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Seat allocation in federal second chambers

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Authors

Allen, Trevor J
Taagepera, Rein

Publication Date

2016-07-13

CSD Center for the Study of Democracy

An Organized Research Unit
University of California, Irvine
www.democ.uci.edu

In federal states, second chambers are commonplace. Although second chambers generally have diverse historical origins and serve a variety of different functions, the second chambers of federal states almost always represent territorial subunits. These chambers are typically paired with a first chamber representing people on a per capita basis (Taagepera and Recchia 2002). Thus, territorial and per capita allocation principles permeate federations, and are entrenched in both mature and fledging assemblies (Griffiths and Nerenberg 2002; Tsebelis and Money 1999). The former principle expresses the idea a territory is a territory, regardless of its population; the latter suggests that a person is a person, regardless of his or her location. While these are not universal rules,¹ surprisingly few second chambers deviate from the territorial norm. Of those federal countries that do deviate from territorial representation in second chambers, only the second chambers of Canada and Germany attempt an explicit compromise between the territorial and per capita allocation principles, allotting more seats to larger units while still falling short of proportionality. It is the formula for this compromise between territorial and per capita representation that is of primary interest in this study.

In what follows we look to the literature on quantitative logical models for a predictive formula for this unusual method of representation. Because of the dual emphasis on proportional and territorial representation, models predicting seat allocation in the European Parliament and Council are hypothesized to be especially effective in forecasting the seat distribution among subunits in federations, as they have similar representative goals (Taagepera and Hosli 2006).² We examine these models in the context of the Canadian Senate and German *Bundesrat*. We consider each case over time to examine how each states' second chamber is informed by the two allocation rules during their various expansions. Finally, as counterexamples, we also consider two unitary countries, France and Italy, where seats in the second chamber are also apportioned on a territorial basis. In these cases, seat allocation by subunit is done for administrative rather than representative purposes, so the quantitative models are hypothesized to underestimate the proportionality of these chambers.

First, however, it is useful to introduce the general logical models used in the analysis below. This allows consideration of allocation rules in a manner consistent with the vocabulary used throughout the study. We also tabulate the prevalence of the various allocation rules

(territorial representation, per capita representation, and some mixed or alternative models) to show the rareness of attempting a compromise between territorial and per capita representation. The study then focuses on Germany and Canada, which attempt a compromise between ‘a territory is a territory,’ (*T is T*) and ‘a person is a person’ (*P is P*). We compare their empirical seat allocations from bivariate regressions of seat number on population share with the allocation predicted by logical models *a priori*. We use two models that have been proposed for allocating seats in international organizations and federations, and which have been also shown to fit seat allocation in the European Parliament and the weighted votes in the EU Council (Taagepera and Hosli 2006). These models are chosen because of the importance of both territorial and per capita norms of representation in European institutions, and the apparent importance of both norms in the Canadian Senate and German *Bundesrat* (Benz and Broschek 2013; Milne 2005). We then present these ideal, empirical, and predictive intermediary options graphically. The models seem to predict the degree to which the second chambers of the compromise cases deviate from proportionality, such that the predictive model gives a slope similar to the regression slope coefficient of logged seats on logged population share. Hence, this model may express what federal countries are intuitively groping for when trying to strike a compromise between representations per capita and per subunit.

Internally consistent allocation, and actual cases

Half a century ago, Henri Theil (1969) showed that the only internally consistent allocation of seats on the basis of population is

$$S_i = SP_i^n / \sum P_k^n. \quad (1)$$

In this model, S is the total number of seats, S_i is the number of seats for the i -th subunit, with population P_i , and k in the summation ranges from 1 to T , the number of territorial subunits. Exponent n expresses how close allocation comes to proportional representation of population. Implicit in this expression is the norm that no smaller population can have more seats than a larger population, and every subunit will have at least a seat.

As n ranges from zero to one, Equation 1 is able to express the entire range of outcomes, from allocation proportional to population, to mixed allocation, to equal allocation for each subunit. That is, when $n=1$, Equation 1 is reduced to $S_i = SP_i / \sum P_k$. In this case, $\sum P_k$ further reduces to P (the total population), and we have $S_i = (S/P)P_i$, meaning that seats are allocated proportionately. Hence $n=1$ expresses proportional allocation of seats, ‘*P is P*’. Conversely, when $n=0$, Equation 1 reduces to $S_i = S/T$, given that $P^0=1$ for any population, and therefore $\sum P_k$ is equal to the number of subunits, T . All subunits have the same number of seats, S/T . Therefore $n=0$ expresses the familiar territorial allocation principle for second chambers, *T is T*. Values of n between 0 and 1 represent compromises between territorial and proportional representations. This model can be extended to circumstances wherever overrepresentation of smaller subunits is desired.

Before discussing intermediary values for n , Table 1 briefly considers how the seat allocation is actually done in federations. Our database consists of all the countries in the *Handbook of Federal Countries* (Griffiths and Nerenberg 2002). Of these, Micronesia, St. Kitts, United Arab Emirates, and Venezuela did not have a second chamber. The remaining 21 federal states are shown in Table 1.

Table 1. Types of seat allocation among territorial subunits in 20 federal second chambers.

T is T	T is T, except	Both T and P, 0<n<1	P is P, except	P is P
Brazil	Argentina ^a	Canada	Belgium ^c	Austria
Comoros	Australia ^b	Germany	Ethiopia ^c	
Russia	Bosnia ^c		India	
South Africa	Malaysia ^a		Spain	
United States	Mexico ^a			
	Nigeria ^b			

Source: *Handbook of Federal Countries* (Griffiths and Nerenberg 2002).

^a Part of second chamber elected by nationwide PR, or appointed (Malaysia).

^b Some subunits have less than standard allocation -- or more (Pakistan capital territory).

^c Allocation to ethnic groups.

From the table it is apparent that most federations allocate their second chamber seats either purely on territorial basis (six cases), corresponding to $n=0$, or with only minor modifications (eight cases). At the opposite extreme, only Austria allocates purely on population basis $n=1$, while four others do so with minor modifications. For instance, India's subunits with population shares of 0.6 to 0.9 percent would deserve two seats, but they actually receive three or even four, in a minor concession to the norm *T is T*. Two subunits in Spain, at 2.7 per cent of the population, are also rounded upwards: two seats rather than one. Belgium, Bosnia, and Ethiopia are somewhat idiosyncratic. The Ethiopian House of Federation (determined principally by population) represents ethnic groups rather than regional states, although the two often coincide (Habtu 2005). Similarly, Belgium also allocates seats by the number of speakers of Flemish, French and German (Griffiths and Nerenberg 2002). Bosnia allocates seats equally to three ethnic groups rather than territorial subunits.

This leaves the two cases referenced above as unequivocal compromises, Canada and Germany, where both per capita and territorial norms are significantly determinative of seat allocation in the second chamber. How close do they come to the logical expectation expressed by Equation 1 with a given value of exponent n ? What features might determine this value?

Combining the subunit and population norms

Here we consider two models for intermediary cases.³ Note that when the total number of seats is small the territorial norm imposes itself. At the extreme, when the number of seats equals the number of subunits ($S=T$), there is no flexibility for allocation on the basis of population. The larger the number of seats becomes, the more the population of individual subunits could be taken into account without depriving the least populous subunits of representation.⁴ The two models can be described as follows.

Rigid-n model. The first possibility is a mechanical middle ground between $n=0$ and $n=1$, respectively, *T is T* and *P is P*): $n=0.5$. This allocation rule approximates the weighted vote shares in the Council of the European Union (Taagepera and Hosli 2006) and has been proposed

repeatedly for weighted voting in international organizations (Penrose 1946; Theil 1969; Richardson 1993). This formula allocates seats to subunits in proportion to the square root of their populations. By this method the seat share S_i of the i -th territorial subunit would always depend on its population share p_i as

$$S_i = S p_i^{1/2} / \sum p_k^{1/2}. \quad (2)$$

By way of example, consider a lopsided two-component federation with populations nine million and one million. Using Equation 2, the seat allocation would be 3-1 when $S=4$. This would indeed be the only way to give the smaller subunit some representation. Similarly, the allocation would be 300-100 for $S=400$ —but here it might be argued that the smaller subunit carries excessive weight, given that even a proportional 360-40 would still provide representation. The total number of seats available makes a difference.

Flexible- n model. An updated version of the model (Taagepera and Hosli 2006; Taagepera 2007, pp. 261-265) is more flexible and proposes that the values of exponent n should take into account the total number of seats (S), the number of territorial subunits (T) and total population (P). In the above example, $S=4$ would still impose a seat allocation 3-1 (if the smaller subunit is to have any representation at all). However, when $S=400$, the larger subunit could be given more than 75% of the seats, to account for the larger disparity in population share.

The model is set up to satisfy three extreme cases. First, when the number of seats equals the number of subunits ($S=T$), one seat per subunit imposes itself regardless of their populations, and $n=0$. Second, when the number of seats equals total population ($S=P$, everyone representing herself), proportionality must prevail: $n=1$. Third, if the number of seats were reduced to one, the seat would go to the subunit with the largest population (see Taagepera and Hosli 2006, p. 371). These three conditions are satisfied when the exponent in Equation 1 takes the value

$$n = [1/\log T - 1/\log S] / [1/\log T - 1/\log P]. \quad (3)$$

Thus n is fully predetermined on the basis of T , S and P . For the hypothetical lopsided case of nine and one million above, $S=4$ leads to $n=0.52$ and seat allocation 3.04-0.96, rounding off to 3-1; while $S=400$ leads to $n=0.92$ and seat allocation 354-46, quite close to the proportional 360-40.

In the case of the EU, investigated by Taagepera and Hosli (2006), the seats in the European Parliament and the voting weights in the Council of the European Union are based on the same member states and populations. This means that T and P are the same in both bodies. The only difference is in the number of seats or voting weights. In 1995, the model yields $n=0.67$ for the Parliament and a clearly lower $n=0.46$ for the Council. The graph S_i vs. P_i (Taagepera and Hosli 2006; Taagepera 2007, p. 264) shows a good fit with the actual allocations, except for Luxembourg with its founding member status.⁵

Indeed, no *postdictive* fit of the form $S_i = s P_i^n$ where s and n can be freely adjusted could do better than this *predictive* model with *no adjustable parameters*. Moreover, this is so for any date during the history of the European Union, from 1964 to 2003, even while the values of S , T , P and P_i increased markedly (Taagepera and Hosli 2006).⁶ The rigid- n model is close to $n=0.46$ for the Council but the EP deviates appreciably from the actual seat shares (where $n=0.67$ is close to optimal). The flexible- n model is hypothesized to perform well in Canada and Germany because the second chambers of these states, like the European Union Council and Parliament, issue seats in service of both popular and territorial representation.

Extreme norms and intermediary patterns: Canada and Germany

We now consider Canada and Germany, the second chambers of which, like the foregoing European institutions, compromise between per capita and territorial representation. We use Canada's second chamber seats (Table 2) as our first example. The Canadian Senate was meant to provide regional representation and 'sober second thought' to counterbalance 'democratic excesses' of the lower house (Docherty 2002). Significantly, Canada's upper house also represents an intentional violation of the territorial allocation principle above, largely to raise the status of Quebec and its linguistic and cultural minority (McKay 1963).⁷ The Constitution Act of 1867 allocated Canadian Senate 24 seats each to Ontario, Quebec, and the Maritime Provinces—subsequently split between Nova Scotia and New Brunswick. This approximates the familiar *T is T* allocation rule. However, with the ascension of other provinces, and the Senate expanded and seat allocation was adjusted. The Prince Edward Island and British Columbia Terms of Union, and the Manitoba Act allocated seats to new provinces for which they were named in the 1870s. Saskatchewan and Alberta gained representation in 1905, followed by Newfoundland and Labrador in 1949. Nunavut, Yukon, and the Northwest Territory gained senators in the latter quarter of the 21st century.

For these later allocations, both territorial and per capita representation norms were apparently determinative of seat share. From Equation 1, the degree of compromise between popular and territorial representation is indicated by the value of the exponent n . The value of n shrinks over time with the admission of new provinces and distribution of the population across more subunits. Thus, we examine the model at multiple time points to correspond with the various expansions of the Canadian Senate and admission of new provinces. Figure 1 charts the predictions for different values of exponent n in Canada after the most recent expansion in 1999. Figure 2 provides historical perspective from six time points using a subset of those values for n . First, Figure 1 shows the number of seats (S_i) received by subunits against their proportion of total population ($p_i=P_i/P$), both on logarithmic scales. Note that $\sum p_i=1$. Then Equation 1 becomes

$$S_i = S p_i^n / \sum p_k^n \quad (4)$$

where p_k indicates the proportion of total population in subunit k . Hence $\log S_i = \log[S / \sum p_k^n] + n \log p_i$, so that $\log S_i$ is linearly related to $\log p_i$ by the slope coefficient n . This also means that the models that use Equation 1 with $n=0$, $n=1$, and any intermediary values of n appear as straight lines on the logarithmic scales in Figure 1. The solid line represents the result of the bivariate regression; its slope is $n=0.49$ ($R^2= 0.68$). Also shown is the theoretically predicted $n=0.53$, which corresponds to the flexible- n model. The actual data for Canada are shown in Table 2. Population data come from the census immediately preceding the dates given in the figures.

Founding provinces are indicated by squares in Figure 1 and marked by a dagger in Table 2. It is clear from both that Quebec and the Maritime Provinces have disproportionately more seats than the later subunits. The greatest positive differences between the number of seats predicted by lines $n=0.49$ and $n=0.53$ and the actual seat allocation occur in the founding provinces. Seats apportionment as an historical artifact appears to be important in explaining seat apportionment in Canada, as well as in Germany, and some of the other cases referenced elsewhere.⁸

Table 2. Actual and predicted seat allocation in Canada, where $n= 0.53$

Subunit	Population Share	Actual	Predicted by $n=0.53$	Difference
† Ontario	0.38	24	24	0
† Quebec	0.24	24	19	5
British Columbia	0.13	6	14	-8
Alberta	0.1	6	12	-6
Manitoba	0.04	6	7	-1
Saskatchewan	0.033	6	7	-1
† Nova Scotia	0.031	10	6	4
† New Brunswick	0.025	10	6	4
Newfoundland & Lab.	0.02	6	5	1
Prince Edward Island	0.004	4	2	2
Northwest Territory	0.0013	1	1	0
Yukon	0.001	1	1	0
Nunavut	0.0009	1	1	0

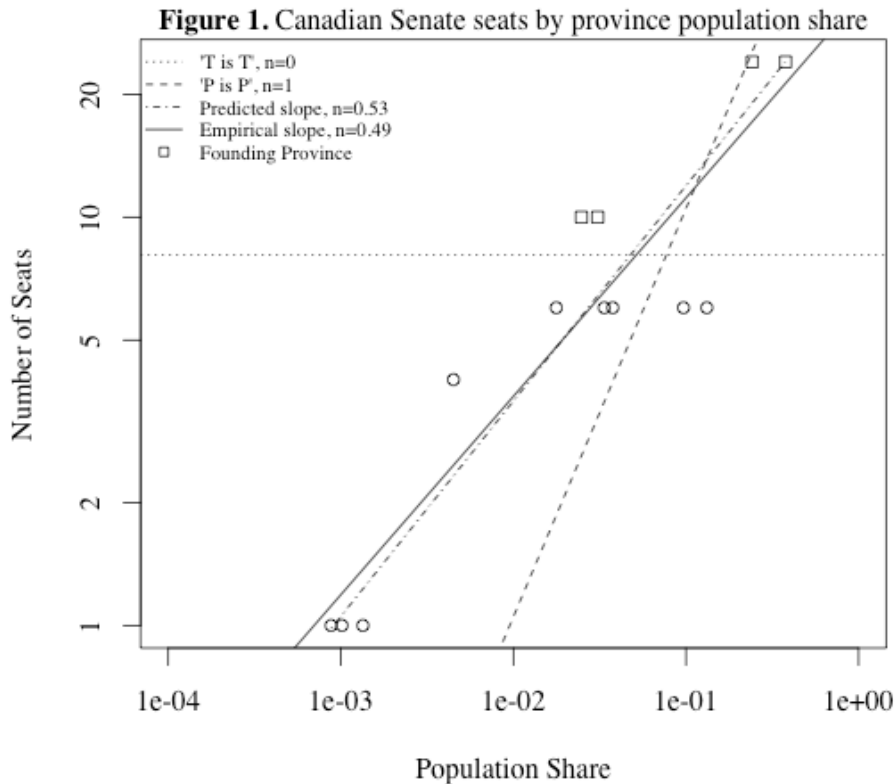
Population data from Canadian census at www.statcan.gc.ca

Table 3. Actual and predicted seat allocation in Germany, where $n= 0.41$ or 0.5 .

Subunit	Population Share, p_i	Actual Seats	Predicted by $n= 0.41$	Difference	Predicted by $n=0.50$	Difference
Nordrhein-Westfalen	0.22	6	8	-2	9	-3
Bayern	0.15	6	7	-1	7	-1
Baden-Wuerttemberg	0.13	6	6	0	7	-1
Niedersachsen	0.095	6	6	0	6	0
Hessen	0.073	5	5	0	5	0
Sachsen	0.06	4	4	0	5	-1
Rheinland-Pfalz	0.05	4	4	0	4	0
Berlin	0.04	4	4	0	4	0
Sachsen-Anhalt	0.033	4	4	0	3	1
Schleswig-Holstein	0.033	4	4	0	3	1
Brandenburg	0.031	4	4	1	3	1
Thuringen	0.031	4	4	0	3	1
Mecklenburg-Vorp.	0.022	3	3	0	3	0
Hamburg	0.021	3	3	0	3	0
Saarland	0.013	3	2	1	2	1
Bremen	0.008	3	2	1	2	1

Population and seats from *Statistisches Bundesamt Deutschland* and Griffiths and Nerenberg (2002)

The other lines in Figure 1 indicate predictions made by the pure models. If all subunits received equal numbers of seats (*T is T*), the horizontal line with slope $n=0$ in Equation 4 would correctly forecast seat distribution.⁹ The resulting (hypothetical) apportionment would fall on the dotted line in Figure 1. If, to the contrary, subunits received seats in proportion to their populations (*P is P*), the dashed line with slope $n=1$ would indicate the resulting allocation. This line would reach the total number of seats ($S=105$, in this case) for $p_i=1$, corresponding to total population. In this case, the smallest subunits would be rounded down and receive 0 seats because of their populations. However, the opposing *T is T* is invoked and even the tiniest distinct subunits receive 1 seat.¹⁰



The line at $n=0.53$ is a theoretically predicted compromise from the flexible- n model. Appropriately, a line of alternating dots and dashes represents this compromise between $n=1$ (dashes) and $n=0$ (dots). The actual data points are located in a roughly linear zone (on the log scale) with slope intermediary between those for $n=0$ and $n=1$. The empirical best-fit line has slope $n=0.49$, reflecting the actual compromise between the two norms of seat allocation. Note that the line $n=0.53$, *predicted* without the benefit of data, fits almost as well as the regression line *postdicted* on the basis of data themselves.

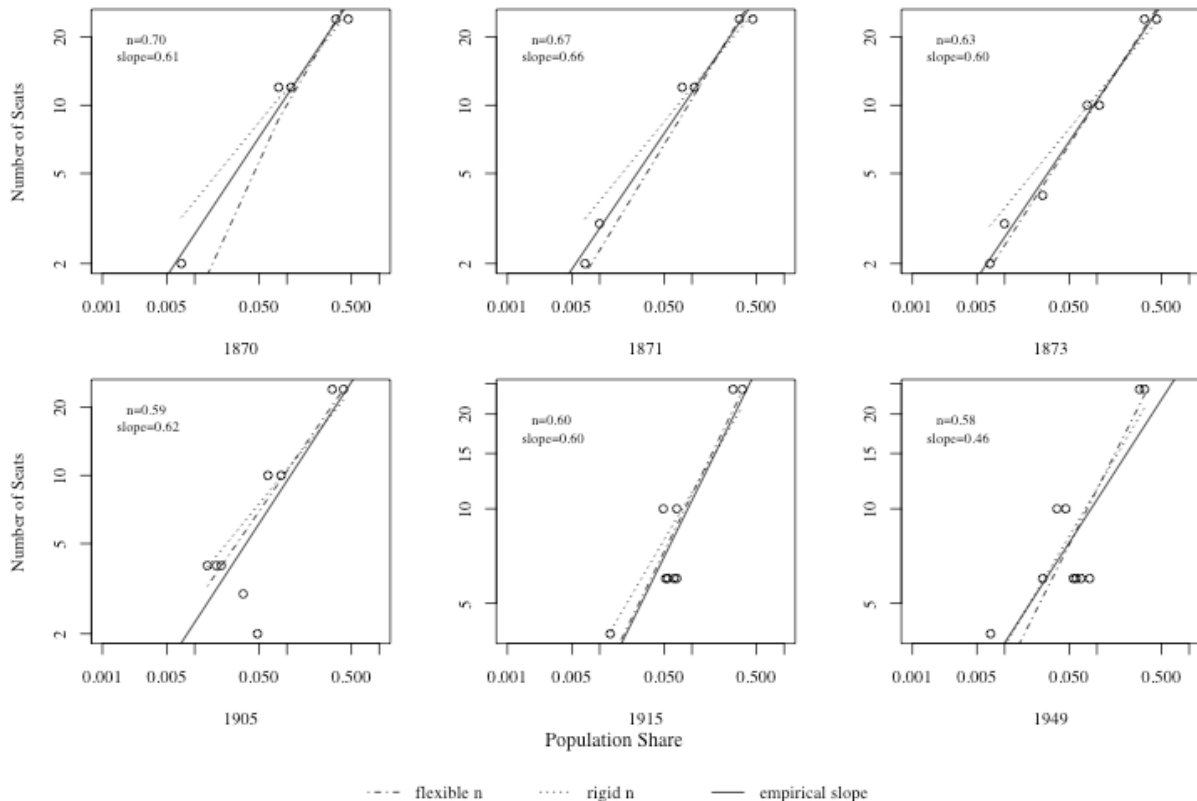
This $n=0.53$ predicted by the model is also close to the more rigid $n=0.5$ in Equation 2, so no obvious preference between the two versions of the model emerge. No simple curve could fit such scattered data cloud any better than the form $S_i = Sp_i^n / \sum p_k^n$, meaning a straight line in Figure

1. Both models, $n=0.5$ and $n=0.53$ from Equation 4, come close to the statistical best fit (where $n=0.49$, $R^2=0.68$).

Although the lines $n=0.49$ and $n=0.53$ in Figure 1 are remarkably close to each other, they are based on completely separate data. The empirical $n=0.49$ results from the actual number of subunit seats – the *output* of the allocation process – and from nothing else. The theoretical $n=0.53$ results from the total number of seats and subunits, plus populations of subunits (and their total) – the *input* of the allocation process – and from nothing else. Hence the predictive power of Equation 3 (jointly with Equation 1) looks impressive, but the same is true of the simpler Equation 2, which needs only the populations. To assess the relative predictive strengths of the rigid- n (Equation 2) and flexible- n (Equation 3) models, we examine the historical allocation of seats.

Figure 2 plots the flexible- n , rigid- n , and empirical regression lines for six historical changes to the composition of the Canadian Senate. The dotted lines represent the rigid model, where n is fixed at 0.5. As in Figure 1, the solid line is empirical data fit, and the alternating dot-dash line is the flexible- n model. In five of these six cases, the flexible- n model is remarkably close to the empirical value, and much closer than the rigid- n model.¹¹ Only in 1949 does rigid model come closer to the empirical slope, when the comparatively sparsely populated Newfoundland and Labrador receive as many seats as the more populous British Columbia. Generally, these figures support the extension of the flexible- n model to the Canadian case. We now turn to seat allocation in the German *Bundesrat*, to further compare the two candidate models.

Figure 2. Empirical seats and two predictions in the Canadian Senate, 1870-1949

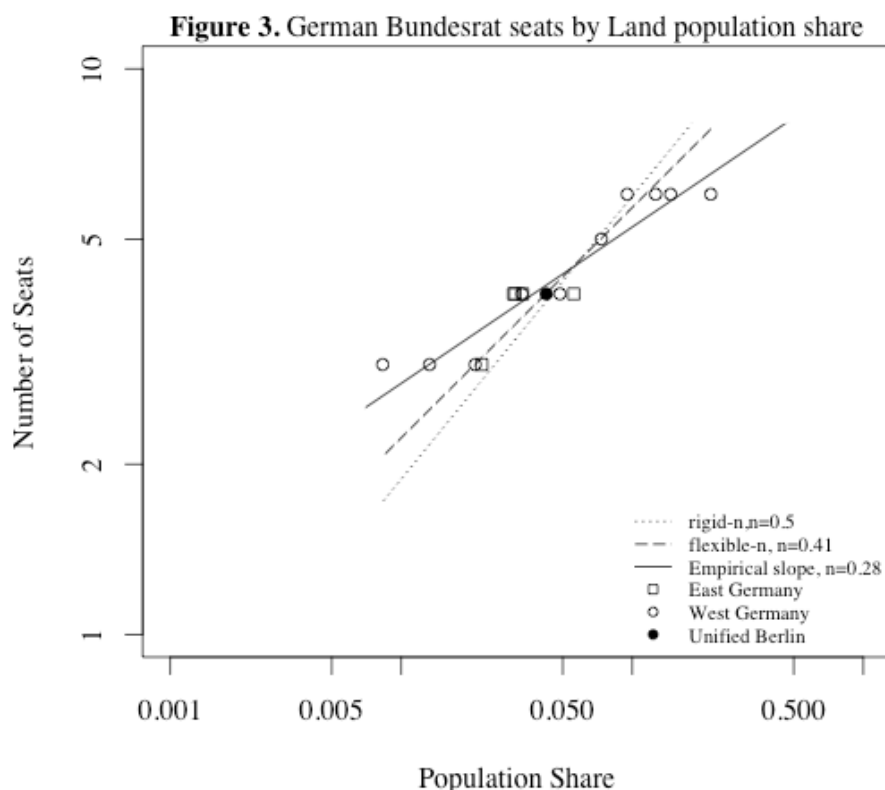


Federalism has been remarkably durable in Germany since the inception of the modern state (Benz and Broschek 2013). The federal tradition, extending from the Holy Roman Empire, served an integrative function for the German Confederation in the 19th century (Broschek 2010). Moreover, territorial representation in Germany has always been weighted by, but not proportional, to population (Benz and Broschek 2013). During unification in 1871, the seats in the first *Bundesrat* were not apportioned by the familiar territorial allocation rule, but rather also accounted for the relative size of Prussia. This federal structure was more palatable to smaller states in the federation (Benz and Broschek 2013). In the aftermath of the Second World War, the allies further encouraged German federalism (Broschek, 2010), emphasizing decentralization.

In some respects, the German case is more tractable than the Canadian one. Firstly, unlike Canada, the population of Germany has changed very since the founding of the current *Bundesrat*. Moreover, there has been only one noteworthy expansion of the *Bundesrat* that occurred after reunification. Secondly, unlike Canada, the population requirements for additional seats in the *Bundesrat* are plainly articulated in Germany's Basic Law. Thus, the compromise between proportional and territorial representation is explicit. With no less than three and no more than six seats per *Land*, Germany perceptibly favors territorial at the expense of per capita representation, although Article 51 clearly weights by population. To an extent, the small, historically autonomous German city states, especially the city of Bremen, which achieved *Land* status during the allied occupation after WWII (see Buse 2002), obscures the allocation rule. However, the flexible- n model still predicts correctly the allocation of all but three *Bundesrat* seats, out of the 69 in the chamber currently. The rigid- n model incorrectly assigns six seats.

For Germany, data and model fits are shown in Table 3 and graphed in Figure 3. With the exception of reunification, German population figures have not changed very much since 1950, so only one figure is necessary. Former East German *Länder* are indicated by squares in Figure 3. Obviously, reunification did affect the size of Berlin, which is indicated by a solid dot. This graph includes three lines with different slopes: the two models, and the statistical best fit. The statistical best linear fit (of logarithms) has an empirical slope $n=0.28$ ($R^2 = .89$) for OLS when all subunits are included. The milder slope reflects the greater emphasis on territorial representation of the German *Länder*. However, overtures toward per capita representation are still in evidence, especially when the unincorporated city-state of Bremen, the leftmost empty circle, is omitted from the sample.

With 69 seats for a population of 82 million spread among 16 subunits, Equation 4 leads to $n=0.41$. This is high compared to $n=0.28$, but it still splits the difference with the $n=0.50$ of the rigid- n model. A comparison of the fifth and seventh columns in Table 3 indicates that the flexible- n model Equation 3 incorrectly predicts second chamber seats only at extreme values, one of which (Bremen) has already been discussed. On the other hand, the rigid- n model from Equation 3 incorrectly forecasts seat allocation for the majority of the 16 German *Länder*. The flexible- n predictions comport with the actual allocation to a surprising degree. Only three seats would be allocated in a different way, taken away from tiny Bremen, Brandenburg, and Saarland and given to the more populous Nordrhein-Westfalen and Bayern. In contrast, the rigid- n model $n=0.5$ would allocate six seats differently.



Seat allocation in non-federal second chambers

Because the number of cases is small, we also briefly consider the seat allocation in *non*-federal second chambers. We abandon the rigid Equation 2 and focus instead on the exponent generated by Equation 3 given its superior results in the German case, and essentially equivalent results in the Canadian case. A number of non-federal countries also allocate all or part of their second chamber seats on the basis of territorial subunits, making them candidate cases at first blush. The second chamber was formed on this basis in Bolivia, Czech Republic, Croatia, Dominican Republic, France, Haiti, Netherlands, Palau, and Poland, and predominantly in Chile, Italy, Japan and Venezuela (until 1991) (see Taagepera and Recchia 2002).

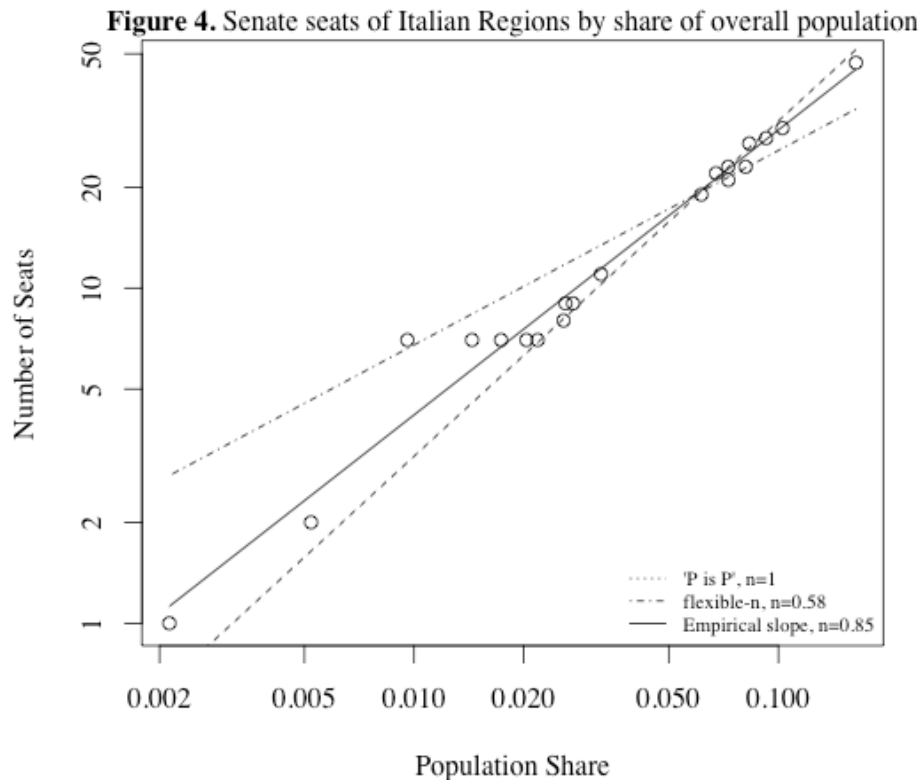
While these non-federal states' territorial subunits are administratively accounted for in a second chamber, there is a substantive difference in the logic of representation. Territorial representation lies at the very foundation of federalism. Conversely, it should be absent in unitary countries. Even if they allocate second chamber seat through administrative units, they can be expected to do so more in accordance with per capita representation.

Out the list above, only Italy and (to an extent) France can be added to the mixed-norm cases.¹² France and Italy apportion seats to their upper houses based on both territorial status and population. As of 2011, the French departments were apportioned somewhere between one and twelve senators corresponding to population, but with an imposed floor and ceiling. As such,

France is a candidate for our model, although seats allotted to French nationals living overseas seemingly cannot be so explained.¹³ The Italian Senate similarly allocates (most of its) seats over twenty districts in proportion to population, although most regions elect a minimum of seven members.^{14,15}

Figures 4 and 5 show the results of our flexible- n model, an OLS data fit, and pure population representation ($n=1$; P is P). The proportional model is included because, as we deal with non-federal states, there is little *a priori* reason to expect considerable overtures toward territorial representation. Significant empirical deviations from $n=1$ support the model's logic, and may reflect increased regional autonomy in both countries.

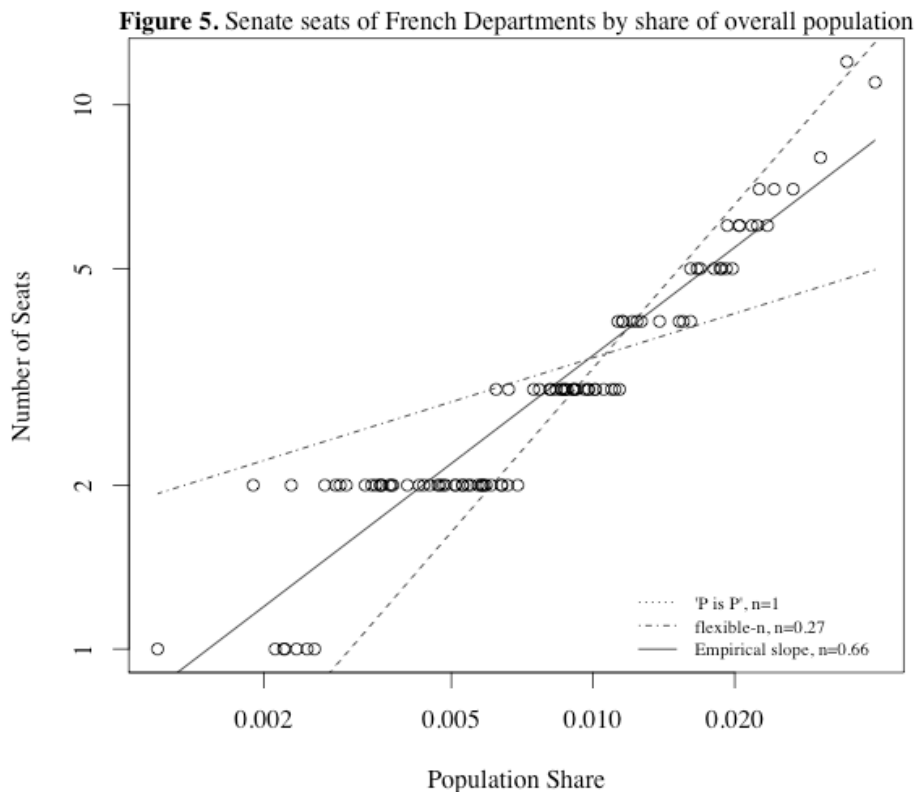
The flexible- n model is a poor fit in both cases. Both empirical slopes are closer to $n=1$ than to those resulting from the flexible- n model. The latter is a better fit in Italy, although the French n deviates from pure per capita representation to a greater extent: $n=0.66$ as compared to Italy's $n=0.85$. This is due to the comparatively small number (20) of Italy's regions as compared to France's departments (101), and similarly sized senates (320 to 348, although not all of these seats are elected by constitutive regions), which means no department contains that great a percentage of the overall population (Figure 4). The rigid- n model would fit even worse than flexible- n for Italy, but it would do better for France.



In both countries, the line $n=1$, which indicates no compromise away from per capita representation, fits the data rather well. In France, deviation occurs because small departments gain one seat to which they would otherwise not be entitled (see above). It is also the case that Paris is not the most populous department, but receives the greatest number of seats, indicating asymmetry produced by uneven population growth. Italy has a few autonomous regions, and one might expect these to have more seats than non-autonomous regions of comparable size. However, they do not appear as outliers.

The results are as hypothesized from the outset. Most federal countries follow 1 fully or with minimal exceptions (see Table 1). Even the few cases (Canada and Germany) that take population into account place at least as much emphasis on territory ($n \leq 0.5$), largely in line with the flexible- n model. The reverse is the case for unitary countries. Even the few cases, Italy and France) that take subunit identity into account do so quite marginally ($n \geq 0.66$), and the flexible- n model does not apply.

The subunits of Germany, Canada, and the European Union all exert significantly more power over the central authority than is the case in Italy, and especially France, which are centralized to a far greater degree. Indeed, the European Union has been—by some measures—treated as qualitatively commensurate with federal states, or as an ‘incipient federal state’ in its own right (Lijphart 1999, p. 34). To the extent that a compromise between territorial and popular representation indicates comparatively influential subunits, the model is expected to perform better in federations. This is indicated by the empirical values for n being greater than the predicted values in France and Italy, whereas the opposite was true in federal Germany, and they are nearly equal in Canada. A stronger commitment to federalism imposes a tension between representation per capita and representation per subunit, potentially captured by the model proposed above.



Prescriptive aspects

This study is an analysis of what countries do, and not what they ought to do. Still, it also offers a conditional prescriptive result. It is the following. If (and only if) a country or other entity wishes to stick to an internally consistent allocation of seats on the basis of population, where all subunits are represented, this has to be Theil's (1969) $S_i = SP_i^n / \sum P_k^n$, with a value of n between 0 and 1.

If (and only if) an entity also wishes to balance territorial and personal representation on an equal basis, the flexible- n model tested here is the way to go. The total number of representatives matters, and this model takes this into account, while the mechanical mean of 0 and 1 ($n=0.5$) does not.

We offer no argument why territorial and personal representation should be balanced, and most countries opt for one or the other. But such balance is crucial for entities like the European Union, and, like Molière's *bourgeois gentilhomme*, EU has largely followed the flexible- n model, without knowing that they did. Depending on the number of seats in the European Parliament and the voting weights in the Council of the European Union, the value of exponent n has been lower than 0.5 for EP and appreciably higher for CEU. The present study has added evidence from Canadian and German upper chambers.

Conclusions

Most federal second chambers follow the norm of equal representation of all subunits, either fully or with minor modifications. A few follow the opposite norm of per capita representation of people, identical to their first chambers (except for differences in the total number of seats). In so doing, they completely deny representation to federal subunits qua subunits. One might expect that compromises between the two norms would be at least as frequent as stark per capita representation. Yet this is not the case: We found only two such cases out of 21: Germany and Canada. Non-federal states attempting some compromise, Italy and France, were also considered.

If such a compromise were sought, the format $S_i = Sp_i^n / \sum p_k^n$ would be the only simple format that does not run into internal inconsistencies. Both Germany and Canada broadly follow this format. Between the two values for n offered on a logical basis, the rigid $n=0.5$ disagrees appreciably with the German allocation, while agreeing with the Canadian case. The more flexible $n = [1/\log T - 1/\log S] / [1/\log T - 1/\log P]$ stipulates variation when the number of subunits (T) and total seats (S) varies, for given total population (P). This model agrees with the Canadian allocation about as well as $n=0.5$, and it also agrees with the German allocation except at extreme sizes. As expected for unitary countries, it disagrees with seat distribution in the Italian and French Senates. This model also agrees with seat distributions in the European Parliament and the weighting votes in the EU Council. Hence this model may express the intuitive formula toward which countries are groping when trying to strike a compromise between representation par capita and per subunit.

References

- Curtis, G. L. (2000). *The logic of Japanese politics: Leaders, institutions, and the limits of change*. Columbia University Press.
- Benz, A., & Broschek, J. (Eds.). (2013). *Federal dynamics: continuity, change, and the varieties of federalism*. OUP Oxford.
- Broschek, J. (2010). Federalism and political change: Canada and Germany in historical-institutionalist perspective. *Canadian Journal of Political Science*, 43(01), 1-24.
- Buse, D. K. (2002). Federalism and identity: Bremen, 1945-1960s. *Debatte*, 10(1), 33-50.
- Docherty, D. C. (2002). The Canadian Senate: Chamber of sober reflection or loony cousin best not talked about. *Journal of Legislative Studies*, 8(3), 27-48.
- Election Resources on the Internet: Elections to the Italian Parliament - The Senate (1994-2001).(n.d.). Retrieved from <http://electionresources.org/it/senate.html> visited on [Accessed on November 2014]
- Griffiths, Ann L., and Karl Nerenberg, eds. (2002). *Handbook of federal countries*. Montreal & Kingston: McGill-Queen's University Press, for Forum of Federations.
- Habtu, A. (2005). Multiethnic federalism in Ethiopia: A study of the secession clause in the constitution. *Publius: The Journal of Federalism*, 35(2), 313-335.
- Koch, K. (1989). *West Germany Today (RLE: German Politics)*. Routledge.
- Lijphart, A. (1999). *Patterns of democracy: Government forms and performance in thirty-six countries*. Yale University Press.
- MacKay, R. A. (1963). *The unreformed senate of Canada* (Vol. 6). McGill-Queen's Press-MQUP.
- Mehlhausen, T. (2016). *European Union enlargement: Material interests, community norms and anomie*. Routledge.
- Milne, D. (2005). *Asymmetry in Canada, past and present* (pp. 479-96). Institute of Intergovernmental Relations, School of Policy Studies, Queen's University.
- Penrose, L. (1946). The elementary statistics of majority voting, *Journal of the Royal Statistical Society* 109(1): 53-7.
- References. (n.d.). Retrieved from http://www.senat.fr/fileadmin/Fichiers/Images/evenement/brochure_essentiel_senat_en/index.htm [Accessed on November 2014]
- Richardson, Lewis F. (1993). Voting in an international organization. In Oliver M. Ashford et al., eds., *Collected Papers of Lewis Fry Richardson, II*. Cambridge: Cambridge University Press, pp. 695-713.
- Statistics Canada: Canada's national statistical agency / Statistique Canada: Organisme statistique national du Canada. (n.d.). Retrieved March 1, 2016, from <http://www.statcan.gc.ca/>
- Statistisches Bundesamt Deutschland. (n.d.). Retrieved March 1, 2016, from www.destatis.de
- Taagepera, R. (2007). *Predicting party sizes: The logic of simple electoral systems*. Oxford: Oxford University Press.
- Taagepera, R., and M. Hosli (2006). National representation in international organizations: The seat allocation model implicit in the European Union Council and Parliament, *Political Studies* 54: 370-98.
- Taagepera, R., and S. P. Recchia (2002). The size of second chambers and European assemblies. *European Journal of Political Research* 41: 165-185.
- Theil, H. (1969). The desired political entropy, *American Political Science Review* 63(2): 521-5.

Endnotes

¹ For instance, Austria allocates its second chamber seats to territorial units purely based on their population, similarly to the first chamber

² Indeed, the European Union has been—by some measures—treated as qualitatively commensurate with federal states, or as an ‘incipient federal state’ in its own right (for example, Lijphart 1999, p. 34). Taagepera and Hosli suggest their model should generalize (2006, p. 370).

³ In the European Union context, this has been called “the principle of degressive proportionality”. It was first mentioned formally in the Lisbon Treaty, in reference to seat allocation in the European Parliament (Mehlhausen, 2016).

⁴ In the trivial case of $S=P$, no subunit can benefit from disproportional allocation of seats by territory.

⁵ In Taagepera (2007, p. 264) the EP line is not visible. It passes through the top of data point ITA-FRA-UK and the bottom of data point IRE (see Taagepera and Hosli 2006, p. 373).

⁶ In 2004 the EU violated the basic norm that no smaller population can have more seats than a larger population. With more population, newcomer Slovakia received fewer seats than Denmark, Finland or Ireland. Deviation from the logical allocation model was bound to increase.

⁷ <http://www.parl.gc.ca/About/Senate/LegisFocus/focus-e.htm>

⁸ Paris continues to receive more seats than the most populous department, and certain Ethiopian nationalities are represented in her upper house despite being less populous than other, unrepresented ethnic groups (Habtu, 2005).

⁹ For $S=105$, each of the 13 subunits would receive 8.08 seats. So S would have to be changed to either 104 or 117. In a slight concession to the norm ‘P is P,’ some territories, usually the least populated, sometimes have less than full status and receive reduced representation, usually one-half of the regular – for example, the Swiss half-cantons.

¹⁰ A frequent further concession is that territories that would be entitled to 1.1 to 1.5 seats on population basis are rounded up to 2 rather than rounded down to 1.

¹¹ Kinks in the line are the result of rounding. We kept the total number of seats fixed at the empirical value, whereas the number of seats predicted by the model occasionally summed to slightly larger or smaller chambers when results were rounded to the nearest integer. In Figure 2, seats are assigned to the largest remainders until all seats are filled.

¹² Because of the (s)election method of the other countries listed, they do not make plausible candidates for compromise cases

¹³ http://www.senat.fr/fileadmin/Fichiers/Images/evenement/brochure_essentiel_senat_en/index.htm

¹⁴ <http://electionresources.org/it/senate.html>

¹⁵ Japan might also be considered as elections to both chambers in the Japanese Diet partake in a degree of compromise between territory and population, although these administrative subunits seem entirely for the sake of filling office (Curtis, 2000).