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Neutrino mass operator renormalization revisited

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Abstract

We re-derive the renormalization group equation for the effective coupling of the dimension five operator which corresponds to a Majorana mass matrix for the Standard Model neutrinos. We find a result which differs somewhat from earlier calculations, leading to modifications in the evolution of leptonic mixing angles and CP phases. We also present a general method for calculating β -functions from counterterms in MS-like renormalization schemes, which works for tensorial quantities.

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1. Introduction

The Standard Model (SM) is most likely an effective theory up to some scale Λ , above which new physics has to be taken into account. The discovery of neutrino masses requires an extension of the SM, which may involve right-handed neutrinos or other new fields. Introducing right-handed neutrinos allows Dirac masses m_D via Yukawa couplings analogous to the quark sector. In general, lepton number need not be conserved, so that Majorana masses are possible. For left-handed neutrinos this can, for example, be achieved with Higgs triplets. Right-handed neutrinos can have explicit Majorana masses M_R of order Λ , since they are gauge singlets and since there are no protective symmetries. This leads to a picture with zero or tiny left-handed Majorana masses M_L , with

m_D similar to the charged lepton masses, and with a huge M_R . Diagonalization of the neutrino mass matrix results in Majorana fermions and eigenvalues $\simeq M_R$ and $M_L - m_D^2/M_R$. For $M_L = 0$ the neutrino masses are thus given by the see-saw relation m_D^2/M_R [1], which provides a convincing explanation for the smallness of neutrino masses.

Another, less model dependent approach is to study the effective field theory with higher-dimensional operators of SM fields. If lepton number is not conserved, some of these generate Majorana neutrino masses. The lowest-dimensional operator of this kind has dimension 5 and couples two lepton and two Higgs doublets. It appears, e.g., in the see-saw mechanism by integrating out the heavy right-handed neutrinos.

As quarks have only small mixings, it is somewhat surprising that neutrinos most likely have two large mixing angles [2–4]. It is interesting to investigate mechanisms which can produce such large or maximal mixings. These mechanisms operate, however, typically at the embedding scale Λ . For a compari-

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son of experimental results with high energy predictions from unified theories, it is thus essential to evolve the predictions to low energies with the relevant renormalization group equations (RGEs). This evolution is related to the running of the leading dimension 5 operator. Therefore, we calculate in this Letter the RGE that governs this running above the electroweak scale at one-loop order in the SM.

2. Lagrangian and counterterms

Let ℓ_L^f , $f \in \{1, 2, 3\}$, be the $SU(2)_L$ -doublets of SM leptons, e_R^f the $SU(2)_L$ -singlet (right-handed) charged leptons, and ϕ the Higgs doublet. The dimension 5 operator that gives Majorana masses to the SM neutrinos is given by

$$\mathcal{L}_\kappa = \frac{1}{4} \kappa_{gf} \bar{\ell}_L^c \varepsilon^{cd} \phi_d \ell_L^f \varepsilon^{ba} \phi_a + \text{h.c.}, \quad (1)$$

where κ is symmetric under interchange of the generation indices f and g , ε is the totally antisymmetric tensor in 2 dimensions, and $\ell_L^C := (\ell_L)^C$ is the charge conjugate of the lepton doublet. $a, b, c, d \in \{1, 2\}$ are $SU(2)$ indices. They will only be written explicitly in terms with a non-trivial $SU(2)$ structure. Summation over repeated indices is implied throughout this Letter.

\mathcal{L}_κ gives rise to the vertex shown in Fig. 1, and an analogous one for the Hermitian conjugate term.

The complete Lagrangian consists of \mathcal{L}_κ , the SM Lagrangian \mathcal{L}_{SM} and proper counterterms \mathcal{C} ,

$$\mathcal{L} = \mathcal{L}_\kappa + \mathcal{L}_{\text{SM}} + \mathcal{C}. \quad (2)$$

In the following, we omit most of those parts that yield only flavour diagonal contributions to the β function and therefore do not contribute to the running of mixing angles, in particular terms involving quarks

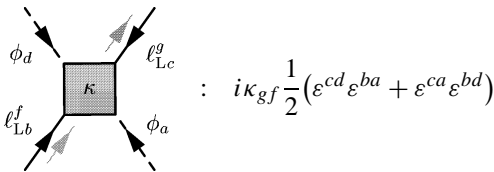


Fig. 1. Vertex from the effective dimension 5 operator and the corresponding Feynman rule. The gray arrow indicates the fermion flow as defined in [5].

and gauge bosons. The remaining ones are

$$\mathcal{L}_{\text{kin}(\ell_L)} = \bar{\ell}_L^f (i \gamma^\mu \partial_\mu) \ell_L^f, \quad (3a)$$

$$\mathcal{L}_{\text{Higgs}} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - m^2 \phi^\dagger \phi - \frac{1}{4} \lambda (\phi^\dagger \phi)^2, \quad (3b)$$

$$\mathcal{L}_{\text{Yukawa}} = -(Y_e)_{gf} \bar{e}_R^g \phi^\dagger \ell_L^f + \text{h.c.}; \quad (3c)$$

$$\mathcal{C}_{\text{kin}(\ell_L)} = \bar{\ell}_L^g (i \gamma^\mu \partial_\mu) (\delta Z_{\ell_L})_{gf} \ell_L^f, \quad (4a)$$

$$\mathcal{C}_{\text{Higgs}} = \delta Z_\phi (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \delta m^2 \phi^\dagger \phi - \frac{1}{4} \delta \lambda (\phi^\dagger \phi)^2, \quad (4b)$$

$$\mathcal{C}_{\text{Yukawa}} = -(\delta Y_e)_{gf} \bar{e}_R^g \phi^\dagger \ell_L^f + \text{h.c.}; \quad (4c)$$

$$\mathcal{C}_\kappa = \frac{1}{4} \delta \kappa_{gf} \bar{\ell}_L^c \varepsilon^{cd} \phi_d \ell_L^f \varepsilon^{ba} \phi_a + \text{h.c.} \quad (5)$$

δZ_i ($i \in \{\ell_L, \phi\}$) determine the wavefunction renormalization constants $Z_i = 1 + \delta Z_i$, defined in the usual way. Note that Z_{ℓ_L} is a matrix in flavour space. $\delta \kappa$ satisfies the relation

$$\kappa_B = Z_\phi^{-1/2} (Z_{\ell_L}^T)^{-1/2} [\kappa + \delta \kappa] \mu^\epsilon Z_{\ell_L}^{-1/2} Z_\phi^{-1/2}, \quad (6)$$

where the factor μ^ϵ is due to dimensional regularization, with μ denoting the renormalization scale and $\epsilon := 4 - d$. The subscript B denotes a bare quantity. Note that the usual ansatz $\kappa_B \sim Z_\kappa \kappa$ is not possible in this case, as it would obviously spoil the symmetry of κ_B or κ with respect to interchange of the flavour indices.

3. Calculation of the counterterms

In the $\overline{\text{MS}}$ scheme, the quantity $\delta \kappa$ can be computed at one-loop order from the requirement that the sum of diagrams in Fig. 2 be ultraviolet finite.

Using FeynCalc [6] we obtain

$$\delta \kappa = -\frac{1}{16\pi^2} [2(Y_e^\dagger Y_e)^T \kappa + 2\kappa (Y_e^\dagger Y_e) - \lambda \kappa + C_\kappa] \frac{1}{\epsilon}, \quad (7)$$

where C_κ denotes the contribution from gauge interactions. The usual calculation of the wavefunction renormalization constants yields

$$\delta Z_\phi = -\frac{1}{8\pi^2} [\text{Tr}(Y_e^\dagger Y_e) + C_\phi] \frac{1}{\epsilon}, \quad (8)$$

$$\delta Z_{\ell_L} = -\frac{1}{16\pi^2} [Y_e^\dagger Y_e + C_{\ell_L} \mathbb{1}] \frac{1}{\epsilon}. \quad (9)$$

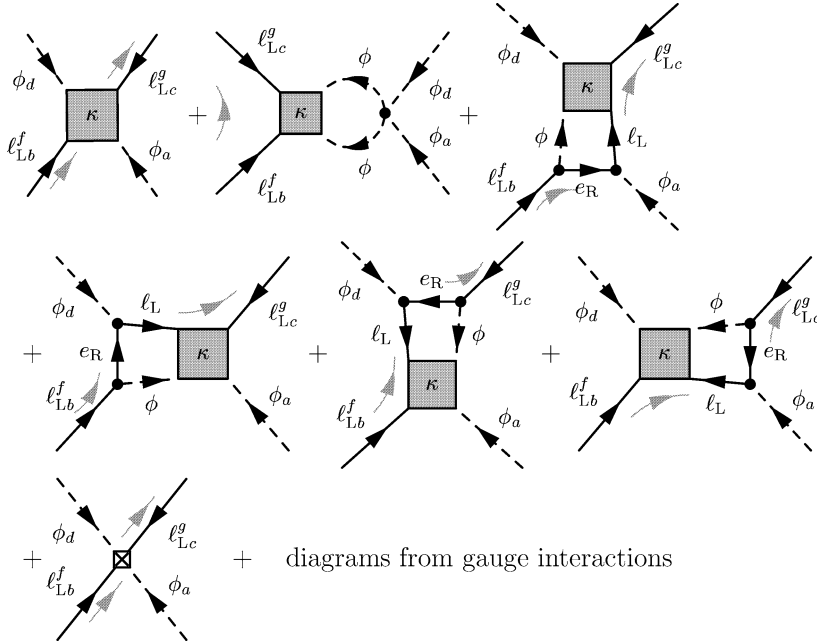


Fig. 2. Diagrams relevant for the renormalization of the vertex from the effective dimension 5 neutrino mass operator. The last diagram represents the counterterm.

Again, C_ϕ and C_{ℓ_L} represent terms from quarks and gauge interactions, which are diagonal in flavour space.

4. Calculating RGEs from counterterms with tensorial structure

The calculation of the β -function involves some subtle points, which are related to the matrix structure of the counterterm Lagrangian. Before presenting our result in Section 5, we provide now some details of the calculation, which should be of general interest and which are essential for verifying our result. In particular, we generalize the usual formalism for calculating β -functions to include tensorial quantities as well as non-multiplicative renormalization.

We are interested in the β -function for a quantity Q , $\beta_Q := \mu \frac{dQ}{d\mu}$. In general, the bare and the renormalized quantity are related by

$$Q_B = Z_{\phi_1}^{n_1} \cdots Z_{\phi_M}^{n_M} [Q + \delta Q] \mu^{D_Q} Z_{\phi_{M+1}}^{n_{M+1}} \cdots Z_{\phi_N}^{n_N}$$

$$= \left(\prod_{i \in I} Z_{\phi_i}^{n_i} \right) [Q + \delta Q] \mu^{D_Q} \left(\prod_{j \in J} Z_{\phi_j}^{n_j} \right), \quad (10)$$

where $I = \{1, \dots, M\}$, $J = \{M+1, \dots, N\}$ and D_Q is related to the mass dimension of Q . δQ and the wavefunction renormalization constants depend on Q and some additional variables $\{V_A\}$,

$$\delta Q = \delta Q(Q, \{V_A\}), \quad (11a)$$

$$Z_{\phi_i} = Z_{\phi_i}(Q, \{V_A\}) \quad (1 \leq i \leq N). \quad (11b)$$

Note that $Q = Q(\mu)$ and $V_A = V_A(\mu)$ are functions of the renormalization scale μ , but δQ and Z_{ϕ_i} do not depend explicitly on μ in an MS-like renormalization scheme. Taking the derivative of Eq. (10) yields

$$\begin{aligned} 0 &\stackrel{!}{=} \mu^{-D_Q} \mu \frac{d}{d\mu} Q_B \\ &= \left(\prod_{i \in I} Z_{\phi_i}^{n_i} \right) \left[\beta_Q + \left\langle \frac{d\delta Q}{dQ} \right\rangle \beta_Q \right] + \sum_A \left\langle \frac{d\delta Q}{dV_A} \right\rangle \beta_{V_A} \\ &\quad + \epsilon D_Q(Q + \delta Q) \left[\prod_{j \in J} Z_{\phi_j}^{n_j} \right] \end{aligned}$$

$$\begin{aligned}
 & + \left(\prod_{i \in I} Z_{\phi_i}^{n_i} \right) [Q + \delta Q] \\
 & \times \left\{ \sum_{j \in J} \left(\prod_{j' < j} Z_{\phi_{j'}}^{n_{j'}} \right) \left[\left\langle \frac{dZ_{\phi_j}^{n_j}}{dQ} \middle| \beta_Q \right\rangle \right. \right. \\
 & \quad \left. \left. + \sum_A \left\langle \frac{dZ_{\phi_j}^{n_j}}{dV_A} \middle| \beta_{V_A} \right\rangle \right] \left(\prod_{j'' > j} Z_{\phi_{j''}}^{n_{j''}} \right) \right\} \\
 & + \left\{ \sum_{i \in I} \left(\prod_{i' < i} Z_{\phi_{i'}}^{n_{i'}} \right) \left[\left\langle \frac{dZ_{\phi_i}^{n_i}}{dQ} \middle| \beta_Q \right\rangle \right. \right. \\
 & \quad \left. \left. + \sum_A \left\langle \frac{dZ_{\phi_i}^{n_i}}{dV_A} \middle| \beta_{V_A} \right\rangle \right] \left(\prod_{i'' > i} Z_{\phi_{i''}}^{n_{i''}} \right) \right\} \\
 & \times [Q + \delta Q] \left(\prod_{j \in J} Z_{\phi_j}^{n_j} \right). \tag{12}
 \end{aligned}$$

Here we have introduced the notation

$$\left\langle \frac{dF}{dx} \middle| y \right\rangle := \begin{cases} \frac{dF}{dx} y & \text{for scalars } x, y, \\ \sum_n \frac{dF}{dx_n} y_n & \text{for vectors } x = (x_m), y = (y_m), \\ \sum_{m,n} \frac{dF}{dx_{mn}} y_{mn} & \text{for matrices } x = (x_{mn}), y = (y_{mn}), \\ \dots \text{ etc.} & \end{cases} \tag{13}$$

We will solve Eq. (12) and the corresponding expression for V_A by expanding all quantities in powers of ϵ . In the MS-scheme the quantities δQ and Z_{ϕ_i} can be expanded as

$$\delta Q = \sum_{k \geq 1} \frac{\delta Q_{,k}}{\epsilon^k}, \tag{14a}$$

$$Z_{\phi_i} = \mathbb{1} + \sum_{k \geq 1} \frac{\delta Z_{\phi_i,k}}{\epsilon^k} =: \mathbb{1} + \delta Z_{\phi_i}, \tag{14b}$$

with higher powers of $1/\epsilon$ corresponding to higher powers in perturbation theory. On the other hand, β -functions are finite as $\epsilon \rightarrow 0$. We can therefore make the ansatz

$$\beta_Q = \beta_Q^{(0)} + \epsilon \beta_Q^{(1)} + \dots + \epsilon^n \beta_Q^{(n)}, \tag{15a}$$

$$\beta_{V_A} = \beta_{V_A}^{(0)} + \epsilon \beta_{V_A}^{(1)} + \dots + \epsilon^n \beta_{V_A}^{(n)}, \tag{15b}$$

where n is an arbitrary integer. Note that in this case the power of ϵ is not related to the order of

perturbation theory. From (14) and (15) we find that

$$\begin{aligned}
 \frac{dZ_{\phi_i}^{n_i}}{dQ} & = n_i Z_{\phi_i}^{n_i-1} \frac{dZ_{\phi_i}}{dQ} \\
 & = n_i \frac{d\delta Z_{\phi_i}}{dQ} + \mathcal{O}\left(\frac{1}{\epsilon^2}\right) = \mathcal{O}\left(\frac{1}{\epsilon}\right), \tag{16}
 \end{aligned}$$

where the lowest possible power of $1/\epsilon$ appearing on the right side of Eq. (16) is 1. An analogous relation holds for $Q \leftrightarrow V_A$. Our analysis of Eq. (12), starting with the inspection of the ϵ^n term, then shows that $\beta_Q^{(n)}$ vanishes. The analog of Eq. (12) for β_{V_A} implies that $\beta_{V_A}^{(n)}$ vanishes as well. Repeating this argument for successively smaller positive powers of ϵ implies that

$$\beta_Q^{(k)} = \beta_{V_A}^{(k)} = 0 \quad \forall k \in \{2, \dots, n\}, \tag{17a}$$

$$\beta_Q^{(1)} = -\epsilon D_Q Q, \tag{17b}$$

$$\beta_{V_A}^{(1)} = -\epsilon D_{V_A} V_A. \tag{17c}$$

Note that these terms do not contribute to the β function in 4 dimensions, i.e., for $\epsilon \rightarrow 0$, but they are necessary to read off $\beta_Q^{(0)}$ from Eq. (12), leading to the result

$$\begin{aligned}
 \beta_Q^{(0)} & = \left[D_Q \left\langle \frac{d\delta Q_{,1}}{dQ} \middle| Q \right\rangle \right. \\
 & \quad \left. + \sum_A D_{V_A} \left\langle \frac{d\delta Q_{,1}}{dV_A} \middle| V_A \right\rangle - D_Q \delta Q_{,1} \right] \\
 & + Q \cdot \sum_{j \in J} n_j \left[D_Q \left\langle \frac{dZ_{\phi_j,1}}{dQ} \middle| Q \right\rangle \right. \\
 & \quad \left. + \sum_A D_{V_A} \left\langle \frac{dZ_{\phi_j,1}}{dV_A} \middle| V_A \right\rangle \right] \\
 & + \sum_{i \in I} n_i \left[D_Q \left\langle \frac{dZ_{\phi_i,1}}{dQ} \middle| Q \right\rangle \right. \\
 & \quad \left. + \sum_A D_{V_A} \left\langle \frac{dZ_{\phi_i,1}}{dV_A} \middle| V_A \right\rangle \right] \cdot Q. \tag{18}
 \end{aligned}$$

Note that for complex quantities Q and V_A we have to treat the complex conjugates Q^* and V_A^* as additional independent variables.

5. Renormalization group equation

The RGE for the effective coupling κ is

$$\mu \frac{d\kappa}{d\mu} = \beta_\kappa. \quad (19)$$

Using Eqs. (18) and (7)–(9), we obtain for the contributions from vertex and wavefunction renormalization (omitting terms from C_κ , C_ϕ and C_{ℓ_L}):

$$16\pi^2 \beta_\kappa^{(v)} = -2[\kappa(Y_e^\dagger Y_e) + (Y_e^\dagger Y_e)^T \kappa] + \lambda\kappa, \quad (20a)$$

$$16\pi^2 \beta_\kappa^{(wf)} = \frac{1}{2}[\kappa(Y_e^\dagger Y_e) + (Y_e^\dagger Y_e)^T \kappa] + 2 \text{Tr}(Y_e^\dagger Y_e)\kappa. \quad (20b)$$

Adding the terms involving quarks and gauge bosons [7,8], we obtain the final result

$$16\pi^2 \beta_\kappa = -\frac{3}{2}[\kappa(Y_e^\dagger Y_e) + (Y_e^\dagger Y_e)^T \kappa] + \lambda\kappa - 3g_2^2\kappa + 2 \text{Tr}(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e)\kappa, \quad (21)$$

where g_2 is the SU(2) gauge coupling constant and where Y_u , Y_d are the Yukawa matrices for the up and the down quarks. Thus, compared to earlier results [7], we find a coefficient $-3/2$ instead of $-1/2$ in front of the non-diagonal term $\kappa(Y_e^\dagger Y_e) + (Y_e^\dagger Y_e)^T \kappa$. Note that the difference in the $\lambda\kappa$ -term is due to a different convention for the Higgs self-interaction used in this work.

We have checked our results by calculating the essential parts of the same β -functions from the finite parts of the relevant diagrams in the framework of an underlying renormalizable theory. This calculation as well as the application to the MSSM and the two Higgs SM will be presented in a future paper [9].

6. Discussion and conclusions

We have calculated in the SM the β -function for the effective coupling κ of the dimension 5 operator which corresponds to a Majorana mass matrix for neutrinos. We have explicitly presented our calculations for the non-diagonal part of the β -function, where our result disagrees with the previous one in [7] by a factor of 3.

This part is responsible for the evolution of neutrino mixing angles and CP phases. Therefore, our result modifies the renormalization group running of these quantities between predictions of models at high energies and experimental data at low energies. Consequently, our work affects the SM results of previous studies based on the existing RGEs, e.g., [10–15].

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