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# THE (N+1)st NOTE ON THE TWIN PARADOX*

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An error is found in H. Lass' discussion of the Twin Paradox. The discussion by R. Muller, disputed by Lass, is maintained to be correct.

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H. Lass, in his recent "N$^{th}$ Note" [1] disputed my resolution of the twin paradox. [2] I will keep my reply short, in the hope that our series will converge.

Lass was confused by what I meant by "John's age as measured by Mary" when John and Mary are distant from each other. He assumed that I meant that Mary had a friend at John's position but in Mary's Lorentz frame. Mary's friend would look at John both before and after the turnaround, and communicate his age to Mary. The introduction of co-moving observers is a standard trick in relativity theory, and it was reasonable for Lass to assume that I meant to employ it. However I did not, because although this trick works well in the usual relativity problems, it harbors a trap(which Lass fell into) when used in problems involving accelerations.

Let me first explain what I did mean by "John's age as measured by Mary." I will start with a simple example. Suppose an astronaut on the moon is hitting a golf ball with a club, and we are home watching it on television. We see him hit the ball, note the time, and then we calculate the time that the event occurred. Such a calculation is particularly easy to do for events that occur within our Lorentz frame: we simply subtract the 1.3 seconds that it took the light to reach us from the moon. (In practice we would also have to subtract the delay in transmission that occurred between the receiving station and our home TV.) If the astronaut were not in our Lorentz frame, but were moving with respect to us with a velocity comparable to that of light, the calculation of the time of the event in the astronaut's frame would be slightly more complicated. We would have to correct for the velocity by using the time transformation formula from special relativity (eq. 3 in ref 2),

which contains a term linear in the velocity. It is this relativistic formula that we must use in the twin problem. It is not necessary to introduce a co-moving observer, although sometimes it is convenient to do so.

The difficulty with the concept of a co-moving observer comes about when acceleration is introduced into the problem. Suppose Mary and her friend turn around simultaneously, as measured in their original frame. Then their turnarounds were not simultaneous in their final Lorentz frame! The concept of "co-accelerating" is not Lorentz invariant. Lass overlooks this important fact in his analysis. In his calculation, he has the friend measure John's age both before and after the turnaround. Mary would claim that her friend's initial measurement was made correctly, but that his second measurement was made at the wrong time, for it was made just after the friend turned around but not just after Mary turned around. By ignoring the non-simultaneity of the turnarounds as observed by Mary, Lass derives the standard incorrect answer, which he then calls "an obvious result."

The twin paradox is indeed perplexing, and it is remarkable that otherwise reasonable scientists are still debating it. The statement that time will eventually resolve all differences is certainly Lorentz invariant; let us hope the statement is also true.

## REFERENCES

1. H. Lass, Amer. Journ. Physics, to be published.
2. R. A. Muller, Amer. Journ. Physics 40, 966 (1972).

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