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Debt Renegotiation and Sovereign Defaults

A Dissertation submitted in partial satisfaction
of the requirements for the degree of

Doctor of Philosophy

in

Computer Science

by

Taghi Farzad

September 2022

Dissertation Committee:

Dr. Marcelle Chauvet, Chairperson
Dr. Urmee Khan
Dr. Jana Grittersova
Dr. DongWon Lee

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The Dissertation of Taghi Farzad is approved:

Committee Chairperson

University of California, Riverside

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I am grateful to my advisors, Professors Chauvet, Khan, Grittersova, and Lee without whose help and support I could not be here.

To my parents, my sisters, and the love of my life.

ABSTRACT OF THE DISSERTATION

Debt Renegotiation and Sovereign Defaults

by

Taghi Farzad

Doctor of Philosophy, Graduate Program in Computer Science
University of California, Riverside, September 2022
Dr. Marcelle Chauvet, Chairperson

This study develops a model of endogenous default with debt renegotiation for emerging economies. A small open economy faces a stochastic stream of income. The government borrows from international financial markets and makes decisions on behalf of the country's residents. Lenders are risk-neutral and operate in a perfectly competitive financial market. Upon default, the borrowing country and the lenders engage in debt restructuring negotiations which are modeled by a Nash bargaining game. The models are calibrated to capture the default episodes in Argentina. This body of work illustrates how adding the endogenous debt negotiation feature to the borrowing-default models can provide results that are closer to the data.

The first chapter studies the maturity structure. The government can issue short- and long-term bonds. Upon default, the borrowing country loses access to the financial markets and will not be able to borrow any longer. The defaulted country has to pay off the principal and interest of the restructured debt to regain access to the credit market. The resulting equilibrium haircut is directly related to the debt level. The resulting interest rate distributions for short- and long-term bonds closely match the observed data. Providing a precise interest rate distribution is crucial as finding the optimal maturity structure relies on it. The paper finds that endogenous debt renegotiation is an important mechanism in generating more realistic fluctuations of the interest rate.

The second chapter explains how the introduction of Brady Bonds was followed by a reduction in the default frequency in Latin American countries during the 1990s. Prior to that, loans from syndicated banks were used as the main debt instrument. This is puzzling since bondholders have lower bargaining power than syndicated banks, hence borrowing countries are expected to default more frequently. This chapter shows that introducing bonds lower interest rates and, consequently, increases the opportunity cost of default.

In the last chapter, we study partial defaults, that is a borrowing country defaulting on some of the outstanding debt while honoring the rest. The model with endogenous debt negotiations provides results that match the frequency and length of default episodes in data.

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Chapter 1

Maturity Structure and Debt Renegotiation in Sovereign Bonds

1.1 Introduction

The Recurrence of financial crises and prolonged debt settlement process in emerging countries induce questions regarding the role of debt structure in sovereign defaults. Thus, debt maturity composition and its impact on the interest rate draw a lot of research attention. It is a well-documented fact that the interest rate plays an important role in emerging economies[1]. Hence, it is crucial to investigate the reciprocal

[1] See Neumeyer and Perri (2005), Aguiar and Gopinath (2006)

effect of debt maturity structure and the interest rate spreads faced by an emerging economy. In particular, Broner, Lorenzoni, and Schmukler (2013) show that for emerging economies short-term debts are generally cheaper than long-term debts. Due to an increase in dilution risks[2], the long-term borrowing is even costlier near financial crises. As a result, countries will shorten the maturity composition in the imminence of a financial crisis. This implies that while long-term borrowing is expensive, the sovereign government suffers massively from rolling over the short-term debt near default episodes.

This paper provides a default model with two debt instruments, short- and long-term bonds, and endogenous debt renegotiation. A risk-averse sovereign faces a stream of income and issues bonds that will be traded in a perfectly competitive market with risk-neutral lenders. Contingent on the state of the economy depending on income shocks and short- and long-term debts, the sovereign may find it optimal to default[3]. Lenders determine the price of a bond taking into account these default probabilities. Upon default, an endogenous haircut will be applied to the short- and long-term bonds. Adding the endogenous debt renegotiation feature to the model allows us to further study the dilution risk.

[2]Dilution risk can be defined as the capital loss for the current long-term lenders after issuance of new bonds. While both short- and long-term bonds are affected by the default risk, only the latter is impacted by the dilution risk.

[3]This paper does not consider the possibility of partial defaults.

This paper is closely related to a set of previous studies. Arellano and Ramanarayanan (2012) proposed a similar model but with no recovery, i.e. 100% haircut. Hatchondo, Martinez, and Sosa-Padilla (2016) proposed a model with exogenous debt dilution that was constant over the states. Bi (2006) applies Yue (2010) renegotiation approach to one- and two-period bonds, representing short- and long-term bonds, respectively. This paper improves Bi's (2006) approach by allowing the country to issue bonds with longer maturities. In order to achieve this, we model bonds as perpetuity contracts with non-state-contingent payments that decay at a constant rate[4]. As in Macaulay (1938), different decay rates can represent bonds with different durations. This allows us to circumvent the curse of dimensionality and study the effect of bonds with longer maturities. Bi (2006) assumes the recovery rate is a function of the total outstanding debt[5]. In this paper, the state-contingent debt is a function of the income and of short- and long-term debts, instead of total debt. We find that expressing the recovery rate as a function of total debt is a specific case of our model when the sovereign is forced to pay the arrears in one period. We conclude that this can lead to suboptimal decision policies.

As explained in Bi (2006), interest rate and debt dilution are the main factors affecting default episodes. Most of the papers that extend Eaton and Gersovitz (1981) address

[4]see e.g. Arellano and Ramanarayanan 2012, Hatchondo et al. 2016, and Hatchondo and Martinez 2009.

[5]As discussed in the second chapter, when the sovereign issues one-period bonds and loans, the recovery rate is a function of total debt. However, in the presence of bonds with different maturity, this approach relies on the presumption that upon default the total debt will be repaid in one period. The default episodes are more likely when the income realization is low and, assuming income persistence, the income is likely to remain low in the future periods. This means the country that is forced to pay back the debt in one period suffers massively from default. This process may artificially reduce the default incentives.

the effect of endogenously determined interest rates. The effect of debt dilution is either discarded (Arellano 2008, Arellano and Ramanarayanan 2012) or modeled by a constant exogenous factor (Hatchondo et al. 2016). This paper models the renegotiation using the Nash Bargaining process, as in Yue (2010) and Bi (2006). As in Bi (2006), the main focus of the proposed model is on *pari passu* debt contracts; all outstanding debts are treated equally upon default. Hence, the same haircut is applied to short- and long-term debts. Bi (2006) argues how lenders tend to hold short-term bonds near the default to prevent debt dilution. Hatchondo et al. (2016) show that without debt dilution the optimal maturity of debt increases by 2 years. Hence, it is crucial to take into account the effect of debt dilution in addition to the interest rate impacts.

We calibrate the model to the Argentina data. We find that the model predictions are closely related to patterns in the data. In particular, the spread dynamics match the observed data: The spread curves (difference between the interest rate of a maturity and corresponding risk-free interest rate) are upwards sloping during tranquil times, but flatten out or even invert closer to the financial crisis. We show that the proposed model with endogenous debt renegotiation delivers spread distribution closer to data compared to models with either no recovery rate or an exogenous one. In short, the equilibrium endogenous recovery rate allows the borrowing country to hold higher levels of debt before declaring a default. Consequently, the observed interest rates are higher than the models without debt renegotiation. At the onset of the crisis, the debt dilution effect reduces the long-term bond prices more than the price of the short-term bonds. Hence borrowing countries shorten the maturity composition. Issuing short-term bonds increases the default probability but does not affect the debt dilution risk. This results in an even larger short-term spread. Studying the behavior of interest rates as done in this paper is important for two main reasons. First, high-interest rates are leading indicators of a financial crisis. Second, finding the optimal maturity depends on analyzing the interest dynamics for different maturities. The proposed model can be used as an important policy tool to predict and understand the dynamics of financial crises related to debt default.

This paper studies the effect of country debt renegotiation on maturity structure and interest rate spread in emerging economies. It proposes a default model with two debt instruments, short- and long-term bonds, and endogenous debt renegotiation. Finding the optimal maturity debt composition relies on information regarding interest rate dynamics. In the data, long-term spreads are higher than short-term spreads during tranquil times. However, in anticipation of a default, spread curves flatten or even become inverted. Nevertheless, borrowing countries will shorten their maturity structure. This shift in debt maturity is attributed to a significant reduction in long-term prices. The traditional literature explains this price reduction introducing two types of risks: default and dilution risks. While default risk impacts both short- and long-term bond prices, dilution risk only affects the latter. Endogenous debt restructuring as proposed in this paper sheds light on the dilution risk: since equilibrium debt recovery rate is negatively related to debt level, dilution risk increases with debt levels. The equilibrium debt recovery rate results in higher bond issuance compared to the models with no debt renegotiation. Consequently, the proposed model with endogenous debt restructuring delivers higher interest rates that closely match observed interest rate dynamics.

Literature Review

This paper is related to several strands of research. It builds on the seminal work of Eaton and Gersovitz (1981) that developed an endogenous default model. This paper also emphasizes the (endogenous) interest rate role in the financial crisis. This is in line with Neumeyer and Perri (2005) conclusion that interest rate spread is an important factor in explaining the business cycle fluctuations in emerging economies. As indicated by Aguiar and Gopinath (2006) models with one-period bonds are not capable of fully capturing the spread fluctuations. Arellano (2008) shows increasing the default cost in a model with one-period bonds can induce higher spreads. To generate a positive spread (even when the economy is not exposed to a high default probability), researchers added debts with longer maturities: Even in tranquil times, the sovereign is exposed to a "bad" income shock over the span of a long-term debt that may lead to a default episode. Taking into account this unpleasant draw, long-term lenders will charge a higher interest rate. Chatterjee and Eyigungor (2008), enriched Arellano (2008) by letting the sovereign issue bonds with longer maturities. The results provide a better fit for the spread behavior in emerging economies.

Broner et al. (2013) studied the effect of debt maturity composition in emerging economies. They document that the risk premium on long-term debts is relatively higher, and would increase during a crisis. This will shift the debt issuance toward short-term debts. To address this, Arellano and Ramanarayanan (2008) and Hatchondo and Martinez (2009) developed models that include both short- and long-term bonds. The long-term bonds are modeled as perpetuity payments with a decay rate. Following Macaulay (1938), finite durations can be modeled with a proper decay rate[6]. This allows us to develop a model in which debt maturity can be changed by choosing another decay rate.

Hatchondo et al. (2016) and Bi (2006) improved the previous works by studying the debt dilution effects. Hatchondo et al. (2016) use a constant exogenous recovery rate that is applied to defaulted debts. Bi (2006) applies the Nash Bargaining (à la Yue (2010)) to find the endogenous recovery rates for one and two-period bonds. Bulow and Rogoff (1989) proposed a model with Rubinstein's (1982) type of bargaining in the debt rescheduling process. As indicated by Bi (2006), the Nash Bargaining approach is more tractable and can be supported by Rubinstein game or other complicated games[7].

[6]Hatchondo and Martinez (2009) explain this approach is used in empirical studies as well as credit rating agencies.

[7]For instance see Bai and Zhang (2009) for the application of stochastic bargaining models.

Sturzenegger and Zettelmeyer (2008) show the inter-creditor equity was ex-post violated for many of the debt restructuring between 1998 and 2005. This is mainly due to higher post-exchange yields, which result in a lower NPV for higher maturity debts. Since there is not enough evidence in favor of ex-ante discrimination, Hatchondo et al. (2016), Bi (2006), and the second chapter of this study all assume the same debt recovery rate for different debt instruments. This is based on the assumption of pari passu debts. Collective action clauses (CACs) and comparability of treatment (in Paris Club principals) can further justify applying the same haircut[8]. Due to the lack of legally-binding rules regarding the seniority of debts, we discarded the priorities in debt repayment.

The outline of the chapter is as follows: Section 2 provides the theoretical model. Section 3 presents the theoretical results and propositions for the chapter. Section 4 contains the quantitative analysis for the model, and finally, section 5 concludes the chapter.

1.2 Model

Time is discrete and is indexed by $t \in \{0, 1, \dots\}$. The economy faces a stream of income $\{y_1, y_2, \dots y_n\}$ that follows a Markov process with transition matrix $f(y_{t+1}|y_t)$.

[8] See Das et al (2012) and the second chapter

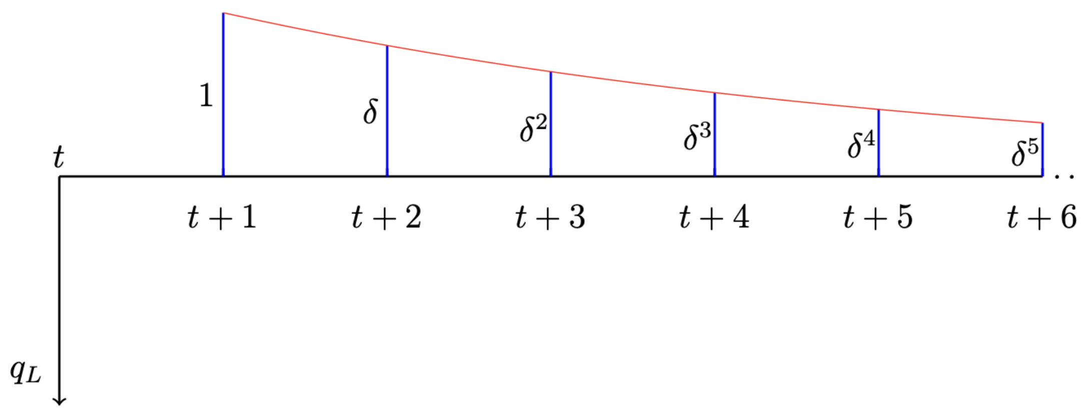


Figure 1.1: Long-Term Bond Representation.

There is a representative agent that lives forever, with preferences represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $0 < \beta < 1$ is the discount factor. Later we assume u is a CARA utility function, hence it is continuous, strictly increasing, and strictly concave. Each period, a benevolent sovereign decides on the level of consumption and borrowings. Government can issue either short-term, B_S, or long-term, B_L, bonds. A short-term bond is a one-period bond sold at the discount price q_S and delivers the face value next period. A long-term bond, as in Arellano and Ramanarayanan (2012) and Hatchondo and Martinez (2009), is modeled by a perpetuity contract with coupon payments that decay geometrically. Figure 1.1 depicts payments associated with the perpetuity contract of a bond issued at time t with a loan face value[9] of one.

[9]Face value of perpetuity does not affect the price.

Appealing to Macaulay's (1938) approach, one is able to use an appropriate decay rate to model a loan contract with finite duration. Macaulay defines the duration as the weighted sum of future payment dates. For instance, duration of a perpetuity with coupon payments c_n and initial price of q is

$$d = \sum_{n=1}^{\infty} n \frac{c_n (1+r)^{-n}}{q}$$

that is, each date n is weighted by the normalized (divided by the initial price $q = \sum_{n=1}^{\infty} \frac{\delta^{n-1}}{(1+r)^n}$) discounted coupon payments on that date. Applying this definition to the risk-free bonds in this paper results in

$$d = \sum_{n=1}^{\infty} n \frac{c_n (1+r)^{-n}}{q} = \frac{1}{\delta q} \sum_{n=1}^{\infty} n \delta^n (1+r)^{-n} = \frac{1}{\sum_{n=1}^{\infty} (\frac{\delta}{1+r})^n} \sum_{n=1}^{\infty} n (\frac{\delta}{1+r})^n$$

Which simplifies as

$$d = \frac{1+r}{1+r-\delta}$$

where r is the risk-free interest rate. With no default, next period, the stock of long-term bonds is decayed by δ; independent of the issuance date each bond is decayed by the factor of δ. In addition, the sovereign's borrowing (lending) this period will add to (subtract from) the next period's stock of the bonds. Hence the law of motion for stock of long-term bonds can be defined as

$$B_{L,t+1} = \delta B_{L,t} + l_t$$

where l_t is current period's issuance or repayment of long-term bonds[10]. The price of new issued bonds, either short- or long-term, depends on the sovereign's state;

$$q_t^S(B_{S,t+1}, B_{L,t+1}, y_t), \;\; q_t^L(B_{S,t+1}, B_{L,t+1}, y_t).$$

Borrowing Country

There is a benevolent government that makes decisions regarding consumption and the issuance (or repayment) of short- and long-term bonds. At a given state (B_S, B_L, y), a sovereign has an option to either service the debt and get the value of $W(B_S, B_L, y)$ or default and get $V^D(B_S, B_L, y)$. The value of the sovereign, $V(B_S, B_L, y)$, is the maximum of these two:

$$V(B_S, B_L, y) = \max\Big\{W(B_S, B_L, y), V^D(B_S, B_L, y)\Big\}$$

Contingent on honoring the debt, current consumption and new issuance (repayment) of each type of bonds will be determined according to the following problem:

[10]For the one period bonds, δ is zero; hence, the stock of short-term bonds will be the same as the amount issued (or bought) in the current period.

$$W(B_S, B_L, y) = \max_{B'_S,\ B'_L,\ 0 \leq C} u(C) + \beta \int_Y V(B'_S, B'_L, y') f(y'|y) dy'$$

$$s.t. \qquad C + B_S + B_L = y + q^S(B'_S, B'_L, y) B'_S + q^L(B'_S, B'_L, y) l'_L$$

$$B'_L = \delta B_L + l'_L$$

Defaulted sovereign will be excluded from the capital market, hence only will be able to consume the current income. The arrears consist of the principal and the due interest. Contingent on the state of the economy, a haircut $1 - \alpha$ will be applied to the outstanding short-term and long-term debt levels.

$$V^D(B_S, B_L, y) = u(y) + \beta \left\{ \int_Y W^D(\alpha(1+r)B_S, \alpha(1+r)B_L, y') f(y'|y) dy' \right\}$$

We can assume the country has the option of permanently leaving the credit market. However, with the assumptions on the utility function and the parameters used in these papers, it is never optimal to do so. Direct output loss, λ, and lacking a smoothed consumption profile make the Autarky a suboptimal choice.

$$V^{AUT}(y) = u((1-\lambda)y) + \beta \int_Y V^{AUT}(y') d\mu(y'|y)$$

Following Yue (2010), it is assumed that the sovereign will regain access the credit market after paying the arrears in full. Until then, the defaulted country has to (weakly) decrease short- and long-term debts, $0 \leq B'_S \leq B_S,\ \ 0 \leq B'_L \leq B_L$. Since the country is a net lender, the discount price for short- and long-term bonds will be the corresponding risk-free interest rate:

$$W^D(B_S, B_L, y) = \max_{0 \leq B'_S \leq B_S,\ 0 \leq B'_L \leq B_L,\ 0 \leq C} u(C) + \beta \int_Y W^D(B'_S, B'_L, y') f(y'|y) dy'$$

$$C + B_S + B_L = (1 - \lambda)y + \frac{B'_S}{1+r} + \frac{l'_L}{1+r-\delta}$$

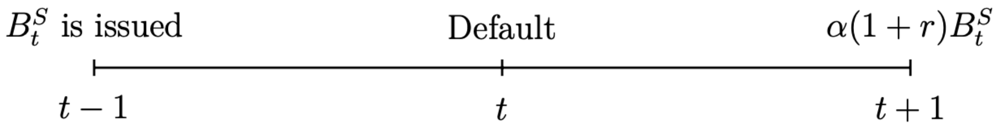
$$B'_L = \delta B_L + l'_L \tag{1.1}$$

When the arrears are fully paid, the sovereign will regain access the credit market and will be able to issue new bonds:

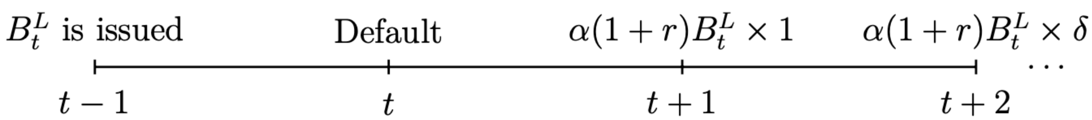
$$W^D(0, 0, y) = V(0, 0, y)$$

Debt Renegotiation Problem

$$PV_{t-1} = \frac{\alpha(\cdot)(1+r)B^L_t}{(1+r-\delta)(1+r)}$$



$$PV_{t-1} = \frac{\alpha(\cdot)(1+r)B_t^S}{(1+r)^2} = \frac{\alpha(\cdot)B_t^S}{(1+r)}$$



$$PV_{t-1} = \frac{\alpha(\cdot)(1+r)B_t^L}{(1+r-\delta)(1+r)}$$

Figure 1.2: Present Value of Defaulted Bonds

Upon default, creditors and debtors engage in a one-round renegotiation process that determines the haircuts. Following Yue (2010), the debt renegotiations will be modeled by a Nash Bargaining game; the optimal recovery rate, α, will maximize the total surpluses of creditors and the debtor. The debtor's surplus is the difference between the default value (implying the debt renegotiations were successful and a haircut $1-\alpha$ will be applied) and the Autarky value (implying the debt renegotiation process has failed):

$$\Delta_D(B_S, B_L, y; \alpha) = V^D(B_S, B_L, y; \alpha) - V^{AUT}(y)$$

Figure 1.2 shows the present value of short- and long-term debts issued at time $t-1$. Present values of the defaulted bonds at time t are calculated in the Figure as well. Creditors' surplus will be the present value of the recovered total debt:

$$\Delta_C(B_S, B_L, y; \alpha) = \frac{\alpha}{1+r}\left[B_S + \frac{B_L(1+r)}{1+r-\delta}\right]$$

Let $\theta \in \Theta \subset [0,1]$ be the debtor's bargaining power. The equilibrium recovery is determined in a Nash-bargaining process and maximizes the total surplus:

$$\begin{aligned} \alpha(B_S, B_L, y) = \arg\max_{\hat{\alpha}\in[0,1]} & \left[\Delta_D(B_S, B_L, y; \hat{\alpha})\right]^{\theta}\left[\Delta_C(B_S, B_L, y; \hat{\alpha})\right]^{(1-\theta)} \\ s.t. \qquad & \Delta_D(B_S, B_L, y; \hat{\alpha}) \geq 0, \;\; \Delta_C(B_S, B_L, y; \hat{\alpha}) \geq 0 \end{aligned} \tag{1.2}$$

Creditors' Problem

Creditors are assumed to be risk-neutral that inelastically access the capital at the risk-free interest rate. Hence bond prices will be the present value of the expected future payments. Assume the short- and long-term debt are B_S and B_L, respectively. Following Arellano and Ramanarayanan (2012), let $R(B_S, B_L)$ denote the set of output levels at which the country services the debt, and $D(B_S, B_L)$ the set of output levels at which default is optimal:

$$R(B_S, B_L) = \left\{y : W(B_S, B_L, y) \geq V^D(B_S, B_L, y)\right\}$$

$$D(B_S, B_L) = \left\{y : W(B_S, B_L, y) < V^D(B_S, B_L, y)\right\} \tag{1.3}$$

One-period bonds are expected to pay back the face value if the sovereign honors the debt, and the recovered value (α fraction of the face value) if the sovereign defaults. Hence, one can write the price of the one-period bond as:

$$q_{S,t} = \int\limits_{R(b^S_{t+1}, b^L_{t+1})} \frac{f(y_{t+1}, y_t)}{(1+r)} dy_{t+1} + \int\limits_{D(b^S_{t+1}, b^L_{t+1})} \alpha(b^S_{t+1}, b^L_{t+1}, y_{t+1}) \frac{f(y_{t+1}, y_t)}{(1+r)} dy_{t+1} \tag{1.4}$$

For the long-term bonds, contingent on servicing the debt till n periods ahead, the present value of the period $t+n$ to creditor is $\frac{\delta^{n-1}}{(1+r)^n}$. The country may defaults in period $t+n$, contingent on servicing the debt in periods $t+1, t+2, \ldots, t+n-1$. Upon default, it is expected to receive α fraction of the the outstanding debt $\delta^{n-1}\frac{1+r}{1+r-\delta}$. Notice this paper assumes after default, the sovereign has to pay back all the outstanding debts before issuing new bonds. This implies countries cannot default twice on the same bond.

$$q_t^L = \sum_{n=1}^{\infty} \frac{\delta^{n-1}}{(1+r)^n}$$

$$\int\limits_{R(b_{t+1}^S, b_{t+1}^L)} \cdots \int\limits_{R(b_{t+n}^S, b_{t+n}^L)} f(y_{t+n}, y_{t+m-1}) \dots f(y_t + 1, y_t) dy_{t+n} \dots dy_{t+1}$$

$$+ \sum_{n=1}^{\infty} \frac{\delta^{n-1}}{(1+r)^n} \int\limits_{R(b_{t+1}^S, b_{t+1}^L)} \cdots \int\limits_{D(b_{t+n}^S, b_{t+n}^L)} \alpha(b_{t+n}^S, b_{t+n}^L, y_{t+n}) \frac{1+r}{1+r-\delta}$$

$$f(y_{t+n}, y_{t+n-1}) \dots f(y_t + 1, y_t) dy_{t+n} \dots dy_{t+1}$$

As in Arellano and Ramanarayanan(2012), one can write the above expression in a recursive form:

$$q_t^L = \int\limits_{R(b_{t+1}^S, b_{t+1}^L)} \frac{f(y_{t+1}, y_t)}{(1+r)} dy_{t+1} + \int\limits_{D(b_{t+1}^S, b_{t+1}^L)} \alpha(b_{t+1}^S, b_{t+1}^L, y_{t+1}) \frac{1+r}{1+r-\delta} \frac{f(y_{t+1}, y_t)}{(1+r)} dy_{t+1}$$

$$+\delta \int\limits_{R(b_{t+1}^S, b_{t+1}^L)} \left[\int\limits_{R(b_{t+2}^S, b_{t+2}^L)} \frac{f(y_{t+2}, y_{t+1})}{(1+r)^2} dy_{t+2} \right] f(y_{t+1}, y_t) dy_{t+1}$$

$$+\delta \int\limits_{R(b_{t+1}^S, b_{t+1}^L)} \left[\int\limits_{D(b_{t+2}^S, b_{t+2}^L)} \frac{(1+r)\alpha(b_{t+2}^S, b_{t+2}^L, y_{t+2})}{1+r-\delta} \frac{f(y_{t+2}, y_{t+1})}{(1+r)^2} dy_{t+2} \right] f(y_{t+1}, y_t) dy_{t+1}$$

$$+ \dots$$

since the expression in the brackets is q_{t+1}^L, the the price function reduces to:

$$q_t^L = \frac{1}{1+r}\Bigg\{ \int\limits_{R(b_{t+1}^S, b_{t+1}^L)} \Big[1 + \delta q_{t+1}^L(b_{t+2}^S, b_{t+2}^L, y_{t+1})\Big] f(y_{t+1}, y_t) dy_{t+1}$$

$$+ \int\limits_{D(b_{t+1}^S, b_{t+1}^L)} \frac{1+r}{1+r-\delta}\alpha(b_{t+1}^S, b_{t+1}^L, y_{t+1}) f(y_{t+1}, y_t) dy_{t+1} \Bigg\} \tag{1.5}$$

For $\delta = 0$, this will reduce to the price of the short-term bond, expressed in 5. With 100% haircuts, the above expression will be reduced to Arellano and Ramanarayanan (2012). Furthermore, with $\alpha = 1$ for all the states, the long-term bond will turn to a risk-free bond:

$$(1+r)q_L = \int\limits_{R(b_{t+1}^S, b_{t+1}^L)} 1 + \delta q_{t+1}^L(b_{t+1}^S, b_{t+1}^L, y_{t+1}) f(y_{t+1}, y_t) dy_{t+1}$$

$$+ \int\limits_{D(b_{t+1}^S, b_{t+1}^L)} \frac{1+r}{1+r-\delta} f(y_{t+1}, y_t) dy_{t+1}$$

$$+ \int\limits_{R(b_{t+1}^S, b_{t+1}^L)} \frac{1+r}{1+r-\delta} f(y_{t+1}, y_t) dy_{t+1} - \int\limits_{R(b_{t+1}^S, b_{t+1}^L)} \frac{1+r}{1+r-\delta} f(y_{t+1}, y_t) dy_{t+1}$$

$$\left(1 + r - \delta \int\limits_{R(b_{t+1}^S, b_{t+1}^L)} f(y_{t+1}, y_t) dy_{t+1}\right) q_L = \left(1 + r - \delta \int\limits_{R(b_{t+1}^S, b_{t+1}^L)} f(y_{t+1}, y_t) dy_{t+1}\right) \frac{1}{1+r-\delta}$$

$$q_L = \frac{1}{1 + r - \delta}$$

1.3 Results

Recursive Equilibrium

A recursive equilibrium for this economy consists of a set of functions defined below; for $s = (B_S, B_L, y)$:

- The country's value functions, $V(s), W(s), V^D(s), W^D(s)$, and $V(y)^{AUT}$ and policy functions of short- and long-term bonds, $B'_S(s), B'_L(s)$, and consumption, $C(s)$,

- Default set, $D(B_S, B_L)$, and the repayment set $R(B_S, B_L)$,

- Price functions $q_S(B'_S, B'_L, y)$ and $q_L(B'_S, B'_L, y)$ in 4 and 5, such that given the recovery rate $\alpha(s)$

  1. The default and the repayment sets are the equilibrium sets defined above,

  2. Next period's bond holdings are in agreement with the country's policy functions:

  $$b^S_{t+1} = B'_S(s), \;\; b^L_{t+1} = B'_L(s)$$

- The recovery rate function $\alpha(s)$ such that

1. Given the bond prices and the recovery rate, country solves the recursive problem

2. Given the bond prices, the value functions, the policy functions, and the default and repayment sets, the recovery rate solves the debt renegotiation problem.

3. Given the recovery rate, bond prices satisfy the zero profit condition for the bondholders.

Proposition 1: $\forall \theta \in \Theta$ recursive equilibrium of the above model exist.

Lemma 1: The debt renegotiation problem is invariant to any transfer of debt that keeps $B_S + \frac{B_L(1+r)}{1+r-\delta}$ unchanged.

Proof: See Appendix.

Proposition 2: The equilibrium recovery rate, $\alpha(B_S, B_L, y)$, satisfies

$$\alpha(B_S, B_L, y) = \begin{cases} 1 & (B_S + d^\delta B_L) \leq \zeta(y) \\ \frac{\zeta(y)}{(B_S + d^\delta B_L)} & (B_S + d^\delta B_L) \geq \zeta(y) \end{cases} \tag{1.6}$$

where $d^\delta = \frac{1+r}{1+r-\delta}$.

Proof: See Appendix.

This result extends Yue (2010) to two dimensions to instruments with different maturities[11]. Upon default, for each level of endowment y, the value function of default is independent of $B_S + d^\delta B_L$, henceforth called the total dated debt. This simplification helps us to derive the same set of results as in Eaton and Gersovitz (1981) and Chatterjee et al. (2007), Arellano (2008), Yue (2010), and Bi (2006) all explained below.

Proposition 3: If default is optimal for a state (B_S^1, B_L^1, y), then it is also optimal for all (B_S^2, B_L^2, y) that $D^2 = B_S^2 + d^\delta B_L^2 \geq B_S^1 + d^\delta B_L^1 = D^1 \geq D(y)$. Hence $D(B_S^1, B_L^1) \subseteq D(B_S^2, B_L^2)$.

Corollary 1: If default is optimal for a state (B_S^1, B_L, y), then it is also optimal for all (B_S^2, B_L, y) that $B_S^2 \geq B_S^1$.

Corollary 2: If default is optimal for a state (B_S, B_L^1, y), then it is also optimal for all (B_S, B_L^2, y) that $B_L^2 \geq B_L^1$.

Corollary 3: Default probability is increasing in B_S and B_L.

Proof: See Appendix.

[11] for one-period debt instruments, $\delta = 0$, the relation reduces to $B_S + B_L$, similar to the second chapter.

Proposition 4: For any level of endowment the equilibrium price of short- (long-) term bond is decreasing in quantity demanded for short- (long-) term bond:

$$q_S(B_S^2, B_L, y) \leq q_S(B_S^1, B_L, y), \;\; \forall B_S^2 \geq B_S^1$$

$$q_L(B_S, B_L^2, y) \leq q_L(B_S, B_L^1, y), \;\; \forall B_L^2 \geq B_L^1$$

Proof: See Appendix.

Proposition 5: Sovereign's value functions are increasing in the realized income. Hence, default incentives are higher at lower income levels.

Proof is the direct application of contraction mapping theorem[12]. Proposition 6 below builds on Yue (2010) to show it might be optimal for the sovereign to pay back the outstanding debt over the span of multiple periods. Let $\tilde{W^D}(D, y) = W^D(B_S, B_L, y)$ when $D = B_S + d^{\delta} B_L$.

Proposition 6: Consider the repayment problem 1, for a given y, if $D' = 0$ is optimum for a state described by $D = B_S + B_L$ and y, then $D' = 0$ is also optimal for all $\hat{D} < D$. Also partial payment, $D' < 0$, is optimal for all $\hat{D} > D$, if

$$\tilde{W^D}(D, y) = u\big((1 - \lambda)y - D\big) + \beta \int_Y v(0, 0, y') f(y', y) dy'.$$

[12]See Stockey, Lucas, and Prescott (1989)

Proof: See Appendix.

Solution Algorithm

First, we need to discretize the asset and the income space. For the asset spaces, short- and long-term bonds, we choose an upper-bound large enough that doesn't distort the optimization solution. For the income space, we use Tauchen (1986) approach to find the Markov chain approximation of the endowment. The solution algorithm is as follows:

1. Start with an arbitrary initial value for the recovery rate, α. For simplicity we can start with $\alpha \equiv 0.50$.

2. Choose arbitrary values for bond prices, q_S and q_B. One can assign the risk-free prices as the initial values.

3. Solve the sovereign's problem and derive the policy functions, repayment, and default sets.

4. Update the prices according to 4 and 5 . Find the difference between the initial prices and the updated ones. Solve the sovereign's problem in step 3 until the updated prices are close enough to the ones calculated before.

5. Find the recovery rate that solves the Nash bargaining game in 2 Find the difference between the initial value and the updated one, and start over from step 3 as long as this difference is above the preset tolerance.

1.4 Quantitative Analysis

Parametrization

Table 1.1 shows the selected parameters for the quantitative analysis. As mentioned before, preferences are modeled with a CARA utility function. Following the literature, the coefficient of risk aversion, σ, is set to two:

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma}$$

.

Endowment process is estimated using Argentina's GDP using data from the Ministry of Finance (MECON). The quarterly data (real, seasonally adjusted) starts from the first quarter of 1980 till the default episode of 2001, the last quarter of 2001. As in Arellano (2008), this paper assumes a log-normal AR(1) process for the GDP:

$$\log(y_t) = \rho \log(y_{t-1}) + \varepsilon_t, \quad E[\varepsilon] = 0 \text{ and } E[\varepsilon^2] = \eta^2$$

The estimated persistence, ρ and error standard deviation, η, are $\rho = 0.95$ and $\eta = 0.02$. Then a Markov chain with 21 discrete endowment states is constructed Using Tauchen (1986).

The annual risk-free interest rate is set to $r = 4\%$, which is the average 1-year yield of the US bonds in that period. Decay rate, $\delta = 0.936$ is selected to represent 10-year default-free duration. Output loss during default is $\lambda = 2\%$, as suggested by Sturzenegger (2002).

Time preference, β, and borrower's bargaining power, θ are calibrated to match the model moments to the observed data. Time preference is calibrated to match the default frequency in data. According to Reinhart, Rogoff, and Savastano (2003), Argentina experienced four default episodes from 1824 to 1999. Including the 2001 default results in an annual (quarterly) default frequency equals 2.8% (0.7%). According to Benjamin and Wright (2009), the average debt recovery rate in the 2001 default was 37%. In order to match this moment, the debtor's bargaining power, θ is calibrated to $\theta = 0.83$.

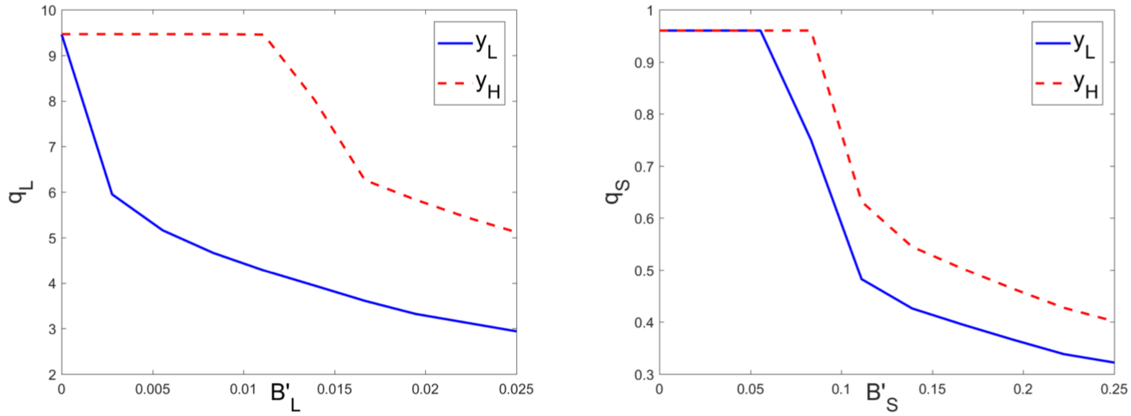
Parameter	Value	Target
one-year risk-free rate	$r = 4\%$	U.S. annual interest rate
Risk Aversion	$\sigma = 2$	Literature
Decay factor	$\delta = 0.936$	10-year Default-free duration
Output loss	$\lambda = 2\%$	Sturzenegger (2002)
Stochastic structure	$\rho = 0.95$, η=0.025	Argentina output (1980Q1: 2001Q4)

Calibration	Value	Target statistics
Borrower's discount factor	$\beta = 0.94$	2.8% Default Probability
Borrower's bargaining power	$\theta = 0.83$	37% Ave. Debt Recovery Rate

Table 1.1: Parameter Values

Simulation Results

Figure 1.3 plots the price functions for short- and long-term bonds as a function of the choice of short-term and long-term debts, respectively. To illustrate the role of endowment, the prices are depicted for high and low values of income, y_H and y_L. As stated by Proposition 4, the price functions are decreasing functions in their corresponding choice variables; which easily can be seen in the Figure. Price functions clearly show the term premium for the low levels of income; for the low levels of short-term debt holding, the price is the same as the risk-free bond, while the price of the long-term bond is below the risk-free price even for a small stock of long-term holdings. Only for high levels of income and low levels of long-term debt, the term premium disappears.



(a) Long Term Bond Price (b) Short Term Bond Price

Figure 1.3: Bond Price Functions. Panel (a) shows the price of the long-term bond as a function of next period's stock of long-term bonds for two income levels y_H and y_L. Panel (b) shows the corresponding values for the short-term bond.

Table 1.2 summarizes the model statistics. As shown in the table, the mean spread for short- and long-term bonds are 4.32% and 6.21%, respectively. These values are close to the corresponding values in Argentina data. Nevertheless, the short-term spreads are higher than the observed values in the data, while the long-term spread is below the observed data. The ratio of trade balance (TB) standard deviation to output standard deviation in the model closely follows the data. The model overestimates consumption standard deviation, which may suggest using a more sophisticated output loss function. The average debt to GDP in the model is 0.36, which explains more than 80% of the debt to GDP in data. This is an improvement compared to the previous models; Arellano and Ramanarayanan (2012) could only explain half of the debt to GDP ratio for Brazil. The model captures the short-term debt to the total debt; 8% in the model versus 11% in the data. Finally, the average default duration in the model matches the data. Allowing the defaulted country to pay back the arrears in more than one period, provides a more realistic model. The last two rows of Table 2 show the calibrated parameters.

Model Statistics

	Model	Data
$Mean\ s_S(\%)$	4.32	4.22
$Mean\ s_L(\%)$	6.21	6.29
$SD(TB)/SD(y)$	0.47	0.48
$SD(C)/SD(y)$	1.11	1.03
$(q_S B_S + q_L B_L)/y$	0.36	0.43
$(q_S B_S)/(q_S B_S + q_L B_L)$	0.08	0.11
Default Duration (years)	4.40	4.50
Default Frequency	2.80%	2.80%
Ave. Haircut Rate	38.35%	37%

Table 1.2: Model Statistics

Spread Curves

To provide a clearer picture regarding the spread behaviors, we provide the spread distribution for different model specifications. In particular, the model in this paper is compared to a model with a zero recovery rate, á la Arellano and Ramanarayanan (2012), and with a constant exogenous recovery rate, á la Hatchondo et al. (2016). The corresponding distributions are reported in Table 1.3.

Spread Curves

	s_S			s_L		
	Overall Mean	$< 50th$	$\geq 50th$	Overall Mean	$< 50th$	$\geq 50th$
Data	4.22	1.12	7.32	6.29	4.65	7.93
Model	4.32	1.09	7.56	6.21	4.38	8.04
$\alpha = 0\%$	3.15	0.03	6.27	3.15	2.24	4.06
$\alpha = 37\%$	3.81	0.68	5.86	5.63	4.20	7.06

Table 1.3: Effect of Endogenous Debt Renegotiation

As the table suggests, the model provides spread distributions that match the data. To put this into perspective, two sets of spread distributions are provided: first spread distributions for a model with no recovery rate ($\alpha = 0\%$) and also for a model with a constant recovery rate ($\alpha = 37\%$), as suggested by Benjamin and Wright (2009). Although the average short- (long-) term bond spread in the model is higher (lower) than the data, the overall distribution of spreads matches the realized spreads.

Discussion

In this section, an intuitive argument is presented to explain the behavior of the interest rate spreads in the models with and without debt renegotiation. First, notice the model with endogenous renegotiation results in larger spread means. In addition, according to Table 1.2, our model delivers higher debt to GDP ratio. The association between these two can be explained by recalling that the high spreads occur at the high levels of the debt. In other words, since in our model the default is optimal at higher levels of debt, the observed spreads (spreads before the default episodes) are larger than the models without debt renegotiation.

Figure 1.4 provides a schematic depiction of the debt renegotiation effect. The optimal recovery rate in 6, makes the default value function decrease at the slope of one, which flattens out after some endogenous level of debt (which is a function of income, $\zeta(y)$). However, for the exogenous recovery rate model, the default value the value function decreases at a milder slope compared to endogenous recovery[13]. As a result, for comparatively the same debt servicing value, W, the default is optimal at higher levels of debt for the endogenous recovery rate model. This in turn will result in higher spreads in the model compared to previous model specifications.

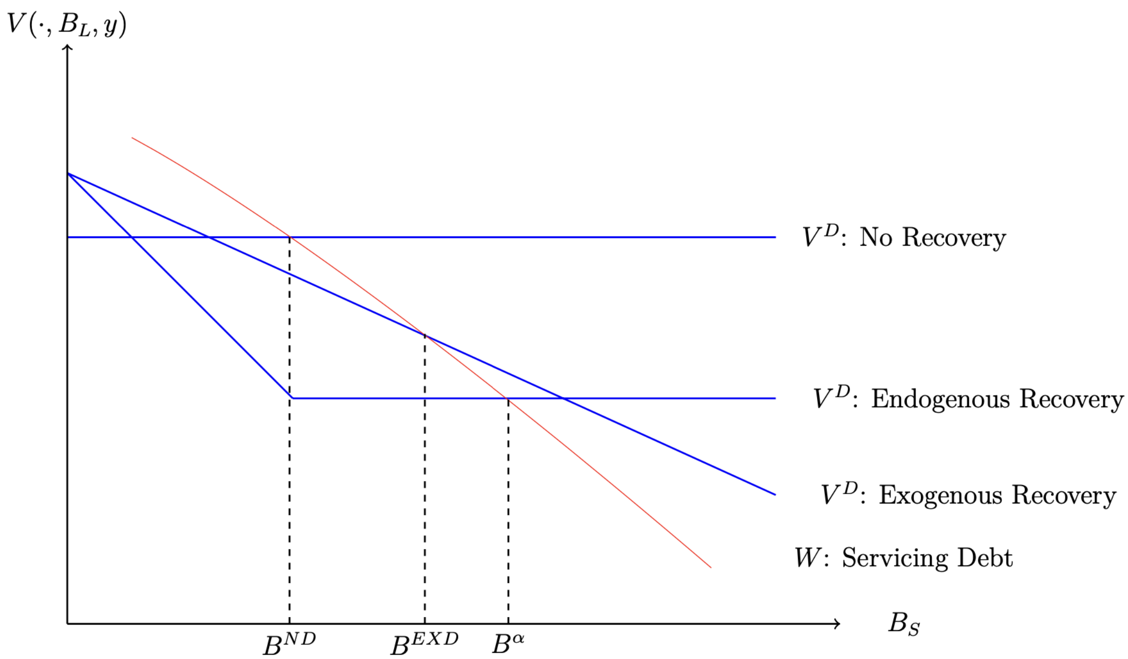


Figure 1.4: Schematic View of Value Functions.

[13] The default value decreases at a constant rate $\alpha < 1$.

1.5 Conclusion

This paper develops a default model with endogenous debt renegotiation for a small open economy. The equilibrium recovery rate in the model allows the borrowing country to hold higher levels of debt. Consequently, near the defaults, the resulting spreads are higher compared to the model specifications without the endogenous restructuring of the debt.

A better understanding of the interest rate behavior will help the countries to select a maturity structure that minimizes the outflow of resources at the onset of a financial crisis. One immediate extension of this problem is to include the possibility of partial defaults; i.e. adding bonds with different maturities to a partial default model (Arellano, Mateos-Planas, and Rios-Rull, 2013). This paper assumes the same haircut for different maturities. Any deviation from this assumption affects the optimal maturity structure. That is, whether the borrowing country is treated differently after default, affects the ex-ante decisions regarding the debt levels. This further sheds light on the liquidity-insurance trade-off between short- and long-term debts.

Appendix

Proof of Proposition 1: The proof follows Yue (2010). We can show in the similar manner that following problems have a fixed point:

1. The bond price functions, given the default and repayment sets, the policy functions of the borrowing country, and the recovery rate,

2. The debt renegotiation problem, given the bond prices,

3. The borrowing country value functions, given the bond prices and recovery rate.

Proof of Lemma 1:

$$W^D(B_S, B_L, y) = \max_{0 \leq B'_S \leq B_S,\ 0 \leq B'_L \leq B_L,\ 0 \leq C} u(C) + \beta \int_Y W^D(B'_S, B'_L, y') f(y'|y) dy'$$

$$C + B_S + B_L = (1-\lambda)y + \frac{B'_S}{1+r} + \frac{B'_L - \delta B_L}{1+r-\delta}$$

$$C = (1-\lambda)y + \left[\frac{B'_S}{1+r} - B_S\right] + \left[\frac{B'_L}{1+r-\delta} - \frac{B_L(1+r)}{1+r-\delta}\right]$$

Letting $X = \frac{B_L(1+r)}{1+r-\delta}$, we can rewrite the above budget constraint as:

$$C = y + \left[\frac{B'_S}{1+r} - B_S\right] + \left[\frac{X'_L}{1+r} - X_L\right]$$

$$C + B_S + X_L = (1-\lambda)y + \left[\frac{B'_S}{1+r} + \frac{X'_L}{1+r}\right]$$

Since $X'_L \leq X_L$ implies $B'_L \leq B_L$, 1 is invariant to any transfer of debt as long as $B_S + \frac{B_L(1+r)}{1+r-\delta}$ is fixed. Since V^D solely depends on W^D, debtor's surplus is invariant to a transformation shown above. The present values show creditors' surplus is invariant to such transformations as well.

Proof of Proposition 2[14]**:** Appealing to Lemma 1, W^D and consequently V^D is a function of $\alpha(B_S + d^\delta B_L)$. Hence $\Delta_D = V^D - V^{Aut}$ is a function of $\alpha(B_S + d^\delta B_L)$. Δ_C in 2 is also a function $\alpha(B_S + d^\delta B_L)$. This means the solution to 1.2 can be written as $\alpha(B_S + d^\delta B_L) = \zeta(y)$. Since $\alpha \in [0,1]$, $\alpha = 1,\ \forall (B_S + d^\delta B_L) \leq \zeta(y)$ and $\frac{\zeta(y)}{(B_S + d^\delta B_L)},\ \forall (B_S + d^\delta B_L) \geq \zeta(y)$.

Proof of Proposition 3: The proof follows Eaton and Gersovitz (1981) and Chatterjee et al. (2007), Arellano (2008), and Yue (2010). Since the value of default is independent of total dated debt level, for any debt level above $D^1 =\leq D^2$, $W(B_S^2, B_L^2, y) \leq W(B_S^1, B_L^1, y) \leq V^D(B^1, L^1, y) = V^D(B^2, L^2, y)$. Proof of the corollaries is by contradiction.

[14] Extension of Yue (2010).

Proof of Proposition 4: First notice by increasing short-term bonds $R(B'_S, B'_L)$ and $D(B'_S, B'_L)$ in 1.3 will (weakly) shrink and expand, respectively. Also α (weakly) decreases according to proposition 2. Hence, the price of short-term bond is (weakly) decreasing.

For the long-term bonds, we can appeal to the contraction mapping characteristics[15]. Using the fact that by increasing long-term bonds $R(B'_S, B'_L)$ and $D(B'_S, B'_L)$ in 3 will (weakly) shrink and expand, respectively, one can show that the price of long-term bond is a decreasing function in B_L.

Proof of Proposition 6:

The proposition is a direct consequence of properties of policy functions; if $D_1 > D_2$, then $D'_1 > D'_2$.

Notice $\tilde{W^D}(D, y)$ is decreasing and concave in D; if $D_1 > D_2$, then $\tilde{W^D}(D_1, y) < \tilde{W^D}(D_2, y)$ and $\frac{\partial}{\partial D}\tilde{W^D}(D_1, y') < \frac{\partial}{\partial D}\tilde{W^D}(D_2, y') < 0$.

[15]See Stokey, Lucas, and Prescott (1989)

By contradiction, let $D_1' < D_2'$ for $D_1 > D_2$. The optimization problem requires:

$$\frac{1}{1+r}u'(y + \frac{D_1'}{1+r} - D_1) = -\beta \int_Y \frac{\partial}{\partial D} \tilde{W^D}(D_1', y') f(y', y) dy'$$

$$\frac{1}{1+r}u'(y + \frac{D_2'}{1+r} - D_2) = -\beta \int_Y \frac{\partial}{\partial D} \tilde{W^D}(D_2', y') f(y', y) dy'$$

However due to concavity of u,

$$u'(y + \frac{D_1'}{1+r} - D_1) \geq u'(y + \frac{D_2'}{1+r} - D_2)$$

Which means,

$$-\beta \int_Y \left[\frac{\partial}{\partial D} \tilde{W^D}(D_1', y') - \frac{\partial}{\partial D} \tilde{W^D}(D_2', y') \right] f(y', y) dy' \geq 0$$

Which is a contraction.

Chapter 2

Debt Instruments and Sovereign Default

2.1 Introduction

There is a well-established body of studies regarding the sovereign debt crisis; in particular endogenous default models [1] show the counter-cyclicality of interest rates can explain the fluctuations in macro-variables in data. To the best of my knowledge, these papers all studied either bonds or loans as debt instruments. As explained by Panizza, Sturzenegger, and Zettelmeyer (2009), loans from syndicated banks were the main debt instrument during the 1970s and 1980s. After the mid-1990s, following

[1] such as Eaton and Gersovitz (1981), Arellano (2008), and Yue (2010)

the Brady deals, [2] bonds have been used as the main debt instrument. Panizza et al. (2009) explained the initial fear due to this change: First, heterogeneity and dispersion of bondholders could make the coordination among the creditors harder in the renegotiation process. Second, the "more fractured creditor side" could result in higher bargaining power for the sovereign. These two may suggest a lower default cost and consequently a higher number of default incidents. Interestingly, switching to bonds was accompanied by a significant drop in the number of defaults (Figure 2.1).

Enderlein, Muller, and Trebesch (2008) studied whether after the mid-1990s, the debtor countries took a more aggressive approach (simply put, whether they were abusing their bargaining power) toward solving the debt crisis. They found, on the contrary, a relatively stable pattern in the bargaining process. The role of the International Monetary Fund, as discussed by Panizza et al. (2009), changed over the pre- and post-Brady bonds. After the pervasiveness of Brady bonds, the IMF distanced itself from the debt renegotiation process. Hence, if anything, the IMF had less impact on the negotiations after the mid-1990s. All in all, the reality was at odds with the predicted number and duration of defaults. To explain these discrepancies, this paper develops a model with both loans and bonds as a borrowing tool and studies the effect of debt instruments on default episodes.

[2] After the 1980s Latin America debt crisis the US treasury Nicholas Brady proposed the idea of converting the nonperforming loans with dollar-denominated bonds issued by the defaulted countries. The bonds mainly used the US treasury 30-year zero-coupon bond as collateral and the whole process was supervised by the IMF and the World Bank.

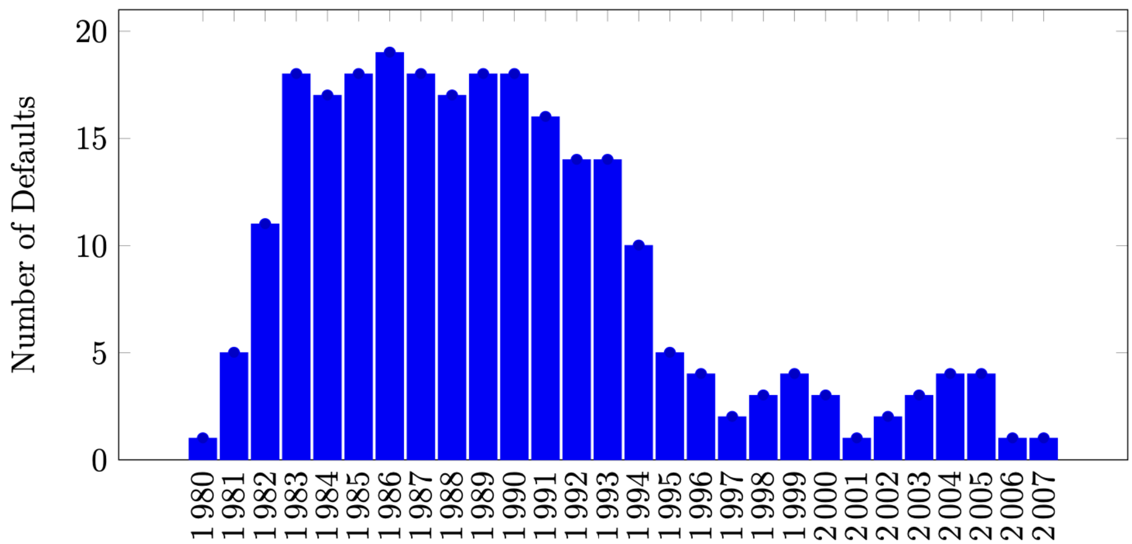


Figure 2.1: Number of Defaults Over Time.

The model in this study is essentially an extension of Yue (2010) to a case with more than one debt instrument: A dynamic equilibrium model with endogenous default risk and endogenous debt recovery rates. The model studies the effect of different debt instruments on the default probability, interest rates paid to each type of creditors, and debt renegotiations. In the quantitative part, following Arellano (2008) and Yue (2010), the model is used to study Argentina's default episode.

As in Arellano (2008) and Yue (2010), the model assumes bond markets to be perfectly competitive. Loans, on the other hand, are priced in a monopolistic market. This follows Kovrijnykh and Szentes (2007) idea of a two-regime economy. Unlike their model, we allow the country to trade in both competitive and monopolistic markets. The amount of haircut after default depends on the bargaining power of the debtors and creditors. Upon default, we assume banks and bondholders will be treated equally; the amount of haircut applied to loans and bonds would be the same. This assumption can be justified by pari passu clauses in the debt contracts. Pari passu, literally meaning "on equal footing", ensures an equal right of payment for the creditors. Pari passu clauses are standard provisions "in public or private international unsecured debt obligations including loan agreements and bond issuance". [3] As stated by Olivares-Caminal (2013), such clauses in sovereign bond issuance have two elements: The internal element that makes the bonds in equal rank with each other, and the external element that gives the bondholders an equal right as creditors of other unsecured debts, such as loans. In recent years the pari passu clauses drew a lot of attention. One reason can be the critical role of such clauses in Argentina's 2005 settlement process. In addition, the pervasiveness of Collective Action Classes (CACs) provides another argument to justify a common bargaining power for different creditors. Succeeding the legal complications of Argentina's debt renegotiations, CACs became more common in debt contracts. CACs prevent a single creditor from halting the debt renegotiations in the hope of extracting a higher surplus, therefore it facilitates the renegotiation process.

After the Brady plan and the waning role of the IMF in debt renegotiations, the Paris Club turned out to be the main institution in the sovereign debt restructuring. Paris Club mainly consists of governments of the largest economies in the world. It first originated after Argentina's 1956 credit crisis and be- came an important player

[3] Olivares-Caminal (2013)

after the 1980 debt crisis. Comparability of Treatment[4] clause is a vital principle of the Paris Club.[5] This clause requires the sovereign to share the default burden equally among all the creditors; including bond holders, banks, and even countries that are not a member of the Paris Club. Comparability of treatment clause shows that it is defensible to assume, even in the absence of pari passu clauses, the debtor sovereign treats the creditors equally after the default. Finally, the empirical work of Sturzenegger and Zettelmeyer (2008), shows the ex-post variation in inter-credit treatment is mainly attributed to the maturity of the debts. For instance in their calculations, in the case of Ukraine's 2000 default, the recovery rate of loans and bonds were very close to each other. The debt recovery rate for ING loans, issued in August 1999 was 62.0%, and debt recovery of international bonds issued in April 2000 was 59.9%. Since our model only studies one-period bonds and loans, assuming the same haircut can be justified by the conclusion of Sturzenegger and Zettelmeyer (2008).

The model is calibrated to Argentina to study the effect of debt instruments on default. The results of the model are in agreement with the previous literature[6] that higher debt increases the probability of default. Knowing this, the creditors will charge a higher interest rate with an increase in the level of outstanding debt (either bond or loan borrowings) or a reduction in the output. Furthermore, the

[4]Comparability of Treatment, one of the six principles of the Paris Club, reads as: "A debtor country that signs an agreement with its Paris Club creditors should not accept from its non-Paris Club commercial and bilateral creditors terms of treatment of its debt less favorable to the debtor than those agreed with the Paris Club."

[5]Das et al. (2012)

[6]Eaton and Gersovitz (1981), Arellano (2008) and Yue (2010)

model delivers results that compare the defaults pre and post Brady plan. First, the sovereign's bargaining power is lower when the creditors are solely composed of banks. Second, having access to both types of debt instruments will decrease the equilibrium interest rate and default probability for the same level of debt and output. Next, upon default, the total outstanding debt is important, not outstanding bonds or loans individually. This result highly relies on the pari passu assumption; since bonds and debts are ranked equally, the sovereign is not concerned about each instrument separately, but the total outstanding debt. Hence, the recovery rate function in the model is comparable to the one in Yue (2010), the functional form is a generalization to two dimensions. The equilibrium recovery rate has the following features: no haircut will be applied to unjustified defaults; i.e. if the outstanding debt is relatively low or endowment is high, the recovery rate would be one. Furthermore, the higher the debt (in both dimensions), the lower the recovery rate. Also, the lower the output, the (weakly) higher the recovery rate.

We can compare the equilibrium outcomes before and after bond issuance. The lower equilibrium interest rate and higher recovery rates upon defaults when the sovereign has both debt instruments, suggest a higher opportunity cost for exclusion from the capital market, which explains the reduction in the number of defaults observed in data. The lower interest rates also reduce the equilibrium haircut; for the same level of debt the haircut applied to the debt would be lower when the sovereign uses both debt instruments. Also due to the higher cost of exclusion from the financial market, the default periods shorten as well. These results are all confirmed by previous empirical works; in particular Sturzenegger and Zettelmeyer (2008).

Literature Review

The sovereign debt problem has been studied from different angles. One set of research studies the debt as a contingent claim. Grossman and Van Huyak (1988) and Grossman and Han (1999) developed a default model interpreting debt as a contingent claim. Following this idea, in one of the earliest attempts to quantify the default models, Alfaro and Kanczuk (2005) developed a model to study sovereign debt. Our model, however, is closely related to Eaton and Gersovitz (1981). They developed a model with endogenous default with the assumption that defaulted sovereign will be permanently excluded from the credit market. Hence, avoiding a bad reputation is the main incentive to service their debt. The assumption of permanent exclusion from the credit market was later challenged by Kovrijnykh and Szentes (2007), where they studied the equilibrium default cycles. Arellano's (2008) model incorporated the assumption that countries can regain access to the credit market and explained many key business cycle fluctuations. Debt renegotiation was studied by Bulow and Rogoff (1989), Fernandez and Rosenthal (1990), and Yue (2010). Bulow and Rogoff (1989) used the Rubinstein's (1982) Bargaining model to study the debt renegotiation between the sovereign and a bank. Fernandez and Rosenthal (1990) develop a game-theoretic model of debt renegotiation in which the penalties (sticks) are ignored by debtors and creditors. Hence the repayment motivation is restricted to the carrots; i.e. improved access to the capital market in the future. In a different approach than Bulow and Rogoff (1989), Yue (2010) and D'Erasmo (2011) developed models that use the Nash bargaining to determine the endogenous recovery rates. All of these studies, including the current paper, ignore the role of vulture creditors and assume a direct negotiation between creditors and debtors.

As in Arellano and Yue (2010), this paper studies a set of incomplete contingent assets, hence default may arise in equilibrium. By contrast, in the optimal contracting studies[7] with a complete set of assets, default never arises in the equilibrium, and as Arellano (2008) discussed, the default incentives are higher when the endowment is higher. Lizzaro (2009) develops a model with risk-averse creditors and shows it can quantitatively improve the business cycle statistics. Nevertheless, in this paper, following Arellano (2008) and Yue (2010), the lenders are modeled as risk-neutral.

The rest of the chapter is structured as follows: The next section provides the theoretical model. Section 3 presents the theoretical results. Section 4 provides the quantitative analysis and section 5 concludes the chapter.

[7] For instance Kocherlakota (1996) and Alvarez and Jermann (2000)

2.2 Model

Consider a small open economy consist of infinitely lived representative household with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where u is assumed to be continuous, strict$u$ is assumed to be continuous, strictly increasing, and strictly concave. The country faces an exogenous stream of income drawn from a compact set and governed by a Markov process with transition matrix $\mu(y_{t+1}|y_t)$. Government is benevolent and maximizes households' lifetime utility. The government accesses the international financial market and issues one-period bonds in a competitive market or asks for one-period loans from monopolistic syndicated banks. At time t the country can borrow $X_{t+1} \geq 0$ in the bonds market ($X = B$) and/or loans market ($X = L$) at a discount price. Let $q_t^X(B', L', y)$ be the price of debt instrument $X \in \{B, L\}$, when the government is asking for bonds with a face value of B' and loans with a face value of L' while the income realization is y. Contingent on servicing the debt, the government will pay back the face value, X_{t+1}, at time $t+1$. The country can also lend to the world by buying bonds at the risk-free interest rate r. Since $q^L \leq q^B \leq \frac{1}{1+r}$, the country will never find it optimal to be in short position in the loan market ($L' > 0$) while being long in the bond market($B' < 0$). Both syndicated banks and buyers of sovereign bonds are assumed to be risk-neutral. The debt contracts are not enforceable and the government may endogenously find it optimal to refuse to service the debt. If so, the country has an option to permanently move to Autarky or to be temporarily banned from the capital market but start a renegotiation process for restructuring the debt. As in Yue (2010), the sovereign goes through an endogenously determined renegotiation. Conditional on outstanding debt (B, L) and country's income (y), a haircut $1 - \alpha(B, L, y)$ is applied to the outstanding debts. To regain access to the international financial market, the country has to fully repay the outstanding debt. Meanwhile, the country is banned from further borrowings and incurs an output loss $(\lambda < 1)$.

Recursive Problem

The government makes decisions on behalf of the households; specifically, the government will decide on debt contracts in the bond and loan markets and decide whether to service the sovereign debt or not. Contingent on the default decision the countries engaging in the financial market may have different credit records. As Yue (2010) a country with a good credit record has serviced the debt in the previous period, while a country with bad credit has debt arrears. Independent of credit record, we can express the state of the economy at time t with the face value of outstanding debts, B_t, L_t, and the realization of income y_t. Contingent on the credit record, a value function $v(B, L, y)$ can be defined to summarize the lifetime utility of the country.

Consider a borrowing country that did not default in the previous period, $\mathcal{I}(D) = 0$. At the beginning of the period t, given the state of the economy (B_t, L_t, y_t) and the

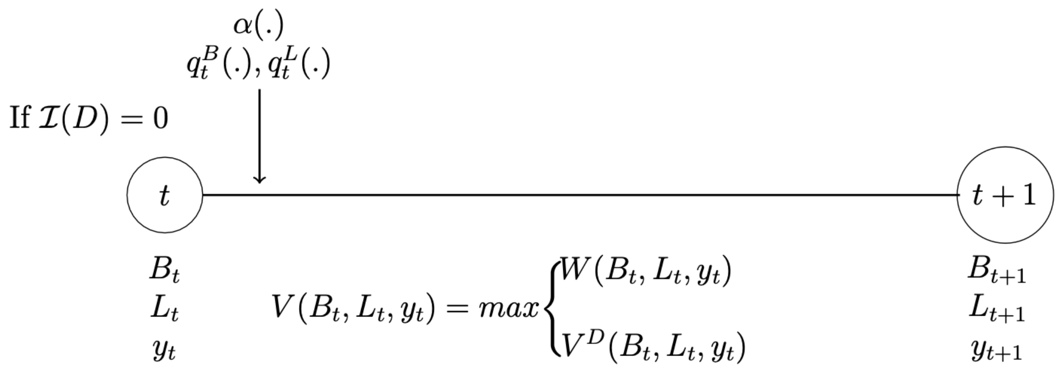


Figure 2.2: Value Function, V

price schedules $q_t^B(B_{t+1}, L_{t+1}, y_t)$ and $q_t^L(B_{t+1}, L_{t+1}, y_t)$ and the recovery rate associated with the states $(\alpha(B_t, L_t, y_t))$, government will decide to either service the debt or default. Let $W(.)$ and $V^D(.)$ be the value functions of servicing the debt and default, respectively. Value function of a country with good credit will be:

$$V(B, L, y) = \max\{W(B, L, y), V^D(B, L, y)\}$$

By honoring the current debt, the government can ask for new loans and/or issue new bonds at the corresponding prices:

$$W(B, L, y) = \max_{0 \leq B',\ 0 \leq L',\ 0 \leq C} u(C) + \beta \int_Y V(B', L', y') d\mu(y'|y)$$

$$C + B + L = y + q^B(B', L', y)B' + q^L(B', L', y)L'$$

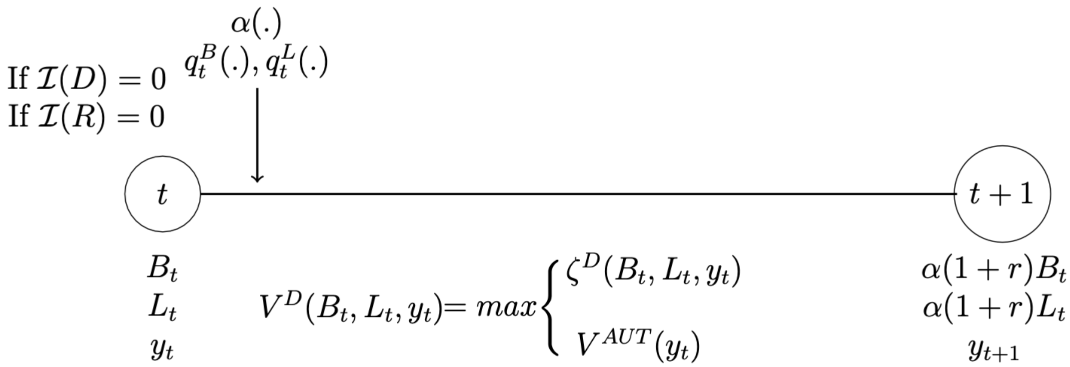


Figure 2.3: Value Function: Default.

The Bellman equation above can represent the value function for the lending country as well. For a lending country, $B' < 0$ and $L' < 0$, that is, the country is in a long position in debts. Furthermore, $q^B(\cdot) = q^L(\cdot) = \frac{1}{1+r}$; independent of the lending levels of the income, the country can buy bonds at the risk-free rate. This assumption is in line with the competitive market structure assumed above.

Upon default, the country will be temporarily dismissed from the credit market; no saving or borrowing is allowed at time t. The country uses all the realized income, y_t, as consumption and a haircut $1 - \alpha(B)$ will be applied to the delinquent debts. Therefore the outstanding debt in the following period is α fraction of the principal plus interest of the delinquent debt:

$$\zeta(B, L, y) = u(y) + \beta \Big\{ \int_Y W^D(\alpha(1+r)B, \alpha(1+r)L, y')d\mu(y'|y) \Big\}$$

The country has the option to permanently leave the international financial market. The corresponding value function can be calculated as below:

$$V^{AUT}(y) = u((1-\lambda)y) + \beta \int_Y V^{AUT}(y')d\mu(y'|y)$$

At the Autarky the country, suffering an output loss λ, will consume the all the resources available currently. With the assumptions regarding the utility function, no consumption smoothing is possible at Autarky and hence the Autarky is never optimal to choose. In other words,

$$V^{AUT}(y) < V^D(B, L, y), \quad \forall \ B, L, y$$

. Hence abusing the notation, we can rewrite the default value function:

$$V^D(B, L, y) = u(y) + \beta \Big\{ \int_Y W^D(\alpha(1+r)B, \alpha(1+r)L, y')d\mu(y'|y) \Big\}$$

After declaring a default($\mathcal{I}(D) = 1$) and debt restructuring($\mathcal{I}(R) = 1$), the government has to pay back the debt. The country can optimally choose the amount of payment. Since the net position on debt is long, the interest on the new borrowings will be the risk-free interest rate. The country will also suffer an output loss λ during the repayment period:

$$W^D(B, L, y) = \max_{0 \le B' \le B, \ 0 \le L' \le L, \ 0 \le C} u(C) + \beta \int_Y W^D(B', L', y')d\mu(y'|y) \tag{2.1}$$

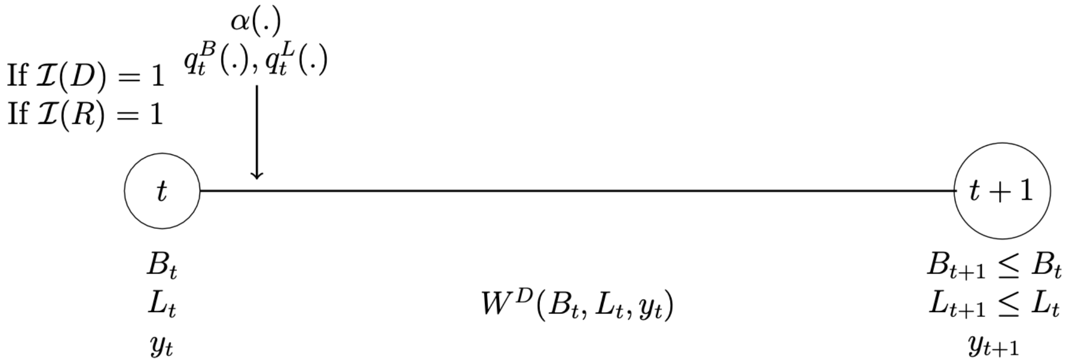


Figure 2.4: Value Function: Repayment Period.

$$C + B + L = y + \frac{B'}{1+r} + \frac{L'}{1+r}$$

When the defaulted country pays the arrears in full, it gets the value function of the borrower with a good credit history:

$$W^D(0, 0, y) = V(0, 0, y)$$

Debt Renegotiation Problem

Following Yue (2010) the defaulted country and creditors will engage in one-round debt restructuring negotiations. As discussed by Yue (2010) allowing multiple rounds of costly renegotiation will not change the results of the model. The debt recovery rate at each state is determined according to a Nash bargaining game. Renegotiation will help the sovereign to pay a fraction of delinquent debt to redeem itself from permanently moving to Autarky. If the country chooses Autarky, the lenders will end up with zero reimbursement. Therefore creditors' and debtor surpluses can be defined as:

$$\Delta_D(B, L, y; \alpha) = V^D(B, L, y; \alpha) - V^{AUT}(y)$$

$$\Delta_C(B, L, y; \alpha) = \frac{\alpha(B + L)}{1 + r} \tag{2.2}$$

Let $\theta \in [0, 1]$ be the debtor's bargaining power. The equilibrium recovery rate will be the maximand of the total surplus:

$$\alpha(B, L, y) = \arg \max_{\hat{\alpha} \in [0,1]} \left[\Delta_D(B, L, y; \hat{\alpha})\right]^{\theta} \left[\Delta_C(B, L, y; \hat{\alpha})\right]^{(1-\theta)} \tag{2.3}$$

$$s.t. \qquad \Delta_D(B, L, y; \hat{\alpha}) \geq 0, \;\; \Delta_C(B, L, y; \hat{\alpha}) \geq 0$$

Creditors' Problem

Following KS(2007), we can assume creditors operate under two opposite market structures: bonds are traded in a competitive market, while loans are priced in a monopolistic market. In the model, at time t, creditors pay the X_{t+1}, for $X \in \{B, L\}$, at the price q_t^X. However they will be reimbursed at period $t+1$, if the country honors the debt, or $t + 2$ if the country defaults:

Independent of the future default status, the present value of the reimbursements is discounted with a factor of $\frac{1}{1+r}$. The only difference is, contingent on default, a haircut will be applied to the reimbursement. To find the expected reimbursement,

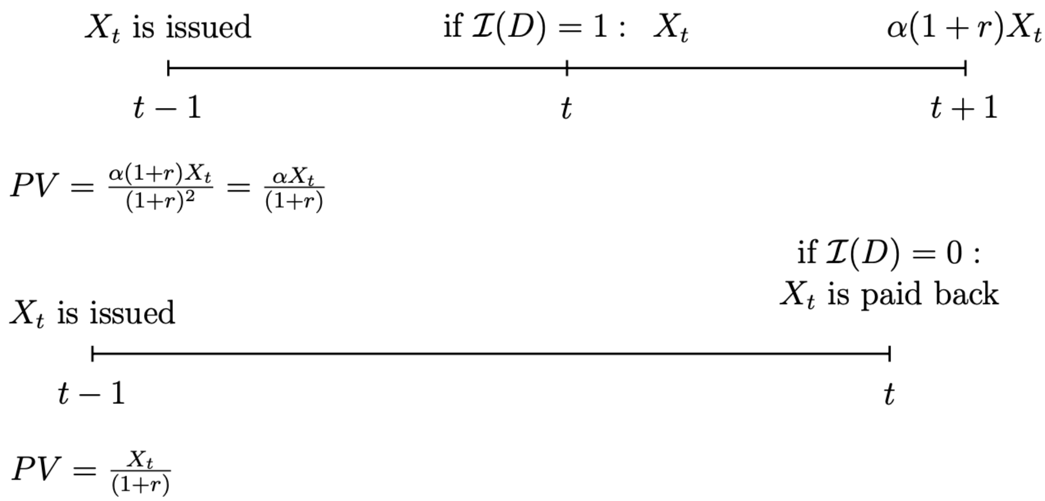


Figure 2.5: Present Value of an Issued Debt X.

creditors have to find the probability of default and the expected recovery rate. To find the expected reimbursement, first, we need to find the probability of default and expected recovery rate. Let $\xi(B', L', y) = \int_{y \in \psi(B',L')} d\mu(y'|y)$ be the probability of default, where $\psi(B, L)$ is the default set defined as:

$$\psi(B, L) = \{y \in Y : W(B, L, y) < V^D(B, L, y)\}.$$

That is, the default set is the set of income realizations under which default is the optimal choice for a country with debt of (B, L). Expected Rate of Recovery at each state can be defined as:

$$\gamma(B', L', y) = \frac{\displaystyle\int_{\psi(B',L')} \frac{\alpha(B',L',y')}{(1+r)} d\mu(y'|y)}{\displaystyle\int_{\psi(B',L')} d\mu(y'|y)}$$

and

$$\gamma(B', L', y) = 1, \ \ if \ \ \xi(B', L', y) = 0$$

This allows us to find the creditor's profit. For an asset $X \geq 0$, the profit of bondholders ($X = B$) or banks ($X = L$) is as follows:

$$\pi^X(B', L', y) = \max_{X'} \ \ -X'q^x(B', L', y) + \Big(1 - \xi(B', L', y) + \xi(B', L', y)\gamma(B', L', y)\Big)\frac{X'}{1+r}$$

Similar to Arellano(2008) and Yue(2010) and following our assumption regarding the structure of the bonds market, we can find the bonds price. In the equilibrium, bondholders should expect a zero profit. Applying zero profit condition to the profit function above will pin down the bond prices:

$$q^B(B', L', y) = \frac{\Big(1 - \xi(B', L', y) + \xi(B', L', y)\gamma(B', L', y)\Big)}{1+r}$$

Observing the state of the sovereign, syndicated banks predict the demand for loans at each discount price, and will choose a price that maximizes the profit. A higher discount price increases the payments at time t (first-order effect), but on the other hand, decreases the probability of default in the next period(second-order effect).

$$\pi^L(B', L', y) = \max_{q^L(.)} \quad -L'(.;q)q^L + \Big(1 - \xi(.;q) + \xi(.;q)\gamma(.;q)\Big)\frac{L'(.;q)}{1+r}$$

$$s.t. \qquad \pi^L(.) \geq 0$$

Notice $\pi^L(.) \geq 0$ implies $q^L(.) \leq q^B(.) \leq \frac{1}{1+r}$, hence there is an endogenous wedge between the interest rate charged by syndicated banks and bondholders. Although theoretically $q \in [0,1]$, quantitative results show the banks will not overcharge the countries; more specifically the model shows the optimal discount price has a lower bound ($q \geq$0.6).

2.3 Results

Recursive Equilibrium

Let $s = \{B, L, y\}$ be the aggregate state of the economy, the following can define the equilibrium for the economy.

A recursive equilibrium is a set of functions for

- The country's value functions, $V\big(V(s), W(s), V^D(s), W^D(s)\big)$, policy functions of asset holdings, $B'(s), L'(s)$, and consumption, $C(s)$,
- Default set, $\psi(B, L)$,
- Price functions $q^B(B', L', y)$ and $q^L(B', L', y)$,

- The recovery rate function $\alpha(s)$ such that

1. Given the bond and loan prices, $q^B(B', L', y)$ and $q^L(B', L', y)$, and recovery rate function, $\alpha(s)$, the country's value functions and the policy functions, $B'(s), L'(s)$, and $C(s)$, will solve the problems described above and $\psi(B, L)$ is compatible with the policy functions.

2. Given the price functions, $q^B(B', L', y)$ and $q^L(B', L', y)$, the value functions, and the policy functions the recovery rate solves the debt renegotiation problem.

3. Given the recovery rate, bond prices, $q^B(B', L', y)$, guarantee the zero profit condition for the bondholders. $q^B(B', L', y)$ is consistent with recovery rate and default probability.

4. Given the recovery rate and policy functions, loan prices, $q^L(B', L', y)$, will satisfy the profit maximization of syndicated banks. $q^L(B', L', y)$ is consistent with recovery rate and default probability.

Proposition 1: $\forall\theta \in \Theta$ recursive equilibrium of the above model exist. **Proof**: See Appendix.

Lemma 1: $W^D(B, L, y)$ is invariant to any transfer of debt as long as $B + L$ remains constant.

Proof: See Appendix.

The fact that the value of W^D only depends on the total outstanding debt, rather than specific values for B and L, allows us to replicate (and generalize) Yue (2010) theorem 2 to two dimensions.

Proposition 2: The equilibrium debt recovery rate satisfies:

$$\alpha(B, L, y) = \begin{cases} 1 & (B + L) \leq D(y) \\ \frac{D(y)}{B+L} & (B + L) \geq D(y) \end{cases}$$

Proof: See Appendix.

This result is parallel to Yue (2010); upon default, for each level of endowment y, the value function of default is independent of total outstanding debt. That is, after the default value function is decreasing in an interval, but remains constant after a debt level (Figure 2.6).

Proposition 3: If default is optimal for a state (B^1, L^1, y), then it is also optimal for all (B^2, L^2, y) that $D^2 = B^2 + L^2 \geq B^1 + L^1 = D^1 \geq D(y)$. Hence $\psi(B^1, L^1) \subseteq \psi(B^2, L^2)$.

Corollary 1: If default is optimal for a state (B^1, L, y), then it is also optimal for all (B^2, L, y) that $B^2 \geq B^1$.

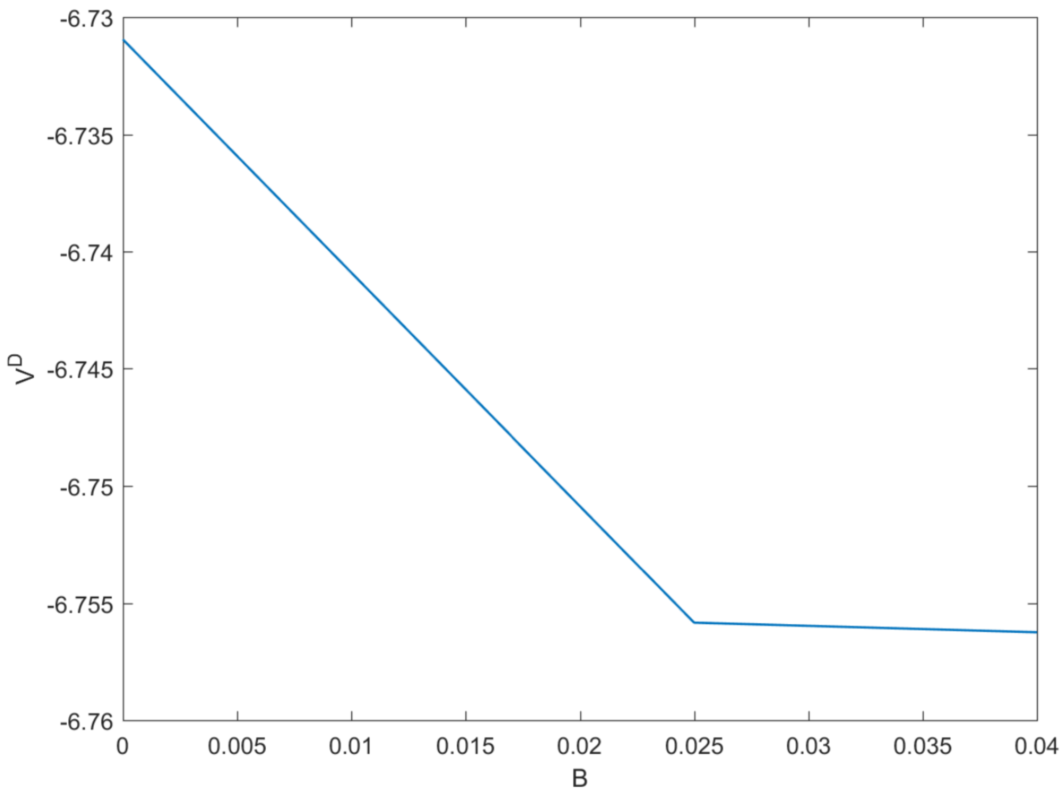


Figure 2.6: Default Value Function: V^D remains the same after a threshold.

Corollary 2: If default is optimal for a state (B, L^1, y), then it is also optimal for all (B, L^2, y) that $L^2 \geq L^1$.

Proof: See Appendix.

Proposition 4: $\forall y$, default probability, $\xi(B, L, y)$, is increasing in B and L. Proof is a direct application of Proposition 3 result that $\psi(B^1, L^1) \subseteq \psi(B^2, L^2)$ and corollaries.

Proposition 5: For any level of endowment and outstanding loan (bond), bond (loan) equilibrium price is decreasing in demanded bonds (loans):

$$q^B(B^2, L, y) \leq q^B(B^1, L, y), \ \ \forall B^2 \geq B^1$$

$$q^L(B, L^2, y) \leq q^L(B, L^1, y), \ \ \forall L^2 \geq L^1$$

Proof: See Appendix.

Proposition 6: Country's value functions are increasing in the endowment,y. As a result, default incentives are higher the lower the endowment.

Proposition 6, generalizes Arellano's(2008) result to all country's value functions.[8]

Proposition 7 shows the country may find it optimal to partially pay back the renegotiated debt, i.e. default episode may take more than one period. The proposition tailors

[8]The proof is a direct application of Stokey, Prescott, and Lucas (1989).

Yue's(2010) proposition 6 to the current model with bonds and loans as debt instruments. For simplicity, let $\tilde{W^D}(D, y) = W^D(B, L, y)$ when $D = B + L$.

Proposition 7: Consider the repayment problem 2.1, for a given y, if $D' = 0$ is the optimum for a state described by $D = B + L$ and y, then $D' = 0$ is also optimal for all $\hat{D} < D$. Also partial payment,$D' < 0$, is optimal for all $\hat{D} > D$, if

$$\tilde{W^D}(D, y) = u((1 - \lambda)y - D) + \beta \int_Y v(0, 0, y') d\mu(y'|y).$$

Proof: See Appendix.

Data and Calibration

Following Arellano (2008) and Yue (2010) we use a CARA utility function, $\left(u(c) = \frac{c^{1-\sigma}}{1-\sigma}\right)$ with a risk aversion coefficient of $\sigma = 2$, following the literature. The annual risk-free interest rate of $r = 4\%$ is selected to match the average 1-year yield of the US bonds in that period. Following Sturzenegger (2002), the output loss during default is selected to be $\lambda = 2\%$. The data (output vector and transition matrix) is the same as Arellano (2008). It is constructed to incorporate Argentina's default incident of 2001. Quarterly, seasonally adjusted output data (from 1983Q3 to 2001Q4) is used to estimate an $AR(1)$ process, $log(y_t) = \rho log(y_t - 1) + \epsilon_t, E[\epsilon] = 0$. Then the estimated ρ, $E[\epsilon^2]$ is used to determine the transition matrix (Tauchen 1986). The risk-free rate is set to %1.7 which is the average of the five-year US Treasury bonds from 1983Q3 to 2001Q4. The estimated persistence, ρ and error standard deviation, $E[\epsilon^2]$, are $\rho = 0.95$ and $E[\epsilon^2] = 0.02$. Then a Markov chain with 21 discrete endowment states is constructed Using Tauchen (1986).

Time preference, β, and borrower's bargaining power, θ, are calibrated to match the model moments to the observed data. The time preference coefficient is calibrated to match the default frequency observed in the data. From 1824 to 2001 Argentina experienced five default episodes.[9] This results in an annual default frequency equal to 2.8%. Debtor's bargaining power is set to match the average recovery rate equal prior to the Brady deal in 1995 (55%) and after it (78%).[10]

Table 2.1 summarizes the selected and calibrated parameters in the model. The calibrated values correspond to the model before and after introducing the bonds.

[9] As documented by Reinhart, Rogoff, and Savastano (2003) four default episodes from 1824 to 1999 and one episode in the last quarter of 2001.

[10] Benjamin and Wright (2009)

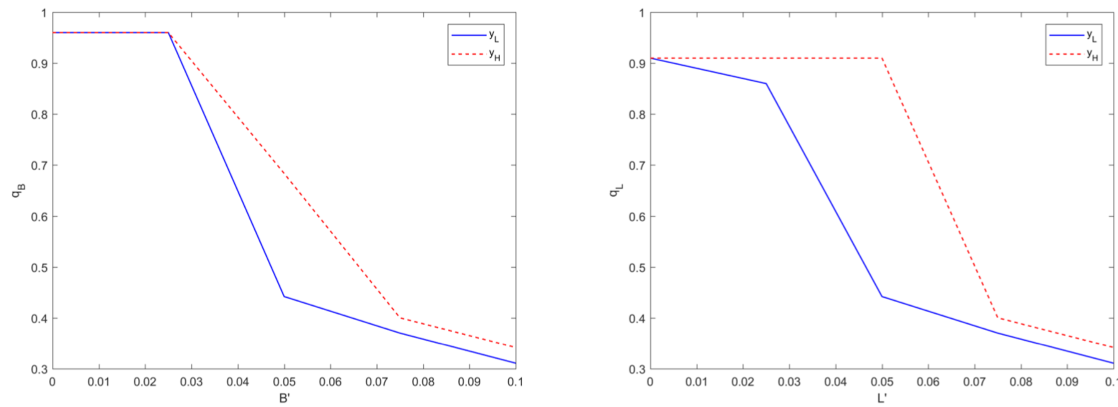
Parameter	Value	Target
one-year risk free rate	$r = 4\%$	U.S. annual interest rate
Risk Aversion	$\sigma = 2$	Literature
Output loss	$\lambda = 2\%$	Sturzenegger (2002)
Stochastic structure	$\rho = 0.95$, η=0.025	Argentina output (1980Q1: 2001Q4)
	Model with Loans and Bonds	
Calibration	Value	Target statistics
Borrower's discount factor	$\beta = 0.93$	2.8% Default Probability
Borrower's bargaining power	$\theta = 0.81$	55% Ave. Debt Recovery Rate
	Model with Loans	
Calibration	Value	Target statistics
Borrower's bargaining power	$\theta = 0.54$	22% Ave. Debt Recovery Rate

Table 2.1: Parameter Values

Simulation

Figure 2.7 depicts the asset prices as a function of debt instruments. As discussed in Proposition 5, the price is negatively related to the demand of the asset. As shown in the graph, the (weakly) decreasing price functions are in line with the results of Proposition 5. In addition, the price is positively related to the endowment. Higher endowment decreases the chance of default, which in turn reduces the prices. This is illustrated in the graph by plotting the price function for high and low levels of income; y_H and y_L respectively.

Next, we will show the effect of adding bonds as a debt instrument. First, it is worth mentioning that the convergence interval, $|Theta$, is not the same for the two cases. More



(a) Bond Price

(b) Loan Price

Figure 2.7: Bond and Loan Price Functions. Panel (a) shows the price of bond as a function of next period's stock of bonds for two income levels y_H and y_L. Panel (b) shows the corresponding values for the loan.

formally, if Θ_L is defined as the convergence interval for the model with the loan as the sole instrument, and $\Theta_{B,L}$ is defined as the convergence interval for the model with both loan and bonds, we have

$$\theta_L < \theta_{B,L}, \ \ \forall \theta_L \in \Theta_L, \forall \theta_{B,L} \in \Theta_{B,L}.$$

This result is in line with the aforementioned intuitive explanation of market structure's role in bargaining power; the higher the market power of creditors, the lower the bargaining power of the debtor country. The increase in bargaining power is accompanied by an increase in the amount of haircuts. Figure 2.7 compares the recovery rates for two cases: when both loans and bonds can be used as an instrument versus the case where loans are the only available debt instrument. For the same level of external debts, the equilibrium level of haircut (recovery rate) is higher (lower) after the introduction of Brady bonds.

We can run the model with bonds as the only available debt instrument; as in Yue (2010). The results are, once again, in agreement with our general intuition about the impact of market power on bargaining power and haircuts: when the creditors have no market power, the debtor country's bargaining power and the recovery rate will increase.

However, higher bargaining power (and equilibrium haircuts) does not mean the borrowing country has more incentives to default. This may seem counterintuitive as the mentioned factors, if anything, facilitate after default negotiations.

Availability of bonds, in addition to loans, prevents the banks from overcharging the borrowing countries. As a result, for a wide range of debt levels, the charged interest rate for the loans is the same as the bonds interest rate. The model confirms lower average interest payments: after introducing the bonds the trade balance reduces by half, normalized trade balance $\frac{T_B}{y}$ drops from 0.67 to 0.36.

Lower interest rate schemes result in lower haircuts, $1 - \alpha > 0$, the same levels of haircuts are observed at higher levels of debt. In other words for the same level of debt, the recovery rate is higher if the sovereign accesses both bonds and loans market. This results in an average higher recovery rate post-1995 period, which was documented by Benjamin and Wright (2009).

Since the haircuts reduce after the availability of bonds, the corresponding default incentives drop as well. Generally, the borrowing country does not find it optimal to default if the corresponding haircut is not a strictly positive number. In sum, the lower interest payments and higher recovery rates will make the default less appealing for the sovereign country.

Lower interest payments after the introduction of bonds make the default duration less attractive as well. Sovereign knows that by paying the arrears in full and regaining access to the financial market, they can smooth their consumption at a very low cost. Therefore they have higher incentives to pay the arrears over the span of a shorter interval. This reduces the default period from 4.58 years in the model with loans, to 2.21 years after introducing the bonds.

Table 2.2 summarizes the corresponding values before and after bond introduction. These results show not only does the frequency of default decrease, but also the severity of default reduces. The defaulted country defaults less frequently and pays back the arrears faster while the creditors suffer less due to lower haircuts.

Model Statistics

	Model w/o Bonds	Model w Bonds
Debtor's Bargaining Power	$\theta = 0.54$	$\theta = 0.81$
Average haircut	$\bar{\gamma} = 0.45$	$\bar{\gamma} = 0.78\%$
Normalized Trade Balance	$\frac{TB}{y} = 0.67$	$\frac{TB}{y} = 0.36$
Default Duration (years)	4.58	2.21
Default Frequency	7.80%	2.80%%

Table 2.2: Model Statistics Pre/Post Introducing Bonds

2.4 Conclusion

This paper provides a theoretical framework to explain how introducing Brady bonds contributed to lower default frequency. As documented by the literature, the severity of defaults decreased after Latin American countries started to issue bonds. That is, default episodes were less severe, in terms of haircuts applied after debt restructuring, and lasted fewer periods. These results may seem puzzling as one may expect the opposite after bond issuance, due to the debtor's higher bargaining power, defaulting on debt may seem more appealing after bond issuance. However, this paper shows issuing bonds will decrease the interest rate the sovereign faces. Also accessing two debt instruments reduces the average equilibrium haircuts; the same level of haircut will be applied at the higher debt levels which occur with a low probability. Low interest payments make the default option less appealing as the sovereign is able to smooth the consumption at a lower cost. This in turn reduces the default duration; countries can enjoy the low cost of consumption smoothing if they pay the arrears in full sooner.

Currently, many African countries rely on loans as their main debt instrument. This paper suggests assisting these countries to issue bonds will help them to experience a lower number of defaults while the default episodes are less severe. Creditors will also benefit from lower levels of haircuts after restructuring the debt.

Appendix

Proof of Proposition 1: The proof follows Yue (2010). We can show in a similar manner that the following problems have a fixed point:

1. The bond price functions, given the default and repayment sets, the policy functions of the borrowing country, and the recovery rate,

2. The debt renegotiation problem, given the loan and bond prices,

3. The borrowing country value functions, given the loan and bond prices in addition to the corresponding recovery rate.

Proof of Lemma 1: We have to show the debtor and creditors' surpluses can be expressed as functions of the total debt. First, notice that the repayment problem 1 is invariant to any transfer of loan to bond that keeps the total debt constant. Since W^D solely depends on total debt, V^D is solely a function of total debt. Hence the debtor surplus only depends on the total debt. The linearity of 2.2 with respect to loans and bonds shows creditors' surplus (as a whole) depends on the total debt.

Proof of Proposition 2: Appealing to Lemma 1, W^D and consequently V^D is a function of $\alpha(B+L)$. Hence $\Delta_D = V^D - V^{Aut}$ is a function of $\alpha(B+L)$. Δ_C in 3.2 is also a function $\alpha(B+L)$. This means the solution to 3.2 can be written as $\alpha(B+L) = D(y)$. Since $\alpha \in [0,1]$, $\alpha = 1,\ \forall (B+L) \leq D(y)$ and $\frac{D(y)}{(B+L)},\ \forall (B+L) \geq D(y)$.

Proof of Proposition 3: The proof follows Eaton and Gersovitz (1981) and Chatterjee et al. (2007), Arellano (2008), and Yue (2010). Since the value of default is independent of total debt level, for any debt level above D^1, $W(B^2, L^2, y) \leq W(B^2, L^2, y) \leq V^D(B^1, L^1, y) = V^D(B^2, L^2, y)$. Proof of the corollaries is by contradiction.

Proof of Proposition 5: since α and ξ are respectively decreasing and increasing in B, q^B is decreasing in B as well:

$$q^B(B', L', y) = \frac{1 - \xi(B', L', y)\Big(1 - \gamma(B', L', y)\Big)}{1 + r}$$

The inverse relation between $q^L$and L can be deduced from the supermodularity of banks' profit function.

Proof of Proposition 7: The proposition is a direct consequence of properties of policy functions; if $D_1 > D_2$, then $D_1' > D_2'$.

Notice $\tilde{W^D}(D, y)$ is decreasing and concave in D; if $D_1 > D_2$, then $\tilde{W^D}(D_1, y) < \tilde{W^D}(D_2, y)$ and $\frac{\partial}{\partial D}\tilde{W^D}(D_1, y') < \frac{\partial}{\partial D}\tilde{W^D}(D_2, y') < 0$.

By contradiction let $D_1' < D_2'$ for $D_1 > D_2$. The optimization problem requires:

$$\frac{1}{1+r}u'(y + \frac{D_1'}{1+r} - D_1) = -\beta \int_Y \frac{\partial}{\partial D}\tilde{W^D}(D_1', y')d\mu(y'|y)$$

$$\frac{1}{1+r}u'(y + \frac{D_2'}{1+r} - D_2) = -\beta \int_Y \frac{\partial}{\partial D}\tilde{W^D}(D_2', y')d\mu(y'|y)$$

However due to concavity of u,

$$u'(y + \frac{D_1'}{1+r} - D_1) \geq u'(y + \frac{D_2'}{1+r} - D_2)$$

Which means,

$$-\beta \int_Y \left[\frac{\partial}{\partial D} \tilde{W^D}(D_1', y') - \frac{\partial}{\partial D} \tilde{W^D}(D_2', y') \right] d\mu(y'|y) \geq 0$$

Which is a contraction.

Chapter 3

Partial Defaults and Debt Renegotiations

3.1 Introduction

It was a well-established fact that the sovereign default incidents are usually partial. Grossman and Huyck (1988) documented this as a stylized fact decades ago. A default starts with a missing payment on an interest payment which may or may not propagate to other debts. For instance, Venezuela's 2005 default incident affected debt payments on as little as 1 or 2 interest payments outstanding debt, while in Argentina 2001, the sovereign defaulted on 95% of the outstanding debts. It was only recently, however, that researchers developed models to study the partial default.

Most of the current theories consider the default as a binary incident. This strain of studies stemmed from the seminal work of Eaton and Gersovitz (1981). Such models assume that upon default, the sovereign will be expelled from the capital markets. Sovereign regains access to the capital market either by an exogenous shock (as in Arellano 2008) or after repaying the arrears in full (as in Yue 2010). This, however, does not fully reflect the reality; as documented by Arellano, Mateos-Planas, and Ros-Rull (2013). They have used a panel data containing the information from 1970-2010 for 99 developing countries. The empirical results show, contingent on positive arrears, on average around 50% of the defaults are partial; i.e. countries on average defaulted on the same amount as the amount they serviced.

Arellano et al. (2013), developed a model of partial default with an exogenous recovery rate after the default. This paper improves the partial default model by endogenizing the recovery rate. Following Yue (2010), the current paper assumes one-shot debt renegotiation according to a Nash bargaining game. As in Arellano et al. (2013), the sovereign can partially default on the outstanding debt. Defaulting on a debt will affect the country in different ways. First, it negatively affects the future realizations of income. Second, it reduces (increases) the required discount price (interest rate) by the lenders. Finally, this chapter introduces another effect of the default; the haircut applied to the defaulted debt is also affected by the defaulted amount. As explained by Arellano (2013), from this perspective, default is essentially a costly borrowing tool, with direct and indirect costs mentioned above.

Debt dilution is an important factor in default decisions; as addressed by Hatchondo, Martinez, and Sapriza (2010). The current work investigates this effect in the context of partial default. The current model studies a small open economy facing a stochastic endowment stream. Each period, the sovereign can issue new bonds and may default on some existing ones. The defaulted debt will be annuitized into perpetuities with decay rate δ and is subject to an endogenous haircut rate $1-\alpha$. Risk-neutral lenders will take decision functions of the sovereign as given and operate in a perfectly competitive capital market, i.e. expect zero economic profit. The endogenous recovery rate α at each state of the economy maximizes the total surplus, consisting of the borrower and lenders' surpluses.

Previous studies implemented the endogenous debt renegotiation through Nash[1] or Rubinstein[2] bargaining in full default models. Partial default in the context of private debts has been studied by Mateos-Planas and Seccia (2007). They have proposed a comparable interpretation for the partial private defaults; they provide the households with an additional insurance tool under the incomplete market framework.

Results of this study present that the default ratio, D/A, is an important factor. Hence, we present the results by normalizing the outstanding liabilities; that is dividing the assets by the maximum asset levels. As expected, bond prices are negatively affected by the default ratio. Also, the higher the income, the higher the bond price. The optimal default level depends on the outstanding debt and income. For high levels of income, default is never justified and will be penalized by no haircuts. For low levels of income, partial default is

[1] Yue (2010), D'Erasmo (2008), Bi (2006), and previous chapters of this study) that built upon Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008).

[2] Bulow and Rogoff (1989)

optimal. The borrowing country will face high haircuts and can repay the debts after the realization of “good” income shocks. Consequently, the model results will better match the default episodes observed in the data. The higher haircuts for the defaulted countries with low income allow them to stay longer in default compared to the case with no endogenous debt renegotiations.

The rest of the chapter is structured as follows: Section 2 provides the theoretical model for partial default with debt renegotiation. Section 3 presents the quantitative analysis and Section 4 concludes the chapter.

3.2 Model

Borrowing Country

Time is discrete and indexed by $t \in \{0, 1, \ldots\}$. The small open economy faces a stochastic income stream $\hat{y}$ that is governed by a Markov process Γ_z. As in Arellano et al. (2013), the index z is itself a Markov process with a known transition matrix. There is a representative agent that lives forever, with preferences characterized by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $0 < \beta < 1$ is the discount factor. The benevolent government decides on behalf of the representative agent. Each period, the government starts with total coupon payments of

A from the previous periods. After the realization of income the index of the distribution, the government decides on the level of consumption (C), new bond issuance (B), and the defaulted amount (D).

Arrears D causes the economy to face reductions in future outputs; $y' = \hat{y}'\Psi(D, z)$, where $\Psi(D, z)$, is a decreasing function in D and represents the direct cost of default modeled by an output loss function Ψ. As in Arellano et al. (2013), this paper assumes $\Psi(0, z) = 1$ and $\lim_{D \to \infty} > 0$.

Government can issue bonds with the face value B and a discount price $q(z, A', D, y)$. Since defaulting is in essence a costly way of borrowing, higher levels of default will reduce the discount price. This characterizes another (indirect) cost of default.

Using Macaulay (1938) approach, we can model different maturities with perpetuities and a proper decay rate. Hence, the law of motion for the debt will be

$$A' = \delta A + B + \alpha(1 - \delta)D$$

Where α the recovery rate. The borrowing country will face the following recursive problem; at each state of the economy, (z, A, y), taking the recovery rate $\alpha(z, A, y)$ as given, government chooses the policy functions $C(z, A, y)$, $B(z, A, y)$, $D(z, A, y)$, and $A'(z, A, y)$.

$$V(z, A, y) = \max_{0 \le C,\ 0 \le B,\ 0 \le D} u(C) + \beta Y E\{V(z', A', y'|z\}$$

$$s.t. \qquad C = y - (A - D) + q(z, A', D)B$$

$$A' = \delta A + B + \alpha(1-\delta)D$$

$$y' = \bar{y'}\Psi(D, z)$$

$$0 \leq D \leq A$$

Lenders

There are many lenders operating in a perfectly competitive market. Each lender, taking the decision functions of the borrower as given, set the discount price of the bond to gain zero expected profits.

A bond with a face value equal to a issued today, is expected to receive $a\big(1 - \frac{D'(z',A',y')}{A'}\big)$ tomorrow[3] and worths the market value[4] $\delta a + \frac{\alpha(1-\delta)D'(z',A',y')}{A'}a$. These two can represent the dividend and capital gain in the usual asset pricing models. The zero expected economic profit condition sets the price of the bond as:

$$q(z, A', y, D) = \frac{1}{1+r} E\left\{ \left(\left(1 - \frac{D'(z', A', y')}{A'}\right) + \delta + \alpha(1-\delta)\frac{D'(z', A', y')}{A'} \right) q(z', A'', D', y') \right\} \tag{3.1}$$

Similar result can be obtained by setting up the value function for the lenders, and applying the zero profit condition, as in Arellano et al. (2013). Let

$$\Omega(z, A, y, a) = aH(z, A, y)$$

[3] Assuming *pari-passu pro-rat*a bonds.

[4] Disregarding the seniority of bonds.

be the value function of a lender holding a coupon debt, when the state of the economy in the borrowing country is captured by (z, A, y). The value function for the risk neutral lender is

$$H(z,A,y) = A\left(1 - \frac{D(z,A,y)}{A}\right) + \frac{1}{1+r}E\left\{\delta + \alpha(z,A,y)(1-\delta)D(z,A,y)\right\} + \frac{1}{1+r}E\left\{H(z',A',y')|z\right\}$$

$$s.t. \qquad A' = \delta A + B(z,A,y) + \alpha(1-\delta)D(z,A,y)$$

$$y' = z'\Psi(D,z)$$

Zero profit condition implies, at the equilibrium:

$$q(z,A',y,D) = \frac{1}{1+r}E\left\{H\left(z',A',\Psi(D,z)\right)|z\right\}$$

Debt Renegotiation Problem

As in Yue (2010), the defaulted country and creditors will engage in one-round debt restructuring negotiations. Negotiation will allow the borrowing country to avoid Autarky:

$$\Delta_D(z,A,y;\alpha) = V(z,A,y;\alpha) - V^{AUT}(z,y)$$

Debt restructuring will help the lenders by compensating part of their loss:

$$\Delta_C(z,A,y;\alpha) = H(z,A,y) - H(z,A-D,y) \tag{3.2}$$

Let $\theta \in [0,1]$ be the debtor's bargaining power. The equilibrium recovery rate will be the maximand of the total surplus:

$$\alpha(z,A,y) = \arg\max_{\hat{\alpha}\in[0,1]} \left[\Delta_D(z,A,y;\hat{\alpha})\right]^{\theta}\left[\Delta_C(z,A,y;\hat{\alpha})\right]^{(1-\theta)} \tag{3.3}$$

$$s.t. \qquad \Delta_D(z,A,y;\hat{\alpha}) \geq 0,\;\; \Delta_C(z,A,y;\hat{\alpha}) \geq 0$$

Equilibrium

Let $s = \{z, A, y\}$ be the aggregate state of the economy, The Recursive Equilibrium for the economy consists of, Borrowing Country's Policy Functions ($C(s)$,$B(s)$,$D(s)$), Borrower and Lenders' Value Functions ($V(s)$ and $H(s)$, respectively), bond price $q(s,D)$, and recovery rate $\alpha(s)$, such that

1. Given the bond price, $q(s,D)$, and recovery rate function, $\alpha(s)$, the country's value functions and the policy functions, $B(s), D(s)$, and $C(s)$, will solve the Borrower's Problem.

2. Given the bond price, $q(s,D)$, and recovery rate function, $\alpha(s)$, value function $H(s)$ solves the lenders' problem.

3. Given the price functions, the value functions, and the policy functions the recovery rate solves the debt renegotiation problem.

4. Given the recovery rate, bond price, $q(s, D)$, is compatible with $D(s)$ and guarantees the zero profit condition for the lenders.

3.3 Quantitative Analysis

The country is assumed to have a constant absolute risk aversion preferences:

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma}$$

where σ is the coefficient of risk aversion. Following the literature, this paper sets the risk aversion coefficient to 2. The endowment stream is calibrated to match Argentina's GDP fluctuations. It is assumed the process follows a log-normal $AR(1)$ process. Then the resulted persistence and error variance is used to discretize the endowment space (Tauchen 1986).

This paper adopts the same output loss function as in Arellano et al. (2013).

$$\psi(D, z) = (1 - \phi_0 D^{\gamma})\big(1 - \hat{\phi}_1(z - z^*)\big)$$

where $\hat{\phi}_1 = \phi_1$ when $d > 0$ and $z > z^*$, otherwise $\hat{\phi}_1 = 0$. To reduce the caculation burden, we use the same parameter values as well: $\phi_0 = 0.04$, $\phi_1 = 0.20$, $\gamma = 1.60$, and $z^* = 0.9\bar{z}$. Table 3.1 summarizes the parameters used in the model.

Parameter	Value	Target
one-year risk free rate	$r = 4\%$	U.S. annual interest rate
Risk Aversion	$\sigma = 2$	Literature
Decay factor	$\delta = 0.936$	10-year Default-free duration
Stochastic structure	$\rho = 0.95$, η=0.025	Argentina output (1980Q1: 2001Q4)
Calibration	Value	Target statistics
Borrower's discount factor	$\beta = 0.97$	2.8% Default Probability
Borrower's bargaining power	$\theta = 0.65$	63% Ave. Recovery Rate

Table 3.1: Parameter Values

3.4 Results

The defaulted ratio turns out to be a key factor determining the price of the bonds. For instance at a very low level of current debt, A, any level of default is penalized by a high level of the discount price. That is the discount price will be set in a way to keep the relative cost of default the same as the borrowing cost; that is, the countries will optimally utilize the same default ratio as an instrument to transfer wealth from one state of the economy to the other.

Figure 3.1 depicts the charged interest rate as a function of the default ratio for two different income levels. As expected, a higher default ratio suggests the country is struggling with more difficulties and casts more doubts on the repayment ability. Hence, the price (interest rate) of future bonds will be lower (higher).

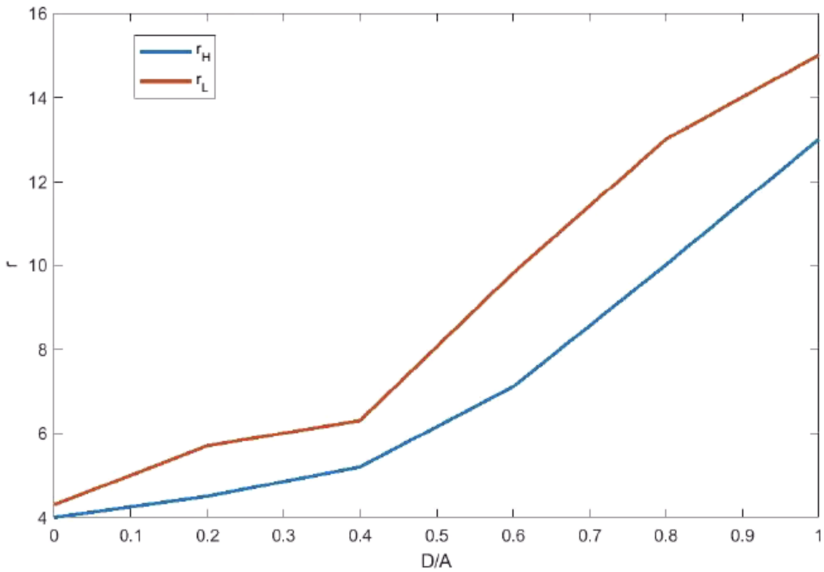


Figure 3.1: Interest Rates as a Function of Default Ratio

Figure 3.2 shows the optimal default ratio as the function of total (normalized) outstanding debt for different income levels. For higher income levels, default is never an optimal option; on the contrary, for low-income levels, partial default is immediately presented as an option in the face of dire straits.

Figure 3.3 illustrates why partial default is optimal for low-income levels. A lower recovery rate associated with low-income levels, allows the borrowing country to go through default as a last resort with no fear of high recovery rates in the future. On the other hand, for a high-income borrowing country, partial default is discouraged with no to low haircut levels.

All in all, the model provides acceptable results. Compared to the partial default model with no debt negotiation possibility, adding the endogenous recovery rate will improve the

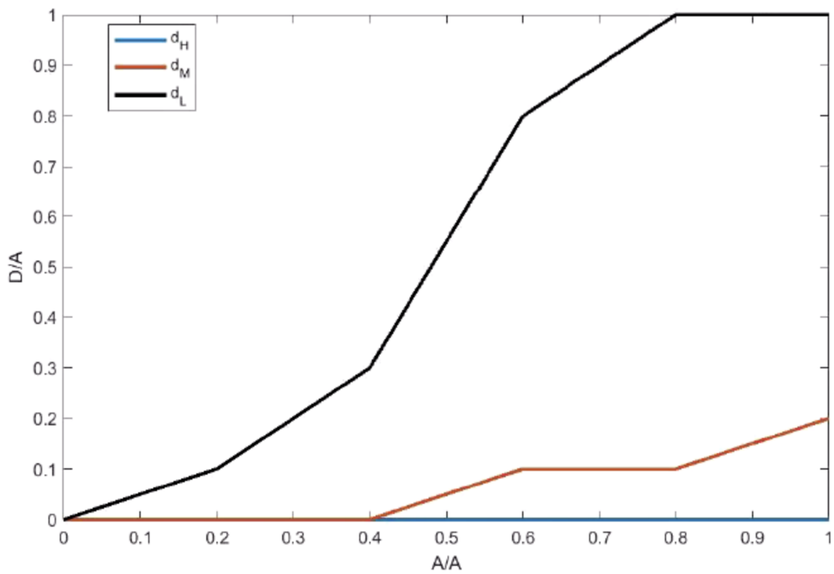


Figure 3.2: Optimal Default Ratio

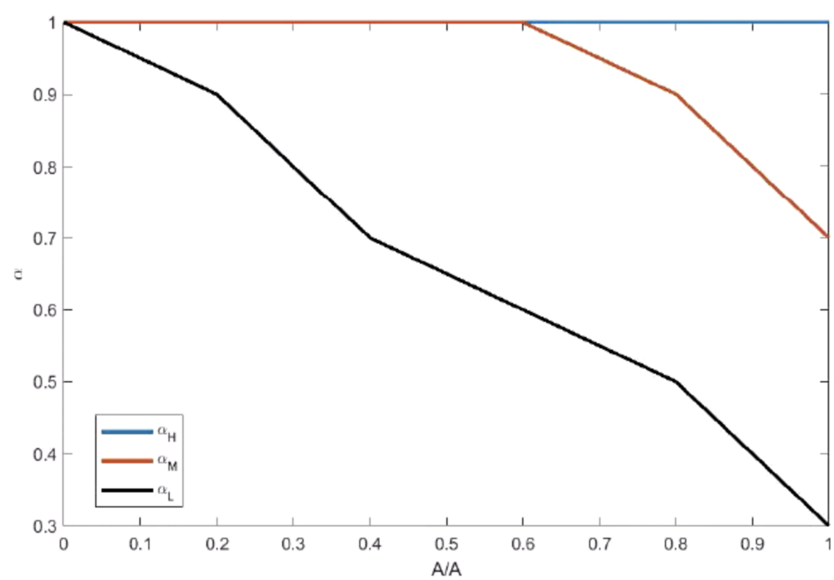


Figure 3.3: Recovery Rate as a Function of Default Ratio

	Model w/o Negotiation	Model with Negotiation	Data
Mean Episode Length (years)	5	10	9
Percentage of Short Episodes ($\leq$ 2 years)	45	35	38

Table 3.2: Default Episodes

frequency and the length of the default. For instance, Arellano et al. (2013) predict underestimates the average default episode lengths. Debt negotiation will allow the defaulted country to postpone the repayment after the occurrence of good shocks. This leads to longer defaults. Defaults only happen in adverse situations, hence in presence of endogenous debt negotiations, the country will face higher haircuts and will not suffer as much. This in turn increases the frequency of long defaults, as shown in Table 3.2.

3.5 Conclusion

This paper provides a government borrowing model with two features. Partial default allows the country to remain in the financial market and even issue new bonds while defaulting on the other payments. Endogenous recovery rates will discourage unnecessary defaults while providing assistance for defaulted low-income countries. In the presence of debt negotiations, defaults episodes are longer, and longer defaults are more frequent. Future works should first focus on a model with less reduced form restrictions. Changing the parameters of the default cost function can affect the result. Second, a partial default model in the presence of long- and short-term bonds will provide more insight into the government borrowings and defaults.

Chapter 4

Conclusions

This dissertation emphasizes the role of debt renegotiations in the government borrowing models. As presented in this study, including debt renegotiations to the sovereign default models provide statistics that better match the data. In the first chapter, we see adding the debt renegotiation feature will lead to a more realistic interest rate behavior. This is helpful, in particular, if our goal is to model the interest rate spreads. The recovery rates resulting from the endogenous debt renegotiations will allow the borrowing country to hold higher levels of debt. The higher debt will further decrease bond prices and helps the model to generate the interest rate spreads seen in the data.

In the second chapter, the debt renegotiation feature helps us to explain the reduction in the number of defaults when countries start to use bonds as a new debt instrument. Despite an increase in the borrowing country's bargaining power, the reduction in interest payments will increase the default's opportunity cost. Hence, the borrowing country may not find default an optimal decision as frequently as before.

Finally, in chapter three, we add the debt negotiation to the partial default models. The equilibrium recovery rates will guarantee that generous haircuts will only be available for a country with a high level of debt or a low level of income. This will help the model to render default episodes that match data. Since default at some states of the economy is "justified" now, the borrowing country may find it optimal to default more frequently and stay in default for a longer time.

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