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A THEORETICAL INVESTIGATION INTO THE RELATIVE ACCURACY OF MANAGEMENT AND ANALYST EARNINGS FORECASTS

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ABSTRACT

Over the past several years many researchers have empirically examined the issue of whether or not managerial earnings forecasts are more accurate than those prepared by security analysts. One result commonly found is that managerial forecasts which are released prior to analyst forecasts are at least as, if not more accurate than the posterior analyst forecasts. On the surface this seems paradoxical since analysts who release their forecasts after managerial forecast announcements are made have just as much, if not more, information with which to make a forecast than does the manager. The purpose of this paper is to provide one possible explanation for this paradox. As shown in the analysis here, analysts may be reluctant to revise their forecasts upon the receipt of new information because of the negative signal such a revision provides concerning the accuracy of their prior information. As a result, the amount of information held by analysts relative to that held by management will not be fully reflected in measures of forecast accuracy.

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INTRODUCTION

Over the past several years many researchers have empirically examined the issue of whether or not managerial earnings forecasts are more accurate than those prepared by security analysts. These studies have generally shown that managerial forecasts are at least as, if not more accurate than those of analysts. For example, Basi, Carey, and Twark (1976) find that a management forecast issued after an analyst's forecast is released is usually more accurate than the analyst's forecast. Ruland (1978) finds that managerial forecasts are as accurate as analyst forecasts issued either prior to or posterior to the managerial forecast. Jaggi (1980) and Waymire (1986) document that managerial forecasts are more accurate than prior analyst forecasts and just as accurate as posterior analyst forecasts. Waymire (1986) and Imhoff and Pare (1982) find that managerial and analyst forecasts issued at approximately the same time are of equal accuracy. Finally, Hassell and Jennings (1986) document that managerial forecasts are of greater accuracy than both prior analyst forecasts and analyst forecasts issued up to four weeks after the managerial forecast.

On the surface it seems paradoxical that managerial forecasts which are released prior to analyst forecasts are at least as, if not more accurate than the posterior analyst forecasts; analysts who release their forecasts after managerial forecast announcements are made have just as much, if not more, information than the manager (including the information contained in the manager's announcement) with which to make a forecast. Because of this, it would be expected that these posterior analyst forecasts would have greater accuracy than the managerial forecasts. The purpose of this paper is to provide one possible explanation for why this is not observed.

As shown by the analysis here, one reason for these puzzling results is that measures of forecast accuracy will not completely reflect the amount of information held by analysts relative to management; analysts in the marketplace will, in general, be holding more private information than the empirical measures of forecast accuracy indicate. This is because analysts have an incentive to only partially revise their forecasts upon receipt of new information. As a result, revised analysts' forecasts, made after the release of a managerial forecast, will not fully reflect all of the new information learned by the analyst at that time; the accuracy of the analyst's forecast will then underestimate the precision of the analyst's information.

The reluctance of an analyst to revise his forecast upon the receipt of new information can be explained by the negative signal such a revision provides concerning the accuracy of his prior information. Because of this, if the analyst's compensation is positively related to both the speed and accuracy with which he obtains information about the earnings of the firm he is following, his compensation may be lower if he revises his forecast. The analyst, as a consequence, may then prefer not to revise his forecast when receiving new information.[1]

An implication of this analysis is that the additional information provided to the market by managerial forecasts may not be as great as indicated from measurement of relative forecast accuracies. Analysts may have much of the information possessed by management but prefer not to reflect it in publicly released forecasts. If the analysts' information does get disseminated, possibly through more private communications, the additional information provided to the market by managerial forecasts may then not be large. But, if the analysts' information does not become widely distributed, then managerial forecasts may still play a significant role in supplying information to the market even though much of that information is already privately possessed by analysts.

The results of this analysis are related to those of Kanodia, Bushman, and Dickhaut (1985) and Trueman (1986) who show that a manager may take actions which increase investors' perceptions of the speed with which he obtains new information affecting his firm. Kanodia, Bushman, and Dickhaut demonstrate that a manager may be unwilling to give up a project which he subsequently finds to be unprofitable since dropping the project would reveal that the manager's information was not accurate at the time of the investment decision. Trueman shows that a manager's motivation to release earnings forecasts may result from his desire to convey to investors that he is

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[1]In principle, this argument could also be made with respect to managerial forecasts. That is, managers may be reluctant to revise their previously released forecasts. However, if present at all, this phenomenon should be of significantly smaller magnitude than for analysts. A major reason for this is that managers rarely issue more than one earnings forecast in any one year. In addition, severe penalties may be imposed on managers by shareholders if it is discovered that the managers are releasing less than truthful forecasts.

up-to-date on any changes in the firm's economic environment which may impact the firm's earnings.

The plan of this work is as follows. In section I the economic setting is described. This is followed in section II by an analysis of the analyst's decision over the nature of the earnings forecast to release at various points in time and, in particular, the extent to which he will revise his forecast upon the receipt of new information. Implications of this for the relative accuracy of managerial and analyst forecasts is discussed in section III. The paper ends with a summary and concluding remarks in section IV.

I. ECONOMIC SETTING

Consider a two period model in which a group of investors pay a fixed amount, at the beginning of each period, to a risk neutral security analyst for him to forecast the current period's earnings of a given firm.[2] The firm's earnings in any period, v, may take on one of two possible values, v_g or v_b, where $v_g > v_b$. There are three relevant dates in each period. At the beginning of each period, date 1, all investors have the same prior beliefs about the probability that each of the two possible earnings levels will be realized. For simplicity, this probability is assumed to be equal to 0.5 for each level of earnings. However, at date 1 the analyst may receive private

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[2]That is, investors' payment to the analyst in any period is assumed to be independent of his subsequent performance that period. This appears to be a common practice. Although payment contingent on the analyst's subsequent performance may lessen some of the problems to be discussed in this paper, there are reasons why a fixed payment at the beginning of the period may be the preferred alternative for analysts. Primary among these reasons may be the minimization of the possibility of default on the part of the analysts' customers; the customers only receive the analyst's services after paying the required fee. Payment contingent on performance may also involve significant litigation costs when customers disagree with the analyst on the nature of his performance during the period.

information about which earnings level will occur that period and at that time will issue a forecast for the current period's expected earnings.[3] Also at date 1, the manager of the firm receives private information about the current period's earnings and will issue his own earnings forecast. To simplify the mathematics it is assumed that neither the manager nor the analyst have knowledge of the other's private information at date 1.[4] Immediately after the manager's announcement, at date 2, the analyst may revise his forecast based on the information contained in the manager's forecast. Finally, at date 3 the analyst may receive additional private information and may revise his forecast further at that time. It is assumed here, however, that if the analyst observes a private signal of the firm's earnings at date 1 he will not obtain any additional information at date 3; only an analyst who did not observe private information earlier may obtain it at that time.[5] At the end of each period the firm's actual earnings are observed by all market participants.

Analysts differ in their ability to collect private information about the firm's earnings. Some analysts, those with greater ability, will receive information early each period, at date 1. Others, with lesser ability, will receive information later each period, at date 3. Still others will not receive any private information during the period. At the beginning of the first period the ex-ante probability, held by all agents (including the analyst), that the analyst will receive a signal at date 1 about the firm's

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[3]Equivalently, the analyst could just release his private information.

[4]The results of this analysis, however, will continue to go through if this assumption is dropped.

[5]This assumption is similar to that made by Kanodia, Bushman, and Dickhaut (1985).

earnings is given by p_1 while the ex-ante probability that the analyst will receive a signal at date 3, given that he did not receive one at date 1, is given by p_3. If a signal is received, it either specifies that earnings will equal v_g or will equal v_b. The precision of the signal is given by s where:

$$\text{prob(realized earnings} = v_i/\text{signal} = v_i) = s \qquad i = g,b \tag{1}$$

The manager's date 2 signal is of a similar nature to that of the analyst. The accuracy of the manager's signal is given by m.

Market investors are assumed competitive in that they pay the analyst each period in an amount directly related to the expected marginal value to them of the analyst's forecasts during the period.[6] Value, in turn, is measured by the extent to which the forecasts allow investors to make more informed (and, presumably, more profitable) investment decisions. Value increases the more precise the analyst's information and the more rapidly the information is collected. The investors who pay the analyst to produce his forecasts are assumed to know the precision of his signal, s, and the prior probabilities p_1 and p_3. Since investors cannot differentiate among analysts as of the beginning of the first period, all analysts will receive the same compensation in that period, based on s, p_1, and p_3.

Investors' payment to the analyst at the beginning of the second period, however, will depend on what they learn about the analyst in the first period. Specifically, since precision, s, is known, compensation will be a function of investors' perception of the speed at which the analyst is able to collect

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[6]The results of this paper would continue to go through if it is alternatively assumed that the analyst sets a price and investors decide whether or not to use his services given that price.

private information. This is captured by investors' posterior probability assessments for the analyst being of the type that receives information at date 1 and being of the type that receives it at date 3, denoted by p_1' and p_3', respectively. Then, the analyst's compensation for the second period, denoted by C, is given by:

$$C = C(p_1', p_3') \tag{2}$$

For mathematical tractibility C will further be assumed to take the following linear form:

$$C = w_1 p_1' + w_3 p_3' \tag{3}$$

with $w_1 > w_3$. This latter condition is consistent with investors placing more value on information which is received earlier in the period. w_1 (w_3) can then be interpreted as the value investors give to the forecasts of an analyst who receives information at date 1 (date 3).

The analyst's compensation could, more generally, also depend on investors' assessment of the probability that, given the receipt of private information, the analyst will release truthful forecasts. But, as will be clear from the analysis below, the analyst will have no motivation not to release truthful forecasts in period 2, it being the last period of the model. Therefore, having the compensation plan also depend on the assessed probability of the analyst issuing truthful forecasts will have no effect on his behavior (since, in either case, only truthful forecasts will be released).[7]

II. THE ANALYST'S AND MANAGER'S FORECASTS

In this section the analyst's decision over what type of forecast to release at each date of the first period will be explored. As an introduction to the analysis consider first a simplified setting in which investors know the time of information receipt by the analyst so that the only information asymmetry concerns the nature of the signal received. Then, the analyst would have no reason not to issue truthful forecasts of expected earnings at each date of the first period (his second period compensation being only a function of p_1' and p_3', which are known to investors). At date 1 he would release a forecast of:

$$E^a(v|g,1) = sv_g + (1 - s)v_b \tag{4}$$

if he received a signal of v_g and a forecast of:

$$E^a(v|b,1) = (1 - s)v_g + sv_b \tag{5}$$

if he received a signal of v_b. If he did not receive any information at date 1 his forecast would be:

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[7]If additional periods were added, however, such a generalized compensation plan would be used by investors. Then, the greater the assessed probability that the analyst would issue truthful forecasts, the greater would be his compensation, everything else the same. This would give the analyst an additional motivation to appropriately revise his forecasts upon the receipt of new information. However, even in such a multi-period setting some incentive would still exist for the analyst not to revise his forecast, in an attempt to raise investors' estimate of the probability that the analyst received information earlier in the period.

$$E^a(v|n,1) = v_g/2 + v_b/2 \tag{6}$$

At date 2 the analyst would revise his forecast based on the information contained in the manager's forecast released at date 1. The manager's forecast would be either:

$$E^m(v|g,1) = mv_g + (1 - m)v_b \tag{7}$$

if he received a signal of v_g or:

$$E^m(v|b,1) = (1 - m)v_g + mv_b \tag{8}$$

if he received a signal of v_b.

If the analyst observed a signal at date 1, his date 2 forecast would appropriately incorporate his signal with that of the manager. If the analyst did not observe a signal at date 1, his date 2 forecast would be the same as the managerial forecast.

At date 3 the analyst would not revise his forecast further if he did not receive additional information at that time. If he did receive a signal at date 3, he would appropriately combine it with that of the manager in determining his revised forecast.

The release of truthful forecasts will not always be the appropriate strategy for the analyst to follow, however, in the case where investors do not know whether or when the analyst received private information. To see this suppose that investors did believe that, in equilibrium, the analyst would only release truthful forecasts. An analyst who did not receive information at date 1 would then have an incentive to mimic the actions of an analyst who did receive information at that date by choosing the following strategy. At date 1 he would issue either the forecast $E^a(v|g,1)$ or $E^a(v|b,1)$, rather than the unbiased forecast $E^a(v|n,1)$. At date 2, after the managerial forecast was released, the analyst would revise his forecast, given the information in the manager's forecast, in a manner consistent with the analyst actually having received the signal implicit in his date 1 forecast. Finally, at date 3 he would not further revise his forecast. Following this strategy the analyst would cause investors to believe that he actually did observe information at date 1. As a result his period 2 compensation would be higher than if he revealed, by releasing the forecast $E^a(v|n,1)$ at date 1, that he had not observed any information at that time.

Therefore, an equilibrium cannot exist in this setting in which the analyst, regardless of his ability to collect information, always truthfully reveals his forecast. How the analyst actually chooses the forecast to be released at each of dates 1, 2, and 3, and the extent of truthful forecast release, depend on the parameters w_1 and w_3 of the analyst's compensation function and on investors' conjectures about the behavior of the analyst in equilibrium. The remainder of this section is devoted to showing that an equilibrium will exist in which the following set of investor conjectures will be fulfilled:

Investors' Equilibrium Conjectures: Investors make the following conjectures about the period 1 behavior of the analyst in equilibrium:

1. An analyst observing signal v_g (v_b) at date 1 will issue the forecast

$E^a(v|g,1)$ $[E^a(v|b,1)]$ at that time. He will appropriately revise his forecast at date 2 given this information and that contained in the manager's forecast and will not revise his forecast at date 3, not receiving any new information at that time.

2. An analyst not observing a signal at date 1 will issue the forecast $E^a(v|g,1)$ with probability 0.5 and the forecast $E^a(v|b,1)$ also with probability 0.5. At date 2 he will revise his forecast, given the information in the manager's forecast, as if the analyst's date 1 forecast were unbiased. At date 3 he will not revise his forecast further if he receives a signal at that time that would result in the same forecast as he released at date 2. (This means that if he releases the forecast $E^a(v|g,1)$ $[E^a(v|b,1)]$ at date 1 and receives the signal v_g (v_b) at date 3, he will not revise his forecast at that time.) However, if he receives a signal at date 3 that, when appropriately combined with the manager's information, would result in a forecast different from the one released at date 2, he will revise his forecast with probability x, where $0 \le x \le 1$. If he does not receive any information at date 3 he will still revise his forecast, with probability y, where $0 \le y \le 1$.

An important implication of an equilibrium in which these conjectures are fulfilled is that the analyst may not fully revise his forecast upon receipt of new information. For example, an analyst who has not received information at date 1 will not completely adjust his forecast at date 2 for the information he learns from the manager's forecast announcement. If he did fully adjust, his date 2 forecast would be identical with that of the manager and would reveal to investors that the analyst did not receive any private information at date 1. This would lower his second period compensation. Similarly, an analyst receiving information at date 3 which would call for him to revise his forecast at that time may prefer not to do so. The reason for this is that if he did revise his forecast, investors would know with certainty that he did not receive any information earlier, at date 1. This might reduce his second period compensation.

To show that an equilibrium exists given these conjectures it must be demonstrated that the analyst's actions will be consistent with the conjectures. In order to do so, and in particular to analyze the analyst's decision as to whether or not to revise his forecast at date 3, investors' posteriors on p_1 and p_3, as of the end of the first period, must first be calculated given these conjectures. For this analysis assume that the analyst has issued a forecast of $E^a(v|g,1)$ at date 1. (Identical calculations can be performed for the case where the date 1 forecast is $E^a(v|b,1)$.) Consider first the case where the analyst does not revise his forecast at date 3. Then, investors' posterior probability that the analyst actually observed private information at date 1 (in this case signal v_g) given that earnings of v_i are actually realized, denoted by $p_1'(nr,i)$, i = g, b (where the functional dependence on x and y is suppressed) is given by the following expression, resulting directly from Bayes' rule:

$$p_1'(nr,i) = \frac{sp_1/2}{sp_1/2 + 0 + sp/4 + (1 - s)(1 - x)p/4 + (1 - y)(1 - p - p_1)/4} \qquad (9)$$

for i = g, where $p = p_3(1 - p_1)$, and:

$$= \frac{(1 - s)p_1/2}{(1 - s)p_1/2 + 0 + (1 - s)p/4 + s(1 - x)p/4 + (1 - y)(1 - p - p_1)/4} \quad (10)$$

for i = b.

The numerator in expressions (9) and (10) is equal to:

prob($E^a(v|g,1)$ released, no revision at date 3, v_i realized|v_g observed at date 1) * prob(v_g observed at date 1)

while the denominator is equal to the sum of:

prob($E^a(v|g,1)$ released, no revision at date 3, v_i realized|v_g observed at date 1) * prob(v_g observed at date 1) +

prob($E^a(v|g,1)$ released, no revision at date 3, v_i realized|v_b observed at date 1) * prob(v_b observed at date 1) +

prob($E^a(v|g,1)$ released, no revision at date 3, v_i realized|v_g observed at date 3) * prob(v_g observed at date 3) +

prob($E^a(v|g,1)$ released, no revision at date 3, v_i realized|v_b observed at date 3) * prob(v_b observed at date 3) +

prob($E^a(v|g,1)$ released, no revision at date 3, v_i realized|nothing observed at dates 1 and 3) * prob(nothing observed at dates 1 and 3)

Similarly, the posterior probability that the analyst observed information at date 3 (either signal v_g or v_b) given that he does not revise his forecast at that time and given that earnings of v_i are actually realized, denoted by $p_3'(nr,i)$, i = g, b (where the functional dependence on x and y is again suppressed) can be calculated in a similar manner and is given by the following expression:

$$p_3'(nr,i) = \frac{sp/4 + (1 - s)(1 - x)p/4}{sp_1/2 + sp/4 + (1 - s)(1 - x)p/4 + (1 - y)(1 - p - p_1)/4} \quad (11)$$

if i = g and:

$$= \frac{(1 - s)p/4 + s(1 - x)p/4}{(1 - s)p_1/2 + (1 - s)p/4 + s(1 - x)p/4 + (1 - y)(1 - p - p_1)/4} \quad (12)$$

if i = b.

Given these expressions, the following can be easily shown and will be used in the later analysis:

Lemma 1: Investors' posteriors on p_1 and p_3 have the following properties:

a. $p_1'(nr,g) > p_1'(nr,b)$ (13)

b. $w_1p_1'(nr,g) + w_3p_3'(nr,g) > w_1p_1'(nr,b) + w_3p_3'(nr,b)$ (14)

given that $w_1 > w_3$.

That is, investors' posterior for the probability that the analyst received information at date 1 given that he did not revise his forecast at date 3 is higher if the firm's actual earnings confirms the analyst's signal (v_g). Further, a weighted average of the date 1 and date 3 posterior probabilities (where more weight is placed on the date 1 probability) is also higher if the actual earnings confirms the analyst's signal.

A similar set of posterior probabilities can be calculated for the case where the analyst does revise his forecast at date 3. Given investors' conjectures, if the analyst revises at date 3, then $p_1' = 0$; any analyst revising his forecast at date 3 is assumed not to have received information at date 1. Further, investors conjecture that an analyst only revises his forecast at date 3 if he receives information at that time inconsistent with the forecast he has already issued (that is, if he receives signal v_b.) Therefore, the probability that the analyst received a private signal at date 3 given that he revised his forecast at that time and given that earnings of v_i are realized is equal to the probability that he received signal v_b at date 3 given these events. This probability, denoted by $p_3'(r,i)$, is easily shown to be given by:

$$p_3'(r,i) = \frac{(1 - s)xp/4}{(1 - s)xp/4 + y(1 - p - p_1)/4} \tag{15}$$

if i = g and:

$$= \frac{sxp/4}{sxp/4 + y(1 - p - p_1)/4} \tag{16}$$

if i = b.

It can easily be shown from these expressions that:

$$p_3'(r,g) < p_3'(r,b) \tag{17}$$

Condition (17) states that it is more likely that the analyst received a private signal at date 3 given that earnings of v_b are realized. This is reasonable given that the analyst will, upon receipt of a signal at date 3, only revise his forecast if signal v_b was observed.

Given these expressions, the expected second period compensation of the analyst, as of date 3 of the first period, can now be calculated, both for the case where the analyst revises his forecast at that time and for the case where he does not revise his forecast. The analyst will then take that action at date 3 which maximizes his expected compensation. Consider first an analyst who observes signal v_i at date t and does not revise his forecast at date 3. His expected compensation, denoted by $E[C(nr,x,y|i,t)]$ (where the dependence on x and y is now explicitly expressed), is given by:

$$E[C(nr,x,y|i,t)] = w_1 E[p_1'(nr,.|i,t)] + w_3 E[p_3'(nr,.|i,t)] \tag{18}$$

where $E[p_1'(nr,.|i,t)]$ ($E[p_3'(nr,.|i,t)]$) is the expected value of p_1' (p_3') from his perspective given that he does not revise his forecast at date 3. The expected compensation for such an analyst if he does revise his forecast at date 3, $E[(C(r,x,y|i,t)]$, is:

$$E[C(r,x,y|i,t)] = w_3 E[p_3'(r,.|i,t)] \tag{19}$$

where $E[p_3'(r,.|i,t)]$ is the expected value of p_3' from the analyst's perspective given that he does revise his forecast.

Similarly, the expected compensation of an analyst who does not observe any signal and does not revise his forecast at date 3, $E[C(nr,x,y|n)]$, is given by:

$$E[C(nr,x,y|n)] = w_1 E[p_1'(nr,.|n)] + w_3 E[p_3'(nr,.|n)] \tag{20}$$

where $E[(p_1'(nr,.|n)]$ $(E[p_3'(nr,.|n)])$ is the expected value of p_1' (p_3') from the analyst's perspective given that he does not revise his forecast at date 3. If he does revise his forecast, his expected compensation, $E[C(r,x,y|n)]$, is given by:

$$E[C(r,x,y|n)] = w_3 E[p_3'(r,.|n)] \tag{21}$$

where $E[p_3'(r,.|n)]$ is the expected value of p_3' from the analyst's perspective given that he does revise his forecast at date 3.

Straightforward calculations reveal the following:

<u>Lemma 2</u>: $E[C(nr,x,y|i,t)]$ and $E[C(nr,x,y|n)]$ are increasing in x and y while $E[C(r,x,y|i,t)]$ and $E[C(r,x,y|n)]$ are decreasing in x and y given that $w_1 > w_3$.

This Lemma states that the expected compensation of an analyst who does not (does) revise his forecast at date 3 increases (decreases) as both x and y increase. The reason for this is that as these probabilities increase the chance increases that an analyst not receiving information at date 1 will revise his forecast at date 3. Therefore, if investors see that an analyst did revise his forecast at date 3 it is relatively more likely that he observed a signal at date 3 rather than at date 1. This causes the expected compensation from revising at date 3 to decrease. Conversely, if investors see that the analyst did not revise his forecast at date 3 it is relatively more likely that he observed a signal at date 1. This causes the expected compensation from not revising at date 3 to increase.

In addition the following can be shown:

Lemma 3: $E[C(nr,x,y|b,3)] < E[C(nr,x,y|n)] < E[C(nr,x,y|g,3)] = E[C(nr,x,y|g,1)]$ and $E[C(r,x,y|b,3)] > E[C(r,x,y|n)] > E[C(r,x,y|g,3)] = E[C(r,x,y|g,1)]$.

Proof: The Lemma follows immediately from conditions (14) and (17) by noting that the probability of earnings of v_g being realized is lower from the viewpoint of an analyst receiving signal v_b at date 3 than from the viewpoint of an analyst not receiving any information. In turn this probability is less than that from the viewpoint of an analyst receiving signal v_g at either date.

Q.E.D.

This Lemma states that an analyst either observing private information at date 1 (signal v_g) or observing signal v_g at date 3, confirming his date 1 forecast, receives the greatest (lowest) expected compensation as a result of not revising (revising) his forecast at date 3. Such an analyst has the greatest incentive not to revise at that time. An analyst observing private information at date 3 inconsistent with his date 1 forecast (signal v_b) receives the greatest (lowest) expected compensation as a result of revising (not revising) his forecast at date 3. He has the greatest incentive to revise at that time.

Given these relations the main Proposition of this section can be proven:

Proposition: An equilibrium will exist in which investors' conjectures are confirmed by the actions of the analyst.

<u>Proof</u>: Given investors' conjectures three possible cases can arise. For each case it will first be shown that there are values of x and y which, if conjectured by investors, will be confirmed in equilibrium. Then it will be demonstrated that the remaining investor conjectures will be confirmed in equilibrium.

Case 1. $E[C(nr,0,0|b,3)] \geq E[C(r,0,0|b,3)]$ and $E[C(nr,0,0|n)] \geq E[C(r,0,0|n)]$. In this case if investors conjecture that $x = 0$ and $y = 0$, these beliefs will be confirmed in equilibrium. The reason for this is that, given these inequalities, the analyst either receiving signal v_b at date 3 or not receiving any information at all will maximize his expected compensation by not revising at date 3 (that is, by setting $x = 0$ and $y = 0$).

Case 2. $E[C(nr,0,0|b,3)] < E[C(r,0,0|b,3)]$ and $E[C(nr,0,0|n)] \geq E[C(r,0,0|n)]$. Then $x = 0$ and $y = 0$ cannot be an equilibrium. (Given these conjectures an analyst receiving signal v_b at date 3 would prefer to revise at that time.) Therefore, fix y at zero and increase x. From Lemma 2 $E[C(nr,x,0|n)]$ will remain greater than $E[C(r,x,0|n)]$. Either $E[C(nr,x,0|b,3)]$ will remain less than $E[C(r,x,0|b,3)]$ for all x, in which case $x = 1$, $y = 0$ will be confirmed beliefs, or there will be a value of x, x^*, for which there is equality. If so, then $x = x^*$ and $y = 0$ will be confirmed beliefs. (x^* will be confirmed because at that value of x the analyst receiving a signal of v_b at date 3 will be indifferent between revising and not revising his forecast at date 3. Then, revising with probability x^* will be consistent with his objective function and therefore will confirm investors' conjectures).

Case 3. $E[C(nr,0,0|b,3)] < E[C(r,0,0|b,3)]$ and $E[C(nr,0,0|n)] < E[C(r,0,0|n)]$. In this case, as in the previous one, $x = 0$, $y = 0$ cannot be equilibrium conjectures. Therefore, fix y at zero and increase x. If $E[C(nr,x^*,0|b,3)] = E[C(r,x^*,0|b,3)]$ for some x^*, then $x = x^*$ and $y = 0$ will be confirmed investor conjectures. (From Lemma 3, since there is equality for an analyst receiving signal v_b at date 3, for an analyst not receiving information at either date the expected compensation from not revising is strictly greater than that from revising. This explains why $y = 0$ represents a conjecture by investors that will be confirmed in equilibrium.) If there is no such x^*, then fix x at one and increase y. Equilibrium conjectures will either be at $y = y^*$ where $E[C(nr,1,y^*|n)] = E[C(r,1,y^*|n)]$ or, if $E[C(nr,1,y|n)]$ remains less than $E[C(r,1,y|n)]$ for all y, at $y = 1$. (In either case $x = 1$ remains an equilibrium conjecture since these conditions imply that $E[C(nr,1,y|b,3)] < E[C(r,1,y|b,3)]$.)

There is a fourth possible case that can be considered, where $E[C(nr,0,0|b,3)] \geq E[C(r,0,0|b,3)]$ and $E[C(nr,0,0|n)] < E[C(r,0,0|n)]$. However, given this setting, this case cannot arise. This is because, given Lemma 3, if $E[C(nr,0,0|b,3)] \geq E[C(r,0,0|b,3)]$ then $E[C(nr,0,0|n)]$ must also be at least as great as $E[C(r,0,0|n)]$.

This proves that there will exist an (x,y) pair for which investors' conjectures over the values of x and y will be confirmed. It remains to be shown that the other conjectures of the investors will also be confirmed in equilibrium. To do so, consider first investors' conjectures about the date 3 behavior of an analyst receiving a signal of v_g (either at date 1 or at date 3). From Lemma 3, if either the analyst receiving a signal of v_b at date 3 or the analyst not receiving one at either date is indifferent between revising and not revising (as in cases 1 and 2 above), the analyst receiving a signal of v_g will prefer not to revise, confirming investors' conjectures. Further, if both the analyst receiving a signal of v_b at date 3 and one not receiving any information at either date 1 or date 3 prefer to revise (as may possibly be true in case 3 above), it is straightforward to calculate that the analyst receiving a signal of v_g will have a higher expected compensation by not revising at date 3. He will therefore not revise at that time, again confirming investors' conjectures.

Consider next the analyst's actions at date 1. An analyst who does not receive any information at date 1 will be indifferent as to whether he issues a date 1 forecast consistent with the signal v_g or with the signal v_b. In either case the analyst expects to have chosen the incorrect forecast half of the time. Then, choosing the date 1 forecast consistent with the signal v_g half of the time and consistent with the signal v_b the remaining half maximizes the analyst's objective function and will again confirm investors' conjectures. Next, consider an analyst who observes signal v_g at date 1. If the analyst issues the appropriate date 1 forecast, $E^a(v|g,1)$, his expected compensation is the same as that of an analyst who issues the forecast $E^a(v|g,1)$ at date 1 and receives signal v_g at date 3, confirming his date 1 forecast. If the analyst issues the inappropriate date 1 forecast, $E^a(v|b,1)$, his expected compensation is the same as that of an analyst who issues $E^a(v|g,1)$ at date 1 but receives signal v_b at date 3, contradicting his date 1 forecast. Straightforward calculation reveals that $E[C(nr,x,y|g,3)]$ is greater than both $E[C(nr,x,y|b,3)]$ and $E[C(r,x,y|b,3)]$. It follows from this that the analyst receiving signal v_g at date 1 will maximize his objective function by issuing the forecast $E^a(v|g,1)$ at that time, an act that will again confirm investors' conjectures.

Finally consider the analyst's action at date 2. At that time the analyst will revise his forecast, given the manager's information, under the premise that his date 1 forecast was unbiased, confirming investors' conjectures about his behavior. Doing so leaves unchanged investors' assessment of p_1' and p_3'; doing otherwise would cause investors to set $p_1' = 0$. This will be true for the analyst regardless of the information, if any, received at either date 1 or date 3. This completes the proof that all of the investors' conjectures are confirmed in equilibrium.

Q.E.D.

This equilibrium has several properties. First, an analyst who receives private information at date 1 will release unbiased forecasts at all three dates of the first period. However, an analyst not receiving any information at date 1 will issue a biased forecast at date 1, either $E^a(v|g,1)$ or $E^a(v|b,1)$, instead of the unbiased forecast $E^a(v|n,1)$. Further the analyst will not fully revise his forecast at date 2, after observing the information contained in the manager's forecast. To fully revise his forecast at that time would mean issuing a forecast identical to that of the manager. But that would reveal to investors that the analyst did not receive any information at date 1 and would result in a lower expected compensation for him. Furthermore, at date 3 the analyst may not revise his forecast at all even if he receives information at that time which indicates that it is appropriate to do so. The reason for this is that when the analyst decides whether or not to revise his forecast at date 3, two factors are involved. On the one hand, if he does revise, investors lower to zero their assessment of the probability that he received information at date 1. On the other hand, such a forecast revision may increase their assessment of the probability that he received information at date 3. If the former effect dominates the latter, the analyst will prefer not to revise his forecast; his expected compensation will be higher as a result. Finally, an analyst not receiving any information at either date 1 or date 3 may or may revise his forecast at date 3. For him too there are the same two factors involved in his decision. If the former effect dominates the latter he will also prefer not to revise his forecast; otherwise, at date 3 he will revise it. He would do so even though he did not receive any information at that date, in order to increase investors' assessment that he did receive information at date 3.

While there is no equilibrium in which the analyst, no matter what the nature of his private information, or the time of information receipt, is guaranteed to issue an unbiased forecast, this Proposition has shown that an equilibrium will exist in which the analyst will issue an unbiased forecast under some conditions. However, as stated above, it has also shown that under other conditions the analyst's forecast will be biased.

The characteristics of this equilibrium depend crucially on investors' conjectures. It is possible that a different equilibrium might also exist in which an alternative set of conjectures are satisfied. For example, one alternative conjecture may be that an analyst, upon receiving information, will issue a forecast inconsistent with that information. Another could be that an analyst will revise his forecast only when he receives information inconsistent with his current forecast. Although these conjectures may not seem intuitive, they still may support an equilibrium. A common thread through all of these equilibria, however, is that there must be circumstances under which the analyst will issue a biased forecast. This has implications for the interpretation of empirical studies which attempt to compare the accuracy of analysts' forecasts with those of management. This is the focus of the next section.

III. IMPLICATIONS OF THE RELATIVE ACCURACY OF THE ANALYST'S AND MANAGER'S FORECASTS

As mentioned in the Introduction there has been much empirical work devoted to testing whether analyst or managerial forecasts are more accurate, with accuracy usually measured by the deviation of actual earnings from forecasted earnings. Based on the results of these tests, researchers have drawn inferences about the relative precision of the private information separately held by the analyst and the manager. However, the analysis presented above shows that it is not straightforward to draw such conclusions based on the observed accuracy of the two sets of forecasts.

To see this, recall that at date 1 the analyst who receives a signal will issue an unbiased forecast at that time. However, an uninformed analyst will issue a biased forecast, randomly choosing between the forecast $E^a(v|g,1)$ and $E^a(v|b,1)$, rather than issuing the unbiased forecast of $(v_g + v_b)/2$. Because of this the average accuracy of the analyst's forecast at date 1 will be lower than it would have been if the analyst always issued unbiased forecasts. Further, at date 2 the analyst will revise his forecast, under the premise that it was unbiased at date 1, given the information in the manager's forecast. For an analyst who did observe a signal at date 1 this revised forecast must necessarily be more accurate than that of the manager. His forecast is based on two signals rather than one. However, this is not true for an analyst who did not receive any information at date 1. If he correctly combined his date 1 private information (that is, null information) with that of the manager, his date 2 forecast would be exactly the same as that of the manager. However, because he is revising his forecast under the premise that he really did receive a private signal at date 1, his date 2 forecast will be biased. (That is, he is not fully revising his forecast given the new information contained in the manager's forecast.) It is easy to show that, as a result, the expected accuracy of the analyst's forecast in this case will be lower than that of the manager's forecast. Because of this, the measured average accuracy of analysts' forecasts issued after a managerial forecast may be lower than that of the managerial forecast. This results despite the fact that, on average, the analyst has more information than the manager. Whether or not the average accuracy of the analyst's forecast will be lower than that of the manager will depend on the probability that an analyst observes a signal at date 1, p_1. At the extreme, if $p_1 = 1$, then the average accuracy of analysts' forecasts issued just after the managerial forecast will be higher than that of the managerial forecast. If $p_1 = 0$, then the average accuracy of analysts' forecasts will be lower than that of the managerial forecast. There will then be a cutoff probability, p_1*, above which the average accuracy of analysts' forecasts will be higher than, and below which it will be lower than that of the managerial forecast.

This result may help explain the empirical results which show that managerial forecasts are at least as, if not more accurate than analysts' forecasts issued at the same time as or after the managerial forecasts. On the surface these empirical results may seem paradoxical since analysts who release their forecasts just after managerial forecast announcements have just as much, if not more, information with which to make a forecast than does the manager. As shown here the reason for managerial forecasts to be at least, if not more, accurate than that of the analysts may be due to the reluctance of the analyst to fully adjust his forecast to new information. The measured accuracy of the analysts' forecasts may then result in a downward biased assessment of the precision of their private information.

One interesting comparative static concerns how the accuracy of the analyst's forecast at date 2 relative to that of the manager's forecast at date 1 changes as the analyst's signal accuracy, s, changes. As the accuracy of the analyst's signal increases it might be expected that the relative accuracy of the analyst's forecast will increase. However, such need not be the case here. As can be formally demonstrated, the relative accuracy might rise or might fall as s increases. The reason for this is that while the relative accuracy of the informed analyst's date 2 forecast improves, the relative accuracy of the uninformed analyst's date 2 forecast decreases. As s increases, the uninformed analyst, in order to mimic the actions of an informed analyst, must change his forecast less at date 2 in response to the release of the manager's forecast. This makes the uninformed analyst's revised forecast relatively less accurate (the most accurate revised forecast for this analyst being the same as the manager's forecast). Increased accuracy of private information therefore need not translate into increased accuracy of the analyst's publicly reported forecast.

At date 3 the average accuracy of the analysts' forecasts, relative to that of the manager, may improve, but may still be lower than that of the manager. Analyst forecast accuracy is unaffected by the actions of an analyst who remains uninformed at date 3 because whether or not he revises his forecast at date 3, the average accuracy of his forecast will be unchanged. (Since he is uninformed, whether he acts as if he has received a signal of v_g or v_b at either date 1 or date 3 does not affect the accuracy of his forecast.) Similarly, if an analyst receives information at date 3 consistent with the forecast he reported at date 1, the average accuracy of his forecast will remain unchanged; he will not revise his forecast at date 3. Using similar reasoning, if he receives information at date 3 inconsistent with the forecast he reported at date 1 but does not revise his forecast, the average accuracy will again be unchanged. However, if he receives information at date 3 inconsistent with the forecast he reported at date 1 and does revise his forecast at that time, the average accuracy of his forecast must increase. Therefore, as long as the probability of such an analyst revising his forecast, x, is greater than zero, the average accuracy of analysts' forecasts relative to that of the managerial forecast will increase at date 3. (Depending on the value of x, however, the accuracy could still be lower than that of the managerial forecast). This prediction is consistent with the empirical evidence which shows that the relative accuracy of analysts' forecasts, as compared to those of management, improves as the time between the management forecast and the subsequent analyst forecast increases.

An additional insight can be gained by focusing on the case where $p_3 = 1$, that is, where the analyst will, with certainty, receive a private signal, either at date 1 or at date 3. In this case it is clear that no analyst would revise his forecast at date 3. To see this note that whether revising his forecast or not, $p_1' + p_3'$ will equal one; investors know that the probability that the analyst received information at date 1 plus the probability that he received it at date 3 equals one. However, if he revises his forecast, investors set $p_1' = 0$; that is, they infer that the analyst did not receive information at date 1. Since the analyst is compensated more highly for observing information at date 1 as opposed to date 3 this implies that the analyst would prefer not to revise his forecast at date 3.

IV. SUMMARY AND CONCLUSIONS

This study has shown that empirical research which has compared the relative accuracy of analyst and managerial forecasts may be underestimating the precision of the analyst's private information. This is due to reluctance on the part of the analyst to revise his forecasts upon the receipt of new information. Because of this, researchers need to be careful in drawing inferences from the empirical results about the relative amount of information supplied to the market by managerial forecasts; the manager may not be providing as much information to the marketplace as the empirical results seem to indicate. This conclusion will hold as long as the analyst's private information does, somehow, get disseminated to the market (possibly through more private channels than that of forecast disclosure). If it does not get disseminated, however, then the managerial forecasts may still provide significant information to the market even though much of that information is already privately possessed by analysts.

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