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Authors
Bergstrom, Ted
Goodman, Robert P.

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Private Demands for Public Goods

By THEODORE C. BERGSTROM AND ROBERT P. GOODMAN*

Goods and services which are publicly supplied generally share two features. Costs of these commodities are divided among members of the community. Decisions about quantities to be supplied are made collectively. Somehow, from a community consisting of many individuals with different tastes, different amounts of wealth, and conflicting interests, a single decision must emerge. Quantities must be determined and costs divided in some way.

In this study we develop a method for estimating demand functions of individuals for municipal public services. Demand will depend on the traditional economic variables, price and income, as well as on certain demographic characteristics of the individual and the city in which he lives. Knowledge of individual demand functions would be useful for many purposes. Such uses include:

1) Prediction of the outcomes of alternative political decision methods and tax structures in a particular city.

2) Computation of tax structures and expenditure levels which satisfy certain preference-based normative criteria such as the Lindahl equilibrium. This would enable one to make statements about whether "too many" or "too few" public goods are actually provided and whether tax burdens are divided "equitably" relative to the norm employed.

3) Investigation of whether there are "scale economies" to city size in the production of public services.

4) Prediction of the effects of projected changes in values of economic and demographic variables on quantities of public goods to be supplied.

In the case of privately purchased commodities, it is in principle possible to observe directly the choices made by individuals belonging to a particular demographic group under alternative price and income situations. This is not in general the case with publicly supplied commodities.[1] One is, however, able to observe the choices made by many distinct municipalities. Employing rather simple assumptions on the nature of the political process in each municipality, this information can be used to make inferences about the responses of individual demands for municipal services to price, income and other variables.

As an illustration of how this may be done, consider the following assumptions:

I. Units of measurement for a given municipally supplied commodity can be chosen in such a way that each municipality j is able to supply the commodity at constant unit cost q_j.[2]

* Associate professor, department of economics, Washington University, St. Louis and economist, department of economics and human development, State of Maryland, respectively. This research was done at Washington University and was in part supported by NSF Research Grant GS-3070. A more detailed presentation of the results is available on request. We are grateful for assistance and helpful comments from James Barr, Banarsi Dhawan, John Legler, and Richard Meadows.

[1] This problem and related issues are well discussed by James Buchanan and Gordon Tullock.

[2] This will be possible even if communities produce public goods using some local inputs whose prices may differ from place to place, so long as a) all municipalities have identical, homothetic production functions and b) all municipalities face horizontal supply curves for inputs. To see this, simply choose output units so that production takes place at constant returns to scale and observe that for any community, a change in output so measured results in a proportional change in total costs. Of course unit costs may differ from place to place, but in each place unit costs are constant with respect to output.

II. For each consumer i there is a tax share τ_i such that i must pay the fraction τ_i of the total cost of municipal expenditures in his community. Consumer i's tax share may depend on his wealth, his income or other individual characteristics, but it does not vary with the size of municipal expenditures nor with the way in which he expresses his desires for municipal services.

III. Any consumer i living in municipality j is aware of his "tax price," $\tau_i q_j$ and is able to determine the quantity of the municipal commodity which he would choose for the community given that he must pay the fraction τ_i of its total cost. To do so he needs only to maximize his own preferences subject to a linear budget constraint where his tax price for the municipal good is $\tau_i q_j$.

IV. In each municipality, the quantity supplied of the municipal commodity is equal to the median of the quantities demanded by its citizens.

V. In each municipality the median of the quantities demanded is the quantity demanded by the citizen with the median income for that municipality.

Observe that if Assumptions IV and V are true, the quantity of municipal commodities chosen by any community is the amount which is desired by the consumer with the median income for that community. Our assumptions about the constancy of τ_i and q_j, enable us to treat the expenditures of any municipality as an observation on the demand curve of a consumer with the median income for this municipality, where the price he pays for public goods is proportional to his tax share.[3] Assumptions I–IV are proposed and studied by Howard Bowen and independently by James Barr and Otto Davis. Assumption IV will hold if, for example, there is a majority voting system in which the quantity selected is such that half the population wants more and half the population wants less than that quantity.

DEFINITION: *An allocation satisfying I–IV will be called a Bowen equilibrium.*

Circumstances under which V is justified will be discussed below. It will also be demonstrated that our estimation procedure applies under somewhat weaker assumptions than V.

I. Estimating Procedure

Our sample consists of 826 municipalities with 1960 populations between 10,000 and 150,000, located in 10 states.[4] Multiple regression is used to fit a function of the form:

$$\log E = c + \alpha \log n + \delta \log \hat{\tau} + \epsilon \log \hat{Y} + \sum_{i=1}^{k} \beta_i X_i$$

The symbols are defined as follows:

E = The expenditures of a municipality on a specified category of municipal service. In particular, separate regressions were run for the following definitions of E.

(a) E = Police expenditures.

[3] A similar approach to the estimation of individual demands for public goods is suggested by Robin Barlow, by James Barr and Otto Davis, and by Thomas Borcherding and Robert Deacon.

[4] The states included and the number of observations from each are shown in Table 1. Our choice of states was determined in part by availability of data and in part by a desire to include states which had a large number of municipalities. The main limitation of data is the availability of figures on equalized assessed valuation by municipality. (Connecticut was excluded because of a lack of other demographic data.) There are several "small" states for which comparable data is available but which were not studied because of limitations of time. Results are reported for all states which were studied. It was believed that cities with populations exceeding 150,000, because of their relative small number and because of the more detailed information available concerning them, would be more appropriately treated in a separate national cross-section study. Places with populations below 10,000 were excluded because of a lack of comparable data.

(b) E= Parks and recreation expenditures.

(c) E= Total municipal expenditures excluding education and welfare.

n= The number of households in a municipality.

$\hat{\tau}$= The tax share of the citizen with the median income for a municipality.

$\hat{Y}$= Median income in a municipality.

The X_i's consist of the values of several descriptive social and economic variables for a municipality. These variables will be discussed below. As argued previously, Assumptions I–V would permit us to use the expenditure of any municipality as an observation of the quantity demanded by one of its citizens whose income is $\hat{Y}$ and whose tax share is $\hat{\tau}$.

If all municipally supplied commodities were pure public goods as defined by Paul Samuelson, the use of a public commodity by any consumer would not reduce its usefulness to others. If this were the case, the size of a city would affect the quantity of municipal commodities only indirectly through the tax share. In larger cities, costs would be divided among more citizens and tax shares would tend to be smaller, hence individuals would tend to vote for more public commodities.

If there is crowding of municipal services, population size may also have a direct effect on individual demands. We suggest here a simple model of crowding which lends a natural interpretation to our population coefficients. Suppose that the usefulness of a public facility to any individual is determined by a function of the form $Z^*=n^{-\gamma}Z$ where n is the number of people sharing the good and Z is the quantity of the public good. Let the utility function of any individual, i, be of the form $u_i(X_i, Z^*)$ where X_i is the quantity of private goods which he consumes. If $\gamma=0$, the public good is a Samuelsonian pure public good. If $\gamma=1$, the individual's preferences are as if he received and enjoyed only the fraction $1/n$ of the total amount of the public good. Suppose the unit cost of the private good is p, the unit cost of the public good is q, and the tax share of consumer i is τ_i. Then his demand function is found by maximizing $u_i(X_i, Z^*)$ subject to

$$X_i + \tau_i qZ \leqq Y_i$$

or equivalently subject to

$$X_i + \tau_i qn^\gamma Z^* \leqq Y_i$$

Thus, determination of his demand function for Z^* is formally equivalent to finding an ordinary demand function where the price is $\tau_i qn^\gamma$. Let us suppose that there are constant income and price elasticities δ and ϵ for the commodity Z^*. Then the demand function for Z^* is of the form,

$$c[\tau_i qn^\gamma]^\delta Y_i^\epsilon$$

The quantity of Z demanded is n^γ times the quantity of Z^* demanded. Hence, his demand for Z is

$$n^\gamma c[\tau_i qn^\gamma]^\delta Y_i^\epsilon = cq^\delta \tau_i^\delta Y_i^\epsilon n^{\gamma(1+\delta)}$$

Thus, the coefficient, α, for the elasticity of demand with respect to population could be interpreted to be $\gamma(1+\delta)$.[5]

If γ were nearly zero, there would be substantial economies to larger city size since in larger cities, more consumers could share in the costs of municipal commodities with only minor crowding effects. Where γ is about one, the gains from sharing the cost of public commodities among persons are approximately balanced by the disutility of sharing the facility among more persons.

The values of the auxiliary variables

[5] A very similar analysis of crowding is presented by Thomas Borcherding and Robert Deacon. Notice that if $\gamma=1$, the demand equation reduces (in *log* form) to $\log(E/n)=\log c+\delta \log(q\tau_i n)+\epsilon \log Y_i$. In this case we could consider demands for *per capita* quantities as a function of tax price *multiplied by population* and of income.

$X_1, \ldots, X_k$, may have two distinct kinds of effects on the expenditure decision of a municipality. Some variables appear to describe characteristics of a city which have fairly uniform effects on the demands of all its citizens for public services. Other variables seem to enumerate a portion of the population which may either have different tastes from the remainder of the population or pay a different tax share than persons with similar income. Examples of the former type are density and employment-residential ratio. Examples of the latter type are percent owner occupied, and percent of population older than 65. In practice, it is difficult to make this distinction in an unambiguous way. (How would one classify population change or percent nonwhite?) The first type of effect presents no difficulties for our method of analysis. The second type presents problems which will be discussed below. Since we have employed only a few of a large complex of interrelated social variables, it is difficult to attribute a clear-cut causal interpretation to the coefficients of these variables. At best, we hope that the variables which we include are adequate to eliminate substantial distortions of our estimates of price, income, and population elasticities.

II. Measurement of Variables

With the exception of the tax share variable, all of our data are available from standard sources. Roughly speaking, the income and demographic data are available from the 1960 census of population, while the expenditure data are from the 1962 census of governments.

We measure the quantity of municipal services by dollars of expenditure, median income in dollars, and the tax price of the median income consumer by his tax share. For this procedure to be literally consistent with our interpretation of the regression coefficients of income and tax share as price and income elasticities, we must assume that prices of private goods and unit costs of public goods are the same in each community observed. It is probably not unreasonable to assume that most of the supplies purchased by municipalities are purchased competitively in the same national market. But some of the inputs of municipal operations will be local labor and local land. Prices of these inputs may differ between communities. Local prices of *private* goods will also depend on local factor prices. Ideally we would like to deflate income by a local price index for all goods and deflate public expenditures by a local price index for public goods. We also would wish to use as our tax price variable the tax share *multiplied by the ratio of local public goods prices to local private goods prices*. Such local price indices are not, however, currently available.[6] It might not be unreasonable to suppose that the ratio of prices of public to private goods differs little between cities. In this case error enters our equation only because of the lack of a local price deflator for our income and expenditure figures. If the income elasticity of demand is close to unity, this will not seriously bias our estimates.[7]

[6] It would be possible in principle to construct such indices, using consumer budget studies, technical information about input requirements, and data on local factor prices.

[7] If q and p represent local prices for public and private goods, respectively, then the equation we would like to estimate is:

$$\log \frac{E}{q} = c + \alpha \log n + \delta \log \left(\hat{\tau} \frac{q}{p}\right) + \epsilon \log \left(\frac{\hat{Y}}{p}\right) + \Sigma \beta_i X_i$$

Rearranging terms we have:

$$\log E = c + \alpha \log n + \delta \log \hat{\tau} + \epsilon \log \hat{Y} + \Sigma \beta_i X_i + (1 + \delta) \log q - (\delta + \epsilon) \log p$$

This expression differs from the equation we estimated only by the final two terms. If q and p were uniform across cities, the omitted terms would affect only our intercept and not our estimates of elasticities. If the ratio of p to q were the same in every community then

Measurement of the tax share variable presents some conceptual and practical difficulties. We should like our demand equation to represent the response of individuals to the prices and incomes which they *perceive*. If one were to ask several individuals what their tax shares are, we suspect that few would be able to answer the question sensibly without more reflection than usually takes place before voting. Nevertheless, they may have knowledge which is for our purposes equivalent. For example, if a citizen knows his municipal tax bill and believes that his taxes will change in proportion to municipal expenditures, he will know the cost to him of a given percentage change in municipal expenditures. Moreover, it is often the case in municipal elections that an expenditure proposal is combined with a proposed change in property tax rates. A citizen who has some knowledge of the assessed value of his property can then determine the cost to him of the proposal.

It seems likely that although individuals have some notion of the cost of a proposal to them, their perceptions of tax shares may be quite imprecise. We formalize this idea by assuming that the perceived tax shares of individuals are independently distributed random variables with expected values equal to the actual tax shares. It turns out that in large populations the effects of independent errors of perception tend to cancel each other and that statistical distortions in our estimates from this cause are likely to be negligible. A formal statement of this proposition and its proof is offered in the Appendix.

In most cities sampled, more than half of locally generated revenue comes from the property tax. Property tax revenue is raised largely from taxes on real property (buildings and land) with the remainder derived from personal property taxes. To estimate the share of the tax on real property which is paid by a citizen with the median income for his community, we assume that in any municipality, the citizen with the median income lives in and owns the house with median value and that his house constitutes his entire holdings of real property. For each municipality, median house value is found in the 1960 U.S. Census of Housing. Tax rates and ratios of assessed to market value are determined for each community from information compiled by the states. The tax bill on the house of median value is then computed. This figure is divided by total property tax revenue for the municipality to produce an estimate of the share of real property taxes paid by the consumer with the median income.[8] It is assumed that the consumer with the median income pays the same share of other municipal revenues as he does of the real property tax. This is purely an assumption of conve-

the last two terms of the above equation would reduce to $(1-\epsilon)$ *log* p (plus a constant). If ϵ is near unity, then this term becomes insignificant.

[8] Those differences in median tax share from place to place which are not attributable to differences in population are largely due to differences in the amount of commercial and industrial property. Implicit in our procedure for estimating the tax share of the median consumer is the assumption that the burden of property taxes paid by commercial and industrial establishments is borne by persons living outside the municipality (or at least by an electorally insignificant portion of the cities' population). The assumption that individuals believe that their tax shares will not be altered if expenditures increase implies that they assume that the percentage of total revenue which can be extracted from commercial and industrial property will not change with changes in total revenue and expenditures. This may be somewhat unrealistic. A more thorough investigation might consider the question of the ability of a municipality to extract locational rents in an economy where the decision of a firm to locate in the municipality depends on levels of taxation and the quantity and composition of municipal taxation. Very little of the revenue for the services we study comes from other governments. The only exception is local highway expenditures for which about 15 to 25 percent of revenue comes from state collected funds. Even here, the transfers seem generally to be of the lump sum type, thus producing only a small income effect and no substitution effect.

nience which should be modified wherever better information is available.[9]

Although independent errors of perception by individuals are unlikely to have substantial effects on the parameter estimates, we are left with the problem that our own estimates of tax prices may differ substantially from the "true" tax prices. There are several reasons for this. One problem is that the accuracy of our estimates depends on the accuracy of the assessment to market value ratios computed by state equalization boards. These may be subject to substantial error. The assumption that the tax share of the consumer with median income is equal to his share of the property tax introduces further possibilities for error, as does the assumption that total unit costs of public goods are the same in all municipalities. Finally, there may be systematic nonindependent errors of perception in a community depending on the information made available to its citizens by the municipal authorities. If there are errors of measurement in the calculation of tax shares, the regression coefficients will be biased estimators and tend to underestimate the absolute value of the price elasticity. We therefore will compute maximum likelihood estimates of the price elasticity under some alternative assumptions about the measurement error.

III. Legitimacy of the Estimating Procedure

Suppose that the tax share of an individual is determined by his income and by the community in which he lives. For a citizen of municipality i, with income Y, the tax share will be written $\tau^i(Y)$. Let the demand for a public commodity by a citizen of municipality i be determined by the function $D(Y, \tau^i(Y), n^i, X^i)$ where n^i is the population and X^i a vector of values of variables describing municipality i.

To examine the effects on demand of differences in income within a given community, compute the total derivative of demand with respect to income. This is $dD/dY = \partial D/\partial Y + (\partial D/\partial \tau)(\partial \tau/\partial Y)$. Transforming this result into elasticity form, we have $(Y/D)(dD/dY) = \epsilon + \delta\xi$ where ϵ is the ordinary income elasticity of demand, δ is the price elasticity of demand, and $\xi \equiv (Y/\tau^i)(\partial \tau^i/\partial Y)$ is the elasticity of the tax share with respect to income. If for all values of Y, $\epsilon + \delta\xi > 0$, then the higher a citizen's income the more he will demand of the municipal commodity, if $\epsilon + \delta\xi < 0$ the quantity demanded decreases with income.[10] In either case, quantity demanded is a monotone function of income. It would then follow that the quantity demanded by the citizen with the median income in any municipality is also the median quantity demanded in that municipality. In such cases if we are willing to assume that the municipality is in Bowen equilibrium, the estimating procedure suggested in the previous section will give us reasonable estimates of price and income elasticities of demand.

If, however, $\epsilon + \delta\xi$ is positive for some levels of income and negative for others, the quantity demanded will not be a monotone function of income. In this case the median quantity demanded will not in general be the quantity demanded by the consumer with the median income.

For example, in Figure 1, if the incomes of half the population lie in the intervals OA and BC while the other half of the population has incomes in the interval AB, then the Bowen equilibrium quantity will be OE rather than OD, which is the quantity desired by the consumer with median

[9] It is generally quite difficult to assign the burden of nonproperty taxes. In the case of Pennsylvania there is a municipal income tax for many cities. We have included this tax in our computations of tax shares.

[10] These notions are also introduced by Buchanan (1964).

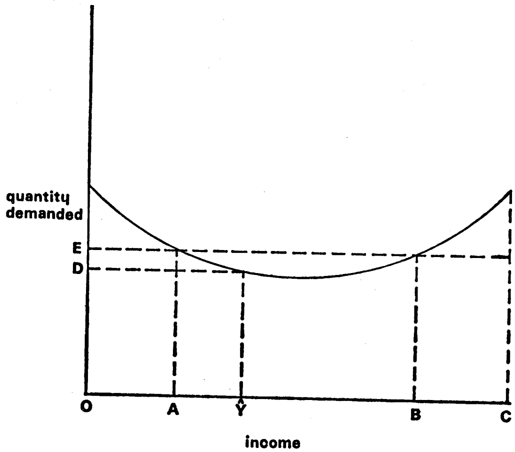

FIGURE 1

income $\hat{Y}$. Obviously, with alternative tax structures, a wide variety of cases can be constructed in which the quantity selected exceeds or falls short of the quantity demanded by the citizen with median income. If there were frequent and substantial variations of this kind, our procedure would not be expected to give reasonable estimates of the income elasticity of demand.

Similar difficulties arise if there are differences in tastes or in tax share functions for different segments of the population of a municipality. Suppose, for example, that renters believe (whether rightly or wrongly) that they do not pay the entire property tax on their housing. Their perceived tax shares will then be lower than those paid by home owners. One would expect them to vote for more public services than their home-owning counterparts. The half of the population desiring the larger amount will consist of proportionally more renters than homeowners. There is again no reason to expect the median quantity desired to be the quantity desired by the consumer with the median income. In general, the Bowen equilibrium quantity will depend on the shape of the entire income distribution functions of each group.

Here, however, things are not quite as chaotic as they may seem. One can make perhaps not grossly unrealistic assumptions which would allow him to use the estimating procedure previously outlined. Matters are greatly simplified if we make certain regularity assumptions about income distributions. We present a theorem which allows us to use the estimating procedure of the previous section even though there may be differences in tastes or in tax share schedules within communities.

Consider a set M of cities. Let P^i be the set of consumers in city i where $i \in M$. Let $\{P_1^i, \ldots, P_n^i\}$ be a partition of P^i, where λ_j^i is the ratio of the number of members of P_j^i to the number of members of P^i. (For each $i \in M$, $\sum_{j=1}^{n} \lambda_j^i = 1$.) Let F_j^i be the cumulative income distribution function for P_j^i. Thus, $F_j^i(Y)$ is the ratio of the number of members of P_j^i with incomes not exceeding Y to the total number of members of P_j^i.

DEFINITION: *Income distributions are said to be proportional in M (with respect to the partition* $\{P_1^i, \ldots, P_n^i\}$*) if there exist functions* F_j*, such that for each* $i \in M$ *there is a real number* $k_i > 0$ *where for all* $j = 1, \ldots, n$, $F_j^i(Y) \equiv F_j(k_i Y)$.

The following remark may help to give intuitive meaning to this definition. Suppose that in municipality 1, the cumulative income distributions for the subsets $P_1^1, \ldots, P_n^1$, are $F_1^1, \ldots, F_n^1$. (These functions need not bear any similarity to each other.) Suppose that income distributions in any other city $i \in M$ are the same as the income distributions in city 1 would be if the income of every citizen of city 1 were multiplied by some number k_i. Income distributions would then be proportional in M.

We can now state Theorem 1. A proof is supplied in the Appendix.

THEOREM 1: *Suppose that for each city*

$i \in M$, the population is partitioned as in the above discussion. Let income distributions be proportional in M with respect to the partition $\{P_i^i, \ldots, P_n^i\}$. Let the tax shares of members of P_j^i with income Y to be determined by a function of the form $\tau_j^i(Y) = \tau^i \tau_j Y^\xi$ (where τ^i and τ_j are constants depending on an individual's community and his subset). Let the demands for a public good of a member of P_j^i with income Y be determined by a function of the form $c_j f(X^i)[\tau_j^i(Y)]^\delta Y^\epsilon$. Suppose also that income distribution functions satisfy certain continuity assumptions (which are made explicit in the proof) and that $\epsilon + \xi\delta \neq 0$. Then the median quantity, $\hat{E}^i$, of a public good demanded by citizens of i is determined by a function of the form

$$\hat{E}_i = h(\overset{i}{\lambda_1}, \ldots, \overset{i}{\lambda_n}) c_1 f(\overset{i}{X}) [\overset{i}{\hat{\tau}_1}]^\delta [\overset{i}{\hat{Y}}]^\epsilon$$

where h is a continuous real valued function of the vector $(\lambda_1^i, \ldots, \lambda_n^i)$, $\hat{Y}^i$ is the median income for the entire city i, and $\hat{\tau}_1^i$ is the tax share of a member of P_1^i whose income is $\hat{Y}^i$.

The point of the theorem is that it allows us to use $\hat{Y}^i$ as the only variable describing income in city i and to use the coefficient ϵ as an estimate of income elasticity. Without the assumption of proportionality of income distributions, the quantity $\hat{E}^i$ would generally depend on the detailed structure of income distribution in each subset, P_j^i.

Nothing is said here about the functional form of h, which in general will be rather complicated. However, if we are willing to approximate the effect of the λ's as either linear or *log*-linear, we can estimate the income and price elasticities from cross-sectional observations of the expenditures of municipalities, their median incomes, and the tax shares of consumers of one type with the median income for the municipality. It is this result which justifies, for example, our use of "percent owner occupied" as an independent variable, our use of the tax share of a home-owner with the community median income as the tax share variable, and median income for the community as the income variable.

IV. Results of the Regressions

We were pleased to discover that our estimates of income elasticity were usually significant and positive and the estimates of price elasticity were usually significant and negative. In no case was an estimated elasticity coefficient both significant and of perverse sign. The results for the individual states are reported in Tables 1–3. Table 4 reports the results when all observations are pooled. For these regressions, dummy variables were used to allow the regression line a different intercept for each state. Although the results for the separate states are roughly similar, and there is considerable regularity in the signs of coefficients, the F-test would lead us to reject the hypothesis that mean square errors of the estimates are reduced by pooling the data. (See C. Toro-Viscarrondo and T. Wallace.) Some of the states could be combined successfully according to this criterion. However, rather than apply *ad hoc* methods, we would prefer to defer further efforts at pooling data until better measurement and perhaps better specification of variables is made. Ideally, we would hope that with proper measurement, it will be discovered that the coefficients of our dependent variables are not significantly different between the states. For the present we must settle for the rough similarity which we do find. It may nevertheless be convenient occasionally to use our estimates of the coefficients from the pooled regressions as crude summaries of our results.

Estimates of γ, the crowding parameter, were made by computing the ratio $\alpha/(1+\delta)$ where α and δ are the estimated elasticities of expenditures with respect to population and tax share, respectively. For most

TABLE 1—DETERMINANTS OF CURRENT GENERAL EXPENDITURES OF MUNICIPALITIES IN 1962[a]
(excluding education and welfare)

Coefficients of other variables	California	Illinois[b]	Michigan	Minnesota	Missouri	New Jersey	New York	Ohio	Pennsylvania	Wisconsin
Income elasticity ϵ	0.28	1.73*	0.88*	1.29*	1.65*	0.79*	1.03*	0.80*	0.37	0.16
	0.17	*0.44*	*0.22*	*0.38*	*0.46*	*0.14*	*0.17*	*0.19*	*0.26*	*0.37*
Tax share elasticity δ	−0.39*	−0.29*	−0.41*	−0.25*	−0.25	−0.13*	−0.50*	−0.21*	−0.15	−0.01
	0.08	*0.08*	*0.13*	*0.11*	*0.31*	*0.05*	*0.14*	*0.10*	*0.10*	*0.07*
Population elasticity α	0.67*	0.46*	0.58*	0.69*	0.94*	0.97*	0.75*	0.85*	0.99*	1.00*
	0.09	*0.12*	*0.14*	*0.19*	*0.33*	*0.06*	*0.15*	*0.11*	*0.12*	*0.11*
Crowding parameter $\gamma=(\alpha/(1+\delta))$	1.10	0.65**	0.98	0.92	1.25	1.11**	1.50**	1.08	1.16**	1.01
Percent population change (1950–60)	−0.06*	−0.10	−0.00	−0.03	−0.03	0.02	−0.03	−0.08*	0.03	−0.07*
	0.01	*0.07*	*0.02*	*0.02*	*0.03*	*0.03*	*0.17*	*0.02*	*0.02*	*0.03*
Employment residential ratio	0.11*	−0.08	0.22	−0.13	−0.02	0.24*	0.10	0.05	0.22*	0.12
	0.03	*0.12*	*0.12*	*0.11*	*0.36*	*0.04*	*0.05*	*0.05*	*0.07*	*0.15*
Percent owner occupied	−0.07	−1.56	−1.71*	−3.21*	0.05	−0.78*	−1.68*	−0.58	−0.35	0.35
	0.28	*0.82*	*0.52*	*1.01*	*1.15*	*0.24*	*0.45*	*0.39*	*0.52*	*0.67*
Percent nonwhite	0.45	1.95*	−0.20	12.79	1.80	1.35*	−0.19	0.75	0.48	3.63
	0.46	*0.92*	*0.54*	*10.92*	*2.23*	*0.33*	*0.72*	*0.89*	*0.57*	*2.58*
Density	−0.07	0.05	−0.02	0.03	−0.30	−0.03	−0.22*	−0.15*	0.08	0.02
	0.05	*0.12*	*0.07*	*0.08*	*0.17*	*0.04*	*0.05*	*0.07*	*0.06*	*0.06*
Percent population 65+	1.69*	4.21	−1.35	1.49	8.85*	1.32	3.70*	0.37	1.10	−0.57
	0.81	*2.68*	*1.76*	*2.01*	*2.67*	*0.96*	*1.47*	*1.15*	*1.72*	*2.37*
Percent living in same house (1955–60)	−1.21*	−.57	0.90	−1.48*	−2.88*	0.08	−1.27	−0.79	−0.88	−0.60
	0.35	*0.88*	*0.58*	*0.66*	*1.00*	*0.43*	*0.64*	*0.51*	*0.56*	*0.75*
Intercept	3.02	−9.81	−1.48	−6.08	−11.45	−7.01	−1.68	−4.09	−4.56	−2.68
Number of observations	160	62	70	36	33	120	74	106	124	41
R^2	.89	.80	.91	.89	.89	.94	.94	.87	.81	.96

[a] Values in italics are the standard errors of the coefficients.
[b] Excluding Cook County.
* Indicates a coefficient that is significant at the 95% confidence level.
** Indicates a value of γ that is significantly different from 1 at the 95% confidence level.

TABLE 2—DETERMINANTS OF MUNICIPAL EXPENDITURES ON POLICE IN 1962[a]

Coefficients of other variables	California	Illinois[b]	Michigan	Minnesota	Missouri	New Jersey	New York	Ohio	Pennsylvania	Wisconsin
Income elasticity ϵ	0.26	1.89*	0.54	0.80	1.04*	0.94*	1.78*	1.08*	0.99*	0.95
	0.16	*0.34*	*0.62*	*0.41*	*0.28*	*0.14*	*0.77*	*0.17*	*0.41*	*0.86*
Tax share elasticity δ	−0.25*	−0.19*	−0.76*	−0.13	−0.29	−0.15*	−0.31	−0.18*	−0.36*	−0.15
	0.07	*0.06*	*0.36*	*0.13*	*0.19*	*0.05*	*0.64*	*0.09*	*0.15*	*0.16*
Population elasticity α	0.75*	0.56*	0.26	0.83*	0.74*	0.81*	1.02	0.77*	0.67*	0.75*
	0.08	*0.09*	*0.40*	*0.20*	*0.21*	*0.07*	*0.70*	*0.10*	*0.18*	*0.25*
Crowding parameter $\gamma=(\alpha/(1+\delta))$	1.00	0.69**	1.08	0.95	1.04	0.95	1.48	0.94	1.05	0.88
Percent population change (1950–60)	−0.04*	−0.06	−0.07	−0.04	0.01	0.04	−2.73*	−0.01	0.06	−0.14*
	0.01	*0.05*	*0.05*	*0.02*	*0.02*	*0.03*	*0.77*	*0.02*	*0.03*	*0.06*
Employment residential ratio	0.08*	−0.04	−0.27	−0.10	0.06	0.21*	−0.14	0.06	0.20	0.19
	0.03	*0.09*	*0.33*	*0.12*	*0.22*	*0.04*	*0.23*	*0.04*	*0.12*	*0.36*
Percent owner occupied	−0.19	−2.25*	−0.51	−3.61*	−0.81	−0.65*	−1.16	−0.59	0.38	−0.08
	0.26	*0.62*	*1.47*	*1.10*	*0.71*	*0.24*	*2.07*	*0.35*	*0.81*	*1.57*
Percent nonwhite	0.69	1.53*	0.13	0.97	0.87	1.21*	2.32	1.43	0.04	−1.55
	0.42	*0.70*	*1.52*	*11.94*	*1.38*	*0.34*	*3.30*	*0.80*	*0.89*	*6.07*
Density	−0.02	0.01	0.03	0.05	−0.09	0.07	−0.46*	0.05	0.55*	0.20
	0.04	*0.09*	*0.21*	*0.08*	*0.10*	*0.04*	*0.23*	*0.07*	*0.09*	*0.15*
Percent population 65+	0.99	−0.97	1.21	−0.85	1.16	0.55	10.38	−1.39	0.77	2.47
	0.74	*2.03*	*4.93*	*2.20*	*1.66*	*0.98*	*6.77*	*1.03*	*2.68*	*5.58*
Percent living in same house (1955–60)	−0.70*	0.53	0.06	−0.74	−0.33	0.47	−9.11*	0.25	−1.50	−2.39
	0.32	*0.67*	*1.64*	*0.72*	*0.62*	*0.44*	*2.96*	*0.46*	*0.87*	*1.76*
Intercept	−0.80	−13.70	4.36	−5.59	−7.12	−9.24	−8.58	−9.90	−11.31	−9.02
Number of observations	160	62	70	36	33	120	74	106	124	41
R^2	.89	.87	.57	.88	.93	.92	.69	.89	.69	.88

[a] Values in italics are the standard errors of the coefficients.
[b] Excluding Cook County.
* Indicates a coefficient that is significant at the 95% confidence level.
** Indicates a value of γ that is significantly different from 1 at the 95% confidence level.

TABLE 3—DETERMINANTS OF MUNICIPAL EXPENDITURES ON PARKS AND RECREATION IN 1962[a]

Coefficients of other variables	California	Illinois[b]	Michigan	Minnesota	Missouri	New Jersey	New York	Ohio	Pennsylvania	Wisconsin
Income elasticity ϵ	0.10	3.54	1.25	2.19*	2.81*	2.28*	1.74*	1.50*	1.76*	−1.87
	0.44	*1.98*	*0.66*	*0.83*	*0.94*	*0.54*	*0.64*	*0.55*	*0.80*	*1.18*
Tax share elasticity δ	−0.68*	0.01	0.25	−0.39	−0.24	−0.00	−0.81	−0.49	0.02	−0.13
	0.20	*0.37*	*0.38*	*0.25*	*0.63*	*0.17*	*0.53*	*0.28*	*0.29*	*0.22*
Population elasticity α	0.67*	1.01	1.49*	0.74	1.11	1.28*	0.69	0.60	1.72*	1.19*
	0.23	*0.52*	*0.43*	*0.41*	*0.69*	*0.25*	*0.58*	*0.31*	*0.36*	*0.35*
Crowding parameter $\gamma=(\alpha/(1+\delta))$	2.09**	1.00	1.19	1.21	1.47	1.28	3.63**	1.18	1.69**	1.37
Percent population change (1950–60)	−0.11*	0.33	0.01	−0.02	−0.22*	−0.15	1.42*	−0.08	0.05	−0.44*
	0.02	*0.30*	*0.05*	*0.04*	*0.07*	*0.12*	*0.64*	*0.06*	*0.07*	*0.08*
Employment residential ratio	0.08	0.05	1.25*	−0.19	1.22	0.41*	0.35	−0.10	0.83*	−0.83
	0.09	*0.51*	*0.35*	*0.24*	*0.75*	*0.16*	*0.19*	*0.14*	*0.23*	*0.49*
Percent owner occupied	0.00	−4.03	0.25	−4.90*	4.08	−2.06*	−3.48*	−0.27	0.23	2.46
	0.73	*3.64*	*1.56*	*2.20*	*2.39*	*0.91*	*1.72*	*1.13*	*1.58*	*2.15*
Percent nonwhite	−2.06	−6.06	−0.33	−1.69	−0.57	2.31	1.66	3.03	−0.23	−5.41
	1.19	*4.09*	*1.61*	*23.95*	*4.61*	*1.28*	*2.75*	*2.62*	*1.74*	*8.35*
Density	0.05	0.49	−0.39	0.15	0.26	0.38*	−0.21	−0.31	0.35*	−0.19
	0.13	*0.52*	*0.22*	*0.17*	*0.35*	*0.16*	*0.19*	*0.21*	*0.17*	*0.21*
Percent population 65+	1.22	25.87*	−8.23	8.99*	11.25*	3.92	14.27*	−2.33	7.11	−4.85
	2.10	*11.91*	*5.25*	*4.15*	*5.53*	*3.68*	*5.64*	*3.38*	*5.24*	*7.67*
Percent living in same house (1955–60)	−1.00	−0.92	0.81	−2.38	−9.27	−0.57	−3.14	−0.90	−4.77*	−5.29*
	0.92	*3.93*	*1.74*	*1.45*	*2.06*	*1.65*	*2.46*	*1.49*	*1.70*	*2.42*
Intercept	3.84	−40.84	−19.33	−15.99	−31.62	−23.20	−8.03	−12.99	−29.73	15.87
Number of observations	160	62	70	36	33	120	74	106	124	41
R^2	.67	.35	.63	.79	.83	.63	.67	.56	.60	.86

[a] Values in italics are the standard errors of the coefficients.
[b] Excluding Cook County.
* Indicates a coefficient that is significant at the 95% confidence level.
** Indicates a value of γ that is significantly different from 1 at the 95% confidence level.

states, the estimates of γ for police expenditures and general expenditures are strikingly close to unity. We applied an F-test to determine whether to reject the hypothesis that the linear restriction $\alpha=1+\delta$ applies. In all states but Illinois and in the pooled regression, the test strongly suggests that the null hypothesis should be accepted for police expenditures. For general expenditures, we would have to reject the null hypothesis at the 5 percent level for about half of the states and for the pooled regression. However, in all cases except Illinois we would reject the hypothesis that $\gamma \leqq 1$. The test also suggests that for parks and recreation expenditures, γ is one or greater.

Some economic implications of the values of the price and income elasticities and the coefficient, γ, will be discussed in the next section.

A few speculations about the effects of some of the auxiliary variables may be justified by the degree of regularity with which the same signs appear. The differences in their sizes and the possibility of correlation with other more "truly explanatory" variables does suggest caution in making causal interpretations. The reader is, of course, invited to make his own speculations.

Percent owner occupied: As argued earlier in this paper, it may be that renters do not believe that they pay the entire property tax on their housing, and tend to vote for more public expenditures than homeowners with the same income. If this is the case, one would expect the negative coefficient which we find for this variable.

Employment-Residential ratio: Communities with high values for this variable would tend to have a large amount of com-

TABLE 4—DETERMINANTS OF MUNICIPAL EXPENDITURES, 1962
ALL OBSERVATIONS POOLED[a]

	General Expenditures	Police Expenditures	Parks & Recreation
Income elasticity ϵ	0.64*	0.71*	1.32*
	0.07	*0.13*	*0.22*
Tax share elasticity δ	−0.23*	−0.25*	−0.19*
	0.03	*0.05*	*0.08*
Population elasticity α	0.84*	0.80*	1.17*
	0.03	*0.06*	*0.11*
Crowding parameter $\gamma=(\alpha/(1+\delta))$	1.09**	1.07	1.44**
Percent population change (1950–60)	−0.04*	−0.04*	−0.08*
	0.01	*0.01*	*0.02*
Employment residential ratio	0.12*	0.01	0.24*
	0.02	*0.04*	*0.06*
Percent owner occupied	−0.77*	−1.12*	−0.78
	0.13	*0.25*	*0.42*
Percent nonwhite	0.84*	0.90*	−0.20
	0.19	*0.36*	*0.60*
Density	−0.07*	0.01	−0.02
	0.02	*0.04*	*0.06*
Percent population 65+	1.75*	1.27	4.94*
	0.45	*0.85*	*1.43*
Percent living in same house (1955–60)	−0.65*	−0.77*	−1.99*
	0.17	*0.32*	*0.53*

[a] Values in italics are the standard errors of the coefficients.
* Indicates a coefficient that is significant at the 95 percent confidence level.
** Indicates a value of γ that is significantly different from 1 at the 95 percent level.

mercial or industrial activity. It may be that larger amounts of public services must be provided in order to attract and retain such activity. This might explain the positive effect which we find.

Population change: Cities which have grown rapidly may not yet have achieved a political equilibrium. Time is required for a rapidly growing city to gain a political consensus for expanding public services. Inertial effects may also result in larger expenditures in places with declining population. This might explain our negative coefficient.

Percent of population over 65: The life cycle hypothesis would predict that persons over 65 years of age tend to spend a larger portion of their current income on current consumption than younger people. If demand for public goods as a fraction of total demand does not diminish with age then one would expect an aged person to demand a larger quantity of public goods than a younger person with the same income and tax share. The effect of this variable would then be positive, which is what we usually find.

The variables, percent nonwhite, percent who lived in the same house from 1955 to 1960, and population density, frequently had coefficients significantly different from zero. We have no particularly compelling explanation for the pattern of signs which emerges. Although these variables are included in the regressions reported, the size and significance of the other coefficients are not substantially altered when we exclude them.

TABLE 5

If 95 percent of the measured values $\hat{T}$ lie in the interval [,] where T^* is the "true" value of median tax share	$[.5T^*, 2T^*]$	$[.57T^*, 1.75T^*]$	$[.66T^*, 1.5T^*]$	$[.75T^*, 1.33T^*]$	$[.9T^*, 1.11T^*]$	No error
then the maximum likelihood estimate of δ is	$-.83$	$-.44$	$-.30$	$-.26$	$-.24$	$-.23$

We experimented with the use of some other variables, including population per household, median school years completed, percent of population less than 18 years old, whether a municipality was a suburb or not, percent of the population with annual incomes below $3,000 and percent of the population with annual incomes exceeding $10,000. Each of these variables proved to have statistically insignificant effects in all or nearly all of our regressions.

The fact that the variables representing percentages of populations at the extreme ends of the income distribution were almost always insignificant provides mild evidence in favor of our hypothesis that the consumer with the median income also demands the median quantity of public goods. It is of some interest to observe however, that for California, the percentage of the application with income under $3,000 has a significant and positive effect on demand. When this variable is included, the coefficient of median income for California rises from values considerably lower than those displayed for other states to values quite similar to the others.

It was thought that perhaps our estimates would be overwhelmed by the effects of population size. In particular there is a strong negative correlation between population and tax share (because in larger cities taxes are divided among more persons). It also seemed possible that the effects of some of our variables might be substantially different in cities of different size, either due to differences in political structure or uncaptured social differences. We therefore stratified our sample into nine population subgroups. Again, our coefficients were almost all reasonable and fairly similar between groups. For no population subgroup were we able to reject the hypothesis that $\gamma = 1$.

As mentioned above, we computed maximum likelihood estimates for the regression coefficients under some alternative assumptions about errors in the measurement of tax share. The only coefficient which was substantially affected was the coefficient of tax share itself. In Table 5 we report the maximum likelihood estimate of the tax share elasticity δ where there is a lognormally distributed measurement error with alternative variances.[11] These values apply to the pooled regression on general expenditures. The effects on the other regressions were similar.

V. Some Implications of the Results

The reasonable values which we find for our coefficients lend at least some support to the hypotheses which we make about political behavior. These hypotheses in turn provide a useful tool for the estimation of the effects of income, price, city size, and other social variables on demand

[11] We follow the maximum likelihood procedure suggested by John Johnston, pp. 168–70.

for public commodities. The reader would, of course, do well to maintain a degree of skepticism about the values of the coefficients which we find. Strong assumptions were made to derive the estimates. It is possible that more precise measurement of the variables and additional tests of the assumptions would result in quite different estimates of demand functions.

Here we examine some implications of the parameter values which we find. The reader is free to apply the analysis below to the implications of any alternative estimates which he prefers.

Suppose that the only real property holdings of voters are their dwellings and that the tax share of a voter is proportional to his holdings of real property. The most commonly accepted estimates of the income elasticity of demand for housing appear to be between 1 and 1.3.[12] This would imply an elasticity, ξ, of tax share with respect to income in the same range. For most of our regressions (exceptions are California and Pennsylvania) we estimate the income elasticity, ϵ, to be substantially greater than $-(1.3)\delta$. Thus, we would expect that $\epsilon+\xi\delta>0$. According to our earlier discussion one would expect that in most cities, citizens with incomes exceeding the median would vote for increased expenditures and citizens with lower incomes would vote against increased expenditures.[13]

The Lindahl solution for allocation of public goods has predictive and normative properties which are attractive to many economists. Under quite reasonable assumptions, Lindahl equilibrium exists, is Pareto optimal, and is in the core.[14] In Lindahl equilibrium, tax shares are arranged in such a way that all consumers demand the same quantity of each public commodity. It has frequently been pointed out that if an individual's stated preferences for public commodities influence his tax share, it is usually in his interest to misrepresent his preferences. This difficulty would seem to greatly reduce the practical usefulness of the Lindahl system as an allocative device. As our paper suggests, it may nevertheless be possible to gain approximate information about the preferences of broadly defined social groups if one observes the voting behavior of individuals under circumstances where it is reasonable to believe that an individual's expressed desires concerning levels of expenditure do not influence his tax share.

A Bowen equilibrium is not, in general, Pareto optimal. For "most" Bowen equilibria one could find an alternative distribution of the tax burden and alternative levels of public expenditure which would result in a Pareto-superior allocation. One might wish to ask whether a certain Bowen equilibrium results in "too little" or "too much" public goods. This question is not generally answerable on the basis simply of the Pareto criterion. As Paul Samuelson stresses, there are an infinity of Pareto optimal allocations corresponding to different points on the "utility possibility" frontier. Generally there will be different quantities of public goods for different Pareto optimal allocations. Some of these quantities may be larger and some smaller than the Bowen equilibrium quantity considered.

A more determinate normative criterion would be to compare a given Bowen equilibrium to the Lindahl equilibrium corresponding to the initial distribution of income. If voters of all incomes are to agree about the quantity of public goods to be supplied, the tax schedule must be

[12] See for example the papers by Richard Muth and Frank de Leeuw.

[13] An interesting piece of independent evidence indicating the same conclusion is provided by Harlan Hahn and W. B. Shephard's study of precinct voting in municipal referenda.

[14] See the papers of Erik Lindahl, Duncan Foley, and Bergstrom (1970).

such that $\epsilon+\xi\delta=0$.[15] For a community with known values of $-\epsilon/\delta$ and ξ, we could compute both the Lindahl equilibrium and the Bowen equilibrium if we had full knowledge of the income distribution. It is in general not possible with knowledge only of $-\epsilon/\delta$ and ξ to determine in which case the quantity of public goods is larger.

If, however, we assume that the elasticities, ξ and $-\epsilon/\delta$ are constant over all incomes, we can provide a partial answer to this question given only limited information about the income distribution. In particular, in either Bowen equilibrium or in Lindahl equilibrium the quantity supplied will be equal to the quantity demanded by the citizen with median income. To determine in which case he demands more, we need only to determine whether his tax share is lower in Lindahl equilibrium or in Bowen equilibrium. Where price elasticities are negative, the Lindahl equilibrium quantity will exceed the Bowen equilibrium quantity if, and only if, the tax share of the consumer with median income is lower in Lindahl equilibrium. In another paper (1973), Bergstrom has shown that if mean income in a community exceeds median income, if $-\epsilon/\delta>1$ and $\xi<-\xi/\delta$, the tax share of the citizen with median income will be lower in Lindahl equilibrium than in Bowen equilibrium. Our estimates suggest that each of these assumptions is likely to hold. It then follows that in the communities we observe, less than the Lindahl equilibrium quantity is provided. One could argue that this result provides some evidence to support John Kenneth Galbraith's contention that in reaching a "truce on income distribution" communities arrive at a quantity of public goods which is less than optimal. On the other hand, our estimates of income elasticities of less than unity for general municipal expenditures would seem "anti-Galbraithian."[16]

Our results suggest that the crowding parameter γ generally has a value of one or greater for cities with populations between 10,000 and 150,000. One could interpret this to mean that as the size of municipalities increases, the advantages of sharing the cost of public services among more persons are countervailed by the costs of sharing the services among more persons. An alternative statement of this proposition would be that over the range of city sizes which we studied, there appear to be no economies of scale to larger municipalities in the provision of public goods. It should, of course, be understood that this statement says nothing about the question of whether there are economies of scale to production of publicly supplied goods measured in some choice of physical units, but relates only to net effects of city size after crowding phenomena are accounted for.

One might reasonably ask why, if there are not increasing returns in the municipal provision of the goods and services which we study, is their provision in the public domain. The answer may be that there are substantial scale economies in the collective provision of these commodities

[15] Where some of the burdens of taxes and benefits of public goods are shared by noncitizens, we might choose to define Lindahl equilibrium in a municipality either as a situation in which there is agreement among all citizens or among all interested parties. If we take the former approach, Lindahl equilibrium is not necessarily Pareto optimal from the viewpoint of the national economy. Even here, one would expect that competition among municipalities for local industry would tend to result in efficient provision of public services. In any case, Lindahl equilibrium does require that there be agreement among citizens of all incomes. Thus, the requirements for income elasticities of tax shares remain unaltered.

[16] Galbraith's position, however, rests at least in part on the claim that expressed demands are a distortion of underlying "true" preferences since greater advertising of private goods reduces relative expressed demands. Hence he might choose to deny any normative significance for the demand functions which we estimate.

for units of smaller size than the cities which we consider.[17] This could provide sufficient reason that they be collectively provided. It may be that once some critical size is achieved, city size becomes largely a matter of indifference from an efficiency viewpoint over a fairly large range of sizes. Some corroborating evidence is provided by the fact that in metropolitan areas, contiguous municipalities frequently differ substantially and seemingly arbitrarily in size.

Appendix

Part I: Individual Errors of Perception

We introduce three lemmas, the first two of which were proved by Mosteller.

LEMMA 1: *If a random sample of size n is drawn from a probability distribution, f, which is continuous and nonvanishing in a neighborhood of its median,* $\tilde{X}$, *then the sample median* $\hat{X}(n)$ *is asymptotically normally distributed (as n becomes large) with expected value* $\tilde{X}$ *and variance* $\sigma^2(n)=1/(4n[f(\tilde{X})]^2)$ *where* $\text{limit}_{n\to\infty}\,\sigma^2(n)=0$.

LEMMA 2: *Let the random variable* $\theta(n)$ *be a function of the values* $X_1, \ldots, X_n$ *in a random sample of size n drawn from a probability distribution f. If* $\theta(n)$ *is asymptotically normally distributed and* $\lim_{n\to\infty}[E\theta(n)]=\tilde{\theta}$ *where the asymptotic variance of* $\theta(n)$ *is zero, then if*

$$g[\theta(n)]$$

is a continuous real-valued function with a nonvanishing derivative in a neighborhood of $\tilde{\theta}$, $g[\theta(n)]$ *is an asymptotically normally distributed random variable such that*

$$\lim_{n\to\infty} E[g(\theta(n))] = g(\tilde{\theta})$$

and the asymptotic variance of g is zero.

[17] In the case of municipal courts and some policing functions it might be argued that these are social reasons for public provision quite distinct from economies of scale.

LEMMA 3: *If* $X_1, \ldots, X_n, Y_1, \ldots, Y_n$ *are symmetrically distributed and mutually independent random variables and if* $Z_i=X_i+Y_i$ *for* $i=1, \ldots, n$, *then* $E(\hat{Z})=E(\hat{X})+E(\hat{Y})$ *where* ^ *denotes the sample median of a random variable.*

PROOF:

Since X, Y, and Z are symmetrically distributed, if we let a bar denote the sample mean, then

$$E(\hat{Z})=E(\overline{Z})=E(\overline{X})+E(\overline{Y})=E(\hat{X})+E(\hat{Y})$$

THEOREM: *In a municipality with n citizens, let the demand of consumer i with income* Y_i *for a public good be a random variable*

$$D_i = c[\tau^*(Y_i)]^{\delta} Y_i^{\epsilon}$$

Let

$$\tau(Y_i) = tY_i^{\xi}$$

be the "true" tax share and $\tau^*(Y_i)=Z_i\tau(Y_i)$ *be his perceived tax share where* Z_i *is a random variable. Suppose that the values of Y and Z for the population of the city are determined by a random sample of size n drawn from a population in which log Z and log Y are independently and symmetrically distributed and where* $E(\log Z)=0$ *and* $E(\log Y)=\log\hat{Y}$ *where* $\hat{Y}$ *is the observed median income in the municipality. Then if* $\hat{D}$ *is the median quantity demanded by citizens of the municipality,* $\log\hat{D}=\log c+\delta\log\tau(\hat{Y})+\epsilon\log\hat{Y}+\gamma$ *where* γ *is asymptotically normally distributed with mean of zero and asymptotic standard deviation equal to zero.*

PROOF:

From the demand equation we have,

$$\log D_i = (\delta\xi + \epsilon)\log Y_i + \delta\log Z_i$$

Applying Lemma 3, we have

$$E(\widehat{\log D}) = E[(\delta\xi + \epsilon)\log Y] + E(\widehat{\delta\log Z}) + \log c$$

where the hat denotes the sample median. But for any monotonic function f, $f(\widehat{x})=f(\hat{x})$.

Hence,

$$E\,(log\,\hat{D}) = E[(\delta\xi + \epsilon)\,log\,\hat{Y}) + E(\delta\,log\,\hat{Z}) + log\,c$$

Lemma 2 implies that

$$\lim_{n\to\infty} E\,(log\,\hat{D}) = (\delta\xi + \epsilon)\,log\,[\lim_{n\to\infty} E(\hat{Y})] + log\,[\lim_{n\to\infty} E(\hat{Z})] + log\,c$$

Applying Lemma 1 and our assumptions about the distributions of Y and Z, we have

$$\lim_{n\to\infty} E\,(log\,\hat{D}) = (\delta\xi + \epsilon)\,log\,\hat{Y} + log\,c = log\,c + \delta\,log\,\tau(\hat{Y}) + \epsilon\,log\,\hat{Y}$$

By Lemma 1, the asymptotic variance of $log\,\hat{D}$ is zero. The theorem follows immediately.

This result provides us with a partial justification for presuming that individual errors of perception will have no statistically unpleasant effects if the population is very large. Two problems remain. The Theorem assumes that $log\,Y$ is symmetrically distributed. If this is not the case, then Lemma 3 does not apply and $\epsilon\,log\,\hat{Y}+\delta\,log\,\tau(\hat{Y}) + log\,c$ may be a biased estimate of $log\,\hat{D}$. This would not be a severe problem unless the asymmetry is substantial and highly variable from city to city. If, for example, income distributions are proportional between cities (as defined in the text) then the income distribution functions in different cities are simply translations of each other. In this case, the bias is the same for each city. The bias would affect the regression intercept but not the estimates of elasticities.

It also should be demonstrated that for cities as large as those which we consider, the asymptotic results are closely approximated. To find the magnitudes of these effects we can use the result that the variance of the median of a random sample of size n drawn from a normal distribution with variance σ^2 will be no larger than $(1/n)(2/\pi)\sigma^2$. (See J. Hodges and E. Lehmann.) If we postulate reasonable values for our parameters, for a community of 2,000 or more families, even very substantial variances of individual perception error result in a variance of the error term σ discussed above which would have negligible statistical effects.

It might also be noted that it is well that these error terms tend to be small, because they are not "well-behaved" in other ways. Although when the error is that of the actor rather than the observer it may be reasonable to assume that the error term is uncorrelated with the observed value of the variable (unlike the usual assumption when the error is that of the observer), it remains the case that the variance of the error term is likely to be smaller as city size increases. This violates the classical assumptions.

Part II: Proof of Theorem 1

For each $i \in M$ and $j=1, \ldots, n$, let I_j^i be the closed interval between the maximum and the minimum income in p_j^i. We make the following continuity assumption. (C) For every $i \in M$, the set $\bigcup_{j=1}^n I_j^i$ is a closed interval, and the functions F_j^i, $j=1, \ldots, n$, are all continuous and strictly increasing on I_j^i.

The cumulative income distribution, F^i, for the entire population P^i is easily seen to be

$$F^i = \sum_{j=1}^{n} \lambda_j^i F_j^i$$

Assumption C insures that F^i is continuous and strictly increasing on $\bigcup_{j=1}^n I_j$. If this assumption were dispensed with, one could prove a result similar to Theorem 1 which applies to discrete distributions. However, for our purposes, the added realism does not appear to justify the added tedium in the statement and proof of the theorem.

Let $\hat{Y}^i$ be the median income for the population P^i. Then

$$1/2 = F^i(\hat{Y}^i) = \sum_{j=1}^{n} \lambda_j^i F_j^i\,(\hat{Y}^i) = \sum_{j=1}^{n} \lambda_j^i F_j(k_i \hat{Y}^i)$$

It follows that the value of $(\lambda_1^i, \ldots, \lambda_n^i)$ determines $k_i\hat{Y}^i$. In fact since F^i is continuous and strictly increasing on $\bigcup_{j=1}^n I_j^i$, there is a continuous function g such that

$$k_i \hat{Y}^i = g(\overset{i}{\lambda}_1, \ldots, \overset{i}{\lambda}_n)$$

Since $\epsilon+\xi\delta \neq 0$, one can choose for each $i \in M$ and for $j=1, \ldots, n$, a $\tilde{Y}^i_j$ such that if $\hat{E}^i$ is the median quantity demanded in city i,

then

$$\hat{E}^i = c_j f(X^i)(\overset{i}{t} t_j)^\delta [\tilde{Y}^i_j]^{\epsilon+\xi\delta}$$

Thus for $j=1, \ldots n$, $\tilde{Y}^i_j = \rho_j \tilde{Y}^i_1$

where

$$\rho_j = \left[\frac{c_j t_j^\delta}{c_1 t_1^\delta}\right]^{1/(\epsilon+\xi\delta)}$$

Since E is the median quantity demanded, it must be that

$$1/2 = \sum_{j=1}^n \overset{i}{\lambda}_j \overset{i}{F}_j(\tilde{Y}^i_j) = \sum_{j=1}^n \overset{i}{\lambda}_j \overset{i}{F}_j(\rho_j \tilde{Y}^i_1)$$

$$= \sum_{j=1}^n \overset{i}{\lambda}_j F_j(k_i \rho_j \tilde{Y}^i_1)$$

$$= \sum_{j=1}^n \overset{i}{\lambda}_j F_j(g(\overset{i}{\lambda}_1, \ldots \overset{i}{\lambda}_n)\rho_j \frac{\tilde{Y}^i_1}{\hat{Y}^i}$$

The latter expression is a continuous strictly increasing function of $\tilde{Y}^i_1/\hat{Y}^i$. It follows that $\tilde{Y}^i_1/\hat{Y}^i$ is determined by the vector $(\lambda^i_1, \ldots \lambda^i_n)$. In fact, there is a continuous function h such that

$$h(\overset{i}{\lambda}_1, \ldots \overset{i}{\lambda}_n) = \left[\frac{\tilde{Y}^i_1}{\hat{Y}^i}\right]^{\epsilon+\xi\delta}$$

But

$$\hat{E}^i = c_1 f(X^i)(\overset{i}{t} t_1)^\delta \tilde{Y}_1^{i\epsilon+\xi\delta}$$
$$= h(\overset{i}{\lambda}_1, \ldots, \overset{i}{\lambda}_n) c_1 f(X^i)(\overset{i}{\tau}_1)^\delta (\hat{Y}^i)^\epsilon$$

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