451	-9781805050
4.3	3074271451	-3177897351	-1105861051	-1218327451
4.4	3397376751	-3281964951	-9711306350	-1479898351
4.5	3730170751	-3371215251	-8091494750	-1763421651
4.6	4071031651	-3442812651	-6177013250	-2069290251
4.7	4418043151	-3493698351	-3945385550	-2397739251
4.8	4768971651	-3520587751	-1373988750	-2748820051
4.9	5121244051	-3519970251	-1559748550	3122374651

$$a = 0$$

X	R ₁	R ₂	R ₃	R ₄
5.0	5471924551	-3488110651	-4878122550	3518006351
5.1	5817693151	-3421053051	-8602901250	3935049351
5.2	6154823151	-3314627951	-1275504051	4372538051
5.3	6479160851	-3164461451	-1735436351	4829172351
5.4	6786105451	-2965988751	-2241921551	5303282751
5.5	7070590551	-2714470251	-2796606851	5792794751
5.6	7327068351	-2405013051	-3400910351	6295189751
5.7	7549494651	-2032595851	-4055974751	6807467751
5.8	7731318151	-1592098451	-4762616551	7326106751
5.9	7865472251	-1078338251	-5521272751	7847023751
6.0	7944370151	-4861096150	-6331941751	8365533351
6.1	7959905751	-1897678650	7194122551	8876309051
6.2	7903458051	-9543970050	8106746251	9373341351
6.3	7765902251	-1812742651	9068109251	9849902051
6.4	7537626751	-2769563751	1007579652	1029850652
6.5	7208557251	-3829340151	1112660352	1071087752
6.6	6768189251	-4996187151	1221646152	1107791352
6.7	6205628151	-6273767251	1334034452	1138966452
6.8	5509640251	-7665189451	1449219152	1163530252
6.9	4668713151	-9172899451	1566481152	1180310552
7.0	3671127551	-1079856552	1684979752	1188044552
7.1	2505041651	-1254294952	1803743352	1185378352
7.2	1158588451	-1440577252	1921660752	1170866852
7.3	3800133650	1638556952	2037470852	1142975652
7.4	2122332951	1847954052	2149755852	1100082952
7.5	4079585551	2068338052	2256930752	1040483052
7.6	6262474151	2299111452	2357236052	9623913351
7.7	8681012851	2539491752	2448730752	8639509051
7.8	1134433452	2788493152	2529284052	7432401051
7.9	1426047252	3044906452	2596570552	5982825751
8.0	1743613652	3307281152	2648065052	4270589351
8.1	2087645352	3573902352	2681038552	2275207851
8.2	2458469552	3842773752	2692556752	2393202049-
8.3	2856198752	4111594252	2679478852	2647385651-
8.4	3280699452	4377738452	2638460252	5615438051-
8.5	3731558652	4638236752	2565955352	8947862351-
8.6	4208047952	4889755452	2458224852	1266365052-
8.7	4709086952	5128576852	2311344752	1678070752-
8.8	5233203252	5350583152	2121218052	2131552052-
8.9	5778492252	5551237352	1883592552	2628278852-
9.0	6342573952	5725572252	1594078852	3169501552-
9.1	6922549452	5868173952	1248175752	3756207352-
9.2	7514956052	5973174452	8412982051	4389072052-
9.3	8115720752	6034246752	3688117251	5068407752-
9.4	8720115052	6044595652	1739294051	-5794109352-
9.5	9322707652	5996964452	7915367751	-6565592752-
9.6	9917318752	5883641152	1488537152	-7381733052-
9.7	1049697553	5696465052	2269315052	-8240798352-
9.8	1105386453	5426846252	3138048252	-9140377352-
9.9	1157929653	5065792752	4098636252	-1007730753-
10.0	1206366453	4603936652	5154625252	-1104759853-

$$a = 0$$

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0.3	2999898850	9998312550	2249983848	-2249962149-
0.4	3999573350	9994666750	5333211448	-3999786849-
0.5	4998698050	9986979550	1041608649	-6249186349-
0.6	5996760150	9973001650	1799791849	-8997570049-
0.7	6992997550	9949984550	2857720649	-1224387350-
0.8	7986348150	9914682350	4265106349	-1598634850-
0.9	8975400250	9863352550	6071441549	-2022232450-
1.0	9958343750	9791759750	8325893749	-2494792550-
1.1	1093292051	9695178550	1107716950	-3015775350-
1.2	1189637351	9568399950	1437334550	-3584453350-
1.3	1284540451	9405737750	1826165950	-4199872350-
1.4	1377612051	9201038950	2278826650	-4860808550-
1.5	1468399151	8947695350	2799794050	-5565723550-
1.6	1556380351	8638659450	3393374350	-6312713350-
1.7	1640961851	8266463250	4063662350	-7099457050-
1.8	1721472951	7823241750	4814498250	-7923160550-
1.9	1797162551	7300760550	5649416250	-8780497350-
2.0	1867195451	6690449550	6571588850	-9667548050-
2.1	1930649251	5983442050	7583764550	-1057973751-
2.2	1986511551	5170620750	8688198350	-1151176551-
2.3	2033677851	4242670250	9886576750	-1245754451-
2.4	2070949751	3190138550	1117993551	-1341012651-
2.5	2097033651	2003506450	1256856851	-1436163551-
2.6	2110541151	6732662649	1405193251	-1530320051-
2.7	2109988851	8099893949	-1562854651	-1622488151-
2.8	2093800551	2455468150	-1729587851	-1711560551-
2.9	2060309051	4272066950	-1905022851	-1796309951-
3.0	2007760551	6268249450	-2088660651	-1875382651-
3.1	1934319051	8451910650	-2279860751	-1947293151-
3.2	1838073351	1083022851	-2477826651	-2010418451-
3.3	1717044551	1340949951	-2681592851	-2062993051-
3.4	1569195951	1619496651	-2890009951	-2103105751-
3.5	1392444451	1919062251	-3101730051	-2128696151-
3.6	1184674151	2239900951	-3315191551	-2137552651-
3.7	9437520150	2582099251	-3528604251	-2127312151-
3.8	6675455850	2945552651	-3739933851	-2095460351-
3.9	3539434250	3329940451	-3946887351	-2039334451-
4.0	8781644047	3734698951	-4146898051	-1956127251-
4.1	3936479250	-4158993851	-4337111951	-1842893351-
4.2	8315343150	-4601689951	-4514373451	-1696557551-
4.3	1314550851	-5061321451	-4675213751	-1513926551-
4.4	1844302651	-5536059251	-4815839051	-1291701751-
4.5	2422192551	-6023678251	-4932121451	-1026497151-
4.6	3049379751	-6521523451	-5019590751	-7148587050-
4.7	3726734751	-7026475751	-5073428851	-3532889050-
4.8	4454791651	-7534916351	-5088466951	-6172640049
4.9	5233696451	-8042692151	-5059185351	-5336857050

$$a = 0$$

X	R_2	R'_2	R''_2	R'''_2
5.0	6063151051	-8545080851	-4979716751	-1066036051
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5.2	7869947351	-9511754151	-4645058251	-2325187951
5.3	8843930951	-9963441951	-4376482451	-3058232051
5.4	9861615051	-1038448752	-4030985951	-3864043851
5.5	1091953952	-1076682952	-3601165351	-4745095551
5.6	1201339852	-1110165552	-3079388351	-5703482351
5.7	1313796752	-1137938152	-2457835551	-6740848551
5.8	1428702652	-1158963152	-1728548351	-7858307551
5.9	1545327352	-1172123152	-8834870050	-9056354051
6.0	1662825252	-1176220152	-8540247549	1033477152
6.1	1780226752	-1169975652	-1186113151	1169253252
6.2	1896430452	-1152032252	-2426489851	1312769052
6.3	2010195452	-1120954952	-3814133051	1463727052
6.4	2120133452	-1075234552	-5356288551	1621715352
6.5	2224701652	-1013291552	-7059728651	1786194352
6.6	2322196152	-9334816651	-8930615851	1956484852
6.7	2410745752	-8341028551	-1097435952	2131754252
6.8	2488306052	-7134031851	-1319545352	2311002552
6.9	2552656052	-5695911551	-1559729952	2493048752
7.0	2601393352	-4008474351	-1818202652	2676516352
7.1	2631933352	-2053386851	-2095027952	2859818352
7.2	2641507852	-1876661150	2390100752	3041143852
7.3	2627166352	-2732800051	2703122852	3218443052
7.4	2585779052	-5599725551	3033578652	3389413052
7.5	2514042252	-8805518351	3380709052	3551485352
7.6	2408485152	-1236636252	3743484552	3701811852
7.7	2265481252	-1629726352	4120575652	3837254352
7.8	2081261152	-2061173652	4510324952	3954372352
7.9	1851928752	-2532146352	4910714452	4049415352
8.0	1573482452	-3043591452	5319336552	4118314552
8.1	1241838152	-3596194252	5733360052	4156677852
8.2	8528585551	-4190334452	6149497352	4159786852
8.3	4023853551	-4826039052	6563974252	4122597552
8.4	1137226651	5502931452	6972495352	4039742352
8.5	6995466551	6220178552	7370211352	3905538352
8.6	1359059352	6976433252	7751691852	3713997552
8.7	2096070252	7769774952	8110891252	3458844052
8.8	2914166052	8597646552	8441122552	3133532552
8.9	3816642852	9456789452	8735033352	2731278552
9.0	4806432952	1034317553	8984580852	2245087952
9.1	5886024652	1125193653	9181014252	1667799552
9.2	7057374752	1217729353	9314860852	9921306351
9.3	8321812552	1311248353	9375915852	2107342851
9.4	9679939552	1404968653	9353238352	6837384051
9.5	1113152053	1497995153	9235155852	1698563852
9.6	1267536253	1589312553	9009276752	2840862552
9.7	1430919653	1677777753	8662505052	4117505552
9.8	1602954253	1762113853	8181079352	5535009352
9.9	1783157553	1840902653	7550606052	7099419052
10.0	1970898353	1912579453	6756113452	8816185052

$$a = 0$$

X_0	R_3	R'_3	R''_3	R'''_3
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0.3	8999878549	5999757050	1999595051	5399959648-
0.4	1599931750	7998976050	1998720051	1279969749-
0.5	2499739650	9996875250	1996875151	2499855449-
0.6	3599222450	1199222451	1993520451	4319481649-
0.7	4898039350	1398319451	1987996351	6858475049-
0.8	6395631250	1596723651	1979523951	1023611750-
0.9	8091143550	1794096151	1967205051	1457114350-
1.0	9983335950	1990002651	1950023151	1998148350-
1.1	1207048151	2183901051	1926844651	2658391850-
1.2	1435025051	2375130051	1896419551	3449366050-
1.3	1681958951	2562898051	1857383851	4382383450-
1.4	1947458251	2746270851	1808261551	5468487050-
1.5	2231030551	2924161451	1747468051	6718376650-
1.6	2532066351	3095319251	1673313751	8142324650-
1.7	2849822651	3258319251	1584008851	9750079250-
1.8	3183404751	3411553251	1477669051	1155075351-
1.9	3531747851	3553219651	1352322751	1355269951-
2.0	3893596651	3681315851	1205919251	1576336851-
2.1	4267485251	3793630851	1036338251	1818915851-
2.2	4651716151	3887738451	8414015650	2083523851-
2.3	5044338051	3960992051	6188864450	2370536051-
2.4	5443123951	4010521251	3665402250	2680166251-
2.5	5845548451	4033230251	8209807649	3012443251-
2.6	6248765451	4025795851	2366976050	-3367188051-
2.7	6649584851	3984671251	5920750050	-3743988051-
2.8	7044451251	3906088851	9862082250	-4142169051-
2.9	7429421551	3786067851	1421189751	-4560767051-
3.0	7800144251	3620423051	1898999851	-4998497251-
3.1	8151839751	3404778851	2421472851	-5453721651-
3.2	8479281851	3134584451	2990260251	-5924416651-
3.3	8776781651	2805135051	3606790851	-6408137251-
3.4	9038172651	2411595851	4272225851	-6901982251-
3.5	9256800051	1949031151	4987412051	-7402556451-
3.6	9425511251	1412438551	5752831251	-7905935251-
3.7	9536652451	7967879050	6568543651	-8407625451-
3.8	9582066551	9706668449	7434130851	-8902529251-
3.9	9553098351	6916694250	-8348632651	-9384906451-
4.0	9440604551	1574238851	-9310481251	-9848338451-
4.1	9234968251	2555269851	-1031743252	-1028569452-
4.2	8926122651	3639129651	-1136648952	-1068909752-
4.3	8503580051	4829844851	-1245383252	-1104989352-
4.4	7956471251	6131015851	-1357473352	-1135862752-
4.5	7273591651	7545721051	-1472348052	-1160501852-
4.6	6443459751	9076415251	-1589328952	-1177793752-
4.7	5454384251	1072481752	-1707622052	-1186540352-
4.8	4294542951	1249179252	-1826309452	-1185456652-
4.9	2952076151	1437721952	-1944340152	-1173171852-

$$a = 0$$

X	R ₃	R' ₃	R'' ₃	R''' ₃
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5.1	3277205450	-1849720152	-2173512852	-1109090552-
5.2	2287943951	-2072532252	-2281812852	-1054134852-
5.3	4476295751	-2305871052	-2383756252	-9816669851-
5.4	6902954951	-2549010452	-2477503852	-8899211451-
5.5	9577279551	-2801031452	-2561036852	-7770684451-
5.6	1250760452	-3060804052	-2632151252	-6412257651-
5.7	1570102152	-3326968052	-2688451452	-4804662051-
5.8	1916313752	-3597914452	-2727348652	-2928320051-
5.9	2289782152	-3871765052	-2746056252	-7634896650-
6.0	2690691752	-4146353452	-2741591052	-1709565151
6.1	3118995252	-4419205052	-2710772452	-4510380251
6.2	3574381752	-4687517252	-2650227052	-7658071251
6.3	4056243052	-4948141052	-2556393252	-1117108652
6.4	4563637052	-5197561852	-2425529052	-1506692952
6.5	5095252452	-5431882452	-2253724052	-1936185352
6.6	5649367452	-5646806452	-2036912652	-2407052252
6.7	6223810452	-5837624652	-1770891952	-2920563652
6.8	6815918052	-5999200852	-1451343752	-3477753052
6.9	7422490452	-6125962252	-1073859952	-4079372852
7.0	8039750052	-6211892652	-6339735051	-4725847252
7.1	8663293252	-6250527052	-1271933651	-5417220452
7.2	9288048952	-6234952452	-4509547451	6153102052
7.3	9908231552	-6157810052	-1104881852	6932608452
7.4	1051729953	-6011306652	-1838883352	7754301052
7.5	1110790753	-5787226652	-2657080452	8616120252
7.6	1167187253	-5476954252	-3563354252	9515317652
7.7	1220012653	-5071500452	-4561273852	1044838453
7.8	1268268953	-4561538252	-5654016252	1141097553
7.9	1310862953	-3937446052	-6844278852	1239783153
8.0	1346604553	-3189360452	-8134184452	1340270653
8.1	1374203453	-2307238652	-9525177852	1441827853
8.2	1392268753	-1280932252	-1101791653	1543607453
8.3	1399307553	-1002712751	-1261214553	1644638653
8.4	1393725953	-1244838252	1430657753	1743819153
8.5	1373829653	-2764303852	1609875453	1839906853
8.6	1337826653	-4467722852	1798489953	1931512853
8.7	1283831453	-6364243052	1995977353	2017093453
8.8	1209869553	-8462408052	2201651053	2094943253
8.9	1113885053	-1076998853	2414646053	2163189853
9.0	9937489152	-1329378953	2633900853	2219787053
9.1	8472695252	-1603945453	2858141253	2262511253
9.2	6722058452	-1901123553	3085860853	2288957853
9.3	4662821352	-2221175453	3315304253	2296539253
9.4	2272056252	-2564174653	3544447653	2282484053
9.5	4731321151	2929978253	3770981453	2243838653
9.6	3595375152	3318197253	3992291453	2177470453
9.7	7116779452	3728165053	4205441453	2080073053
9.8	1105863253	4158904653	4407156253	1948174853
9.9	1544107753	4609094253	4593805653	1778149853
10.0	2028275453	5077031853	4761390253	1566230853

$$a = 0$$

X	R_4	R'_4	R''_4	R'''_4
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0.2	3218840650	-6093565650	-5001461951	2498140852-
0.3	<u>3611692050</u>	<u>-2036289350</u>	<u>-3337354751</u>	<u>1107839752-</u>
0.4	3664331050	-8464239249	2507997751	6203352751-
0.5	3463487950	-3088403650	2013295551	3941067951-
0.6	3059950950	-4928033150	1686357851	2709344351-
0.7	2486991350	-6492766350	1455429951	1966591251-
0.8	<u>1767995850</u>	<u>-7858691550</u>	<u>1284385951</u>	<u>1486985751-</u>
0.9	9202351149	-9074641250	1152899251	1162957151-
1.0	4303966848	1017347951	1048514351	9381032050-
1.1	1111322550	1117794951	9630459850	7806330050-
1.2	2276019350	1210395551	8907808450	6715974050-
1.3	<u>3529868850</u>	<u>1296249951</u>	<u>8275100550</u>	<u>5991908050-</u>
1.4	4866517050	1376088251	7699784050	5557954050-
1.5	6280188650	1450348751	7155542650	5363541050-
1.6	7765422850	1519228551	6620247950	5374311050-
1.7	9316851250	1582719051	6074641650	5566470050-
1.8	<u>1092900351</u>	<u>1640629451</u>	<u>5501471650</u>	<u>5923246050-</u>
1.9	1259613351	1692603851	4884910050	6432601050-
2.0	1431206451	1738133551	4210161850	7085718050-
2.1	1607003751	1776566251	3463198450	7875926050-
2.2	1786256851	1807111951	2630583250	8797969050-
2.3	<u>1968132551</u>	<u>1828849051</u>	<u>1699358050</u>	<u>9847436050-</u>
2.4	2151698251	1840728651	6569810049	1102034351-
2.5	2335910651	1841577751	5086960049	-1231279551-
2.6	2519603251	1830104451	1809427050	-1372069651-
2.7	2701473951	1804901151	3256534050	-1523951951-
2.8	<u>2880075151</u>	<u>1764449651</u>	<u>4860857050</u>	<u>-1686408151-</u>
2.9	3053801451	1707125751	6632675050	-1858834351-
3.0	3220880651	1631205651	8581612050	-2040522351-
3.1	3379362351	1534873451	1071653151	-2230641651-
3.2	3527111551	1416228551	1304538451	-2428221251-
3.3	<u>3661798751</u>	<u>1273296051</u>	<u>1557508551</u>	<u>-2632132051-</u>
3.4	3780893251	1104038251	1831132351	-2841068551-
3.5	3881659151	9063667050	2125840051	-3053533951-
3.6	3961148751	6781582050	2441899951	-3267818851-
3.7	4016201551	4172717050	2779400251	-3481987751-
3.8	<u>4043442551</u>	<u>1215672050</u>	<u>3138221651</u>	<u>-3693858851-</u>
3.9	4039283751	2110721050	-3518014551	-3900989751-
4.0	3999927951	5827148050	-3918171051	-4100659351-
4.1	3921373951	9953557050	-4337797351	-4289854351-
4.2	3799427251	1450883151	-4775681951	-4465251551-
4.3	<u>3629709451</u>	<u>1951048951</u>	<u>-5230266451</u>	<u>-4623207951-</u>
4.4	3407676551	2497428851	-5699610051	-4759744951-
4.5	3128637151	3091384951	-6181357551	-4870540351-
4.6	2787776251	3734022051	-6672704051	-4950917651-
4.7	2380183551	4426136051	-7170360051	-4995842351-
4.8	1900884951	5168178051	-7670511051	-4999916651-
4.9	1344882051	5960176051	-8168790051	-4957379151-
5.0	7071980050	6801707051	-8660231051	-4862109651-

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5.3	1744361051	-9610867051	-1003422952	-4193493151-
5.4	2756303051	-1063467052	-1043557052	-3819253351-
5.5	3872565051	-1169659052	-1079513652	-3356714051-
5.6	5096738051	-1279199752	-1110370552	-2797933051-
5.7	6431897051	-1391529952	-1135124552	-2134777051-
5.8	7880508051	-1505985052	-1152690652	-1358971051-
5.9	9444319051	-1621789452	-1161900452	-4621630050-
6.0	1112423852	-1738045152	-1161502652	-5640050050
6.1	1292021952	-1853725152	-1150161452	-1727827051
6.2	1483111052	-1967665352	-1126460052	-3037421051
6.3	1685453252	-2078555352	-1088901052	-4500647051
6.4	1898671852	-2184931452	-1035909952	-6124988051
6.5	2122235252	-2285168252	-9658409051	-7917403051
6.6	2355441152	-2377473352	-8769808051	-9884210051
6.7	2597399952	-2459878952	-7675572051	-1203092752
6.8	2847015652	-2530238452	-6357474051	-1436207452
6.9	3102968552	-2586220452	-4796891051	-1688103752
7.0	3363696552	-2625304652	-2974938051	-1958982252
7.1	3627376652	-2644784552	-8725820050	-2248887052
7.2	3891903652	-2641758952	-1529144051	2557681352
7.3	4154875252	-2613140952	-4248970051	2885023552
7.4	4413569752	-2555658252	-7305182051	3230340552
7.5	4664929952	-2465857852	-1071536752	3592800752
7.6	4905543352	-2340115252	-1449612352	3971285952
7.7	5131629252	-2174648652	-1866280752	4364358752
7.8	5339017952	-1965526852	-2322911552	4770235252
7.9	5523144652	-1708694252	-2820681052	5186746052
8.0	5679028152	-1399987052	-3360530052	5611311052
8.1	5801271452	-1035163252	-3943113052	6040897052
8.2	5884047352	-6099299051	-4568761052	6471993052
8.3	5921099552	-1199770051	-5237427052	6900565052
8.4	5905743052	-4389730051	5948638052	7322031052
8.5	5830863252	-1071136052	6701424052	7731221052
8.6	5688937052	-1780597052	7494279052	8122343052
8.7	5472037052	-2571258052	8325075052	8488964752
8.8	5171858052	-3446784052	9191020052	8823963852
8.9	4779740052	-4410516052	1008855853	9119517552
9.0	4286716052	-5465403052	1101332853	9367068852
9.1	3683541052	-6613914052	1196006453	9557315452
9.2	2960746052	-7857943052	1292254453	9680189652
9.3	2108725052	-9198711052	1389349253	9724839652
9.4	1117765052	-1063664853	1486451253	9679655252
9.5	2183000050	1217129853	1582601353	9532231052
9.6	1319670052	1380117253	1676711953	9269415052
9.7	2785157052	1552363153	1767559353	8877319052
9.8	4427351052	1733475053	1853778753	8341331052
9.9	6254881052	1922915253	1933855853	7646193052
10.0	8275735052	2119986353	2006118653	6776015052

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0.3	9998842950	1542846448	-1542832249	-1028521650
0.4	9996342950	3657062748	-2742717149	-1371218550
0.5	9991071750	7142475848	-4285180249	-1713644850
0.6	9981486750	1234149049	6169833649	2055547950
0.7	9965703550	1959597849	-8395977849	-2396552750
0.8	9941496050	2924690149	-1096246750	-2736136250
0.9	9906297750	4163378549	-1386754750	-3073603850
1.0	9857203950	5709402349	-1710867450	-3408064350
1.1	9790973750	7596198149	2068230650	3738405650
1.2	9704033950	9856789349	-2458366450	-4063272150
1.3	9592483650	1252364950	-2880648750	-4381039050
1.4	9452100750	1562853750	-3334274250	-4689792250
1.5	9278349850	1920231050	-3818232850	-4987301550
1.6	9066392350	2327469950	4331274450	5271004050
1.7	8811099150	2787404950	-4871876350	-5537977950
1.8	8507065750	3302703150	-5438203250	-5784927250
1.9	8148630650	3875830750	-6028071750	-6008161550
2.0	7729897350	4509016850	-6638908650	-6203577850
2.1	7244759950	5204211950	7267709650	6366648850
2.2	6686933350	5963042150	-7910995250	-6492408050
2.3	6049988150	6786761150	-8564767650	-6575442050
2.4	5327390150	7676194250	-9224464350	-6609880050
2.5	4512546950	8631680050	-9884913550	-6589397050
2.6	3598858850	9653008250	1054028751	6507207050
2.7	2579777450	1073935251	-1118405751	-6356074050
2.8	1448871250	1188919151	-1180894851	-6128317050
2.9	1998978449	1310024451	-1240689751	-5815836050
3.0	1173114650	-1436937851	-1296900851	-5410118150
3.1	2675779250	1569252651	1348551751	4902283150
3.2	4313252150	-1706460651	-1394575451	-4283110450
3.3	6090126050	-1847941551	-1433811151	-3543085450
3.4	8010314050	-1992954851	-1465001851	-2672456650
3.5	1007692451	-2140628751	-1486791851	-1661296950
3.6	1229212251	2289951351	1497725851	4995802049
3.7	1465699051	-2439760251	-1496248051	-8227344049
3.8	1717136651	-2588732051	-1480702651	-2315603150
3.9	1983368051	-2735373451	-1449334951	-3988786750
4.0	2264078051	-2878010151	-1400294351	-5851724850
4.1	2558774651	3014778551	1331637951	7913396950
4.2	2866769551	-3143616351	-1241336051	-1018216751
4.3	3187158151	-3262253951	-1127278951	-1266561351
4.4	3518797851	-3368207751	-9872859950	-1537033851
4.5	3860286951	-3458771951	-8191162850	-1830176751
4.6	4209941751	3531015351	6204813550	2146392651
4.7	4565774151	-3581775351	-3890605050	-2485919351
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X	R ₁	R ₁ '	R ₁ ''	R ₁ '''
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5.2	6343754151	-3386373351	-1340852051	4526659351
5.3	6674913351	-3228876751	-1816946151	4998377151
5.4	6987863051	-3021381351	-2341127351	5488078451
5.5	7277360051	-2758991751	-2915089851	5993606651
5.6	7537663351	2436655251	3540289551	6512347151
5.7	7762520051	-2049186551	-4217896451	7041184451
5.8	7945153451	-1591300751	-4948741951	7576464651
5.9	8078254651	-1057648451	-5733263151	8113950151
6.0	8153978551	-4428587950	-6571444251	8648781451
6.1	8163943651	2584118650	7462748551	9175434051
6.2	8099237951	-1051424851	8406051851	9687675051
6.3	7950429751	-1941296751	9399571051	1017853052
6.4	7707586151	-2932929951	1044078652	1064023752
6.5	7360297751	-4030933851	1152636252	1106421952
6.6	6897712151	5239540551	1265205952	1144103952
6.7	6308576851	-6562509951	1381265452	1176038852
6.8	5581291151	-8003025951	1500184852	1201104352
6.9	4703969351	-9563585451	1621217052	1218085852
7.0	3664515351	-1124587552	1743489052	1225675252
7.1	2450710551	1305064252	1865992252	1222469452
7.2	1050314151	-1497755452	1987573252	1206972052
7.3	5488211450	1702504752	2106924152	1177593152
7.4	2358618051	1919017052	2222573352	1132653252
7.5	4390627751	2146841452	2332877252	1070386052
7.6	6655865551	2385353752	2436011452	9889437051
7.7	9164627951	2633737852	2529962752	8864035051
7.8	1192629252	2890966852	2612522652	7607761051
7.9	1494909352	3155782352	2681280152	6100157851
8.0	1823988552	3426675652	2733618352	4320328151
8.1	2180387952	3701864752	2766708752	2247075451
8.2	2564435952	3979274552	2777510652	1409243050
8.3	2976237652	4256515752	2762770252	2864931551
8.4	3415642952	4530862052	2719022152	5945920651
8.5	3882211252	4799231152	2642593652	9404329251
8.6	4375175052	5058162752	2529611852	1325978052
8.7	4893401052	5303799552	2376012852	1753076252
8.8	5435349852	5531870052	2177554652	2223428952
8.9	5999033152	5737668652	1929834552	2738551152
9.0	6581969952	5916043352	1628308852	3299729852
9.1	7181141152	6061381552	1268319152	3907978252
9.2	7792943052	6167600152	8451206151	4563986052
9.3	8413139252	6228138952	3539188851	5268065352
9.4	9036814052	6235957352	2100912251	-6020093852
9.5	9658323252	6183535052	8516781451	-6819452852
9.6	1027124753	6062879552	1575520852	7664962252
9.7	1086834353	5865537652	2386149352	-8554809852
9.8	1144150253	5582612552	3287877952	-9486481352
9.9	1198170353	5204790752	4284730652	-1045667953
10.0	1247897653	4722371552	5380363152	-1146124953

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0.3	8999877249	5999754550	1999590951	5454503648-
0.4	1599931050	7998965750	1998707151	1292897949-
0.5	2499737050	9996843550	1996843551	2525102949-
0.6	3599214650	1199214651	1993455051	4363100049-
0.7	4898019550	1398302451	1987875151	6927714749-
0.8	6395587150	1596690451	1979317151	1033941650-
0.9	8091054150	1794036551	1966873751	1471810850-
1.0	9983167650	1989901651	1949518451	2018285950-
1.1	1207018251	2183738451	1926105851	2685155150-
1.2	1434974751	2374878951	1895373651	3484044250-
1.3	1681877751	2562523351	1855943851	4426362950-
1.4	1947331651	2745728251	1806325951	5523242350-
1.5	2230838951	2923395651	1744919151	6785457850-
1.6	2531784251	3094262351	1670017051	8223343050-
1.7	2849416951	3256889051	1579811951	9846689650-
1.8	3182833451	3409651251	1472400951	1166463151-
1.9	3530957951	3550729451	1345792851	1368551351-
2.0	3892522651	3678100951	1197916651	1591675251-
2.1	4266047051	3789532551	1026631551	1836467051-
2.2	4649816351	3882573751	8297381850	2103432251-
2.3	5041859951	3954551751	6049923650	2392929651-
2.4	5439928251	4002567951	3501200250	2705150751-
2.5	5841470851	4023495351	6283536049	3040095951-
2.6	6243612751	4013978651	2591400050	-3397550851-
2.7	6643132551	3970436051	6180543050	-3777059051-
2.8	7036439151	3889063251	1016100251	-4177893751-
2.9	7419550451	3765841151	1455387951	-4599028151-
3.0	7788071851	3596544351	1937912851	-5039104851-
3.1	8137176451	3376755551	2465522651	-5496399351-
3.2	8461586851	3101881351	3039879951	-5968790551-
3.3	8755558451	2767173251	3662419451	-6453719951-
3.4	9012864551	2367752151	4334307051	-6948158451-
3.5	9226785851	1898638251	5056387851	-7448567051-
3.6	9390100951	1354785251	5829136351	-7950858451-
3.7	9495082351	7311198650	6652601651	-8450358951-
3.8	9533495951	2258821949	7526346351	-8941769951-
3.9	9496604951	7757932050	-8449384051	-9419130551-
4.0	9375180251	1668879051	-9420111951	-9875778651-
4.1	9159516551	2661329851	-1043624152	-1030431852-
4.2	8839455251	3757539951	-1149472352	-1069658252-
4.3	8404414551	4961557751	-1259167152	-1104360652-
4.4	7843429151	6276997451	-1372228252	-1133559652-
4.5	7145197551	7706944051	-1488075352	-1156191152-
4.6	6298140951	9253848551	-1606019452	-1171103552-
4.7	5290471551	1091941552	-1725254952	-1177058052-
4.8	4110273651	1270448052	-1844849952	-1172726152-
4.9	2745596351	1460888152	-1963738552	-1156691552-

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5.2	2572997351	-2101853652	-2303304852	-1022912152-
5.3	4791753451	-2337366152	-2405715452	-9441959851-
5.4	7251011651	-2582718552	-2499774952	-8454470651-
5.5	9960161051	-2836974852	-2583436552	-7247826751-
5.6	1292754852	-3098985052	-2654466252	-5802662351-
5.7	1616025752	-3367366252	-2710436452	-4099179851-
5.8	1966386252	-3640483152	-2748723552	-2117285851-
5.9	2344216952	-3916427652	-2766503852	-1632553150
6.0	2749693052	-4192999752	-2760755052	-2762641751
6.1	3182754452	-4467687152	-2728255452	-5700831451
6.2	3643073252	-4737645452	-2665587152	-8997310551
6.3	4130019652	-4999678952	-2569143152	-1267083152
6.4	4642626452	-5250222152	-2435133952	-1673912752
6.5	5179550052	-5485321752	-2259599552	-2121858252
6.6	5739031352	-5700620652	-2038423652	-2612388252
6.7	6318855152	-5891343452	-1767351552	-3146763352
6.8	6916307852	-6052282252	-1442012552	-3725990952
6.9	7528131352	-6177788352	-1057945752	-4350782752
7.0	8150481252	-6261763152	-6106311251	-5021502252
7.1	8778881352	-6297654852	-9552596050	-5738110452
7.2	9408176752	-6278457752	-4918944951	6500113252
7.3	1003248953	-6196716752	-1156089652	7306492852
7.4	1064517253	-6044535852	-1901399952	8155650952
7.5	1123877153	-5813593852	-2731988152	9045334552
7.6	1180497653	-5495164652	-3651772252	9972567752
7.7	1233458453	-5080145852	-4664351152	1093357453
7.8	1281747053	-4559094152	-5772925052	1192370153
7.9	1324254453	-3922269952	-6980206452	1293734153
8.0	1359773253	-3159690952	-8288322652	1396784553
8.1	1386995253	-2261195452	-9698711552	1500744453
8.2	1404509953	-1216517252	-1121200953	1604716153
8.3	1410804053	-1537164250	-1282792953	1707672953
8.4	1404261853	-1352445852	1454512953	1808450153
8.5	1383166153	-2896947252	1636108053	1905738053
8.6	1345701553	-4627826252	1827191653	1998072453
8.7	1289957553	-6554316452	2027228053	2083828553
8.8	1213934353	-8685035052	2235517053	2161212753
8.9	1115549853	-1102780953	2451176453	2228256953
9.0	9926482852	-1358948753	2673125353	2282812753
9.1	8430113552	-1637572753	2900066153	2322547053
9.2	6643708352	-1939077653	3130466453	2344938653
9.3	4544242752	-2263722153	3362540153	2347276153
9.4	2108526252	-2611573053	3594229053	2326658153
9.5	6865905451	2982477153	3823185453	2279994453
9.6	3863981152	3376030853	4046752253	2204009853
9.7	7445977252	3791548853	4261944953	2095250053
9.8	1145407053	4228030353	4465436153	1950091253
9.9	1590857953	4684123653	4653538153	1764751153
10.0	2082829353	5158091053	4822187853	1535303953

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X ₀	R ₀	R' ₀	R'' ₀	R''' ₀
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0.3	1643133250	8215204750	1368355751	2293268551-
0.4	2529653950	9484520150	1183252151	1505085351-
0.5	3534960150	1060028951	1054978351	1100431551-
0.6	4646015650	1160461051	9574908750	8696028150-
0.7	5852968050	1252120651	8779758950	7325126250-
0.8	7147805650	1336407851	8090937350	6528040650-
0.9	8523601550	1414135051	7461286050	6119481750-
1.0	9974031050	1485719151	6857551750	5996934050-
1.1	1149303651	1551287451	6254398950	6100126150-
1.2	1307456951	1610740151	5631253450	6391936450-
1.3	1471238751	1663787051	4970524550	6848622750-
1.4	1639985951	1709973151	4256561950	7454484250-
1.5	1812984251	1748693051	3475017350	8198763250-
1.6	1989450951	1779203351	2612453850	9073767850-
1.7	2168522251	1800629451	1656102050	1007366051-
1.8	2349240851	1811971851	5937187649	1119365551-
1.9	2530543351	1812110951	5864888749	-1242944851-
2.0	2711248651	1799811451	1895889850	-1377679951-
2.1	2890046251	1773726051	3345417150	-1523119851-
2.2	3065485951	1732401051	4945530450	-1678761151-
2.3	3235966651	1674280351	6706147450	-1844023451-
2.4	3399727051	1597711951	8636559650	-2018229951-
2.5	3554836151	1500954551	1074532551	-2200587451-
2.6	3699184651	1382185351	1304014551	-2390169351-
2.7	3830476151	1239508951	1552771451	-2585896651-
2.8	3946223951	1070969851	1821357151	-2786521651-
2.9	4043741351	8745623450	2110191451	-2990611251-
3.0	4120139451	6482473850	2419540651	-3196529751-
3.1	4172325351	3899672350	2749496751	-3402421351-
3.2	4196998751	9766447449	3099954651	-3606194751-
3.3	4190656051	2306964250	3470587251	-3805507651-
3.4	4149591251	5971067550	3860820751	-3997749951-
3.5	4069902551	1003485951	4269805451	-4180029251-
3.6	3947500851	1451654751	4696388451	-4349156851-
3.7	3778121851	1943301051	5139082251	-4501633851-
3.8	3557340251	2479946751	5596033451	-4633641351-
3.9	3280588651	3062908351	6064990451	-4741027451-
4.0	2943178451	3693256251	6543268651	-4819301051-
4.1	2540331551	4371768851	7027718851	-4863624951-
4.2	2067204651	5098884451	7514689551	-4868811751-
4.3	1518932451	5874651651	7999992051	-4829324351-
4.4	8906650950	6698669851	8478869251	-4739278851-
4.5	1776194350	7570032851	8945956051	-4592449351-
4.6	6248706650	8487268651	9395248751	-4382281851-
4.7	1521293451	9448267751	9820071951	-4101908051-
4.8	2515891051	1045022252	1021305152	-3744168751-
4.9	3612584751	1148954752	1056608052	-3301639451-
5.0	4814899551	1256181252	1087029852	-2766664651-

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X	R_3	R'_3	R''_3	R'''_3
5.1	6125867551	-1366166052	-1111606952	-2131400151-
5.2	7547939051	-1478273152	-1129296852	-1387862051-
5.3	9082874651	-1591758352	-1138976452	-5279819450-
5.4	1073163152	-1705761552	-1139441852	-4563231350
5.5	1249424452	-1819296952	-1129408452	-1573077251
5.6	1436969852	-1931247652	-1107512452	-2830157251
5.7	1635579452	-2040355852	-1072312152	-4235196051
5.8	1844899652	-2145216352	-1022291052	-5795471651
5.9	2064429152	-2244267352	-9558624651	-7517785151
6.0	2293502152	-2335786752	-8713744551	-9408331851
6.1	2531272652	-2417883352	-7671168951	-1147254252
6.2	2776696152	-2488492652	-6413295151	-1371493152
6.3	3028513552	-2545371552	-4922127051	-1613891752
6.4	3285232452	-2586096652	-3179386251	-1874663152
6.5	3545107952	-2608059552	-1166653251	-2153871352
6.6	3806126152	-2608468052	-1134468851	2451408852
6.7	4065984152	-2584347652	-3742175751	2766974552
6.8	4322073052	-2532543552	-6674256051	3100046952
6.9	4571459052	-2449724552	-9947852951	3449858552
7.0	4810867552	-2332393852	-1357920852	3815369252
7.1	5036666052	-2176897252	-1758338052	4195235652
7.2	5244848552	-1979437752	-2197391052	4587782452
7.3	5431024152	-1736092252	-2676249552	4990969552
7.4	5590402752	-1442829952	-3195859252	5402362652
7.5	5717788952	-1095539152	-3756900552	5819097652
7.6	5807570952	-6900554151	-4359746852	6237848552
7.7	5853721552	-2221949351	-5004410852	6654797152
7.8	5849792952	-3122086351	5690502052	7065593752
7.9	5788924652	-9172579051	6417162152	7465328452
8.0	5663852852	-1596947352	7183016052	7848499752
8.1	5466920152	-2355102452	7986107452	8208981652
8.2	5190096052	-3195324052	8823830352	8539995252
8.3	4825009352	-4120913552	9692874852	8834086452
8.4	4362970352	-5134808052	1058914653	9083095452
8.5	3795021252	-6239487252	1150769653	9278140352
8.6	3111981652	-7436894452	1244265553	9409605552
8.7	2304506552	-8728336952	1338715453	9467126152
8.8	1363160552	-1011438053	1433324753	9439588152
8.9	2784868151	-1159473853	1527184353	93151120652
9.0	9588912351	1316815553	1619262153	9081122752
9.1	2358170352	1483228153	1708397153	8724270752
9.2	3928254352	1658353853	1793291453	8230554352
9.3	5677616852	1841697153	1872502953	7585310152
9.4	7614172852	2032611953	1944441653	6773283952
9.5	9745096252	2230285053	2007360453	5778688452
9.6	1207666553	2433720353	2059352653	4585294952
9.7	1461406353	2641722753	2098348453	3176527052
9.8	1736116553	2852882153	2122109353	1535568852
9.9	2032033153	3065556753	2128229453	3544897651-
10.0	2349215453	3277854353	2114133253	2510488152-

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X	R ₄ 0	R' ₄ ∞	R'' ₄ -∞	R''' ₄ ∞
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0.2	4472082850	1117914851	2797172151	-2093705052
0.3	5476897150	9123780050	1527202651	-7559333351
0.4	6323356150	7892201850	1000017251	-3632014051
0.5	7067794150	7041606450	7277290650	-2018222551
0.6	7738530350	6399205450	5704418850	-1209291251
0.7	8351721050	5880498950	4746814050	-7440371750
0.8	8917137850	5437559350	4161337450	-4466890950
0.9	9440736950	5040025350	3823771450	-2395321150
1.0	9925950050	4666870250	3665140650	-8432517049
1.1	1037439751	4302407350	3644693650	-3932305949-
1.2	1078630451	3934176950	3737171350	-1429760050-
1.3	1116075851	3551757250	3926299550	-2335058050-
1.4	1149587851	3146060450	4201234050	-3151381050-
1.5	1178892151	2708901650	4554504850	-3905263550-
1.6	1203635851	2232727850	4980770650	-4613456450-
1.7	1223392951	1710448750	5476024050	-5286344150-
1.8	1237668651	1135330550	6037068050	-5929988550-
1.9	1245901851	5009340949	6661149150	-6547378750-
2.0	1247469051	1989147949	-7345693950	-7139219550-
2.1	1241685751	9701337649	-8088106550	-7704431450-
2.2	1227809651	1818372850	-8885609650	-8240479950-
2.3	1205043651	2748989250	-9735106450	-8743584850-
2.4	1172538451	3767010150	-1063306751	-9208858350-
2.5	1129396651	4877082850	-1157542151	-9630397850-
2.6	1074675951	6083417550	-1255746151	-1000134251-
2.7	1007394951	7389717650	-1357374551	-1031391251-
2.8	9265377650	8799100450	-1461800951	-1055943451-
2.9	8310609650	1031401451	-1568307951	-1072835551-
3.0	7199000750	1193613851	-1676078151	-1081026551-
3.1	5919779950	1366628451	-1784185851	-1079390651-
3.2	4462147050	1550427651	-1891588551	-1066718751-
3.3	2815371450	1744883751	-1997118751	-1041721151-
3.4	9689096049	1949745751	-2099476351	-1003030151-
3.5	1087460350	-2164625351	-2197220351	-9492034750-
3.6	3363501750	-2388983251	-2288762251	-8787292650-
3.7	5868352350	-2622113251	-2372358851	-7900313350-
3.8	8610357050	-2863126851	-2446106651	-6814765650-
3.9	1159687551	-3110937251	-2507936251	-5513830550-
4.0	1483406651	-3364242151	-2555607751	-3980303750-
4.1	1832668151	-3621506651	-2586708551	-2196710150-
4.2	2207780851	-3880945451	-2598649651	-1454450149-
4.3	2608861751	-4140506051	-2588666651	-2191076950
4.4	3035808551	-4397850251	-2553819451	-4830234950
4.5	3488270151	-4650337251	-2490996451	-7789009350
4.6	3965615751	-4895007751	-2396919551	-1108374951
4.7	4466901751	-5128565151	-2268149851	-1472992551
4.8	4990837051	-5347362051	-2101101451	-1874184951
4.9	5535748651	-5547385451	-1892050451	-2313236351
5.0	6099542751	-5724243751	-1637155151	-2791250851

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X	R ₄	R' ₄	R'' ₄	R''' ₄
5.1	6679666751	-5873157151	-1332472051	-3309115751
5.2	7273069851	-5988945151	-9739818850	-3867461051
5.3	7876165351	-6066024151	-5576187350	-4466617351
5.4	8484785351	-6098402851	-7929895749	-5106567551
5.5	9094143051	-6079683251	-4650414250	5786901951
5.6	9698792551	-6003061051	-1079399951	6506759851
5.7	1029258652	-5861339351	-1767665851	7264777151
5.8	1086863852	-5646939451	-2533564451	8059028251
5.9	1141928652	-5351921751	-3380595251	8886961851
6.0	1193605952	-4968008451	-4311971451	9745336751
6.1	1240963952	-4486620051	-5330544951	1063015952
6.2	1282983952	-3898912551	-6438724951	1153660552
6.3	1318557652	-3195824451	-7638397351	1245896052
6.4	1346485152	-2368139751	-8930826151	1339054052
6.5	1365474052	-1406548151	-1031655652	1432361952
6.6	1374138752	-3017266250	-1179530652	1524936052
6.7	1371000352	-9555738750	1336585152	1615773952
6.8	1354488652	-2374429551	1502590252	1703747252
6.9	1322943952	-3963625051	1677197352	1787595252
7.0	1274620452	-5731531251	1859925452	1865917852
7.1	1207691452	-7685973651	2050145252	1937169752
7.2	1120255352	-9834058651	2247067252	1999654852
7.3	1010343852	-1218202252	2449722652	2051520752
7.4	8759313951	-1473502952	2656950652	2090755752
7.5	7149464951	-1749699352	2867380552	2115184952
7.6	5252866051	-2047032752	3079416452	2122469152
7.7	3048327151	-2365574752	3291219052	2110103652
7.8	5146877950	-2705199352	3500692952	2075419852
7.9	2368984651	3065556852	3705464752	2015587752
8.0	5623144251	3446047452	3902872952	1927622052
8.1	9267499651	3845787552	4089947352	1808387552
8.2	1332073652	4263583052	4263399052	1654610452
8.3	1780017352	4697893052	4419604652	1462890852
8.4	2272139652	5146797152	4554596052	1229715952
8.5	2809786352	5607961452	4664046652	9514833552
8.6	3394048152	6078599552	4743267952	6245214851
8.7	4025713052	6555438852	4787203052	2451132451
8.8	4705217152	7034682052	4790418452	1904672751
8.9	5432586052	7511971952	4747112552	6859221151
9.0	6207381952	7982349952	4651117052	1244889952
9.1	7028641552	8440222052	4495899752	1870899752
9.2	7894801952	8879327152	4274588752	2567315752
9.3	8803649752	9292697752	3979985052	3337289852
9.4	9752227252	9672631852	3604584452	4183683452
9.5	1073677953	1001066253	3140617952	5109019252
9.6	1175265553	1029753753	2580078852	6115385352
9.7	1279424753	1052318553	1914785052	7204359352
9.8	1385489253	1067672253	1136421352	8376923652
9.9	1492680053	1074642153	2366025851	9633367752
10.0	1600096353	1071971553	7930318451	-1097317353

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X	R ₁	R ₁ '	R ₁ ''	R ₁ '''
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0.1	9999985350	5882352546	-1764705648	-3529409649-
0.2	9999764750	4705876047	-7058801048	-7058756049-
0.3	9998808850	1588224248	-1588209549	-1058772050-
0.4	9996235350	3764623048	-2823384549	-1411547450-
0.5	9990809150	7352546548	-4411212049	-1764042650-
0.6	9980942250	1270446849	-6351291049	-2115996650-
0.7	9964694850	2017230949	-8642897249	-2467021250-
0.8	9939775350	3010705049	-1128484550	-2816575250-
0.9	9903541950	4285818449	-1427532050	-3163939750-
1.0	9853004350	5877299949	-1761169050	-3508192550-
1.1	9784826650	7819565349	-2129028850	-3848183350-
1.2	9695330450	1014660250	-2530617150	-4182509550-
1.3	9580500550	1289182950	-2965286150	-4509489850-
1.4	9435990850	1608792750	-3432204650	-4827142650-
1.5	9257133150	1976664250	-3930327250	-5133162150-
1.6	9038947250	2395855550	-4458361150	-5424894850-
1.7	8776154050	2869281250	-5014729750	-5699318950-
1.8	8463191550	3399683150	-5597534950	-5953023950-
1.9	8094233550	3989594450	-6204516950	-6182190250-
2.0	7663212450	4641304050	-6833013850	-6382574150-
2.1	7163845750	5356811650	-7479916750	-6549491050-
2.2	6589667650	6137781650	-8141625650	-6677802050-
2.3	5934064250	6985493150	-8814004150	-6761906050-
2.4	5190315950	7900781250	-9492331450	-6795732050-
2.5	4351643750	8883980050	-1017125751	-6772731050-
2.6	3411263450	9934852350	-1084475051	-6685887050-
2.7	2362445850	1105252451	-1150605751	-6527715050-
2.8	1198584050	1223540751	-1214765151	-6290266050-
2.9	8673102248	-1348112051	-1276118851	-5965162050-
3.0	1499626950	-1478640551	-1333746851	-5543601750-
3.1	3045858050	-1614703651	-1386638851	-5016396650-
3.2	4730704450	-1755773651	-1433690951	-4374010950-
3.3	6558861250	-1901207251	-1473702351	-3606606850-
3.4	8534318850	-2050236251	-1505372751	-2704100750-
3.5	1066023251	-2201957151	-1527299851	-1656228950-
3.6	1293878251	-2355321251	-1537978151	-4526272049-
3.7	1537102351	-2509124351	-1535798651	-9170824049
3.8	1795672251	-2661995551	-1519048551	-2463188450
3.9	2069419051	-2812387951	-1485913251	-4195774350
4.0	2358009651	-2958568351	-1434478951	-6124588650
4.1	2660928151	-3098606851	-1362736451	-8258901450
4.2	2977455351	-3230368651	-1268586951	-1060734351
4.3	3306647951	-3351504651	-1149849651	-1317772551
4.4	3647316651	-3459443851	-1004269951	-1597684951
4.5	3998003851	-3551386951	-8295314450	-1901028751
4.6	4356959951	-3624299451	-6232691350	-2228216851
4.7	4722120551	-3674909951	-3830850250	-2579490551
4.8	5091081751	-3699705451	-1065664850	-2954894351
4.9	5461076251	-3694932051	-2086923750	3354247251

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X	R ₁	R ₁ '	R ₁ ''	R ₁ '''
5.0	5828949551	-3656596551	-5650669550	3777111551
5.1	6191136051	-3580470551	-9648745650	4222760251
5.2	6543635751	-3462099151	-1410343951	4690142951
5.3	6881992151	-3296809351	-1903581151	5177851851
5.4	7201271351	-3079725551	-2446532051	5684079651
5.5	7496042351	-2805787551	-3040940151	6206585151
5.6	7760359751	-2469772151	-3688301751	6742649551
5.7	7987749051	-2066321651	-4389814651	7289037351
5.8	8171194351	-1589975451	-5146327051	7841952051
5.9	8303130051	-1035207851	-5958276751	8396993751
6.0	8375436951	-3964726850	-6825629451	8949117051
6.1	8379441751	-3317464550	7747811951	9492587051
6.2	8305923651	-1154878951	8723643051	1002093552
6.3	8145126051	-2078202051	9751254751	1052692252
6.4	7886774851	-3106769151	1082801652	1100249852
6.5	7520105351	-4245328251	1195044452	1143876052
6.6	7033897051	-5498234251	1311413152	1182592252
6.7	6416517451	-6869349751	1431363752	1215328852
6.8	5655977151	-8361938951	1554240952	1240922252
6.9	4739994751	-9978549451	1679268252	1258112452
7.0	3656075151	-1172089152	1805538252	1265542852
7.1	2391599951	-1358969352	1932003152	1261759152
7.2	9339327750	-1558456552	2057464552	1245209252
7.3	7294612250	1770383652	2180564552	1214245652
7.4	2610879351	1994439952	2299775252	1167127252
7.5	4722228451	2230152452	2413390552	1102023152
7.6	7074854251	2476868652	2519516652	1017018252
7.7	9679351951	2733736752	2616064552	9101203051
7.8	1254535752	2999685952	2700742352	7792683051
7.9	1568131752	3273406252	2771049052	6223427151
8.0	1909424052	3553326452	2824269552	4371792751
8.1	2278942252	3837593352	2857470552	2215828151
8.2	2677015952	4124048452	2867498052	2665462050
8.3	3103742552	4410207952	2850978152	3097337351
8.4	3558953752	4693239552	2804317652	6298250451
8.5	4042179852	4969940252	2723708952	9890428951
8.6	4552611352	5236716152	2605137052	1389416252
8.7	5089059152	5489561952	2444389852	1832856052
8.8	5649912052	5724042152	2237071552	2321119152
8.9	6233093052	5935272252	1978619452	2855768652
9.0	6836013752	6117904852	1664327152	3438130252
9.1	7455527152	6266115352	1289368152	4069244452
9.2	8087879752	6373592452	8488290151	4749815352
9.3	8728661752	6433529852	3377453851	5480155452
9.4	9372758152	6438624252	2488568651	-6260123452
9.5	1001429953	6381076452	9159123951	-7089064052
9.6	1064661053	6252598452	1668261552	-7965737852
9.7	1126216353	6044424452	2510588652	-8888248552
9.8	1185253153	5747332552	3447352452	-9853969052
9.9	1240834453	5351667352	4482709252	-1085946353
10.0	1291924553	4847376052	5620430352	-1190039953

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X_0	R_2	R_2'	R_2''	R_2'''
0.1	9999998348	1999999050	2000000051	2040816247
0.2	3999989149	3999967450	2000048951	1632650648
0.3	8999876049	5999752050	2000248051	5510162448
0.4	1599930350	7998955150	2000783751	1306091349
0.5	2499734350	9996811350	2001913251	2550871949
0.6	3599206650	1199206651	2003967051	4407631049
0.7	4897999250	1398285151	2007349051	6998434149
0.8	6395542150	1596656751	2012535751	1044499350
0.9	8090962850	1793975651	2020076751	1486845750
1.0	9982995850	1989798651	2030593851	2038915150
1.1	1206987851	2183572451	2044779751	2712621850
1.2	1434923451	2374622651	2063398151	3519719150
1.3	1681794851	2562141051	2087280851	4471746150
1.4	1947202351	2745174551	2117327351	5579965250
1.5	2230643451	2922614351	2154501051	6855285850
1.6	2531496351	3093183951	2199826951	8308179850
1.7	2849002951	3255429751	2254387851	9948579450
1.8	3182250351	3407710551	2319319751	1178576551
1.9	3530151851	3548188451	2395805551	1382823751
2.0	3891426751	3674820451	2485069551	1608357151
2.1	4264579451	3785350551	2588368751	1855825851
2.2	4647877851	3877303751	2706984551	2125752951
2.3	5039331251	3947980251	2842211451	2418516551
2.4	5436667451	3994452451	2995345151	2734327951
2.5	5837309951	4013562151	3167669551	3073209751
2.6	6238355051	4001920651	3360440051	3434970351
2.7	6636548651	3955910751	3574866751	3819178551
2.8	7028263651	3871690851	3812094151	4225135051
2.9	7409478051	3745202351	4073179551	4651842051
3.0	7775753251	3572179251	4359069651	5097973151
3.1	8122214351	3348161651	4670572151	5561840051
3.2	8443531451	3068512451	5008327351	6041356951
3.3	8733902751	2728438851	5372776751	6534007151
3.4	8987040951	2323016751	5764127551	7036804251
3.5	9196160451	1847220851	6182315451	7546256051
3.6	9353969751	1295960351	6626964151	8058326351
3.7	9452666451	6641181150	7097342851	8568395651
3.8	9483937351	5340201949	-7592318651	9071225051
3.9	9438962551	8616234950	-8110309451	9560913051
4.0	9308425951	1765437751	-8649231651	1003086852
4.1	9082531851	2769538051	-9206445951	1047376152
4.2	8751027651	3878346751	-9778700051	1088150252
4.3	8303235951	5095933751	-1036207252	1124520552
4.4	7728093451	6425928151	-1095190652	1155515852
4.5	7014199751	7871420451	-1154275452	1180080552
4.6	6149877051	9434857651	-1212830852	1197072352
4.7	5123239451	1111792952	-1270134152	1205261252
4.8	3922275251	1292144252	-1325363652	1203328152
4.9	2534941751	1484519252	-1377592752	1189866552

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5.2	2863798351	-2131760352	-1505305552	1064944652
5.3	5113565151	-2369488652	-1533950752	9895913151
5.4	7606070851	-2617096352	-1553176252	8944247051
5.5	1035073552	-2873630252	-1561303152	7775698051
5.6	1335591952	-3137920052	-1556512152	6370964951
5.7	1662869552	-3408559652	-1536839852	4710304751
5.8	2017460452	-3683886552	-1500178352	2773663251
5.9	2399738752	-3961962252	-1444273652	5408244450
6.0	2809870352	-4240552652	-1366728252	2008411251-
6.1	3247781852	-4517106452	-1265003752	4894029551-
6.2	3713127952	-4788736552	-1136425052	8135574451-
6.3	4205257152	-5052199852	-9781885051	1175189152-
6.4	4723175452	-5303878552	-7873699051	1576084852-
6.5	5265506852	-5539762452	-5609379351	2017900552-
6.6	5830454652	-5755431852	-2957676451	2502128952-
6.7	6415758752	-5946043552	-1134089050	-3030058252-
6.8	7018652652	-6106317352	-3636412751	-3602732652-
6.9	7635820352	-6230525852	-7644155851	-4220905752-
7.0	8263349752	-6312487052	-1216945352	-4884991652-
7.1	8896688552	-6345560152	-1724477852	-5595012652-
7.2	9530596352	-6322645052	-2290188052	-6350542252-
7.3	1015910053	-6236187452	-2917135452	-7150645252-
7.4	1077544953	-6078187352	-3608216152	-7993812552-
7.5	1137207253	-5840215152	-4366109052	-8877897052-
7.6	1194053253	-5513432052	-5193214952	-9800037852-
7.7	1247149153	-5088620052	-6091593352	-1075659453-
7.8	1295467053	-4556217052	-7062889252	-1174305953-
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8.0	1373169253	-3128953952	-9228265752	-1378291753-
8.1	1400001653	-2213707252	-1042283053	-1482227353-
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8.3	1422477653	-7186100750	1303130253	-1689596853-
8.4	1414948853	-1462901452	1444076753	-1790889253-
8.5	1392622753	-3033003752	1591566453	-1888925253-
8.6	1353659253	-4791959452	1745095653	-1982271553-
8.7	1296122853	-6749090152	1904024753	-2069335153-
8.8	1217987753	-8913087252	2067565653	-2148357753-
8.9	1117145153	-1129183753	2234766453	-2217407753-
9.0	9914120652	-1389222753	2404498753	-2274376253-
9.1	8385426752	-1671993553	2575440853	-2316971053-
9.2	6562412352	-1977920153	2746064853	-2342714553-
9.3	4421780252	-2307257553	2914620153	-2348940453-
9.4	1940071452	-2660065653	3079118653	-2332794253-
9.5	9061227351	3036180453	3237320553	-2291234253-
9.6	4139922252	3435183953	3386719053	-2221034653-
9.7	7783888552	3856371053	3524527453	-2118792453-
9.8	1185972153	4298716353	3647664853	-1980934253-
9.9	1638791853	4760838153	3752745653	-1803729753-
10.0	2138741553	5240962453	3836066753	-1583303153-

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0.3	1280508550	7286247550	1716810851	1685150151-0000000000
0.4	2092423750	8928807850	1576737151	1174168351-0000000000
0.5	3062318950	1045206851	1474331051	8995545450-0000000000
0.6	4179820850	1188445051	1393097251	7388716250-0000000000
0.7	5436735350	1324241051	1324407551	6434323450-0000000000
0.8	6826150650	1453567751	1263004051	5905331850-0000000000
0.9	8341896750	1576963551	1205279651	5684379750-0000000000
1.0	9978179050	1694654851	1148509051	5706506250-0000000000
1.1	1172930051	1806622651	1090466251	5934132250-0000000000
1.2	1358944251	1912640951	1029216551	6345001250-0000000000
1.3	1555246451	2012300151	9630001450	6925789950-0000000000
1.4	1761173251	2105020251	8901608950	7668626450-0000000000
1.5	1975994551	2190058451	8091028550	8568960550-0000000000
1.6	2198898851	2266514751	7182652650	9624258050-0000000000
1.7	2428976551	2333334051	6161058950	1083313351-0000000000
1.8	2665204651	2389307551	5010935050	1219474751-0000000000
1.9	2906431251	2433073651	3717042250	1370836551-0000000000
2.0	3151362051	2463118551	2264227550	1537300351-0000000000
2.1	3398542451	2477778351	6374568249	1718715351-0000000000
2.2	3646344451	2475238351	1178112050	-1914852251-0000000000
2.3	3892951451	2453538051	3197045650	-2125380351-0000000000
2.4	4136343251	2410572351	5433520250	-2349846451-0000000000
2.5	4374282451	2344096851	7901186850	-2587653551-0000000000
2.6	4604299951	2251734551	1061302151	-2838038851-0000000000
2.7	4823682851	2130981951	1358114351	-3100053651-0000000000
2.8	5029462851	1979220351	1681662251	-3372542151-0000000000
2.9	5218402751	1793725451	2032925951	-3654117951-0000000000
3.0	5386989851	1571683351	2412734551	-3943143851-0000000000
3.1	5531424951	1310205351	2821739851	-4237707451-0000000000
3.2	5647618051	1006347151	3260387351	-4535600351-0000000000
3.3	5731183151	6571324150	3728886951	-4834293551-0000000000
3.4	5777432751	2595763850	4227177251	-5130915051-0000000000
3.5	5781387951	1892844550	-4754893351	-5422228551-0000000000
3.6	5737769251	6923598050	-5311325551	-5704607951-0000000000
3.7	5641014251	1252470651	-5895383051	-5974020351-0000000000
3.8	5485284051	1872307451	-6505549251	-6226000651-0000000000
3.9	5264478151	2554385351	-7139838751	-6455634451-0000000000
4.0	4972255151	3300996451	-7795752851	-6657542551-0000000000
4.1	4602060151	4114155951	-8470228251	-6825860851-0000000000
4.2	4147150051	4995539751	-9159593651	-6954231851-0000000000
4.3	3600634351	5946427151	-9859514951	-7035792651-0000000000
4.4	2955520151	6967626251	-1056494652	-7063167351-0000000000
4.5	2204756451	8059407051	-1127007952	-7028466551-0000000000
4.6	1341297451	9221413051	-1196829152	-6923291951-0000000000
4.7	3581670250	1045258552	-1265209252	-6738737051-0000000000
4.8	7514642250	-1175107152	-1331307952	-6465412251-0000000000
4.9	1994201551	-1311413052	-1394188852	-6093454651-0000000000
5.0	3376321651	-1453803452	-1452814152	-5612564451-0000000000

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X	R ₃	R' ₃	R'' ₃	R''' ₃
5.1	4903676051	-1601795952	-1506041252	-5012036651
5.2	6581580051	-1754790152	-1552619152	-4280814551
5.3	8414679951	-1912052852	-1591184052	-3407537251
5.4	1040682052	-2072710752	-1620257852	-2380613651
5.5	1256089252	-2235735152	-1638245052	-1188302251
5.6	1487868152	-2399932952	-1643433052	-1812011650
5.7	1736069652	-2563934352	-1633991352	-1739650851
5.8	2000596652	-2726178652	-1607971652	-3498633251
5.9	2281188552	-2884907552	-1563312552	-5469425651
6.0	2577395452	-3038147652	-1497841052	-7662850751
6.1	2888562052	-3183706052	-1409279352	-1008909352
6.2	3213800052	-3319154852	-1295251852	-1275751052
6.3	3551967352	-3441825152	-1153293852	-1567643852
6.4	3901642152	-3548797852	-9808641151	-1885294452
6.5	4261098252	-3636895352	-7753577851	-2229257852
6.6	4628277652	-3702678452	-5341227351	-2599910952
6.7	5000766452	-3742439852	-2544798751	-2997423152
6.8	5375766052	-3752204652	-6625531250	3421724752
6.9	5750066752	-3727730752	-4307468851	3872472252
7.0	6120020852	-3664509152	-8416097251	4349013652
7.1	6481518252	-3557777852	-1301376652	4850352452
7.2	6829959752	-3402523352	-1812461652	5375104652
7.3	7160231452	-3193499152	-2377118052	5921456052
7.4	7466685452	-2925245552	-2997396852	6487124252
7.5	7743117452	-2592106752	-3675094452	7069309852
7.6	7982749552	-2188262752	-4411697852	7664645752
7.7	8178214552	-1707764952	-5208330052	8269161252
7.8	8321546252	-1144572152	-6065682852	8878221452
7.9	8404170952	-4925969851	-6983948952	9486485952
8.0	8416903852	-2542362851	7962750952	1008785353
8.1	8349961452	-1101937752	9001061952	1067542553
8.2	8192959352	-2056374652	1009711953	1124144653
8.3	7934938552	-3123201452	1124834253	1177725953
8.4	7564393252	-4307768252	1245124753	1227327853
8.5	7069294052	-5615023352	1370132053	1271892453
8.6	6437150052	-7049417852	1499296453	1310261353
8.7	5655043352	-8614778152	1631936053	1341170753
8.8	4709725652	-1031418253	1767236653	1363249753
8.9	3587666252	-1214982253	1904242453	1375018553
9.0	2275178652	-1412286453	2041843553	1374887953
9.1	7585128951	-1623328653	2178766853	1361158453
9.2	9760133051	1847968953	2313562153	1332022153
9.3	2941860252	2085915653	2444593853	1285565253
9.4	5152126552	2336701653	2570030553	1219770253
9.5	7619327052	2599667653	2687833253	1132524953
9.6	1035522953	2873938653	2795746853	1021625253
9.7	1337060553	3158404953	2891293453	8847877352
9.8	1667498553	3451695253	2971762053	7196591552
9.9	2027639353	3752156953	3034202453	5238313452
10.0	2418103853	4057828953	3075423053	2948549852

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X	R_4 0	R_4' 0	R_4'' -∞	R_4''' ∞
0.1	5094555950	1492140851	1055188852	-1801037153
0.2	6241222050	9138095450	3235015651	-2754325952
0.3	7027773950	6853835350	1626133251	-9136175151
0.4	7644284950	5578171450	1006724351	-4126612051
0.5	8157623550	4738538850	7045462450	-2178841651
0.6	8599413750	4124784850	5383825450	-1244980951
0.7	8986797050	3638872350	4420039650	-7279766350
0.8	9329647050	3227359850	3863186650	-4086849450
0.9	9633648350	2857562450	3568558350	-1932034350
1.0	9901813050	2507567150	3457392450	-3654793049
1.1	1013528951	2161517750	3483773350	-8471458349-
1.2	1033383851	1807181050	3619420050	-1836007850-
1.3	1049611651	1434606350	3846079650	-2677050550-
1.4	1061986851	1035345250	4151452550	-3416199350-
1.5	1070204851	6019830449	4526881250	-4081857650-
1.6	1073890551	1278484349	4965970950	-4691701450-
1.7	1072605451	3931658149	-5463726450	-5256541850-
1.8	1065852051	9667133049	-6015989950	-5782600950-
1.9	1053078551	1598057150	-6619057050	-6272895750-
2.0	1033681851	2292100750	-7269401850	-6728099850-
2.1	1007012251	3053395250	-7963471250	-7147091150-
2.2	9723757550	3886128350	-8697529250	-7527314850-
2.3	9290388050	4794099050	-9467518050	-7865009250-
2.4	8762317550	5780681050	-1026895451	-8155367950-
2.5	8131534650	6848771650	-1109682351	-8392645050-
2.6	7389766050	8000739750	-1194549451	-8570225150-
2.7	6528531250	9238354050	-1280863251	-8680681450-
2.8	5539203250	1056271251	-1367912051	-8715810250-
2.9	4413084650	1197415951	-1454898651	-8666665650-
3.0	3141484550	1347219351	-1540932351	-8523590250-
3.1	1715806650	1505537651	-1625022851	-8276244050-
3.2	1276516749	1672122651	-1706073051	-7913642350-
3.3	1631074450	-1846610751	-1782872951	-7424186050-
3.4	3568042150	-2028511451	-1854093551	-6795715850-
3.5	5690360250	-2217195151	-1918281651	-6015557150-
3.6	8004434250	-2411880751	-1973855351	-5070591650-
3.7	1051580951	-2611622151	-2019099251	-3947328850-
3.8	1322899151	-2815294651	-2052161551	-2631989550-
3.9	1614727151	-3021582051	-2071052351	-1110615950-
4.0	1927252151	-3228961751	-2073640351	-6308226449
4.1	2260498151	-3435691251	-2057655651	-2606285050
4.2	2614303251	-3639793951	-2020688751	-4829518550
4.3	2988295451	-3839044851	-1960194751	-7313883550
4.4	3381868051	-4030959651	-1873497851	-1007216751
4.5	3794151451	-4212778151	-1757798151	-1311636551
4.6	4223985851	-4381455251	-1610180651	-1645746051
4.7	4669893351	-4533649951	-1427626751	-2010516751
4.8	5130045451	-4665712851	-1207028251	-2406764151
4.9	5602232751	-4773680951	-9452035350	-2835117751
5.0	6083835651	-4853271251	-6389201850	-3295990351

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X	R ₄	R ₄ '	R ₄ ''	R ₄ '''
5.1	6571788351	-4899874851	-2849157650	-3789539751
5.2	7062546751	-4908555051	-1200728650	4315630551
5.3	7552059851	-4874052551	-5792798550	4873796751
5.4	8035731751	-4790785651	-1095874051	5463196151
5.5	8508395651	-4652862851	-1672918951	6082566551
5.6	8964277051	-4454090151	-2313328951	6730173351
5.7	9396968951	-4187993651	-3019820551	7403764351
5.8	9799407951	-3847840951	-3794854951	8100518951
5.9	1016383752	-3426665851	-4640581351	8816982451
6.0	1048180152	-2917308451	-5558771351	9549027551
6.1	1074411652	-2312451851	-6550745751	1029178252
6.2	1094086152	-1604672551	-7617297051	1103957652
6.3	1106137152	-7864990650	-8758612751	1178588952
6.4	1109423252	-1495260350	9974178351	1252328552
6.5	1102729252	-1210769351	1126269352	1324335352
6.6	1084766552	-2404423651	1262195952	1393666152
6.7	1054176252	-3737414351	1404878652	1459269052
6.8	1009531952	-5216290651	1553886652	1519978452
6.9	9493438951	-6847112651	1708668352	1574511352
7.0	8720649551	-8635324551	1868536552	1621462352
7.1	7760971051	-1058560652	2032657252	1659299052
7.2	6597998551	-1270172652	2200037252	1686361152
7.3	5215006451	-1498638052	2369510652	1700856552
7.4	3595056351	-1744099752	2539725752	1700861352
7.5	1721139451	-2006556352	2709131552	1684319352
7.6	4236687750	2285840652	2875965652	1649043752
7.7	2856033251	2581597752	3038239052	1592720452
7.8	5592163851	2893262552	3193726552	1512911052
7.9	8647597151	3220034652	3339952952	1407060852
8.0	1203692152	3560853452	3474182552	1272509452
8.1	1577353252	3914370852	3593405752	1106495752
8.2	1986934152	4278924752	3694337352	9061770151
8.3	2433439852	4652510152	3773401552	6686434951
8.4	2917658452	5032747752	3826727552	3909370851
8.5	3440118552	5416858752	3850149752	7007352850
8.6	4001052652	5801632252	3839204452	2969294751-
8.7	4600345152	6183395152	3789128152	7130234151-
8.8	5237493152	6557985552	3694866752	1181096652-
8.9	5911547652	6920717852	3551083252	1703934552-
9.0	6621067452	7266364152	3352166152	2284174852-
9.1	7364057352	7589118852	3092252652	2924266652-
9.2	8137915352	7882578652	2765244952	3626408452-
9.3	8939363352	8139720952	2364838652	4392500652-
9.4	9764395352	8352883552	1884560652	5224078352-
9.5	1060819853	8513750952	1317803652	6122239252-
9.6	1146510153	8613338052	6578733451	7087577052-
9.7	1232850253	8641995152	1019518651	-8120100952-
9.8	1319079353	8589394652	9683657351	-9219144352-
9.9	1404330653	8444549752	1947952552	-1038327653-
10.0	1487624253	8195819252	3047116452	-1161022253-

$$a = 3/4$$

X	R ₀	R' ₀	R'' ₀	R''' ₀
0	1000000051	0	0	0
0.1	9999984850	6060605646	-1818181548	-3636361449
0.2	9999757650	4848478047	-7272703848	-7272657049
0.3	9998772750	1636352248	-1636337049	-1090855850
0.4	9996121350	3878702148	-2908940849	-1454320450
0.5	9990530650	7575348848	-4544882149	-1817495050
0.6	9980364750	1308944449	-6543745249	-2180109150
0.7	9963625050	2078356949	-8904781249	-2541760450
0.8	9937950450	3101932749	-1162676150	-2901888750
0.9	9900619150	4415678849	-1470780550	-3259749950
1.0	9848550350	6055373049	-1814518950	-3614388150
1.1	9778306950	8056469249	-2193511350	-3964612750
1.2	9686099450	1045397850	-2607246150	-4308970450
1.3	9567791150	1328232150	-3055052150	-4645720550
1.4	9418904650	1657515550	-3536068650	-4972811150
1.5	9234630650	2036517050	-4049213150	-5287853650
1.6	9009838850	2468384750	-4593146650	-5588100550
1.7	8739091350	2956118650	-5166236350	-5870421850
1.8	8416658750	3502538850	-5766516450	-6131286250
1.9	8036540250	4110250750	-6391646750	-6366737650
2.0	7592486850	4781605250	-7038870850	-6572382050
2.1	7078029050	5518654250	-7704968350	-6743367050
2.2	6486508850	6323104350	-8386211950	-6874372050
2.3	5811117450	7196259450	-9078317150	-6959598050
2.4	5044937950	8138968150	-9776396650	-6992754050
2.5	4180994050	9151553850	-1047490751	-6967065050
2.6	3212305850	1023375651	-1116760651	-6875262050
2.7	2131952250	1138464951	-1184749751	-6709593050
2.8	9331408249	1260257051	-1250678951	-6461836050
2.9	3907141649	-1388503451	-1313684051	-6123308050
3.0	1845905350	-1522864751	-1372812251	-5684902050
3.1	3438336650	-1662901851	-1427017551	-5137105650
3.2	5173419250	-1808066251	-1475156851	-4470047750
3.3	7055956650	-1957689651	-1515986951	-3673545850
3.4	9090020650	-2110975151	-1548161551	-2737164250
3.5	1127881751	-2266985151	-1570228651	-1650281950
3.6	1362453851	-2424632151	-1580629751	-4021753049
3.7	1612821251	-2582666851	-1577698851	-1017891050
3.8	1878952951	-2739668351	-1559663151	-2620558650
3.9	2160667251	-2894033551	-1524644251	-4416253150
4.0	2457612151	-3043965351	-1470661051	-6415052350
4.1	2769246051	-3187464351	-1395634251	-8626533750
4.2	3094817051	-3322318351	-1297391651	-1105960951
4.3	3433340651	-3446093551	-1173676451	-1372234051
4.4	3783577951	-3556126751	-1022155851	-1662173351
4.5	4144011551	-3649518251	-840433550	-1976353051
4.6	4512822351	-3723125351	-6260629450	-2315195051
4.7	4887864451	-3773559851	-3765640550	-2678945251
4.8	5266640851	-3797183951	-8944204849	-3067644051
4.9	5646278251	-3790111051	-2377902950	3481097551

$$a = 3/4$$

X	R ₁	R ₁ '	R ₁ ''	R ₁ '''
5.0	6023502351	-3748207051	-6075875550	3918845151
5.1	6394613051	-3667094851	-1022344151	4380126851
5.2	6755460851	-3542162751	-1484361251	4863846051
5.3	7101423951	-3368574451	-1995812751	5368534051
5.4	7427385951	-3141285251	-2558704851	5892310451
5.5	7727716551	-2855058751	-3174833851	6432840951
5.6	7996253151	-2504492451	-3845737751	6987298651
5.7	8226285751	-2084044051	-4572646051	7552317351
5.8	8410544451	-1588066951	-5356422351	8123950651
5.9	8541191151	-1010848751	-6197505151	8697625651
6.0	8609815051	-3466568650	-7095841251	9268100051
6.1	8607433251	-4102084050	8050819751	9829413051
6.2	8524497151	-1265354551	9061196551	1037484752
6.3	8350904451	-2224229851	1012501152	1089688452
6.4	8076019451	-3292047251	1123951452	1138716252
6.5	7688699951	-4473702551	1240107852	1183643652
6.6	7177334351	-5773679451	1360509752	1223454852
6.7	6529886951	-7195949651	1484591252	1257038952
6.8	5733955451	-8743862451	1611669852	1283188152
6.9	4776838751	-1042002152	1740937652	1300594952
7.0	3645617751	-1222615452	1871450652	1307851052
7.1	2327250351	-1416297552	2002119152	1303447252
7.2	8086800550	-1623003152	2131697752	1285772852
7.3	9230392350	1842553952	2258774552	1253118552
7.4	2880602051	2074621952	2381762052	1203677752
7.5	5076292151	2318711152	2498887352	1135551052
7.6	7521805851	2574138652	2608183252	1046752352
7.7	1022805552	2840014652	2707479952	9352151051
7.8	1320494852	3115222252	2794397152	7988020051
7.9	1646115752	3398395052	2866337952	6353156251
8.0	2000385152	3687896952	2920483052	4425118751
8.1	2383842052	3981797252	2953787252	2181155251
8.2	2796816752	4277850852	2962976852	4016137050-
8.3	3239398252	4573472252	2944549552	3345990451-
8.4	3711399052	4865715952	2894776652	6674456051-
8.5	4212318252	5151251552	2809707052	1040890252-
8.6	4741301652	5426342252	2685175152	1457032952-
8.7	5297100252	5686825552	2516811252	1917850252-
8.8	5878026652	5928091552	2300055252	2425158152-
8.9	6481909352	6145066152	2030175752	2980570652-
9.0	7106045552	6332193552	1702291852	3585454252-
9.1	7747152052	6483424052	1311399852	4240878652-
9.2	8401316052	6592203252	8524067051	4947563452-
9.3	9063942852	6651464052	3201674551	5705819352-
9.4	9729706052	6653623152	2904715651	-6515487352-
9.5	1039249553	6590585652	9846212351	-7375872352-
9.6	1104536453	6453748052	1767293852	-8285670252-
9.7	1168048253	6234014652	2643337152	-9242896652-
9.8	1228908553	5921814152	3617363452	-1024480453-
9.9	1286142953	5507132052	4693668752	-1128780653-
10.0	1338674953	4979539052	5876147952	-1236738553-

$$a = 3/4$$

X_0	R_2	R_2'	R_2''	R_2'''
0.1	99999998348	19999998750	20000000051	
0.2	39999989049	39999958450	2000002851	2040816247
0.3	8999874749	5999684450	2000044551	1632650648
0.4	1599929650	7998670250	2000712551	5510162448
0.5	2499731550	9995941650	2001739351	1306091249
0.6	3599198450	1198990251	2003606451	2550871449
0.7	4897978650	1397817451	2006680851	4407629349
0.8	6395496150	1595744951	2011396051	6998429349
0.9	8090869750	1792332651	2018251251	1044498050
1.0	9982820550	1987016551	2027811651	1486843050
1.1	1206956851	2179092551	2040707051	2038909250
1.2	1434871151	2367702151	2057630851	2712610350
1.3	1681710251	2551817051	2079338951	3519697950
1.4	1947070451	2730224651	2106647851	4471709050
1.5	2230443251	2901513651	2140432151	5579902650
1.6	2531202651	3064060451	2181621951	6855184650
1.7	2848580551	3216015551	2231198551	8308020850
1.8	3181655351	3355291551	2290190751	9948336650
1.9	3529329251	3479550951	2359669351	1178540351
2.0	3890308351	3586195951	2440740451	1382770951
2.1	4263081651	3672358651	2534538251	1608281451
2.2	4645899451	3734893451	2642216551	1855719451
2.3	5036750451	3770370451	2764938251	2125605751
2.4	5433339351	3775070951	2903864051	2418315651
2.5	5833063351	3744985951	3060139151	2734057851
2.6	6232988951	3675815151	3234878151	3072850351
2.7	6629829051	3562971351	3429148851	3434498051
2.8	7019919851	3401586051	3643952551	3818564251
2.9	7399198251	3186519151	3880203751	4224343451
3.0	7763180951	2912373151	4138706351	4650831651
3.1	8106944151	2573510651	4420129051	5096694451
3.2	8425104151	2164077451	4724976051	5560233951
3.3	8711801551	1678030851	5053557951	6039355451
3.4	8960685951	1109172751	5405956651	6531530051
3.5	9164905051	4511901450	5781991851	7033758651
3.6	9317095251	3022986250	-6181180951	7542535351
3.7	9409378251	1157690151	-6602697751	8053807251
3.8	9433360051	2121334551	-7045329551	8562938851
3.9	9380135751	3199465851	-7507429151	9064670051
4.0	9240300551	4398123851	-7986867451	9553080651
4.1	9003966451	5723066951	-8480979451	1002155352
4.2	8660785451	7179675051	-8986512851	1046273652
4.3	8199982251	8772842851	-9499570051	1086851352
4.4	7610393351	1050686152	-1001555352	1122996652
4.5	6880518151	1238529252	-1052909652	1153735652
4.6	5998577351	1441082552	-1103402252	1178009152
4.7	4952585551	1658513152	-1152326252	1194671352
4.8	3730433151	1890869652	-1198881052	1202488652
4.9	2319983551	2138065152	-1242165052	1200138252
				1186209252

$$a = 3/4$$

X	R ₂	R ₁	R ₀	R ₋₁
5.0	7091815050	2399858552	-1281170852	1159203452
5.1	1113820651	-2675835252	-1314778552	1117536952
5.2	3160523951	-2965385452	-1341751652	1059543952
5.3	5441926351	-3267683852	-1360730152	9834811651
5.4	7968347651	-3581665552	-1370228352	8875333651
5.5	1074924052	-3906002752	-1368628952	7698212751
5.6	1379297552	-4239080452	-1354181152	6284104951
5.7	1710661652	-4578971252	-1324998252	4613226351
5.8	2069566952	-4923408952	-1279056452	2665483651
5.9	2456380952	-5269763252	-1214195452	4206260950
6.0	2871259752	-5615012852	-1128119252	2141577451
6.1	3314116152	-5955719352	-1018400152	5041137551
6.2	3784587552	-6288001752	-8824836051	8297617951
6.3	4282000152	-6607510652	-7176956051	1192987552
6.4	4805331852	-6909404552	-5212519051	1595577452
6.5	5353174352	-7188326552	-2902700151	2039187552
6.6	5923691352	-7438383552	-2178460050	2525307452
6.7	6514576952	-7653127452	-2872350451	-3055222152
6.8	7123012652	-7825538652	-6398634051	-3629970052
6.9	7745620452	-7948014452	-1039194052	-4250297752
7.0	8378420852	-8012358952	-1488310152	-4916610452
7.1	9016781852	-8009780252	-1990252152	-5628918652
7.2	9655377352	-7930890752	-2547979652	-6386780552
7.3	1028813753	-7765714352	-3164328552	-7189243852
7.4	1090820353	-7503701452	-3841965052	-8034780752
7.5	1150788553	-7133749852	-4583330452	-8921218952
7.6	1207861753	-6644235652	-5390585752	-9845671152
7.7	1261092053	-6023053052	-6265544052	-1080446353
7.8	1309436153	-5257665352	-7209602952	-1179305453
7.9	1351752553	-4335166652	-8223667052	-1280595953
8.0	1386798753	-3242356052	-9308065652	-1383666653
8.1	1413228853	-1965827452	-1046246053	-1487755453
8.2	1429592253	-4920712351	-1168575653	-1591980853
8.3	1434333253	-1192405552	1297599253	-1695333653
8.4	1425791253	-3100943552	1433023653	-1796668853
8.5	1402202253	-5246577452	1574447253	-1894696653
8.6	1361701553	-7641862052	1721347253	-1987975653
8.7	1302327753	-1029867853	1873068053	-2074904253
8.8	1222028653	-1322802053	2028807253	-2153714253
8.9	1118667853	-1643975753	2187602653	-2222464153
9.0	9900348952	-1994237153	2348318153	-2279034153
9.1	8338557952	-2374268153	2509628553	-2321120653
9.2	6478067252	-2784552853	2670007053	-2346233453
9.3	4295300452	-3225344353	2827708753	-2351693453
9.4	1766525052	-3696629553	2980757553	-2334632753
9.5	1131931952	4198090553	3126931153	-2291995453
9.6	4423442152	4729064153	3263747453	-2220541453
9.7	8130800952	5288498653	3388451753	-2116852253
9.8	1227591853	5874908353	3498003753	-1977339653
9.9	1687947753	6486326253	3589065853	-1798257153
10.0	2196056453	7120255653	3657995653	-1575714053

$$a = 3/4$$

X ₀	R ₀ ³	R ₀ ^{'3}	R ₀ ^{''3}	R ₀ ^{'''3}
0.1	1361364749	2540340250	2199992951	2947730651-
0.2	4962527049	4630070250	2004789751	1345133851-
0.3	1057519150	6577638650	1898371251	8547448850-
0.4	1808891350	8437701850	1825510051	6273183650-
0.5	2742951850	1023419851	1769520351	5042586050-
0.6	3854041350	1197982651	1722876851	4354858850-
0.7	5137460050	1368162551	1681298651	4007719050-
0.8	6589026150	1534314351	1641903951	3907571850-
0.9	8204782350	1696542051	1602481351	4007903350-
1.0	9980771050	1854747151	1561157451	4285161850-
1.1	1191284551	2008653151	1516226951	4727978450-
1.2	1399649851	2157817951	1466061351	5331846050-
1.3	1622670251	2301637551	1409055151	6096267350-
1.4	1859773951	2439347251	1343594751	7023118850-
1.5	2110305251	2570020151	1268040651	8115648350-
1.6	2373507151	2692563151	1180716451	9377816250-
1.7	2648505151	2805713851	1079904751	1081383951-
1.8	2934289251	2908035851	9638463150	1242783851-
1.9	3229697651	2997914951	8307421450	1422356351-
2.0	3533397751	3073554851	6787586950	1620414951-
2.1	3843868251	3132975351	5060351550	1837188851-
2.2	4159379751	3174007951	3106928050	2072801851-
2.3	4477978651	3194297151	9084677849	2327251351-
2.4	4797463351	3191298051	1553804150	-2600385651-
2.5	5115371051	3162279751	4298427050	-2891882551-
2.6	5428955451	3104327951	7343520550	-3201225951-
2.7	5735169151	3014349251	1070658551	-3527681851-
2.8	6030648651	2889080551	1440426651	-3870273151-
2.9	6311694651	2725096451	1845209751	-4227752651-
3.0	6574258551	2518824351	2286422451	-4598578251-
3.1	6813926851	2266556251	2765308551	-4980881651-
3.2	7025909351	1964471651	3282907751	-5372441951-
3.3	7205030451	1608657351	3840019351	-5770655551-
3.4	7345716851	1195133151	4437160651	-6172504151-
3.5	7442000051	7198839150	5074526651	-6574528151-
3.6	7487502351	1788923250	5751941151	-6972791351-
3.7	7475453351	4318200050	-6468811551	-7362855251-
3.8	7398685551	1116150351	-7224075051	-7739748151-
3.9	7249643751	1877862051	-8016138751	-8097931951-
4.0	7020414151	2720533451	-8842834651	-8431283351-
4.1	6702727851	3647490951	-9701343651	-8733059951-
4.2	6288003851	4661745251	-1058814752	-8995883051-
4.3	5767379951	5765920351	-1149895452	-9211715551-
4.4	5131751751	6962165851	-1242863452	-9371843551-
4.5	4371823551	8252076651	-1337115452	-9466866651-
4.6	3478181051	9636593751	-1431950352	-9486689651-
4.7	2441345551	1111590552	-1526562652	-9420518151-
4.8	1251862251	1268933952	-1620035852	-9256869951-
4.9	9960616149	-1435525252	-1711334652	-8983578951-
5.0	1622180651	-1611089952	-1799299252	-8587824751-

$$a = \frac{3}{4}$$

X	R_3	R'_3	R''_3	R'''_3
5.1	3324648251	-1795231252	-1882638352	-8056158451
5.2	5215325651	-1987415852	-1959922652	-7374539351
5.3	7301934351	-2186961652	-2029580652	-6528402051
5.4	9591422851	-2393021052	-2089892452	-5502704751
5.5	1208980652	-2604566452	-2138984952	-4282019951
5.6	1480198052	-2820376352	-2174830252	-2850628351
5.7	1773150452	-3039018152	-2195242052	-1192624851
5.8	2088040752	-3258832152	-2197874252	-7079447250
5.9	2424893452	-3477917152	-2180221952	-2866939451
6.0	2783528052	-3694111752	-2139621952	-5299951851
6.1	3163538452	-3904983152	-2073258852	-8022114651
6.2	3564256852	-4107805752	-1978167652	-1104790352
6.3	3984730252	-4299555052	-1851243352	-1439086752
6.4	4423687752	-4476886252	-1689251752	-1806338952
6.5	4879505052	-4636124852	-1488840352	-2207639952
6.6	5350175752	-4773258752	-1246557952	-2643903452
6.7	5833274752	-4883922552	-9588707951	-3115831752
6.8	6325921952	-4963399252	-6221876751	-3623875852
6.9	6824746152	-5006607052	-2328859451	-4168197352
7.0	7325852752	-5008102352	-2126558651	4748624852
7.1	7824783252	-4962081752	-7180243451	5364607652
7.2	8316480252	-4862385752	-1286730452	6015163552
7.3	8795254552	-4702511152	-1922161452	6698831652
7.4	9254749452	-4475620852	-2627534752	7413617952
7.5	9687907452	-4174572152	-3405836852	8156932752
7.6	1008694453	-3791933052	-4259764652	8925535252
7.7	1044331853	-3320021152	-5191655852	9715471852
7.8	1074770953	-2750939452	-6203413352	1052201253
7.9	1098999953	-2076628652	-7296427252	1133958253
8.0	1115925653	-1288917852	-8471482652	1216170853
8.1	1124372953	-3795917251	-9728671652	1298092853
8.2	1123084953	-6595314651	1106729053	1378875853
8.3	1110722753	-1836527452	1248572453	1457559553
8.4	1085868453	-3159251252	1398135353	1533068053
8.5	1047026653	-4635248552	1555040553	1604201753
8.6	9926287152	-6271613752	1718785453	1669632253
8.7	9210379952	-8074884752	1888728153	1727897953
8.8	8305553552	-1005087253	2064072553	1777398153
8.9	7194283952	-1220451153	2243857853	1816389953
9.0	5858596952	-1453968853	2426937553	1842984053
9.1	4280203352	-1705904153	2611973053	1855142753
9.2	2440609252	-1976376953	2797412153	1850681453
9.3	3212867751	-2265341653	2981477553	1827264853
9.4	2096161852	2572559553	3162150653	1782412453
9.5	4829772452	2897582653	3337157753	1713499853
9.6	7897034052	3239718553	3503957153	1617766453
9.7	1131459853	3598005953	3659720953	1492322253
9.8	1509801653	3971190653	3801330753	1334156853
9.9	1926141453	4357687653	3925357653	1140156053
10.0	2381718353	4755552353	4028060253	9071141052

$$a = 3/4$$

X	R ₀ ⁻⁴	R _∞ ^{'4}	R _{-∞} ^{"4}	R _∞ ^{'''4}
0.1	7345559750	9840816250	8523915251	-1590108453
0.2	8060223450	5396103150	2343243151	-2175836552
0.3	8509490850	3788700650	1109529451	-6736044951
0.4	8841932650	2933193750	6642943650	-2869899351
0.5	9106056550	2383026950	4598224250	-1421085151
0.6	9323395150	1981021950	3553854050	-7426029550
0.7	9504784850	1655736850	3013286050	-3715046850
0.8	9655803050	1368753550	2764327550	-1427438150
0.9	9779023550	1096642150	2703724750	-1269622249-
1.0	9875096050	8235962049	2776229950	-1271798650-
1.1	9943322350	5380329049	2950021150	-2172397650-
1.2	9981980750	2308725849	3205611950	-2918852950-
1.3	9988526150	1053914849	-3530392550	-3562656450-
1.4	9959716450	4772224849	-3915747050	-4134179950-
1.5	9891704850	8903503749	-4355432050	-4651571650-
1.6	9780096850	1349958050	-4844618550	-5125533450-
1.7	9620005150	1860790050	-5379292650	-5562000450-
1.8	9406085550	2427212950	-5955862250	-5963701950-
1.9	9132575150	3053244450	-6570891350	-6331098550-
2.0	8793326850	3742557350	-7220899250	-6662961850-
2.1	8381842550	4498468150	-7902214850	-6956736850-
2.2	7891314150	5323911450	-8610856350	-7208779050-
2.3	7314658850	6221404350	-9342428550	-7414507250-
2.4	6644563350	7192999350	-1009203451	-7568502950-
2.5	5873537450	8240230850	-1085419951	-7664585250-
2.6	4993963550	9364053850	-1162279151	-7695857850-
2.7	3998162650	1056477651	-1239095951	-7654753250-
2.8	2878457750	1184197851	-1315105751	-7533058250-
2.9	1627257650	1319443651	-1389459051	-7321951750-
3.0	2371325549	1462003151	-1461215451	-7012030850-
3.1	1299085650	-1611565651	-1529337251	-6593351650-
3.2	2988195750	-1767711651	-1592685151	-6055463350-
3.3	4836525050	-1929902151	-1650012751	-5387456650-
3.4	6849793350	-2097468351	-1699962851	-4578022950-
3.5	9032983950	-2269599751	-1741062851	-3615509850-
3.6	1139019451	-2445333151	-1771723251	-2488006350-
3.7	1392447851	-2623539751	-1790232951	-1183418150-
3.8	1663766551	-2802913551	-1794760551	-3104224949
3.9	1953019751	-2981960551	-1783352951	-2005649750
4.0	2260091351	-3158983951	-1753936151	-3914234050
4.1	2584685551	-3332074351	-1704318351	-6047836850
4.2	2926304951	-3499097651	-1632192351	-8417645650
4.3	3284224351	-3657682451	-1535142951	-1103420251
4.4	3657473051	-3805211851	-1410653351	-1390719251
4.5	4044802951	-3938811651	-1256115351	-1704523651
4.6	4444666651	-4055343651	-1068840551	-2045564951
4.7	4855189151	-4151396751	-8460749950	-2414417051
4.8	5274140751	-4223282351	-5850171150	-2811468651
4.9	5698907951	-4267029251	-2828353650	-3236894351
5.0	6126467351	-4278384051	-6330558549	3690618051

$$a = 3/4$$

X	R ₄	R' ₄	R'' ₄	R''' ₄
5.1	6553353951	-4252809151	-4562199750	4172281651
5.2	6975637451	-4185488651	-8986707250	4681206851
5.3	7388890951	-4071334451	-1393335851	5216356551
5.4	7788164151	-3904995251	-1942768651	5776288851
5.5	8167963851	-3680874851	-2549357451	6359122351
5.6	8522220151	-3393146351	-3215278351	6962483651
5.7	8844272051	-3035777551	-3942442251	7583459551
5.8	9126847751	-2602562751	-4732441451	8218557251
5.9	9362044351	-2087153551	-5586486851	8863646351
6.0	9541323251	-1483104151	-6505343951	9513913751
6.1	9655495351	-7839155250	-7489258851	1016380852
6.2	9694718651	-1690527549	8537883251	1080699452
6.3	9648511651	-9257848950	9650195451	1143629652
6.4	9505750951	-1949008851	1082441352	1204365852
6.5	9254694651	-3092644251	1205789852	1262008552
6.6	8883014551	-4362446151	1334706852	1315560552
6.7	8377823351	-5763760851	1468729152	1363922652
6.8	7725718251	-7301414451	1607278152	1405889152
6.9	6912855451	-8979593651	1749650052	1440146452
7.0	5925001351	-1080171052	1895003152	1465267652
7.1	4747628951	-1277026452	2042347752	1479712952
7.2	3366016151	-1488668952	2190534952	1481827352
7.3	1765355651	-1715117952	2338243552	1469840552
7.4	6911525549	1956251552	2483970152	1441868652
7.5	2151952951	2211789152	2626017652	1395915552
7.6	4497343351	2481269352	2762484152	1329877952
7.7	7118922351	2764029952	2891250752	1241549552
7.8	1002953752	3059185852	3009974352	1128627852
7.9	1324105052	3365604452	3116076952	9887248151
8.0	1676404252	3681886252	3206737852	8193777351
8.1	2060755052	4006335252	3278886752	6180622951
8.2	2477876052	4336936352	3329199152	3822084151
8.3	2928269252	4671327952	3354091152	1092219551
8.4	3412178152	5006779552	3349719552	2034956851-
8.5	3929556052	5340160052	3311981452	5585215651-
8.6	4480022752	5667918752	3236520452	9583822351-
8.7	5062819452	5986053952	3118729252	1405520152-
8.8	5676764052	6290092452	2953763152	1902258152-
8.9	6320202752	6575064552	2736552252	2450759852-
9.0	6990959852	6835484952	2461820052	3052982352-
9.1	7686281952	7065329152	2124108052	3710631152-
9.2	8402787152	7258020952	1717799652	4425101852-
9.3	9136409852	7406413252	1237159052	5197423252-
9.4	9882336852	7502780752	6763664551	6028195852-
9.5	1063495653	7538818052	2956789250	6917522452-
9.6	1138779353	7505633652	7090746251	-7864934352-
9.7	1213345953	7393750552	1545318852	-8869314452-
9.8	1286358753	7193129852	2484774952	-9928814952-
9.9	1356877453	6893175052	3532830552	-1104076553-
10.0	1423854053	6482772952	4694559052	-1220159653-

$$\alpha = 2$$

X	R ₁	R ₁ '	R ₁ ''	R ₁ '''
0	1000000051	0	0	0
0.1	9999982150	7142856646	-2142856748	-4285711649-
0.2	9999714350	5714277647	-8571400248	-8571344049-
0.3	9998553650	1928557548	-1928539149	-1285649650-
0.4	9995428650	4571324848	-3428389949	-1714013450-
0.5	9988839650	8928076848	-5356450549	-2142026050-
0.6	9976858550	1542679949	-7712217649	-2569360650-
0.7	9957129650	2449478649	-1049478650	-2995530350-
0.8	9926870450	3655815049	-1370266750	-3419857550-
0.9	9882873450	5204114449	-1733358950	-3841441250-
1.0	9821507750	7136525649	-2138425350	-4259125850-
1.1	9738723250	9494804849	-2585006350	-4671470550-
1.2	9630054650	1232017250	-3072483050	-5076716050-
1.3	9490627750	1565313550	-3600043250	-5472758050-
1.4	9315167850	1953327650	-4166645650	-5857113050-
1.5	9098010150	2399900650	-4770980450	-6226890950-
1.6	8833112450	2908727850	-5411427550	-6578768950-
1.7	8514071950	3483324550	-6086011750	-6908960050-
1.8	8134144950	4126989250	-6792354950	-7213192050-
1.9	7686270250	4842760550	-7527626250	-7486680050-
2.0	7163097950	5633368450	-8288489650	-7724108050-
2.1	6557022650	6501182350	-9071050850	-7919609050-
2.2	5860222450	7448152150	-9870800150	-8066748050-
2.3	5064704250	8475743350	-1068255551	-8158509050-
2.4	4162355350	9584866950	-1150040351	-8187293050-
2.5	3145002950	1077580451	-1231764251	-8144900050-
2.6	2004480850	1204812451	-1312671951	-8022550050-
2.7	7327053449	1340059551	-1391917451	-7810874050-
2.8	6782402049	-1483109251	-1468558551	-7499932050-
2.9	2236011250	-1633649751	-1541550451	-7079242050-
3.0	3947897350	-1791259351	-1609740851	-6537799050-
3.1	5820707050	-1955395851	-1671865651	-5864121050-
3.2	7860641250	-2125384451	-1726542951	-5046296050-
3.3	1007315551	-2300406251	-1772270151	-4072041350-
3.4	1246280851	-2479486051	-1807420351	-2928771850-
3.5	1503310051	-2661479251	-1830239551	-1603691650-
3.6	1778629851	-2845060351	-1838845151	-8388800048-
3.7	2072324551	-3028707751	-1831225151	-1643555150
3.8	2384316251	-3210693151	-1805239251	-3591425150
3.9	2714343151	-3389067951	-1758620151	-5772235850
4.0	3061936951	-3561649851	-1688977651	-8198062450
4.1	3426399351	-3726012651	-1593803951	-1088035651
4.2	3806776351	-3879472451	-1470480151	-1382974151
4.3	4201832451	-4019079651	-1316286751	-1705579251
4.4	4610023251	-4141607451	-1128414451	-2056678051
4.5	5029467851	-4243543651	-9039787250	-2436941251
4.6	5457919251	-4321086451	-6400376550	-2846852951
4.7	5892735651	-4370135251	-3336107050	-3286680251
4.8	6330850151	-4386292351	-1829644849	3756438151
4.9	6768741451	-4364860551	-4186658050	4255853451

$$a = 2$$

X	R_1	R_1'	R_1''	R_1'''
5.0	7202403151	-4300845851	-8704361550	4784326651
5.1	7627315351	-4188964351	-1376467951	5340889251
5.2	8038415251	-4023651851	-1939504851	5924163451
5.3	8430070551	-3799077051	-2562130251	6532312051
5.4	8796052851	-3509159951	-3246719151	7162995151
5.5	9129514551	-3147596951	-3995384951	7813318851
5.6	9422967451	-2707886851	-4809921851	8479783551
5.7	9668265151	-2183369151	-5691741851	9158232051
5.8	9856588451	-1567263251	-6641805651	9843796051
5.9	9978436451	-8527179450	-7660551751	1053084452
6.0	1002362252	-3286834149	-8747810451	1121292452
6.1	9981272651	-8991002850	9902727251	1188270752
6.2	9839841251	-1949878851	1112366852	1253194252
6.3	9587120151	-3125952051	1240812452	1315139652
6.4	9210267651	-4433505951	1375261452	1373081052
6.5	8695842851	-5878325351	1515257452	1425885052
6.6	8029850751	-7465680051	1660225652	1472306552
6.7	7197800451	-9200200651	1809460852	1510984952
6.8	6184775551	-1108574352	1962115552	1540441352
6.9	4975518951	-1312523952	2117188452	1559075752
7.0	3554532951	-1532053952	2273512652	1565165452
7.1	1906196351	-1767223552	2429742052	1556865252
7.2	1489907949	-2017948252	2584340552	1532207252
7.3	2134804251	2283979552	2735568752	1489103152
7.4	4558020151	2564884652	2881472852	1425347352
7.5	7269320851	2860024252	3019872652	1338622652
7.6	1028252652	3168528852	3148351052	1226506452
7.7	1361046252	3489275252	3264243752	1086480452
7.8	1726469352	3820860852	3364630152	9159411051
7.9	2125523152	4161578452	3446325852	7122140051
8.0	2559021852	4509388152	3505875752	4725699251
8.1	3027557652	4861891352	3539550452	1942445751
8.2	3531464152	5216302452	3543342552	1255394051
8.3	4070775552	5569421452	3512968452	4895462351
8.4	4645184452	5917605452	3443868452	9004974051
8.5	5253995752	6256743752	3331215752	1361038352
8.6	5896078652	6582227852	3169923352	1873701152
8.7	6569815752	6888929152	2954658752	2440862552
8.8	7273049952	7171175152	2679861352	3064697452
8.9	8003028252	7422725352	2339765752	3747128452
9.0	8756344852	7636753652	1928428252	4489769752
9.1	9528881352	7805833552	1439762752	5293866352
9.2	1031574653	7921922352	8675787251	6160228052
9.3	1111121253	7976356852	2056306151	7089158052
9.4	1190865353	7959847852	5523299951	-8080377952
9.5	1270048753	7862484052	1412490652	-9132942952
9.6	1347810553	7673741252	2380907752	-1024516053
9.7	1423181953	7382499152	3463427352	-1141449153
9.8	1495079953	6977067052	4665594852	-1263745753
9.9	1562301953	6445219352	5992557152	-1390954253
10.0	1623520753	5774237952	7448956952	-1522508653

$\alpha = 2$

X	R_2	R_2'	R_2''	R_2'''
0	0	0	20000000051	0
0.1	9999998248	1999998950	2000003351	2173912947
0.2	3999988449	3999965250	2000052251	1739127848
0.3	8999868049	5999735950	2000264251	5869520048
0.4	1599925850	7998887050	2000834851	1391270549
0.5	2499717050	9996603350	2002037951	2717230149
0.6	3599154850	1199154851	2004225851	4695074549
0.7	4897868750	1398173351	2007828251	7454822449
0.8	6395251350	1596438651	2013353251	1112610750
0.9	8090373550	1793582751	2021386051	1583795550
1.0	9981886950	1989133351	2032588751	2171849450
1.1	1206791451	2182501251	2047699451	2889457150
1.2	1434592451	2372967851	2067531151	3749128250
1.3	1681259751	2559672551	2092970151	4763140650
1.4	1946367851	2741600151	2124973651	5943470850
1.5	2229381351	2917569451	2164567551	7301713350
1.6	2529638051	3086221351	2212843151	8848986150
1.7	2846330351	3246007551	2270952351	1059582451
1.8	3178486051	3395180451	2340103751	1255205251
1.9	3524947651	3531783251	2421555551	1472664851
2.0	3884351351	3653640451	2516608951	1712759051
2.1	4255104451	3758350551	2626599651	1976167451
2.2	4635362351	3843279151	2752887551	2263433351
2.3	5023005351	3905553251	2896846351	2574942451
2.4	5415614251	3942057551	3059849051	2910899251
2.5	5810446151	3949432451	3243254051	3271303651
2.6	6204409951	3924074051	3448387751	3655923851
2.7	6594041951	3862136651	3676525951	4064266751
2.8	6975482051	3759537551	3928871551	4495548851
2.9	7344450551	3611964651	4206531851	4948663751
3.0	7696225151	3414888251	4510491751	5422147351
3.1	8025620451	3163574751	4841584851	5914143051
3.2	8326968451	2853106451	5200461251	6422364251
3.3	8594100751	2478403251	5587553851	6944055851
3.4	8820334251	2034250851	6003039951	7475954151
3.5	8998458751	1515333551	6446800751	8014246951
3.6	9120729551	9162722250	6918378351	8554533051
3.7	9178861851	2316679750	7416927851	9091776951
3.8	9164032551	5438467550	-7941167651	9620273251
3.9	9066885951	1415551651	-8489327651	1013360152
4.0	8877544851	2388573851	-9059092351	1062458852
4.1	8585630251	3467816851	-9647543051	1108526652
4.2	8180287051	4657879851	-1025109352	1150684452
4.3	7650219451	5962970751	-1086542952	1187966852
4.4	6983734051	7386809051	-1148544852	1219319352
4.5	6168794451	8932520251	-1210517752	1243595752
4.6	5193085951	1060252252	-1271771752	1259556852
4.7	4044093251	1239839952	-1331516952	1265868352
4.8	2709188651	1432077052	-1388856652	1261100352
4.9	1175737651	1636914652	-1442780052	1243728152

$$a = 2$$

X	R_2	R'_2	R''_2	R'''_2
5.0	5687841050	-1854177752	-1492155852	1212132952
5.1	2536655751	-2083548852	-1535726052	1164604652
5.2	4739760851	-2324551252	-1572098852	1099344952
5.3	7189419251	-2576530852	-1599743652	1014472952
5.4	9896200451	-2838637052	-1616985452	9080309451
5.5	1286971652	-3109804052	-1622001352	7779934451
5.6	1611839752	-3388729352	-1612814952	6222767351
5.7	1964924452	-3673854452	-1587295852	4387511151
5.8	2346756352	-3963342052	-1543156952	2252553451
5.9	2757667452	-4255055352	-1477955352	2038669650-
6.0	3197760052	-4546536252	-1389093352	3003467051-
6.1	3666873952	-4834983252	-1273821752	6167699851-
6.2	4164550452	-5117230452	-1129246452	9717504151-
6.3	4689995452	-5389726452	-9523352051	1367302352-
6.4	5242039152	-5648513852	-7399290051	1805329852-
6.5	5819095552	-5889211052	-4887539751	2287590552-
6.6	6419118352	-6106994352	-1954398251	2815659152-
6.7	7039555152	-6296581552	-1434616351	-3390883952-
6.8	7677303552	-6452219752	-5314524551	-4014342052-
6.9	8328660152	-6567673852	-9720604551	-4686789352-
7.0	8989274952	-6636218752	-1468807252	-5408604852-
7.1	9654100352	-6650636552	-2025171152	-6179736452-
7.2	1031734153	-6603216252	-2644545052	-6999636252-
7.3	1097240953	-6485759952	-3330189252	-7867195052-
7.4	1161187153	-6289593352	-4085178752	-8780674552-
7.5	1222740153	-6005583652	-4912342852	-9737631052-
7.6	1280974153	-5624164252	-5814200752	-1073483953-
7.7	1334865453	-5135366152	-6792889352	-1176821353-
7.8	1383289053	-4528859952	-7850085252	-1283272053-
7.9	1425014853	-3794005552	-8986916452	-1392229453-
8.0	1458705153	-2919913352	-1020387453	-1502975353-
8.1	1482912153	-1895515552	-1150070653	-1614670153-
8.2	1496077253	-7096509651	-1287631553	-1726344153-
8.3	1496529753	-6488387151	1432864353	-1836888353-
8.4	1482488153	-2190997452	1585454653	-1945045753-
8.5	1452061453	-3927629052	1744966953	-2049401953-
8.6	1403252553	-5869156052	1910830553	-2148377053-
8.7	1333962353	-8025461352	2082326253	-2240217153-
8.8	1241996753	-1040571353	2258570253	-2322986953-
8.9	1125073953	-1301817053	2438500753	-2394563353-
9.0	9808353252	-1586997153	2620860353	-2452628353-
9.1	8068572352	-1896690153	2804180653	-2494665453-
9.2	6006659752	-2231314353	2986766253	-2517955553-
9.3	3597553552	-2591099853	3166678953	-2519574953-
9.4	8160656251	-2976060553	3341718953	-2496394753-
9.5	2362884052	3385961753	3509412053	-2445083553-
9.6	5964002652	3820287453	3666991753	-2362110653-
9.7	1001132753	4278204653	3811384753	-2243753053-
9.8	1452788453	4758526653	3939198253	-2086105253-
9.9	1953532653	5259673653	4046704153	-1885091753-
10.0	2505352253	5779633253	4129830953	-1636482953-

$a = 2$

X O	R ³ O	R ^{'3} O	R ^{"3} O	R ^{'''3} O
0.1	3852887548	9301691349	1315456551	5448730651
0.2	2053696149	2479022150	1752906251	3629649851
0.3	5465809149	4398467350	2073294351	2859676951
0.4	1094645450	6606407050	2335136651	2410248351
0.5	1875919550	9056527650	2560017051	2103939151
0.6	2912976250	1171765751	2758398851	1872880651
0.7	4225700550	1456642151	2935990851	1684185551
0.8	5831878550	1758380051	3096019051	1519194651
0.9	7747526550	2075319351	3240218451	1365977451
1.0	9987071050	2405921351	3369324751	1216130951
1.1	1256342351	2748682151	3483342151	1063245351
1.2	1548797751	3102068151	3581697651	9021029250
1.3	1877056451	3464465051	3663337951	7282336550
1.4	2241933151	3834129851	3726787251	5376549850
1.5	2644061251	4209154451	3770189951	3267167950
1.6	3083872451	4587426451	3791337951	9200721549
1.7	3561576651	4966595951	3787691151	1697040450-
1.8	4077134751	5344043251	3756394251	4614970150-
1.9	4630231251	5716848251	3694290851	7863154450-
2.0	5220241451	6081759951	3597936451	1146969951-
2.1	5846200151	6435170051	3463613151	1546129151-
2.2	6506759351	6773084551	3287343251	1986305851-
2.3	7200153751	7091100151	3064906651	2469835551-
2.4	7924155751	7384379551	2791859851	2998847951-
2.5	8676029551	7647630451	2463557651	3575235651-
2.6	9452488651	7875088451	2075179251	4200617251-
2.7	1024964752	8060498751	1621757351	4876293451-
2.8	1106296352	8197102551	1098212551	5603200551-
2.9	1188719952	8277631651	4993916950	6381860551-
3.0	1271636252	8294299751	1798870550	-7212324151-
3.1	1354365352	8238800051	9447843150	-8094106651-
3.2	1436142252	8102318451	1800383951	-9026133751-
3.3	1516110652	7875535251	2751628551	-1000665852-
3.4	1593319152	7548646051	3803249951	-1103319852-
3.5	1666715952	7111391451	4959693351	-1210245452-
3.6	1735143452	6553077651	6225025851	-1321022952-
3.7	1797336852	5862634251	7602851151	-1435135252-
3.8	1851917952	5028656151	9096202251	-1551958052-
3.9	1897392952	4039464451	1070742652	-1670751252-
4.0	1932150552	2883189151	1243807452	-1790650552-
4.1	1954459752	1547847451	1428876352	-1910657452-
4.2	1962470152	2144839649	1625904452	-2029630352-
4.3	1954211952	1707894051	-1834725052	-2146275852-
4.4	1927596752	3651832851	-2055033552	-2259138252-
4.5	1880422552	5821640251	-2286372552	-2366592752-
4.6	1810375752	8228047351	-2528112152	-2466834852-
4.7	1715039152	1088106052	-2779432852	-2557874252-
4.8	1591902052	1378976652	-3039307452	-2637527152-
4.9	1438364852	1696211052	-3306479552	-2703410452-
5.0	1251759052	2040466052	-3579444752	-2752934352-

$$a = 2$$

X	R ₃	R' ₃	R'' ₃	R''' ₃
5.1	1029354652	2412234152	-3856427952	-2783300852-
5.2	7683843251	2811817052	-4135365752	-2791501452-
5.3	4660611451	3239293652	-4413881952	-2774312752-
5.4	1196013851	3694489952	-4689268452	-2728302552-
5.5	2737457551	-4176941452	-4958460852	-2649828652-
5.6	7166692351	-4685861652	-5218024352	-2535048952-
5.7	1211762352	-5220098352	-5464126352	-2379927252-
5.8	1761481652	-5778096552	-5692523752	-2180246152-
5.9	2368108052	-6357857852	-5898544552	-1931624052-
6.0	3033696752	-6956890352	-6077067152	-1629532852-
6.1	3760027752	-7572169152	-6222513452	-1269321752-
6.2	4548552452	-8200088852	-6328836052	-8462452851-
6.3	5400326252	-8836415052	-6389507152	-3555004751-
6.4	6315953152	-9476236052	-6397519352	-2077408951
6.5	7295503152	-1011391453	-6345381752	-8482818651
6.6	8338449952	-1074304153	-6225127352	-1570855052
6.7	9443586452	-1135638753	-6028322752	-2380057652
6.8	1060893553	-1194585853	-5746080952	-3280294652
6.9	1183166353	-1250244553	-5369090252	-4275702352
7.0	1310799753	-1301619553	-4887641452	-5370070752
7.1	1443311053	-1347615753	-4291665052	-6566756252
7.2	1580103953	-1387036553	-3570784452	-7868586552
7.3	1720455753	-1418579653	-2714369452	-9277745852
7.4	1863510353	-1440836153	-1711608152	-1079568253
7.5	2008263053	-1452287753	-5515851751	-1242296753
7.6	2153553353	-1451307253	-7766169851	1415917653
7.7	2298052353	-1436158553	-2283829252	1600273253
7.8	2440251753	-1404998553	-3980647552	1795077453
7.9	2578454053	-1355879753	-5877324652	1999900253
8.0	2710760553	-1286753353	-7983592952	2214148053
8.1	2835066753	-1195478953	-1030851153	2437051053
8.2	2949044553	-1079827153	-1286026053	2667640453
8.3	3050142153	-9374935052	-1564595953	2904732553
8.4	3135574553	-7661086652	-1867142253	3146908353
8.5	3202315753	-5632516852	-2194092153	3392493653
8.6	3247094053	-3264681552	-2545690253	3639541253
8.7	3266395753	-5329008851	-2921974453	3885805853
8.8	3256456653	-2587432752	3322740053	4128728753
8.9	3213270653	-6120584952	3747513253	4365416253
9.0	3132590253	-1009020153	4195507353	4592616853
9.1	3009936553	-1451896553	4665599753	4806710853
9.2	2840609853	-1942826553	5156281053	5003681953
9.3	2619703853	-2483775753	5665619653	5179108353
9.4	2342128553	-3076493353	6191222153	5328144953
9.5	2002628253	-3722465253	6730194253	5445516953
9.6	1595820753	-4422862453	7279079353	5525495853
9.7	1116214753	-5178480953	7833840653	5561912253
9.8	5582734352	-5989672453	8389789953	5548132053
9.9	8356697351	6856300953	8941557853	5477071953
10.0	8148121752	7777642253	9483045353	5341201553

$$\alpha = 2$$

X	R_4	R_4'	R_4''	R_4'''
0	∞	$-\infty$	∞	$-\infty$
0.1	2595445851	1075108052	-1520292653	3670672354-
0.2	1947586851	4035923551	-2849635652	3445235953-
0.3	1646081951	2279438551	-1066561352	8651770552-
0.4	1460208051	1526136251	-5269839551	3261988052-
0.5	1329446851	1126351751	-3006515651	1544372652-
0.6	1229633251	8889421750	-1855900351	8498608051-
0.7	1148767751	7394322750	-1188321451	5226084851-
0.8	1079979951	6434357250	-7599783750	3511628651-
0.9	1018898051	5831588950	-4616340250	2541540651-
1.0	9624946050	5486143550	-2389042650	1960018851-
1.1	9085183250	5338477450	-6247793149	1595605851-
1.2	8551910650	5351422050	-8443819849	-1359076551-
1.3	8010355750	5500919750	-2119302150	-1200981751-
1.4	7447715050	5770923850	-3262735950	-1092414851-
1.5	6852523950	6150432550	-4314669750	-1015666851-
1.6	6214239850	6631676850	-5300860050	-9594122350-
1.7	5522988450	7208968850	-6237784250	-9161032050-
1.8	4769391050	7877935550	-7135618850	-8804984750-
1.9	3944465750	8634983550	-8000073850	-8488012650-
2.0	3039565450	9476920650	-8833559650	-8181380250-
2.1	2046355050	1040066051	-9635931950	-7862340450-
2.2	9568126949	1140299851	-1040498451	-7512073950-
2.3	2367483249	-1248041751	-1113676351	-7114344550-
2.4	1541641150	-1362892751	-1182578551	-6654613850-
2.5	2964750850	-1484392151	-1246517051	-6119459050-
2.6	4512464150	-1612003551	-1304673851	-5496172050-
2.7	6190588150	-1745102751	-1356105851	-4772508650-
2.8	8004258550	-1882965151	-1399749551	-3936520650-
2.9	9957828950	-2024753851	-1434422651	-2976461150-
3.0	1205475551	-2169507351	-1458826451	-1880739450-
3.1	1429746951	-2316129951	-1471547251	-6379179949-
3.2	1668722651	-2463376851	-1471057051	-7632612249
3.3	1922396651	-2609846551	-1455717251	-2333820550
3.4	2190613951	-2753967451	-1423780351	-4084465950
3.5	2473053351	-2893987251	-1373393151	-6025485850
3.6	2769210351	-3027966251	-1302603551	-8166629150
3.7	3078374451	-3153760251	-1209363551	-1051695551
3.8	3399611251	-3269019151	-1091539651	-1308467851
3.9	3731741151	-3371174651	-9469209650	-1587697951
4.0	4073316351	-3457434251	-7732312950	-1889980651
4.1	4422597951	-3524774551	-5681414650	-2215764451
4.2	4777532851	-3569937151	-3292855150	-2565327951
4.3	5135729951	-3589426451	-5428020949	-2938753151
4.4	5494438551	-3579509451	-2592559650	3335898051
4.5	5850519551	-3536212651	-6136769550	3756364051
4.6	6200428251	-3455334151	-1011283851	4199467851
4.7	6540186351	-3332442651	-1454292351	4664202951
4.8	6865361651	-3162892051	-1944800751	5149206551
4.9	7171048151	-2941833751	-2484751151	5652723251
5.0	7451843251	-2664234951	-3075889551	6172560051

$$a = 2$$

X	R ₄	R' ₄	R'' ₄	R''' ₄
5.1	7701837151	-2324899651	-3719718851	6706055451
5.2	7914588451	-1918493851	-4417450851	7250027951
5.3	8083119551	-1439583151	-5169950351	7800740451
5.4	8199904051	-8826622350	-5977678851	8353852351
5.5	8256867351	-2422058450	-6840634851	8904384851
5.6	8245377451	-4872880350	7758281051	9446662451
5.7	8156260851	-1311235651	8729479651	9974285551
5.8	7979806751	-2234909751	9752414051	1048008252
5.9	7705785951	-3263358551	1082450352	1095606252
6.0	7323482251	-4401335451	1194233652	1139339152
6.1	6821721851	-5653206751	1310156952	1178235052
6.2	6188915451	-7022853251	1429684152	1211229952
6.3	5413115751	-8513563151	1552168052	1237165952
6.4	4482080751	-1012791952	1676840352	1254788652
6.5	3383350051	-1186767452	1802803452	1262746152
6.6	2104339151	-1373361452	1929019352	1259587752
6.7	6324329050	-1572540352	2054298752	1243764752
6.8	1044886151	1784145152	2177295052	1213630952
6.9	2939900151	2007873152	2296490452	1167446352
7.0	5064520951	2243260252	2410188352	1103379052
7.1	7430094751	2489664952	2516505952	1019511652
7.2	1004724352	2746246352	2613364152	9138475651
7.3	1292563052	3011946652	2698480552	7843194251
7.4	1607374852	3285468452	2769361952	6287985851
7.5	1949865852	3565254552	2823301552	4451094151
7.6	2320573452	3849466452	2857371252	2310422351
7.7	2719835652	4135961852	2868422452	1562792950-
7.8	3147759952	4422271952	2853085052	2971222851-
7.9	3604189752	4705579252	2807764652	6156369951-
8.0	4088669752	4982698752	2728651852	9733137251-
8.1	4600403652	5250049252	2611726952	1372214052-
8.2	5138221352	5503641952	2452769752	1814285952-
8.3	5700527052	5739056352	2247370352	2301328352-
8.4	6285264752	5951416152	1990950252	2834950352-
8.5	6889864152	6135387352	1678782552	3416528952-
8.6	7511202252	6285152852	1306008952	4047161452-
8.7	8145544952	6394404852	8676875351	4727613152-
8.8	8788506152	6456338952	3588135851	5458263152-
8.9	9434991852	6463648352	2256357951	-6239043652-
9.0	1007915553	6408525552	8906487151	-7069376552-
9.1	1071434053	6282668652	1641125852	-7948105752-
9.2	1133303853	6077293052	2481822752	-8873420952-
9.3	1192684153	5783144252	3417279852	-9842790152-
9.4	1248638553	5390535652	4451739152	-1085286653-
9.5	1300132853	4889368452	5589067652	-1189942353-
9.6	1346029353	4269181052	6832664752	-1297724853-
9.7	1385083753	3519196152	8185351852	-1408007153-
9.8	1415943853	2628401552	9649267152	-1520045353-
9.9	1437144753	1585590452	1122574153	-1632972153-
10.0	1447110953	3794851351	1291517353	-1745785053-

$$a = 4$$

X	R ₁	R ₁ '	R ₁ ''	R ₁ '''
0	1000000051	0	0	0
0.1	9999975050	9999999246	-2999999348	-5999996349-
0.2	9999600050	7999988047	-1199995849	-1199987650-
0.3	9997975050	2699979848	-2699952849	-1799905550-
0.4	9993600150	6399848348	-4799734449	-2399601950-
0.5	9984375550	1249927749	-7498987549	-2998784850-
0.6	9967601950	2159740849	-1079697650	-3596976150-
0.7	9939981750	3429237649	-1469237550	-4193464550-
0.8	9897619450	5118058249	-1918301050	-4787258450-
0.9	9836024850	7285571649	-2426555950	-5377040750-
1.0	9750115750	9990741849	-2993519650	-5961122050-
1.1	9634223150	1329195950	-3618520450	-6537394550-
1.2	9482097650	1724683050	-4300653250	-7103288050-
1.3	9286918950	2191191850	-5038730150	-7655724050-
1.4	9041307650	2734243550	-5831229050	-8191070050-
1.5	8737340250	3359188450	-6676234850	-8705106050-
1.6	8366568750	4071162450	-7571378650	-9192968050-
1.7	7920044050	4875039650	-8513772250	-9649124050-
1.8	7388345050	5775376550	-9499938850	-1006732551-
1.9	6761613750	6776349450	-1052574051	-1044058251-
2.0	6029595950	7881684050	-1158629951	-1076112351-
2.1	5181690850	9094578950	-1267592251	-1102037351-
2.2	4207007850	1041761951	-1378801851	-1120893151-
2.3	3094432250	1185268151	-1491501151	-1131654951-
2.4	1832701150	1340083151	-1604826051	-1133212351-
2.5	4104903849	1506221651	-1717797051	-1124368451-
2.6	1183488050	-1683594151	-1829310451	-1103840851-
2.7	2960375050	-1871994051	-1938130251	-1070262251-
2.8	4931041150	-2071084551	-2042879551	-1022182751-
2.9	7105948750	-2280383651	-2142032551	-9580730050-
3.0	9494998850	-2499248551	-2233906451	-8763291050-
3.1	1210736451	-2726860951	-2316654851	-7752784050-
3.2	1495130151	-2962208451	-2388260751	-6531855050-
3.3	1803395451	-3204068751	-2446531051	-5082638050-
3.4	2136112951	-3450991151	-2489091651	-3386837050-
3.5	2493706051	-3701278251	-2513383851	-1425865450-
3.6	2876415651	-3952967751	-2516662551	-8190062049
3.7	3284271851	-4203812751	-2495995051	-3366489450
3.8	3717065451	-4451264651	-2448262351	-6235071650
3.9	4174316351	-4692453251	-2370163051	-9442808050
4.0	4655240651	-4924169351	-2258217751	-1300707151
4.1	5158715951	-5142847051	-2108778351	-1694427751
4.2	5683244651	-5344548151	-1918038251	-2126958751
4.3	6226915951	-5524946651	-1682047551	-2599655551
4.4	6787366751	-5679313751	-1396729451	-3113678951
4.5	7361740151	-5802508751	-1057903051	-3669952451
4.6	7946644451	-5888968951	-6613087950	-4269119751
4.7	8538110151	-5932701651	-2026382450	-4911498851
4.8	9131546851	-5927283351	-3224298250	5597029451
4.9	9721700351	-5865859151	-9181901350	6325219851

$$a = 4$$

X	R ₁	R ₁ '	R ₁ ''	R ₁ '''
5.0	1 0 3 0 2 6 0 9 5 2	- 5 7 4 1 1 4 8 0 5 1	- 1 5 8 8 8 6 3 8 5 1	7 0 9 5 0 9 1 5 5 1
5.1	1 0 8 6 7 5 6 4 5 2	- 5 5 4 5 4 5 2 7 5 1	- 2 3 3 8 5 4 7 4 5 1	7 9 0 5 1 1 3 7 5 1
5.2	1 1 4 0 9 0 6 4 5 2	- 5 2 7 0 6 7 4 1 5 1	- 3 1 7 1 1 5 4 2 5 1	8 7 5 3 1 4 2 7 5 1
5.3	1 1 9 1 8 7 8 1 5 2	- 4 9 0 8 3 3 5 1 5 1	- 4 0 9 0 3 4 9 6 5 1	9 6 3 6 3 5 5 0 5 1
5.4	1 2 3 8 7 5 1 9 5 2	- 4 4 4 9 6 0 5 8 5 1	- 5 0 9 9 4 7 9 7 5 1	1 0 5 5 1 1 7 1 5 2
5.5	1 2 8 0 5 1 8 4 5 2	- 3 8 8 5 3 4 2 4 5 1	- 6 2 0 1 4 9 1 5 5 1	1 1 4 9 3 1 8 8 5 2
5.6	1 3 1 6 0 7 5 6 5 2	- 3 2 0 6 1 2 8 8 5 1	- 7 3 9 8 8 4 8 2 5 1	1 2 4 5 7 1 0 2 5 2
5.7	1 3 4 4 2 2 5 7 5 2	- 2 4 0 2 3 3 1 0 5 1	- 8 6 9 3 4 3 3 1 5 1	1 3 4 3 6 6 3 1 5 2
5.8	1 3 6 3 6 7 4 3 5 2	- 1 4 6 4 1 6 0 1 5 1	- 1 0 0 8 6 4 5 1 5 2	1 4 4 2 4 4 3 7 5 2
5.9	1 3 7 3 0 2 8 1 5 2	- 3 8 1 7 4 4 8 4 5 0	- 1 1 5 7 8 3 1 5 5 2	1 5 4 1 2 0 4 0 5 2
6.0	1 3 7 0 7 9 5 5 5 2	- 8 5 4 7 8 2 5 6 5 0	1 3 1 6 8 5 3 1 5 2	1 6 3 8 9 7 4 5 5 2
6.1	1 3 5 5 3 8 6 2 5 2	- 2 2 5 5 1 9 0 0 5 1	1 4 8 5 5 5 7 0 5 2	1 7 3 4 6 5 6 1 5 2
6.2	1 3 2 5 1 1 3 5 5 2	- 3 8 2 9 0 3 5 5 5 1	1 6 6 3 6 7 3 6 5 2	1 8 2 7 0 1 2 0 5 2
6.3	1 2 7 8 1 9 6 6 5 2	- 5 5 8 5 5 4 2 7 5 1	1 8 5 0 8 0 2 5 5 2	1 9 1 4 6 6 0 0 5 2
6.4	1 2 1 2 7 6 4 5 5 2	- 7 5 3 3 4 6 4 4 5 1	2 0 4 6 3 9 7 6 5 2	1 9 9 6 0 6 6 1 5 2
6.5	1 1 2 6 8 6 2 0 5 2	- 9 6 8 0 9 2 6 7 5 1	2 2 4 9 7 5 1 5 5 2	2 0 6 9 5 3 6 8 5 2
6.6	1 0 1 8 4 5 6 3 5 2	- 1 2 0 3 5 2 6 1 5 2	2 4 5 9 9 7 8 8 5 2	2 1 3 3 2 1 3 4 5 2
6.7	8 8 5 4 4 5 9 5 5 1	- 1 4 6 0 2 8 1 8 5 2	2 6 7 6 0 0 0 2 5 2	2 1 8 5 0 6 6 7 5 2
6.8	7 2 5 6 7 1 8 3 5 1	- 1 7 3 8 8 7 6 5 5 2	2 8 9 6 5 2 4 7 5 2	2 2 2 2 8 9 2 2 5 2
6.9	5 3 6 9 2 9 9 6 5 1	- 2 0 3 9 6 8 6 4 5 2	3 1 2 0 0 3 1 9 5 2	2 2 4 4 3 0 7 2 5 2
7.0	3 1 6 9 8 6 7 5 5 1	- 2 3 6 2 9 2 3 7 5 2	3 3 4 4 7 5 3 9 5 2	2 2 4 6 7 4 8 2 5 2
7.1	6 3 5 9 6 6 7 9 5 0	- 2 7 0 8 6 1 0 3 5 2	3 5 6 8 6 5 7 9 5 2	2 2 2 7 4 7 1 0 5 2
7.2	2 2 5 4 7 7 3 7 5 1	3 0 7 6 5 5 1 1 5 2	3 7 8 9 4 2 6 9 5 2	2 1 8 3 5 5 1 7 5 2
7.3	5 5 2 4 4 0 9 2 5 1	3 4 6 6 3 0 4 2 5 2	4 0 0 4 4 4 3 4 5 2	2 1 1 1 8 8 9 3 5 2
7.4	9 1 9 4 4 1 6 7 5 1	3 8 7 7 1 5 0 5 5 2	4 2 1 0 7 7 1 1 5 2	2 0 0 9 2 1 2 3 5 2
7.5	1 3 2 8 5 4 0 2 5 2	4 3 0 8 0 6 0 2 5 2	4 4 0 5 1 3 7 8 5 2	1 8 7 2 0 8 5 4 5 2
7.6	1 7 8 1 6 7 7 1 5 2	4 7 5 7 6 5 8 7 5 2	4 5 8 3 9 2 0 4 5 2	1 6 9 6 9 2 1 2 5 2
7.7	2 2 8 0 6 3 6 9 5 2	5 2 2 4 1 9 2 0 5 2	4 7 4 3 1 3 0 0 5 2	1 4 7 9 9 9 3 5 5 2
7.8	2 8 2 7 0 0 8 1 5 2	5 7 0 5 4 8 7 5 5 2	4 8 7 8 3 9 8 8 5 2	1 2 1 7 4 5 4 8 5 2
7.9	3 4 2 2 1 3 9 4 5 2	6 1 9 8 9 1 5 7 5 2	4 9 8 4 9 6 9 0 5 2	9 0 5 3 5 6 9 0 5 1
8.0	4 0 6 7 0 9 2 2 5 2	6 7 0 1 3 5 3 0 5 2	5 0 5 7 6 8 4 2 5 2	5 3 9 6 7 6 4 0 5 1
8.1	4 7 6 2 5 8 8 9 5 2	7 2 0 9 1 3 8 9 5 2	5 0 9 0 9 8 2 7 5 2	1 1 6 3 4 3 8 5 5 1
8.2	5 5 0 8 9 5 7 8 5 2	7 7 1 8 0 3 7 2 5 2	5 0 7 8 8 9 5 2 5 2	3 6 8 7 2 1 9 8 5 1 -
8.3	6 3 0 6 0 7 2 4 5 2	8 2 2 3 1 9 3 3 5 2	5 0 1 5 0 4 5 2 5 2	9 1 9 5 7 4 9 7 5 1 -
8.4	7 1 5 3 2 8 8 7 5 2	8 7 1 9 0 9 5 7 5 2	4 8 9 2 6 5 4 3 5 2	1 5 4 0 1 9 7 0 5 2 -
8.5	8 0 4 9 3 7 6 8 5 2	9 1 9 9 5 3 4 0 5 2	4 7 0 4 5 5 1 4 5 2	2 2 3 4 4 4 6 8 5 2 -
8.6	8 9 9 2 4 4 9 5 5 2	9 6 5 7 5 6 3 7 5 2	4 4 4 3 1 8 7 8 5 2	3 0 0 6 0 0 4 5 5 2 -
8.7	9 9 7 9 8 8 6 3 5 2	1 0 0 8 5 4 6 7 5 3	4 1 0 0 6 5 8 7 5 2	3 8 5 8 3 0 7 6 5 2 -
8.8	1 1 0 0 8 2 5 5 5 3	1 0 4 7 4 7 1 6 5 3	3 6 6 8 7 3 0 0 5 2	4 7 9 4 4 8 1 5 5 2 -
8.9	1 2 0 7 3 2 3 0 5 3	1 0 8 1 5 9 4 9 5 3	3 1 3 8 8 7 4 9 5 2	5 8 1 7 2 6 3 5 5 2 -
9.0	1 3 1 6 9 5 0 4 5 3	1 1 0 9 8 9 3 4 5 3	2 5 0 2 3 1 4 5 5 2	6 9 2 8 9 1 6 4 5 2 -
9.1	1 4 2 9 0 7 0 6 5 3	1 1 3 1 2 5 5 6 5 3	1 7 5 0 0 7 1 8 5 2	8 1 3 1 1 4 0 8 5 2 -
9.2	1 5 4 2 9 3 0 4 5 3	1 1 4 4 4 7 8 9 5 3	8 7 3 0 3 1 6 1 5 1	9 4 2 4 9 7 2 7 5 2 -
9.3	1 6 5 7 6 5 2 1 5 3	1 1 4 8 2 6 9 6 5 3	1 3 7 9 8 6 3 3 5 1	- 1 0 8 1 0 6 7 6 5 3 -
9.4	1 7 7 2 2 2 3 8 5 3	1 1 4 1 2 4 2 0 5 3	1 2 9 2 1 4 6 4 5 2	- 1 2 2 8 7 6 3 3 5 3 -
9.5	1 8 8 5 4 9 0 7 5 3	1 1 2 1 9 1 9 3 5 3	2 5 9 8 5 0 1 7 5 2	- 1 3 8 5 4 2 1 4 5 3 -
9.6	1 9 9 6 1 4 5 7 5 3	1 0 8 8 7 3 5 1 5 3	4 0 6 5 8 8 5 9 5 2	- 1 5 5 0 7 6 5 6 5 3 -
9.7	2 1 0 2 7 2 0 7 5 3	1 0 4 0 0 3 6 5 5 3	5 7 0 2 7 9 4 8 5 2	- 1 7 2 4 3 9 1 9 5 3 -
9.8	2 2 0 3 5 7 8 1 5 3	9 7 4 0 8 7 4 3 5 2	7 5 1 7 2 4 9 7 5 2	- 1 9 0 5 7 5 4 0 5 3 -
9.9	2 2 9 6 9 0 2 8 5 3	8 8 9 0 7 4 9 2 5 2	9 5 1 6 6 4 9 2 5 2	- 2 0 9 4 1 4 9 1 5 3 -
10.0	2 3 8 0 6 9 5 1 5 3	7 8 3 1 1 5 4 2 5 2	1 1 7 0 7 6 0 8 5 3	- 2 2 8 8 7 0 1 5 5 3 -

$$a = 4$$

X	R ₂ ⁰	R ₂ ¹	R ₂ ²	R ₂ ³
0.1	9999998048	2000006250	2000000051	2380952247
0.2	3999987349	4000200050	2000176251	1904758948
0.3	8999855449	6001518650	2000891951	6428520548
0.4	1599918750	8006400050	2002819051	1523771549
0.5	2499690050	1001953151	2006882351	2976008949
0.6	3599074350	1204859851	2014270651	5142206649
0.7	4897665850	1410503551	2026436851	8164752849
0.8	6394799150	1620477351	2045097051	1218560350
0.9	8089456650	1836897851	2072229451	1734602750
1.0	9980162050	2062480051	2110073351	2378628550
1.1	1206485851	2300609551	2161127051	3164519250
1.2	1434077451	2555416651	2228145951	4105959750
1.3	1680427451	2831845451	2314137251	5216373450
1.4	1945069851	3135725651	2422357251	6508844050
1.5	2227418151	3473838651	2556303751	7996026450
1.6	2526747351	3853982151	2719708851	9690037550
1.7	2842172951	4285032951	2916530351	1160233751
1.8	3172630451	4777003451	3150939251	1374358951
1.9	3516852451	5341094251	3427306451	1612350051
2.0	3873345551	5989739251	3750185851	1875065051
2.1	4240366051	6736648451	4124296051	2163229251
2.2	4615894651	7596835451	4554496551	2477414351
2.3	4997611051	8586641551	5045762351	2818014151
2.4	5382867251	9723745451	5603153551	3185219451
2.5	5768661451	1102716252	6231781251	3578989751
2.6	6151611451	1251722352	6936768451	3999024051
2.7	6527927651	1421555052	7723206451	4444727751
2.8	6893387751	1614499852	8596105351	4915179051
2.9	7243311451	1832958952	9560339351	5409092451
3.0	7572535551	2079441452	1062058752	5924780151
3.1	7875392251	2356552052	1178126352	6460113751
3.2	8145688051	2666975952	1304644452	7012479851
3.3	8376684751	3013461552	1441979052	7578738551
3.4	8561085151	3398799552	1590445952	8155178351
3.5	8691019951	3825798952	1750300352	8737471451
3.6	8758039651	4297258052	1921727752	9320626451
3.7	8753112151	4815933352	2104832152	9898942651
3.8	8666622951	5384502052	2299624852	1046596352
3.9	8488384851	6005521252	2506011252	1101443052
4.0	8207651251	6681381552	2723778352	1153624052
4.1	7813139751	7414256252	2952579852	1202240252
4.2	7293062451	8206044252	3191922052	1246299552
4.3	6635166151	9058306952	3441148352	1284713552
4.4	5826783851	9972202252	3699422752	1316294252
4.5	4854896851	1094840853	3965713452	1339750852
4.6	3706207951	1198704253	4238775552	1353688552
4.7	2367231351	1308757753	4517133652	1356606252
4.8	8243932050	1424874553	4799064152	1346896752
4.9	9358490850	-1546844153	5082576352	1322846552

$$\alpha = 4$$

X	R ₂	R' ₂	R'' ₂	R''' ₂
5.0	2926873551	-1674361453	5365395352	1282638452
5.1	5161740751	-1807015953	5644943352	1224352952
5.2	7653023951	-1944279753	5918322452	1145974152
5.3	1041261852	-2085495453	6182297252	1045394352
5.4	1345152952	-2229863153	6433278552	9204222251
5.5	1677964152	-2376427453	6667307652	7687923951
5.6	2040546252	-2524063253	6880042952	5881770851
5.7	2433584852	-2671462853	7066746952	3762000651
5.8	2857569952	-2817120653	7222276052	1304532651
5.9	3312763652	-2959319853	7341072352	1514839951-
6.0	3799165052	-3096117653	7417158052	4720227751-
6.1	4316473552	-3225331753	7444133852	8335391951-
6.2	4864047752	-3344526353	7415179452	1238345852-
6.3	5440865552	-3450999153	7323060052	1688659652-
6.4	6045477352	-3541769553	7160135952	2186565852-
6.5	6675961852	-3613566653	6918378652	2733978352-
6.6	7329876552	-3662820053	6589391752	3332596152-
6.7	8004206652	-3685650653	6164438152	3983854152-
6.8	8695314652	-3677865053	5634476352	4688872652-
6.9	9398886752	-3634950653	4990201452	5448398352-
7.0	1010987853	-3552074453	4222096952	6262745452-
7.1	1082246053	-3424084153	3320494252	7131727152-
7.2	1152996453	-3245513053	2275643152	8054587052-
7.3	1222482553	-3010589553	1077791252	9029921752-
7.4	1289853553	-2713249453	2827238051	-1005560653-
7.5	1354158453	-2347154653	1815371152	-1112870353-
7.6	1414341653	-1905716653	3529310652	-1224538353-
7.7	1469238453	-1382126353	5433264152	-1340082353-
7.8	1517570453	-7693908352	7535371152	-1458912353-
7.9	1557942353	-6037627151	9843033452	-1580319853-
8.0	1588839053	-7521400852	-1236274153	-1703468353-
8.1	1608623153	-1675409853	-1509988253	-1827383353-
8.2	1615534053	-2716644153	-1805854353	-1950941353-
8.3	1607687353	-3882934953	-2124129453	-2072860353-
8.4	1583076153	-5181164553	-2464894353	-2191689053-
8.5	1539573253	-6617905753	-2828028253	-2305797553-
8.6	1474934953	-8199309053	-3213183653	-2413367153-
8.7	1386806853	-9930977353	-3619756753	-2512381353-
8.8	1272730853	-1181782954	-4046857953	-2600617853-
8.9	1130155553	-1386394754	-4493280353	-2675640753-
9.0	9564472752	-1607241354	-4957467853	-2734793553-
9.1	7489054452	-1844512954	-5437480953	-2775195053-
9.2	5047789252	-2098262454	-5930962153	-2793734453-
9.3	2212870352	-2368384154	-6435099853	-2787070153-
9.4	1043576452	2654592954	-6946593653	-2751629853-
9.5	4749220652	2956398954	-7461616153	-2683612253-
9.6	8931222752	3273082654	-7975777053	-2578993453-
9.7	1361589053	3603669554	-8484087553	-2433533953-
9.8	1882828953	3946901254	-8980924053	-2242791053-
9.8	2459181753	4301206554	-9459993553	-2002133353-
10.0	3092775153	4664672454	-9914302453	-1706759453-

$a = 4$

X_0	R_3	R_3'	R_3''	R_3'''
0.1	9999998947	29999999449	59999996250	59999981251
0.2	7999989048	11999996050	11999988051	59999700151
0.3	2699980649	26999954550	17999908951	59998481451
0.4	6399853049	47999743750	23999615951	59995199651
0.5	1249930250	74999023250	29998828151	59988281351
0.6	2159750250	1079708551	3597084351	5975701951
0.7	3429264650	1469264751	4193697851	5954986751
0.8	5118127650	1918361751	4787713351	5923215651
0.9	7285729450	2426678851	5377860651	5877021351
1.0	9991072050	2993751051	5962510351	5812593051
1.1	1329260451	3618930451	6539630351	5725680751
1.2	1724801551	4301343951	7106741351	5611599951
1.3	2191399251	5039846751	7660874651	5465239851
1.4	2734592051	5832970251	8198529551	5281072251
1.5	3359752951	6678868351	8715631551	5053164151
1.6	4072049251	7575256151	9207493951	4775192651
1.7	4876394751	8519348351	9668778551	4440465351
1.8	5777397751	9507792051	1009346152	4041941051
1.9	6779299351	1053659752	1047479652	3572261951
2.0	7885906851	1160105852	1080528752	3023781651
2.1	9100517251	1269568552	1107666352	2388608551
2.2	1042583852	1381411852	1127984952	1658649751
2.3	1186389252	1494905252	1140495852	8256651850
2.4	1341592352	1609215152	1144127152	1186734150-
2.5	1508228252	1723396352	1137723452	1182710951-
2.6	1686231852	1836384052	1120046252	2374731551-
2.7	1875425552	1946985752	1089774952	3702869551-
2.8	2075505152	2053872652	1045508752	5175010051-
2.9	2286027152	2155571752	9857695051	6798679551-
3.0	2506392852	2250459352	9090066051	8580923451-
3.1	2735832652	2336751852	8136010751	1052816552-
3.2	2973391952	2412502052	6978742751	1264606952-
3.3	3217911852	2475591852	5600949651	1493937452-
3.4	3468011752	2523726652	3984898951	1741170552-
3.5	3722076452	2554435552	2112560351	2006542052-
3.6	3978229452	2565063052	3426011849	-2290134552-
3.7	4234324652	2552774352	2473755951	-2591862252-
3.8	4487920852	2514552552	5223939151	-2911443352-
3.9	4736264752	2447201452	8302428251	-3248374752-
4.0	4976278152	2347352852	1172622552	-3601908752-
4.1	5204534552	2211472152	1551144352	-3971023352-
4.2	5417247652	2035869052	1967302752	-4354392752-
4.3	5610256252	1816711752	2422442652	-4750360352-
4.4	5779006352	1550042052	2917727352	-5156902352-
4.5	5918546152	1231796652	3454095652	-5571601252-
4.6	6023508952	8578306351	4032224852	-5991606452-
4.7	6088114252	4239466451	4652484052	-6413606452-
4.8	6106160152	7407152350	-5314886152	-6833792752-
4.9	6071022652	6404228151	-6019038652	-7247826752-
5.0	5975659852	1279242952	-6764083752	-7650802252-

$\alpha = 4$

X	R_3	R_3'	R_3''	R_3'''
5.1	5812623152	1994557252	-7548646652	-8037221052-
5.2	5574069252	2790225052	-8370771952	-8400954352-
5.3	5251779052	3669878052	-9227856252	-8735213852-
5.4	4837184452	4636852852	-1011658553	-9032529652-
5.5	4321399652	5694114452	-1103286053	-9284716052-
5.6	3695266152	6844178352	-1197172753	-9482859152-
5.7	2949402452	8089018352	-1292730953	-9617298952-
5.8	2074259052	9429968252	-1389271653	-9677604752-
5.9	1060185052	1086762553	-1485998753	-9652591052-
6.0	1024833751	-1240172553	-1581999353	-9530294252-
6.1	1423334452	-1403103853	-1676238053	-9297998852-
6.2	2911786752	-1575322753	-1767547853	-8942245952-
6.3	4576960052	-1756472153	-1854623953	-8448857752-
6.4	6427545852	-1946058153	-1936016653	-7802984652-
6.5	8471673852	-2143433053	-2010123953	-6989143652-
6.6	1071673753	-2347781453	-2075186053	-5991287652-
6.7	1316922953	-2558104753	-2129281153	-4792891252-
6.8	1583454053	-2773202853	-2170319153	-3377027852-
6.9	1871675753	-2991658153	-2196039753	-1726494352-
7.0	2181842753	-3211818853	-2204008853	-1760554151
7.1	2514033653	-3431782053	-2191620653	-2347970552
7.2	2868120353	-3649372053	-2156094253	-4806416652
7.3	3243746853	-3862130653	-2094480553	-7568189752
7.4	3640294253	-4067295253	-2003665553	-1064948853
7.5	4056850753	-4261781753	-1880374453	-1406565853
7.6	4492180653	-4442173353	-1721188653	-1783093253
7.7	4944690453	-4604704353	-1522549453	-2195811153
7.8	5412389153	-4745245353	-1280783153	-2645823653
7.9	5892857853	-4859298553	-9921119452	-3134022153
8.0	6383203353	-4941976653	-6526838552	-3661044453
8.1	6880032453	-4988012453	-2585965552	-4227234653
8.2	7379397853	-4991743553	-1940686752	4832593453
8.3	7876768753	-4947115453	-7092144752	5476728953
8.4	8366993553	-4847690553	-1290680953	6158807753
8.5	8844254653	-4686644453	-1942197053	6877493953
8.6	9302029853	-4456795753	-2667336153	7630890353
8.7	9733067553	-4150611853	-3469449453	8416481753
8.8	1012934654	-3760238953	-4351600753	9231062953
8.9	1048204054	-3277531853	-5316496453	1007067554
9.0	1078149754	-2694104353	-6366413053	1093053354
9.1	1101721654	-2001355953	-7503089653	1180495754
9.2	1117783354	-1190555253	-8727667553	1268730254
9.3	1125109854	-2528772252	-1004055954	1356987254
9.4	1122388654	-8204899552	1144135954	1444386654
9.5	1108218854	-2038278253	1292871854	1529927854
9.6	1081113154	-3409036353	1450022154	1612485354
9.7	1039500654	-4941010553	1615227454	1690799854
9.8	9817295653	-6642014953	1787994154	1763471854
9.9	9060726653	-8519306053	1967682754	1828956554
10.0	8107339053	-1057942154	2153489154	1885556454

$a = 4$

X	R ₈ ⁴	R ₋₈ ⁴	R ₈ ^{''4}	R ₋₈ ^{'''4}
0.1	10000032452	9999112052	-20000152754	59998897555-
0.2	5002128551	2497140652	-2502359553	3749320554-
0.3	3339604751	1105589852	-7436714752	7402641453-
0.4	2513331051	6163355051	-3158315552	2340429353-
0.5	2023711651	3878576651	-1636052552	9579084752-
0.6	1704355751	2619368351	-9636898751	4517051752-
0.7	1484007051	1844152751	-6217006651	2494420952-
0.8	1327037051	1327122351	-4293240151	1467442052-
0.9	1213613651	9607338950	-3124527751	9236011951-
1.0	11311773251	6886239050	-2368778251	6151402051-
1.1	1073817651	4790555550	-1853169851	4306230351-
1.2	1034514351	3131520850	-1483955551	3155342351-
1.3	1010126651	1792034650	-1207251251	2413189951-
1.4	9978610750	6971448649	-9905594350	1921195251-
1.5	9955336750	2021826749	8134225350	1586472651-
1.6	1001362251	9384018849	6625377550	1351619651-
1.7	1013830451	1532986550	5289252950	1179949451-
1.8	1031596951	1999915050	4063598350	1041807651-
1.9	1053432651	2347895350	2904022650	9195330450-
2.0	1078175151	2581830550	1778059350	7943595350-
2.1	1104696351	2703811950	6513966240	6423826450-
2.2	1131878151	2713796450	4645411440	-4671284650-
2.3	1159593551	2610108750	1514206450	-2284605750-
2.4	1183691751	2389783350	2798972050	-2831238049
2.5	1205987251	2048842450	4027844850	-5032164450
2.6	1234250651	1582505450	5307932750	-1060322351
2.7	1237201351	9853625949	6644748950	-1772407251
2.8	1243502251	25152260749	8042407950	-2701578851
2.9	1241755551	6252417249	9503723850	-3874857551
3.0	1230499451	1651395350	-1103027151	-5349707851
3.1	1208207751	2833482950	-1262237951	-7184498051
3.2	1173288251	4178026650	-1427911551	-2444871951
3.3	1124084351	5691387650	-1599821551	-1220399052
3.4	1058877651	7379627650	-1777601651	-1554261952
3.5	9758899750	9248377150	-1960736051	-1954901752
3.6	8732915650	1130266151	-2148548851	-2431856752
3.7	7492041150	1354675351	-2340192451	-2995310352
3.8	6017123250	1598398351	-2534634551	-3655986552
3.9	4288729550	1861654951	-2730645551	-4425002352
4.0	2287271050	2144531951	-2926783851	-5313666752
4.1	6852439047	-2446960551	-3121382751	-6333219352
4.2	2613084050	-2768693851	-3312536051	-7494499052
4.3	5550525150	-3109280951	-3498083351	-8807533452
4.4	8837706850	-3468042551	-3675596451	-1028103653
4.5	1249236151	-3844042751	-3842366251	-1192179753
4.6	1653113651	-4236058351	-3995392051	-1373397453
4.7	2096930951	-4642552651	-4131363851	-1571823353
4.8	2582044451	-5061641951	-4246659251	-1727076153
4.9	3109494151	-5491063751	-4337328651	-2013214153
5.0	3680512851	-5928144651	-4399090651	-2263602553

$$\alpha = 4$$

X	R_4	R'_4	R''_4	R'''_4
5.1	4295384751	-6369764951	-4427330251	-2520766053
5.2	4954497951	-6812328151	-4417089051	-2786221653
5.3	5657745551	-7251724251	-4363071151	-3055289953
5.4	6404582851	-7683295951	-4259647251	-3321888253
5.5	7193970051	-8101807851	-4100861651	-3578299753
5.6	8024314851	-8501414151	-3880440751	-3814921053
5.7	8893407051	-8875623351	-3591817551	-4019989553
5.8	9798355051	-9217276251	-3228149051	-4179287653
5.9	1073551652	-9518514951	-2782340051	-4275825053
6.0	1170042652	-9770762751	-2247093051	-4289506753
6.1	1258772652	-9964703151	-1614933951	-4196777653
6.2	1369108852	-1009026752	-8782700050	-3970260153
6.3	1470314052	-1013662052	-2944714349	-3578375553
6.4	1571538652	-1009216152	-9391832850	-2984967753
6.5	1671813452	-9944535551	-2035190651	2148923353
6.6	1770042052	-9680631751	-3265973051	1023807153
6.7	1864992452	-9286611251	-4638663651	4424921152
6.8	1955291952	-8747942651	-6160018151	2308071053
6.9	2039418652	-8049442351	-7836297851	4637286953
7.0	2115695652	-7175332151	-9673126951	7500977553
7.1	2182285552	-6109302751	-1167536052	1097659154
7.2	2237186152	-4834613651	-1384690352	1514822854
7.3	2278225552	-3334184851	-1619056252	2010656154
7.4	2303059652	-1590712651	-1870782752	2594861554
7.5	2309171752	-4131692050	2139869552	3277734454
7.6	2293870752	-2694753951	2426143452	4070105954
7.7	2254294052	-5271063951	2729236552	4983257454
7.8	2187411852	-8158639151	3048560952	6028804654
7.9	2090030652	-1137332452	3383282952	7218553954
8.0	1958805552	-1492997352	3732295652	8564324354
8.1	1790245952	-1884221052	4094192652	1007772555
8.2	1580735252	-2312207952	4467235052	1176987255
8.3	1326545152	-2777969952	4849323352	1365108555
8.4	1023856052	-3282291252	5237966352	1573049355
8.5	6687839951	-3825685152	5630248452	1801560455
8.6	2574097951	-4408350052	6022799052	2051177255
8.7	2141897151	-5030124452	6411757152	2322162655
8.8	7499000651	-5690435852	6792740852	2614438355
8.9	1353527452	-6388243752	7160821552	2927509055
9.0	2028747152	-7121986852	7510482752	3260377255
9.1	2779051352	-7889522152	7835601052	3611447655
9.2	3607685852	-8688063052	8129410752	3978418055
9.3	4517581852	-9514113852	8384488952	4358167555
9.4	5511283052	-1036340153	8592723452	4746622955
9.5	6590866652	-1123080253	8745307452	5138617755
9.6	7757847252	-1211029053	8832713552	5527748155
9.7	9013093752	-1299463253	8844693852	5906196155
9.8	1035671453	-1387634853	8770278652	6264569555
9.9	1178795753	-1474561753	8597778852	6591702955
10.0	1330508353	-1559222153	8314802052	6874472355

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APPENDIX

The differential equations for deflection are determined from the requirement of equilibrium and the relationships between stress and strain.

For symmetrical bending, moments and shear are related by the equilibrium equation:

$$\frac{d}{dr} (r M_r) - M_t - r Q_r = 0 \quad (a)$$

Where yielding is in the tangential direction only, moments and curvatures are related by:

$$M_r = - D_1 (w^{ii} + a\nu \frac{1}{r} w^i) + \nu M_c \quad (b)$$

$$M_t = M_c - a D_1 (\frac{1}{r} w^i + \nu w^{ii})$$

where $D_1 = \frac{Et^3}{12(1-a\nu^2)}$; ν is the elastic value of Poisson's ratio;

and primes denote differentiation with respect to r .

From Eqns. a and b, the shear Q_r may be expressed as

$$Q_r = - D_1 (w^{iii} + \frac{1}{r} w^{ii} - a \frac{1}{r^2} w^i) - \frac{1-\nu}{r} M_c \quad (c)$$

Alternatively, by integrating loads and reactions from the origin to any ordinate r , Q_r , in the case of a plate on an elastic foundation, is found to be

$$Q_r = \frac{1}{2\pi r} \int_0^r (q - \gamma w) 2\pi r dr \quad (d)$$

Equating the expressions for Q_r in Eqns. c and d, and differentiating with respect to r , gives

$$w^{iv} + \frac{2}{r} w^{iii} - a \frac{1}{r^2} w^{ii} + a \frac{1}{r^3} w^i = (q - \gamma w)/D_1 \quad (5)$$

Where yielding is in the radial direction only,

$$M_r = M_c - a D_1 (w^{ii} + \nu \frac{1}{r} w^i)$$

$$M_t = - D_1 (\frac{1}{r} w^i + a \nu w^{ii}) + \nu M_c \quad (e)$$

and from Eqns. a and e,

$$Q_r = - D_1 (a w^{iii} + a \frac{1}{r} w^{ii} - \frac{1}{r^2} w^i) + \frac{1-\nu}{r} M_c \quad (f)$$

Equating the expressions for Q_r in Eqns. d and f, differentiating,

and simplifying, gives

$$w^{iv} + \frac{2}{r} w^{iii} - \frac{1}{ar^2} w^{ii} + \frac{1}{ar^3} w^i = (q - \gamma w)/aD_1 \quad (6)$$

Where yielding occurs in each direction,

$$M_r = M_c - a D (w^{ii} + \nu \frac{1}{r} w^i)$$

$$M_t = M_c - a D (\frac{1}{r} w^i + \nu w^{ii}) \quad (g)$$

and from Eqns. a and g,

$$Q_r = a D (w^{iii} + \frac{1}{r} w^{ii} - \frac{1}{r^2} w^i) \quad (h)$$

Hence from Eqns. d and h,

$$w^{iv} + \frac{2}{r} w^{iii} - \frac{1}{r^2} w^{ii} + \frac{1}{r^3} w^i = (q - \gamma w)/aD \quad (7)$$

For an edge supported plate, $\gamma = 0$, and Eqns. 5, 6 and 7 reduce

respectively to

$$w^{iv} + \frac{2}{r} w^{iii} - a \frac{1}{r^2} w^{ii} + a \frac{1}{r^3} w^i = q/D_1 \quad (9)$$

$$w^{iv} + \frac{2}{r} w^{iii} - \frac{1}{ar^2} w^{ii} + \frac{1}{ar^3} w^i = q/aD_1 \quad (10)$$

$$w^{iv} + \frac{2}{r} w^{iii} - \frac{1}{r^2} w^{ii} + \frac{1}{r^3} w^i = q/aD \quad (11)$$