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Authors
McLaughlin, GC
Fuller, GM

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EFFECTS OF DARK MATTER ON THE STABILITY OF SUPERMASSIVE STARS

GAIL C. MCLAUGHLIN AND GEORGE M. FULLER
Department of Physics, University of California, San Diego, La Jolla, CA 92093-0319
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ABSTRACT

The stability of nonrotating supermassive stars ($M > 10^5\ M_\odot$) in the presence of a small dark matter component is examined. We find that the destabilizing effects of first-order nonlinear corrections from general relativity can be neutralized when a significant amount of nonrelativistic dark matter is present. In this case, it is possible for supermassive stars to be stabilized through the hydrogen-burning stage. Significant mass loss in this epoch might result in the ejection of the nucleosynthesis products of hot hydrogen burning. Ultimately, however, the post-Newtonian instability can only be delayed. If this instability occurs after the hydrogen-burning epoch, collapse of the star to a black hole is guaranteed.

Subject headings: dark matter — gravitation — instabilities — nuclear reactions, nucleosynthesis, abundances — relativity — stars: interiors

1. INTRODUCTION

In this paper we examine the effects of a small component of "dark matter" particles on the stability of supermassive stars. We assume that these dark matter particles interact only gravitationally with each other and with ordinary baryonic matter. Massive radiation dominated stars are known to be subject to instability induced either by electron-positron pair formation or by the nonlinear nature of gravitation in general relativity. Stars with masses in the range $\sim 200\ M_\odot$ to $\sim 10^4\ M_\odot$, very massive objects (VMOs), are destabilized by the pair instability during core oxygen burning. Stars with masses in excess of $\sim 10^4\ M_\odot$ are termed supermassive objects (SMOs) and generally experience the post-Newtonian instability before, during, or at the onset of hydrogen burning.

When a supermassive star is embedded in a dark matter halo (e.g., something like a galactic halo), some dark matter particles are present in the star itself. Here we will assume that the dark matter particles interact only gravitationally with the star, and we will consider the effect of this dark matter component on the stability of the object.

Supermassive stars will encounter a general relativistic instability before they ignite hydrogen (Feynman 1963; Iben 1963; Hoyle & Fowler 1963; Chandrasekhar 1964; Zeldovich & Novikov 1971; Shapiro & Teukolsky 1983). Once such stars become unstable, they can either collapse or explode, depending on their initial metallicity and their angular momentum content and distribution (see Fricke 1973, 1974). As these stars collapse following the post-Newtonian instability, they burn hydrogen dynamically. Their fate, collapse to a black hole or explosion, depends on the rate at which this nuclear energy is released. Hydrogen burning by the *p-p* chain produces insufficient energy to halt the collapse (Fricke 1973; Fuller, Woosley, & Weaver 1986), so that objects with primordial initial abundances would collapse to black holes. Objects with initially higher metallicity will be more likely to explode, since they could generate nuclear energy more rapidly from the beta-limited CNO cycle or, conceivably, from breakout into the rp-process (see Fuller et al. 1986). The other important factor, rotation, will tend to favor explosions, since rotation has the effect of providing a centrifugal restoring force.

Supermassive objects may be relevant in several different astrophysical scenarios. In the early universe, for example, a nonlinear $10^6\ M_\odot$ density fluctuation could come into hydrostatic equilibrium and subsequently encounter the post-Newtonian instability (Hogan 1993). Supermassive objects have been invoked as the dark matter itself (Gnedin & Ostriker 1992). In this scenario, such objects could form soon after decoupling and collapse to black holes as a result of the post-Newtonian instability. Carr & Rees (1984) and Loeb (1993) discuss scenarios in which the gravitational potential of a dark matter cluster causes clouds $\sim 10^6\ M_\odot$ to collapse. However, such objects would be unlikely to form hydrostatic supermassive stars when Thomson drag on background photons provides an efficient cooling mechanism. As a result of these Compton cooling effects, these objects can ultimately have temperatures and entropies which are too low for hydrostatic equilibrium. Salati & Silk (1989) suggest that large objects could form at the center of dense dark matter galactic cores through stellar collisions. Hogan & Rees (1988) discuss the formation of stars in the context of scenarios in which baryons fall into axion clusters. In addition, models of quasars and active galactic nuclei usually invoke supermassive black holes with accretion disks as the "central engine" (e.g., Salpeter 1964). The progenitors of these black holes could be supermassive stars formed by the disruption and coalescence of low-mass stars (Begelman & Rees 1978).

In all these cases, the object could be in the presence of dark matter. For example, in the early universe, a fluctuation in baryons could reasonably be expected to be accompanied by a fluctuation in dark matter particles. Also, a galactic core is thought to be surrounded by a dark matter halo.

Once a supermassive star has formed, its subsequent evolution hinges on the post-Newtonian instability. If the instability is postponed by the dark matter component, then the star has a chance to burn hydrogen stably and experience significant mass loss during this period (Men'shchikov & Tutukov 1988). This scenario might result in the ejection of the nucleosynthesis products of hot hydrogen burning and the rp-process (e.g., intermediate-mass elements up to the iron group; see Wallace & Woosley 1981). Such nucleosynthesis might allow us to constrain, for example, primordial isocurvature baryon models for structure formation (Peebles 1987; Cen, Ostriker, & Peebles 1993).

An interesting question concerns the evolution of the dark matter surrounding a supermassive object. Is it possible to concentrate enough dark matter inside the supermassive object to affect the instability? A dark matter halo would presumably form by the process of violent relaxation and settle into a distribution which resembles an isothermal gas sphere. A higher concentration of dark matter particles would have to be achieved by subsequent evolution. Prospects for achieving high concentrations of dark matter will be discussed in § 6.

In the next section we discuss general characteristics of supermassive stars. Section 3 gives a physical description of the instability, and § 4 details the calculations. In § 5 we present the results of the calculations. In § 6, possibilities for the enhancement of dark matter in the supermassive object will be discussed, as well as other issues relevant to the evolution of such stars.

2. GENERAL CHARACTERISTICS OF SUPERMASSIVE STARS

The pressure in supermassive stars is dominated by radiation. The parameter β measures the ratio of gas pressure P_g to total pressure $P = P_r + P_g$, where P_r is the radiation pressure. This parameter is nearly constant throughout a supermassive star and is given approximately by

$$\beta = \frac{P_g}{P} \approx \frac{4.3}{\mu}\left(\frac{M}{M_\odot}\right)^{-1/2} . \tag{1}$$

The latter approximation in equation (1) employs an index $n = 3$ polytrope to describe the density and temperature run in the star. In this expression, μ, the mean molecular weight, is 0.5 for pure hydrogen. A typical value of the parameter β in a supermassive star with a mass $M = 10^6\, M_\odot$ is about $\beta \approx 10^{-2}$.

The pressure-averaged adiabatic index is defined as

$$\bar{\Gamma}_1 \equiv \int \Gamma_1 P 4\pi r^2\, dr \Big/ \int P 4\pi r^2\, dr , \tag{2}$$

where the local adiabatic index is defined as $\Gamma_1 \equiv (d \ln P/d \ln \rho)_s$. The derivative is taken at constant entropy per baryon, s. Here ρ is the total mass energy density and P is the total pressure ($P = P_g + P_r$). This quantity, $\bar{\Gamma}_1$, is crucial to determining stability in supermassive objects.

Supermassive stars with small β are nearly unique objects, in that their local mass-energy density is dominated by the baryon rest mass density, while the pressure is dominated by radiation. The structure (density and pressure runs) of these stars is essentially completely Newtonian and is well represented by an index $n = 3$ polytrope. Assuming a Newtonian polytropic structure and an ideal gas equation of state, we obtain

$$\bar{\Gamma}_1 \approx \frac{4}{3} + \frac{\bar{\beta}}{6} , \tag{3}$$

where $\bar{\beta}$ is the pressure-averaged value of β. Here the pressure average inherent in $\bar{\beta}$ is calculated in the same manner as $\bar{\Gamma}_1$ in equation (2). Since β is approximately constant in the stars considered here, we can set $\bar{\beta} \approx \beta$ without appreciable loss of accuracy. If the only contribution to the total pressure in a supermassive star was from radiation, then the pressure-averaged adiabatic index would be $\bar{\Gamma}_1 = 4/3$.

The Newtonian criterion for stability of a star against radial perturbations is

$$\bar{\Gamma}_1 > \tfrac{4}{3} . \tag{4}$$

Since these supermassive stars are so close to $\bar{\Gamma} = 4/3$, a small change in the adiabatic index, or in the stability criterion, can cause dramatic changes in stability. While gas pressure contributes to stability by increasing the adiabatic index, general relativity destabilizes these objects by lowering the stability condition on the adiabatic index. In this way, an object which is nearly Newtonian in structure can be destabilized by corrections to gravity from general relativity. In the absence of rotation and dark matter, instability in supermassive stars will occur before the onset of hydrogen burning.

Supermassive stars are almost completely convective, since their temperature gradients in hydrostatic equilibrium are very close to the adiabatic gradient. This means that the entropy per baryon, s, may be approximated as constant throughout the object. The entropy in these stars is dominated by photons. For an object with mass $M = 10^6\, M_\odot$, the entropy per baryon is about $s/k \approx 1000$ in units of Boltzmann's constant, k. Another consequence of convection is that there will be composition mixing in these stars.

Supermassive stars radiate at almost the Eddington limit, $L_{\rm Edd} \approx 1.3 \times 10^{38} (M/M_\odot)$ ergs s^{-1}. The evolution of a supermassive object begins with contraction toward the main sequence. The object will slowly contract and radiate away energy and entropy until it encounters the post-Newtonian instability. Once this occurs, the star will either explode or collapse to a black hole. The Kelvin-Helmholtz timescale, $\tau_{\rm KH}$, for contraction to the instability point is about $\tau_{\rm KH} \approx 3 \times 10^{16} (M/M_\odot)^{-1}$ s. For an object with mass $10^6\, M_\odot$, we therefore estimate $\tau_{\rm KH} \approx 10^3$ yr.

Another characteristic of supermassive stars is that they have a very small binding energy. An index $n = 3$ polytrope with only radiation pressure contributing to total pressure has a Newtonian binding energy $E = 0$. It is only the presence of the nonrelativistic component of pressure (i.e., the gas pressure) that lowers the energy slightly. It is this nonrelativistic pressure component which stabilizes the star.

3. STABILITY ANALYSIS: PHYSICAL DESCRIPTION

In hydrostatic equilibrium a fluid element in the star has gravitational force, F_g, and balancing (equal and opposite) pressure force, F_p. If this fluid element is displaced, it will experience a net force which is the sum of the new forces at the displaced point, $\delta F_g + \delta F_p$. If the fluid element is displaced inward, then we would require that $|\delta F_g| < |\delta F_p|$ for stability. Any perturbed fluid element must experience a net restoring force if the star is to be stable. By considering the effects of Newtonian gravitational forces only, it can be shown that the criterion for stability is that the pressure-averaged adiabatic index satisfies $\bar{\Gamma}_1 > 4/3$. If only Newtonian gravity is considered, then supermassive stars are always stable, since as mentioned in the previous section $\bar{\Gamma}_1 \approx 4/3 + \beta/6$.

The criterion for stability changes when the effects of general relativity are included. Consider perturbing a star in hydrostatic equilibrium as above, but with first-order general relativistic corrections to Newtonian gravity. If a fluid element is now displaced inward, then the restoring force will be different than that for the purely Newtonian case. In fact, the net restoring force will be reduced. The reason for this is that the first-order nonlinear corrections of general relativity cause the gravitational force to grow more quickly with decreasing radius than in the case of purely Newtonian gravity. In other words, the change in the general relativistic gravitational force is greater than the change in the purely Newtonian gravitational force, $|\delta F_g|_{\mathrm{GR}} > |\delta F_g|_{\mathrm{newt}}$. The gravitational field itself generates more gravitational field. Therefore, expanding or contracting the stellar configuration will have a greater effect on the gravitational force when the effect of general relativity is considered.

Clearly, these effects will tend to push the object closer to instability. The new stability criterion with general relativity is $\bar{\Gamma}_1 > 4/3 + \delta_{\mathrm{gr}}$. Here $\delta_{\mathrm{gr}} \sim (R_s/R)$, where R is the radius of the supermassive star and R_s is the corresponding Schwarzschild radius for the object. For a supermassive star with $M \approx 10^6\ M_\odot$, we would have $R_s \sim 10^{11}$ cm and $R \sim 10^{14}$ cm, when the star is close to instability. This radius is for a nonrotating supermassive star that does not contain dark matter and has not arrived at the main sequence. For this case, the general relativistic correction will be of order $\delta_{\mathrm{gr}} \sim 10^{-3}$. This correction grows as the object shrinks, until eventually dynamical instability sets in. The instability point can be estimated analytically to occur when the object attains a central density $\rho_c \approx 2 \times 10^{-3}(10^6\ M_\odot/M)^{7/2}$ g cm^{-3}. Figure 1 shows the general relativistic correction (*upper curved line*) as a function of central density for a supermassive star of mass $M = 10^6\ M_\odot$. The horizontal line is the quantity $\beta/6$. The intersection of the $\beta/6$ line and the general relativistic correction line is the instability point.

General relativistic stability for supermassive stars in the presence of dark matter particles may be analyzed in similar fashion. The dark matter particles are assumed to be present throughout the supermassive star and are assumed to interact only gravitationally with the baryons, electrons, and photons in the star. For the purposes of our study, we shall consider a supermassive star of mass $M \sim 10^6\ M_\odot$ located in the center of a spherical distribution or "cluster" of noninteracting dark matter particles. We assume that the overall mass and radius of this "cluster" is very large compared to both the mass and radius of the star. Furthermore, we will assume that the density of mass-energy contributed by the dark matter particles is small compared to the rest mass–energy density of the baryons in the region inside the supermassive star.

The timescale for radial oscillations in a supermassive star is parameterized by the central density and the mass. In the case of a $10^6\ M_\odot$ object, the oscillation timescale is $\tau_o > 10^7$ s. The oscillation timescale becomes shorter as the central density of the star increases. The time it takes for a dark matter particle to cross the supermassive star is important in determining the stability condition. The radius of an $M \approx 10^6\ M_\odot$ supermassive object with central density $\rho_c \approx 10^{-3}$ g cm^{-3} is about $R \approx 4 \times 10^{13}$ cm. For example, a dark matter particle traveling at about 400 km s^{-1} takes about $t \sim 10^6$ s to cross this star. Of course, dark matter particles with higher velocities will have shorter crossing times.

If the dark matter particles are moving slowly when they are far from the supermassive star, then their velocities inside the star will be determined largely by the local gravitational potential. On the other hand, the trajectories of very rapidly moving dark matter particles will be relatively unaffected by the gravitational field produced by matter in the supermassive star. If a dark matter particle were to free fall to the center of the supermassive star, then it would have a crossing time comparable to the free-fall timescale, $\tau_{\mathrm{ff}} \approx 10^6$ s $(10^{-3}$ g cm$^{-3}/\rho_c)^{(1/2)}$, where ρ_c is the central density of the star. This timescale represents an upper limit on the time that it takes a dark matter particle to pass through the star. Since this time is short in comparison with the oscillation timescale, the dark matter particles pass through the star too quickly to have their phase-space densities appreciably altered by the changing

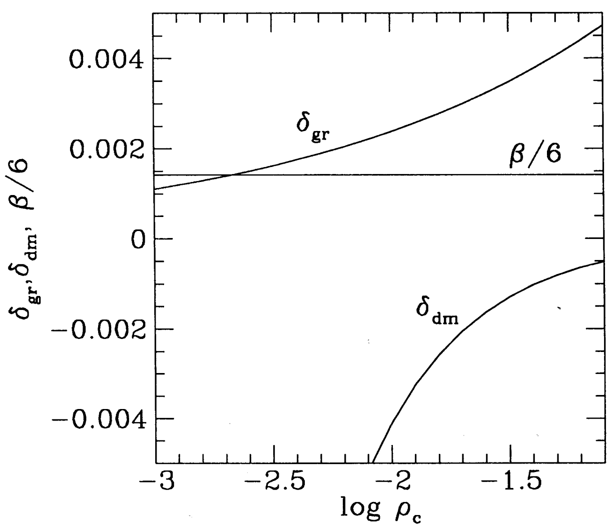

FIG. 1.—The correction terms δ_{gr} and δ_{dm} and the quantity $\beta/6$ are plotted against the central density of the supermassive star. The mass of this star is taken to be $M = 10^6\ M_\odot$, and it is assumed to contain a constant dark matter density of 10^{-5} g cm^{-3}. The horizontal line gives $\beta/6$ as a function of central density, while the lower and upper lines give δ_{dm} and δ_{gr} as functions of central density, respectively.

gravitational potential of the star. In other words, the changing gravitational potential of the baryons from radial stellar oscillations has a negligible effect on the motion, and therefore distribution of the dark matter particles. Based on these considerations, in what follows we will approximate the distribution of dark matter particles to be fixed. This fixed dark matter distribution provides a static background gravitational potential in which the evolution of the supermassive star is to be viewed. This is a crucial assumption since, as we shall show, it leads to increased stability for supermassive stars. If astrophysical environments suggest other conditions for the dark matter distribution where our approximation breaks down, then we would have to modify our conclusions accordingly.

When a fluid element in the supermassive star is displaced slightly inward, the change in the gravitational force is smaller than it would have been if dark matter was not present, $|\delta F_g|_{\rm dm+baryons} < |\delta F_g|_{\rm baryons\ only}$. This will make the star more stable than it would have been without the dark matter. The new stability criterion on the pressure-averaged adiabatic index is

$$\bar{\Gamma}_1 > \tfrac{4}{3} + \delta_{\rm gr} + \delta_{\rm dm} + \delta_{\rm grdm}\,, \tag{5a}$$

where $\delta_{\rm gr}$, $\delta_{\rm dm}$, and $\delta_{\rm grdm}$ represent the effects of first-order general relativistic corrections, dark matter, and a cross term between these effects, respectively. The cross term, $\delta_{\rm grdm}$, is presented in § 5. The effect of the dark matter stress-energy on the stability criterion can be shown to be

$$\delta_{\rm dm} = -\frac{4\pi}{9}\frac{\int \rho_0 \rho_1 r^4\, dr}{\int p_0 r^2\, dr}\,. \tag{5b}$$

In this expression, ρ_0 is the density of the baryons (matter), and ρ_1 is the density of the dark matter. Figure 1 shows the dark matter correction as a function of central matter density ρ_c, as the lower curved line. In calculating this curve, we have assumed a constant dark matter density of 10^{-5} g cm^{-3}.

We have chosen this particular dark matter density since it best illustrates the stabilizing effect for a $10^6\ M_\odot$ star. For a smaller dark matter density, the effect will be smaller; at 10^{-7} g cm^{-3} it is negligible. For a larger dark matter density, the effect is greater. However, when the dark matter density becomes comparable to the baryon density in the supermassive star, then structural changes in the star must be taken into account. For supermassive stars smaller than $10^6\ M_\odot$, the instability density is higher, and so a higher dark matter density is required for stabilization to occur. For example, for a $10^5\ M_\odot$ star, the instability occurs at $\rho_c \approx 10$ g cm^{-3}, and a density of dark matter of about 10^{-2} g cm^{-3} is required for significant stabilization. Similarly, for higher mass supermassive stars, the instability will occur at a lower baryon density, and so a lower dark matter density will be required for stabilization.

In deriving the expression for $\delta_{\rm dm}$ in equation (5b), we have assumed that the dark matter distribution is constant in time. However, in estimating the magnitude of $\delta_{\rm dm}$ in Figure 1, we also have assumed that the spatial distribution of the dark matter particles can be taken as constant across the length scale of the supermassive star. We would expect this latter approximation to be valid for the case in which the radius and mass of the dark matter "cluster" are very large compared to the radius and mass of the supermassive star.

Note that as the central density (ρ_c) of the supermassive star increases, the effect of the dark matter decreases. The last term in equation (5a) is very small, and if it is neglected then an equation for the instability point can be obtained analytically by approximating the matter density run as being given by an index $n = 3$ polytrope, and by taking the spatial distribution of dark matter to be constant:

$$3.5 \times 10^{-4}\left(\frac{10^6\ M_\odot}{M}\right)^{1/2}\rho_c - 2.7 \times 10^{-3}\rho_c^{4/3}\left(\frac{M}{10^6\ M_\odot}\right)^{2/3} - \rho_1 \approx 0\,. \tag{6}$$

In this equation, all densities are understood to be expressed in units of g cm^{-3}. Figure 2 gives a graphical representation of stability for the assumptions inherent in equation (6). The horizontal line in this figure is the quantity $\beta/6 \approx \bar{\Gamma}_1 - 4/3$. The upper curved line is the general relativistic correction term $\delta_{\rm gr}$ as a function of ρ_c. In the absence of dark matter, stellar configurations above the line for $\delta_{\rm gr}$ are stable, while those below are unstable. The instability point for the supermassive star occurs at the intersection of the two lines. The third line in this figure represents the sum of the two corrections $\delta_{\rm gr}$ and $\delta_{\rm dm}$, again shown as a function of ρ_c. The new instability point (intersection of this last line with the $\beta/6$ line) has been moved toward higher density owing to the presence of dark matter. This means that the star will be able to evolve to a higher central density and temperature before becoming unstable.

4. CALCULATION

Our calculation of the terms $\delta_{\rm dm}$ and $\delta_{\rm grdm}$ is based on the analysis of adiabatic, radial pulsations of a nonrotating, relativistic star in Misner, Thorne, & Wheeler (1973, hereafter MTW, pp. 688–699), where the term $\delta_{\rm gr}$ is calculated. We follow MTW and consider small-amplitude radial perturbations of a spherically symmetric distribution of perfect fluid. Our analysis departs from that of MTW by considering two perfect fluid components: a spherically symmetric and static distribution of dark matter particles and a pulsating spherically symmetric distribution of baryons, electrons, and photons (hereafter the "baryons"). We assume that the local spacetime geometry is spherically symmetric and describable by generalized Schwarzschild coordinates (t, r, θ, ϕ) with time-time and radial-radial metric components given by $g_{tt} = -\exp[2\Phi(t, r)]$ and $g_{rr} = \exp[2\Lambda(t, r)]$, respectively. The unperturbed equilibrium solution has pressure p_0 from baryons and pressure p_1 from dark matter, mass-energy density ρ_0 from baryons and ρ_1 from dark matter, baryon number density, n_0, and metric functions Λ_0 and Φ_0. The perturbed solution is described by the metric perturbation functions $\delta\Phi$ and $\delta\Lambda$, the thermodynamic perturbation functions δp, $\delta\rho$, and δn, and the radial displacement function, ξ. The evolution of these functions with time can be obtained from Einstein's field equations and conservation of energy and momentum. Euler's equation can be obtained, once the perturbation functions have been expressed in terms of the displacement, ξ.

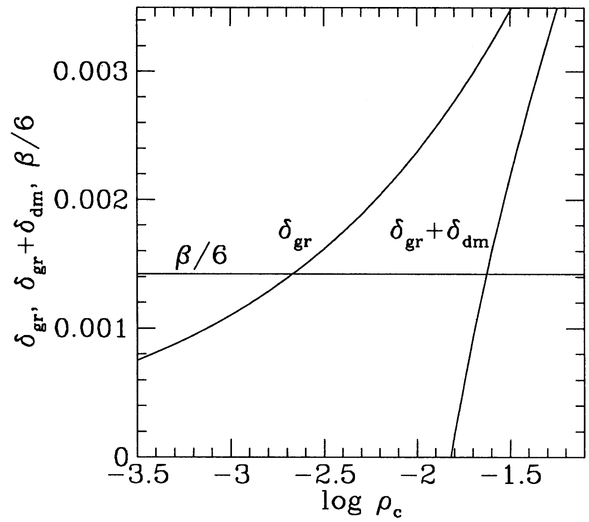

FIG. 2.—The correction terms δ_{gr} and $\delta_{dm} + \delta_{gr}$ and the quantity $\beta/6$ are plotted against the central density of the star. As in Fig. 1, the mass of the star is taken to be $10^6\ M_{\odot}$, and it is assumed to contain a dark matter density of 10^{-5} g cm^{-3}. The instability points occur at the intersection of the correction term curves with the line for $\beta/6$.

The thermodynamic perturbation functions may be obtained for the case with dark matter with the following three-step prescription. To begin with, since the number density of baryons is still conserved, the expression for the Lagrangian variation in baryon number density, Δn, is the same as in the non–dark matter case. Secondly, the equation for the Eulerian variation in pressure, δp, which comes from the definition of the adiabatic index, is the same as in the case without dark matter. This latter condition stems from the fact that the adiabatic index depends only on the baryonic quantities. Note that $\delta \rho \equiv \delta \rho_0 + \delta \rho_1 \approx \delta \rho_0$ and $\delta p \equiv \delta p_0 + \delta p_1 \approx \delta p_0$, since we have assumed a static dark matter distribution. Thirdly, the equation for $\delta \rho$ can be obtained from energy conservation. This equation, which differs from the case without dark matter, is given by

$$\delta \rho = -(\rho_0 + p_0)\left(\xi' + \xi \Lambda_0' + \frac{2\xi}{r} + \delta \Lambda\right) + \xi[p_1' + \Phi_0'(\rho_1 + p_1)] - \rho_0' \xi \,, \tag{7}$$

where the primed quantities indicate derivatives with respect to radial coordinate r. The term which includes dark matter quantities in this equation results from the inseparability of the Tolman-Oppenheimer-Volkoff (TOV) equation for hydrostatic equilibrium for two perfect fluids. The Eulerian variation in the metric function, $\delta \Lambda$, can be obtained from the Einstein field equations. Written in terms of metric functions, this variation also has a term involving dark matter quantities,

$$\delta \Lambda = [-(\Lambda_0' + \Phi_0') + 4\pi r(\rho_1 + p_1)e^{2\Lambda_0}]\xi \,. \tag{8}$$

The field equations also give an expression for the metric perturbation function $\delta \Phi'$,

$$\delta \Phi' = -8\pi r(\rho_0 + p_0)e^{2\Lambda_0}\xi \Phi' - 4\pi(\rho_0 + p_0)e^{2\Lambda_0}\xi - 4\pi \bar{\Gamma}_1 p_0 r\left[\frac{2}{r}\,\xi + \xi' - \Phi_0'\,\xi + 4\pi r(\rho_1 + p_1)e^{2\Lambda_0}\xi\right]e^{2\Lambda_0} - 4\pi \xi p_0'\, e^{2\Lambda_0} r \,. \tag{9}$$

In terms of these metric functions, Euler's equation is the same in this situation as it is in the one without dark matter. However, the effect of dark matter becomes evident once substitutions are made for the perturbed metric functions. With some rearrangement, Euler's equation describing the pulsation can be put in the form

$$W\ddot{\alpha} = (P\alpha')' + Q\alpha \,, \tag{10a}$$

where $\ddot{\alpha}$ denotes the second derivative of α with respect to time and α, W, P, and Q are defined below:

$$\alpha = r^2 \xi \exp(-\Phi_0) \tag{10b}$$

$$W = (p_0 + \rho_0)r^{-2} \exp(3\Lambda_0 + \Phi_0) \tag{10c}$$

$$P = \Gamma_1 p_0 r^{-2} \exp\left[\Lambda_0 + 3\Phi_0 + \int \frac{p_0' + (\rho_0 + p_0)\Phi_0'}{p_0 \Gamma_1}\, dr\right] \tag{10d}$$

$$\begin{aligned} Q = \{&\Phi_0'[(\rho_0 + p_0)4\pi r(p_1 + \rho_1)e^{2\Lambda_0} - p_1' - \Phi_0'(\rho_1 + p_1) + \rho_0']e^{\Phi_0} r^{-2} + \Phi_0'[\Gamma_1 p_0\, 4\pi r(p_1 + \rho_1)e^{2\Lambda_0} + p_0']e^{\Phi_0} r^{-2} \\ &+ [\Gamma_1 p_0\, e^{\Phi_0} r^{-2} 4\pi r(\rho_1 + p_1)e^{2\Lambda}]' + (p_0'\, e^{\Phi_0} r^{-2})' + (\rho_0 + p_0)e^{\Phi_0 + 2\Lambda_0} r^{-2}[8\pi r(\rho_0 + p_0)\Phi_0' + 4\pi p_0' r \\ &+ 4\pi(\rho_0 + p_0) + 16\pi^2 \Gamma_1 p_0 r^2(\rho_1 + p_1)e^{2\Lambda_0}]\} \exp\left[\Lambda_0 + 3\Phi_0 + \int \frac{p_0' + (\rho_0 + p_0)\Phi_0'}{p_0 \Gamma_1}\, dr\right]. \end{aligned} \tag{10e}$$

If $\alpha = \xi(r)e^{-i\omega t}$, then this equation will lead to an eigenvalue problem for ω, as described in MTW. Using the variational principle, a solution for ω can be obtained. A homologous function $\xi = \epsilon r$ (with ϵ a constant) leads to the eigenvalue $\omega = 0$ in the Newtonian case. Inserting this particular expression for ξ into the minimum variational principle results in a condition on $\bar{\Gamma}_1$ for instability. All formulas given in this paper are given in units of $G = c = 1$.

5. RESULTS

Employing the homologous displacement function allows us to find an expression for ω^2 in terms of $\bar{\Gamma}_1$ and the critical value of the pressure-averaged adiabatic index required for stability, $\Gamma_{\rm crit}$:

$$\omega^2 = 3(\bar{\Gamma}_1 - \Gamma_{\rm crit})\frac{|\Omega|}{I}. \tag{11a}$$

Here $|\Omega|$ is the Newtonian gravitational binding energy of the supermassive star, and I is the trace of the second moment of its mass distribution function:

$$|\Omega| = \int 3p_0 4\pi r^2\,dr\,, \tag{11b}$$

$$I = \int \rho_0 r^4 4\pi\,dr\,. \tag{11c}$$

The condition on $\bar{\Gamma}_1$ for stability is that

$$\bar{\Gamma}_1 > \Gamma_{\rm crit} \approx \tfrac{4}{3} + \delta_{\rm dm} + \delta_{\rm gr} + \delta_{\rm grdm}\,, \tag{11d}$$

where the correction terms are defined below. The Newtonian term for the dark matter correction is

$$\delta_{\rm dm} = -\frac{4\pi}{3}\frac{\int \rho_0\rho_1 r^2 4\pi r^2\,dr}{|\Omega|}. \tag{11e}$$

The solely general relativistic correction term in this expression for $\Gamma_{\rm crit}$ can be written as

$$\delta_{\rm gr} = \frac{1}{3|\Omega|}\int\left(\frac{3\rho_0 m^2}{r^2} + \frac{4p_0 m}{r}\right)dV\,, \tag{11f}$$

where $dV = 4\pi r^2\,dr$ is the coordinate volume element. Finally, the cross term correction factor for dark matter and general relativity is

$$\begin{aligned}\delta_{\rm grdm} = {}& \frac{1}{3|\Omega|}\int\left(\frac{18p_0 m_1}{r} + \frac{6m_0 p_1}{r} - \frac{\rho_0 m_0 m_1}{r^2} - \frac{3\rho_1 m_0^2}{r^2} - 8\pi m_0\rho_0\rho_1 r\right)dV \\ &+ \frac{1}{3|\Omega|}\int\left[\frac{2}{3}(4\pi\rho_1)^2\rho_0 r^4 - \frac{3m_0 m_1\rho_1}{r^2} - \frac{4m_1^2\rho_0}{r^2} - 8\pi m_1\rho_1\rho_0 r\right]dV \\ &- \frac{1}{3|\Omega|}\int 2r\left(1 - \frac{2}{\Gamma_1}\right)[p_0' + (\rho_0 + p_0)\Phi_0']dV\,. \end{aligned} \tag{11g}$$

Most of the terms in $\delta_{\rm grdm}$, and also in $\delta_{\rm gr}$, become more amenable to physical interpretation on replacing the Newtonian gravitational constant G (which we have set to 1) in the nonrelativistic expression for $\Gamma_{\rm crit}$ by an effective "general relativistic coupling constant" for small R_s/R,

$$G_{\rm rel} \equiv G\left(1 + \frac{P}{\rho c^2} + \frac{2GM}{rc^2} + \frac{4\pi P r^3}{Mc^2}\right), \tag{12}$$

where $M = \int_0^r 4\pi r^2\rho\,dr$ is the total mass of the star (this mass is well approximated by the baryonic rest mass of the star), and R is the star's radius. The magnitude of the first term for $\delta_{\rm grdm}$ in equation (11g) is determined mostly by $G_{\rm rel}$. The second term in equation (11g) contains contributions which are second order in the dark matter quantities. The last term in equation (11g) results from the inseparability of the dark matter and the baryonic components in the TOV equation.

If the length scale of the dark matter spatial distribution is much greater than the radius of the supermassive star, then the dark matter density inside the star can be approximated as constant. The correction terms may be calculated using an index three polytrope for the structure of the supermassive star. Because both general relativity and the dark matter represent very small perturbations on the equilibrium non–dark matter solution, the structure of the supermassive star will be virtually unchanged from the Newtonian, non–dark matter case. Table 1 shows the various correction terms for $\Gamma_{\rm crit}$ for a $10^6\ M_\odot$ star near the instability point. Note that the dark matter has the largest effect on stability when the central density of the supermassive star is low. However, as the central density increases, the general relativistic destabilizing effect begins to dominate. This result shows that the presence of dark matter delays the post-Newtonian instability. However, the star eventually (inevitably) will become unstable at higher central density and temperature. We find that the dark matter density must be a fraction 10^{-3} of the central baryon rest mass density in

TABLE 1

CORRECTION TERMS[a]

ρ_c	δ_{gr}	δ_{dm}	δ_{grdm}	δ_{total}	$\beta/6$
10^{-1}......	5.0×10^{-3}	-4.1×10^{-5}	4.8×10^{-6}	5.0×10^{-3}	1.4×10^{-3}
10^{-2}......	2.4×10^{-3}	-4.1×10^{-4}	5.7×10^{-5}	2.0×10^{-3}	1.4×10^{-3}
10^{-3}......	1.1×10^{-3}	-4.1×10^{-3}	4.8×10^{-4}	-2.4×10^{-3}	1.4×10^{-3}

[a] In the first column, ρ_c is expressed in units of g cm^{-3}. This table shows the correction terms for a $10^6\ M_\odot$ star with a constant dark matter density of $\rho_1 = 10^{-6}$ g cm^{-3}. The dark matter correction overwhelms the general relativistic correction until a central density of $\rho_c \approx 10^{-2}$ g cm^{-3} is reached.

order to have an appreciable effect on the stability of a supermassive star. The effect of dark matter and general relativity on stability is shown in Figures 1 and 2. The cross term, δ_{grdm}, is small, as can be seen in Table 1. Therefore, it was neglected in the analysis in § 3 and in Figures 1 and 2.

6. EVOLUTION

In the absence of any extra stabilizing influence (like dark matter), a supermassive star will typically become dynamically unstable before, or at the onset of, hydrogen burning. In other words, the post-Newtonian instability would preclude a stable main-sequence phase for these stars. However, if a distribution of dark matter particles were to provide a hedge against general relativistic instability, then it is possible that supermassive stars could burn hydrogen in a stable main-sequence stage. This stage would last for about 10^6 yr. Since a supermassive star radiates at very near the Eddington limit, it will experience substantial mass loss. For example, a $10^6\ M_\odot$ object will lose mass at the rate of at least $10^{-3}\ M_\odot$ yr^{-1} (Men'shchikov & Tutukov 1988). Since a supermassive star is expected to be completely convective, nucleosynthesis from hot hydrogen burning could be released during this period. A $10^6\ M_\odot$ supermassive object could then be expected to eject some $10^3\ M_\odot$ of processed material.

Once the star becomes dynamically unstable it will either collapse or explode, depending on how quickly nuclear energy is added. At the onset of instability the binding energy of the star is close to zero, so the determining factor in its fate is the competition between buildup of infall kinetic energy and nuclear energy generation. Initially, the star would be collapsing close to the free-fall rate and nearly homologously (Fuller et al. 1986; Goldreich & Weber 1980). Therefore, the acceleration as a function of radius, $a(r)$, is close to

$$a(r) \approx \alpha \frac{GM}{r^2} , \tag{13}$$

where α is a parameter related to both the structure of the star and the deficit in pressure relative to that required for hydrostatic equilibrium.

This expression for the inward acceleration of the star can be used in calculating the infall kinetic energy,

$$E_{KE} \approx \tfrac{1}{2} \int v^2 4\pi r^2 \rho_0\, dr . \tag{14}$$

Here $v(r)$ is the velocity field of the infalling star as computed from $a(r)$. Assuming polytropic distribution functions for the structure of the supermassive star allows us to express the infall kinetic energy in terms of the instantaneous central density ρ_c,

$$E_{KE} \approx 1.3 \times 10^{58} \alpha \rho_c^{1/3} \left[1 - \left(\frac{\rho_{bc}}{\rho_c}\right)^{1/3}\right] \left(\frac{M}{10^6\ M_\odot}\right)^{5/3} \text{ ergs} , \tag{15}$$

where ρ_c is to be expressed in units of g cm^{-3}. In this expression, ρ_{bc} is the central density at which the supermassive star first begins to collapse. Note that ρ_c is time dependent in the collapsing stellar configuration.

Hydrogen burning on the *p-p* chain produces nuclear energy at an insufficient rate to halt a collapsing, nonrotating supermassive star. If an explosion is to be produced, then at the very least there must be sufficient catalyst carbon, nitrogen, or oxygen present to ensure efficient hydrogen burning on the beta-limited CNO cycle. For temperatures relevant for supermassive stars, the beta-limited CNO cycle produces energy at a rate (see Wallace & Woosley 1981)

$$\epsilon_{CNO} \approx 5.86 \times 10^{15} Z \text{ ergs g}^{-1} \text{ s}^{-1} . \tag{16}$$

Here Z is the "metallicity," or mass fraction of the elements carbon, nitrogen, and oxygen. If the object begins with zero metallicity, then it must produce the heavier element catalysts needed in the CNO cycle by the 3α process. The 3α process generates energy at a rate (Clayton 1983, p. 414)

$$\epsilon_{3\alpha} \approx 3.9 \times 10^{11} \frac{\rho^2 X_\alpha^3}{T_8^3} f \exp\left(-\frac{42.94}{T_8}\right) \text{ ergs s}^{-1} \text{ g}^{-1} . \tag{17}$$

Here T_8 is the temperature in units of 10^8 K, X_α is the mass fraction of α particles, and f is a screening factor. Assuming an initial alpha particle mass fraction of $X_\alpha \approx 0.25$, we can calculate the "metallicity" generated from the 3α process during the infall of a supermassive star. This is the lower curve shown in Figure 3. In generating this curve we have assumed an initial metallicity $Z \approx 0$.

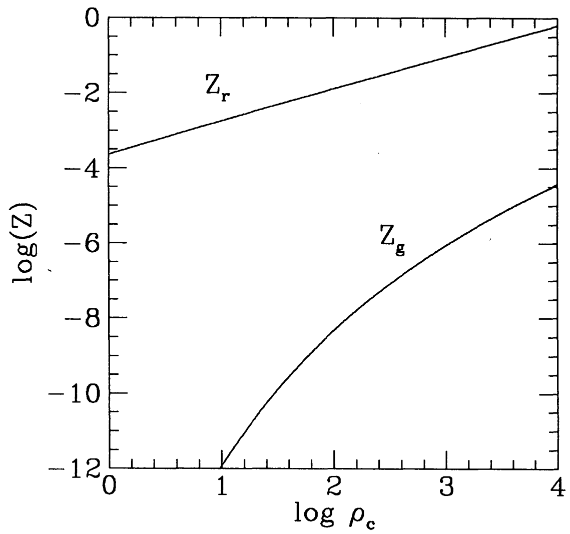

FIG. 3.—Metallicity is plotted against central density. *Lower curve:* metallicity generated by the infalling supermassive star, Z_g; *upper curve:* metallicity required for an explosion, Z_r, if hydrogen burning were ignited at that density. The star is again assumed to have mass $10^6\,M_\odot$ and is taken to start from zero metallicity. We have used a homologous collapse parameter $\alpha = 0.01$ in this example.

In the calculations for this figure we have assumed a homologous collapse parameter $\alpha \approx 0.01$ and a stellar mass $M \approx 10^6\,M_\odot$. This value of α was chosen since it represents a slow collapse (Fuller et al. 1986). As shown in this figure, the rate of the 3α process is too slow to facilitate an explosion on the beta-limited CNO cycle. Since explosion was impossible in this case, it will be impossible for faster collapses as well. The upper curve in Figure 3 shows the metallicity necessary for such an explosion as a function of the time-dependent central density. The star picks up more infall kinetic energy as the central density increases. This is why the 3α reaction rate is critical to the prospects for an explosion: if this rate is too slow, then by the time the requisite CNO catalyst has been produced, the infall kinetic energy will exceed the total possible hydrogen-burning energy yield. From this argument, it is clear that a nonrotating supermassive star with primordial initial abundances will likely become a black hole if its only nuclear energy source is beta-limited CNO cycle hot hydrogen burning.

It is possible that hydrogen burning could "break out" of the beta-limited CNO cycle via $^{15}O(\alpha, \gamma)^{19}Ne$ and other reactions, allowing much more rapid incorporation of protons into nuclei (more rapid energy generation) through (p, γ) reactions in the rp-process (Wallace & Woosley 1981; Wiescher et al. 1989). However, by the time the temperature is high enough for breakout into the rp-process to occur, the radius of the supermassive star already will be of order $\sim 20R_s$. At this point the supermassive star is probably too deep in its gravitational potential well to explode (Fuller et al. 1986), but recently measured reaction rates conceivably could change this conclusion (Wiescher et al. 1989). Clearly, supermassive stars with metallicity higher than primordial values have a better chance of exploding.

If the star could be stabilized through the hydrogen-burning stage by, for example, a dark matter component, then it would burn hydrogen on the main sequence for about 10^6 yr. After this time, it would contract quasi-statically and would ultimately encounter the post-Newtonian instability and begin to collapse. In this situation, its fate would again be determined by the competition between infall kinetic energy buildup and nuclear energy generation. This time, most of the energy would have to be generated by the 3α process. However, in this situation very little nuclear energy is available for the star. For example, a $10^6\,M_\odot$ star would have only $\sim 10^{56}$ ergs potentially available from helium burning. As can be seen from equation (15), the infall kinetic energy will increase above this number after only a relatively small change in central density.

Another possibility for the supermassive star is that it becomes unstable during hydrogen burning. As hydrogen is burned to helium, the mean molecular weight increases and the nonrelativistic (gas) pressure decreases. In the context of our stability analysis, the parameter β decreases. In Figures 1 and 2 the horizontal line representing $\beta/6$ falls, resulting in new instability points at lower central density. Converting hydrogen to helium brings the star closer to instability. If the supermassive object were to become unstable under these circumstances, its fate would be determined by the amount of hydrogen and the mass fraction of catalyst elements contained in the star at the beginning of collapse.

Finally, there is the question of how the dark matter "cluster" could evolve to the relatively high densities required to counteract the destabilizing effects of general relativity. Studies of accretion of dark matter in the Sun have shown that this process is slow (Press & Spergel 1985). It is unlikely that sufficient dark matter can be accreted through gravitational capture to prevent collapse of a supermassive star, since there are only 10^3 years before such a star would quasi-statically contract to the instability point.

The spatial distribution function of stars in a cluster will evolve with time as stars "evaporate" from the surface of the cluster and two-body gravitational interactions redistribute energy. Star clusters evolve to higher central core densities through these processes (see Binney & Tremaine 1987, p. 190), albeit very slowly. However, the interaction between dark matter particles in the dark matter "cluster" would proceed far too slowly for an appreciable concentration of dark matter particles to accumulate in the core. This is because of the assumed small mass of the dark matter particles relative to the stellar masses characteristic of the "particles" in a star cluster. The effectiveness of gravitational scattering in redistributing energy decreases as the mass of the "particles" decreases. The relaxation time for a cluster is $t_{relax} \approx N/(8 \ln N) t_{cross}$, where N is the number of particles in the cluster and t_{cross} is the time for a "particle" to cross the cluster (Binney & Tremaine 1987). Therefore, for a particle with the mass of a nucleon, the relaxation time is $\sim 10^{55}$ times the relaxation time for a star cluster! For primordial black holes, with masses between 10^{15} g and 10^{20} g, we find that, the relaxation time would be between 10^{18} and 10^{12} times the relaxation time of a star cluster. Clearly, ordinary gravitational interactions between dark matter particles could not possibly bring about the relatively large density of such particles necessary to influence the stability of supermassive stars.

We do not know how to arrange to have a supermassive star in a relatively dense "cluster" of dark matter particles. From the timescale arguments presented above, we do not feel that such a configuration could arise naturally from nonlinear fluctuations (e.g., Hogan 1993; Jedamzik & Fuller 1995) in the early universe.

A variant of this scenario has been examined for the case of annihilating cold dark matter particles in main-sequence stars by Salati & Silk (1989). In a dense galactic core, dark matter particles become trapped and annihilate, releasing energy in the stars. The stars expand, enhancing the stellar collision rate and facilitating the formation of a black hole. The authors find that a density of as little as 10^{-17} g cm^{-3} will provide the enhanced collision rate, whereas we require a dark matter density on the order of 10^{-5} g cm^{-3} to influence the evolution of a $10^6\ M_\odot$ supermassive star. In our analysis, we have assumed that there is no annihilation between dark matter particles. However, for a particular model of dark matter, such effects may need to be considered. Stephenson & Goldman (1993, 1995) find that neutrino clusters with a density of $\sim 10^{-16}$ g cm^{-3} may form as a consequence of a theory in which a scalar boson is weakly coupled to neutrinos. Although this density is too low to influence the evolution of supermassive stars, other particle physics models built along these lines may give rise to clusters with a larger dark matter density.

7. CONCLUSION

We have shown that supermassive stars possessing a dark matter component can be stabilized against the post-Newtonian instability under some circumstances. Stabilization could occur when the ratio of central density of the supermassive star to dark matter density is $\sim 10^{-3}$. In other words, the ratio of the dark matter component mass inside the star to the baryon rest mass must be of order 10^{-2}. A key assumption utilized in reaching this conclusion is that the spatial distribution of dark matter particles can be taken to be nearly fixed on the characteristic oscillation time of the star. This assumption is most likely to be correct for the example we have considered: a "cluster" of dark matter particles with a core radius large compared to the radius of the supermassive star at its center. We have discussed the difficulties inherent in forming such an object, given that it must have a relatively large central density of dark matter particles.

If such an object could form, then the supermassive star at its center could be stabilized through the hydrogen-burning epoch. Significant mass loss during a stable main-sequence phase could eject the products of hot hydrogen burning. These nucleosynthesis products may give a signature for, or constraint on, the former existence of supermassive stars at one time in, for example, active galactic nuclei or quasars. However, gravity always wins in the end, and the star eventually becomes unstable. If this occurs after the hydrogen-burning phase, then the star is virtually guaranteed to become a black hole. Ironically, the extra stabilization from a dark matter component mitigates *against* an explosion for the supermassive star and so helps to ensure the formation of a black hole.

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