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Beam ion driven instabilities in the National Spherical Tokamak Experiment[a]

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Recent progress in the analysis of the low and high frequency beam ion driven instabilities in the National Spherical Tokamak Experiment (NSTX) [S. Kaye *et al.*, Fusion Technol. **36**, 16 (1999)] plasma is reported. The low Alfvén speed with respect to the beam ion injection velocity in NSTX offers a window in the plasma parameter space to study instabilities driven by super-Alfvénic fusion alphas, which are expected in the International Tokamak Experimental Reactor—ITER [D. J. Campbell, Phys. Plasmas **8**, 2041 (2001)]. Low frequency magnetic field activities identified as an instability of toroidicity-induced Alfvén eigenmodes (TAEs) have been observed in NSTX and analyzed with the linear hybrid kinetic magnetohydrodynamic stability code NOVA-K [C. Z. Cheng, Phys. Rep. **1**, 211 (1992)]. The comparison between the TAE analysis and observations in NSTX and DIII–D [J. L. Luxon, Nucl. Fusion **42**, 614 (2002)] similarity experiments confirms that the toroidal mode number of the most unstable TAE modes scales with q^{-2} and is independent of plasma major radius, where q is the safety factor. This scaling helps validate the predictive capability of the NOVA-K code for studying TAE stability in future burning plasma devices. The subion cyclotron frequency magnetic activities in NSTX are identified as compressional and global shear Alfvén eigenmodes (AEs) (CAEs and GAEs). CAE and GAE instabilities are driven by beam ions via the Doppler shifted cyclotron resonance by the velocity space bump-on-tail distribution function in the perpendicular velocity. Results of the GAE/CAE theoretical and numerical analysis are presented. [DOI: 10.1063/1.1689667]

I. INTRODUCTION

Collective instabilities associated with superthermal ions in plasmas have been of interest to fusion researchers during the last four decades (see for instance Refs. 1 and 2, and references therein). Low frequency instabilities of Alfvén eigenmodes (AE) driven by the spatial energetic particle pressure gradient can limit the energetic particle confinement in toroidal fusion devices and thus may be a principal concern in a fusion tokamak reactor such as International Tokamak Experimental Reactor (ITER).[3,4] Among many AEs observed in tokamaks,[1,2] the toroidicity-induced Alfvén eigenmodes (TAEs)[5,6] are considered to be ones of the most efficient in transporting fast ions across the plasma minor radius.[7] AEs are also of interest since they can be used for the diagnostics of the plasma density, safety factor, and fast ion parameters.[8,9]

A low-field, low aspect-ratio device such as National Spherical Tokamak Experiment (NSTX)[10] is an excellent testbed for the study of ITER relevant physics of fast particle confinement, an issue of major importance for burning plasmas. The low Alfvén speed in NSTX offers a window to the super-Alfvénic regime expected in ITER. Effects such as the large ion Larmor radii, finite orbit widths, strong plasma shaping, and high thermal and fast-ion betas make this effort a greater challenge.

Low frequency magnetohydrodynamic (MHD) activities in ~100 kHz frequency range on NSTX are often observed and identified as the TAEs driven by beam ions.[11] TAEs in low-aspect ratio tokamaks [or spherical tokamaks (STs)] driven by beam ions have been observed in Small Tight Aspect Ratio Tokamak (START),[12–14] NSTX,[11] and Mega-Amp Spherical Tokamak (MAST) experiments[15] and have been studied theoretically in application for NSTX[16–18] and START.[14] Typically the range of NSTX operational parameters are: toroidal current $I_p = 0.7–1$ MA, vacuum toroidal field at the geometrical axis $B_0 = 0.3–0.5$ T, central electron density $n_{e0} = 1–5\times 10^{19}\,\mathrm{m}^{-3}$, central electron temperature $T_{e0} \lesssim 1$ keV. The plasma is heated with a deuterium beam with power of $P_b = 1.5–3$ MW and an injection energy of usually $E_{b0} = 80$ keV. With such a low magnetic field, the typical ratio of beam ion velocity to the Alfvén velocity satisfies $1 < v_{b0}/v_A \lesssim 3$ (in ITER $1 < v_{\alpha 0}/v_A < 2$, where $v_{\alpha 0}$ is the birth velocity of alpha particle). This parameter is critical in determining the stability of TAEs, i.e., typically when fast ions are super-Alfvénic, their TAE drive is largest. In NSTX, the neutron flux drops by as much as 10%–15% due to TAEs. Sometimes they are accompanied by beam ion losses in high confinement mode (*H* mode), high central safety factor, q_0, plasmas.[11] TAEs often produce bursting behavior of the mode amplitude which sharply increases on a time scale

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TABLE I. Plasma parameters in the NSTX/DIII–D similarity experiments and in the planned ITER experiment.

	R (m)	a (m)	B_0 (T)	β_{i0}(%)	q_0	$\beta_{b,\alpha 0}$(%)	v_{b0}/v_{A0}	$a/\rho_{b,\alpha}$	$n_{\max}$
NSTX	0.77	0.6	0.5	1.1	1.69	4.2	1.85	6	1
DIII–D*	1.63	0.6	0.63	2.6	0.89	4.4	1.5	6	4
ITER–FEAT, beam ions	6.2	2	5.3	2.2	1	1.2	1.4	50	15
Alphas						0.7	1.8	40	15

of ~1 ms. The measured TAE amplitude at the plasma edge is up to $\delta B/B = 3\times 10^{-4}$. Multiple modes are often present in the experiments. Hence, it is clear that TAE instabilities may lead to profound changes in the predicted transport of energetic particles, such as α particles in a burning plasma experiment.

Linear theory of TAE instability predicts that the most unstable TAE mode numbers are determined by finite orbit width effects[19–21] where $k_\perp \Delta_f \sim 1$ holds for the most unstable modes, where $k_\perp$ is the perpendicular wave vector and Δ_f is the radial width of fast ion drift orbit. A plateau in the dependence of the TAE drive on toroidal mode number n is achieved in the range

$$n_{\min} < n < n_{\max}, \tag{1}$$

where $n_{\min} \simeq r n_{\max}/R$, and $n_{\max} \simeq r\omega_{\mathrm{cf}}/q^2 v_A$, r is the minor radius of the plasma magnetic surface, R is the major plasma radius, ω_{cf} is the fast ion cyclotron frequency. This agrees with numerical NOVA code[6] calculations where finite orbit width (FOW) and finite Larmor radius (FLR) effects are included.[22] By changing the parameter $n/z_f \sim k_\perp \Delta_f$ through the variation of the energetic particle charge z_f (note that m scales with nq), it was found that at $n < n_{\min}$ fast ion driven growth rate scales like $\gamma_f \sim n$, whereas at $n > n_{\max}$, it decreases as $\gamma_f \sim n^{-1}$. The latter scaling confirms the results of Ref. 20 but is different from the analytical work of Ref. 21, where $\gamma_f \sim n^{-2}$ was predicted for $n > n_{\max}$.

One can see from Eq. (1) that the range of toroidal mode numbers of the most unstable TAEs is shifted toward high n's in ITER which will have a larger machine size. The machine size scaling of the most unstable mode numbers was verified in the specially designed similarity experiments on NSTX and DIII–D,[23] in which similar plasma parameters were established with the exception of their major radii and the safety factors.[24] In NSTX, TAEs are observed with $n = 1-2$, whereas in DIII–D $n = 2-7$. We analyze these experiments in Sec. II with the help of the ideal MHD code NOVA[6] and hybrid kinetic perturbative code NOVA-K[22,25] in an attempt to recover the observed scaling. This helps validate the application of NOVA and NOVA-K codes to future burning plasma experiments.

In this paper we show that medium-n TAE instability thresholds as follows from the hybrid kinetic-MHD NOVA-K analysis are in agreement with the experimental results. For the low-n TAE instabilities such analysis shows that there is a rather large uncertainty in the experimental plasma parameters upon which the numerical model is based. In NSTX NOVA-K computes damping rate of $n=1$ TAE instability which is several times smaller than the damping required for consistent theoretical predictions. This echoes the problem raised in the recent publication[26] that the $n=1$ damping rate measured in Ohmic-heated Joint European Torus (JET) experiments is higher than the one predicted by the NOVA-K code.

High frequency AEs, at or above the ion cyclotron frequency, driven by the beam ions have been observed in NSTX and predicted to be excited by the positive velocity space gradients.[27–30] Initial observations of high frequency modes and their analysis showed that the instability dispersion is consistent with the dispersion of compressional Alfvén eigenmodes (CAE). Since the CAE can be driven by the gradient in the velocity space one expects that it will result in fast ion energy diffusion. If many high frequency AEs are excited at sufficiently large amplitude they can interact with the bulk ions via the stochastic heating mechanism with the result of perhaps opening a channel for energy transfer from fast ions to plasma ions without heating electrons.[31]

Recently some new features of the high frequency magnetic fluctuation spectrum were observed in NSTX, which suggests new instabilities associated with the shear Alfvén branch, the so-called global Alfvén eigenmodes (GAEs). Similar to the compressional AEs (CAEs), GAEs can be destabilized by beam ions via the Doppler shifted cyclotron resonance. To simulate GAE/CAEs in realistic NSTX plasma conditions we have developed a nonlinear hybrid kinetic-MHD simulation code, HYM,[32,33] which is capable of simulating the mode structure, saturation, and energetic particle transport. In Sec. III the experimental observations, development of analytical theory, and numerical tools for the analysis of these new instabilities are presented.

II. TOROIDICITY-INDUCED ALFVÉN EIGENMODES IN NSTX AND DIII–D

A specially designed similarity experiment between NSTX and DIII–D[24] was performed to verify the theoretical predictions for the scaling of the most unstable toroidal mode numbers with the machine size. In order to do that the plasmas in both tokamaks were created to be similar. However, the major radii and safety factors were considerably different in NSTX and DIII–D. Table I shows the main plasma and fast ion parameters for NSTX and DIII–D and how they are compared to those of ITER, where a is the plasma column minor radius, β is the ratio of the ion pressure to the pressure of the magnetic field, subscripts here and below i, b, α refer to background ions, beam ions, and fusion alpha-particles, respectively, whereas subscript 0 means that the value is taken at the magnetic axis, ρ_j is the Larmor radius of specie j. Critical for the TAE stability is the ratio of fast ion velocity

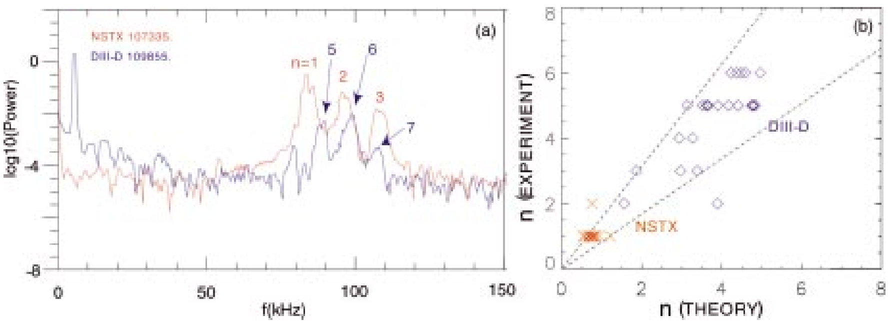

FIG. 1. (Color) Comparison of the Mirnov signal spectrum in the NSTX and DIII–D similarity experiments shows higher observed mode numbers of unstable TAEs in DIII–D in (a). Observations of the most unstable TAE toroidal mode numbers vs those theoretically predicted are summarized in (b).

to the Alfvén velocity, which is similar in all the compared plasmas. Also shown in the table is the estimate of the maximum toroidal mode number expected for each machine [Eq. (1)]. The difference in estimates for NSTX and DIII–D comes primarily from the different values of the safety factor.

As predicted, the observed most unstable mode numbers are higher in DIII–D. This is shown in Fig. 1(a) for two representative discharges in which several modes have been observed: $n=1-3$ in NSTX and $n=4-7$ in DIII–D (Fig. 1 is reproduced from Ref. 24). The instability spectrum peaks in each discharge are separated by the Doppler frequency shift approximately equal to the plasma rotation, which is induced by the unbalanced neutral beam injection. The dependence of the observed toroidal mode numbers versus predicted ones is shown in Fig. 1(b), where the theoretically predicted value was computed according to $n_{\rm theory}=1.3a\,\omega_{\rm cb}/q^2\nu_A$, $q=0.8q_0+0.2q_{95}$, q_{95} is the safety factor of the magnetic surface that encloses 95% of the poloidal magnetic flux, ν_A is the Alfvén velocity calculated with the magnetic field on the plasma axis and with the averaged electron density. In Fig. 1(b) the cone in which the majority of observations fall is also shown. It implies that the TAE toroidal mode numbers are expected within the range $n_{\rm exp}=(1.2\pm0.35)n_{\rm theory}$. Based on these results, it was concluded in Ref. 24 that the most unstable mode numbers is independent on the plasma major radius and scale with q^{-2}. The machine size comes into the scaling through minor radius dependence $n_{\rm theory}\sim a$. Note that the experimental toroidal mode numbers shown in Fig. 1 are for TAEs with nearly constant frequencies; the toroidal mode number with the largest edge magnetic field amplitude is shown. Another difference between NSTX and DIII–D is that the instabilities chirp rapidly in frequency more often in NSTX, whereas this phenomenon is rare in DIII–D.

Application of NOVA to similarity experiments

Linear TAE stability is analyzed with the NOVA and NOVA-K codes. The NOVA-K code implements a perturbative method in which the ideal MHD mode structure of the TAEs is calculated first by the NOVA code.[6] The plasma parameters are modeled with the TRANSP code.[34] Then the mode structure is analyzed with the NOVA-K code,[22,25] in which the mode damping and driving mechanisms are evaluated. Fast-ion drive includes FOW and FLR effects. In the calculations the following damping mechanisms are incorporated: ion and electron Landau damping, radiative damping, and trapped electron collisional damping. Note that it follows from the Table I that the pressure of fast beam ions can be larger than the background pressure. The majority of fast ions do not drive the TAE instability, because the Alfvén velocity is much lower than the beam injection velocity. The nonresonant beam ions are included in the equilibrium and in the adiabatic response of the plasma. The justification for the use of the perturbative approach is supported by the calculations hereafter in which it is obtained that the growth rate due to beam ions is much smaller than the mode frequency $\gamma_b/\omega\lesssim10\%$. TAE growth rates become smaller if the damping is included. The beam ion contribution to the growth rate is relatively large for the TAE type instability, which suggests that a nonperturbative code may be needed to obtain an improved calculation.

Since the interactions of trapped and passing beam ions with TAEs are different, it is important to have the ratio of

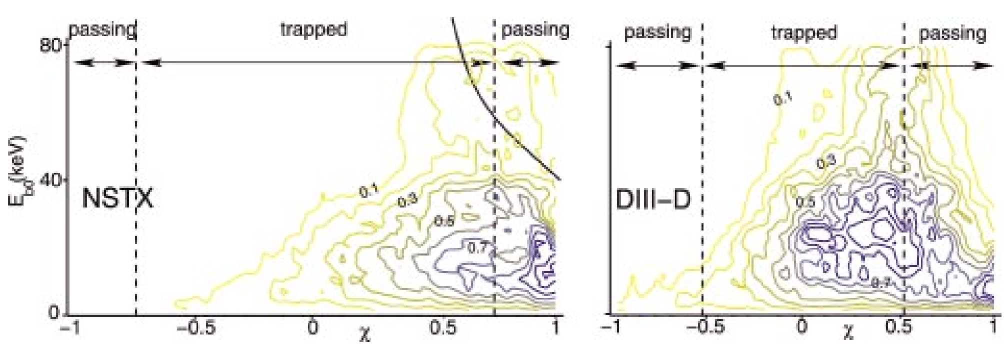

FIG. 2. (Color) Contours of the beam ion distribution function in NSTX (left) and DIII–D (right) in the plane of ion energy and pitch angle taken at the midplane and at the minor radius $r/a=1/2$. The dashed lines show the boundaries between the passing and trapped regions. The solid black line in the distribution function for NSTX shows schematically the GAE-beam ion resonance line $\nu_\parallel=\text{const}$.

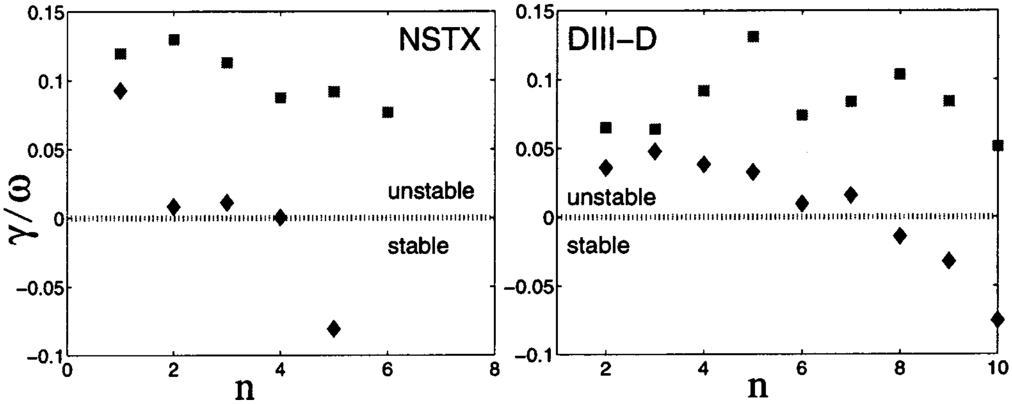

FIG. 3. The results of NOVA/NOVA-K simulations of the TAE growth rates in NSTX and DIII–D (as indicated). The fast ion contribution to the growth rates of the most unstable modes are indicated with a ■ whereas the net growth rate with the damping terms included is shown as ◆.

trapped to passing particle densities similar in both experiments. This was achieved by adjusting the angle of neutral beam injection (NBI).[24] The distribution function of beam ions is calculated by TRANSP. As shown in Fig. 2, the ratio of trapped/passing particles is approximately the same in both experiments. Particles injected at 80 keV into a relatively narrow pitch angle window slow down and scatter in pitch angle, so that the pitch angle width is increasing. The beam distribution function that is obtained from the TRANSP Monte-Carlo code has statistical errors that are too large to be used directly in the NOVA code where derivatives of the distribution function need to be calculated. Thus a numerical distribution function is fitted into the following analytical form:

$$f_b(\psi,\nu,\chi)=n_b(\psi)C\,\frac{e^{-(\chi-\chi_0)^2/\delta\chi(\psi,\nu)^2}}{\nu^3+\nu_*(\psi)^3},$$

where C is a normalization constant (details of the distribution function model are published elsewhere),[35] n_b is the beam ion density, $\chi=\nu_\parallel/\nu$ is its pitch angle, $\nu_\parallel$ is the component of the particle velocity parallel to the magnetic field, χ_0 is the central value of the Gaussian pitch angle distribution function defined by the injection geometry, and ψ is the magnetic field poloidal flux marker of the magnetic surface.

The results of the stability analysis using NOVA and NOVA-K codes are shown in Fig. 3. The TAE unstable range of toroidal mode numbers is shifted to higher n's in DIII–D due to $n_{\text{theory}}\sim q^{-2}$ dependence as predicted by the theory. The beam ion drive term peaks at $n=2$ for NSTX and at $n=5$ in DIII–D. Note that the anisotropic distribution is important for reproducing the observed unstable mode numbers, whereas the NOVA-K calculations with an isotropic distribution function do not predict unstable modes observed in the experiment. The main damping mechanisms in the calculations turned out to be the ion Landau damping and radiative damping. In ITER the ion Landau damping is expected to be the main damping mechanism.[36]

The radiative damping is very sensitive to how close the TAE frequency is to the Alfvén continuum. This is the reason for the jump in the damping rate of the $n=2$ TAE in NSTX as compares to the $n=1$ mode. In the case of the $n=2$ mode the most unstable TAE is positioned close to the lower toroidicity induced continuum, which enhances the damping. This is a typical situation for low-n TAE modes in tokamaks.

From the calculations of the TAE beam ion driven growth rate and the sum of the background damping rates, γ_d, in DIII–D it follows that $\gamma_d/\gamma_b<1$ for low n modes. Since these low-n ($n=2-3$ for the analyzed shot) modes were not observed at the edge it is possible that the calculated damping rates are underestimated or there are extra linear or nonlinear damping terms that are not accounted for in the calculations. Additional nonlinear damping may be due to the dynamical change in the distribution function of fast ions as an effect of TAE saturation.

Can we estimate the uncertainties of the NOVA-K model. The linear analysis of the TAE instabilities is based on both direct measurements of the plasma parameters and the TRANSP analysis code, which in turn includes several theoretical models such as models for plasma transport and equilibrium. This implies that the final results for the plasma profiles and parameters may have experimental and model uncertainties, with quantitative errors that are hard to assess. In addition, the NOVA-K code does not include the continuum damping and the radiative damping is calculated[37] by using an analytical model.[38] Thus for the purpose of validating the predictive capabilities of the NOVA-K code we need to provide a way to estimate the uncertainties of the model. One way to approach this problem is to employ a nonlinear saturation theory of TAEs. Such theory, if applicable, can predict TAE saturation amplitude if the linear growth rate and damping rates are known.[39,40] Therefore, it maybe used to check the consistency of the linear calculations and should provide an information if there is additional TAE damping.

Let us introduce *ad hoc* additional damping term γ_{dx} and estimate it based on the following arguments. (i) Numerical simulations show that the TAE amplitudes at the plasma edge is close, within an order of magnitude, to the peak mode amplitude in the plasma. This can be seen from Fig. 14 of Ref. 24 for both devices. (ii) Low amplitudes $\delta B/B\sim0.5\times10^{-5}$ at the plasma edge were measured in both devices for the largest amplitude TAEs in the discharges of interest. The amplitudes of other TAEs with lower signal can be es-

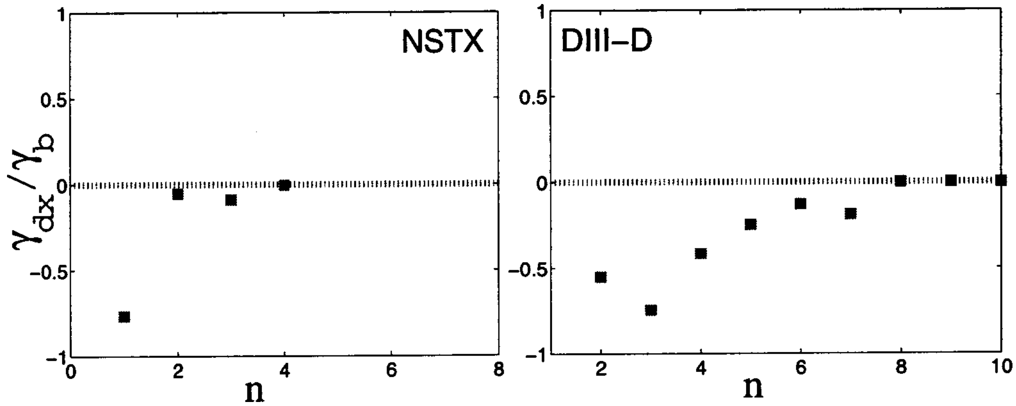

FIG. 4. Model of extra damping required to make a prediction of near threshold TAE excitation.

timated from Fig. 1. (iii) TAE amplitudes are measured outside the plasma, which requires the extension of the solution into the vacuum region. To estimate the amplitude in the vacuum it is sufficient to apply the cylindrical approximation, which results in the poloidal harmonic amplitude attenuation in the vacuum from the plasma edge value $\sim(a/r_{\rm det})^m$, where $r_{\rm det}$ is the minor radius of the Mirnov coil location. In DIII–D $r_{\rm det}/a\simeq 1.25$, whereas in NSTX $r_{\rm det}/a\simeq 1.7$. In DIII–D the highest poloidal mode harmonic for each n, $m\simeq nq_a$, decreases less than two orders of magnitudes (for n's of interest) from its value at the plasma edge, where q_a is the safety factor at the edge ($q_a=4$ in DIII–D and $q_a\simeq 8$ in NSTX were measured). In NSTX, due to low aspect ratio and strong plasma shaping, a strong poloidal harmonic coupling even for low-m harmonics are present at the edge as the calculations show. Typically in NSTX the amplitude attenuates within an order of magnitude from its plasma edge value. (iv) In the NOVA-K, a nonlinear mode saturation theory is included which predicts the saturated amplitude of a steady state mode.[39,40] It follows from this theory that the saturated amplitude inside the plasma for the modes of interest would be considerably larger, $\delta B/B \gtrsim 10^{-3}$, than the measured amplitudes unless the modes being excited are close to the marginal stability condition. (v) In experiments there were observations of nearly steady TAE amplitude evolution in both NSTX and DIII–D.

Thus, if we assume that the nonlinear saturation theory is applicable we conclude that TAEs with the amplitude observed in the experiments should be near threshold. Therefore, the *ad hoc* extra damping term γ_{dx} should satisfy the near threshold excitation condition $\mathcal{R}\equiv-(\gamma_d+\gamma_{dx})/\gamma_b\simeq 1$, i.e., $\gamma_{dx}=-\mathcal{R}\gamma_b-\gamma_d$. Note that for the measured amplitudes NOVA-K calculations predict $(1-\mathcal{R})\simeq 10^{-2}$–$10^{-3}$. As simulations show, γ_{dx} is less than 20% of the drive for medium n's, which is illustrated in Fig. 4. However low-n modes are very sensitive to the details of the q profile. For example, in the case of DIII–D $n=2$ Alfvén continuum gap is open, which means that there is no continuum damping. A small change in q or density profile may increase the damping of low-n modes and reduce the value of γ_{dx}.

For the medium n numbers, the NOVA-K code does fairly well in predicting the threshold of TAE instability in both NSTX and DIII–D. Since in ITER high-n modes are expected, this helps validate the study of burning plasmas in ITER using the NOVA/NOVA-K codes, such as the study done recently (Ref. 36). This work shows that TAEs are expected to be weakly (marginally) unstable in nominal ITER plasmas with a monotonic q profile, ion central temperatures of $T_{i0}=20\,$keV and fusion alpha central beta $\beta_{\alpha 0}=0.7\%$ when only α particles are included in the drive. The plasmas with $T_{i0}>20\,$keV are predicted to be unstable in ITER with respect to TAEs due to fusion α's. In addition there may be 1 MeV tangentially injected beams which are being planned to drive plasma current. Beam ions are super-Alfvénic and can drive TAE with the growth rates that are comparable to α-particle growth rates. In Ref. 36 it is shown that including both NBI and α-particle drives TAEs are expected to be unstable over a much wider toroidal mode number range than for the α-particle drive alone. Lowering NBI energy to 0.5 MeV reduces the TAE drive, and it appears to still allow the 0.5 MeV energy beam to penetrate the ITER plasma.

III. HIGH SUBION CYCLOTRON FREQUENCY MODES IN NSTX

A. Observations of high frequency modes in NSTX

A new type of MHD activity with frequencies below the ion cyclotron frequency have been observed in NSTX.[27] It correlates with the NBI and was initially identified as compressional Alfvén eigenmodes based on the observed spectrum and theoretical and numerical analysis.[27–29] It was shown that CAEs can be excited by the super-Alfvénic beam ions with the distribution function which has a positive gradient in $v_\perp$. The beam ions are in the Doppler shifted cyclotron resonance with the CAEs.

Further study showed that high frequency Mirnov signal spectra in NSTX sometimes contain signatures of a different instability.[30] Typical for these new instabilities is that the frequency spectrum peaks intersect each other as plasma parameters evolve as shown in Fig. 5 (it is published in Ref. 30). Recall that the CAE spectrum peaks evolve parallel to each other.[27,29] It is known that the shear Alfvén modes with different combinations of (m,n) can have frequencies with different time dependence. This is due to the shear Alfvén dispersion $\omega=k_\parallel v_A$, $k_\parallel\simeq(m-nq)/qR$ and the fact that the q

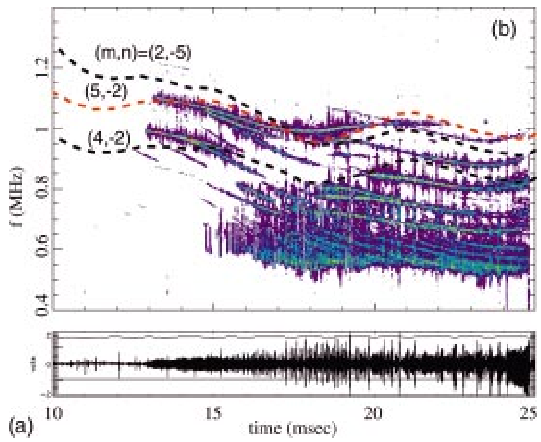

FIG. 5. (Color) Time evolution of the Mirnov signal (a), and its frequency spectrum (b) in NSTX shot No. 108 236. The mode frequency time dependences of three GAEs are shown as dashed lines.

profile and density are evolving during the discharge. It was suggested that the global shear Alfvén eigenmodes are responsible for such MHD activity.

Global Alfvén eigenmodes are formed below the frequency minimum of the Alfvén continuum with the frequency given approximately by $\omega \simeq \omega_{A\,\mathrm{min}} = [k_{\parallel}(r) v_A(r)]_{\mathrm{min}}$.[41] The subscript min means that the value is taken at the minimum $\omega_{A\,\mathrm{min}}$, $k_{\parallel}$ is the parallel wavevector. The GAE eigenfrequency is slightly below $\omega_{A\,\mathrm{min}}$, and depends on the q and density profiles. GAEs are localized radially near $\omega_{A\,\mathrm{min}}$ and are dominated by one poloidal harmonic m. With a typical flat q profiles, the Alfvén continuum has a minimum at the plasma center, so that $\omega \simeq \pm v_{A0}(m/q_0 - n)/R_{\mathrm{ax}}$, where R_{ax} is the major radius of the magnetic axis.

As an example we show the GAE frequencies for different sets of mode numbers in Fig. 5(b). The time evolution of the plotted peaks is qualitatively similar to the observed ones. Due to the uncertainty in the measurements of m and q it is difficult to make one to one comparison.

The $n=3$ Alfvén continuum and the mode structure of a GAE in NSTX calculated by the NOVA code are shown in Fig. 6. The frequencies are normalized to the Alfvén frequency $\omega_{A0} = v_{A0}/q_a R_0$, where R_0 is the major radius of the plasma geometrical center. Note that as expected the GAE frequency is very close to the minimum of the Alfvén continuum $\omega \simeq \omega_{A\,\mathrm{min}}$.

B. Theory of GAE instability

GAE instabilities are driven by NBI ions and have been recently studied in detail.[30] This theory is outlined here. Due to the tangential NBI injection in NSTX, the distribution function of beam ions has a positive gradient in the velocity space. It provides an energy source for the instability and forms a "bump on tail" in the $v_{\perp}$ direction. In Fig. 2(a) the black solid line shows schematically the location of beam ions, which are in resonance with GAE modes. Particles resonate with GAEs via the Doppler shifted cyclotron resonance $\omega - \omega_{\mathrm{cb}} - k_{\parallel} v_{\parallel} \simeq 0$. The theory predicts that GAEs are unstable if $2 < (\omega/\omega_{\mathrm{cb}})(v_{\perp b0}/v_A)(k_{\perp}/k_{\parallel}) < 4$, where v_{b0} is the beam ion injection velocity, and the bump-on-tail width in the $v_{\perp}$ direction satisfies $\delta v_{\perp b} < 2 v_A \omega_{\mathrm{cb}}/\omega$. The main GAE damping is the continuum damping.[42] For high-m GAEs it is small[30]

$$\frac{\gamma_d}{\omega} \sim \left(\frac{r_2}{r_s}\right)^{2m+\delta},$$

where r_2 is the mode location, r_s is the minor radii of the $m+1$ continuum branch where the GAE has a singularity, and $\delta = O(1)$. r_s at the singularity is always larger than r_2 so that $r_2/r_s < 1$ and $\gamma_d/\omega \ll 1$.

C. HYM nonlinear hybrid code for GAE study

Three-dimensional (3D) hybrid simulations using the HYM code[32,33] has been employed to study the excitation of AEs by energetic ions in NSTX. The HYM code is a nonlinear, kinetic-MHD simulation code in toroidal geometry. In the numerical model the δf particle simulation method is employed to describe the beam ions with their full orbits. A one-fluid resistive MHD model is used to represent the background plasma. These two plasma components are coupled using a current coupling scheme. It is assumed that the fast ion pressure can be comparable to the pressure of the thermal plasma, but the beam ions have low density $n_b \ll n_e$. The

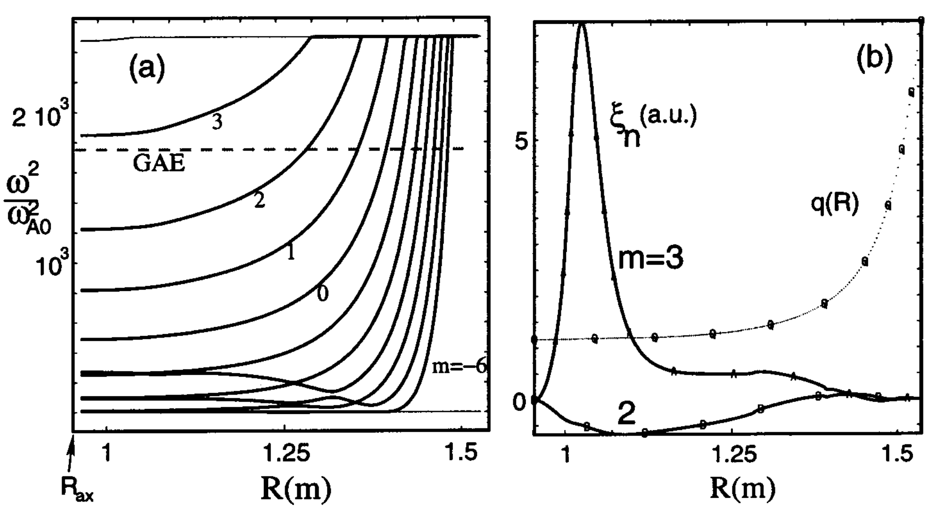

FIG. 6. Alfvén continuum (a) and a GAE mode structure (b) shown for $n = -3$ in a NSTX plasma. The frequency of GAE is shown as the dashed line in the Alfvén continuum. Only two dominant poloidal harmonics are shown for GAE.

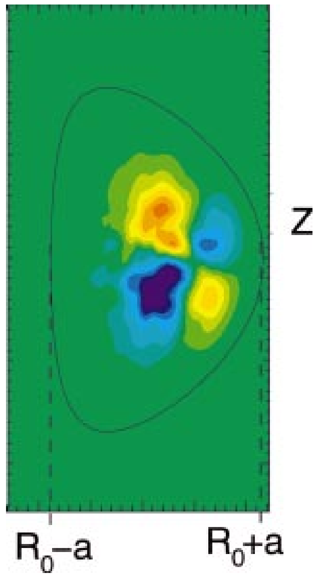

FIG. 7. (Color) Contours of the fluid pressure perturbation of $n=-4$, $m=2$ GAE mode plotted in the poloidal R,Z plane. Shown also is the last closed magnetic surface of the NSTX plasma. The red color regions indicate the high perturbed pressure, whereas dark blue ones correspond to the negative value of the pressure perturbation.

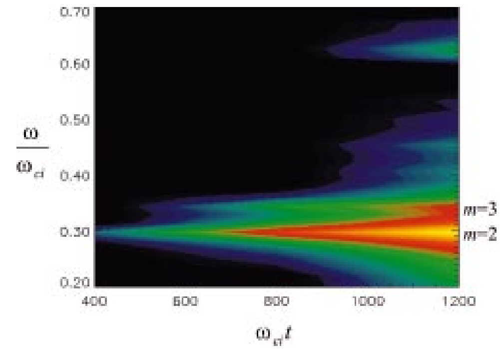

FIG. 8. (Color) The amplitude evolution of the spectrum of unstable modes with $n=-4$ as calculated by the HYM code. Lower frequency peaks of the spectrum correspond to GAEs with $m=2$ and 3.

effects of the beam ion toroidal and poloidal currents are included nonperturbatively to account for the anisotropic fast ion pressure tensor and to calculate self-consistent equilibria, which serves as an initial condition for the 3D simulations. The HYM code includes finite viscosity, which allows for the effective damping of plasma oscillations. In particular this introduces the continuum damping discussed earlier if the radial grid step is small enough.

The HYM simulations for typical NSTX parameters confirm that for large injection velocities of beam ions, $v_{b0}>3v_A$, and strong anisotropy in the pitch-angle distribution, there are unstable Alfvén modes.[43] It is found that the most unstable modes for low toroidal mode numbers, $2<|n|<7$, are GAE modes.[41] These modes are found to be localized near the magnetic axis, and have large $k_\parallel$ (with $nm<0$), so that $\omega\sim|k_\parallel v_\parallel|\sim\omega_{cb}/2$. Note that in the HYM model the background ions are treated as fluid, which is justified because the GAE frequency is well below the ion cyclotron frequency, and the thermal ion cyclotron and transit damping can be neglected. Indeed only ions which satisfy the condition $\omega-\omega_{cb}-k_\parallel v_\parallel\simeq 0$ and $\omega-k_\parallel v_\parallel\simeq 0$ can be in resonance. Therefore, both types of resonant thermal ions should have velocities $v_i\simeq v_{b\parallel}\simeq v_A$, which means that an exponentially small number of ions will contribute to an exponentially small damping $\sim\exp(-2/\beta_{i0})$, where typically $\beta_{i0}\lesssim 5\%$. The perturbed plasma pressure in the poloidal cross section of a NSTX plasma due to a GAE with $n=-4$ and $m=2$ is shown in Fig. 7. The poloidal velocity has a vortex-like structure, which is characteristic for shear Alfvén waves. However, in NSTX these modes have a significant compressional component, $\delta B_\parallel\sim\delta B_\perp/3$, due to strong coupling to the compressional Alfvén wave. Linearized simulations for different n's show that for most unstable modes, a condition $m-n\simeq 6$ is satisfied (i.e., approximately same $k_\parallel$). Several Alfvén modes (with different dominant m) can be excited for each toroidal mode number n as illustrated in Fig. 8. In the HYM simulations for $n_b/n_p\simeq 3\%$, the growth rates of the unstable GAEs are found to be of the order $\gamma/\omega_{cb}=0.002-0.01$ with frequencies $\omega/\omega_{cb}=0.3-0.5$, where ω_{cb} is evaluated at the magnetic axis. For the mode shown in Figs. 7 and 8, the growth rate was $\gamma/\omega_{cb}=0.005$ and, as in the case of NOVA calculations, its frequency is below the central Alfvén continuum value $\omega/\omega_{cb}=0.3\lesssim\omega_{A\,\min}/\omega_{cb}=(2+4q_0)v/q_0R_0\omega_{cb}=0.33$.

It is difficult to make a direct comparison between NOVA and HYM results, which are build on different physical models. The qualitative comparison of the GAE frequencies shows that the results of both codes are consistent and, as it is shown, agree with the theory.[41] In both codes the mode structure of GAEs is similar to the cylindrical GAEs. It is represented by one dominant poloidal harmonic, which is peaked near the center.

IV. CONCLUSIONS

Perturbative linear numerical tools such as NOVA/NOVA-K hybrid kinetic codes predict TAE instabilities in the NSTX and DIII–D similarity experiment in which such instabilities have been observed. A detail study shows that the unstable low-n TAEs are reproduced for NSTX, whereas for DIII–D the modeling predicts $n=2-7$ TAEs to be unstable. It is shown that the model underestimates the damping at the low end of the unstable n-mode number range so that $n=2-3$ modes in DIII–D are predicted to be unstable while in the experiments they were seen stable. For the medium-n mode number range the uncertainty of the damping rate calculations seems to be reasonable within $<20\%$. It also follows from the comparison of numerical modeling with experiments that the toroidal number of the most unstable mode does not depend on the plasma major radius and is proportional to a/q^2. The numerical analysis helps validate the NOVA/NOVA-K predictions that TAEs will be unstable in ITER if driven by fusion alphas in a plasma with $T_{i0}>20$ keV. Calculations also predict that TAEs will be unstable in ITER if the NBI drive is included even for $T_{i0}<20$ keV.

New features of the subion cyclotron frequency spectrum in NSTX, which reveals frequency peaks intersecting in time, have been identified as due to the excitation of global shear Alfvén eigenmode instabilities driven by NBI ions. GAEs interact with ions via the Doppler shifted cyclotron resonance and are driven by the positive gradient in the $v_\perp$ velocity space direction of the confined ion beam. Modeling with both hybrid HYM and NOVA codes agree with the analytical theory on GAE mode structure and dispersion.

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[1]W. W. Heidbrink and G. J. Sadler, Nucl. Fusion **34**, 535 (1994).
[2]K.-L. Wong, Plasma Phys. Controlled Fusion **41**, R1 (1999).
[3]D. J. Campbell, Phys. Plasmas **8**, 2041 (2001).
[4]ITER Physics Basis Editors, Nucl. Fusion **39**, 2471 (1999).
[5]C. Z. Cheng, L. Chen, and M. S. Chance, Ann. Phys. (N.Y.) **161**, 21 (1985).
[6]C. Z. Cheng and M. S. Chance, Phys. Fluids **29**, 3695 (1986).
[7]W. W. Heidbrink, Plasma Phys. Controlled Fusion **37**, 937 (1995).
[8]J. P. Goedbloed, H. A. Holties, S. Poedts, G. T. A. Huysmans, and W. Kerner, Plasma Phys. Controlled Fusion **35**, B277 (1993).
[9]G. J. Kramer, C. Z. Cheng, Y. Kusama, R. Nazikian, S. Takeji, and K. Tobita, Nucl. Fusion **41**, 1135 (2001).
[10]S. M. Kaye, M. Ono, Y.-K. M. Peng, and D. B. Batchelor, Fusion Technol. **36**, 16 (1999).
[11]E. D. Fredrickson, C. Z. Cheng, D. Darrow, G.-Y. Fu, N. N. Gorelenkov, G. J. Kramer, S. S. Meadley, J. Menard, L. Roquemore, D. Stutman, and R. B. White, Phys. Plasmas **10**, 2852 (2003).
[12]S. E. Sharapov, M. Gryaznevich, and the START Team, "Alfven eigenmodes driven by super-Alfvenic NBI in tight aspect ratio tokamak," in Proceedings of International Workshop on Spherical Torus and US-Japan Workshop for Low Aspect Ratio Tokamaks, 4–6 December, 1996, edited by A. Sykes, Culham, UK, Vol. 1, pp. 177–192.
[13]M. Gryaznevich, K. Morel, S. E. Sharapov *et al.*, "Observations of MHD modes in the Alfven frequency range in beam-heated START discharges," in Proceedings of 5th IAEA Technical Commitee Meeting in "Alpha Particles in Fusion Research," 8–11 September 1997, JET Joint Undertaking, Abingdon, UK, edited by J. Jacquinot, B. E. Keens, and G. Sadler, pp. 53–56.
[14]K. G. McClements, M. P. Gryaznevich, S. E. Sharapov, R. J. Akers, L. C. Appel, G. F. Counsell, C. M. Roach, and R. Majeski, Plasma Phys. Controlled Fusion **41**, 661 (1999).
[15]B. Lloyd, J.-W. Ahn, R. J. Akers *et al.*, Nucl. Fusion **43**, 1665 (2003).
[16]N. N. Gorelenkov, C. Z. Cheng, Y. Chen, G. Y. Fu, R. Nazikian, R. B. White, M. V. Gorelenkova, and H. L. Berk, *Proceedings of the 17th International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Yokohama, Japan, 1998* (International Atomic Energy, Vienna, Austria), paper IAEA-F1-CN-69/THP2/21.
[17]N. N. Gorelenkov, C. Z. Cheng, G. Y. Fu, S. Kaye, R. B. White, and M. V. Gorelenkova, Phys. Plasmas **7**, 1433 (2000).
[18]C. Z. Cheng, N. N. Gorelenkov, E. Fredrickson *et al.*, *Proceedings of the 19th International Conference on Fusion Energy, Lyon, France, 2002* (International Atomic Energy, Vienna, Austria), paper IAEA-CN-94/TH/7-1Rb.
[19]H. L. Berk, B. N. Breizman, and H. Ye, Phys. Lett. A **162**, 475 (1992).
[20]G. Y. Fu and C. Z. Cheng, Phys. Fluids B **4**, 3722 (1992).
[21]B. N. Breizman and S. E. Sharapov, Plasma Phys. Controlled Fusion **37**, 1057 (1995).
[22]N. N. Gorelenkov, C. Z. Cheng, and G. Y. Fu, Phys. Plasmas **6**, 2802 (1999).
[23]J. L. Luxon, Nucl. Fusion **42**, 614 (2002).
[24]W. W. Heidbrink, E. D. Fredrickson, N. N. Gorelenkov, A. W. Hyatt, G. J. Kramer, and Y. Luo, Plasma Phys. Controlled Fusion **45**, 983 (2003).
[25]C. Z. Cheng, Phys. Rep. **1**, 211 (1992).
[26]D. Testa, G.-Y. Fu, A. Jaun, A. Fasoli, and O. Sauter, Nucl. Fusion **43**, 479 (2003).
[27]E. Fredrickson, N. N. Gorelenkov, C. Z. Cheng *et al.*, Phys. Rev. Lett. **87**, 145001 (2001).
[28]E. Fredrickson, N. N. Gorelenkov, C. Z. Cheng *et al.*, Phys. Plasmas **9**, 2069 (2002).
[29]N. N. Gorelenkov, C. Z. Cheng, E. Fredrickson, E. Belova, D. Gates, S. M. Kaye, G. J. Kramer, R. Nazikian, and R. B. White, Nucl. Fusion **42**, 977 (2002).
[30]N. N. Gorelenkov, E. Fredrickson, E. Belova, C. Z. Cheng, D. Gates, S. Kaye, and R. White, Nucl. Fusion **43**, 228 (2003).
[31]D. A. Gates, N. N. Gorelenkov, and R. B. White, Phys. Rev. Lett. **87**, 205003 (2001).
[32]E. V. Belova, S. C. Jardin, H. Ji, M. Yamada, and R. Kulsrud, Phys. Plasmas **7**, 4996 (2000).
[33]E. V. Belova, N. N. Gorelenkov, and C. Z. Cheng, Phys. Plasmas **10**, 3240 (2003).
[34]R. V. Budny, D. C. Mccune, M. H. Redi, J. Schivell, and R. M. Wieland, Phys. Plasmas **3**, 4583 (1996).
[35]J. P. Graves, O. Sauter, and N. N. Gorelenkov, Phys. Plasmas **10**, 1034 (2003).
[36]N. N. Gorelenkov, H. L. Berk, C. Z. Cheng, G.-Y. Fu, W. W. Heidbrink, G. J. Kramer, D. Meade, and R. Nazikian, Nucl. Fusion **43**, 594 (2003).
[37]G. Y. Fu, C. Z. Cheng, R. Budny, Z. Chang, D. S. Darrow, E. D. Fredrickson, E. Mazzucato, R. Nazikian, K. L. Wong, and S. Zweben, Phys. Plasmas **3**, 4036 (1996).
[38]H. L. Berk, R. R. Mett, and D. M. Lindberg, Phys. Fluids B **5**, 3969 (1993).
[39]H. L. Berk, B. N. Breizman, and M. S. Pekker, Plasma Phys. Rep. **23**, 778 (1997).
[40]N. N. Gorelenkov, Y. Chen, R. B. White, and H. L. Berk, Phys. Plasmas **6**, 629 (1999).
[41]K. Appert, R. Gruber, F. Troyuon, and J. Vaclavic, Plasma Phys. **24**, 1147 (1982).
[42]J. W. Van Dam, C. Z. Cheng, and G. Y. Fu, Fusion Technol. **18**, 461 (1990).
[43]E. V. Belova, N. N. Gorelenkov, C. Z. Cheng, R. C. Davidson, and E. D. Fredrickson, "Numerical study of instabilities driven by the energetic neutral beam ions in NSTX," paper presented by the 30th European Physical Society Conference on Controlled Fusion and Plasma Physics, St. Petersburg, Russia, 7–11 July 2003, P-3.102.