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DISCLAIMER

COMPARISON OF NEUTRON RESONANCE SPACINGS WITH MICROSCOPIC THEORY FOR SPHERICAL NUCLEI*

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Abstract

The nuclear level spacings determined from neutron resonance experiments for nuclei with $20 \leqslant A \leqslant 148$ and $181 \leqslant A \leqslant 209$ are compared with spacings calculated for spherical nuclei with a microscopic theory which includes the nuclear pairing interaction. Single particle levels of Seeger *et al.* and Nilsson *et al.* are used in the calculations. The gross features of the experimental data due to nuclear shells are reproduced with the microscopic theory. In addition, the absolute agreement between experiment and theory is reasonable (67% of the 151 cases examined agree to within a factor of 2) in view of uncertainties in the experimental data, the theoretical single particle levels and the pairing strength. Values of the spin cutoff parameter $\sigma^2(E)$, calculated with a microscopic theory, are included also for several even-even nuclei and discussed in terms of nuclear shells.

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1. Introduction

Neutron resonance data are the most extensive source of information on nuclear level densities. In this type of experiment the nuclear energy levels are observed at an energy just exceeding the neutron binding energy, and the number of levels is obtained by counting the resonances in a particular neutron energy interval. The levels excited by neutron-resonance spectroscopy have narrowly selected values of angular momentum I and parity π quantum numbers.

Level spacing information has been obtained from slow-neutron-resonance (s-wave) data for nuclei with A values across the whole periodic table. Hence, it is possible to investigate trends and systematics of the nuclear level density as a function of A. Although the technique of neutron resonance spectroscopy is an important one in terms of level density information, it suffers from a number of sources of experimental error. First of all, the strengths of resonances of a particular spin and parity vary greatly from one resonance to another and one cannot be certain that all the s-wave resonances have been detected. Secondly, if positive means of identification have not been used, one cannot be certain that some of the resonances detected are not of p-wave character. Fortunately, these two errors are to a certain extent compensatory. Finite instrumental resolution may lead also to an underestimate of the number of resonances in cases where close-lying resonances are unresolved. For the present comparison of experimental data with theory, we have used three compilations[1-3]) of the nuclear level spacings from neutron resonance data. These level spacings, based on different analyses of the neutron resonance data, are tabulated in Tables 1-4 according to nuclear type.

A number of authors[2-10]) have analyzed the neutron resonance data with a Bethe type formula containing various phenomenological modifications to account for nuclear pairing and shells. Although several of these comparisons were reasonably successful, the degree of their success depends in large part on adjustable parameters. The purpose of this article is to make a comparison of the nuclear level spacings determined from neutron resonance experiments with spacings calculated for a large number of nuclei with a microscopic theory of interacting Fermions. Realistic sets of single particle levels for spherical nuclei are used in the calculations[11,12]). The use of single particle levels obtained from a shell-model calculation in the evaluation of nuclear state densities has been discussed by several authors[13-30]). However, there have been only a limited number of comparisons[17-20,30]) of experimental level densities with theoretical results which are based on realistic sets of single particle levels including the nuclear pairing interaction. In the present paper the A (mass number) dependence of the level density is investigated for nuclei with $20 \leqslant A \leqslant 148$ and $181 \leqslant A \leqslant 209$ at an essentially constant excitation energy, namely the neutron binding energy. In a previous paper[30]) the excitation energy dependence of the level density was investigated for several nuclei near A = 60.

2. Calculational Procedure

The level density of a spherical nucleus for a particular value of the angular momentum I is given by

$$\rho(E,I) = \omega(E,M=I) - \omega(E,M=I+1) \quad (1)$$

where M is the projection of I on a space-fixed axis and $\omega(E,M)$ is the density of states of a particular projection M. Since many independent degrees of freedom contribute to M, the density of states $\omega(E,M)$ is expected to approach a normal distribution,

$$\omega(E,M) = [\omega(E)/(2\pi\sigma^2(E))^{1/2}] \exp[-M^2/2\sigma^2(E)] \quad (2)$$

where $\omega(E)$ is the total state density and $\sigma^2(E)$ is defined as a spin cutoff factor. From eqs. (1) and (2), one obtains to a good approximation the spin dependent level density,

$$\rho(E,I) = [(2I+1)/(8\pi)^{1/2}\sigma^3(E)]\,\omega(E) \exp[-I(I+1)/2\sigma^2(E)] \quad . \quad (3)$$

The state density $\omega(E)$ is calculated with realistic sets of single particle levels[11,12]) by the grand partition function method for a system of interacting Fermions. The Hamiltonian describing a system of paired Fermions has been discussed by various authors[14,17,19,25,29,30]). Such a Hamiltonian is approximately diagonalized by means of a transformation where the quasi-particle excitations are considered to be independent Fermions with energy[31]

$$E_k = \left[(\varepsilon_k - \lambda)^2 + \Delta^2\right]^{1/2} \tag{4}$$

where λ is the chemical potential, ε_k the single particle energy and Δ the gap parameter which gives a measure of the pairing correlation. For a paired system the logarithm of the grand partition function for one type of Fermion is given by[14])

$$\ln Z(\alpha,\beta) = -\beta \sum_k (\varepsilon_k - \lambda - E_k) + 2 \sum_k \ln\left[1 + \exp(-\beta E_k)\right] - \beta \frac{\Delta^2}{G} \tag{5}$$

where G is the pairing strength and β is related to the temperature of the system, $\beta = 1/T$. The summation is over doubly degenerate orbitals designated by k. Equation (5) is valid only if the quantities Δ, λ and β satisfy the gap equation,

$$\frac{2}{G} = \sum_k \frac{1}{E_k} \tanh\left(\frac{1}{2}\beta E_k\right) \quad . \tag{6}$$

The statistical properties of a nucleus defined in terms of its neutron and proton numbers N and Z, respectively, and the total energy are given in the grand partition function $Z(\alpha_n, \alpha_p, \beta)$. The quantities α_n, α_p and β are Lagrangian multipliers associated with the particle numbers and energy. In the framework of statistical mechanics the state density which is the inverse Laplace transform of the grand partition function, is given by

$$\omega(N,Z,E) = \left(\frac{1}{2\pi i}\right)^3 \int_{-i\infty}^{i\infty} d\alpha_n \int_{-i\infty}^{i\infty} d\alpha_p \int_{-i\infty}^{i\infty} d\beta \; Z(\alpha_n,\alpha_p,\beta) \; \exp\,(-\alpha_n N - \alpha_p Z + \beta E) \quad . \quad (7)$$

This integral is evaluated by the method outlined previously[30].

The spin cutoff factor $\sigma^2(E)$ is calculated also with the microscopic theory and is given by[17,30],

$$\sigma^2(E) = \frac{1}{2}\left\{\sum_k m_{pk}^2 \; \mathrm{sech}^2\left(\frac{1}{2}\beta E_{pk}\right) + \sum_k m_{nk}^2 \; \mathrm{sech}^2\left(\frac{1}{2}\beta E_{nk}\right)\right\} \quad (8)$$

The additional quantity needed to solve eqs. (7) and (8) is the ground state gap parameter Δ which is used to fix the pairing strength G. In the present calculations we have used the functional forms of Δ_p and Δ_n given in figs. 1 and 2. These relations are similar to the smooth functions reported in the literature, except for a slight reduction of 15% in their magnitude.

For the odd particle system, the statistical functions were calculated for the adjacent even-even nucleus and then the energy scale was shifted by an energy equivalent to that required to produce one quasiparticle.

3. Results and Discussion

The results of the theoretical calculations are summarized in Tables 1 to 4. In these tables both the experimental data and the theoretical results are presented in terms of level spacings. The level spacing D in eV for one parity is defined in relation to eq. (3) by

$$D(\mathrm{eV}) = 2\times10^6/[\rho(E,I_0-\tfrac{1}{2}) + \rho(E,I_0+\tfrac{1}{2})] \qquad (9)$$

where each level density is defined in terms of the number of levels per MeV of a particular angular momentum and both parities and I_0 refers to the target spin. When the target spin is non-zero, s-wave neutron capture excites levels of a single parity and two values of I. For even-even targets, levels of a single angular momentum and parity, $I = 1/2^+$, are excited. The factor of 2 in eq. (9) is a result of the assumption that the number of levels of each parity are equal. Although this assumption is not expected to be valid at low excitation energies, present theoretical calculations indicate that the assumption is reasonably good at an energy corresponding to the neutron binding energy[19,23]).

The ratios of $D_{theo.}/D_{exp.}$ for 151 nuclei are plotted in fig. 3. The agreement between experiment and theory is within a factor of 2 for 67% of the cases studied. In the preparations of fig. 3, the most favorable single value of $D_{exp.}$ was used. For some nuclei, the average value of $D_{exp.}$ from the three analyses of the experimental data gives a better agreement with $D_{theo.}$. For the values of the proton and neutron pairing gaps given in figs. 1 and 2, respectively, the values of $D_{theo.}/D_{exp.}$ scatter about the value of unity.

Although this outcome is interesting, it is too early to attach particular significance to it because uncertainties in the various parameters discussed below may shift the absolute values of the ratio up or down. The important results from the comparison of the microscopic theory with experimental data are, (1) that no systematic structure due to nuclear shells is evident in this ratio as a function of A and (2) that more than 2/3 of the points are within a factor of 2 of some average value of $D_{theo.}/D_{exp.}$.

The average value of $D_{theo.}/D_{exp.}$ depends on the magnitude of the pairing gaps. Increasing the pairing gap by 15% increases the value of $D_{theo.}$ (and, hence, also $D_{theo.}/D_{exp.}$) by a factor of approximately 3. Hence, deviations in the values of the pairing gaps from the smooth trends of figs. 1 and 2 will cause fluctuations in the ratio $D_{theo.}/D_{exp.}$. In addition, we have assumed that the residual interaction matrix elements $G_{kk'}$ are equal to a constant G which is called the pairing strength (see eq. 6). There is, however, some evidence that this is a rather good approximation[33]).

It is not possible to predict the error in $D_{theo.}$ associated with a particular set of single particle levels. Some estimate of the uncertainty is obtained by comparing results obtained with different sets of single particle levels. In Table 5 such a comparison is made for several nuclei for single particle levels of Seeger et al.[11]) and Nilsson et al.[12]). The overall agreement, for all nuclei, between experiment and theory for each set of single particle levels is comparable. However, the single particle levels of Nilsson et al. give a much superior agreement with experiment for nuclei very near ^{208}Pb. As shown in fig. 3, the values of $D_{theo.}$ for nuclei with A = 206 - 208 are too small as calculated with the levels of Seeger and Perisho.

This is associated with the 82 proton and 126 neutron shell gaps being smaller for the Seeger single particle levels than the Nilsson *et al.* single particle levels.

The theoretical values of the level spacings which are reported here were calculated with single particle levels for spherical nuclei. Although the well known statically deformed nuclei in the lanthanide and actinide regions of the periodic table are not included in this survey, some nuclei are included which may have small deformations and others which are in transition regions between spherical and deformed nuclei. This subject of the effect of deformation on the level density is discussed in the following paper[34]). For the nuclei included here, no enhancement in the level density due to either rotational or vibrational levels is assumed. The general agreement between the experimental and theoretical spacings for the included nuclei as a function of A may be interpreted to mean that no enhancement due to collective excitation is justified. This conclusion is, however, not warranted since uncertainties in the pairing energy and single particle density mentioned earlier allow for the possibility of some contribution to the level density due to collective excitations. If such an enhancement exists, the present results indicate that it is rather independent of A.

Values of $\sigma(E)$, the square root of the spin cutoff factor, for several even-even nuclei are listed in Table 1 for excitation energies just exceeding the neutron binding energy. In fig. 4, values of $\sigma^2(E)$ for a number of even-even nuclei are plotted at a fixed excitation energy of 7 MeV. The values of $\sigma^2(E)$ do not increase smoothly with A as expected on the basis of the macroscopic theory with a rigid-body moment of inertia. Instead the values of $\sigma^2(E)$ show structure reflecting the angular momenta of the shell model orbitals near the Fermi energy. The total magnitude of $\sigma^2(E)$ is made up of a sum of a neutron and proton component. The trends in the values of $\sigma^2(E)$ with A calculated with the microscopic theory are in general agreement with experimental information.

In summary, the values of the level spacings and spin cutoff factors calculated with the microscopic theory including nuclear pairing for realistic sets of single particle levels are in good agreement with experiment. In particular, gross features of the experimental data due to nuclear shells are reproduced.

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FOOTNOTES AND REFERENCES

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Table 1. Experimental and theoretical level spacings of even Z-even N spherical nuclei.

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV[d]	σ
^{43}Ca	$\frac{7}{2}$ -	^{44}Ca	3-,4-	11.16		2900	3300	3850	4.30
^{47}Ti	$\frac{5}{2}$ -	^{48}Ti	2-,3-	11.67		2820	1600	3120	4.45
^{49}Ti	$\frac{7}{2}$ -	^{50}Ti	3-,4-	11.07		3600	6000	9100	3.97
^{53}Cr	$\frac{3}{2}$ -	^{54}Cr	1-,2-	9.85		3200	5700	9090	4.01
^{57}Fe	$\frac{1}{2}$ -	^{58}Fe	0-,1-	10.05		5900	1500	10,000	4.07
^{61}Ni	$\frac{3}{2}$ -	^{62}Ni	1-,2-	10.63		2300	1400	3300	4.06
^{73}Ge	$\frac{9}{2}$ +	^{74}Ge	4+,5+	10.20		77	62	260	5.57
^{77}Se	$\frac{1}{2}$ -	^{78}Se	0-,1-	10.50	100		120	240	5.81
^{83}Kr	$\frac{9}{2}$ +	^{84}Kr	4+,5+	10.52	200			60	5.61
^{87}Sr	$\frac{9}{2}$ +	^{88}Sr	4+,5+	11.11		210	250	100	5.42
^{91}Zr	$\frac{5}{2}$ +	^{92}Zr	2+,3+	8.64	315	250	110	280	5.28
^{95}Mo	$\frac{5}{2}$ +	^{96}Mo	2+,3+	9.15		100	102	60	5.75
^{97}Mo	$\frac{5}{2}$ +	^{98}Mo	2+,3+	8.64	170	120	80	110	5.80
^{99}Ru	$\frac{5}{2}$ +	^{100}Ru	2+,3+	9.67		200	34	80	6.05

(continued)

Table 1. (continued)

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV[d]	σ
^{101}Ru	$\frac{5}{2}$ +	^{102}Ru	2+,3+	9.22	16	15	18	20	6.22
^{105}Pd	$\frac{5}{2}$ +	^{106}Pd	2+,3+	9.56	13	11	9	11	6.56
^{111}Cd	$\frac{1}{2}$ +	^{112}Cd	0+,1+	9.40	26	34	26	17	6.96
^{113}Cd	$\frac{1}{2}$ +	^{114}Cd	0+,1+	9.04	25	27	25	22	7.15
^{115}Sn	$\frac{1}{2}$ +	^{116}Sn	0+,1+	9.57	50	50		36	7.01
^{117}Sn	$\frac{1}{2}$ +	^{118}Sn	0+,1+	9.33	25	65	45	33	7.11
^{119}Sn	$\frac{1}{2}$ +	^{120}Sn	0+,1+	9.10	30	62	70	20	7.12
^{129}Xe	$\frac{1}{2}$ +	^{130}Xe	0+,1+	9.26	82		35	40	7.14
^{131}Xe	$\frac{3}{2}$ +	^{132}Xe	1+,2+	8.94	31	31	35	40	6.65
^{135}Xe	$\frac{3}{2}$ +	^{136}Xe	1+,2+	7.99		500		1360	5.50
^{135}Ba	$\frac{3}{2}$ +	^{136}Ba	1+,2+	9.11	51	35	36	19	6.57
^{137}Ba	$\frac{3}{2}$ +	^{138}Ba	1+,2+	8.61	200	460	230	160	6.16
^{143}Nd	$\frac{7}{2}$ -	^{144}Nd	3-,4-	7.82	72	19	36	63	6.73
^{145}Nd	$\frac{7}{2}$ -	^{146}Nd	3-,4-	7.57	33	25	1.9	3	6.83

(continued)

Table 1. (continued)

Target	I_o,π	Compound Nucleus	$I_o \pm \frac{1}{2},\pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV[d]	σ
^{147}Sm	$\frac{7}{2}$ -	^{148}Sm	3-,4-	8.14	8.0	7.9	7.3	3	7.38
^{183}W	$\frac{1}{2}$ -	^{184}W	0-,1-	7.41	15	16	12	3	8.73
^{187}Os	$\frac{1}{2}$ -	^{188}Os	0-,1-	7.99		14	9.1	3	8.18
^{189}Os	$\frac{3}{2}$ -	^{190}Os	1-,2-	7.79		5.0	4.3	5	7.53
^{195}Pt	$\frac{1}{2}$ -	^{196}Pt	0-,1-	7.92	18	19	12	15	6.28
^{199}Hg	$\frac{1}{2}$ -	^{200}Hg	0-,1-	8.03	70	84	75	24	5.89
^{201}Hg	$\frac{3}{2}$ -	^{202}Hg	1-,2-	7.76	100	110	90	20	5.48
^{207}Pb	$\frac{1}{2}$ -	^{208}Pb	0-,1-	7.67	8000	22,000	60,000	500	5.48

[a] Data compiled by Lynn[1]).

[b] Data compiled by Baba[2]).

[c] Data compiled by Vonach *et al.*[3]).

[d] Calculated with spherical single particle levels of Seeger and Perisho[11]). The theoretical spacing is for two spin states.

Table 2. Experimental and theoretical level spacings of even Z-odd N spherical nuclei.

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV^d
^{24}Mg	0+	^{25}Mg	$\frac{1}{2}$ +	7.63		170,000		1,000,000
^{32}S	0+	^{33}S	$\frac{1}{2}$ +	8.94	350,000	87,000		77,000
^{40}Ar	0+	^{41}Ar	$\frac{1}{2}$ +	6.41			90,000	286,000
^{40}Ca	0+	^{41}Ca	$\frac{1}{2}$ +	8.56	49,000	50,000	45,000	74,000
^{42}Ca	0+	^{43}Ca	$\frac{1}{2}$ +	8.25	28,500	28,500	28,000	59,000
^{44}Ca	0+	^{45}Ca	$\frac{1}{2}$ +	7.71	50,000	55,000	33,000	148,000
^{46}Ti	0+	^{47}Ti	$\frac{1}{2}$ +	8.98	30,000	45,000	22,000	40,000
^{48}Ti	0+	^{49}Ti	$\frac{1}{2}$ +	8.24	15,000	20,000	25,000	200,000
^{50}Ti	0+	^{51}Ti	$\frac{1}{2}$ +	6.48	120,000	18,000		380,000
^{50}Cr	0+	^{51}Cr	$\frac{1}{2}$ +	9.44	16,500	19,000	21,000	76,000
^{52}Cr	0+	^{53}Cr	$\frac{1}{2}$ +	8.11	44,000	46,000	47,000	39,000
^{54}Cr	0+	^{55}Cr	$\frac{1}{2}$ +	6.44	23,500	48,000	66,000	95,000
^{54}Fe	0+	^{55}Fe	$\frac{1}{2}$ +	9.55	25,000	21,000	20,000	20,000
^{56}Fe	0+	^{57}Fe	$\frac{1}{2}$ +	7.97	29,000	21,000	25,000	62,000

(continued)

Table 2. (continued)

Target	I_o,π	Compound Nucleus	$I_o \pm \frac{1}{2},\pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV[d]
^{58}Ni	0+	^{59}Ni	$\frac{1}{2}$ +	9.30	27,000	21,000	22,000	21,000
^{60}Ni	0+	^{61}Ni	$\frac{1}{2}$ +	8.12	23,000	21,000	17,000	39,000
^{62}Ni	0+	^{63}Ni	$\frac{1}{2}$ +	7.14	19,500	19,500	19,000	39,000
^{64}Ni	0+	^{65}Ni	$\frac{1}{2}$ +	6.40	28,500	28,500	28,000	64,000
^{64}Zn	0+	^{65}Zn	$\frac{1}{2}$ +	7.99	1800	3400	3600	9000
^{66}Zn	0+	^{67}Zn	$\frac{1}{2}$ +	7.06	5000	5600	6000	6200
^{68}Zn	0+	^{69}Zn	$\frac{1}{2}$ +	6.60		20,000	10,000	27,000
^{70}Ge	0+	^{71}Ge	$\frac{1}{2}$ +	7.43	1700	2000	1330	4000
^{72}Ge	0+	^{73}Ge	$\frac{1}{2}$ +	6.80	2100	3900	1550	10,000
^{74}Ge	0+	^{75}Ge	$\frac{1}{2}$ +	6.50	8500	8500	3900	14,000
^{76}Ge	0+	^{77}Ge	$\frac{1}{2}$ +	6.03	8000	8000	4200	43,000
^{74}Se	0+	^{75}Se	$\frac{1}{2}$ +	8.05	250	200	370	170
^{76}Se	0+	^{77}Se	$\frac{1}{2}$ +	7.42	1200	1200	700	1800
^{78}Se	0+	^{79}Se	$\frac{1}{2}$ +	6.99	3700	4500	1000	4500

(continued)

Table 2. (continued)

Target	I_o,π	Compound Nucleus	$I_o \pm \frac{1}{2},\pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV[d]
^{80}Se	0+	^{81}Se	$\frac{1}{2}$ +	6.71	4300	1600	1200	10,000
^{82}Se	0+	^{83}Se	$\frac{1}{2}$ +	5.94	7000	6900	6700	69,000
^{80}Kr	0+	^{81}Kr	$\frac{1}{2}$ +	7.86	530			1100
^{84}Sr	0+	^{85}Sr	$\frac{1}{2}$ +	8.53		350	400	950
^{86}Sr	0+	^{87}Sr	$\frac{1}{2}$ +	8.43		2100	1000	4800
^{88}Sr	0+	^{89}Sr	$\frac{1}{2}$ +	6.51	55,000	12,000	12,000	12,000
^{90}Zr	0+	^{91}Zr	$\frac{1}{2}$ +	7.24	4500	3300	5000	2100
^{92}Zr	0+	^{93}Zr	$\frac{1}{2}$ +	6.78	1200	3400	2500	4200
^{94}Zr	0+	^{95}Zr	$\frac{1}{2}$ +	6.48	2400	3300	2400	7500
^{92}Mo	0+	^{93}Mo	$\frac{1}{2}$ +	8.07			700	900
^{94}Mo	0+	^{95}Mo	$\frac{1}{2}$ +	7.38			430	670
^{96}Mo	0+	^{97}Mo	$\frac{1}{2}$ +	6.82		1200	290	1400
^{112}Cd	0+	^{113}Cd	$\frac{1}{2}$ +	6.54	200	200	198	200
^{114}Cd	0+	^{115}Cd	$\frac{1}{2}$ +	6.15	160		157	560

(continued)

Table 2. (continued)

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV[d]
^{112}Sn	0+	^{113}Sn	$\frac{1}{2}$ +	7.74	108	140	20	330
^{114}Sn	0+	^{115}Sn	$\frac{1}{2}$ +	7.53	150	320	300	230
^{116}Sn	0+	^{117}Sn	$\frac{1}{2}$ +	6.94	180	250	250	350
^{118}Sn	0+	^{119}Sn	$\frac{1}{2}$ +	6.48	180	730	600	290
^{120}Sn	0+	^{121}Sn	$\frac{1}{2}$ +	6.18	200	240	180	1700
^{124}Sn	0+	^{125}Sn	$\frac{1}{2}$ +	5.73	400	250	2500	8500
^{130}Ba	0+	^{131}Ba	$\frac{1}{2}$ +	7.49		120	55	140
^{134}Ba	0+	^{135}Ba	$\frac{1}{2}$ +	6.98		380	140	230
^{136}Ba	0+	^{137}Ba	$\frac{1}{2}$ +	6.90	8000	3800	600	1850
^{138}Ba	0+	^{139}Ba	$\frac{1}{2}$ +	4.82	10,000	9600	10,000	18,000
^{136}Ce	0+	^{137}Ce	$\frac{1}{2}$ +	7.64			58	63
^{140}Ce	0+	^{141}Ce	$\frac{1}{2}$ +	5.43		3000	3000	5500
^{142}Ce	0+	^{143}Ce	$\frac{1}{2}$ +	5.18	1000	1000	1000	8900
^{142}Nd	0+	^{143}Nd	$\frac{1}{2}$ +	6.14			415	1700

(continued)

Table 2. (continued)

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV[d]
^{144}Nd	0+	^{145}Nd	$\frac{1}{2}$ +	5.77			537	1200
^{180}Hf	0+	^{181}Hf	$\frac{1}{2}$ +	5.70		140	125	100
^{180}W	0+	^{181}W	$\frac{1}{2}$ +	6.65			18	8
^{182}W	0+	^{183}W	$\frac{1}{2}$ +	6.20	55	56	66	24
^{184}W	0+	^{185}W	$\frac{1}{2}$ +	5.76		93	89	140
^{186}W	0+	^{187}W	$\frac{1}{2}$ +	5.47	150	87	123	890
^{186}Os	0+	^{187}Os	$\frac{1}{2}$ +	6.30			30	33
^{192}Pt	0+	^{193}Pt	$\frac{1}{2}$ +	6.25		140		90
^{198}Hg	0+	^{199}Hg	$\frac{1}{2}$ +	6.65	99	100	90	120
^{200}Hg	0+	^{201}Hg	$\frac{1}{2}$ +	6.22	2200	1300	1300	440
^{202}Hg	0+	^{203}Hg	$\frac{1}{2}$ +	6.00	2400			1600
^{204}Pb	0+	^{205}Pb	$\frac{1}{2}$ +	6.76			2700	530
^{206}Pb	0+	^{207}Pb	$\frac{1}{2}$ +	7.11		24,000	50,000	800
^{208}Pb	0+	^{209}Pb	$\frac{1}{2}$ +	4.84		110,000	105,000	41,000

(continued)

Table 2. (continued)

[a]Data compiled by Lynn[1]).

[b]Data compiled by Baba[2]).

[c]Data compiled by Vonach *et al.*[3]).

[d]Calculated with spherical single particle levels of Seeger and Perisho[11]). The theoretical spacing is for one spin state.

Table 3. Experimental and theoretical level spacings of odd Z-even N spherical nuclei.

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV[d]
^{50}V	6+	^{51}V	$\frac{11}{2}+, \frac{13}{2}+$	11.05	1100	2610	1300	1800
^{138}La	5−	^{139}La	$\frac{9}{2}-, \frac{11}{2}-$	8.78	23	41	23	6

[a]Data compiled by Lynn[1]).

[b]Data compiled by Baba[2]).

[c]Data compiled by Vonach _et al_.[3]).

[d]Calculated with spherical single particle levels of Seeger and Perisho[11]). The theoretical spacing is for two spin states.

Table 4. Experimental and theoretical level spacings of odd Z-odd N spherical nuclei.

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV[d]
^{19}F	$\frac{1}{2}$ +	^{20}F	0+,1+	6.90		55,000		300,000
^{23}Na	$\frac{3}{2}$ +	^{24}Na	1+,2+	7.00	270,000	66,000		57,000
^{27}Al	$\frac{5}{2}$ +	^{28}Al	2+,3+	7.73	54,000	26,000		23,000
^{31}P	$\frac{1}{2}$ +	^{32}P	0+,1+	7.95		21,000		18,000
^{35}Cl	$\frac{3}{2}$ +	^{36}Cl	1+,2+	8.58	5600	40,000		6000
^{37}Cl	$\frac{3}{2}$ +	^{38}Cl	1+,2+	6.11		37,000		31,000
^{39}K	$\frac{3}{2}$ +	^{40}K	1+,2+	7.80		10,000		15,000
^{41}K	$\frac{3}{2}$ +	^{42}K	1+,2+	7.53		10,000		8500
^{45}Sc	$\frac{7}{2}$ -	^{46}Sc	3-,4-	8.92		1600	1100	1500
^{51}V	$\frac{7}{2}$ -	^{52}V	3-,4-	7.38	3600	4390	4900	2900
^{55}Mn	$\frac{5}{2}$ -	^{56}Mn	2-,3-	7.37	2100	2970	1900	2000
^{59}Co	$\frac{7}{2}$ -	^{60}Co	3-,4-	7.55	960	1530	1300	1700
^{63}Cu	$\frac{3}{2}$ -	^{64}Cu	1-,2-	7.93	1200	1060	580	450
^{65}Cu	$\frac{3}{2}$ -	^{66}Cu	1-,2-	7.08	2000	1170	1000	930

(continued)

Table 4. (continued)

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV^d
^{69}Ga	$\frac{3}{2}$ -	^{70}Ga	1-,2-	7.66	340	320	320	160
^{71}Ga	$\frac{3}{2}$ -	^{72}Ga	1-,2-	6.52	170	190	370	700
^{75}As	$\frac{3}{2}$ -	^{76}As	1-,2-	7.33	87	87	87	65
^{79}Br	$\frac{3}{2}$ -	^{80}Br	1-,2-	7.88	57	61	60	50
^{81}Br	$\frac{3}{2}$ -	^{82}Br	1-,2-	7.60	51	52	80	220
^{85}Rb	$\frac{5}{2}$ -	^{86}Rb	2-,3-	8.65	130	1100	130	160
^{87}Rb	$\frac{3}{2}$ -	^{88}Rb	1-,2-	6.08	1200	1800		680
^{89}Y	$\frac{1}{2}$ -	^{90}Y	0-,1-	6.87	1000	1600	3000	500
^{93}Nb	$\frac{9}{2}$ +	^{94}Nb	4+,5+	7.23	70	36	64	22
^{99}Tc	$\frac{9}{2}$ +	^{100}Tc	4+,5+	6.59	24	26		33
^{103}Rh	$\frac{1}{2}$ -	^{104}Rh	0-,1-	7.00	19	10	27	60
^{107}Ag	$\frac{1}{2}$ +	^{108}Ag	0+,1+	7.27	14	50	23	45
^{109}Ag	$\frac{1}{2}$ +	^{110}Ag	0+,1+	6.80	13	19	18	40
^{113}In	$\frac{9}{2}$ +	^{114}In	4+,5+	7.31	6.5	7.1	11	9

(continued)

Table 4. (continued)

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV[d]
^{115}In	$\frac{9}{2}$ +	^{116}In	4+,5+	6.78	6.7	9.5	11	4
^{127}I	$\frac{5}{2}$ +	^{128}I	2+,3+	6.83	13	19	14	5
^{129}I	$\frac{7}{2}$ +	^{130}I	3+,4+	6.46	18	21		18
^{133}Cs	$\frac{7}{2}$ +	^{134}Cs	3+,4+	6.89	20	21	20	14
^{139}La	$\frac{7}{2}$ +	^{140}La	3+,4+	5.17	73	110	260	140
^{141}Pr	$\frac{5}{2}$ +	^{142}Pr	2+,3+	5.85	51	84	90	50
^{147}Pm	$\frac{7}{2}$ +	^{148}Pm	3+,4+	5.90	5.2	5.7		5
^{181}Ta	$\frac{7}{2}$ +	^{182}Ta	3+,4+	6.06	4.4	4.3	4.3	0.7
^{185}Re	$\frac{5}{2}$ +	^{186}Re	2+,3+	6.18	3.8	3.2	3.3	2
^{187}Re	$\frac{5}{2}$ +	^{188}Re	2+,3+	5.87	4.5	6.4	3.8	3
^{191}Ir	$\frac{3}{2}$ +	^{192}Ir	1+,2+	6.20	3.1	3.2	2.8	6
^{193}Ir	$\frac{3}{2}$ +	^{194}Ir	1+,2+	6.07	8.2	8.5	8.0	12
^{197}Au	$\frac{3}{2}$ +	^{198}Au	1+,2+	6.51	17	16	16	12
^{203}Tl	$\frac{1}{2}$ +	^{204}Tl	0+,1+	6.65	2000	2200	2000	1250
^{205}Tl	$\frac{1}{2}$ +	^{206}Tl	0+,1+	6.54	10,000	19,000	4000	300

(continued)

Table 4. (continued)

[a]Data compiled by Lynn[1]).

[b]Data compiled by Baba[2]).

[c]Data compiled by Vonach *et al.*[3]).

[d]Calculated with spherical single particle levels of Seeger and Perisho[11]). The theoretical spacing is for two spin states.

Table 5. Comparison of theoretical level spacings calculated with spherical single particle levels of Seeger and Perisho[11] and Nilsson *et al.*[12].

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV (d)	(e)
^{49}Ti	$\frac{7}{2}$ -	^{50}Ti	3-,4-	11.67		3600	6000	9100	23,000
^{61}Ni	$\frac{3}{2}$ -	^{62}Ni	1-,2-	10.63		2300	1400	3300	2600
^{101}Ru	$\frac{5}{2}$ +	^{102}Ru	2+,3+	9.22	16	15	18	20	13
^{115}Sn	$\frac{1}{2}$ +	^{116}Sn	0+,1+	9.57	50	50		36	470
^{117}Sn	$\frac{1}{2}$ +	^{118}Sn	0+,1+	9.33	25	65	45	33	350
^{119}Sn	$\frac{1}{2}$ +	^{120}Sn	0+,1+	9.10	30	62	70	20	130
^{137}Ba	$\frac{3}{2}$ +	^{138}Ba	1+,2+	8.61	200	460	230	160	1300
^{185}Re	$\frac{5}{2}$ +	^{186}Re	2+,3+	6.18	3.8	3.2	3.3	2	8
^{187}Re	$\frac{5}{2}$ +	^{188}Re	2+,3+	5.87	4.5	6.4	3.8	3	7
^{187}Os	$\frac{1}{2}$ -	^{188}Os	0-,1-	7.99		14	9.1	3	10
^{189}Os	$\frac{3}{2}$ -	^{190}Os	1-,2-	7.79		5	4.3	5	8
^{193}Ir	$\frac{3}{2}$ +	^{194}Ir	1+,2+	6.07	8.2	8.5	8.0	12	12
^{195}Pt	$\frac{1}{2}$ -	^{196}Pt	0-,1-	7.92	18	19	12	15	15

(continued)

Table 5. (continued)

Target	I_o, π	Compound Nucleus	$I_o \pm \frac{1}{2}, \pi$	E^* MeV	D_{exp} in eV (a)	(b)	(c)	D_{Theo} in eV (d)	(e)
^{197}Au	$\frac{3}{2}$ +	^{198}Au	1+,2+	6.51	17	16	16	12	25
^{199}Hg	$\frac{1}{2}$ -	^{200}Hg	0-,1-	8.03	70	84	75	24	60
^{201}Hg	$\frac{3}{2}$ -	^{202}Hg	1-,2-	7.76	100	110	90	20	100
^{205}Tl	$\frac{1}{2}$ +	^{206}Tl	0+,1+	6.54	10,000	19,000	4000	300	5000
^{206}Pb	0+	^{207}Pb	$\frac{1}{2}$ +	7.11		24,000	50,000	800	11,400
^{207}Pb	$\frac{1}{2}$ -	^{208}Pb	0-,1-	7.67	8000	22,000	60,000	500	6700

[a] Data compiled by Lynn[1]).

[b] Data compiled by Baba[2]).

[c] Data compiled by Vonach et al.[3]).

[d] Calculated with spherical single particle levels of Seeger and Perisho[11]).

[e] Calculated with spherical single particle levels of Nilsson et al.[12]).

Figure Captions

Fig. 1. Proton pairing gap Δ_p as a function of proton number used in the microscopic state density calculations.

Fig. 2. Neutron pairing gap Δ_n as a function of neutron number used in the microscopic state density calculations.

Fig. 3. The ratio of the theoretical level spacing to the experimental level spacing as a function of mass number. The different symbols refer to the different types of nuclei. Closed symbols refer to theoretical spacings calculated with single particle levels of Seeger and Perisho[11]) and the open symbols refer to theoretical spacings calculated with single particle levels of Nilsson et al.[12]). The ratios $D_{theo.}/D_{exp.}$ plotted in this figure are calculated with the most favorable single value of $D_{exp.}$ listed in Tables 1 to 4.

Fig. 4. The spin cutoff parameter, σ^2, for an excitation energy of 7 MeV is plotted as a function of the mass number A for even-even spherical nuclei.

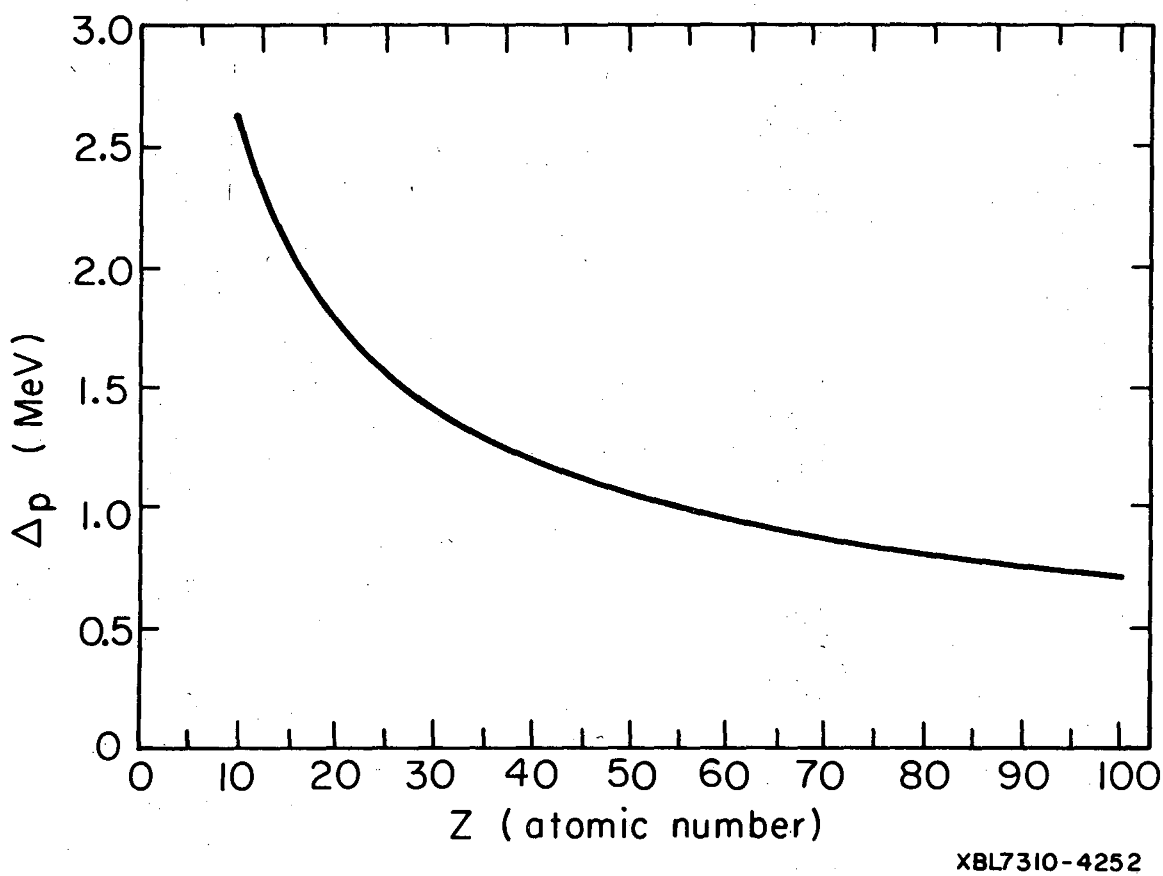
3.0
2.5
2.0
1.5
1.0
0.5
0
Δp (MeV)
0
10
20
30
40
50
60
70
80
90
100
Z (atomic number)
XBL7310-4252

Fig. 1

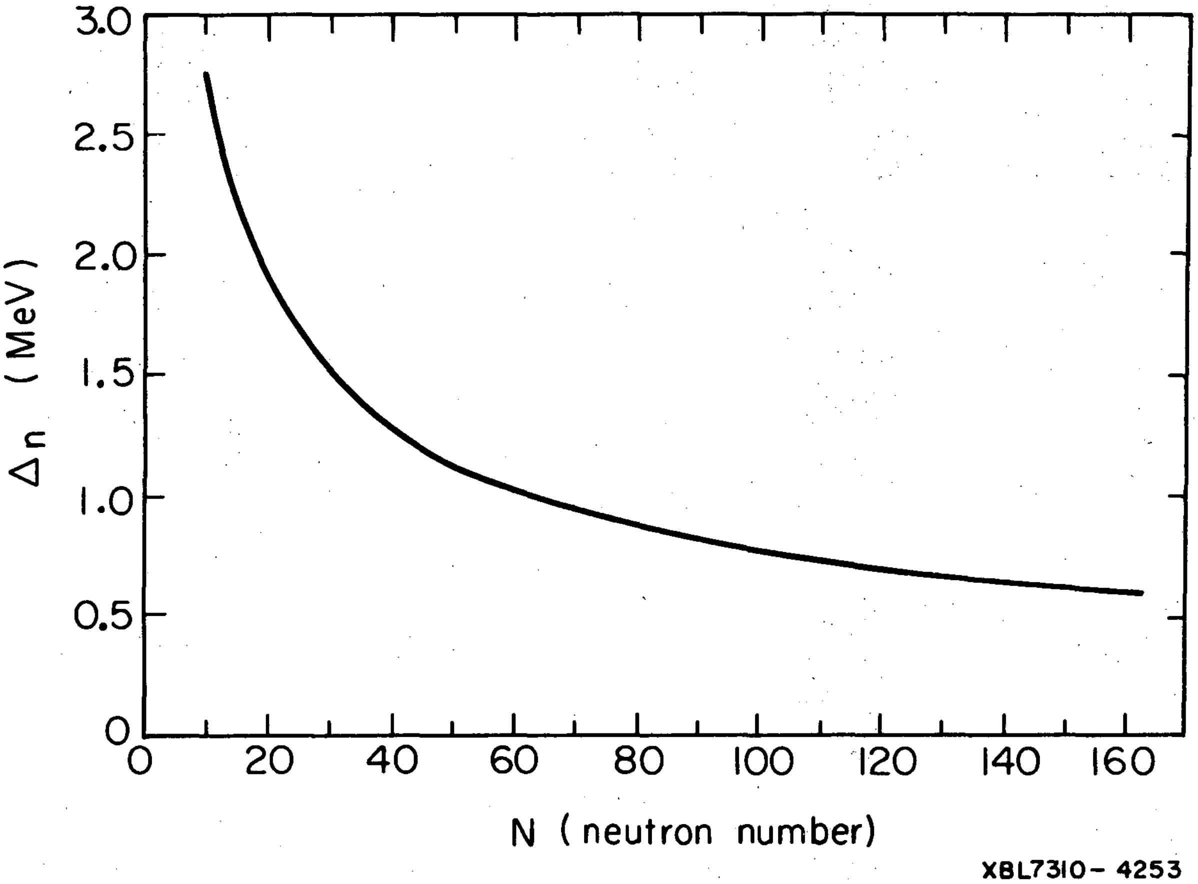
3.0
2.5
2.0
1.5
1.0
0.5
0
Δn (MeV)
0
20
40
60
80
100
120
140
160
N (neutron number)
XBL7310-4253

Fig. 2

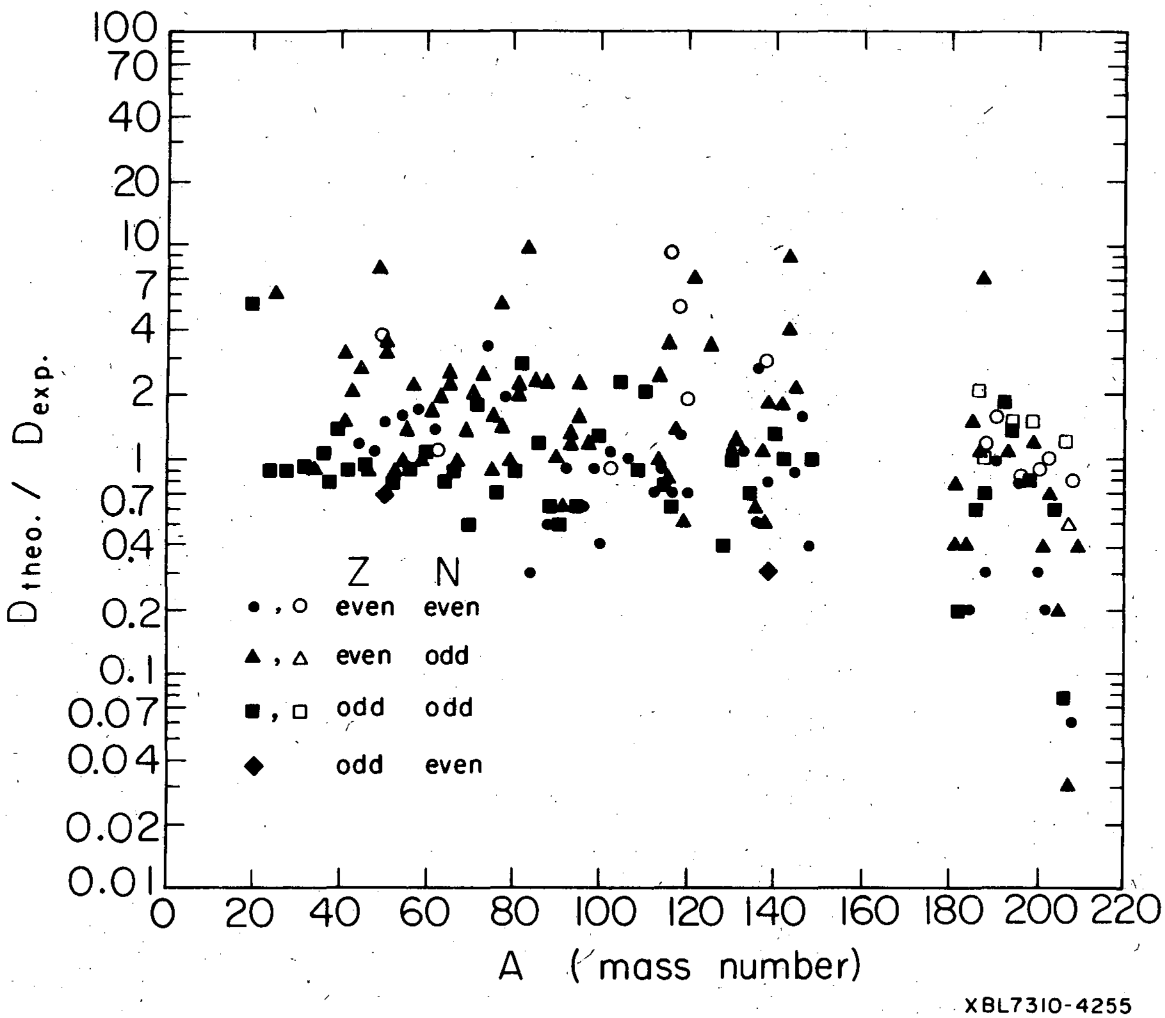
100
70
40
20
10
7
4
2
1
0.7
0.4
0.2
0.1
0.07
0.04
0.02
0.01
0
20
40
60
80
100
120
140
160
180
200
220
$D_{theo.}$ / $D_{exp.}$
A (mass number)
Z N
●, ○ even even
▲, △ even odd
■, □ odd odd
◆ odd even
XBL7310-4255

Fig. 3

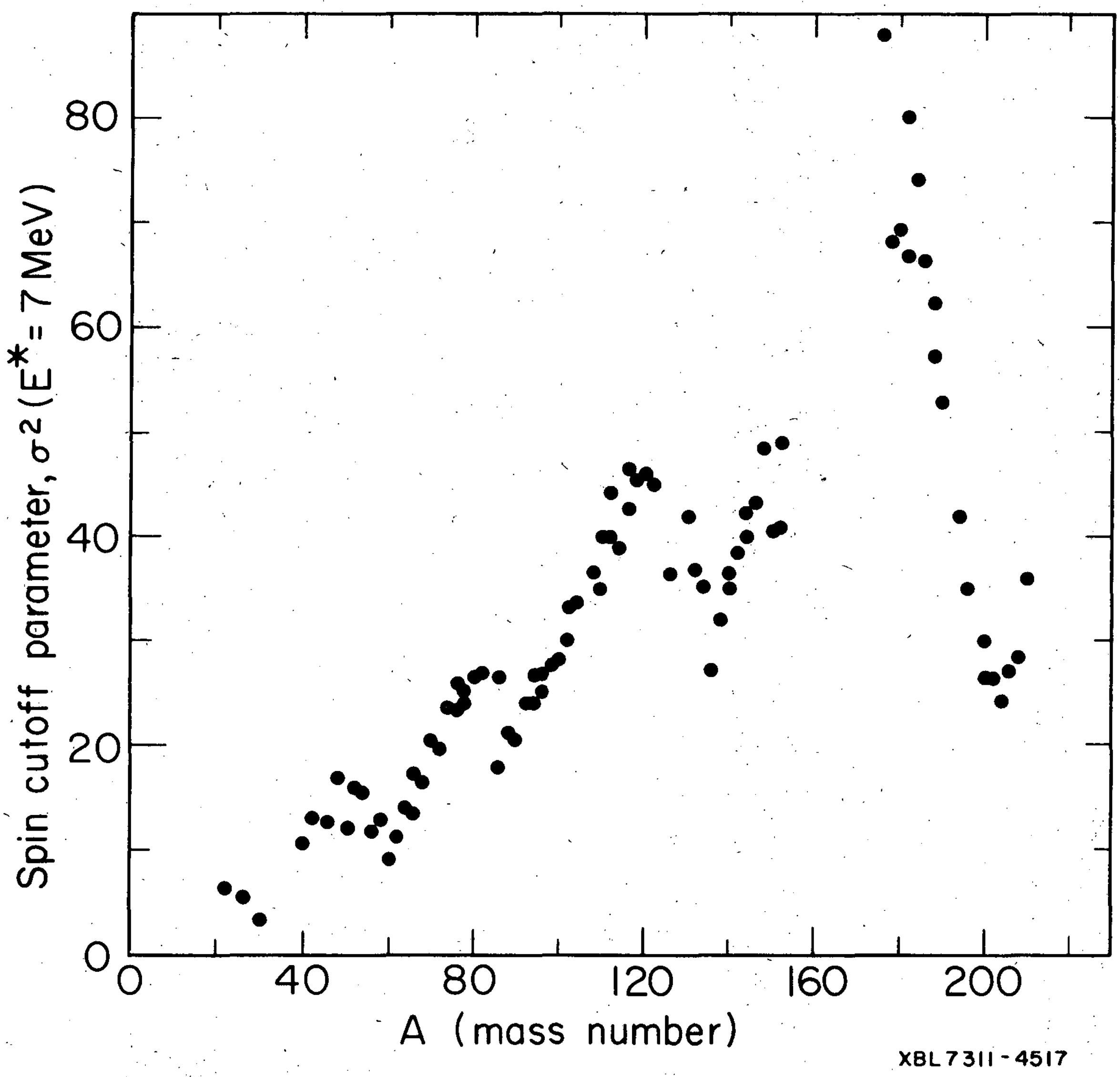
Spin cutoff parameter, σ² (E* = 7 MeV)
0
20
40
60
80
0
40
80
120
160
200
A (mass number)
XBL 7311-4517

Fig. 4

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