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ON THE z^3 CORRECTIONS TO ENERGY LOSS AND RANGE*

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ABSTRACT

Higher order corrections to the stopping power, proportional to z^3, are evaluated. Both close and distant collisions are considered. The energy loss formula can be written as

$$\frac{dE}{dx} = z^2 I + z^3 (J_c + J_d)$$

where I is the customary lowest order energy loss and J_c and J_d are the close and distant collision parts of the z^3 term, respectively. The close collision contribution J_c is a relativistic effect, first estimated in unpublished work by Fermi. It has the simple form, $J_c = \pi\alpha C/2\beta$, where C is the standard constant multiplying β^{-2} times the Bethe-Bloch logarithm in I [see Eq. (2)] and α is the fine structure constant. At high energies J_c gives a constant z^3 contribution to the energy loss and causes a range difference ΔR roughly proportional to the range R for stopping particles of the same mass and energy, but opposite charge. For

$2 < P/Mc < 20$, $\Delta R/R$ changes by less than $\pm 6\%$ and depends only slightly on the stopping material, varying from 1.9×10^{-3} for carbon to 2.5×10^{-3} for lead. The distant collision effect is important only at low velocities. The calculation of this contribution is patterned after a recent work of Ashley, Ritchie, and Brandt, but differs importantly from it. Using a statistical model for the atom it is found that at low velocities the relative z^3 contribution can be written $J_d/I = F(V)/(Z)^{\frac{1}{2}}$ where Z is the atomic number of the stopping medium and $F(V)$ is a universal function of the reduced velocity variable, $V = 137\gamma\beta/(Z)^{\frac{1}{2}}$. In the region where J_d/I is appreciable $(1 < V < 10)$, $F(V)$ varies as V^{-n} with $n \simeq 2.0$-2.5. These results on the z^3 effect at low velocities are in good agreement with available data on comparison of the energy loss of helium ions and protons of the same velocities.

Range differences are calculated for carbon, copper, lead, and emulsion absorbers, including the effects of both close and distant collisions. The results are in rough agreement with data on slow stopping pions and sigma hyperons in emulsions and in good agreement with very recent measurements of fast positive and negative muons.

The upper limit of the range of validity of the results is examined in some detail. It is found that the approximations begin to fail for dynamic reasons above $\gamma \simeq 20$ for muons, and presumably also for other heavy particles.

I. INTRODUCTION

For a heavy particle of charge ze and velocity $v = \beta c$ passing through a medium of atomic number Z, the standard expression for energy loss in MeV cm^2/gm is[1-3]

$$\frac{dE}{dx} = z^2 I = C \frac{z^2}{\beta^2} L(\beta,Z) \tag{1}$$

where

$$C = \frac{4\pi N_0 e^4}{mc^2} \left(\frac{Z}{A}\right) = 0.307 \frac{Z}{A} \tag{2}$$

and $L(\beta,Z)$ is given at velocities well above the orbital velocities of the atomic electrons by

$$L(\beta,Z) \simeq \ln\left(\frac{2\gamma^2 mv^2}{I_0 Z}\right) - \beta^2 . \tag{3}$$

The parameter I_0 characterizes the medium; empirically it varies somewhat over the periodic table, ranging from 12.0 for $Z = 13$ to 9.9 for $Z = 82$. At high energies (3) must be modified by the density effect, while at low velocities the inner shell corrections enter. Equation (1), based on the first Born approximation, gives an energy loss proportional to z^2. As a consequence, particles of opposite charge are predicted to lose energy at the same rate, helium ions are predicted to lose energy four times as rapidly as protons of the same velocity, and so on.

In the stopping of slow particles, small charge dependent effects at variance with Eq. (1) have been known experimentally for a long time, usually in connection with precise measurements of Q values and the masses of particles.[4,5] For example, Barkas, Dyer,

and Heckman[5] found that the fractional difference in range in emulsions for negative and positive Σ hyperons of $\beta \simeq 0.14$ amounted to slightly more than 3×10^{-2}, the negative sigma having the greater range. Direct observation of differences in energy loss for slow positive and negative pions $(0.05 < \beta < 0.18)$ in emulsions have been made by Heckman and Lindstrom.[6] They found a 14% greater loss by positive pions at $\beta \simeq 0.05$, but no difference at the level of accuracy of 1% for $\beta > 0.14$. In comparisons of energy losses and ranges of hydrogen and helium ions with kinetic energies of the order of a few MeV, discrepancies with the z^2 dependence in Eq. (1) have also been known for some time, but systematic errors have prevented identifying the source of the difficulty.[7] In 1969, however, careful experiments with an absolute accuracy of 0.3% were made by Andersen, Simonsen, and Sørensen.[8] In a comparison of the energy loss in aluminum and tantalum by hydrogen and helium ions of the same velocity they found that helium ions lost energy at a rate slightly larger than four times that of the hydrogen ions. For $\beta = 0.073$ the fractional excess was 2.6% in tantalum, 1.3% in aluminum, and varied roughly as β^{-2} over the range $0.07 < \beta < 0.12$. The inference from these experiments is that the energy loss formula should read

$$\frac{dE}{dx} = z^2 I + z^3 J \tag{4}$$

where J/I is a small positive quantity that decreases with increasing velocity, being of the order of a few percent for $\beta \simeq 0.1$.

The idea of z^3 (and higher) corrections to the basic energy loss formula is, of course, fairly obvious. Higher order Born approximations bring in such terms. But only recently has there been theoretical work specifically directed at a calculation of the z^3 effect at low velocities for energy loss. Hill and Merzbacher[9] and Ashley, Ritchie, and Brandt[10] have considered the contribution from distant collisions, treating the heavy incident particle as a classical source of a Coulomb potential. Merzbacher treats the atom (a harmonically bound electron) quantum-mechanically, while Ashley, Ritchie, and Brandt treat it classically. The two calculations agree, as is expected for harmonic oscillators. Ashley, Ritchie, and Brandt use a Thomas-Fermi statistical model to generalize their prototype calculation to actual atoms. With one adjustable parameter they obtain good agreement with the data of Ref. 8.

Less well known and certainly less well documented experimentally are z^3 effects in energy loss and range for <u>fast</u> particles. Systematic comparisons of the energy losses of fast positive and negative muons have shown equality at the level of 1% precision.[11,12] Expected relative differences from higher order electromagnetic effects are of the order of the fine structure constant or smaller and so need an order of magnitude improvement in accuracy for their verification. There are, however, high-precision experiments for which knowledge of energy loss and range differences between positive and negative particles may be important. One such experiment is the measurement of the charge asymmetry in the $K_L^0 \to \pi\mu\nu$ decay mode.[13,14] A muon range difference of a few tenths of a percent in these particular experiments, where none has been assumed, would necessitate an appreciable correction to the quoted asymmetry.[15] We show that the fractional difference in range at high energies is indeed of this order of magnitude.

Calculations of the differences in energy loss and range for <u>fast</u> muons or other heavy particles do not seem to exist in the published literature. Higher order electromagnetic corrections have been considered in connection with the density effect at ultrahigh energies.[16-18] Zhdanov et al.[17,18] report a "Tsytovich effect" of the order of 5-8% for electrons with $\gamma > 200$. Crispin and Fowler[19] discuss the existing data on the density effect and conclude that the work of Ref. 17 is the only evidence for as large an effect as Tsytovich[16] predicted. None of these authors discusses the question of <u>differences</u> in energy loss dependent on the sign of the incident charge.

The calculations of Refs. 9 and 10 on the z^3 effect at low velocities give results that fall off rapidly with increasing velocity and become quite negligible for $P/Mc = \gamma\beta > 1$. There are two questions that arise here. One is whether a relativistic generalization of these calculations of the effect of distant collisions does or does not give a nonvanishing z^3 contribution as $\beta \to 1$. The other is whether there is a z^3 contribution from close collisions. The first question is answered in Sec. III where it is shown that the distant collision contribution to J varies as $(1/\gamma^2\beta^5)$ times a logarithm and so is confined exclusively to the low velocity domain. What about close collisions where the atomic electrons can be treated as free and the energy loss computed from the scattering of the electrons by the incident particle? The usual argument[10] is that there is no z^3 term from close collisions because the Rutherford scattering formula is strictly proportional to z^2. This argument is valid at low velocities, but is not correct at relativistic speeds. This fact was recognized over eighteen years ago by Enrico Fermi and

"[27] Communicated in a letter dated October 8, 1953, to W. H. Barkas. Professor Fermi pointed out that the Mott theory of scattering [see G. Wentzel, Handbuch der Physik (Verlag, Julius Springer, Berlin, 1933), Vol. 24, Sec. 1, p. 708] may be applied to the scattering of negative electrons by both negative and positive mesons (in the coordinate frame in which the meson is at rest). He found in this way that the average impulse transmitted to the negative meson is less than that received by the positive meson."[4]

We repeat the simple and elegant calculation of Fermi in Sec. II and obtain the close-collision contribution to the z^3 part of the energy loss.[20] In Sec. III the relativistic generalization of the calculation of Ref. 10 is given. A different treatment of the minimum impact parameter leads to a universal function $F(V)$ for $(Z)^{\frac{1}{2}}J_d/I$, where $V = 137\gamma\beta/(Z)^{\frac{1}{2}}$ is a reduced velocity variable. In Sec. IV the numerical results for energy loss and range differences are given and compared with available data. In Sec. V a number of factors are considered including the proper quantum-electrodynamic calculation of muon-electron scattering to order α^3, in order to identify the range of incident momenta over which the Fermi calculation for the close-collision part of the difference in energy loss is a good approximation.

II. THE CALCULATION OF FERMI

As indicated in the introduction, both close and distant collisions contribute to the z^3 term in the energy loss (4). In contrast to the z^2 term, in which close and distant collisions contribute roughly equally at all but the lowest velocities,[1,2] the z^3 term is dominated by the effects of distant collisions at low velocities and by the effects of close collisions at high velocities. For the close collisions the binding of the atomic electrons can be neglected. We will see below that for the z^3 contribution the collisions are "harder" than for the z^2 term. The neglect of binding is therefore even better justified for the calculation of the z^3 term at high energies than for the z^2 part. For this close collision contribution we follow the path clearly spelled out by Fermi.

If the fast incident particle is much heavier than an electron, it is advantageous to consider the collision in the rest frame of the incident particle. Then, provided the momentum of the electron in that frame is small compared to the mass of the incident particle, the collision can be treated as the elastic scattering of the electron by a fixed center of force and the energy loss simply related to the momentum and scattering angle. Specifically, if the fast heavy particle has a laboratory momentum and energy $E = \gamma Mc^2$, $p = \gamma Mv$ then the energy loss per collision, in which the electron is deflected by an angle θ in the rest frame of the incident particle, is

$$\epsilon = 2\gamma^2 v^2 m \sin^2 \frac{\theta}{2} . \tag{5}$$

Here m is the mass of the electron. Equation (1) is valid provided $\gamma << M/m$. This restriction on the kinematics is easily removed and will be removed in Sec. V. But for the present we are doing the calculation à la Fermi.

The probability of a given energy loss ϵ is given by the differential scattering cross section $d\sigma/d\epsilon$. In the limit $M \to \infty$, $\gamma\beta m \ll M$, this cross section for the scattering of electrons by a fixed center of force of charge ze is the well-known result of Mott.[21] Mott's formulas have been expanded in powers of ze^2 by McKinley and Feshbach.[22] Their result, correct to third order in ze^2, is

$$\frac{d\sigma}{d\Omega} = \frac{z^2 e^4}{4\gamma^2 m^2 v^4 \sin^4 \frac{\theta}{2}} \left[1 - \beta^2 \sin^2 \frac{\theta}{2} + \pi z\alpha\beta \sin \frac{\theta}{2}\left(1 - \sin \frac{\theta}{2}\right)\right] \tag{6}$$

where $\beta = v/c$, $\gamma = (1 - \beta^2)^{-\frac{1}{2}}$, $\alpha = 1/137$ and the charge convention is such that the proton has $z = 1$. Use of Eq. (5) allows us to transform (6) into $d\sigma/d\epsilon$:

$$\frac{d\sigma}{d\epsilon} = \frac{2\pi e^4}{mv^2} \cdot \frac{z^2}{\epsilon^2}\left\{1 - \beta^2 \frac{\epsilon}{\epsilon_m} + \pi z\alpha\beta \left[\left(\frac{\epsilon}{\epsilon_m}\right)^{\frac{1}{2}} - \frac{\epsilon}{\epsilon_m}\right]\right\} \tag{7}$$

where $\epsilon_m = 2\gamma^2 v^2 m$ is the maximum energy transfer in the collision.

The energy loss from close collisions, expressed as energy loss per gm/cm^2, is

$$\left(\frac{dE}{dx}\right)_{close} = N_0 \frac{Z}{A} \int_{\epsilon_0}^{\epsilon_m} \epsilon \frac{d\sigma}{d\epsilon}\, d\epsilon \tag{8}$$

where N_0 is Avagadro's number, Z and A are the atomic number and atomic weight of the absorber, and ϵ_0 is a minimum energy loss, below which binding effects become important. The first two terms in

(7) give the standard result for the close-collision contribution to the z^2 term in Eq. (4):

$$I_c = \frac{C}{2\beta^2}\left[\ell n\left(\frac{\epsilon_m}{\epsilon_0}\right) - \beta^2\left(1 - \frac{\epsilon_0}{\epsilon_m}\right)\right] \tag{9}$$

where C is given by Eq. (2). When combined with the contribution from the distant collisions, this leads to the Bethe-Bloch result, Eq. (1).

The last terms in (7) contribute to J in Eq. (4). The presence of the factors of $\epsilon^{\frac{1}{2}}$ and ϵ, or equivalently the factors of $\sin(\theta/2)$ and $\sin^2(\theta/2)$ in Eq. (6), shows that these terms correspond to "hard" collisions. They vanish relative to the leading term as $\epsilon \to 0$ $(\theta \to 0)$. In fact, in the energy loss expression, (8), they lead to a finite result in the limit $\epsilon_0 \to 0$. This is in contrast to the logarithmic divergence of the leading contribution. Since ϵ_m/ϵ_0 is typically 10^3 to 10^5 we make a negligible error in taking the lower limit in (8) to be zero in calculation of the z^3 part. The close-collision contribution to J in Eq. (4) is therefore

$$J_c = \frac{\pi\alpha C}{2\beta} \int_0^{\epsilon_m} \left[\left(\frac{\epsilon}{\epsilon_m}\right)^{\frac{1}{2}} - \frac{\epsilon}{\epsilon_m}\right] \frac{d\epsilon}{\epsilon} = \frac{\pi\alpha C}{2\beta} \tag{10}$$

where C is given by Eq. (2). This is the basic Fermi result.[20] It is a purely relativistic effect.[23] Its β^{-1} dependence compared with the $\beta^{-2}L(\beta,Z)$ variation of the z^2 term makes it unimportant at low velocities. But as $\beta \to 1$, J_c contributes a roughly constant fractional amount in the region of minimum energy loss $(3 < \gamma < 30)$. This means that at high energies the fractional difference in range

$\Delta R/R$ from the z^3 term in (4) is more or less energy-independent and given in magnitude by $2J_c/I \simeq \pi\alpha/L$. With $L \sim 10$-15, $\Delta R/R$ is of the order of $(1.5$-$2.3) \times 10^{-3}$. This estimate is borne out by detailed numerical calculations--see Fig. 4.

The mode of derivation of J_c implies that its validity is restricted to momenta such that $\gamma \ll M/m$. This limitation on the kinematics is shown in Sec. V to be not required, but one might be concerned that a proper quantum-electrodynamic calculation, including radiative corrections consistently, would lead to appreciable modifications. This question is also explored in Sec. V where it is found that the Fermi result is a reasonable approximation for muons of momenta up to several GeV/c.

III. THE CONTRIBUTION OF DISTANT COLLISIONS

The effect of distant collisions in higher approximation has been considered by Hill and Merzbacher[9] and Ashley, Ritchie, and Brandt.[10] These authors assume nonrelativistic motion and treat the incident heavy particle of charge ze and speed v as moving classically. The struck atom is thus acted upon by a time-dependent external field. It is well known that, provided $M/m \gg 1$, this method is mathematically equivalent to a quantum-mechanical description of both atom and incident particle, at least for the first Born approximation.[24] In Ref. 9 the atom is approximated by a quantum-mechanical harmonic oscillator with level spacing $\hbar\omega_0$, while in Ref. 10 the electronic oscillator is treated classically. Hill and Merzbacher[9] have shown that for a straight-line path of the incident particle the two methods yield the same result for the z^3 contribution. This exact agreement undoubtedly follows from the special properties of the quantum oscillator, but it is plausible that for more realistic models of the atom the equivalence of the classical and quantum treatments follows upon summing over all possible transitions (as occurs in the z^2 energy loss via the dipole and generalized oscillator strength sum rules).

Since one of our concerns is a possible distant-collision contribution to J at high energies, we repeat the classical calculation of Ashley, Ritchie, and Brandt[10] without the approximation of nonrelativistic motion of the incident particle. In addition, we make a different choice of the minimum impact parameter a and are led to a simpler, universal form for J_d/I with no adjustable parameters. Since the calculation is described in detail in their paper and the only modifications in the expressions for a single electron atom are appropriate factors of γ occurring in the Lorentztransformed fields, we merely state the results. The distant collision contribution to I in Eq. (4) is well known[25] to be

$$I_d = \frac{C}{\beta^2} \int_\xi^\infty x \left[K_1^2(x) + \frac{1}{\gamma^2} K_0^2(x)\right] dx \tag{11}$$

where $\xi = \omega_0 a/\gamma v$. For $\xi \ll 1$, this can be written approximately as

$$I_d = \frac{C}{2\beta^2} \left[\ln\left(\frac{1.123}{\xi}\right)^2 - \beta^2\right]. \tag{12}$$

The corresponding contribution to J in (4) is

$$J_d = \frac{\alpha C}{\gamma^2\beta^5} \cdot \frac{\hbar\omega_0}{mc^2} \left[I_1(\xi) + \frac{1}{\gamma^2} I_2(\xi)\right] \tag{13}$$

where $I_1(\xi)$ and $I_2(\xi)$ are integrals defined and tabulated by Ashley, Anderson, Ritchie, and Brandt.[26] For small values of their arguments the integrals are given numerically by $I_1(\xi) \simeq \frac{3\pi}{2} \ln(3/8\xi)$ and $I_2(\xi) \simeq 2.175$. For large argument they vanish exponentially. In the nonrelativistic limit $(\gamma \to 1)$, Eq. (13) reduces to the corresponding result of Ref. 10, with their integral $I = I_1 + I_2$. At high energies $(\gamma \gtrsim 2)$ the presence of the factor γ^{-2} rapidly suppresses J_d compared to J_c, Eq. (10).

To make the distant-collision contributions to I and J well defined it is necessary to specify the minimum impact parameter $\underline{a}$. Our choice is

$$a = (\hbar/2m\omega_0)^{\frac{1}{2}}, \qquad \xi = \left(\frac{\hbar\omega_0}{2m\gamma^2 v^2}\right)^{\frac{1}{2}}. \tag{14}$$

The justification of this value for $\underline{a}$ is that it is the magnitude of the dipole matrix element x_{10} for the harmonic oscillator, or equivalently a measure of the amplitude of a classical oscillator with energy $E = \hbar\omega_0$. It is thus the impact parameter where the expansion of the interaction energy into multipoles fails and the dipole approximation for I (and dipole plus quadrupole for J) can no longer be trusted. It is completely analogous to the momentum transfer $K_0 = a^{-1}$ that divides the soft collisions from the hard collisions in Bethe's calculation of I.[27] With the choice (14) in Eq. (12) for I_d and $\epsilon_0 = \hbar\omega_0$ in Eq. (9) for I_c, the sum yields the standard result (1) for the total z^2 energy loss with $I_0 Z$ in (3) given by $I_0 Z = \hbar\omega_0/1.123$.

The result (13) for a single, harmonically bound electron (multiplied by Z) is too stylized for immediate comparison with experiment. Like Ashley, Ritchie, and Brandt, we use the Thomas-Fermi statistical model of the atom in the manner described in detail by Lindhard and Scharff[28] to give an approximate description of a many-electron atom. The basic idea is to specify the number of electrons per unit volume in the atom by means of the number density $\rho(r)$ of the statistical model and to relate the effective oscillator frequency ω_0 for the various electrons to the plasma frequency $\omega_p(r)$ corresponding to $\rho(r)$. Thus, for example, the logarithm in (12) is replaced by an integral,

$$\ln\left(\frac{2\gamma^2 mv^2}{\hbar\omega_0}\right) \to \int d^3r\, \rho(r) \ln\left(\frac{2\gamma^2 mv^2}{\chi\hbar\omega_p(r)}\right) \tag{15}$$

where

$$\omega_p^{\,2}(r) = \frac{4\pi Ze^2}{m}\,\rho(r) \qquad (16)$$

and $\rho(r)$ is the statistical model number density for atomic number Z, normalized to unity. If the electrons in the atom acted independently, the parameter χ in the argument of the logarithm would be expected to be unity. Inside an atom, however, the electrons respond both individually and collectively (polarization effects). Lindhard and Scharff[28] present arguments that $\chi \simeq \sqrt{2}$ in heavy atoms.

With the choice (14) for ξ and the Lindhard-Scharff ansatz $\omega_0 \to \chi\omega_p(r)$ we have the statistical model generalization of (13):

$$J_d = \frac{2\alpha C}{\beta^3}\int d^3r\ \rho(r)\ \xi^2\left[I_1(\xi) + \frac{1}{\gamma^2}I_2(\xi)\right] \qquad (17)$$

where $\xi = (\chi\hbar\omega_p(r)/2m\gamma^2v^2)^{\frac{1}{2}}$. The corresponding expression for the function $L(\beta,Z)$ in Eq. (1) is

$$L = \int d^3r\ \rho(r)\ [\ell n(1/\xi^2) - \beta^2]\ . \qquad (18)$$

For the logarithm term in (18) it is necessary to put a lower limit $r = r_0$, defined by $\xi(r_0) = 1$, in order to avoid spurious negative contributions to the integral. A nonzero r_0 represents the statistical approximation to the inner shell corrections.[10] At large velocitites, $r_0 \to 0$, and (18) becomes equal to (3). With the Lenz-Jensen approximation[29,30] for $\rho(r)$ and $\chi = \sqrt{2}$, Lindhard and Scharff found $I_0 = 10.7$ eV in (3), in reasonable agreement with empirical values for all but the lightest elements.

Because the statistical model has a length scale proportional to $Z^{-\frac{1}{3}}$, $\rho(r)$ scales like Z and so does the plasma frequency $\omega_p(r)$. This scaling property has as its consequence that the integrals in (17) and (18) are not functions of $\gamma\beta$ and Z separately, but depend only on the combination $[\gamma\beta/(Z)^{\frac{1}{2}}]$. It is convenient therefore to introduce a reduced velocity variable V defined by

$$V = 137\ \gamma\beta/(Z)^{\frac{1}{2}}\ . \qquad (19)$$

In the low-velocity region where J_d is important, Eq. (17) can then be written in nonrelativistic approximation as

$$J_d^{\,NR} = \frac{C}{\beta^2}\ \frac{2}{(Z)^{\frac{1}{2}}}\cdot\frac{1}{V}\int d^3r\ \rho(r)\ \xi^2[I_1(\xi) + I_2(\xi)] \qquad (20)$$

while

$$I^{NR} = \frac{C}{\beta^2}\int d^3r\ \rho(r)\ \ell n(1/\xi^2)\ . \qquad (21)$$

Since the integrals are functions only of V, at low velocities the fractional difference in energy loss (J_d/I) is given for all pure substances in universal form,

$$\frac{J_d^{\,NR}}{I^{NR}} = \frac{F(V)}{(Z)^{\frac{1}{2}}} \qquad (22)$$

where F(V) is the appropriate ratio of integrals from (20) and (21). Ashley, Ritchie, and Brandt[10] did not obtain a universal dependence on V for $(Z)^{\frac{1}{2}} J_d/I$. This can be traced to their different choice of the minimum impact paramter $\underline{a}$. They identified $\underline{a}$ with the radius $\underline{r}$ associated with the plasma frequency $\omega_p(r)$, writing $\underline{a} = \eta\underline{r}$ with

η a parameter expected to be of order unity. As a result their expression for $(Z)^{\frac{1}{2}}J_d/I$ depends on the parameter η and $Z^{1/6}$, as well as V.

IV. NUMERICAL RESULTS AND COMPARISON WITH EXPERIMENT

A. Energy Loss at Low Velocities

In the velocity range $\beta < 0.2$ the z^3 term in the energy loss is given almost entirely by the contribution from distant collisions. With the description of the atom by means of the statistical model, the energy loss (4) at low velocities for a stopping material of atomic number Z can be written by means of (22) in the form,

$$\frac{dE}{dx} = z^2 \frac{C}{\beta^2} L(V) \left[1 + \frac{z}{(Z)^{\frac{1}{2}}} F(V)\right] \tag{23}$$

where L(V) is defined by (18) or (21) and V is given by (19). The functions L(V) and F(V) were calculated numerically using the Lenz-Jensen approximation for $\rho(r)$ and the numerical values of I_1 and I_2 of Ref. 26. The results are shown graphically in Fig. 1. The logarithm function L(V) rapidly approaches the form given by Eq. (3) with the β^2 term missing. For the reader's convenience, we plot $L(V) - 2\ell n\, V - 1.6$. This quantity rapidly approaches a constant value of 0.0244, corresponding to (3) with $I_0 = 10.72$ eV. The rise near $V = 1$ reflects the presence of the inner shell corrections at low velocities.

The function F(V) decreases rapidly with increasing V. It is found to vary as V^{-n}, with $n \simeq 2.0$ for $0.5 < V < 1.5$, $n \simeq 2.3$ for $1.5 < V < 4$, and $n \simeq 2.5$ for $4 < V < 10$. We have therefore plotted the more slowly varying function, $V^2F(V)$, in Fig. 1. Because of the rather crude description of the inner atomic shells, the results for L(V) and F(V) are not reliable below $V \simeq 0.8$.

The solid curves in Fig. 1 are calculated for the preferred value, $\chi^2 = 2$. The dashed curves for $V^2F(V)$ correspond to $\chi^2 = 1$ and $\chi^2 = 3$. The differences are of the order of 15-20%. We adopt the viewpoint that $\chi^2 \simeq 2$ is determined empirically by $L(V)$, and that $F(V)$ is thereby specified completely within the framework of the model.

In the experiments of Andersen, Simonsen, and Sørensen[8] comparison was made between the energy loss of helium ions and four times the energy loss by protons of the same velocity stopping in tantalum ($Z = 73$) and in aluminum ($Z = 13$). The results are displayed as the fractional difference, $(\mathrm{He} - 4\mathrm{H})/\mathrm{He}$. From Eq. (23) we see that this fractional difference is $F(V)/(Z)^{\frac{1}{2}}$. A comparison of these data with $F(V)$ is presented in Fig. 2. The velocity interval covered in the experiment is $0.07 < \beta < 0.12$ for both absorbers, but the use of the reduced variable V separates the tantalum and aluminum data. Within errors the data are in excellent agreement with the calculated curve. Again, the dashed curves correspond to $\chi^2 = 1$ and 3. If the error bars are ignored, one might argue that the aluminum data require $\chi^2 > 2$. If this tendency is real, it probably reflects the fact that the empirical value of I_0 in (3) for light elements tends to be larger than the statistical value of 10.7 eV. In any event, the existing data on the z^3 effect in Al and Ta are in very satisfactory agreement with the theory. Other comparisons of the stopping powers of protons with kinetic energies from 0.4 to 1.0 MeV and alpha particles of the same speeds, in copper and gold,[31] and in argon,[32] are in general agreement with the curves

in Figs. 1 and 2, although the errors are so large that only the order of magnitude and a rough energy dependence can be established.

For emulsion (or other mixtures) the simple result (23) must be properly averaged over the various ingredients. For the standard nuclear emulsion[33] we have evaluated the appropriate averages of $L(V)$ and $L(V)F(V)$ and computed the quantity $(2J_d^{NR}/I^{NR})$ as a function of $p/Mc = \gamma\beta$. This is shown in Fig. 3 for the range, $0.02 < \gamma\beta < 0.16$, along with the data of Heckman and Lindstrom[6] on the difference in energy loss for slow positive and negative pions in emulsion. The agreement here is not as satisfactory as that shown in Fig. 2, although the velocity dependence comparison is reasonable and the errors are large. An added consideration is the fact that the conversion from observed grain density differences to differences in energy loss does involve a model of that phenomenon.

The comparisons shown in Figs. 2 and 3 indicate that the z^3 contribution to stopping power at low speeds is reasonably well described by the Eq. (23). As already mentioned, because of the relative crudeness of the statistical model, its validity is restricted to $V \gtrsim 0.8$. An additional limitation is the neglect of the complicated effects of capture and loss of electrons by the incident particle when its speed is near the orbital speeds in the atoms. Note also that we have implicitly assumed that $(ze^2/\hbar v) < 1$, or equivalently, $V > z/(Z)^{\frac{1}{2}}$, and so have excluded slow, highly charged ions from consideration. At the high energy end, the nonrelativistic approximations ($\gamma \simeq 1$) that led to (22) and (23) must be abandoned. Equation (17) must be used for the distant collision contribution and the close collision term (10) must be included. For such speeds, however, the z^3 term is extremely small. It is probably observable only indirectly via range differences (see the next section).

B. Range Differences

The mean range of a particle of initial energy $E = \gamma Mc^2$ stopping in matter is defined by

$$R(E) = \int_{Mc^2}^{E} \frac{dE}{\left(\frac{dE}{dx}\right)} . \tag{24}$$

Since dE/dx is a function of the speed of the particle, the range of a particle of a given speed is proportional to its mass. The quantity R/M is thus a function only of speed and charge. Because the dominant energy loss is proportional to z^2, the reduced range, z^2R/M, is approximately a function only of speed β and the properties of the stopping medium. The presence of a z^3 term in the energy loss causes departures from this standard behavior. In particular, it leads to range differences for particles of the same mass and initial energy, but opposite charge. For definiteness, we shall consider the range difference ΔR for particles of the same mass and initial speed and $z = \pm 1$. To first order in small quantities, the calculated $\Delta R/R$ can be used in an obvious way to evaluate departures from the z^2R/M form for other ions.[34] The first order difference in range follows from substitution of (4) into (24):

$$\Delta R \equiv R_- - R_+ = \int_{Mc^2}^{E} \frac{2J}{I^2} dE . \tag{25}$$

For comparison, the z^2 range is

$$R_0(E) = \int_{Mc^2}^{E} \frac{dE}{I} . \tag{26}$$

Equations (25) and (26) show that, to the extent that (J/I) is constant in energy, ΔR is proportional to R. This is true at high energies in the region of minimum ionization, but is far from true at low speeds. We thus expect that $\Delta R/R$ will be relatively large at low speeds, will decrease rapidly with increasing speed, and will ultimately level off to a more or less constant plateau. The numerical calculations shown in Fig. 4 bear out this behavior.

The integration of (25) and (26) was done numerically using relativistic kinematics and Eqs. (10) and (17) for the close and distant collision contributions to J. For I, the statistical model Eq. (21) was used for $L(\beta,Z)$ at low speeds, and augmented by the $(-\beta^2)$ term in (3) and corrections for the density effect at high energies. Figure 4 presents values of $\Delta R/R$ for carbon, copper, lead, and emulsion as a function of $p/Mc = \gamma\beta$. The calculated values of $\Delta R/R$ are probably not reliable for values greater than about 10%. Available data from emulsions indicate that the calculated ratios of $\Delta R/R$ are in rough agreement with observation. For example, in Ref. 5 a value of $\Delta R/R$ of $3.6 \pm 0.7\%$ was found for stopping Σ^- hyperons compared to Σ^+ at $\beta = 0.144$. The value from Fig. 4 is 2%. Similar agreement is found for stopping π^+ and π^-.[6]

It can be seen from Fig. 4 that the values of $\Delta R/R$ for a given substance are constant within 10% over the range $2 < p/Mc < 10$, and then fall slowly as the logarithmic rise in I begins to occur.

In the plateau region values for other materials can be estimated by a simple recipe based on (10) for J:

$$\frac{\Delta R}{R} \simeq \frac{3.52(2Z/A) \times 10^{-3}}{\left(\frac{dE}{dx}\right)_{min}} \tag{27}$$

where $(dE/dx)_{min}$ is the minimum energy loss in $\text{MeV-cm}^2/\text{gm}$. Equation (27) gives the following estimates:

Material	H	C	Al	Fe	Cu	Pb
$10^3\ \Delta R/R$	1.67	1.98	2.10	2.21	2.24	2.45

The agreement of these values with the curves in Fig. 4 and the similarity in shapes indicates that satisfactory estimates of $\Delta R/R$ for any substance can be made using (27) to interpolate between the curves.

Very recently, measurements have been made of the range difference for stopping positive and negative muons with initial momenta from 0.5 to 1.6 GeV/c.[35] In a Pb/C/Fe absorber (mostly Fe at the highest momentum), the values of $10^3(\Delta R/R)$ are 2.46 ± 0.30, 2.19 ± 0.30, and 2.09 ± 0.40 for the $\gamma\beta$ intervals (4.8, 7.5), (7.5, 10.2), and (10.2, 14.7), respectively. These results are in good agreement with the predictions shown in Fig. 4, although they are not precise enough to establish the shape of $\Delta R/R$ vs $\gamma\beta$ with any accuracy.

V. LIMITATIONS OF THE FERMI CALCULATION

The high-energy range of validity of the Fermi calculation presented in Sec. II appears to be limited by both kinematic and dynamic considerations. We first show that the kinematic restrictions are not real and that Eq. (10) holds for arbitrary incident velocities provided that the c.m.s. scattering is described by the Mott formula, suitably interpreted. We then address ourselves to the question of whether or not the Mott formula is an adequate dynamic description of the scattering of electrons by the incident particle. As a typical example, we consider the α^3 QED calculation of muon-electron scattering.

A. Removal of the Kinematic Restriction, $\gamma m \ll M$

The energy loss expression (5) is valid for $\gamma m \ll M$. The exact expression is[36]

$$\epsilon = \frac{2\gamma^2 v^2 m \sin^2 \theta/2}{1 + \frac{2m}{M}\gamma + \frac{m^2}{M^2}} \tag{5'}$$

where θ is now the scattering angle in the c.m.s. and all other quantities are the same as in Sec. II. Once we drop the restriction $\gamma m \ll M$, the meaning of the Mott formula (6) becomes ambiguous. Can it be interpreted as the cross section in the incident particle-electron c.m.s.? Are the factors of γ and β to be interpreted as c.m.s. quantities for the electron? Dynamics apart, it seems obvious that the touchstone should be agreement at small momentum transfers (where spins are unimportant) with the relativistic form of Rutherford's c.m.s. scattering cross section. This relativistic Rutherford formula is

$$\left(\frac{d\sigma}{d\Omega}\right)_{c.m.s.} = \frac{z^2\alpha^2\hbar^2}{4\beta^2 q^2 \sin^4(\theta/2)} \tag{28}$$

where βc is the velocity of the incident particle in the laboratory, as before, and

$$q = \frac{\gamma m\beta c}{\left(1 + \frac{2m}{M}\gamma + \frac{m^2}{M^2}\right)^{\frac{1}{2}}} \tag{29}$$

is the electron's c.m.s. momentum. Since $\beta \simeq 1$ when $\gamma m << M$ no longer holds, we see that the Mott formula agrees with (28) at small angles provided we interpret the factors of γ in (6) as γ_e of the electron in the c.m.s. With this ansatz for the interpretation of the Mott formula, it is elementary to show that $d\sigma/d\epsilon$ is still given by (7), with $\beta = 1$ and ϵ_m given by Eq. (5') with $\theta = \pi$. There is thus no change in the calculation of the z^3 energy loss from close collisions when the kinematics are treated exactly, rather than in the approximation of Sec. II, provided the Mott formula describes the c.m.s. scattering accurately.

B. Dynamic Limitations for Incident Muons

While it is amusing that the kinematics can be treated exactly, the real question is one of dynamics. As soon as the momentum of the electron in the c.m.s. becomes comparable to the mass of the incident particle, dynamic effects beyond the static Coulomb approximation begin to come into play. The incident particle not only has finite mass, it may have spin and a magnetic moment; a consistent treatment beyond the one-photon-exchange approximation must include radiative corrections. Only for an infinite mass scattering center can one make

separate expansions in the strength Ze of the external potential and in the coupling e of the electron to the electromagnetic field. As an important special case, we consider the incident particle to be a muon.

Quantum-electrodynamic calculations of muon-electron scattering, correct to order α^3, have been published by Eriksson,[37] Nikishov,[38] and Eriksson, Larsson, and Rinander.[39] The last reference is the most complete and explicit, with care taken to exhibit clearly the differences between positive and negative muons scattering from electrons. Numerical tables of cross sections and radiative correction factors are given for representative incident muon momenta.

For muons of 0.2 GeV/c incident momentum comparison of the Mott formula with the results of Ref. 39 shows that the difference in cross section is given accurately (to a few parts in 10^4 or better) by Eq. (6) for the angular range, $0 < \theta < 120°$. At larger angles, the difference given by Eq. (6) begins to underestimate the actual difference somewhat. At such large angles the cross section is so small that these departures are of negligible importance for the energy loss difference. For orientation on the importance of various angular regions to J_c, Eq. (10), we note that 50% of J_c comes from $\theta < 34°$ $(\epsilon/\epsilon_m < 0.086)$ and over 90% comes from $\theta < 90°$ $(\epsilon/\epsilon_m < 0.5)$. At 0.2 GeV/c incident momentum, the radiative correction to the difference in cross section increases with increasing angle, but is only at the relative level of 2×10^{-4} at its largest, and so is quite insignificant.

The next higher incident muon momentum for which results are tabulated in Ref. 39 is 10 GeV/c, corresponding to $\gamma\beta \simeq 95$. At

10 GeV/c with exact kinematics, the lowest order Mott cross section agrees with the lowest order QED cross section to an accuracy of 6% or better for $\theta \leq 90^\circ$, but is a factor of two smaller at 105°. The cross section difference from the Mott formula is in error by 15% at 90° and has a somewhat different angular variation from the QED result. As a consequence the Mott difference in energy loss, which involves an integral over angles, is actually only in error by approximately 4%. The radiative correction difference, not included above, is not more than 15 or 20 percent in integrated effect. This is because it is largest fractionally at backward angles where the cross section is very small.

The net conclusion from these comparisons with the α^3 QED calculations is that, provided exact kinematics and the interpretation of part A above are employed, the use of the Mott formula is perfectly adequate for muon momenta up to 2 GeV/c ($\gamma\beta \simeq 20$). At higher momenta the neglect of dynamic effects becomes more important, but even at 10 GeV/c for (muons) the Fermi expression (10) for the z^3 energy loss from close collisions is probably reliable to 30 percent.[40] At still higher muon momenta, radiative effects, including emission of hard photons, become so important that the values computed here are only of order of magnitude validity.

For other incident particles, for example pions, the dynamic effects are different. A rough rule of thumb, based on the examination of muon-electron interactions, might be that the Fermi result (10) can be trusted to 25-30% for $20 < \gamma < M/2m$. For hadrons, finite size effects manifest through electromagnetic form factors will enter eventually. These will not be important, however, until $\gamma \sim 10^3$, corresponding to $(\Delta^2)_{max} \sim 1(\mathrm{GeV/c})^2$.

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FOOTNOTES AND REFERENCES

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$$\frac{z^2 R}{M} = R_1(\beta)\left[1 - \frac{z}{2}\frac{\Delta R}{R}\right]$$

where $R_1(\beta)$ is the z^2 range of a particle of unit charge and unit mass, and $\Delta R/R$ is the quantity plotted in Fig. 4.

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FIGURE CAPTIONS

Fig. 1. The functions $L(V)$ and $F(V)$ for evaluation of the z^2 and z^3 contributions to the energy loss. For convenience in use of the figure, $L(V) - 2\ell nV - 1.60$ and $V^2F(V)$ are displayed, rather than L and F. Because of inadequacies in the statistical model of the atom, the curves are not reliable for $V \lesssim 0.8$. The solid curves are calculated for the Lindhard-Scharff parameter $\chi^2 = 2$; the dashed curves labelled 1 and 3 are for $\chi^2 = 1$ and 3.

Fig. 2. Comparison of the data of Ref. 8 with $F(V)$ as a function of the reduced velocity variable V, Eq. (19). The plotted quantities are $(Z)^{\frac{1}{2}}$ times the fractional difference between the energy loss of helium ions and four times the energy loss of hydrogen ions of the same speed. The triangles are He^4 and the solid dots are He^3. The dashed curves are the same as in Fig. 1.

Fig. 3. Fractional difference in energy loss in emulsion for singly-charged particles of opposite charge as a function of $p/Mc = \gamma\beta$. The data are for slow positive and negative pions (Heckman and Lindstrom, Ref. 6).

Fig. 4. Fractional difference in range $\Delta R/R$ for singly-charged particles of the same mass and velocity, but opposite charge, as a function of $p/Mc = \gamma\beta$. The negatively charged particle has the greater range. The curves are for carbon, copper, and lead absorbers, as indicated; the dashed curve is for photographic emulsion. The inset shows the high energy part of the graph on an expanded linear ordinate. The curves are not reliable at low velocities for $\Delta R/R \gtrsim 0.1$.

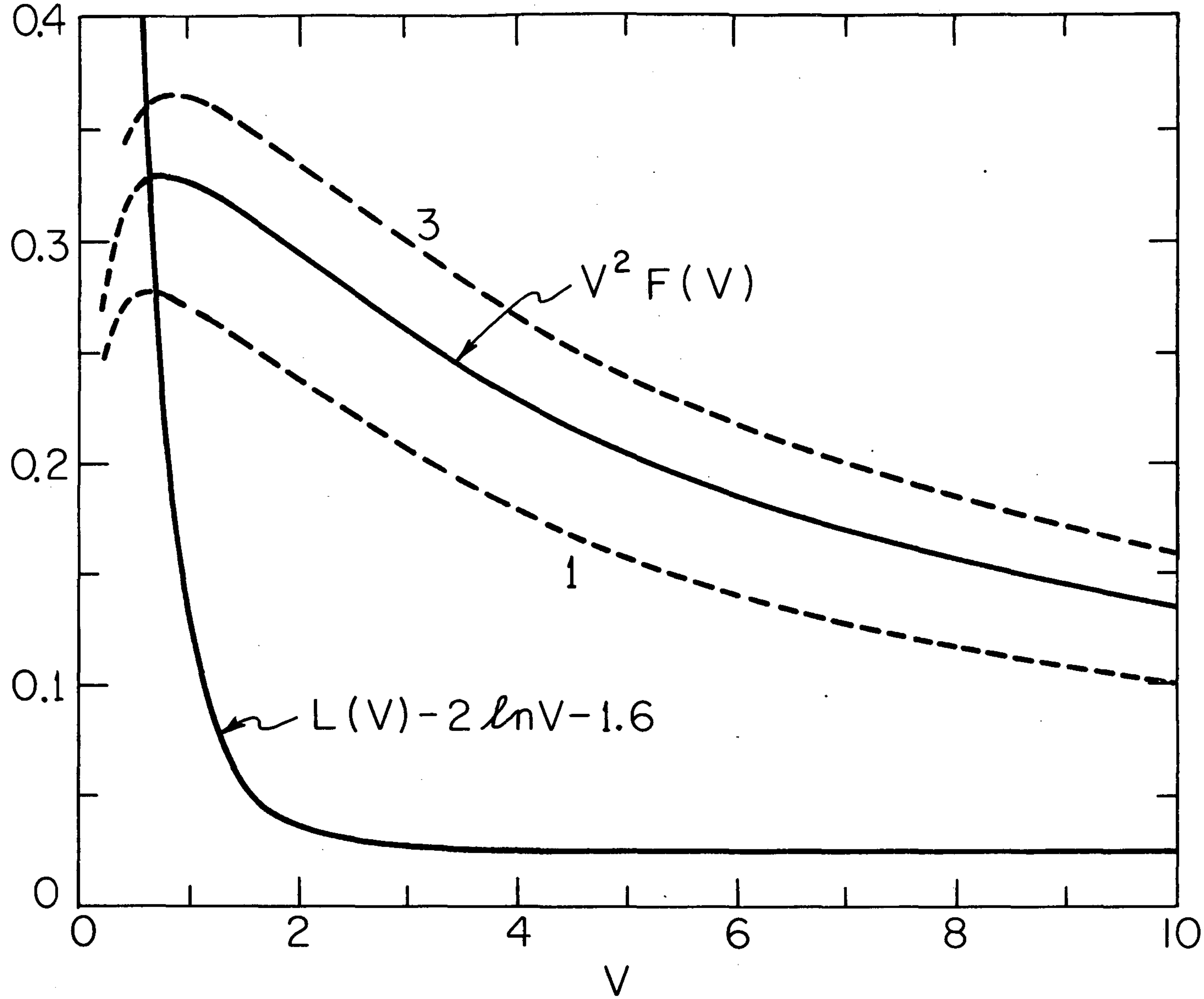

Fig. 1

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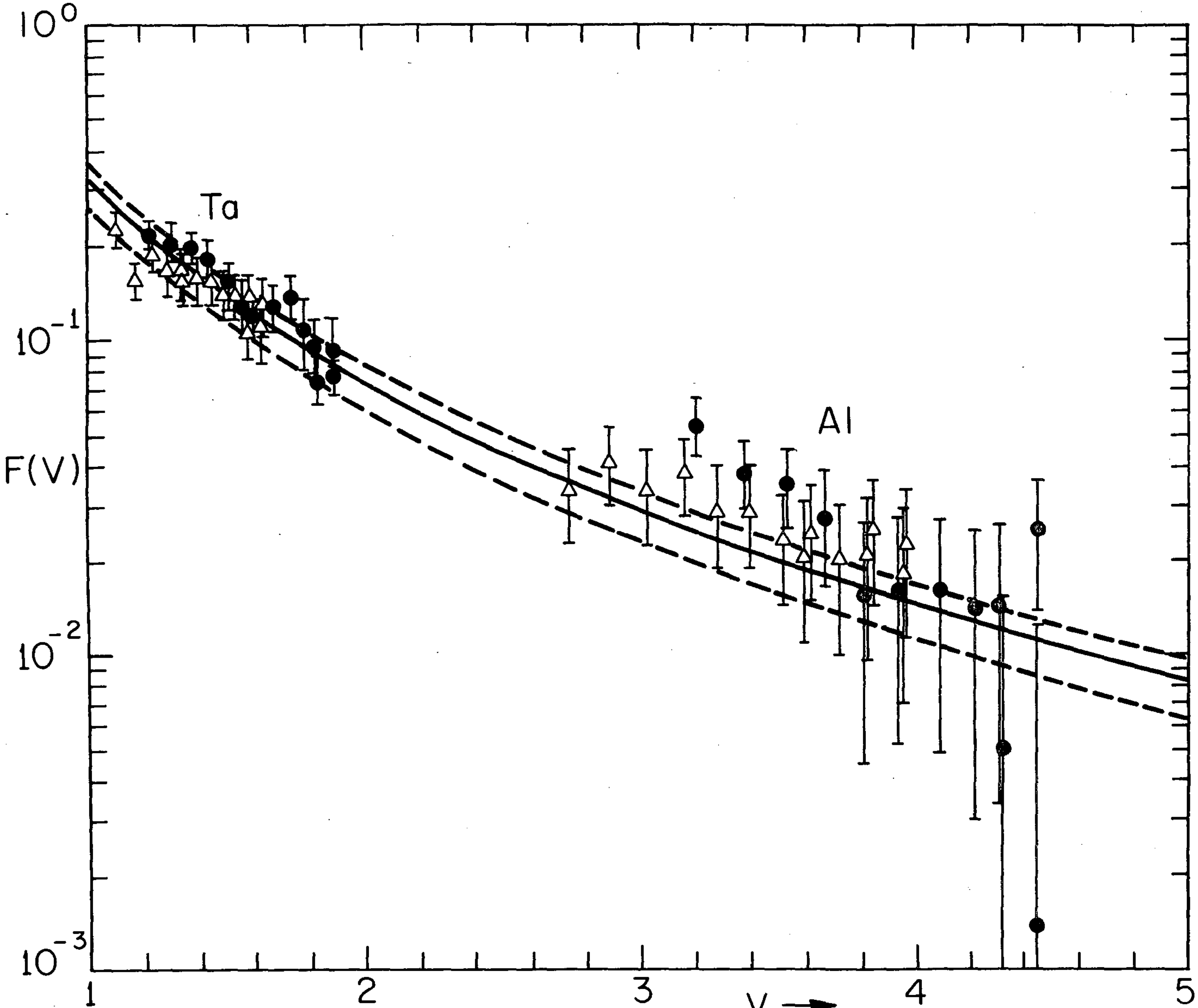

Fig. 2

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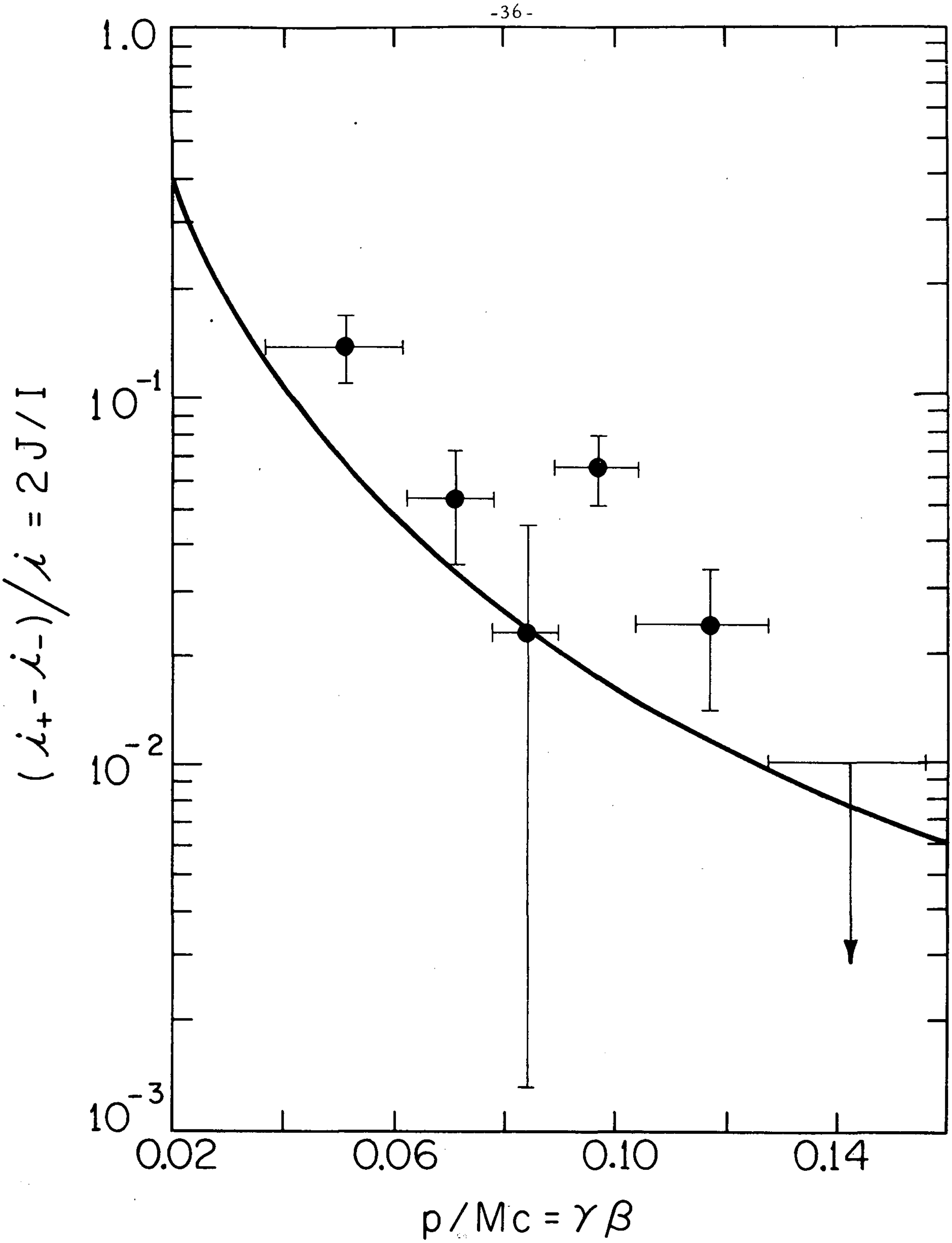

Fig. 3

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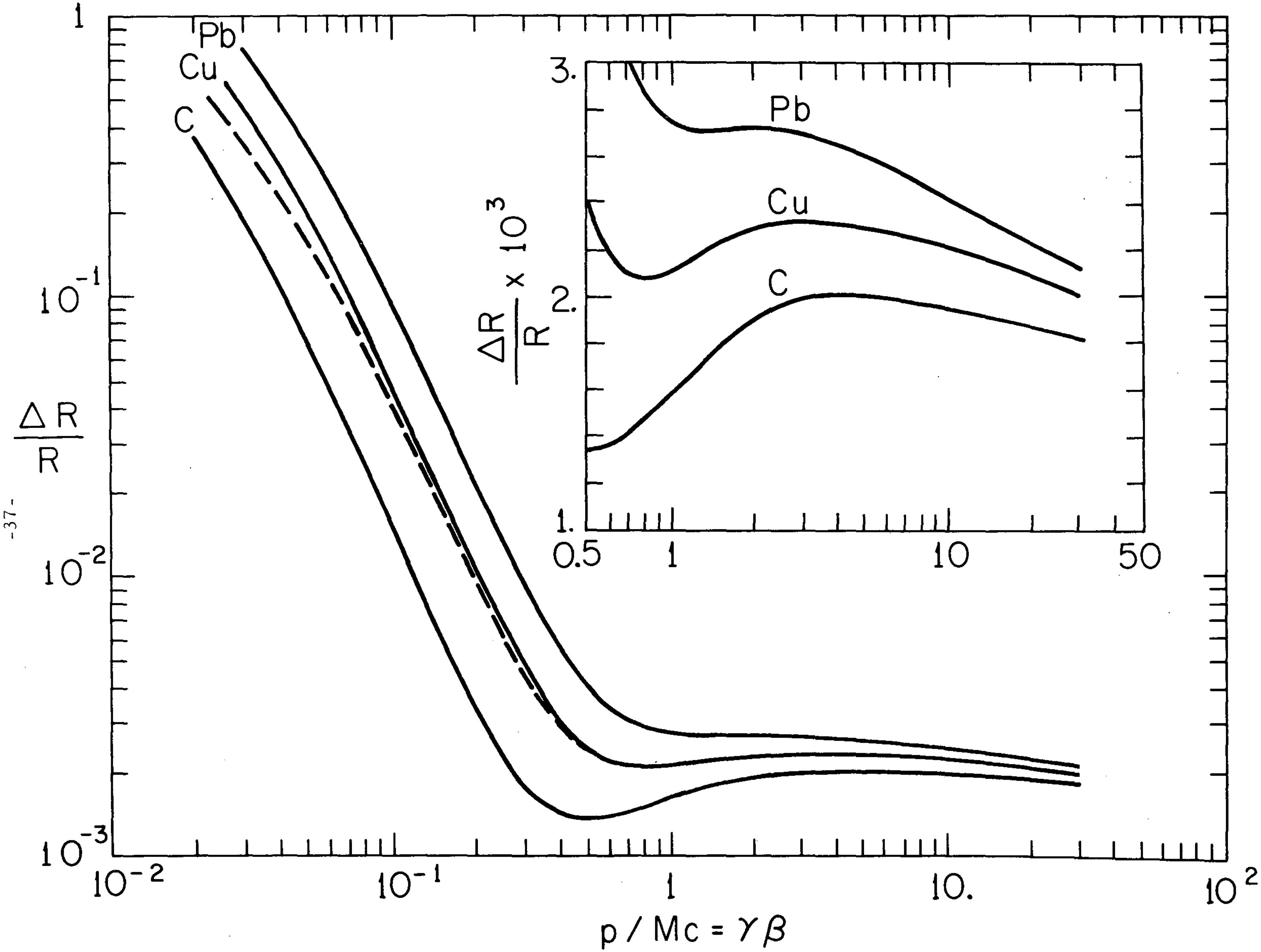

Fig. 4

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