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Essays on Macroeconomics and Finance

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by

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ABSTRACT OF THE DISSERTATION

Essays on Macroeconomics and Finance

by

Huifeng Chang
Doctor of Philosophy in Economics
University of California, Los Angeles, 2022
Professor Pierre-Olivier Weill, Chair

This dissertation consists of three chapters on macroeconomics and finance. In Chapter 1, I study how disruptions in secondary bond market liquidity affect the macroeconomy. I introduce search-based secondary markets for long-term corporate bonds into a dynamic general equilibrium model. In the model, with borrowing constraints and incomplete insurance, firms restrict hiring ex-ante when default risk increases. A worsening of bond market liquidity, by affecting bond prices and thus the borrowing limits for firms, has aggregate negative impact on firms' labor choices. A positive default-liquidity spiral further amplifies these effects. In the quantitative analysis of my model, I show that a liquidity shock calibrated to match the observed increase in the bid-ask spread could explain about 20% of the employment losses in the Great Recession. I also provide a structural estimate of the impacts of the Fed's corporate bond purchasing program on the real economy during the COVID-19 crisis. By improving bond market liquidity, the Fed's interventions avoided a 2 percentage point drop in employment.

In Chapter 2 (joint with Adrien d'Avenas and Andrea Eisfeldt), we explain why credit spreads explain firm-level investment better than equity volatility does. While credit spreads

always predict lower investment, the sensitivity of investment to equity volatility changes sign in the cross section of firms depending on their distance to default. Higher equity volatility predicts greater investment for firms far from their default threshold, consistent with a larger option value of investment at higher levels of volatility. On the other hand, higher equity volatility predicts lower investment for firms with high credit spreads, consistent with debt overhang. Opposite effects at the firm level wash out and confound aggregate inference. We provide clean intuition using a simple model.

In Chapter 3 (joint with Lucyna Gornicka, Federico Grinberg, and Marcello Miccoli), we study how the introduction of Central Bank Digital Currency (CBDC) might disintermediate the banking sector. Using a simple portfolio choice model we find that CBDC reduces bank credit only in special cases and when it does, the effect is quantitatively small. In the model, households allocate their wealth between an illiquid asset and three liquid assets: cash, bank deposits and CBDC. An imperfectly competitive banking sector provides deposits and lending. When all liquid assets are costless to access, the introduction of CBDC does not lead to bank disintermediation, as banks increase the return on deposits to fight off the competition from CBDC. However, if the access to deposits and CBDC is costly, the introduction of the latter may lead to bank disintermediation under specific conditions. The conditions are that CBDC is much cheaper to access than bank deposits and that the wealth distribution is very unequal. Under these conditions, poorer households will stop holding deposits in favor of CBDC, but banks will not aggressively fight the outflow of customers due to their relatively small wealth. Still, the impact on lending turns out quantitatively small if banks have access to other forms of funding.

The dissertation of Huifeng Chang is approved.

Lee Ohanian

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Pierre-Olivier Weill, Committee Chair

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2022

To my parents

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CHAPTER 1

A Macroeconomic Model with Bond Market Liquidity

1.1 Introduction

Liquidity in U.S. corporate bond market dried up during the financial crisis of 2007-2009. Two measures of illiquidity for corporate bonds, the bid-ask spread and bond-CDS spread[1], surged dramatically. In mid-March of 2020, the bid-ask spread and bond-CDS spread also surged, but the surge disappeared almost immediately after the March 23 announcement of the Fed to purchase corporate bonds directly. How do disruptions in market liquidity of bonds interact with the macroeconomy? Do policies improving bond market liquidity have effects on the real economy?

Existing macro models are not suitable for answering these questions. In most dynamic stochastic general equilibrium (DSGE) models, bond prices depend on credit risk only and yield spread spikes in recessions are attributed to increases in default rates and credit risk premia. Meanwhile, a long-standing literature in finance argued that both illiquidity and credit risk contribute to bond prices, but much of this literature does not precisely investigate the real effects of bond illiquidity.

This paper incorporates secondary bond markets as search-based over-the-counter (OTC) markets à la Duffie, Gârleanu, and Pedersen (2005) into a quantitative DSGE model. Firms

[1]Following Longstaff et al. (2005), the difference between bond yield spreads and credit-default swap rates (bond-CDS spread) is often used as a measure of illiquidity, see Chen et al. (2017) and Goldberg and Nozawa (2021) for example. Section 1.2.1 provides detailed discussion on this measure.

are financed by long-term bonds.[2] Investors who buy the bonds in the primary market might have liquidity needs and want to liquidate their holdings before the bonds mature. They can sell the bond in secondary markets, which is an OTC market featuring search and bargaining frictions. Exogenous liquidity needs, combined with the risk of delayed trade, give rise to the liquidity premium in the yield spread of firms' bonds. When investors are desperate for liquidity, the prices of outstanding bonds and new issuance will be negatively affected. This will impact firms' borrowing constraints and distort their investment and hiring decisions.

I provide empirical evidence supporting the quantitative importance of the liquidity channel. I first show that the liquidity premium in corporate bond spreads surged during the financial crisis of 2007-2009. Comparing the 2001 recession to this period, the default premium in bond spreads observed a similar increase, but the liquidity premium experienced little change in the 2001 recession. Then I establish that the liquidity shock in the financial crisis had large and significant effects on firms' real activities through refinancing risk. In particular, I exploit the ex-ante variation in firms' long-term debt maturity structures at the end of 2007 to infer the firms' refinancing needs in 2008 and 2009.[3] I find that firms with large refinancing needs in the current year experienced a drop in employment growth rate in 2008 that is 5.2-percentage-point larger than otherwise similar firms, and the difference is significant. Firms with large refinancing needs in the next year experienced a 6.4-percentage-point larger drop in employment growth rate in 2008 than otherwise similar firms. Moreover, I perform placebo tests using non-crisis periods and find that such refinancing effects don't hold for any placebo periods, including the 2001 recession. In sum, if a firm has refinancing needs or expect to have refinancing needs in the near future, it will cut back more on real

[2] Corporate bonds are an important source of financing for US firms. According to Flow of Funds, the U.S. nonfinancial corporate sector had outstanding corporate bonds of \$6.5 trillion at the end of 2020; in comparison, the outstanding bank loans and equity were \$3.8 and \$39.7 trillion.

[3] Almeida et al. (2011) use the same methodology to identify heteorgeneity in financial contracting at the onset of the 2007 crisis. They assess the effects on investment while I examine the effects on employment and output. Also, I study the effects of not only the current refinancing needs but also the expected refinancing needs in the future.

activities following a liquidity shock.

I build the macro model based on Arellano et al. (2019) (ABK henthforce). This is a model in which credit frictions matter for macro aggregates through the combination of borrowing constraints and incomplete insurance. The key idea is that firms borrow to hire their labor before they learn about their productivity. If productivity turns out to be low they cannot pay back all maturing liabilities given the borrowing limit. When this happens, they have to default and lose the firm's future value. This risk is uninsurable and has real consequences, which induces firms to restrict hiring ex-ante in order to reduce the risk of default. Bond market liquidity, by affecting bond prices and thus the borrowing limits for firms, will affect firms' labor choices.

The model also has an endogenous amplification mechanism through a positive liquidity-default spiral similar to that in He and Milbradt (2014) and Chen et al. (2017). The post-default bonds are assumed less liquid than pre-default bonds. As a firm gets closer to default, investors will put a larger weight on the default scenario so that the liquidity premium increases. This lowers bond prices and thus borrowing limits, and pushes firms even closer to default.

Since the model is highly nonlinear and features default and occasionally binding borrowing constraints, I solve the model using projection-based numerical methods. I then simulate a liquidity shock calibrated to match the observed increase in bid-ask spread during the financial crisis of 2007-2009 and explain about 20% of the employment losses in data.The endogenous amplification mechanism through the liquidity-default spiral contributes about one-fourth of the effects. There are also other general equilibrium (GE) effects: wages and aggregate demand fall after a liquidity shock. Quantitatively, the wage effect dominates the demand effect and the GE effects dampen the fall in employment by almost 40%. Certain key parameters are particularly important for the quantitative results. With higher persistence of the liqudity shock, shorter bond maturity (i.e. more frequent refinancing needs), and larger trading frictions, the model predicts larger responses to the liquidity shock.

The contribution of this paper is threefold. First, from a modeling perspective, I develop a framework to study the interactions between bond market liquidity and the macroeconomy, using mostly standard assumptions for OTC bond markets and introducing them into a macro model. Second, with this framework, I study a new source of aggregate disturbances that hasn't been studied precisely in existing macro-financial crisis models, that is, fluctuations in the market liquidity of corporate bonds. I show that disruptions in bond market liquidity can have quantitatively important impacts on macro aggregates.

Third, this paper also provides a structural estimate of the impacts of Fed's corporate bond purchasing program (PMCCF/SMCCF) on the real economy during the COVID-19 crisis. During the recent COVID-19 crisis, the bid-ask spread of corporate bond surged dramatically, but different from what happened in the financial crisis of 2007-2009, the surge disappeared almost immediately this time, as can be seen from panel (a) of Figure 1.1. Panel (b) of Figure 1.1 shows that the drop in the bid-ask spread coincided with the Fed's announcement on March 23 to purchase corporate bonds. To quantify the effects of the Fed's interventions, I consider a liquidity shock that generates the peak value of bid-ask spread in mid-March of 2020, it would have generated a 2 percentage point drop in employment and output, as predicted by the calibrated version of the model. In other words, the Fed's interventions in corporate bond market avoided a 2 percentage point drop in employment and output during the COVID-19 crisis.

Related Literature

This paper relates to three strands of research. First, this paper belongs to the large literature that studies the effect of financial disturbances in DSGE models (e.g. Bernanke et al., 1999; Christiano et al., 2014; Gertler and Karadi, 2011). In particular, asset liquidity and liquidity (financial) shocks have been studied extensively in this line of research (Jermann and Quadrini, 2012; Kiyotaki and Moore, 2019; Del Negro et al., 2017). Kurlat (2013) and Bigio (2015) endogeneize the asset liquidity through adverse selection [4] and Cui and Radde

[4]Eisfeldt (2004) and Guerrieri and Shimer (2014) are also examples on this, but they don't focus on the

(2020) endogeneize asset liquidity through costly search. Unlike these studies, I emphasize the market liquidity of the long-term debts issued by firms, which arise from the liquidity need of bond investors coupled with the risk of delayed trade due to search frictions. The model provides clear link between parameters driving illiquidity at a micro level in OTC model with the macro aggregates. The liquidity shock in my model is calibrated to target standard measure of bond illiquidity (bid-ask spreads), and the results have implications on policies that improve bond market liquidity.

In particular, this paper contributes to a growing body of DSGE models with heterogeneity, like Khan and Thomas (2013), Gilchrist et al. (2014), Gomes and Schmid (2021), and ABK, to name a few. The macro model in this paper is built on ABK. While they study how volatility shocks can lead to quantitatively sizable contractions in labor and output, as well as an increase in firms' interest rate spread, my paper is intended to quantify how much of the decline in labor and output could be explain by fluctuations in market liquidity of long-term bonds.

Second, this paper is related to the vast literature on OTC market frictions and bond liquidity premium. The setup for OTC markets in the paper is based on the seminal work of Duffie et al. (2005) and thus can speak to many research on search-based OTC models.[5] Bond markets affect firms' borrowing constraint and thus a natural idea is to relate the OTC asset pricing model to corporate financing choices (He and Milbradt, 2014, Chen et al., 2017, Bruche and Segura, 2017, Bethune et al., 2019, Kozlowski, Forthcoming). Kozlowski (Forthcoming) shows that trading frictions in the secondary market can affect firms' maturity choices and their choices over investment projects with different horizons. In particular, the endogenous amplification mechanism in my model works through a positive liquidity-default spiral as in He and Milbradt (2014) and Chen et al. (2017). They study how the interactions between liquidity and default affect corporate bond pricing and firms' issuance cost, and

effects of liquidity on production.

[5]See Weill (2020) for a review of this literature.

leave it as an open question the general equilibrium implications of bond market liquidity. My paper provides an answer to this question, showing that liquidity in corporate bond market has important real macroeconomic impacts. A recent paper by Chaumont (2020) also embeds search-based secondary markets into a general equilibrium model, but he uses a model of sovereign default and studies the implications for government bonds.

Finally, this paper is related to a number of recent papers evaluating the Fed's corporate bond purchasing program during the Covid-19 crisis. A growing literature emerged after the Fed's unprecedented interventions in corporate bond markets during the pandemic (Kargar et al., 2020; Gilchrist et al., 2020; Haddad et al., 2020, O'Hara and Zhou, 2021). These studies show that the Fed's intervention greatly improved market liquidity and significantly reduced credit spreads. While these are mostly empirical studies focusing on asset prices and transaction costs, my model provides a structural estimate of its impact on macro aggregates.

The rest of the paper is organized as follows. Section 1.2 presents the empirical evidence. Section 1.3 describes the model, while Section 1.4 calibrates the model and conducts quantitative analysis. Section 2.5 concludes the paper. Data descriptions, computational algorithm, and additional results are gathered in the Appendices.

1.2 Empirical Evidence

This section provides empirical evidence to support the mechanisms studied in this paper. Subsection 1.2.1 shows that illiquidity is a key driver of corporate bond yield spreads, and played an important role during the financial crisis of 2007-2009. Subsection 1.2.2 provides suggestive evidence that the liquidity shock had large and significant impacts on firms' real decisions through the refinancing risk.

1.2.1 Liquidity premium in corporate bond spreads

It is well known that both liquidity and credit risk are important determinants of asset prices. Collin-Dufresn et al. (2001) first documented a large common variation of changes in the yield spread of corporate bonds over Treasury bonds that cannot be explained by conventional determinants suggested structural models of credit risk. Huang and Huang (2012) indicate that levels of the yield spread also cannot be fully explained by these credit risk determinants. Market liquidity due to search frictions à la Duffie et al. (2005) has been emphasized to be an explanation for these "puzzles". Empirical papers like Edwards et al. (2007) document that the secondary markets for corporate bonds are highly illiquid and the illiquidity is reflected by a large bid-ask spread. A number of papers show that liquidity is an important price factor in the US corporate bond market (Chen et al., 2007; Bao et al., 2011; Friewald and Nagler, 2019; He et al., 2019)[6], and its importance is more pronounced in periods of financial crises (Dick-Nielsen et al., 2012).

Longstaff et al. (2005) first used information from credit-default swaps (CDS) to decompose corporate spreads into a default and a non-default component. They argue that the CDS spread should mainly reflect the default risk of a bond, and the gap between the CDS spread and the credit spread represents a non-default component. They show that the non-default component is time varying and strongly related to measures of illiquidity. Following Longstaff et al. (2005), the difference between the bond credit spread and the corresponding CDS spread (bond-CDS spread) is often use as a measure of liquidity premium in credit spread[7], that is, the CDS spread captures a default component in bond spread, while the bond-CDS spread reflects a liquidity component in bond spread.

I plot time series of credit spreads and CDS spreads in panel (b) of Figure 1.2. The

[6] Huang et al. (2019) document evidence that liquidity is also an important pricing factor in corporate bond markets outside the US. They empirically test and support the He and Milbradt (2014) corporate bond pricing model.

[7] See for example Chen et al. (2017) and Goldberg and Nozawa (2021).

construction of the time series are as follows. I use month-end option-adjusted spreads from ICE database as the credit spreads. I limit the sample to only senior unsecured bonds with a fixed coupon schedule. Only bonds with maturity between 4 and 6 year and a credit rating of BBB are kept. The CDS spreads are from Markit dataset. I use month-end 5-year CDS spreads on the quoted modified restructuring clause, which is the most commonly traded type. Then I match the credit spread data and the CDS spread data to the merged CRSP/Compustat dataset using a set of identifiers detailed in Appendix 1.8. For a firm with multiple observations on credit spread, I keep the one with maturity closest to 5 years. For a firm with multiple observations on CDS spread, I use the average of these observations.[8] The time series are constructed by averaging across all firms in the merged sample at each date.

As can be seen from Figure 1.2, the bond-CDS spread which represents the liquidity premium accounts for a significant portion of the credit spread and is time-varying. The liquidity premium of corporate bonds increased sharply during the financial crisis of 2007-2009. In particular, if we compare the 07-09 financial crisis with the 2001 recession, increases in the default premium which is represented by the CDS spread are similar, while the increase in the liquidity premium is much larger during the financial crisis.

1.2.2 Refinancing risk and firm outcomes

In this section, I provide empirical evidence that refinancing risk has large and significant effects on firms' real activities. In particular, I show that in the year of 2008, (i) firms whose long-term debt was largely maturing in the *current* year experienced larger drops in employment and output growth than other firms; (ii) firms whose long-term debt was expected to be largely maturing in the *next* year experienced larger drops in employment and output growth than other firms. Moreover, such refinancing effects don't hold for other

[8] More than 90% of the firms in the sample have unique CDS spreads after applying the data filters previous described.

periods, including the 2001 recession.

I exploit the ex-ante variation in firms' long-term debt maturity structures at the end of 2007, which is plausibly exogenous. I use data from COMPUSTAT's North America Fundamentals Annual, exclude financial and public service firms, and apply standard data filters to remove outliers (see Appendix 1.8). In this dataset, the items *dd1* and *dd2* represent the amount of long-term debt maturing during the first and the second year after the report, respectively . The data item *dltt* represents the amount of long-term debt that matures in more than one-year. Therefore, a firm's total long-term debt can be calculated as *dd1+dltt*. I focus on firms that have long-term debt maturing beyond one year (*dltt*) represents at least 5% of total assets (*at*), indicating that long-term debt is an important source of financing for the firm. From the financial report of fiscal year 2007[9], the firms for which the ratio of *dd1* (long-term debt maturing in 2008) to *dd1+dltt* (total long-term debt) is larger than 20% is assigned to the treatment group.

The outcome variables are the annual growth rate of firms' employment (*emp*) and output (*sale*). In Panel A of Table 1.1, we see that the employment growth rate for treated firms is 3.32% in 2007, and dropped to −5.34% in 2008; while the rate for non-treated firms is 2.28% in 2007 and dropped to −0.64% in 2008, so the drop in employment growth in 2008 is 8.66 percentage points for treated firms, which is larger than that for the non-treated firms by 5.2 percentage points. The difference is significant at the 5% level. Also, the employment growth rates for treated and non-treat firms in 2007 are not significantly different, which is a hint that these two groups of firms are not different prior to the shock. In Panel B of Table 1.1, I provide results for a placebo test using the year prior to the crisis. Just as before I classify the firms according to their debt maturity profiles at the end of 2006 and compare their performance in 2007, and find no significant differences between treated and non-treated firms. In Panel C, I show the results for the 2001 recession, and find that the

[9]I use firms that have fiscal year-end months in September, October, November, December, January and Februry, which covers more than 80% of the universe of firms in fiscal year 2007.

refinancing effects don't hold in this period as well. In addition to the placebo tests for the year of 2007 and 2001, I perform a complete set of placebo tests for the years 2002-2006 in Appendix 1.10, and confirm that there is no such refinancing effect in all these placebo periods.

I perform similar analysis on firms' output growth and find similar results. I relegate the details to Appendix 1.10. In Almeida et al. (2011), they use a differences-in-differences matching estimator, and find similar effects of refinancing needs on firms' investment rate during the financial crisis of 2007-2009. So far we have established that if a firm has to largely refinance its debt in the crisis period, it would experience larger drops in employment, investment, and output. Next, I will show that during the crisis period, if a firm expect to have large refinancing needs in the near future, it will also cut back more on employment and output.

In this experiment, I include only firms that don't have long-term debt largely maturing in the current period, that is, the non-treated firms in Table 1.1. Among these firms, the firms for which the ratio of *dd2* (long-term debt maturing in 2009) to *dd1+dltt* (total long-term debt) is greater than 20% in the year-end report for 2007 is assigned to the treatment group. Notice that I didn't use the ratio of *dd1* to *dd1+dltt* from the financial report of 2008 to measure the refinancing need in 2009. This is to make sure that the measured variation in maturity is exogenous to the financial crisis.

Panel A of Table 1.2 summarizes the employment growth rates of these treated firms and non-treated firms in 2007 and 2008. The employment growth rate of treated firms dropped by 9.33 percentage points from 2007 to 2008, while the growth rate of non-treated firms dropped by 2.97 percentage points. In 2008, if a firm expected itself to have large refinancing needs next period, its employment growth would decrease by 6.36 percentage points more than firms that don't have large refinancing needs in the next period. Similarly, Panel B and Panel C show results for placebo periods. In Panel C, notice that both groups of firms experienced significant and large drop in employment growth during the 2001 recession, yet there is no significant difference between these two groups of firms. [10]

In subsection 1.2.1, I establish that the liquidity premium in corporate bond spreads surged dramatically during the financial crisis of 2007-2009. In this subsection, I show that the shock in this crisis period has large and significant effects on firms' real activities through firms' refinancing risk. These facts support the main channel in my model: an aggregate liquidity shock increases the liquidity premium of firms' long-term debt, so firms will get less proceeds from bond issuance when they refinance their debt, and thus they cut their employment in order to reduce risk.

1.3 Model

To study the interactions between bond market liquidity and the aggregate economy, I introduce a search-based secondary corporate bond market à la Duffie et al. (2005) into a dynamic general equilibrium model based on ABK. Specially, I consider a discrete-time model with an infinite time horizon. The model consists of four main types of agents: final goods firms, intermediate goods firms, households and bond investors.

The final goods firms are competitive and have a technology that converts intermediate goods into a final good. There are idiosyncratic shocks to the productivity of intermediate goods in producing the final good, which affect the demand of final good firms for these intermediate goods. The intermediate goods firms are monopolistically competitive and use labor to produce differentiated intermediate goods. They can issue long-term corporate bond and are allowed to default on their debt and wage payments. These corporate bonds are bought by bond investors that face idiosyncratic distressed shocks. Once hit by the shock, a distressed investor need to search for a dealer to trade in a secondary bond market featuring OTC frictions, otherwise she incurs a periodic holding cost. The holding cost for investors in the secondary market is stochastically time-varying, which is the *only* aggregate shock in this

[10] See Appendix 1.10 for a complete set of placebo tests for the years 2002-2006.

model. Households can insure themselves against the aggregate shock using state-contingent assets and there is perfect risk sharing among its members. Households own all firms and supply labor to intermediate goods firms.

Figure 1.3 summarizes the model timeline. At the beginning of each period, intermediate goods firms enter the period with labor and debt. Bond investors enter with bond holding, and distressed investors incur the periodic holding cost. Then, all shocks are realized, and production take place. Then, intermediate goods firms choose whether to default. If it defaults, the firm is sold in a competitive market. Bond investors receive the recovered value and equity holders walk away with zero value. If it doesn't default, the firm pays its debt and wage payment as well as dividends. They hire labor and issue bond for next period. The newly issued bonds are bought by bond investors in the primary market. Households receive wage and payments on their assets. They consume, invest in stage-contingent assets for next period, and buy the defaulting firms. Then, the investors receive the distress shock, and after that the secondary markets for bonds are open.

1.3.1 Intermediate goods firms

As is standard, I assume that each of the intermediate good enters as a separate variety in a constant elasticity of substitution (CES) final good production function with elasticity γ. This implies that the demand faced by an intermeidate good firm is

$$y_t = \left(\frac{z_t}{p_t}\right)^{\gamma} Y_t, \tag{1.1}$$

where z_t is the idiosyncratic shock to this intermediate good in producing the final good Y_t, and p_t is the price set by the intermediate good firm. Notice z_t here need not be technological; they can also be interpreted as demand shock. The idiosyncratic shock z_t follows a Markov process with transition function $\pi_z(z_t|z_{t-1})$.

The intermediate goods firm produces intermediate good y_t using labor as the only input via the technology $y_t = l_t^{\alpha} l_{mt}(i)^{\omega}$, where l_t is the input of workers and $l_{mt} = 1$ is the input of a single manager. The labor share $0 < \alpha < 1$. After the productivity shock z_t is realized, the firm chooses the price p_t . Given the demand function in equation (1.1), the firm sets $p_t = z_t \left(Y_t / l_t^{\alpha}\right)^{1/\gamma}$, so the price can be eliminated as a choice variable. Also, from now on I will refer to intermediate goods firms as firms.

Firms can issue defaultable long-term debt b_t. The debt takes the form of a consol bond that pays coupon c in every period until random expiration. The process governing the firm's refinancing need is captured by an i.i.d. random variable ζ_t.[11] If $\zeta_t = 1$, the firm receives a refinancing shock; all its existing debt must be retired and the firm chooses how much new debt b_{t+1} to issue. If $\zeta_t = 0$, the firm cannot change its debt level. I assume that ζ_t takes the value 1 with probability λ and 0 with probability $1 - \lambda$, so that the expected debt maturity is $\frac{1}{\lambda}$.

Firms are also subject to idiosyncratic revenue shocks ϵ_t that have a distribution $F(\epsilon)$ and are independent across firms and time. Firms enter period t with l_t and b_t, then all idiosyncratic shocks $\{z_t, \epsilon_t, \zeta_t\}$ and the aggregate shock s_t are realized. Production takes place, firms sell the intermediate goods to final goods firms and receive the revenue. Then firms choose whether to default, decide l_{t+1} and b_{t+1} for next period, and pay dividends. The dividend payout to equity holders d_t is the revenue net of wage and coupon payments plus gains from debt rollover:

$$d_t = z_t Y_t^{\frac{1}{\gamma}} l_t^{\frac{\alpha(\gamma-1)}{\gamma}} - \epsilon_t - w_t l_t - w_{mt} - c b_t + \zeta_t \left(q_t b_{t+1} - b_t\right), \tag{1.2}$$

where w_t is wage for workers, $\bar{w}_m$ is wage for managers, and q_t is the price of newly issued bond. The equity payout d_t is restricted to be non-negative. Therefore the firm value is always non-negative, and the equity holder will choose to default if and only if there is no

[11] In practice, debt has a deterministic maturity. This is a convenient assumption so as to avoid the leverage "ratchet" effect as discussed in DeMarzo and He (2021) when the firm can continuously adjust leverage and cannot commit to a policy ex ante. This assumption is also used in Gomes and Schmid (2021).

feasible choice for them to make non-negative equity payout. Define the maximal borrowing M_t, which is the largest proceeds the firm could get from new bond issuance:

$$M_t(S_t, z_t) = \max_{l_{t+1}, b_{t+1}} q(S_t, z_t, l_{t+1}, b_{t+1}) b_{t+1}, \tag{1.3}$$

where $S_t = \{s_t, \Upsilon_t\}$ denote the aggregate state where Υ_t is the distributions of firms. The bond price $q_t = q(S_t, z_t, l_{t+1}, b_{t+1})$, which will be described in section 1.3.2.

Then there exist a cutoff value of the revenue shock $\hat{\epsilon}_t$ such that the firm will default if and only the revenue shock ϵ_t exceeds this level. This cutoff satisfies

$$\hat{\epsilon}_t = z_t Y_t^{\frac{1}{\gamma}} l_t^{\frac{\alpha(\gamma-1)}{\gamma}} - w_t l_t - \bar{w}_m - c b_t + \zeta_t \left(M_t - b_t \right) \tag{1.4}$$

It is convenient for the discussion to define the cash-on-hand x_t as follows:

$$x_t = z_t Y_t^{\frac{1}{\gamma}} l_t^{\frac{\alpha(\gamma-1)}{\gamma}} - w_t l_t - \bar{w}_m - c b_t - \epsilon_t - \zeta_t b_t. \tag{1.5}$$

Then the value function of a firm with state variable $\{S_t, z_t, x_t, b_t, \zeta_t\}$ such that $x_t + \zeta_t M_t(S_t, z_t) \geq 0$ can be expressed as

$$V(S_t, z_t, x_t, b_t, \zeta_t) = \max_{l_{t+1}, b_{t+1}, d_t} d_t + \beta E_t \left[\iint^{\hat{\epsilon}_{t+1}} V(S_{t+1}, z_{t+1}, x_{t+1}, b_{t+1}, \zeta_{t+1}) dF(\epsilon_{t+1}) d\psi(\zeta_{t+1}) \right] \tag{1.6}$$

subject to the non-negative equity payout condition

$$d_t = x_t + \zeta_t q(S_t, z_t, l_{t+1}, b_{t+1}) b_{t+1} \geq 0 \tag{1.7}$$

The expectation in equation (1.6) is taken over $\{S_{t+1}, z_{t+1}\}$ conditional on current $\{S_t, z_t\}$ and β is the discount rate for equity holders. I will discuss this in detail in households' problem in Section 1.3.3. The firm's value is

$$V(S_t, z_t, x_t, b_t, \zeta_t) = 0, \tag{1.8}$$

if $x_t + \zeta_t M_t(S_t, z_t) < 0$.

When a firm defaults, its equity holders walks away with zero value. The debt holders seize the firm and collect the operating income $z_t Y_t^{\frac{1}{\gamma}} l_t^{\frac{\alpha(\gamma-1)}{\gamma}} - \epsilon_t - w_t l_t - w_{mt}$. Then they sell the unlevered firm in a competitive market. This firm will start to hire labor and issue bond in next period. Its productivity in next period is drawn conditional on its productivity level when it defaults. Therefore, the price of the defaulting firm in a competitive market is equal to the expectation value of a firm with state variable $z = z_t$, cash-on-hand $x = 0$, outstanding debt $b = 0$, and refinancing shock $\zeta = 1$ in next period. That is,

$$v^b(S_t, z_t) = \beta E_{S_{t+1}|S_t} V(S_{t+1}, z_t, 0, 0, 1) \tag{1.9}$$

However, $1 - \kappa$ of this value is lost in the bankruptcy proceedings so that the firm recovery rate is κ. Therefore, the recovered rate for debt holders ρ is[12]

$$\rho(S_t, z_t, l_t, b_t, \epsilon_t) = \frac{z_t Y_t^{\frac{1}{\gamma}} l_t^{\frac{\alpha(\gamma-1)}{\gamma}} - \epsilon_t - w_t l_t - \bar{w}_m + \kappa v^b(S_t, z_t)}{b_t}. \tag{1.10}$$

The firm's problem gives the policy rules $l(S, z, x, b, \zeta)$, $b(S, z, x, b, \zeta)$ and $d(S, z, x, b, \zeta)$, as well as the recovery rate $\rho(S, z, l, b, \epsilon)$.

[12] Notice that it is possible for ρ to be greater than 1 in the current model setup. To avoid such unrealistic scenario, I assume that the firm will not default even if $x_t + \zeta_t M_t(S_t, z_t) < 0$ as long as $\rho(S_t, z_t, l_t, b_t, \epsilon_t) > 1$, which requires a small adjustment to the cutoff in equation (1.4). This assumption implies that equity holders will absorb the loss in such scenario, which can be thought of as a seasoned equity offering. This assumption is strongly supported by empirical evidence, which indicates that a near-term cash need is the primary motive for seasoned equity offering (DeAngelo et al., 2010). In the baseline quantitative model, this almost never occurs, that is, $\rho < 1$ when ϵ is below the cutoff defined in equation (1.4).

1.3.2 Bond investors

A large number of risk-neutral international investors with discount rate $\hat{\beta}$ invest in corporate bonds issued by firms. The discount rate of bond investors $\hat{\beta}$ is assumed to be larger than that of equity holders β. This is to justify the use of debt. The existence of financial frictions in the model makes external finance more costly than internal finance. Absent this assumption, firms would have built up savings and avoided using debt.[13]

Following Duffie et al. (2005), each investor can hold $a_t \in \{0, 1\}$ unit of bond. As assumed in this literature, there are two types of investors, high and low valuation investors. I assume an infinite mass of high type investors on the sideline can enter the bond market freely. At the end of each period, a high type investor receives a distress shock with probability π and becomes a low type investor. In each period, a low type investor with debt holding incur a holding cost h (h_b if this is a defaulted bond). The individual distress shock is uninsurable and results type dependent valuations, which creates gain from trade between high and low type investors in the secondary bond market featuring search frictions. For simplicity, I assume that low type investors exit the market forever after they get rid of their debt holdings.

In each period, an investor can meet and trade with a dealer with probability ξ in the secondary market. When they meet, the terms of trade are set accroding to the generalized Narsh bargaining solution. The bargaining weight is η for the investor and $1-\eta$ for the dealer, across all dealer-investor pairs. Assume that there is a competitive inter-dealer market. Dealers cannot hold any inventory and are just pass-through intermediaries.

The holding cost for defaulted bonds is assumed to be larger, that is, $h_b > h$. The liquidity premium is thus dependent on firms' fundamentals, which generates a positive feedback loop between default and liquidity similar to He and Milbradt (2014). As a firm gets closer to

[13] In the literature, there have been various ways to deal with this issue, such as finite lifetimes and tax advantages of debt. ABK uses the Jensen effect (Jensen, 1986) to modify the effective discount rate of entrepreneurs.

default, investors put a higher weight on the default scenario so that the liquidity premium of its bond increases, which pushes the firm even closer to default.

I assume that there is a delay in the payout of the recovery value in case of default, otherwise bankruptcy will be actually providing liquidity for low type investors. In practice bankruptcy leads to a freezing of firm assets and length court proceedings, and bond holders often receive payouts after years. This delay in the bankruptcy payout is captured by the parameter θ, which is the probability that the recovery value will be paid in each period. On average, an investor will receive the recovery value $\frac{1}{\theta}$ periods after the default occurs.

Consider the problem of investor with one share of non-defaulting bond. Since firms are heterogeneous, the valuation of their bonds are also heterogeneous. In each period after the distress shock realizes, when high and low type investors trade in the secondary market, their valuation of a bond depends on aggregate state S, and the current productivity level z and choices for next period $\{l', b'\}$ of the issuing firm. Denote the valuation function of a high type investor by $V_H = V_H(S, z, l', b')$ and that of a low investor by $V_L = V_L(S, z, l', b')$, the value function of a high type investor can be expressed recursively as follows,

$$V_H = \hat{\beta} E_t \left\{ \begin{array}{l} \lambda F(\bar{\epsilon}_1')(c+1) \\ +(1-\lambda)F(\bar{\epsilon}_0')\left[c + (1-\pi)\, V_H' + \pi\left(\xi \tilde{q}' + (1-\xi) V_L'\right)\right] \\ + \int \left[\int_{\bar{\epsilon}'} (1-\pi) V_H^b(\rho, s') + \pi(\xi \tilde{q}_b' + (1-\xi) V_L^b(\rho, s')) dF(\epsilon')\right] d\psi(\zeta') \end{array} \right\}. \quad (1.11)$$

The expectation is taken over S', z' conditional on S, z. In the next period, with probability λ, the bond matures and the investor receives coupon c and principal 1 if the firm doesn't default, which is summarized in the term in the first line in equation (1.11). The default cutoff $\bar{\epsilon}'$ is given by equation (1.4) and depends on ζ'. For ease of notation, I use $\bar{\epsilon}_1'$ and $\bar{\epsilon}_0'$ to denote the default cutoff when the firm receives the refinancing shock (the bond matures) and when it doesn't (the bond doesn't mature).

The term in the second line corresponds to the scenario when the bond doesn't mature and doesn't default. With probability $1-\lambda$, the bond doesn't mature. If the firm doesn't default, the investor receives coupon c and continue to hold the bond. With probability $1-\pi$, the investor doesn't receive the distress shock and its valuation of the bond in next period is denoted by V_H' which implicitly takes into consideration the firm's policy function. With probability π, the investor receives the distress shock. The investor will be able to trade in the secondary market and sell it with probability ξ at price $\tilde{q}$, otherwise the investor will stay in the market as a low type investor and has valuation V_L'.

The last line in equation (1.11) reflects the value of the bond when the first default. Depending on whether the firm receives the refinancing shock ($\zeta' = 0, 1$), the firm's default cutoff $\bar{\epsilon}'$ is different. Depending on the revenue shock ϵ' the firm receives next period, the recovered value ρ is given by equation (1.10). $V_H^b(\rho, s)$ and $V_L^b(\rho, s)$ denote the valuation of the bond if it defaults next period for a high type investor and a low type investor, respectively. The valuation of defaulted bonds will be discussed in detail later and the expressions of $V_H^b(\rho, s)$ and $V_L^b(\rho, s)$ are given by equation (1.14) and (1.15). Similar to the case in which the bond doesn't default (the second line), with probability $1-\pi$, the investor doesn't receive the distress shock and remains as a high type, and with probability π, the investor receives the distress shock. The investor can sell the defaulted bond at price $\tilde{q}_b'$ with probability ξ and stays in the market as a low type otherwise.

The value function of a low type investor $V_L(S, z, l', b')$ is given by

$$V_L = \hat{\beta} E_t \left\{ \begin{array}{l} -h \\ +\lambda F(\bar{\epsilon}_1')(c+1) \\ +(1-\lambda) F(\bar{\epsilon}_0') \left[c + (\xi \tilde{q}' + (1-\xi) V_L')\right] \\ +\int \left[\int_{\bar{\epsilon}'} \xi \tilde{q}_b' + (1-\xi) V_L^b(\rho, s') dF(\epsilon')\right] d\psi(\zeta') \end{array} \right\}. \tag{1.12}$$

Comparing equation (1.12) to equation (1.11), the low type receives an additional periodic holding cost h and the payoff structure is very similar otherwise. The investor receives coupon c and principal 1 if the bond matures and doesn't default (the second line). If the bond doesn't mature and doesn't default, the investor receives coupon c, sells the bond at price $\tilde{q}$ with probability ξ and stays on the market otherwise (the third line). If the bond defaults, the low type investor can sell the bond at price $\tilde{q}_b'$ with probability ξ and stays in the market as a low type with valuation $V_L^b(\rho, s')$ otherwise.

Since dealers have a frictionless competitive market among themselves, a dealer who buy from the low type investor can immediately sell this bond to a dealer who trade with a high type investor at the inter-dealer market price. With free entry of high type, I can assume that the flow of high type buyers contacting dealers is greater than the flow of low type sellers contacting dealers, which generate excess demand from the buy side and drives the inter-dealer price to be equal to the valuation of high type investors. When the low type investor trade with the dealer, they split the surplus with Nash-bargaining weights η for the investor and $1 - \eta$ for the dealer. Therefore, the selling price $\tilde{q}$ is

$$\tilde{q}' = V_L' + \eta(V_H' - V_L') \tag{1.13}$$

Next consider the problem of investors with one share of defaulted bond. In each period after the distress shock realizes, the high and low type investors with defaulted bonds can trade in the secondary market. The valuation of a defaulted bond only depends on the recovered value ρ and the aggregate bond market liquidity state s. The value function for a high type investor $V_H^b(\rho, s)$ can be written as follows,

$$V_H^b(\rho, s) = \hat{\beta} E_{s'|s} \left\{ \theta\rho + (1-\theta) \left[(1-\pi) V_H^b(\rho, s') + \pi \left(\xi \tilde{q}_b' + (1-\xi) V_L^b(\rho, s') \right) \right] \right\}. \tag{1.14}$$

The investor will receive the recovered value ρ with probability θ in next period, and with probability $1 - \theta$, the bankruptcy is still processing and the investor continues to hold the bond. With probability $1 - \pi$, the investor doesn't receive the distress shock and remains the high type so that its valuation in next period is $V_H^b(\rho, s')$. With probability π, the investor receives the distress shock and becomes a low type. Then she can sell of the defaulted bond at price $\tilde{q}_b'$ in the secondary market with probability ξ, and with probability $1-\xi$ she stays in the market as a low type investor so that the valuation is $V_H^b(\rho, s')$. The selling price $\tilde{q}_b'$ is given by equation (1.16) and will be discussed later.

The valuation function for a low type investor $V_L^b(\rho, s)$ is given by

$$V_L^b(\rho, s) = \hat{\beta} E_{s'|s} \left\{ -h_b + \theta\rho + (1-\theta)\left[\xi \tilde{q}_b' + (1-\xi) V_L^b(\rho, s')\right] \right\}. \tag{1.15}$$

The low type investor will incur a holding cost every period. For defaulted bonds, the holding cost is h_b as introduced above. Similar to equation (1.14), the investor receives the recovered value ρ with probability θ in each period and continues to hold the bond otherwise. She can sell off the bond at price $\tilde{q}_b'$ with probability ξ, or remain a low type with valuation $V_L^b(\rho, s')$ with probability $1-\xi$.

Similar to equation (1.13), the selling price in the secondary market for defaulted bonds is given by

$$\tilde{q}_b' = V_L^{b\prime} + \eta(V_H^{b\prime} - V_L^{b\prime}). \tag{1.16}$$

Define the value wedge between high and low type investor for defaulted and non-defaulting bonds $\Delta V^b = V_H^b - V_L^b$ and $\Delta V = V_H - V_L$. Their expressions are given by the following equations:

$$\Delta V = \hat{\beta} E_t \left\{ \begin{array}{l} h + (1-\lambda)(1-\pi)(1-\xi\eta)\left[F(\bar{\epsilon}_0')\Delta V'\right] \\ +(1-\pi)(1-\xi\eta)\left[1 - \lambda F(\bar{\epsilon}_1') - (1-\lambda)F(\bar{\epsilon}_0')\right]\Delta V^b(s') \end{array} \right\}, \tag{1.17}$$

and

$$\Delta V^b(s) = \hat{\beta}\left\{h_b + (1-\theta)(1-\pi)(1-\xi\eta)E_{s'|s}[\Delta V^b(s')]\right\} \tag{1.18}$$

.

Then I can write the bond price function $q(S, z, l', b')$ as follows:

$$q = \hat{\beta} E_t \left\{ \begin{array}{l} \lambda F(\bar{\epsilon}_1')(c+1) + (1-\lambda) F(\bar{\epsilon}_0')\,(c+q') \\ + \int \left[\int_{\bar{\epsilon}'} V_H^b(\rho, s) - \pi(1-\xi\eta)\Delta V^b(s') dF(\epsilon') \right] d\psi(\zeta') \end{array} \right\} - \pi(1-\xi\eta)\Delta V, \quad (1.19)$$

The bond price function in equation (1.19) denotes the price at which a firm can sell its bond to a (high type) investor in the primary bond market when the aggregate state is S, the firm has current productivity level z and it chooses $\{l', b'\}$ for next period. Terms in the large bracket is the valuation of the bond if the investor were not subject to the distress shock this period. However, the investor will receive the distress shock with probability π this period after she purchases the bond. Then with probability ξ she could sell the bond in the secondary market and receives a fraction of η of the trade surplus. Otherwise, the investor will stay in the market as a low type. As a result, there is an additional term $-\pi(1-\xi\eta)\Delta V$ in the bond pricing function arising from the risk of a distress shock combined with the search frictions.

In the large bracket in equation (1.19), the first line reflects the cash flow when the bond doesn't default. In this case, the investor receives coupon c and principal 1 when the bond matures. She receives coupon c and continues to hold this bond with price q' when it doesn't mature. The second line in the large bracket corresponds to the scenario where the bond defaults. Because of the risk of a distress shock and the search frictions, this term is the valuation of the defaulted bond for a high type investor adjusted by $-\pi(1-\xi\eta)\Delta V^b$.

Note that q' implicitly involves bond investors' rational expectation of firm's future choices:

$$q' = q\left(S', z', b(S', z', x', b', \zeta'), l(S', z', x', b', \zeta')\right) \quad (1.20)$$

1.3.3 Households

Households supply workers and managers to intermediate goods firms and own all firms in the economy. There are a measure μ_w of workers and a measure μ_m of managers. I normalize $\mu_w + \mu_m = 1$ and assume that the measure of managers μ_m is larger than the measure of intermediate goods firms μ_f, so that there is no shortage of managers. For managers who are not matched with a firm, they also work one unit and earn $\bar{w}_m$ in home production. There is perfect risk sharing among household members. Households are allowed to invest in state-contingent arrow security provided by international intermediaries. They discount the future value with discount factor β. These assumptions is to make the problem of households as simple as possible.

In each period, the household receive wage $(\mu_w w_t L_t + \mu_m \bar{w}_m)$ and dividends D_t from firms. They also pay E_t to buy the defaulted firms and receive payments on their asset holding A_t. Then they choose consumption C_t, labor supply in next period L_{t+1} and the state-contingent assets to invest $A_{t+1}(s_{t+1})$. Households have preference over consumption and leisure. The utility function has the additive separable form $U(C_t) - \mu_w G(L_t) - \mu_m G(1)$, where $\mu_m G(1)$ is a constant and thus can be dropped in the maximization problem.

In period $t-1$, the household will choose labor L_t. The state variable is the aggregate state S_{t-1} and its state-contingent asset holdings for next period $A_t = \{A_t(s_t)\}$. The recursive problem for families is the following:

$$V^f(A_t, S_{t-1}) = \max_{L_t} \left\{ E_{s_t|S_{t-1}} \left\{ \max_{C_t(s_t), \{A_{t+1}(s_{t+1})\}} \left[U(C_t) - \mu_w G(L_t) + \beta V^f(A_{t+1}, S_t) \right] \right\} \right\} \tag{1.21}$$

subject to their budget constraint for each s_t

$$C_t(s_t) + \sum_{s_{t+1}} A_{t+1}(s_{t+1}) Q_{t,t+1}(S_{t+1}|S_t) = \mu_w w_t(S_{t-1}) L_t + \mu_m \bar{w}_m + A_t(s_t) + D_t(s_t, S_{t-1}) - E_t(s_t, S_{t-1}), \tag{1.22}$$

where $S_{t+1} = (\Upsilon_{t+1}, s_{t+1})$ and $\Upsilon_{t+1} = H(S_t)$, $Q_{t,t+1}(S_{t+1}|S_t)$ is the price on the stage-contingent bond provided by risk-neutral international financial intermediaries with discount rate β. Since the economy is small in the world, these prices are given by $Q_{t,t+1}(S_t|S_{t+1}) = \beta\pi(s_{t+1}|s_t)$, where $\pi(s_{t+1}|s_t)$ is the transition probabilities on the aggregate shock in the Markov chain.

$$D_t(s_t, S_{t-1}) = \int d\left(S, z, x, b, \zeta\right) d\Upsilon_t(z, x, b, \zeta), \tag{1.23}$$

$$E_t(s_t, S_{t-1}) = \int \left[\int_{\hat{\epsilon}_t} v^b\left(S, z\right) dF(\epsilon_t)\right] d\Upsilon_t(z, x, b, \zeta), \tag{1.24}$$

where $\Upsilon_t = H(S_{t-1})$ and $S = (\Upsilon_t, s_t)$.

1.3.4 Equilibrium

In this section, I specify the market clearing conditions in $t+1$ and describe the dynamic competitive equilibrium in this economy.

The clearing of the labor market requires that the amount of labor demanded by firms equals the amount of labor supplied by the household:

$$\int l_{t+1}(S_t, z, x, b, \zeta) d\Upsilon_t(z, x, b, \zeta) = \mu_w L_{t+1}(S_t). \tag{1.25}$$

Final goods in $t+1$ satisfy

$$Y(S_t) = \left[\int_{z,x,b,\zeta} \int_{z'} \pi_z(z'|z) z' y'^{\frac{\gamma-1}{\gamma}} dz' d\Upsilon_t(z, x, b, \zeta)\right]^{\frac{\gamma}{\gamma-1}}, \tag{1.26}$$

where $y' = [l(S_t, z, x, b, \zeta)]^{\alpha}$.

The transition function for the measure of firms from t to $t+1$ is given by

$$\Upsilon_{t+1}(x', z', b', \zeta') \equiv H(S_t) = \int \Lambda(x', z', b', \zeta', x, z, b, \zeta | S_t) d\Upsilon_t(z, x, b, \zeta)$$

where

$$\Lambda(x', z', b', \zeta', x, z, b, \zeta | S_t) = \pi_z(z'|z) f(\tilde{\epsilon}') \psi(\zeta') 1_{\{b' = b(S_t, z, x, b, \zeta)\}}, \tag{1.27}$$

the corresponding $\tilde{\epsilon}'$ in equation (1.27) satisfies

$$\tilde{\epsilon}' = \left(z' Y(S_t)^{\frac{1}{\gamma}} l'^{\alpha(\frac{\gamma-1}{\gamma})} - w(S_t) l' - \bar{w}_m - cb' \right) - \zeta' b' - x', \tag{1.28}$$

and $l' = l(S_t, z, x, b, \zeta)$.

Notice that wage $w_{t+1} = w(S_t)$ and output $Y_{t+1} = Y(S_t)$, that is, the wage and output in $t+1$ are determined at the end of period t. This because they are based on the choices at the end of period t and the distribution of idiosyncratic states at t.

Given the initial measure Υ_0 and initial aggregate shock s_0, an equilibrium consists of policy and value functions of intermediate goods firms $\{d(S_t, z_t, x_t, b_t, \zeta_t), l(S_t, z_t, x_t, b_t, \zeta_t), b(S_t, z_t, x_t, b_t, \zeta_t), V(S_t, z_t, x_t, b_t, \zeta_t)\}$; of households $\{C(s_t, S_{t-1}), L(S_{t-1}), A(s_{t+1}|s_t, S_{t-1})\}$; the wage rate $w(S_{t-1})$, and state-contingent prices $Q_{t,t+1}(S_{t+1}|S_t)$; bond price schedules $q(S_t, z_t, l_{t+1}, b_{t+1})$; and the evolution of aggregate states Υ_t governed by the transition function $H(S_{t-1})$, such that for all t (*i*) the policy and value funtions of intermediate goods firms satisfy their optimization problem, (*ii*) households' decisions are optimal, (*iii*) the bond price schedules are consistent with the investor's problem, (*iv*) the labor market clears, and (*v*) the evolution of the measure of firms is consistent with the policy functions of firms, households and shocks.

1.3.5 Characterization of first-order conditions

In this subsection, I analyze firms' first order conditions for hiring and financing policies. Consider the firm's firm-order conditions for labor l' when it doesn't receive the refinancing shock ($\zeta = 0$),

$$\frac{E_{s',z',\zeta'}\left[z'\int^{\hat{\epsilon}'}(1+\chi')\,dF(\epsilon')\right]}{E_{s',z',\zeta'}\left[\int^{\hat{\epsilon}'}(1+\chi')\,dF(\epsilon')\right]}Y^{\frac{1}{\gamma}}\alpha(l')^{\frac{\alpha(\gamma-1)}{\gamma}-1} = \frac{\gamma}{\gamma-1}\left[w + \frac{E_{s',z',\zeta'}\left[V'^{*}f(\hat{\epsilon}')\left(-\frac{\partial\hat{\epsilon}'}{\partial l'}\right)\right]}{E_{s',z',\zeta'}\left[\int^{\hat{\epsilon}'}(1+\chi')\,dF(\epsilon')\right]}\right], \tag{1.29}$$

where χ' is the multiplier associated with the non-negative equity payout constraint.

In a frictionless environment, the first-order condition for the firm's labor choice would be

$$E_{s',z'}[z']Y^{\frac{1}{\gamma}}\alpha(l')^{\frac{\alpha(\gamma-1)}{\gamma}-1} = \frac{\gamma}{\gamma-1}w, \tag{1.30}$$

In equations (1.29) and (1.30), the left-hand side is the expected marginal benefit of labor, and the right-hand side represents the expected marginal cost of labor times a markup $\frac{\gamma}{\gamma-1}$. Comparing these two equations, the distortions are mainly driven by three forces. First, equity holders receive the profits only if the firm doesn't default, so they put weight only on states in which the firm doesn't default. Second, the profits are more valuable to a firm if the firm faces a binding constraint next period, captured by the shadow price $1+\chi' > 1$. These two forces make the marginal benefit of labor on the left-hand side of equation (1.29) different from that of equation (1.30).

Third, an additional unit of labor input will increase the default probability of a firm by $-f(\hat{\epsilon}')\frac{\partial\hat{\epsilon}'}{\partial l'}$. Let V'^{*} denote the firm's future value evaluated at the default cutoff. Then $V'^{*}f(\hat{\epsilon}')\left(-\frac{\partial\hat{\epsilon}'}{\partial l'}\right)$ is the loss of value because from default incurred by one additional unit of labor. Such default cost shows up as a wedge on the right-hand side of equation (1.29). Note that $\frac{\partial\hat{\epsilon}'}{\partial l'} < 0$ in most states, at least for low values of z', so the wedge is positive and acts like a tax on labor.

With these distortions, firms' labor choice will be dependent on its default risk. Since a worsening of market liquidity lowers bond prices, thereby limiting the maximal borrowing $M(S, z)$ and pushes firms closer to default, it will affect firms' labor choices. Under most circumstances, the last two channels dominant, so firms cut their labor input in bad times.

The first order condition for the firm's labor choice when it receives the refinancing shock ($\zeta = 1$) can be written as follows:

$$\frac{E_{s',z',\zeta'}\left[z'\int^{\hat{\epsilon}'}(1+\chi')\,dF(\epsilon')\right]}{E_{s',z',\zeta'}\left[\int^{\hat{\epsilon}'}(1+\chi')\,dF(\epsilon')\right]}Y^{\frac{1}{\gamma}}\alpha(l')^{\frac{\alpha(\gamma-1)}{\gamma}-1} = \frac{\gamma}{\gamma-1}\left[w + \frac{E_{s',z',\zeta'}\left[V'^{*}f(\hat{\epsilon}')\left(-\frac{\partial\hat{\epsilon}'}{\partial l'}\right)\right] + \frac{1+\chi}{\beta}\left(-\frac{\partial q}{\partial l'}\right)b'}{E_{s',z',\zeta'}\left[\int^{\hat{\epsilon}'}(1+\chi')\,dF(\epsilon')\right]}\right]. \tag{1.31}$$

Comparing it with equation (1.29), there is an additional source of distortion, captured by $\frac{1+\chi}{\beta}\left(-\frac{\partial q}{\partial l'}\right)b'$. The term $-\frac{\partial q}{\partial l'}b' = -\frac{\partial(qb')}{\partial l'}$ measures the marginal decrease in proceeds from bond issuance (qb') incurred by an additional unit of labor, and the shadow price $1+\chi$ measures the marginal value of these funds to the firm. The intuition is that when the firm choose labor and issue bonds at the same time, it will take into account how its labor choice impact the benefit from bond issuance.

The first order condition for the firm's financing policy b' when the firm receives the refinancing shock is given by

$$\begin{aligned}(1+\chi)\left(\frac{\partial q}{\partial b'}b' + q\right) &= \beta(1-\lambda)E_{s',z'}\left[F(\hat{\epsilon}_0')\left(c - \frac{\partial V}{\partial b'}\right)\right] \\ &\quad + \beta\lambda E_{s',z'}\left[F(\hat{\epsilon}_1')E[1+\chi'(x_1')|\epsilon' \le \hat{\epsilon}_1']\,(c+1)\right] \\ &\quad + \beta E_{s',z'}\left[(1-\lambda)cV_0^{*\prime}f(\hat{\epsilon}_0') + \lambda\,(c+1)\,V_1^{*\prime}f(\hat{\epsilon}_1')\right]\end{aligned} \tag{1.32}$$

In equation (1.32), the left-hand side represents the marginal benefit from one unit of debt issuance, and the right-hand side represents the expected marginal cost. The term $\frac{\partial q}{\partial b'}b' + q = \frac{\partial(qb')}{\partial b'}$ is the marginal increase in proceeds from bond issuance associated with one unit of bond. Multiplying this term by $1+\chi$, which is marginal value of one unit of funds to the firm, the left-hand side is the marginal benefit from an additional unit of bond issuance.

There are three terms on the right-hand side. The firm term captures the marginal cost if the firm doesn't default and the bond doesn't mature in next period ($\zeta' = 0$), in which case the firm has to pay coupon c and the debt continues to next period and lowers the future value ($\frac{\partial V}{\partial b'}$). The second term represents the marginal cost if the firm doesn't default and the bond matures in next period ($\zeta' = 1$). In this case, the firm has to repay coupon and principal $c+1$, and the marginal value of these funds to the firm is $1+\chi'$ because of the possible binding constraint. The third term represents the marginal cost if the firm default in next period, in which case the firm will lose its future firm value. If the firm doesn't receive the refinancing shock tomorrow, it has to pay c for an additional unit of b', which increases the default probability by $cf(\bar{\epsilon}_0')$, so the marginal cost from losing future value in case of default is $cf(\bar{\epsilon}_0')V_0^{*\prime}$. Similarly, it the firm receives the refinancing shock, it has to pay $c+1$ for an additional unit of b' and the marginal cost is $(c+1)f(\bar{\epsilon}_1')V_1^{*\prime}$.

Notice that when the firm doesn't receive the refinancing shock, the cash-on-hand x can be dropped from the state variables and the policy function can be denoted by $l_0(S, z, b)$. When the firm receives the refinancing shock, current debt level b can be dropped from the state variables and the policy functions can be expressed as $l_1(S, z, x)$ and $b_1(S, z, x)$. Furthermore, the following lemma holds for policy functions $l_1(S, z, x)$ and $b_1(S, z, x)$. These properties help simplify the numerical solutions.

Lemma. There exists a cutoff level of cash-on-hand, $\hat{x}$ such that for $x < \hat{x}$, the non-negative equity payout constraint is binding, and for $x \geq \hat{x}$, the constraint is slack and firm's choices do not vary with x. Denote the policy functions when the non-negative equity payout constraint is slack as $\hat{l}_1(S, z)$ and $\hat{b}_1(S, z)$, the cutoff level of cash-on-hand $\hat{x} = -q(S, z, \hat{l}, \hat{b})\hat{b}$.

1.4 Quantitative Analysis

This section takes the model to the data. I show that disruptions in secondary bond market could have quantitatively important impacts on macro aggregates and thus policies improving bond market liquidity have a sizable impact on the macroeconomy.

1.4.1 Parameterization

I assume the utility function has the additively separable form

$$U(C) - \mu_w G(L) = \frac{C^{1-\sigma}}{1-\sigma} - \chi \frac{L^{1+\upsilon}}{1+\upsilon}, \tag{1.33}$$

where χ captures both the share of workers and the weight on the disutility of labor. Consider next the parameterization of the Markov processes over idiosyncratic shocks and aggregate shocks to holding cost. I assume a discrete process for idiosyncratic productivity shocks that approximates the auto-regressive process in equation (1.34) using the method in Tauchen (1986)

$$\log z_t = \mu + \rho_z \log z_{t-1} + \sigma_z \epsilon_t, \tag{1.34}$$

where the innovations $\epsilon_t \sim N(0, 1)$ are independent across firms. We choose $\mu = -\sigma_z^2/2$ (so as to keep the mean level of z across firms unchanged as σ varies). We assume that holding cost shock h takes on two values, a high value h^B and a low value h^G, with the transition probabilities determined by the probabilities of remaining in the good and bad states, π_{GG} and π_{BB}. The revenue shock ϵ is assumed to be normal with mean μ_ϵ and standard deviation σ_ϵ.

1.4.2 Solution method

Since the model is highly nonlinear and features default and occasionally binding borrowing constraints, I solve the model using projection-based numerical methods. In this subsection, I will explain the globally nonlinear algorithm in some detail and relegate the detailed description to Appendix 1.9.

The model is solved with four nested loops. First, the value wedge for bond investors in secondary bond market $\Delta V(S, z, l', b')$ is solved iteratively according to equation (1.17) in the innermost loop. Second, an outer loop solves the bond price schedule by iterating on the maximal borrowing $M(S, z)$ and $q(S, z, l', b')$ according to equation (1.3) and (1.19). Third, an outer loop solves the firms' value function $V(S, z, x, b, \zeta)$ and policy functions $\{l_0(S, z, b), l_1(S, z, x), b_1(S, z, x)\}$. The firm's problem is characterized by equation (1.6) and the first-order conditions are given by equations (1.29), (1.31) and (1.32). Finally, the outermost loop solves the aggregate variables $\{w(S), Y(S)\}$ using the market clearing conditions for labor market and final goods market, expressed in equations (1.25) and (1.26), respectively.

I solve the function for a set of knots on state variables and interpolate these functions using multivariate piece-wise polynomial cubic splines for values off the knot points. In each inter loop, I use the rules from outer loop, and iterate until converge. So the full model is solved when the outermost loop converges. In particular, in the outermost loop, for each iteration, I simulate the economy for $T = 1000$ periods, project the simulated values of wage and output along the time series on a set of dummy variables corresponding to aggregate state S, and use the fitted value as the updated aggregate rules for $w(S)$ and $Y(S)$. This procedure is repeated until the aggregate rules converge.

Note that the distribution of firms (Υ) is part of the aggregate state S, while in practice it is impossible to include the entire measure. I follow a version of Krusell and Smith (1998) to approximate the distribution with lags of aggregate shocks. Therefore, the aggregate state variable S actually refers to the current and lags of aggregate shocks in the algorithm. I choose to use $S = (s, s_{-1}, s_{-2}, k)$, where $k = 1, ..., 12$ indicates how many periods the aggregate state have been unchanged. Also, I assume there are two realization of state $s \in \{s^G, s^B\}$, therefore, the total number of aggregate state S is 30. Appendix 1.9 describes the numerical method in greater detail, presents specific equations and choices for state grids, and discusses accuracy.

1.4.3 Calibration

The model is calibrated to reproduce the state of firms in US economy before the financial crisis. I assume $s = s^G$ in calibrating the steady state. Later I will calibrate parameters for $s = s^B$ so that the model generates the same increase in bid-ask spread as in the financial crisis of 2007-2009.

The assigned parameters are

$$\{\hat{\beta}, \nu, \sigma, \alpha, \gamma, \rho_z, \sigma_z\}, \{\lambda, \theta, \eta\}, \{\pi_{BB}, \pi_{GG}\}$$

The model is calibrated at a quarterly frequency. The discount factor for foreign investors $\hat{\beta}$ is set to be 0.995 so that the annual risk-free rate is 2%. I set $\nu = 0.5$ which implies a labor elasticity of 2. The elasticity is in the range of elasticities used in macroeconomic work, as reported by Rogerson and Wallenius (2009). I set $\sigma = 2$, a common estimate in the business cycle literature. For the intermediate goods production, I set parameter α equal to the labor share of 0.7 and think of there being two other fixed factors, managerial input and capital, which receive a share of 0.3. For the final goods production function, I choose the elasticity of substitution parameter $\gamma = 5.76$ so as to generate a markup of about 21%, which is estimated by Basu and Fernald (1997). I choose the serial correlation of the firm-level productivity shock $\rho_z = 0.9$. This value is consistent with the estimates of Foster et al. (2008) for their traditional TFP index, which measures output as the deflated dollar value using the four-digit industry-level deflator from the NBER Productivity Database. The parameter on aggregate shocks to volatility σ_z follows ABK and is set as 0.09.

I set $\lambda = 0.05$, which implies the average maturity of corporate bond is 5 years. This same number is used by Gomes et al. (2016) for long-term corporate debt in their model.

For the delay in receiving debt payments after bankruptcy, Franks and Torous (1994) report that the average bankruptcy takes 2.7 years and Weiss (1990) estimates 2.5 years from the filing the filing of the bankruptcy petition to resolution. For the sample in Bris et al. (2006), which is more recent and contains smaller firms, the average Chapter 7 proceeding lasts 2 years and the average Chapter 11 proceeding lasts 2.3 years.[14] Based on these evidence, I choose $\theta = 0.1$ so that it takes 2.5 years on average after the bankruptcy for the investors to receive payments. The Nash-bargaining weights η for the investor is assumed to be 0.5, which implies that the dealer and the seller split the trade surplus equally.

The transition probability for the aggregate bond market liquidity state s is as follows,

$$\Pi^s = \begin{bmatrix} \pi_{GG} & 1 - \pi_{GG} \\ 1 - \pi_{BB} & \pi_{BB} \end{bmatrix}. \tag{1.35}$$

In the transition matrix in equation (1.35), π_{GG} is the probability of continuing in ordinary liquidity conditions, $Pr\{s' = s^G | s = s^G\}$, while $1 - \pi_{BB}$ is the probability of escape from crisis conditions, $Pr\{s' = s^G | s = s^B\}$. Following Khan and Thomas (2013) I choose the parameters of the matrix using evidence on banking crises from Reinhart and Rogoff (2009). In the postwar period 1945-2008, the U.S. only has had two banking crises (the 1989 savings and loan crisis and the 2007 subprime lending crisis), but since the incidence and number of crises is similar across the extensive set of advanced countries, they instead rely on the data for advanced economies to infer the transition probabilities. The average number of banking crises across advanced economies over 1945 - 2008 was 1.4, while the share of years spent in crisis was 7 percent. Combining these observations, we set $\pi_{BB} = 0.922$ and $\pi_{GG} = 0.994$ so that the average duration of a liquidity crisis is 3.2 years, and the economy spends 7 percent of time in the crisis state.[15]

[14] In the example of Lehman Brothers bankruptcy, after much legal uncertainty, payouts to the debt holders only started trickling out after about 3.5 years.

[15] The choices of these transition probabilities are different from what is used in Chen et al. (2017), which corresponds to 10 years of expansions and 2 years of recessions. The choices are also different from ABK,

The parameters from moment-matching exercise are

$$\{\beta, w_m + \mu_\epsilon, \sigma_\epsilon, \kappa, h, h_b, \xi, \pi, c\}.$$

The model is highly nonlinear so all parameters affect all the moments. The nine parameters are calibrated to target nine moments: (1) an average credit spread of 100 bps, (2) an aggregate default rate of 1%, (3) an average leverage ratio of 40%, (4) an average bond recovery rate of 42%, (5) an average bid-ask spread for non-defaulting bonds of 35 bps, (6) an average bid-ask sprad for defaulted bonds of 140 bps, (7) an average liquidity premium of 50 bps, (8) an annual turnover rate of 100%, and (9) an average bond price at issuance of 1. Some moments are more sensitive to some parameters, and I will discuss them below.

Credit spread, default rate and leverage Credit spread, default rate and leverage are mainly determined by $\{\mu_\epsilon + w_m, \sigma_\epsilon, \beta\}$. The targeted credit spread is set as 100 bps, which is the average credit spread for BBB bonds from 2003 to 2006. I target an annual default rate of 1%, which is consistent with Moody's average default rate of 0.26% per quarter (Moody's Investor Service 2014), as used by Gomes et al. (2016).[16] The leverage ratio is defined as the ratio of debt to market assets. I compute the average leverage ratio for a sample of non-financial firms in COMPUSTAT and the average is about 40%.

Bond recovery rate Bond recovery rate is largely driven by firm recovery rate κ. Bond recovery rate is a widely-used measure defined as the defaulted bond price divided by its promised face value. I choose to target a bond recovery rate of 42%, which is the average

which assume $\pi_{GG} = 0.94$ and $\pi_{BB} = 0.84$. Therefore, the liquidity shock in this paper corresponds financial crises which are more rare and persistent compared with business cycles. Later I will show how the quantitative effects differ for different values of transition probabilities.

[16]Kozlowski et al. (2020) and Khan and Thomas (2013) target a 2% annual default rateGourio (2013) chooses 0.5%.

issue-weighted bond recovery rate in Moody's recovery data over 1982-2012.[17]

Bid-ask spread and liquidity premium The parameters on holding cost $\{h, h_b\}$ and trading probability ξ are important in matching the liquidity premium and the bid-ask spread of non-defaulting bonds, and bid-ask spread of defaulted bonds. The liquidity premium in data is approximated using the bond-CDS spread, and I target a liquidity premium of 50 bps for the calibration. The bid-ask spread for non-defaulting bonds in TRACE data is about 35 basis points before the crisis. The information on bid-ask spreads of defaulted bonds is scarce. In Chen et al. (2017), the median bid-ask spread for pre-default bonds in good and bad times are 50 and 125 bps, respectively; the bid-ask spread for defaulted bonds are 200 and 620 bps in good and bad times, respectively. So I assume the bid-ask spread for post-default bonds is higher than that for pre-default bonds by a factor of 4 in good times and by a factor of 5 in bad times, which implies a bid-ask spread for defaulted bonds of 140 bps.

Finally, the probability of distressed shocks π is chosen to target an annual turnover rate of 100% (i.e. holding time of 1 year) following He and Milbradt (2014) and Bao et al. (2011). The coupon rate c is normalized so that the bond is issued at par on average.

Table 1.3 summarizes the parameters values used for the numerical solutions. Table 1.4 presents the targeted moments in the model and in the data. In Appendix 1.8.3, I provide details on the definitions of these moments in the model and in the data.

[17] ABK assume 100% cost of default and Zeke (2016) argues that if using a more "reasonable" cost of default the drop in employment won't be as large. He uses estimates for the cost of default from corporate finance literature directly and chooses 30% in his baseline model, which generates a relationship between credit spread and default probability that is in line with data. In my calibration, the cost of default $1 - \kappa$ is selected to generate the targeted bond recovery rate. The calibrated cost of default is about 80%, which is larger than what is used in Zeke (2016).

1.4.4 Firms' decision rules

In Figure 1.4, I plot firms' policy functions $l_0(S,z,b)$, $l_1(S,z,x)$ and $b_1(S,z,x)$ in Panel (a), (b) and (c), respectively. I consider a firm that receives the median level of productivity shock z, and plot its choices against b or x. The black line is the policy function when the aggregate bond market liquidity condition is good ($s = s^G$), while the red line shows the policy function when the bond market liquidity condition is bad ($s = s^B$). I keep the aggregate variables $\{w, Y\}$ at the same level to focus on the direct effects of the liquidity shock on firm's choices.

Panel (a) of Figure 1.4 shows how firm's labor choice changes with its current debt outstanding when the firm doesn't receive a refinancing shock. First notice that the firm's labor choice is not monotonic in debt level. This is because here are different forces that distort firm's labor choices as shown in equation (1.29). When leverage is not too high, firms tend to cut labor to reduce loss from a possible default when they have higher level of debt and more default risk. However, when firms have a very high level of debt outstanding and thus the default probability is very high, risk-shifting could be the dominating force and firm increase labor to gamble as they are closer to default. Next compare the policy function when the aggregate bond market liquidity condition is good and bad. The policy function in bad state is almost an inward shifting of the policy function in good state. The idea is that the worsening of bond market liquidity lowers bond prices, and pushes the firms with all levels of leverage closer to default. In sum, a liquidity shock generates little change in labor choice for firms with low leverage, leads to decreases in labor input for firms with middle leverage, and leads to increases in labor input for firms with very high leverage. Most firms belongs to the middle range and thus a liquidity shock leads to a decrease in aggregate employment.

In Panel (b) and (c) of Figure 1.4 shows how firm's labor and debt choices change with its current cash-on-hand when the firm receives a refinancing shock. The characterization of the corresponding first-order conditions have been discussed in Section 1.3.5. When there are abundant cash-on-hand so that non-negative equity payout constraint is not binding, the firm's optimal choices of labor and debt are not affected by its cash-on-hand. This can be seen from the right part of the policy functions in the figure, which is a horizontal line. When the constraint is binding, the dominating incentive for the firm is to get more proceeds from bond issuance to cover its shortage of cash. This induces firms to cut the labor in order to reduce default risk and increase the bond price at issuance. Comparing the red line with the black line, firms with low cash-on-hand cut their labor input dramatically following a liquidity shock and firms with very low cash-on-hand defaults following the shock. For firms with abundant cash-on-hand, their labor input in bad times could be even slightly larger than that in good times. This is because the firm lowers its financial leverage a lot in response to the shock and thus might increase its operational leverage a bit.

1.4.5 Impulse responses

I now analyze how the model could explain the dynamics of credit spreads and macro aggregates during the financial crisis of 2007-2009.

General equilibrium dynamics Figure 1.5 reports the model impulse responses of real and financial variables to a liquidity shock that mimics the liquidity dry-up after the Lehman collapse in 2008:Q4. The red line shows the model predictions, while the actual path of data is in black. In this experiment, I first simulate the model for 500 periods, and then assume the economy stay in good state for 50 periods, switch to bad state in period 0 and stays there afterwards.

As shown in panel (a) of Figure 1.5, the size of the shock is calibrated to deliver an immediate jump of bid-ask spread to 160 bps, which is the peak level of the value-weighted average of bid-ask spreads during the Great Recession. In panel (b)-(d) of Figure 1.5, we can see that this shock generates an increase in credit spread of 268 bps, including 128 bps from the liquidity premium and 140 bps from the default premium. The increase in default premium is more transitory as the firms are gradually deleveraging. Panel (e)-(f) present the drop in aggregate labor and output following the shock. Notice that the labor decline is 1.3 percentage point, which is about 19% of the employment losses in the data. The output drop is about 1.4 percentage point, which is almost one-third of the actual drop in data.

This experiment shows that a disruptions in the market liquidity of long-term bonds could have sizable effects on the real economy. Note that this is not arguing that such liquidity shock is the only source of aggregate disturbances, or even the main source of disturbances. The idea is that this is a new channel that could be quantitatively important, and is ignored in previous studies.

Partial equilibrium responses In this part, I keep the aggregate variables, wage w and aggregate demand Y unchanged along the transition, and study the partial-equilibrium response of the economy to a liquidity shock. The results are represented by the blue dashed line in Figure 1.5.

In response to a liquidity shock, firms cut their labor so that wage and aggregate output both fall. A lower wage induces firms to hire more, while a lower output means lower aggregate demand and induces firms to hire less. In Figure 1.5 we can see that the partial equilibrium impacts on real variables are larger that the general equilibrium results. This means in the baseline model, quantitatively the wage effects dominates the aggregate demand effects, and these two general equilibrium effects dampen the drop in labor compared with the partial equilibrium results. If we rely on the partial equilibrium results, we would overestimate the liquidity effects by about 40%.

Sensitivity to parameters There are several parameters that are important for the quantitative predictions of the model: the persistence of the shock π_{BB}, the frequency of refinancing shock λ, and the trading probability ξ. Table 1.5 summarizes how the model predictions would change when each of these parameters changes, everything else unchanged.

The effects of a liquidity shock are larger when it is more persistent. This is intuitive as the the shock is at heart a result of exogenous liquidity need combined with the risk of delayed trade. If the liquidity shock is transitory, the holding cost will return to normal immediately, and the liquidity premium won't increase a lot. Thus the effects on bond prices, default risk and firm's choices on real variables will also be negligible. When λ is larger, the bonds have shorter maturity, one might expect the liquidity concerns to be less important in this case. However, a higher λ also implies that the firm will face a refinancing shock more often, which make fluctuations in bond market more important. When trading probability ξ is smaller, the results are larger. Again, market liquidity arises from liquidity need coupled with search frictions. So when the frictions are stronger, as represented by slower trade from a lower ξ, the responses to an increase in liquidity need are stronger.

1.4.6 COVID-19 crisis

After the Federal Reserve's unprecedented corporate bond purchasing program during the recent COVID-19 crisis, there has been many papers study the effects of such interventions. Kargar et al. (2020) show that it greatly improved market liquidity and Gilchrist et al. (2020) show that it reduced credit spreads significantly. My model provide a structural estimate of its impact on real aggregates.

As seen in panel (b) of Figure 1.1, there was sharp increase in bid-ask spreads and immediate recovery coincided with Fed's interventions in March 2020. I calibrate a set of liquidity shock $\{h^G, h^B\}$ to match a bid-ask spread of 25 bps in good times, which correspond to the average level of before the Covid-19 crisis and 180 bps in bad times, which correspond to the peak value in mid-March. Other parameters, in particular the transition probabilities of the shock, are kept at the same value as in the baseline model. Then the model predicts a 2% drop in employment and output following such a liquidity shock. In other words, the Fed's interventions, by improving market liquidity of bonds, avoided a 2% drop in employment and output.

The liquidity shock considered in my model is a demand shock which increases the demand for liquidity by assuming a higher holding cost h in crisis times. I ignore potential supply shocks by assuming that the trading probability ξ is the same in normal and in crisis times. This may be a concern because there might be both demand and supply shocks during the COVID-19 crisis. I argue that the assumption of a demand shock is reasonable because during this crisis, the price (transaction cost) and quantity (trading volume) of transaction services increased at the same time, which is suggestive of demand shocks instead of supply shocks (Kargar et al., 2020). Kargar et al. (2020) also model a demand shock instead of a supply shock, arguing that this captures the key feature of the COVID-19 crisis in the corporate bond market, i.e., the surge in selling pressure.

1.5 Conclusion

A long-standing literature in finance has made a convincing argument that a significant portion of credit spreads can be attributed to bond illiquidity. However, existing general equilibrium macro models that study the disruptions in credit markets and their correlation with macroeconomic aggregates largely abstract from the secondary bond market and the bond illiquidity. Early models attribute movements in credit spreads to variations in default rates, and more recent work starts to put more emphasis on risk premia.

In this paper, I introduce a fairly standard secondary OTC bond market into a macro model to fill this gap and link bond illiquidity at a micro level to the macroeconomy. I attempt to use the simplest model setup — two types of agents, risk neutral investors, exogenous trading probability — to determine endogenously the mapping between bid-ask spread and yield spread, so that I can calibrate the model to micro data on secondary market liquidity.

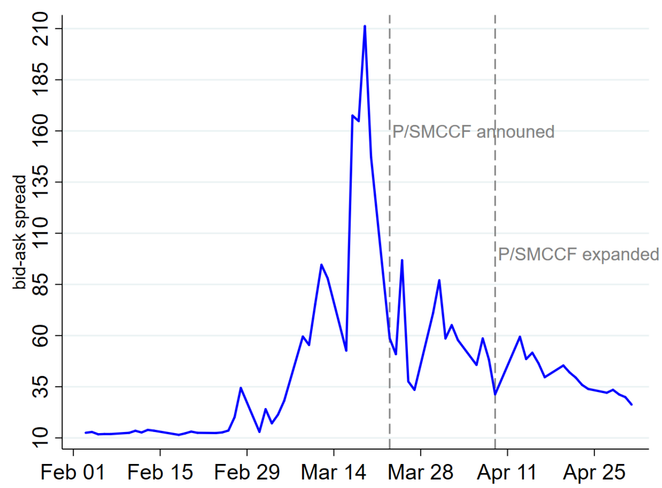
Using the model, I show that disruptions in secondary bond market liquidity can be a quantitatively important source of aggregate disturbances in macro-financial crisis models, and policies improving bond market liquidity can have sizable real effects.

I think the framework developed in this paper can be used for studying many interesting questions concerning the interactions between secondary bond markets and the macroeconomy. One interesting application is to use the framework to shed light on the regulatory discussions of the micro-structure of secondary bond market. There have been such discussions in the OTC literature and the model I propose can help link it to the macroeconomy. Future research may also extend the model by allowing heterogeneous bond maturities across firms or different selling pressures in the bond market.

1.6 Appendix: Figures

Figure 1.1: Time series of bid-ask spreads

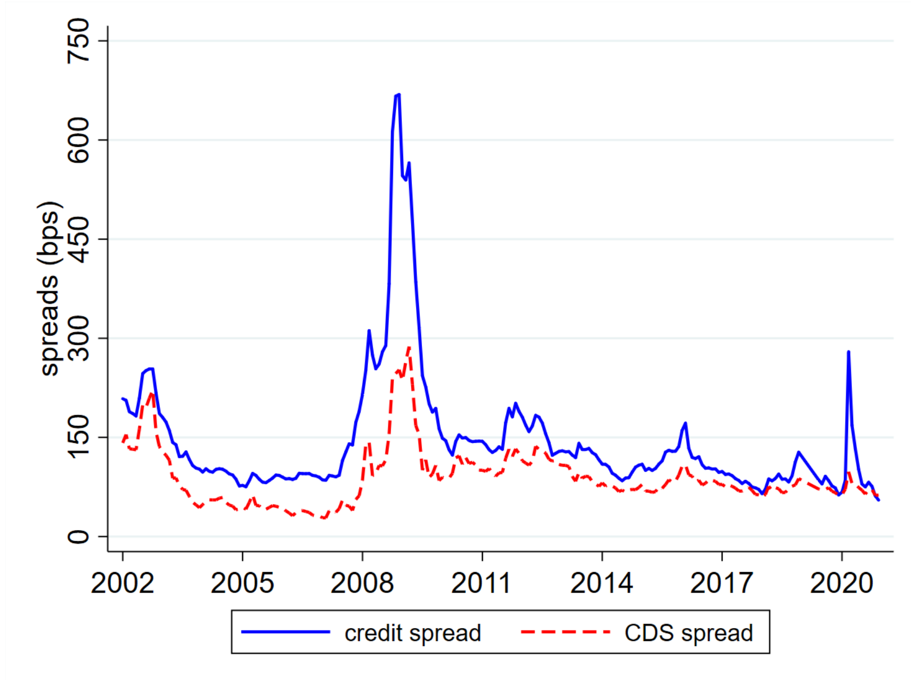
(a) 2007-2020



(b) COVID-19 Crisis

Notes: This figure depicts the time series of bid-ask spreads. The bid-ask spreads are computed customer-buy prices minus customer-sell prices in TRACE database. The time series is constructed as the median across all risky-principal trades at each date. Panel (a) reports the 10-day moving average of bid-ask spreads from 2007 to 2020. Panel (b) reports the daily time series from Feburary 2020 to April 2020.

Figure 1.2: Credit spread vs. CDS spread for BBB bonds: 2002-2020



Notes: This figure depicts the month-end average credit spread and CDS spread for BBB bonds from 2002 to 2020. The blue solid line represents the credit spread and the red dashed line represents the CDS spread.

Figure 1.3: Model timeline

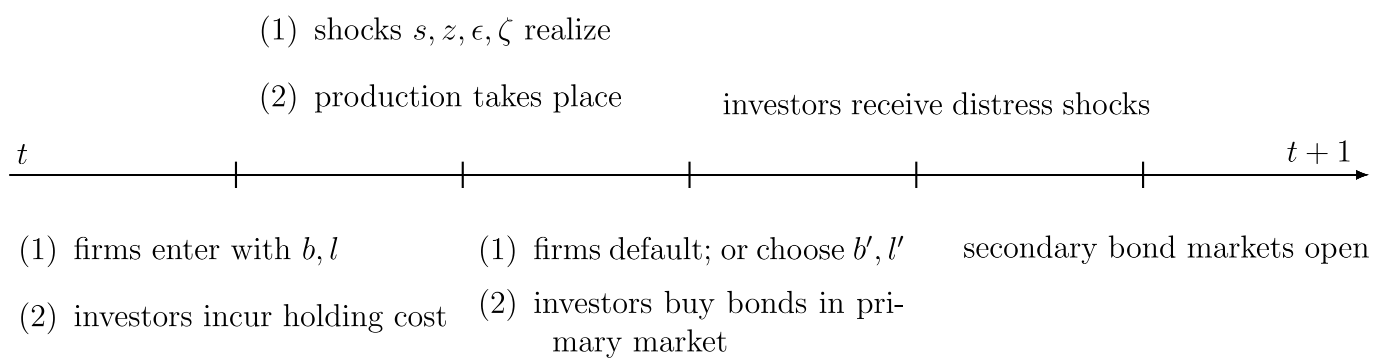
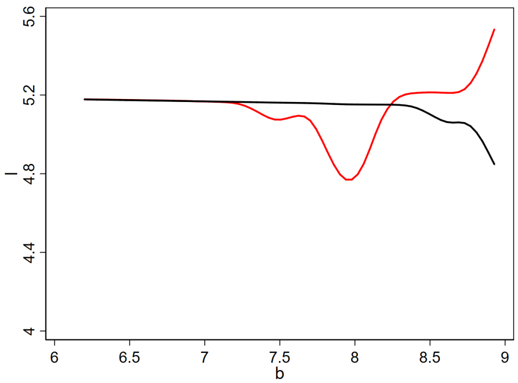
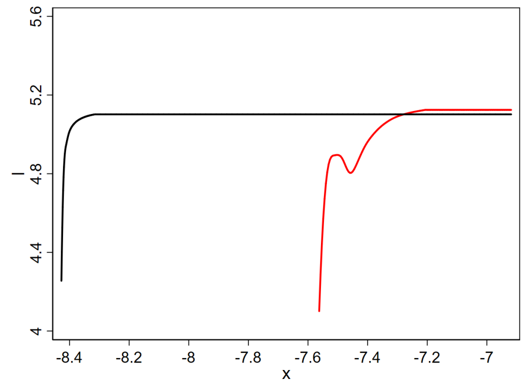


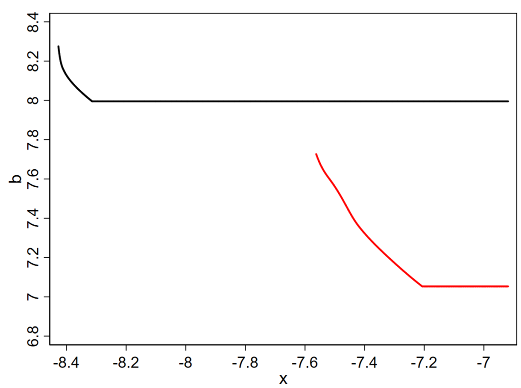
Figure 1.4: Firms' policy functions



(a) Labor choice when $\zeta = 0$



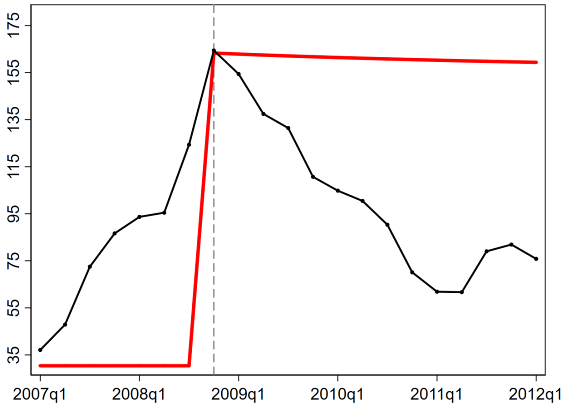
(b) Labor choice when $\zeta = 1$



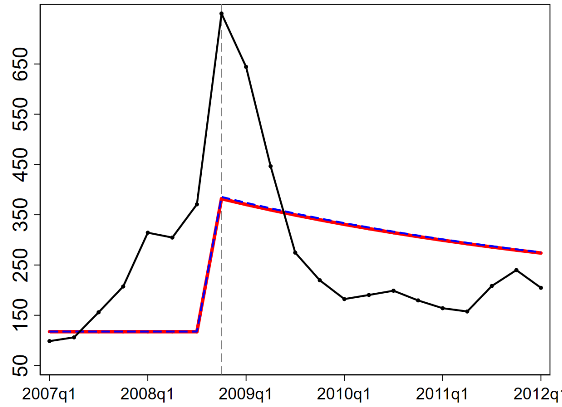
(c) Bond Issuance when $\zeta = 1$

Notes: This figure depicts the firms' policy functions. Panel (a) plots the firm's labor choice as a function of debt level b when $\zeta = 0$ at median level of productivity. Panel (b) and panel (c) plot the firm's labor choice and debt choice as a function of debt level b when $\zeta = 1$ at median level of productivity, respectively. The black line represents the policy function when the economy is in good state, and the red line represents the policy function when the economy is in bad state while keeping the aggregate variables $\{w, Y\}$ unchanged.

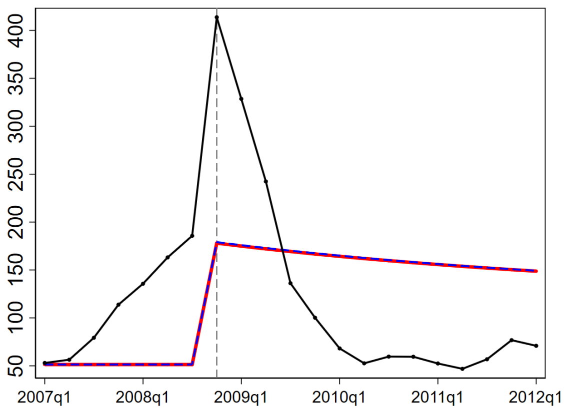
Figure 1.5: Impulse responses to liquidity shock



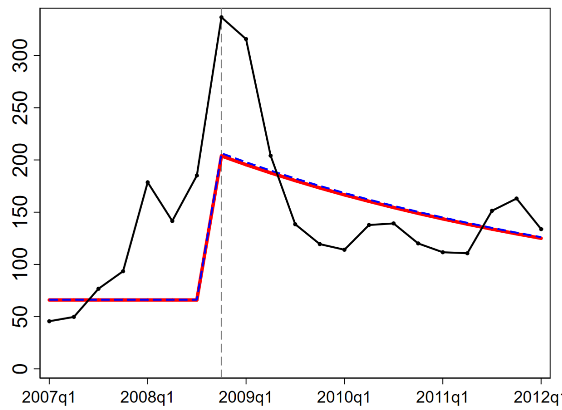
(a) Bid-ask spread



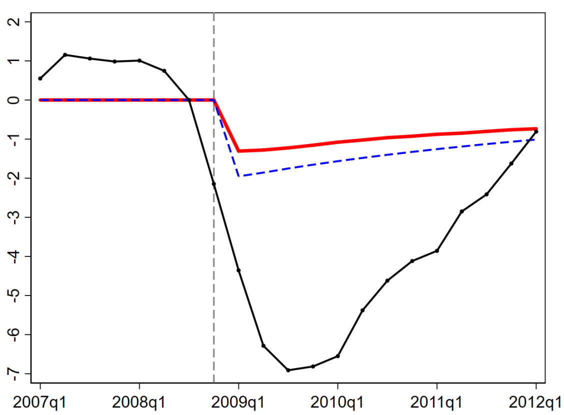
(b) Credit spread



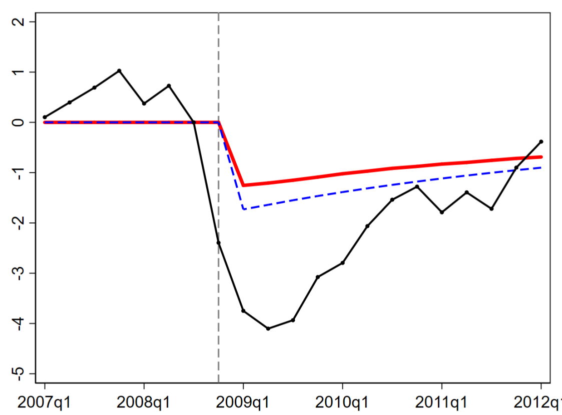
(c) Liquidity premium



(d) Default premium



(e) Labor



(f) Output

1.7 Appendix: Tables

Table 1.1: Annual employment growth rates for firms with different current refinancing needs

	Panel A: Drop in employment growth in 2008		
	2007	2008	2008-2007
Treated Firms	3.32	-5.34	-8.66***
	(2.10)	(2.51)	(3.28)
Non-treated Firms	2.82	-0.64	-3.46***
	(0.59)	(0.51)	(0.78)
Difference	0.50	-4.70**	-5.20**
	(2.03)	(1.84)	(2.57)
	Panel B: Drop in employment growth in 2007		
	2006	2007	2007-2006
Treated Firms	3.87	3.18	-0.70
	(1.87)	(1.66)	(2.50)
Non-treated Firms	3.38	2.89	-0.48
	(0.54)	(0.53)	(0.76)
Difference	0.50	0.28	-0.21
	(1.89)	(1.84)	(2.44)
	Panel C: Drop in employment growth in 2001		
	2000	2001	2001-2000
Treated Firms	-2.46	-5.18	-2.72
	(1.81)	(2.20)	(2.84)
Non-treated Firms	3.33	-3.00	-6.33***
	(0.70)	(0.63)	(0.94)
Difference	-5.79***	-2.18	3.60
	(2.09)	(1.96)	(2.66)

Notes: This table presents the annual employment growth rates for firms with current refinancing needs (treated-firms) and firms without (non-treated firms). In each panel, the last row computes the differences between treated firms and non-treated firms, and the last column computes the changes between two consecutive years. The standard errors are reported in the parenthesis. ***, **, and * indicate significance at 1%, 5% and 10% levels from two tailed t-tests.

Table 1.2: Annual employment growth rates for firms with different future refinancing needs

	Panel A: Drop in employment growth in 2008		
	2007	2008	2008-2007
Treated Firms	4.07	-5.27	-9.33***
	(1.95)	(1.78)	(2.64)
Non-treated Firms	2.72	-0.25	-2.97***
	(0.61)	(0.53)	(0.81)
Difference	1.35	-5.02***	-6.36**
	(2.19)	(1.91)	(2.76)
	Panel B: Drop in employment growth in 2007		
	2006	2007	2007-2006
Treated Firms	0.35	1.86	1.51
	(1.77)	(2.16)	(2.80)
Non-treated Firms	3.55	2.14	-1.41*
	(0.59)	(0.56)	(0.82)
Difference	-3.20*	-0.29	2.92
	(1.90)	(1.85)	(2.47)
	Panel C: Drop in employment growth in 2001		
	2000	2001	2001-2000
Treated Firms	2.77	-4.17	-6.95**
	(2.12)	(1.71)	(2.72)
Non-treated Firms	3.43	-2.77	-6.21***
	(0.73)	(0.68)	(1.00)
Difference	-0.66	-1.40	-0.74
	(1.90)	(1.72)	(2.38)

Notes: This table presents the annual employment growth rates for firms with future refinancing needs (treated-firms) and firms without (non-treated firms). In each panel, the last row computes the differences between treated firms and non-treated firms, and the last column computes the changes between two consecutive years. The standard errors are reported in the parenthesis. ***, **, and * indicate significance at 1%, 5% and 10% levels from two tailed t-tests.

Table 1.3: Parameter Values

Parameters from Moment Matching		
Discount rate	$\beta = 0.95$	Leverage
Revenue shock process	$\mu_\epsilon + w_m = 0.1, \sigma^\epsilon = 0.1$	Credit spread & default rate
Firm recovery rate	$\kappa = 0.17$	Bond recovery rate
Coupon rate	$c = 0.0087$	Bond price
Liquidity shock prob.	$\pi = 0.26$	Turnover rate
OTC frictions	$h^G = 0.0042, h_b^G = 0.006, \xi = 0.8$	Liq. prem, & bid-ask spread
Assigend Parameters		
Volatility of z	$\sigma_z = 0.09$	ABK
Persistence of z	$\rho_z = 0.9$	Foster et al. (08)
Labor elasticity	$\upsilon = 2$	Rogerson and Wallenius (09)
Labor share	$\alpha = 0.7$	National accounts
Markup	$\gamma/(\gamma - 1) = 1.21$	Basu and Fernald (97)
Discount rate investor	$\hat{\beta} = 0.995$	Annual risk free rate
Curvature	$\sigma = 2$	Business cycle literature
Bankruptcy delay	$\theta = 0.1$	Bankruptcy delay
Maturity	$1/\lambda = 20$	Maturity
Bargaining power investor	$\eta = 0.5$	Bargaining power

Table 1.4: Moment-Matching Exercises

Moments	Model	Data
Leverage	0.38	0.4
Credit spread (bps)	100	100
Liquidity premium (bps)	51	50
Default rate	1%	1%
Bond recovery rate	0.41	0.42
Bid-ask spread (bps)	34	35
Bid-ask spread of defaulted bonds (bps)	145	140
Turnover rate	100%	100%

Table 1.5: Parameter Sensitivity

	Δbas	Δliq prem	Δdef prem	Δlabor
$\pi^{\mathbf{BB}} = \mathbf{0.922}$: avg duration of crisis = 3.2 yrs				
0.875	+130	+64	+44	-0.7%
0.922	+142	+130	+ 133	-1.6%
0.95	+153	+239	+247	-2.1%
$\lambda = \mathbf{0.05}$: avg maturity of debt = 5 yrs				
0.025	+136	+60	+28	-0.4%
0.05	+142	+130	+133	-1.6%
0.083	+145	+201	+245	-2.6%
$\xi = \mathbf{0.8}$: trading prob in one period = 80%				
0.5	+187	+282	+203	-2.1%
0.8	+142	+130	+133	-1.6%
0.95	+127	+89	+99	-0.9%

1.8 Appendix: Data

1.8.1 Link ICE bond data and Markit CDS data

Bond spreads data from ICE database and CDS spreads data from Markit database are first matched to merged CRSP/Compustat data at firm-month level and then linked to each other. When matched with merged CRSP/Compustat, I use TICKER, TSYMBOL and 6-digit CUSIP as candidate identifiers sequentially and then append all the matched observations.

1.8.2 Data filters for Compustat data

Following Almeida et al. (2011), starting from COMPUSTAT's North America Fundamentals Annual, I first disregard observations from financial institutions (SICs 6000-6999) and public service firms (SICs greater than 8000), as well as ADRs. I drop firms with missing or negative values for total assets (*at*), capital expenditures (*capx*), property, plant and equipment (*ppent*), cash holdings (*che*), or sales (*sale*). I also drop firms for which cash holdings, capital expenditures or property, plant and equipment are larger than total assets. I then discard observations for which the value of total assets is less than 10 million, and those displaying asset growth exceeding 100%. I also require firms' sales to be positive, and winsorize employment (*emp*) at 0.5%. I remove observations for which the employment growth rate or sales growth rate is greater than 100%. I delete firms with total long-term debt (*dd1+dltt*) greater than assets and firms for which debt maturing in more than one year (*dltt*) is lower than the sum of debt maturing in two, three, four and five years (*dd2+dd3+dd4+dd5*). Finally, I focus on firms that have long-term debt maturing beyond one period that represents at least 5% of assets.

1.8.3 Definitions of variables in data and model

Credit Spread In data, the credit spread in calculated as the average option-adjusted spread for bonds with BBB credit ratings. The targeted value for calibration is set as the average from 2003 to 2006 .In the model, the credit spread is given by

$$cs = (1 + \frac{\lambda + c}{q} - \lambda)^4 - \left(\frac{1}{\hat{\beta}}\right)^4 \tag{1.36}$$

Then I compute the average of the credit spread for all the firms.

Liquidity premium In data, the liquidity premium in calculated as the average option-adjusted spread minus the average CDS spread for bonds with BBB credit ratings. The targeted value for calibration is set as the average from 2003 to 2006. In the model, to calculate the liquidity premium, I first compute the bond prices when there is no liquidity risk, that is, $\Delta V = 0$ and $\Delta V^b = 0$ in the bond price function given by equation (1.19). Denote the bond price function for the perfect liquid bond as q^l, the liquidity premium is given by

$$liq = (1 + \frac{\lambda + c}{q} - \lambda)^4 - (1 + \frac{\lambda + c}{q^l} - \lambda)^4 \tag{1.37}$$

Then I compute the average of the liquidity premium for all the firms.

Bid-ask spread In data, the bid-ask spread is computed as customer-buy prices and customer-sell prices in TRACE database. The target value for calibration is the average of the median across all risky-principal trades in 2007:Q1-Q3.[18] In the model, the percentage bid-ask spread is computed as follows,

$$bas = \frac{(1 - \eta)\Delta V}{V_H - (1 - \eta)\Delta V/2} \tag{1.38}$$

[18] I thank Shuo Liu for sharing the data with me.

Leverage In data, the leverage is computed as the ratio of debt ($dlttq + dlcq$) to equity ($cshoq \times prcc_f$). The target value for calibration is the median across all firms in the sample. Correspondingly, the defintion of leverage in the model is defined as b/V.

Default rate The target default rate is set as 1% as in the literature. In the model, the default rate is computed as the ratio of the measure of defaulting firms to the total measure of firms. In Section 1.9, I discussed how I deal with the movement in distribution and the measure of defaulting firms is summarized by a vector F.

Bond recovery rate The targeted bond recovery rate is set as 42% from Moody's data. In the model, it is computed as the value of bond at default for high-type investor (V_H^b) divided by the face value 1.

Turnover rate The targeted annual turnover rate is set as 100% from literature. let m denote the measure right before the secondary market opens, the movement of the measure follows:

$$\pi\left(\delta_0(1-\lambda)\left(m(H_1)+\xi m(L_1)\right)+\int b(S,z,x,b,\zeta)d\Upsilon\right)+\delta_0(1-\lambda)(1-\xi)m(L_1)=m(L_1),$$

where δ_0 is average probability that a bond doesn't default given by

$$\delta_0=\frac{\int b'\left[E_{z'|z}F(z'Y^{\frac{1}{\gamma}}(l')^{\alpha(\frac{\gamma-1}{\gamma})}-wl'-w_m-cb')\right]d\Upsilon,}{\int b'd\Upsilon}$$

$$l'=l(S,z,x,b,\zeta),b'=b(S,z,x,b,\zeta).$$

In equilibrium

$$m(H_1)+m(L_1)=\int b(S,z,x,b,\zeta)d\Upsilon$$

So I can solve $m(L_1)$ and compute turnover rate as $\frac{\xi m(L_1)}{\int b(S,z,x,b,\zeta)d\Upsilon}$

Employment The employment data is the HP-filtered log of total hours worked for the period from 1985 to 2012 from the FRED database. The employment in the model is the aggregate labor supply L.

Output The output data is the HP-filtered log of real GDP for the period from 1985 to 2012 from the FRED database. The output in the model is the aggregate output Y.

1.9 Appendix: Computational Algorithm

This appendix describes the algorithm used to compute the model.

The model is solved with four nested loops. First, the value wedge for bond investors in secondary bond market $\Delta V(S, z, l', b')$ is solved in the innermost loop. Second, an outer loop solves the bond price schedule by iterating on the maximal borrowing $M(S, z)$ and $q(S, z, l', b')$. Third, an outer loop solves the firms' value function $V(S, z, x, b, \zeta)$ and policy functions $l_0(S, z, b)$, $l_1(S, z, x)$, and $b_1(S, z, x)$. Finally, the outermost loop solves the aggregate variables $w(S)$ and $Y(S)$.

1.9.1 Solve $\Delta V^b(s)$ and $V_H^b(\rho, s)$

Before turning to numerical methods, note that the value wedge for defaulted bonds $\Delta V^b(s)$ and the valuation of defaulted bonds for H-type investors $V_H^b(\rho, s)$ can be derived analytically since s follows a two-state Markov chain with $s = G, B$.

The law of motion for the value wedge $\Delta V^b(s)$ is given by

$$\Delta V^b(s) = \hat{\beta} \left\{ h_b + (1-\theta)(1-\pi)(1-\xi\eta) E_{s'|s}[\Delta V^b(s')] \right\} \tag{1.39}$$

Write $\boldsymbol{h_b} = [h_b^G, h_b^B]^\top$, $\boldsymbol{\Delta V^b} = [\Delta V^b(G), \Delta V^b(B)]^\top$. Then from equation (1.39), the solution for $\boldsymbol{\Delta V^b}$ is given by

$$\boldsymbol{\Delta V^b} = \frac{\hat{\beta}}{A_1}\boldsymbol{h_b} + \frac{\hat{\beta}(1-\theta)(1-\pi)(1-\xi\eta)}{A_1} \begin{pmatrix} -\pi_{BB} & 1-\pi_{GG} \\ 1-\pi_{BB} & -\pi_{GG} \end{pmatrix} \boldsymbol{h_b}, \tag{1.40}$$

where

$$\begin{aligned} A_1 = & \left(1 - \hat{\beta}(1-\theta)(1-\pi)(1-\xi\eta)\pi_{GG}\right)\left(1 - \hat{\beta}(1-\theta)(1-\pi)(1-\xi\eta)\pi_{BB}\right) \\ & - \hat{\beta}^2(1-\theta)^2(1-\pi)^2(1-\xi\eta)^2(1-\pi_{GG})(1-\pi_{BB}). \end{aligned}$$

Similarly, write $\boldsymbol{V_H^b(\rho)} = [V_H^b(\rho,G), V_H^b(\rho,B)]^\top$, $\boldsymbol{V_H^b(\rho)}$ will have the following analytically expression:

$$\boldsymbol{V_H^b(\rho)} = \frac{\hat{\beta}\theta\,(1-A_2)}{A_3}\rho - \frac{\hat{\beta}(1-\theta)\pi(1-\xi\eta)}{A_3}\begin{pmatrix}\pi_{GG}-A_2 & 1-\pi_{GG}\\ 1-\pi_{BB} & \pi_{BB}-A_2\end{pmatrix}\boldsymbol{\Delta V^b}, \tag{1.41}$$

where

$$A_2 = \hat{\beta}(1-\theta)(\pi_{GG}+\pi_{BB}-1),$$

$$A_3 = \left(1-\hat{\beta}(1-\theta)\pi_{GG}\right)\left(1-\hat{\beta}(1-\theta)\pi_{BB}\right) - \hat{\beta}^2(1-\theta)^2(1-\pi_{GG})(1-\pi_{BB}).$$

1.9.2 Solve $\Delta V(S,z,l',b')$

The value wedge $\Delta V(S,z,l',b')$ for bond investors is solved in the innermost loop.

Starting with an initial guess for $\Delta V^0(S,z,l',b')$, update $\Delta V^1(S,z,l',b')$ according to

$$\begin{aligned}\Delta V^1(S,z,l',b') =&\hat{\beta}h + \hat{\beta}(1-\lambda)(1-\pi)(1-\xi\eta)E_{S',z'|S,z}\left[F(\bar{\epsilon}_0')\Delta V^0(S',z',l_0(S',z',b'),b')\right]\\ &+\hat{\beta}(1-\pi)(1-\xi\eta)\left[1-\lambda F(\bar{\epsilon}_1')-(1-\lambda)F(\bar{\epsilon}_0')\right]E_{s'|S}\Delta V^b(s')\end{aligned}$$

This is repeated until convergence. In this equation, $\Delta V^b(s)$ is already given by equation (1.40). Default cutoffs $\bar{\epsilon}_0'$ and $\bar{\epsilon}_1'$ are given by

$$\bar{\epsilon}_0' = z'Y(S)^{\frac{1}{\gamma}}(l')^{\frac{\alpha(\gamma-1)}{\gamma}} - w(S)l' - w_m - cb',$$

$$\bar{\epsilon}_1' = z'Y(S)^{\frac{1}{\gamma}}(l')^{\frac{\alpha(\gamma-1)}{\gamma}} - w(S)l' - w_m - cb' - b' + M(S',z').$$

When iterating over $\Delta V(S,z,l',b')$ in the innermost loop, the functions from outer loops $w(S), Y(S)$, $l_0(S,z,b)$, and $M(S,z)$ are taken as given.

1.9.3 Solve $q(S, z, l', b')$ and $M(S, z)$

Next, an outer loop solves $q(S, z, l', b')$ and $M(S, z)$ iteratively. In this loop, I start with an initial guess for $q^0(S, z, l', b')$, and repeat steps 1-2 until convergence.

Step 1: given $q(S, z, l', b')$, solve $\{\bar{l}, \bar{b}\}$ according to

$$\frac{\partial q}{\partial l'}(S, z, \bar{l}, \bar{b}) = 0,$$

$$\frac{\partial q}{\partial b'}(S, z, \bar{l}, \bar{b})\bar{b} + q(S, z, \bar{l}, \bar{b}) = 0.$$

Update $M(S, z) = q(S, z, \bar{l}, \bar{b})\bar{b}$.

Step 2: Given $M(S, z)$, update $q(S, z, l', b')$ according to

$$q = \hat{\beta} E \left\{ \begin{array}{c} \lambda F(\bar{\epsilon}_1')(c+1) + (1-\lambda) F(\bar{\epsilon}_0')\left(c + q'(S', z', l_0'(S', z', b'), b')\right) - \pi(1-\xi\eta)\Delta V \\ +\lambda \int_{\bar{\epsilon}_1'} V_H^b(\rho, s) dF(\epsilon') + (1-\lambda)\int_{\bar{\epsilon}_0'} V_H^b(\rho, s) dF(\epsilon') \end{array} \right\}.$$

In this equation, note that $V_H^b(\rho, s)$ is already expressed analytically as a function of ρ in equation (1.41). The recovered value ρ is given by. Within each step of iteration, I solve the converged $\Delta V(S, z, l', b')$ using an inner loop as described in subsection 1.9.2. Again, functions from outer loops $w(S), Y(S)$ and $l_0(S, z, b)$ are taken as given.

1.9.4 Solve firm's problem

Next, an outer loop solves the firms' value function $V(S, z, x, b, \zeta)$ and policy functions $l_0(S, z, b)$, $l_1(S, z, x)$, and $b_1(S, z, x)$. For simplicity, define a value function $W(S, z, l', b')$ as the expectation of future value if the firm choose l', b'.

$$W(S, z, l', b') = E_{S', z'|S, z}\left[\iint^{\bar{\epsilon}'} V(S', z', x', b', \zeta') dF(\epsilon') d\psi(\zeta')\right]$$

Then the firm's problem can be written as

$$V(S,z,x,b,\zeta) = \max_{l',b'} x + \zeta q(S,z,l',b')b' + \beta W(S,z,l',b')$$

subject to

$$x + \zeta q(S,z,l',b')b' \geq 0$$

In this loop, I will start with an initial guess for $W(S,z,l',b')$ and repeat step 1-2 until converge.

Step 1: Given $W(S,z,l',b')$, solve $l_0(S,z,b)$ according to

$$\frac{\partial W(S,z,l',b)}{\partial l'} = 0,$$

As discussed in the main text, there exists a cutoff level of cash-on-hand, $\hat{x}$ such that for $x < \hat{x}$, the non-negative equity payout constraint is binding, and for $x \geq \hat{x}$, the constraint is slack and firm's choices do not vary with x, denoted by $\hat{l}_1(S,z)$ and $\hat{b}_1(S,z)$.

Solve $\{\hat{l}_1(S,z), \hat{b}_1(S,z)\}$ according to

$$\frac{\partial q(S,z,l',b')}{\partial l'}b' + \beta\frac{\partial W(S,z,l',b')}{\partial l'} = 0$$

$$\frac{\partial q(S,z,l',b')}{\partial b'}b' + q(S,z,l',b') + \beta\frac{\partial W(s,z,l',b')}{\partial b'} = 0$$

Then the cutoff level of cash-on-hand for the constraint to be binding is $\hat{x} = -q(S,z,\hat{l},\hat{b})\hat{b}$. For $-M(S,z) < x < \hat{x}$, solve $\{l_1(S,z,x), b_1(S,z,x)\}$ according to

$$\frac{\frac{\partial q(S,z,l',b')}{\partial l'}b'}{\frac{\partial q(S,z,l',b')}{\partial b'}b' + q(S,z,l',b')} = \frac{\frac{\partial W(S,z,l',b')}{\partial l'}}{\frac{\partial W(S,z,l',b')}{\partial b'}}$$

$$x + q(S,z,l',b')b' = 0$$

Step 2: Given policy functions $l_0(S,z,b)$, $l_1(S,z,x)$, and $b_1(S,z,x)$, update $W(S,z,l',b')$ according to

$$W(S,z,l',b') = E_{S',z'|S,z}\left\{\begin{array}{l} (1-\lambda)\int^{\bar{\epsilon}_0'}\left[x_0' + \beta W(S',z',l_0(S',z',b'),b')\right]dF(\epsilon') \\ +\lambda\int^{\bar{\epsilon}_1'}\left[\begin{array}{l} x_1' + q(S',z',l_1(S',z',x_1'),b_1(S',z',x_1'))b_1(S',z',x_1') \\ +\beta W(S',z',l_1(S',z',x'),b_1(S',z',x')) \end{array}\right]dF(\epsilon') \end{array}\right\}$$

Within each step of iteration, I solve the converged $q(S,z,l',b')$ and $M(S,z)$ using an inner loop as described in subsection 1.9.3. Again, functions from outer loops $w(S), Y(S)$ are taken as given.

1.9.5 Solve $w(S), Y(S)$

Finally, the outermost loop solves the aggregate variables $\{w(S), Y(S)\}$. Starting with an initial guess of $\{w^0(S), Y^0(S)\}$, in each iteration of this loop, I take as given the current aggregate rules, and solve the firm's optimization problem using an inner loop as described in subsection 1.9.4.

Then starting from an initial distribution of firms $\Upsilon_0(z,x,b,\zeta)$, I simulate the economy for $T = 1000$ periods using the solved policy functions. In each period, I use equations (1.42) and (1.43) to compute the wage and output.

$$\bar{C}^{\sigma}\left(\int l'(S,z,b,x,\zeta)d\Upsilon\right)^{\nu} = W(S) \tag{1.42}$$

$$Y(S) = \left[\int\left(\sum_{z'}\pi_z(z'|z)z'l(S,z,x,b,\zeta)^{\frac{\alpha(\gamma-1)}{\gamma}}\right)d\Upsilon\right]^{\frac{\gamma}{\gamma-1}} \tag{1.43}$$

The distribution of firms over state $\{z,x,b,\zeta\}$ when they are making choices for next period, $\Upsilon(S)$, is summarized by three arrays U_0, U_1 and F, which correspond to firms with $\zeta = 0$ and $\zeta = 1$, and defaulted firms, respectively. Matrix U_0 is two-dimension in which the first dimension correspond to productivity level z and the second dimension correspond to debt level b. Matrix U_1 is also two-dimension in which the first dimension correspond to productivity level z and the second dimension correspond to debt level x. Array F is one-dimension, corresponding to productivity level z. The distribution function $\Upsilon(S)$ is updated in each period as follows.

From U_0 For each grid (z_k, b_j) in U_0, I can compute its choice for next period $l' = l_0(S, z_k, b_j)$ and $b' = b_j$. Suppose the mass on this grid is g_0. I will first allocate it to each $(z_{k'}, b_j), k' = 1, ..., N_z$ in next period distribution matrix U'_0 according to

$$g_0(1-\lambda)\pi_{z_{k'}|z_k} F(\vec{\epsilon}'_0).$$

Then I will allocate it to each $(z_{k'}, x_{i'}), k' = 1, ..., N_z, i' = 1, ..., N_x$ in next period distribution matrix U'_1. At each level of $z_{k'}$, I compute the an array of $\{\tilde{\epsilon}_{i'}\}$ corresponding to $\{x_{i'}\}$ according to

$$\tilde{\epsilon}_i = z_{k'} Y^{\frac{1}{\gamma}} (l')^{\alpha(\frac{\gamma-1}{\gamma})} - wl' - w_m - cb' - b' - x_{i'}.$$

Then I assign to each $(z_{k'}, x_{i'})$ the mass given by

$$g_0 \lambda \pi_z(z_{k'}|z_k) \times \left(\int_{\tilde{\epsilon}_{i'+1}}^{\tilde{\epsilon}_{i'}} \frac{\epsilon' - \tilde{\epsilon}_{i'+1}}{\tilde{\epsilon}_{i'} - \tilde{\epsilon}_{i'+1}} dF(\epsilon') + \int_{\tilde{\epsilon}_{i'}}^{\tilde{\epsilon}_{i'-1}} \frac{\tilde{\epsilon}_{i'-1} - \epsilon'}{\tilde{\epsilon}_{i'-1} - \tilde{\epsilon}_{i'}} dF(\epsilon') \right).$$

Finally, I assign mass to each grid $z_{k'}$ in the distribution of defaulted firms in next period F' given by

$$g_0 \pi_z (z_{k'}|z_k) (1 - (1-\lambda)F(\vec{\epsilon}'_0) - \lambda F(\vec{\epsilon}'_1)).$$

From U_1 Similarly, for each grid (z_k, x_i) in U_1, I can compute its choice for next period $l' = l_1(S, z_k, x_i)$ and $b' = b_1(S, z_k, x_i)$. Suppose the mass on this grid is g_1. I will first allocate it to each $(z_{k'}, b_{j'}), k' = 1, ..., N_z, j' = 1, ..., N_b$ in next period distribution matrix U'_0

according to

$$g_1(1-\lambda)\pi_{z_{k'}|z_k}F(\bar{\epsilon}_0') \times \max\left(1-\left|\frac{b'-b_{j'}}{\Delta b}\right|,0\right).$$

Then I assign to each $(z_{k'}, x_{i'})$ the mass given by

$$g_1\lambda\pi_z(z_{k'}|z_k) \times \left(\int_{\tilde{\epsilon}_{i'+1}}^{\tilde{\epsilon}_{i'}} \frac{\epsilon'-\tilde{\epsilon}_{i'+1}}{\tilde{\epsilon}_{i'}-\tilde{\epsilon}_{i'+1}}dF(\epsilon') + \int_{\tilde{\epsilon}_{i'}}^{\tilde{\epsilon}_{i'-1}} \frac{\tilde{\epsilon}_{i'-1}-\epsilon'}{\tilde{\epsilon}_{i'-1}-\tilde{\epsilon}_{i'}}dF(\epsilon')\right)$$

. Finally I assign a mass to each grid $z_{k'}$ in the distribution of defaulted firms in next period F' given by

$$g_1\pi_z\left(z_{k'}|z_k\right)\left(1-(1-\lambda)F(\bar{\epsilon}_0')-\lambda F(\bar{\epsilon}_1')\right).$$

From F For each grid z_k in the distribution of defaulted firms, these are the defaulted firms in this period. They will be bought by households and enter as new entrants in next period. These new entrants have zero debt in place and abundant cash. Therefore, the mass on this grid, denoted by g_f will be assigned to the grid $(z_{k'}, x_{N_x})$ a mass of $g_f\pi_z\left(z_{k'}|z_k\right)$.

In this loop, I obtain a time series of $\{w(S), Y(S)\}$ and S in each period. I project $\{w(S), Y(S)\}$ onto a set of dummy variables corresponding to the state S, and update the aggregate rules using the fitted values. This procedure is repeated until the aggregate variables $\{w(S), Y(S)\}$ converged.

1.9.6 State grids and function interpolation

In the iterations described above, I iterate on a set of arrays. The policy functions for firms with $\zeta = 0$ are computed on grids (z_k, b_j), the policy functions for firms with $\zeta = 1$ are computed on grids (z_k, x_i), and the value functions $W(S, z, l', b')$ and bond price function $q(S, z, l', b')$ are computed on grids (z_k, l_n, b_j), where $k = 1, ..., N_z$, $j = 1, ..., N_b$, $i = 1, ..., N_x$ and $n = 1, ..., N_l$. I choose $N_z = 7$, $N_b = 30$, $N_x = 30$, and $N_l = 15$.

The policy function $l_0(S, z, b)$ is defined continuously on b, the policy functions $\{l_1(S, z, x), b_1(S, z, x)\}$ are defined continuously on x, and the value functions $W(S, z, l', b')$ and bond price function $q(S, z, l', b')$ are defined continuously onf $\{l', b'\}$. They are interpolated between grids using cubic splines.

The aggregate state S is approximated with lags of aggregate shocks $\{s, s_{-1}, s_{-2}, k\}$ following a version of Krusell and Smith (1998), where $k = 1, ..., \bar{k}$ denotes how many periods the aggregate state hasn't changed. I choose $\bar{k} = 12$, so that the total number of points for aggregate state S is $N_S = (2^3 - 2) + 2 \times 12 = 30$.

1.10 Appendix: Additional Results

This section provides additional results from the empirical experiments. Table 1.6 and Table 1.7 perform analysis similar to that in Table 1.1 and Table 1.2, respectively. In Table 1.1 and Table 1.2, I show how current refinancing needs and future refinancing needs impact firms' *employment* growth, while in Table 1.6 and Table 1.7, I show how current refinancing needs and future refinancing needs impact firms' *output* growth. In Table 1.8 and 1.9, I present the results from placebo tests for the years 2002-2006. In Table 1.1 and Table 1.2, I present the results from tests for the years 2001, 2007 and 2008. Combined with the results summarized in Table 1.8 and 1.9, we can see that in contrast to the strong refinancing effects in 2008, there is no such effects in all placebo period from 2001 to 2007.

Table 1.6: Annual sales growth rates for firms with different current refinancing needs

	Panel A: Drop in sales growth in 2008		
	2007	2008	2008-2007
Treated Firms	4.70	-1.90	-6.60*
	(2.07)	(2.65)	(3.37)
Non-treated Firms	7.12	7.05	-0.06
	(0.59)	(0.64)	(0.87)
Difference	-2.42	-8.96***	-6.54***
	(2.04)	(2.25)	(2.43)
	Panel B: Drop in sales growth in 2007		
	2006	2007	2007-2006
Treated Firms	8.08	4.86	-3.22
	(2.18)	(2.86)	(3.60)
Non-treated Firms	9.11	6.32	-2.80***
	(0.65)	(0.58)	(0.87)
Difference	-1.03	-1.46	-0.42
	(2.27)	(2.13)	(2.83)
	Panel C: Drop in sales growth in 2001		
	2000	2001	2001-2000
Treated Firms	8.43	-2.83	-11.26***
	(2.22)	(1.89)	(2.91)
Non-treated Firms	13.14	2.80	-10.34***
	(0.78)	(0.68)	(1.04)
Difference	-4.72**	-5.63***	-0.91
	(2.36)	(2.05)	(2.87)

Notes: This table presents the annual sales growth rates for firms with current refinancing needs (treated-firms) and firms without (non-treated firms). In each panel, the last row computes the differences between treated firms and non-treated firms, and the last column computes the changes between two consecutive years. The standard errors are reported in the parenthesis. ***, **, and * indicate significance at 1%, 5% and 10% levels from two tailed t-tests.

Table 1.7: Annual sales growth rates for firms with different future refinancing needs

	Panel A: Drop in sales growth in 2008		
	2007	2008	2008-2007
Treated Firms	10.38	1.30	-9.08***
	(2.21)	(2.45)	(3.30)
Non-treated Firms	6.84	7.54	0.70
	(0.61)	(0.66)	(0.90)
Difference	3.53	-6.24***	-9.77***
	(2.21)	(2.38)	(2.59)
	Panel B: Drop in sales growth in 2007		
	2006	2007	2007-2006
Treated Firms	5.78	4.80	-0.98
	(2.54)	(1.90)	(3.17)
Non-treated Firms	9.47	6.48	-2.99***
	(0.66)	(0.61)	(0.90)
Difference	-3.69*	-1.68	2.01
	(2.18)	(1.97)	(2.65)
	Panel C: Drop in sales growth in 2001		
	2000	2001	2001-2000
Treated Firms	11.72	1.47	-10.25***
	(2.16)	(1.87)	(2.86)
Non-treated Firms	13.42	3.06	-10.36***
	(0.84)	(0.73)	(1.11)
Difference	-1.70	-1.59	0.11
	(2.13)	(1.85)	(2.62)

Notes: This table presents the annual sales growth rates for firms with future refinancing needs (treated-firms) and firms without (non-treated firms). In each panel, the last row computes the differences between treated firms and non-treated firms, and the last column computes the changes between two consecutive years. The standard errors are reported in the parenthesis. ***, **, and * indicate significance at 1%, 5% and 10% levels from two tailed t-tests.

Table 1.8: Annual employment growth rates for firms with different current refinancing needs

	2005	2006	2006-2005
Treated Firms	3.87	3.18	-0.70
	(1.87)	(1.66)	(2.50)
Non-treated Firms	3.38	2.89	-0.48
	(0.54)	(0.53)	(0.76)
Difference	0.50	0.28	-0.21
	(1.89)	(1.84)	(2.44)
	2004	2005	2005-2004
Treated Firms	1.37	0.71	-0.67
	(1.16)	(1.73)	(2.08)
Non-treated Firms	2.44	3.07	0.63
	(0.55)	(0.53)	(0.76)
Difference	-1.07	-2.36	-1.29
	(1.65)	(1.65)	(2.12)
	2003	2004	2004-2003
Treated Firms	-0.82	2.58	3.41
	(1.42)	(1.54)	(2.11)
Non-treated Firms	0.10	2.44	2.34***
	(0.47)	(0.51)	(0.70)
Difference	-0.92	0.15	1.07
	(1.50)	(1.63)	(2.11)
	2002	2003	2003-2002
Treated Firms	-3.32	-0.32	3.00*
	(1.30)	(1.06)	(1.68)
Non-treated Firms	-0.77	-0.14	0.63
	(0.54)	(0.49)	(0.73)
Difference	-2.54	-0.17	2.36
	(1.61)	(1.45)	(2.01)
	2001	2002	2002-2001
Treated Firms	-4.52	-2.62	1.90
	(1.67)	(1.62)	(2.33)
Non-treated Firms	-2.24	-2.13	0.12
	(0.62)	(0.57)	(0.84)
Difference	-2.27	-0.49	1.78
	(1.78)	(1.63)	(2.23)

Notes: This table presents the annual employment growth rates for firms with current refinancing needs (treated-firms) and firms without (non-treated firms). In each panel, the last row computes the differences between treated firms and non-treated firms, and the last column computes the changes between two consecutive years. The standard errors are reported in the parenthesis. ***, **, and * indicate significance at 1%, 5% and 10% levels from two tailed t-tests.

Table 1.9: Annual employment growth rates for firms with different future refinancing needs

	2005	2006	2006-2005
Treated Firms	2.72	4.46	1.73
	(1.24)	(1.51)	(1.95)
Non-treated Firms	3.44	2.73	-0.71
	(0.58)	(0.56)	(0.81)
Difference	-0.72	1.73	2.45
	(1.86)	(1.82)	(2.40)
	2004	2005	2005-2004
Treated Firms	3.36	4.17	0.82
	(1.51)	(1.40)	(2.06)
Non-treated Firms	2.32	2.92	0.60
	(0.59)	(0.57)	(0.82)
Difference	1.04	1.25	0.21
	(1.70)	(1.64)	(2.15)
	2003	2004	2004-2003
Treated Firms	-0.17	3.72	3.89**
	(1.11)	(1.56)	(1.92)
Non-treated Firms	0.14	2.22	2.08***
	(0.52)	(0.54)	(0.75)
Difference	-0.31	1.50	1.81
	(1.34)	(1.46)	(1.89)
	2002	2003	2003-2002
Treated Firms	-1.46	0.60	2.06
	(1.63)	(1.14)	(1.99)
Non-treated Firms	-0.64	-0.28	0.36
	(0.57)	(0.55)	(0.79)
Difference	-0.81	0.88	1.69
	(1.49)	(1.35)	(1.86)
	2001	2002	2002-2001
Treated Firms	-5.27	-1.92	3.34
	(1.49)	(1.51)	(2.12)
Non-treated Firms	-1.62	-2.17	-0.55
	(0.68)	(0.61)	(0.92)
Difference	-3.64**	0.24	3.89*
	(1.66)	(1.50)	(2.05)

Notes: This table presents the annual employment growth rates for firms with future refinancing needs (treated-firms) and firms without (non-treated firms). In each panel, the last row computes the differences between treated firms and non-treated firms, and the last column computes the changes between two consecutive years. The standard errors are reported in the parenthesis. ***, **, and * indicate significance at 1%, 5% and 10% levels from two tailed t-tests.

CHAPTER 2

Bonds vs. Equities: Information for Investment

with Adrien d'Avenas and Andrea L. Eisfeldt

2.1 Introduction

Why do credit spreads predict economic activity better than equity market measures? We argue that this is because of the precise non-linear transformation of leverage and asset volatility that credit spreads represent. Both credit spreads and equity market measures are driven by leverage and asset volatility. In particular, equity volatility is levered asset volatility. However, while higher credit spreads predict lower investment for all firms, equity volatility is an ambiguous signal for investment. This is due to the position of equity relative to debt in firms' capital structure. For healthy firms, higher equity volatility signals greater option value and better investment opportunities. But for more distressed firms greater equity volatility exacerbates the debt overhang problem because not all of the marginal returns to investment accrue to equity holders.

Our study speaks to two important literatures linking investment decisions to asset prices, namely the literatures on the predictive power of credit spreads and on the role of uncertainty in determining investment. Economists and practicioners alike have long argued that there is a tight connection between bond markets and the macroeconomy. Friedman and Kuttner (1992) show that the spread between commercial paper and Treasury bills forecasts recessions. Gilchrist and Zakrajšek (2012) use firm-level data to construct a credit spread measure with substantial predictive power for consumption, inventories, and output. Philippon (2009) constructs a credit-spread measure of Tobin's q and shows that it outperforms traditional q in predicting firm-level investment.[1] At the same time, a large and growing literature emphasizes the link between equity volatility and investment. Bloom (2009) shows that shocks to uncertainty measured using implied equity volatility forecast lower investment.[2]

We show why credit spreads perform better than equity volatility in predictive regressions for firm-level investment. In doing so, we explain the finding in Gilchrist, Sim, and Zakrajšek (2014) that controlling for credit spreads substantially reduces the predictive power of equity volatility for investment.[3] We also clarify the distinct information in different measures of firm-level volatility that have been used extensively in the literature on uncertainty and investment. We show that *asset* volatility is an unambiguously *positive* signal for investment. However, our results are not a challenge to the wait-and-see mechanism of Bloom (2009) or Alfaro, Bloom, and Lin (2018). Those papers study the effect of a change in volatility. In the data, it appears that firms' sensitivity to the level of asset volatility is positive, while shocks to volatility have a temporary negative effect. It is crucial to distinguish between equity and asset volatility, and between levels and changes in studies of the relation between volatility and investment.

The link between equity and debt in structural models of credit risk can be used to understand the difference between debt and equity market signals for investment. We build on the the seminal work of Merton (1974) and Leland (1994). In these models bond spreads and equity volatility are tightly related and it may be surprising that credit spreads and

[1]See also the important contributions by Friedman and Kuttner (1998), Stock and Watson (1989), Bernanke (1990), Gertler and Lown (1999), and Gilchrist, Yankov, and Zakrajšek (2009), Giesecke, Longstaff, Schaefer, and Strebulaev (2014), Krishnamurthy and Muir (2017).

[2]See Panousi and Papanikolaou (2012) for related evidence that the negative relation between idiosyncratic equity volatility and investment is stronger when managerial ownership is higher.

[3]Gilchrist, Sim, and Zakrajšek (2014) emphasize the role of financial frictions in exacerbating negative effects from uncertainty on investment. See also Christiano, Motto, and Rostagno (2014) and Arellano, Bai, and Kehoe (2019).

equity volatility have different predictions. Indeed, Atkeson, Eisfeldt, and Weill (2017) show theoretically that under very minimal assumptions the inverse of equity volatility is bounded above by distance to insolvency and below by distance to default. Empirically that paper shows that the (the inverse of) equity volatility and credit spreads have a tight log-linear relationship.[4] We emphasize that even if credit spreads and equity volatility contain similar information about firms' financial soundness, as long as debt and equity holders are not united option values and debt overhang problems drive a wedge between debt and equity market signals for investment.

The model and empirical motivation in Philippon (2009) also recognizes the structural relationship between debt and equity claims in predicting investment. That paper emphasizes the relationship between Tobin's q and credit spreads whereas our focus is on equity volatility and credit spreads. The model in Philippon (2009) cannot be used to understand our findings that the sensitivity of investment to equity volatility changes sign in the cross section because in that model leverage does not affect firm value or investment (the Modigliani and Miller (1958) theorem holds). On the other hand, our model can be used to explain why credit spreads negatively predict investment (and, the inverse of credit spreads positively predicts investment).[5]

Our model embraces the fact that bonds capture downside risk better while equity prices are more affected by growth options. We study the relationship between equity volatility and credit spreads—the two most widely-used measures of risk based on equity and bond markets data—and provide robust empirical evidence and a model of investment with debt overhang that support the fact that it is precisely this difference that explains why bond market data appears to have better forecasting power for real outcomes than equity market

[4] See also Campbell and Taksler (2003) which shows that idiosyncratic equity volatility explains as much of the cross-sectional variation in bond yields as credit spreads do.

[5] See Proposition 2 in Philippon (2009) expressing q as approximately equal to $\frac{\psi}{\delta(1+r)}\frac{1+r_t}{1+y_t}$ where r is the risk free rate, y is the coprorate bond yield, ψ is leverage and δ is the risk-neutral default rate. Figure I presents numerical results for the full model.

data. We also empirically rule out the notion that bond markets predict investment better because they have more "smart money".

We establish four main empirical facts. First, as documented by Gilchrist, Sim, and Zakrajšek (2014), credit spreads drive out equity volatility in an empirical model of the sensitivity of firm-level investment to equity volatility and credit spreads. But, this result is due to systematic heterogeneity in the elasticity of investment to equity volatility in the cross section of firms. The elasticity of investment to equity volatility is positive for firms far enough away from default, and negative otherwise. These different signs in the cross section drive the pooled effect of equity volatility to be less significant than credit spreads. By contrast, the elasticity of investment to credit spreads is always negative. Our model is consistent with the interpretation in Gilchrist, Sim, and Zakrajšek (2014) that financial frictions are important for understanding the equity and bond market information. Our model and empirical evidence support the idea that the fact that the interests of debt and equity holders are not aligned is the key friction driving our results.

Second, we show that reason credit spreads predict investment better than equity volatility is not due to bond markets having more "smart money." To do this, we repeat the above analysis using credit spreads constructed using equity market data, leverage ratios and historical default rates as inputs into a structural model.[6] The results using these fair-value spreads based on equity-market data are virtually identical to those using bond-market spreads.

Third, both equity volatility and credit spreads are in large part driven by asset volatility and leverage, as predicted by structural models of credit risk.[7] However, credit spreads have higher loadings on leverage, while equity volatility loads more on asset volatility. This is

[6]See Arora, Bohn, and Zhu (2005) and Nazeran and Dwyer (2015).

[7]Collin-Dufresn, Goldstein, and Martin (2001) show that *changes* in credit spreads also have a common component that appears unrelated to structural determinants, however we show the majority of variation in credit spread levels, and about one third of credit spread changes, can be explained by asset volatility and financial leverage.

intuitive given the priority of debt versus equity in firms' capital structures and, together with our fourth fact, is helpful for understanding why equity volatility might positively impact investment decisions.

Fourth, the sensitivity of investment to asset volatility is positive for all firms. At least two interpretations of that novel result are possible. First, as in our model, asset volatility can boost the option value of equity, alleviate the debt overhang effect, and incentivize equity holders to invest more (an causal channel). Alternatively, the uncertainty from future investment could feed back into the volatility of current asset values (an endogeneity channel). We show using a Granger causality test, using subsamples of R&D-intensive firms, and using instrumental variables, that the first explanation is more likely.[8]

We build a simple model of investment to study the option value of asset volatility and the debt overhang channel of credit spreads. Because of the debt overhang effect, equity holders choose a suboptimal level of investment. An increase in asset volatility has the potential to boost the option value for equity holders as, in the presence of debt with limited liability, they face limited downside but unlimited upside. We show that, controlling for credit spreads, an increase in asset volatility always has a positive effect on investment. In contrast, equity volatility is an ambiguous signal for investment, as an increase in equity volatility can reflect an increase in leverage or an increase in asset volatility, which have opposite impacts on equity holders' incentives to invest. Interestingly, we show that controlling for asset volatility and leverage instead of asset volatility and credit spreads also leads to asset volatility being an ambiguous signal for investment, a prediction of the model that we confirm in the data.

To document the importance of our findings for understanding the role of uncertainty and credit spreads on aggregate activity, we plot the time series and cross section of the estimated firms' elasticity of investment with respect to equity volatility in Figure 2.4. Firms with lower credit spreads which are further away from default display a positive elasticity of

[8] We follow the IV strategy in Alfaro, Bloom, and Lin (2018).

investment, while firms with higher credit spreads display a negative elasticity. Aggregate effects are driven by the movement of the entire cross section of firms away from and closer to their respective default boundaries. Thus, a positive shock to equity volatility has a particularly dire impact on investment when the entire cross section of firms is closer to default. In contrast, Figure 2.5 shows that the elasticity of investment to credit spreads is negative for all firm-quarters. We also confirm that our micro-results aggregate with a recursive vector autoregression model of the aggregate time series of investment, asset volatility, and credit spreads. As expected, the aggregate investment response to a positive shock to asset volatility is positive while the response to a positive shock to credit spreads is negative.

The remainder of the paper is organized as follows. Section 2.2 presents our firm-level empirical results. In Section 2.3, we show that our results hold at the aggregate level. Section 2.4 presents our model to build economic intuition. Section 2.5 concludes.

2.2 Firm-level Panel Regressions

In this section, we establish our four main stylized facts. First, we show that credit spreads drive out equity volatility in a horse-race to predict firm-level investment rates. Second, we show that this result is not due to information in bond markets, but is instead due to the non-linear transformation of asset volatility and leverage that credit spreads represent. To do this, we use credit spreads constructed from equity market data and a structural model. Third, we show that credit spreads load more on leverage while equity volatility loads more on asset volatility. Finally, we show that the sensitivity of investment to asset volatility is positive for all firms.

We use S&P's Compustat quarterly database from 1984 to 2018. To compute equity volatility, we use daily returns from the Center for Research in Security Prices (CRSP) database or implied volatility from Option Metrics (1996-2018). Bond prices come from the Lehman/Warga (1984-2005) and ICE databases (1997-2018). Appendix 2.8 contains detailed definitions for each variable we study. Our main sample contains 1,273 unique firms and 42,580 firm-quarter observations. Table 2.1 provides summary statistics.

To establish our four key stylized facts, we start by presenting a set of firm-level panel regressions of investment rates on measures of volatility and credit spread:

$$\log[I/K]_{i,t} = \beta_1 \log X^{\sigma}_{i,t-1} + \beta_2 \log X^{cs}_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}, \tag{2.1}$$

where $\log[I/K]_{i,t}$ is the log of investment rate of firm i in period t, $X^{\sigma}_{i,t-1}$ denotes measures of volatility (such as idiosyncratic equity volatility $\sigma^{e}_{i,t-1}$ or idiosyncratic asset volatility $\sigma_{i,t-1}$), and $X^{cs}_{i,t-1}$ denotes measures of credit risk (such as credit spread $cs_{i,t}$, fair value spreads $\hat{cs}_{i,t-1}$, or market leverage $[MA/ME]_{i,t-1}$), all lagged by one quarter. We control for firm and time fixed effects by including η_i and λ_t. Following Alfaro, Bloom, and Lin (2018), our control variables $\mathbf{X}_{i,t-1}$ include the lag of firm i's return on equity, log tangibility, log sale ratio, log income ratio, and log Tobin's q.[9]

Equity Volatility and Credit Spread We first replicate the results in Gilchrist, Sim, and Zakrajšek (2014) that the adverse effect of idiosyncratic equity volatility on investment is dampened when controlling for credit spreads. Table 2.2 presents the estimation results of equation (2.1) using equity volatility and credit spread as control variables.

As shown in Columns 1-3 of Table 2.2, the coefficient on idiosyncratic equity volatility and credit spread are statistically significant and economically important on their own (Columns 1-2). However, when both measures are included in the regression, the coefficient on equity volatility is substantially reduced both in terms of magnitude and statistical significance while the coefficient on credit spread is unaffected (Column 3).

[9]Controlling for return on assets instead of return on equity when using asset volatility measures does not change any of our results.

To see why bond spreads can drive out equity volatility, we sort firms into tercile groups based on credit spread each quarter and run the same regression for these subsamples (Columns 4-6).[10] We find that the coefficient on equity volatility changes sign in the cross section: it is significantly positive among firms with low credit spread and significantly negative among firms with high credit spread. Column 7 shows results from the regression with an interaction term and confirms our findings from Columns 4-6. The last column shows that the result in Column 7 is robust to adding control variables. A simple back-of-the-envelope calculation using estimations in Column 7 suggests that the sign flip happens at a credit spread level of 181 basis points.

In Table 2.3, we replace credit spreads with fair value spreads. The results are qualitatively identical to Table 2.2. The coefficient on equity volatility goes from significantly positive to significantly negative as firm's credit spread goes up, while the coefficient on the fair value spread remains significantly negative across the subgroups. As the fair value spreads are constructed with only equity market information and does not contain bond market information, the results from Table 2.3 cannot be driven by differences in the investor base or information about financial frictions only reflected in credit spreads.

Asset Volatility and Credit Spread Equity volatility can be decomposed into asset volatility and market leverage. Here we construct asset volatility by first unlevering equity returns with market leverage and then computing the idiosyncratic volatility of these unlevered returns. Later we will show that our results are robust to using other measures. In Table 2.4, we run the same regression but we replace idiosyncratic equity volatility $\sigma^e_{i,t}$ with idiosyncratic asset volatility $\sigma_{i,t}$. The coefficient on asset volatility is always positive and statistically significant in the full sample and in all subgroups.

In our model, equity holders make investment decisions given uncertainty about future

[10]This method of splitting uses quarter-specific cutoffs. Using fixed cutoffs to sort all firm-quarter observations leads to similar results.

returns. In Table 2.5, we replicate the same exercise but with implied asset volatility from equity options. The results are economically stronger[11] than the results with volatility derived from past equity return observations, lending support to the idea that it is the expectation of future asset volatility that drives changes in investment, not past uncertainty.[12]

Given the decomposition of equity volatility, a natural question is whether the coefficient on asset volatility is also positive when market leverage is used as an additional explanatory variables for investment. As shown in Table 2.6, the coefficient on asset volatility changes sign in the cross-section when the level of credit spread is not controlled for. In the model section, we rationalize this finding by showing that, together, leverage and asset volatility are not unambiguous signals of debt overhang and option value.

Loadings Asset volatility and leverage are also important drivers for bond spreads. To understand why there is no such sign flip for credit spreads, we consider the loadings of credit spreads and equity volatility on asset volatility and leverage and estimate the following equation:

$$\log y_{i,t} = \beta_1 \log \widetilde{\sigma}_{i,t} + \beta_2 \log[MA/ME]_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t},$$

where $y_{i,t}$ is either the equity volatility ($\sigma^e_{i,t}$) or the credit spread ($cs_{i,t}$), and $[MA/ME]_{i,t}$ is market leverage.[13] We estimate the equation both in levels and in first differences.

Table 2.7 summarizes the results. Columns 1-2 show how the levels of equity volatility and credit spread load on levels of asset volatility and leverage, and columns 3-4 show how the changes load on corresponding changes. Both specifications imply that changes in bond spreads are mainly driven by leverage, while changes in equity volatility are driven by both

[11]The coefficient is twice as large and statistically more significant when we restrict the regression in Table 2.4 to firms with observable implied asset volatility. See Table 2.22.

[12]Lettau and Ludvigson (2002) emphasize the information about future investment returns contained in asset prices.

[13]We use asset volatility derived from Merton's model $\widetilde{\sigma}_{i,t}$ instead of $\sigma_{i,t}$ such that the decomposition in levels is not mechanical.

asset volatility and leverage. Since shocks to asset volatility and leverage impact investment differently, bond spreads and equity volatility contain different information for investment. In Appendix 2.10, instead of using the firm's asset volatility and market leverage directly, we use the industry-level regressors, constructed as a simple average of all firms in the same industry excluding the firm itself. This exercise shows similar patterns that equity volatility loads more on asset volatility while credit spread loads more on leverage.

Thus, an increase in equity volatility could either signal an increase in asset volatility (positive for investment) or in leverage (negative for investment). Whether one force dominates the other changes in the cross-section. As seen in Table 2.4, the asset volatility effect weakens as firms' credit spreads increase, while the leverage effect strengthens.[14] Thus, the sensitivity of investment to equity volatility becomes negative when the leverage effect dominates for firms with higher credit spreads. Although credit spreads are also a combination of asset volatility and leverage, the loading of credit spreads on asset volatility is not large enough to ever drive a positive relation between credit spreads and investment.

In light of this decomposition, we consider another reduced-form measure of asset volatility: the residual of a panel regression of the log of idiosyncratic equity volatility on the log of market leverage with time and firm fixed effects, denoted $\mathring{\sigma}_{i,t}$. Thus, this measure captures the changes in equity volatility that are orthogonal to changes in leverage. Table 2.8 confirms again that once we control for changes in leverage, an increase in volatility is associated with an increase in future investments.

Our third alternative measure of asset volatility is given by the asset volatility derived from Merton's model. Table 2.9 confirms again that our results are robust to using different measures of asset volatility.[15]

[14] The statistical significance of not only the credit spreads coefficient in Table 2.4 but also the leverage coefficient in Table 2.6 strengthens when credit spreads are higher.

[15] All of our regression results hold with asset volatility derived from Merton's model.

Tobin's q The q theory of investment predicts a strong relation between firms' market values and investment. While earlier work has shown that a simple regression of investment on Tobin's q performs quite poorly.[16] Philippon (2009) proposes a new aggregate measure of Tobin's q derived from bond credit spreads and shows that it dramatically outperforms the equity market based measure of Tobin's q. In Table 2.10, we compare the performance of Tobin's q to predict investment with the performance of credit spread and asset volatility. At the firm level, Tobin's q is a strong predictor of investment rates and is not subsumed by credit spreads. As more than 80% of our observations come from after 1995, our results are aligned with Andrei et al. (2019) that show that the relation between aggregate investment and Tobin's q has become remarkably tight after 1995.

Zero-Leverage Firms An important restriction of our analysis so far is that we focus on the subset of firms with observable credit spreads. In Columns 1, 2, 4, and 5 of Table 2.11, we show that our results are still valid for firms without observable credit spreads: the elasticity of investment with respect to equity volatility switches sign in the cross-section when controlling for their distance-to-default. In Columns 3 and 6 of Table 2.11, we see that for zero-leverage firms (for which equity volatility and asset volatility are equivalent), the impact of equity/asset volatility is either negative or insignificant, consistent with our results being driven by the presence of debt overhang.

2.2.1 Endogeneity

We hypothesize that the positive correlation between investment and asset volatility is most likely driven by two mechanisms: (i) due to higher investments, the value of the assets of the firm become more uncertain and (ii) an increase in business risk makes the value of assets in place more volatile and incentivizes firms to invest more. In this section, we document four

[16] For examples, see Fazzari, Hubbard, and Petersen (1988), Kaplan and Zingales (1997), Gilchrist and Himmelberg (1995), Erickson and Whited (2000), Gomes (2001), Cooper and Ejarque (2003), Moyen (2004), Abel and Eberly (2011), and Peters and Taylor (2017) among others.

tests that lend support for the later mechanism dominating the former.

Lags and Investment Intensity If asset volatility increases because of uncertainty driven by more investment in place, we would expect that (i) an increase in investment to Granger-cause an increase in asset volatility and (ii) that the correlation is more pronounced for firms with higher levels of research and development or investment. Table 2.12 shows that the coefficients on asset volatility are statistically and economically larger to explain the variation in investment with lags rather than leads. This pattern is even stronger when using Merton's asset volatility measure or implied asset volatility (see Table 2.24 and Table 2.25). Table 2.13 shows that our results are weaker for firms with higher levels of research and development or investment. Thus both exercises suggest that this potential mechanism is not driving our results.

Instrumental Variables We follow closely the instrumentation strategy of Alfaro, Bloom, and Lin (2018) to address endogeneity in estimating the impact of equity and asset volatility on investment. First, we estimate sensitivities to energy, currencies, treasuries, and policy at the industry level as the factor loadings of a regression of a firm's daily stock return on the price growth of energy, 7 currencies, return on treasury bonds, and changes in daily policy uncertainty from Baker, Bloom, and Davis (2016). That is, for firm i in industry j, the sensitivity β_j^c is estimated as follows:

$$r_{i,t} = \alpha_j + \sum_c \beta_j^c \cdot r_t^c + \varepsilon_{i,t},$$

where $r_{i,t}$ is the daily risk-adjusted return on firm i, r_t^c is the change in the price of commodity c, and α_j is industry j's intercept.

The risk-adjusted returns $r_{i,t}$ are the residuals from running firm-level time-series regressions of daily CRSP stock returns on the classical Carhart (1997) four-factor asset pricing model. When constructing instruments for equity volatility, we use equity risk-adjusted returns, while we use delevered equity risk-adjusted returns when constructing instruments for asset volatility. We estimate the betas for the risk-adjusted returns and the sensitivity β_j^c yearly and using the same 10-year window.

Next, for these 10 aggregate market price shocks (oil, 7 currencies, treasuries, and policy) we multiply the absolute value of their time-varying sensitivities $|\beta_j^c|$ by their implied volatilities σ_t^c. This provides 10 instruments for lagged firm-level uncertainty, as follows:

$$z_{i,t-1}^c = |\beta_j^c| \cdot \sigma_{t-1}^c.$$

We refer the reader to Alfaro, Bloom, and Lin (2018) for further details on the construction of the instrumental variables. The key differences with their regressions is that we estimate the impact of the level of volatility on the level of investment and not the impact of shocks to volatility. Thus, we construct instruments for the level of volatility rather than volatility shocks.

Importantly, our results—a higher level of asset volatility is associated with a higher level of investment—are compatible with the results of Alfaro, Bloom, and Lin (2018)—a positive shock to uncertainty is associated with a lower level of investment. Indeed, while firms might postpone their investment following a large temporary uncertainty shocks, they can still invest more when the level of uncertainty is persistently higher. We illustrate that point in Table 2.14 by adding the shock to implied asset volatility $\Delta \log \widehat{\sigma}_{i,t-1} = \log \widehat{\sigma}_{i,t-1} - \log \widehat{\sigma}_{i,t-2}$ as an additional control variable similar to the uncertainty shock of Alfaro, Bloom, and Lin (2018).[17]

In Table 2.15, we show the results of our instrumental variable regression. Our main results hold: asset volatility has an unconditional positive impact on investment. Note however that the low Kleibergen-Paap F-statistic indicates that the excluded instruments

[17] When using asset volatility instead of implied asset volatility, the shock to asset volatility is not significant anymore as shown in Table 2.26.

are correlated with the endogenous regressors, but only weakly.

Covenant Tightness As we are attributing the main force driving our results to debt overhang, we expect this coefficient to be stronger when debt holders have tighter control over cash flows resulting from investment. To test this hypothesis, we split the sample into two groups according to its covenant tightness following the measurements of Kermani and Ma (2020). The observations with the overall measure of distance between actual financial ratios and covenant thresholds below median are placed in the "Tight Covenant" group, and the remaining are assigned to the "Slack Covenant" group. We estimate equation (2.1) with the interaction term using the two subsamples. The results are summarized in Table 2.16. For the subsample with tight covenant, all the coefficients are larger in absolute value, and are statistically more significant. This exercise provides empirical support for our model with debt overhang as a key distorting force.

2.2.2 Excess bond premium

Our discussion emphasizes that credit spreads are in large part driven by asset volatility and leverage, while a long-standing literature points out that a nontrivial fraction of credit spreads cannot be explained by credit risk. In particular, Gilchrist and Zakrajšek (2012) decompose the aggregate credit spread into two components: a component that captures the movements in default risk based on fundamentals (the predicted component) and a residual component (the excess bond premium). They show that, in the aggregate, the excess bond premium has substantial predictive content for future economic activity and outperform the predicted component of credit spreads.

To speak to this literature, we construct the excess bond premium following the methodology of Gilchrist and Zakrajšek (2012). First, we estimate the following panel regression:

$$\log cs_{i,m}[k] = \boldsymbol{\gamma}'\mathbf{X}_{i,m}[k] + \epsilon_{i,m}[k],$$

where the log of credit spread on bond k issued by firm i in month m is regressed on a vector of bond-specific characteristics $\mathbf{X}_{i,m}[k]$ for bond k issued by firm i.[18] We then build a firm-level quarterly excess bond premium as the quarterly average of the residuals for all bonds issued by the firm during that quarter: $\log(ebp_{i,q}) = \frac{1}{3}\sum_{m=q_1}^{q_3} \frac{1}{N_{i,m}^k} \sum_{k=1}^{N_{i,m}^k} \epsilon_{i,m}[k]$, where q_n is the nth month of quarter q and $N_{i,m}^k$ is the number of bonds of firm i in month m.

As shown in Table 2.17, the firm-level excess bond premium is in itself a strong predictor of investment. This is consistent with the notion that an increase in the firm-level excess bond premium reflects an increase in the cost of capital of the firm and, as a result, a contraction in future investments. Interestingly, the impact of asset volatility is marginally stronger when controlling for the firm-level excess bond premium.

2.3 Aggregates

Time Series To understand the implications of our findings for time series, we plot the elasticity of investment rate with respect to equity volatility, asset volatility, and credit spread across time and across firms using the estimates from the regressions with interaction terms. In Figure 2.4, we compute the overall coefficient on equity volatility at each credit spread level using estimates on equity volatility ($\log \sigma_{i,t}^e$) and the interaction term ($\log \sigma_{i,t}^e \times \log s_{i,t}$) reported in the last column of Table 2.2. We repeat the procedure for credit spreads in Figure 2.5. Figure 2.4 shows that the cross section of elasticities of investment with respect to equity volatility varies a lot over time. In particular, this coefficient is negative for the whole cross-section of firms during the Great Recession, while it is mainly positive in the late 1980s. By contrast, in Figure 2.5, the elasticity of investment to credit spreads remains

[18] The bond characteristics $\mathbf{X}_{i,m}[k]$ include the firm's distance-to-default, bond's amount outstanding, duration, coupon rate, and an indicator variable for callable bonds. It also includes the interactions of callability with these bond characteristics, firm's distance-to-default, the level, slope, and curvature of the Treasury yield curve, as well as the realized monthly volatility of the daily ten-year Treasury yield, reflecting the value of the call option embedded in callable bonds. The industry fixed effects and credit rating fixed effects are included as well.

negative, both in the cross-section and over time.

VAR Analysis Using an identified vector autoregression (VAR) framework, we confirm that our micro-level result—asset volatility has positive impact on investment—still holds at the macro-level. We aggregate the variables in our sample and estimate a simple VAR consisting of the three endogeneous variables: the log of idiosyncratic asset volatility ($\log \sigma_t$), the log of credit spread ($\log s_t$), and the log of investment rate ($\log[I/K]_t$).[19] We employ a standard recursive ordering technique and consider two identification schemes, one in which credit spread has an immediate impact on asset volatility and the other where asset volatility has immediate impact on credit spread.

Figure 2.6 reports the impulse responses of investment rate to credit spread and asset volatility using the two specifications. Credit spread has a negative impact on investment while asset volatility has a positive impact. As shown in panel (a) and (c) of Figure 2.6, the positive impact of asset volatility on investment is economically and statistically larger in the first specification, highlighting the importance of controlling for credit spread for asset volatility to be a strong positive signal for investment in the aggregate.

2.4 Investment Decisions with Debt Overhang

In this section, we develop a simple but general credit risk model to analyze the investment choices of a firm with outstanding debt already in place. Two forces drive the investment decision: debt overhang and the option value of equity. We demonstrate that credit spreads and asset volatility are jointly unambiguous signals of these two forces. However, the signals provided by leverage and equity volatility are ambiguous and can change in the cross-section. All proofs are relegated to Appendix 2.9. For ease of notation, we sometimes write $f_x(x) \equiv$

[19] We use the value-weighted average of $\sigma_{i,t}$, $s_{i,t}$ and $[I/K]_{i,t}$ to generate the corresponding aggregate time series. We seasonally adjust the investment rate time series by subtracting a seasonal average computed over the previous five years. All variables are detrended using the HP filter with weight 1600.

$\frac{\partial f(x)}{\partial x}$.

Consider a firm has funded itself partly with debt. In the first period, shareholders choose how much capital to invest in the firm subject to convex costs. In the second period, a random productivity shock is realized, and, after observing the payoff of their investment, shareholders decide whether to file for bankruptcy or not. For our basic argument, we make the following assumptions regarding the firm and its investments.

Assumption 1 (Investments). *The firm has the option to invest in capital with a final value of iz, which is a function of investment i and a random productivity shock z realized in the second period. The convex function $\phi(i)K$ captures the total cost of investment.*

Assumption 2 (Firm Liabilities). *The firm is funded by equity, together with a debt claim with total face value B that is due in the second period when the asset returns are realized. In the second period, shareholders decide whether to default. Upon bankruptcy, the entirety of the firm's value is lost. Furthermore, shareholders cannot liquidate the firm ($i \geq 0$).*

We show that our results are robust to a relaxation of Assumption 2 featuring complete or partial recovery of the firms' assets upon bankruptcy in Appendix 2.9.

Assumption 3 (Pricing). *All securities are traded in perfect Walrasian markets. We normalize the risk-free interest rate to zero and set prices of securities equal to their expected payoff with respect to a risk-neutral distribution $F(z;\sigma)$ of firm's asset productivity z with full support on $[0,\infty)$.*

Given our assumptions about payouts and pricing, it follows that the value of equity e and debt d are given by:

$$e(b,\sigma) = \max_{i,\underline{z}} \int_{\underline{z}}^{\infty} (iz - b)dF(z;\sigma) - \phi(i),$$
$$d(b,\sigma) = (1 - F(\underline{z}(b,\sigma);\sigma))b.$$

The first order conditions for investment i and the default threshold $\underline{z}$ imply that, at an optimum, i and $\underline{z}$ satisfy:

$$\int_{\underline{z}}^{\infty} z dF(z;\sigma) = \phi_i(i), \tag{2.2}$$

$$i\underline{z} = b. \tag{2.3}$$

The credit spread of the firm is defined as $cs(\underline{z},\sigma) \equiv F(\underline{z};\sigma)/(1-F(\underline{z};\sigma))$.[20] We define book leverage as b.[21] To streamline our analysis, we also make assumptions on the distribution of productivity shocks, $F(z;\sigma)$, which are satisfied with the Black–Scholes–Merton model used in the calculation of the Greek risk measures and most risk distributions usually considered in finance.

Assumption 4 (Vega)**.** *The distribution of the productivity shock $F(z;\sigma)$ is such that vega is always positive:*

$$\nu(\underline{z},\sigma) = \frac{\partial}{\partial\sigma}\mathbb{E}\left[(z-\underline{z})^{+}\right] > 0$$

for $\underline{z} > 0$. Furthermore, the standard deviation σ of z is a finite moment of the distribution $F(z;\sigma)$ and we normalize the size of the productivity shock with $\mathbb{E}[z] = 1$.

The model has two free parameters, leverage b and asset volatility σ. The model has two endogenous decision variables, investment i and the default threshold $\underline{z}$. We use this simple model to study the behavior of investment following changes in the key observable variables from our empirical section: asset volatility σ, leverage b, credit spreads cs, equity volatility σ^e, and Tobin's q. Without measurement error, in our model it is sufficient to observe only

[20] The credit spread is the difference between the yield of the corporate bond y and the risk-free rate. As the risk-free rate is assumed to be 0 in this simple model, and the yield is given by $y = b/D(K,B,\sigma) - 1$, we get $cs = F/(1-F)$.

[21] We already normalized the size of the firm by assuming that there is not capital in place in the first time period and that $\mathbb{E}[z] = 1$.

two non-perfectly correlated functions of the parameters and endogenous variables to identify these parameters.

Proposition 1 (Credit Spread and Asset Volatility)**.** *Holding asset volatility constant, the partial derivative of investment with respect to credit spread is given by:*

$$\frac{\partial i}{\partial cs} = -\frac{\underline{z}(1 - F(\underline{z}; \sigma))^2}{\phi_{ii}(i)} \leq 0. \tag{2.4}$$

Holding credit spread constant, the partial derivative of investment with respect to asset volatility is given by:

$$\frac{\partial i}{\partial \sigma} = \frac{\nu(\underline{z}, \sigma)}{\phi_{ii}(i)} \geq 0. \tag{2.5}$$

In Proposition 1, we provide the elasticities of investment when observing asset volatility and credit spread. Given Assumptions 1-4, the sign of these partial derivatives match our empirical results. When the credit spread increase, the debt overhang problem intensifies and there are less incentives for equity holders to invest. As asset volatility increase, the option value of equity dilutes the debt overhang problem and incites equity holders to invest more. When there is no debt ($b = 0$) and therefore no credit risk ($\underline{z} = 0$), these partial derivatives are equal to 0 and the firm does not underinvest anymore.

In terms of the magnitude of the negative impact of credit spreads on investment, the nominator of equation (2.4), $\underline{z} \times (1 - F(\underline{z}; \sigma))^2$, represents the marginal product lost in default $\underline{z}$ times a term that arises due to the nonlinearity of credit spreads with respect to the default probability. If cs was approximated with F instead of $F/(1 - F)$, that term would disappear. In the denominator, we note the role of the convexity of the adjustment cost function. If the cost of adjusting the stock of capital is more convex in investment, the impact is attenuated as firms do not have to adjust the stock of capital that much to reduce the marginal cost of investment.

By contrast, investment reacts positively to an increase in volatility as the payout to shareholders is non-linear with limited downside and unlimited upside, that is, vega $\nu(\underline{z}, \sigma)$ is positive.

Thus, in this simple model with fairly general and standard assumptions, the signs of the effects of credit spreads and asset volatility on investment are unambiguous. Changes in credit spreads cs signal changes in the debt-overhang burden and changes in asset volatility σ signal changes in the option value of equity. In Figure 2.1, we illustrate the optimal investment function with a log-normal distribution of risk.

We now compare the straightforward roles of credit spreads and asset volatility in determining investment with the more intricate relation between *leverage* and asset volatility in investment decisions. This analysis exemplifies why credit spreads and asset volatility are clean empirical measures of the effects of financial soundness and option value on investment decisions.

Proposition 2 (Leverage and Asset Volatility)**.** *Holding asset volatility constant, the partial derivative of investment with respect to leverage is given by:*

$$\frac{\partial i}{\partial b} = -\frac{\underline{z} f(\underline{z}; \sigma)}{\varphi(i, \underline{z}, \sigma)} \leq 0, \tag{2.6}$$

where

$$\varphi(i, \underline{z}, \sigma) \equiv \phi_{ii}(i) i - \underline{z}^2 f(\underline{z}; \sigma) > 0.$$

Holding leverage constant, the partial derivative of investment with respect to volatility is given by:

$$\frac{\partial i}{\partial \sigma} = \frac{i}{\varphi(i, \underline{z}, \sigma)} \big(\nu(\underline{z}, \sigma) - \underline{z} F_\sigma(\underline{z}; \sigma)\big). \tag{2.7}$$

Proposition 2 shows that if, instead of controlling for credit spreads cs, we observe leverage

b, the elasticities of investment become more intricate. In equation (2.6), the numerator still represents the marginal product lost to default. In the denominator, the term φ captures the feedback loop between investment and default decisions. Following a decrease in investment, shareholders default more often as output and incentives to pay back the debt decrease. That additional force was not present in Proposition 1, since changing credit spreads $cs(\underline{z};\sigma)$ controls for the default decision $\underline{z}$ directly. Holding leverage constant instead controls for $b = i\underline{z}$) (see the first order condition for $\underline{z}$ in equation (2.3)), which is a function of both i and $\underline{z}$. This term φ is always positive due to the second-order conditions for a maximum, and the sign of the effect of leverage on investment holding asset volatility constant is always negative.

Turning to the effect of asset volatility on investment holding leverage constant, the sign now becomes ambiguous. Intuitively, there are two effects to increasing asset volatility holding leverage constant. The first is that the option value of investment increases. The second is that the debt overhang problem also increases. To hold leverage $b = i\underline{z}$ constant as asset volatility increases, the default threshold $\underline{z}$ must change and the distance to default could shrink faster than the increase in the option value. The term $\nu(\underline{z}, \sigma) - \underline{z}F_\sigma(\underline{z};\sigma)$ captures this horse race between option value and what is lost in default as asset volatility increases. If the option value effect is strong, this term will be positive. If the increase in asset volatility moves a large probability mass into the default region ($\underline{z}F_\sigma(\underline{z};\sigma) > 0$), this term can be negative. In other words, when the marginal increase in investment returns lost to default $zF_\sigma(\underline{z};\sigma)$ dominates the marginal increase in the option value $\nu(\underline{z}, \sigma)$, shareholders reduce investment following an increase in volatility.

Which effect dominates is highly dependent on the shape of the distribution $F(z;\sigma)$. In Figure 2.2, we plot the optimal investment decision as a function of asset volatility σ when holding leverage b constant assuming a log-normal distributions for z. The monotonic relation between leverage and investment holding asset volatility constant is clear. However, the relation between investment and asset volatility holding leverage constant is non-monotonic.

When leverage is high, the option-value effect dominates while the debt overhang effect dominates when leverage is low.

Next, we consider the changes in investment when observing credit spreads and equity volatility, and illustrate the intuition our model suggests for the empirical finding that the sign of the elasticity of investment with respect to equity volatility changes sign in the cross section of more and less distressed firms. First, we define equity volatility as

$$\sigma^e(\underline{z}, \sigma) \equiv \frac{\sigma}{\mathbb{E}\left[(z - \underline{z})^+\right]}.$$

Thus, equity is simply levered asset volatility,[22] where the denominator represents the impact of leverage on equity volatility. If the debt burden from leverage b increases, then the default threshold $\underline{z}$ increases as well and equity's expected payoff per unit of capital $\mathbb{E}\left[(z - \underline{z})^+\right]$ decreases. Conversely, if the firm is funded entirely by equity ($b = 0$), then $\underline{z}$ is equal to zero—the lower bound of the support. In that case, equity volatility is equal to asset volatility ($\sigma^e(\underline{z}, \sigma) = \sigma$) since $\mathbb{E}[z] = 1$.

Proposition 3 (Credit Spread and Equity Volatility)**.** *Holding equity volatility constant, the partial derivative of investment with respect to credit spreads is given by:*

$$\frac{\partial i}{\partial cs} = -\frac{\underline{z}(1 - F(\underline{z}; \sigma))^2}{\phi_{ii}(i)} \xi_{cs}(\underline{z}, \sigma), \tag{2.8}$$

[22] Given our model, equity volatility could include the impact of investment and the truncation of equity volatility above the default threshold and be given by

$$\frac{\sqrt{\mathbb{V}ar\left[i(z - \underline{z})^+ - \phi(i)\right]}}{\mathbb{E}\left[i(z - \underline{z})^+ - \phi(i)\right]}.$$

In this case, our key insight—equity volatility is an ambiguous signal for investment—still holds but the elasticities become undecipherable.

where

$$\xi_{cs}(\underline{z},\sigma) \equiv \frac{\int_{\underline{z}}^{\infty} z/\underline{z} dF_\sigma(z;\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma) + f(\underline{z};\sigma)\sigma^e_\sigma(\underline{z},\sigma)}{f(\underline{z};\sigma)\sigma^e_\sigma(\underline{z},\sigma) - F_\sigma(\underline{z};\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma)}.$$

Holding credit spread constant, the partial derivative of investment with respect to equity volatility is given by:

$$\frac{\partial i}{\partial \sigma^e} = \frac{\nu(\underline{z},\sigma)}{\phi_{ii}(i)}\xi_{\sigma^e}(\underline{z},\sigma), \tag{2.9}$$

where

$$\xi_{\sigma^e}(\underline{z},\sigma) \equiv \frac{f(\underline{z};\sigma)}{f(\underline{z};\sigma)\sigma^e_\sigma(\underline{z},\sigma) - F_\sigma(\underline{z};\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma)}.$$

We defines the wedges ξ_{cs} and ξ_{σ^e} to make the distinction between Propositions 3 and 1 clear. It is easiest to start with the relation between investment and equity volatility holding credit spread constant. To understand the additional complication when using equity volatility as a signal of uncertainty, it is useful to look at the partial derivative of equity volatility with respect to asset volatility σ and the default threshold $\underline{z}$:

$$\sigma^e_\sigma(\underline{z},\sigma) = \frac{1}{\mathbb{E}\left[(z-\underline{z})^+\right]} - \frac{\sigma\nu(\underline{z},\sigma)}{\mathbb{E}\left[(z-\underline{z})^+\right]^2} \qquad \text{and} \qquad \sigma^e_{\underline{z}}(\underline{z},\sigma) = \frac{\sigma\left(1-F(\underline{z};\sigma)\right)}{\mathbb{E}\left[(z-\underline{z})^+\right]^2} \geq 0.$$

As shown in these equations, when the option value impact of asset volatility $\nu(\underline{z},\sigma)$ is large, equity volatility decreases following a positive shock to asset volatility. Indeed, the increase in the payoff to equity holders (denominator of σ^e) gets larger than the relative increase in asset volatility (numerator of σ^e). Add to that effect that to keep the credit spread cs constant, the default threshold $\underline{z}$ needs to decrease, and it is not surprising anymore that following a positive asset volatility shock, equity volatility might decrease. Corollary 1 makes that argument explicit.

Corollary 1 (Equity Volatility and Asset Volatility). *If the total derivative of the default threshold with respect to asset volatility is such that:*

$$\frac{d\underline{z}}{d\sigma} < \frac{\sigma\nu(\underline{z},\sigma) - \mathbb{E}\left[(z-\underline{z})^{+}\right]}{\sigma(1-F(\underline{z};\sigma))},$$

then the total derivative of equity volatility with respect to asset volatility is negative:

$$\frac{d\sigma^{e}(\underline{z},\sigma)}{d\sigma} < 0.$$

These additional forces are captured by the wedges ξ_{cs} and ξ_{σ^e} such that the signs of the elasticities of Proposition 3 are highly dependent on the shape of the risk distribution F and the level of leverage and volatility of the firm, contrarily to the robust signs of the elasticities of Proposition 1.

Lemma 1 (Existence of Credit Spread and Equity Volatility Pair). *Given* $(cs, \sigma^e) \in [0,1] \times \mathbb{R}^+$*, there does not always exist a solution* $(\underline{z}, \sigma) \in \mathbb{R}^+ \times \mathbb{R}^+$ *to the following system of two equations:*

$$cs = \frac{F(\underline{z};\sigma)}{1-F(\underline{z};\sigma)}, \qquad \sigma^e = \frac{\sigma}{\mathbb{E}\left[(z-\underline{z})^{+}\right]}.$$

Furthermore, the solution might not be unique.

Following Lemma 1, these non-monotonicities also complicate the mapping of investment decisions in the (cs, σ^e)-space. Thus, in Figure 2.3, instead of directly plotting investment as a function of cs and σ^e, we show the sign of the wedges in the (cs, σ)-space for two distributions: a log-normal distribution and a log-normal mixture distribution. In the case of the log-normal distribution, the wedges are either both positive (white area), such that the signs of the elasticities are identical to Proposition 1, or both negative (light gray area), such that the signs of the elasticities are opposite to Proposition 1.

The mixture distribution is a mixture of two log-normal distributions (see caption of Figure 2.3) and therefore bimodal. This risk distribution could correspond to a technology where the productivity shock is drawn from either a bad (low mean) or a good (high mean) distribution. In this case, an increase in uncertainty could have a large effect on the option value without substantially impacting default risk. Thus, a third area (dark grey) appears, where the elasticities with respect to credit spread and equity volatility are both negative. In the example of Figure 2.3, fixing asset volatility to 0.3, the elasticity with respect to equity volatility is positive for low credit spread level ($cs \leq 0.15$) and negative for high level of leverage ($0.30 \leq cs \leq 0.8$) while the elasticity with respect to credit spread is negative in both of these interval. Thus, in that example, we observe the same change of sign for equity volatility in the cross-section as in our empirical setting.

Finally, we can study the model's prediction for the relationship between Tobin's q and investment. As in Philippon (2009), we define Tobin's q by the market value of the firm scaled by it's end-of-period assets:

$$q = \int_{\underline{z}}^{\infty} z dF(z;\sigma) = \phi_i(i).$$

As it is the case in most models of investment, Tobin's q equates to the marginal cost of investment, $\phi_i(i)$, as implied by the first order condition for investment in equation (2.2). Thus, observing q directly pins down the investment level i and credit spread and asset volatility have no additional predictive information for investment.[23] Of course, in the presence of measurement error, other signals for investment incentives not perfectly correlated with q can have additional predictive content, as in our empirical analysis.

[23] This result also holds if debt holders can retrieve a fraction α of the capital after bankruptcy. Indeed, in that case Tobin's q becomes

$$q = \int_{\underline{z}}^{\infty} z dF(z;\sigma) + \alpha \int_{0}^{\underline{z}} z dF(z;\sigma) = (1-\alpha)\phi_i(i) + \alpha$$

since $\int_0^{\infty} z dF(z;\sigma) = 1$.

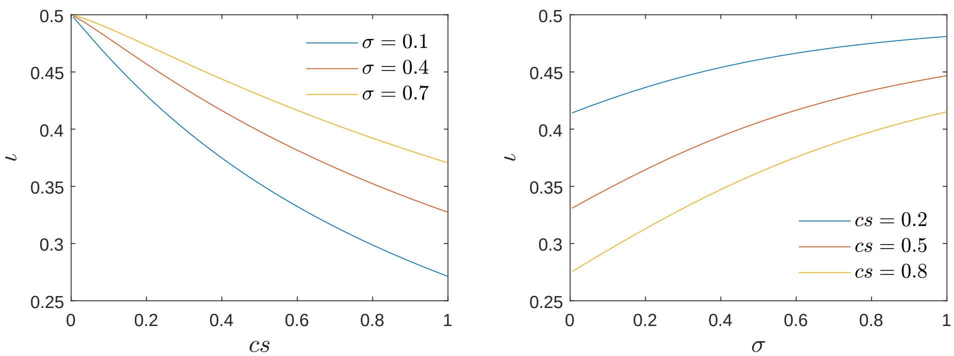
In Appendix 2.11, we show that our results hold in a setting with leverage dynamics with endogenous debt issuance. We extend the framework of DeMarzo and He (2020) to include an investment function and show that Proposition 1 still holds.

2.5 Conclusion

A large literature has focused on measuring the impact of uncertainty on real economic activity. In this paper, we show that equity volatility—a measure often used to proxy for uncertainty both at the firm and the aggregate level—is an ambiguous signal for investment decisions and might not be a good proxy for uncertainty. Intuitively, if a positive uncertainty shock causes a large increase in the option value of equity, equity volatility might go down. Furthermore, we find that uncertainty can have a positive impact on real economic activity by alleviating the debt overhang problem. Overall, our model and evidence provide support for the idea that the close connection between bond markets and the macroeconomy is due to the unique non-linear transformation of asset volatility and leverage that credit spreads represent.

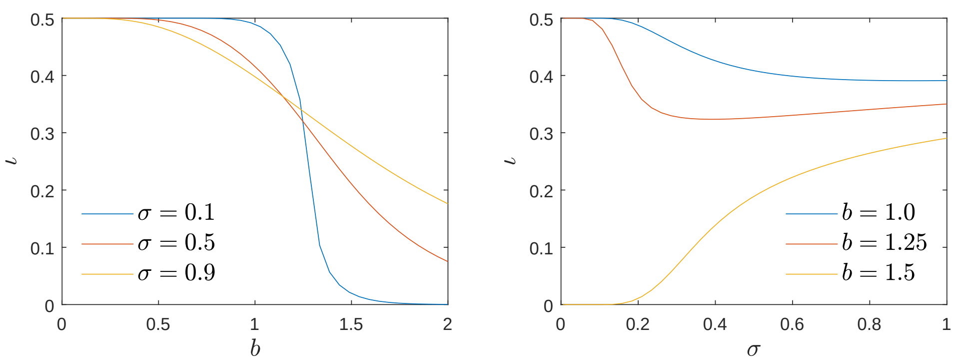
2.6 Appendix: Figures

Figure 2.1: Optimal investment as a function of credit spread and asset volatility



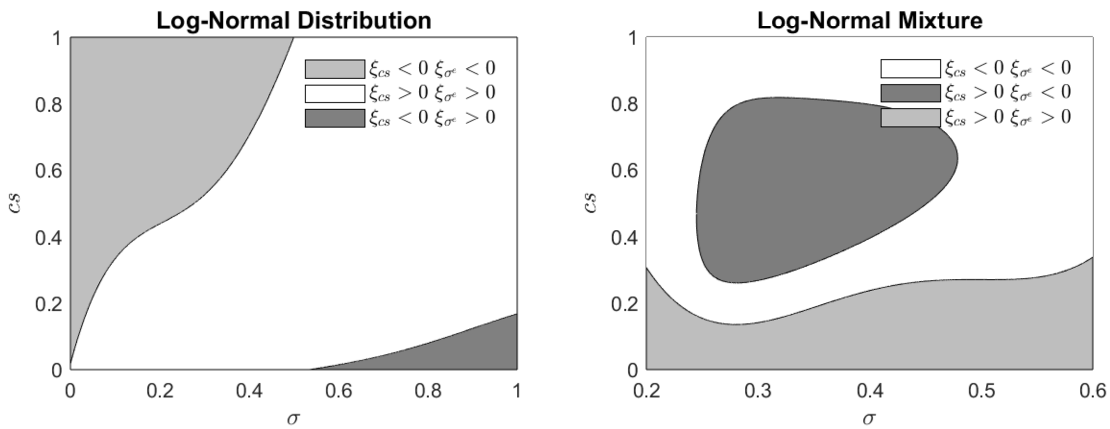
Notes: The left picture shows the level of investment i as a function of credit spreads cs for different levels of asset volatility σ, while the right figure shows the level of investment i as a function of asset volatility σ for different levels of credit spreads cs. The adjustment cost function is given by: $\phi(i) = i^{\gamma}$ with $\gamma = 2$.

Figure 2.2: Optimal investment as a function of leverage and asset volatility



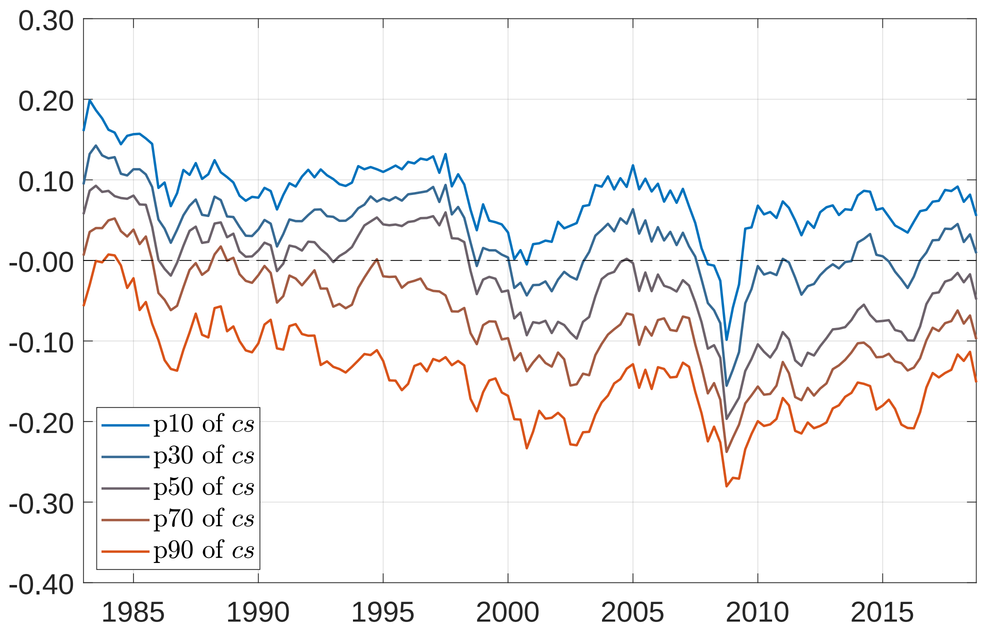
Notes: The left picture shows the level of investment i as a function of leverage b for different levels of asset volatility σ, while the right figure shows the level of investment i as a function of asset volatility σ for different levels of leverage b. The adjustment cost function is given by: $\phi(i) = i^{\gamma}$ with $\gamma = 2$.

Figure 2.3: Sign of wedges for log-normal and log-normal mixture distributions



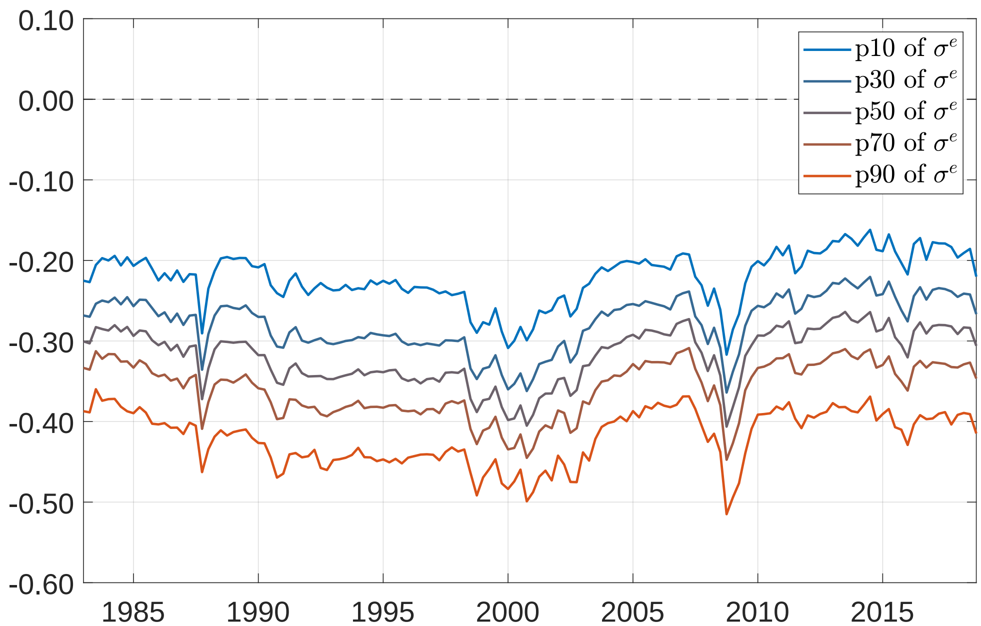
Notes: These pictures shows the sign of the wedges of Proposition 3 in the (cs,σ)-space for the log-normal distribution (left) and a log-normal mixture distribution (right). The mixture distribution is a mixture of two log-normal distributions drawn with 50% probability with parameters $(\mu_1, \hat{\sigma})$ and $(\mu_2, \hat{\sigma})$ such that the unconditional mean of z is 1 and the standard deviation of z is σ. We set $\hat{\sigma} = 0.2$ in this example.

Figure 2.4: Elasticity of investment with respect to equity volatility



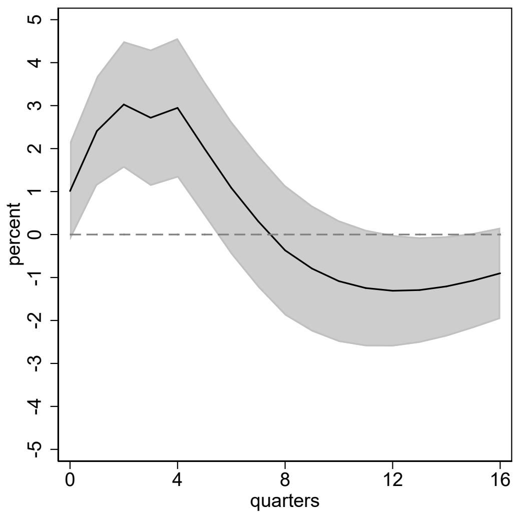
Notes: This figure presents the elasticity of investment with respect to equity volatility across time and across firms using the estimates from the regressions with interaction terms. In each quarter we generate five cutoffs in the cross-section of log credit spread: $\{p_{10}, p_{30}, p_{50}, p_{70}, p_{90}\}$. Using the estimates in column 7 of Table 2.2 on $\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t} + \beta_2 \log cs_{i,t} + \gamma \log \sigma^e_{i,t} \times \log cs_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$, the elasticity at each cutoff point is computed as $\beta_1 + \gamma p_n$, $n = 10, 30, 50, 70, 90$.

Figure 2.5: Elasticity of investment with respect to credit spread

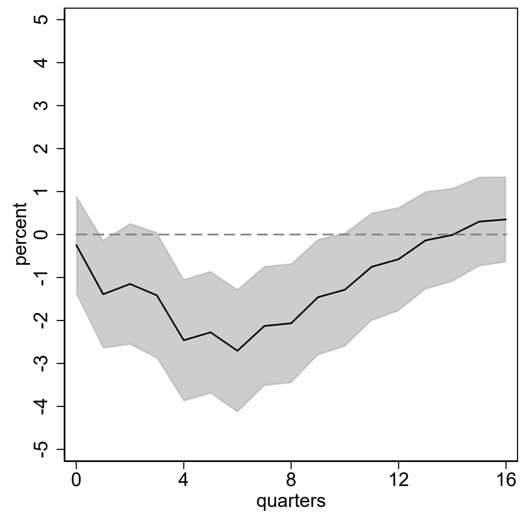


Notes: This figure presents the elasticity of investment with respect to credit spread across time and across firms using the estimates from the regressions with interaction terms. In each quarter we generate five cutoffs in the cross-section of log equity volatility: $\{p_{10}, p_{30}, p_{50}, p_{70}, p_{90}\}$. Using the estimates in column 7 of Table 2.2 on $\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t} + \beta_2 \log cs_{i,t} + \gamma \log \sigma^e_{i,t} \times \log cs_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$, the elasticity at each cutoff point is computed as $\beta_2 + \gamma p_n$, $n = 10, 30, 50, 70, 90$.

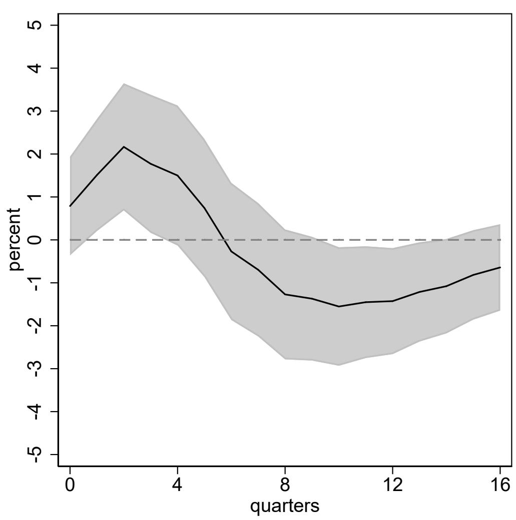
Figure 2.6: Impulse responses of investment to shocks to asset volatility and credit spread



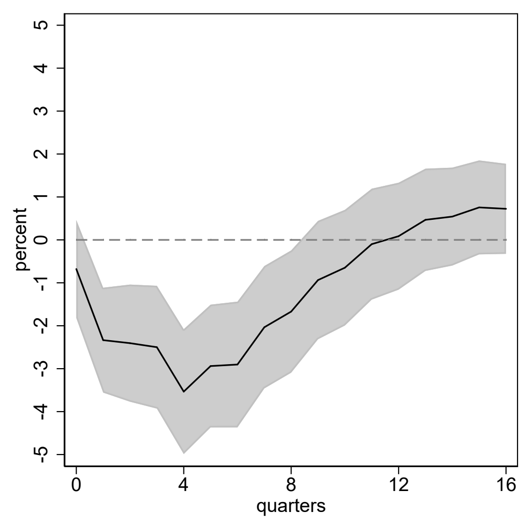
(a) Asset volatility



(b) Credit spread



(c) Asset volatility



(d) Credit spread

Notes: This figure plots the impulse responses of investment to an orthogonalized 1 standard deviation shock to asset volatility and credit spread. The VAR is estimated using four lags of each endogenous variable. Subfigures (a) and (b) correspond to the recursive ordering: $(cs,\, \sigma,\, I/K)$. Subfigures (c) and (d) correspond to the recursive ordering: $(\sigma,\, cs,\, I/K)$. The shaded bands represent the 95% confidence interval.

2.7 Appendix: Tables

Table 2.1: Summary statistics

	count	mean	sd	min	max
idiosyncratic equity volatility σ^e	42747	0.28	0.16	0.07	2.12
implied equity volatility $\widehat{\sigma}^e$	18228	0.34	0.15	0.12	1.58
idiosyncratic asset volatility σ	42263	0.14	0.07	0.01	0.82
Merton's idiosyncratic asset volatility $\widetilde{\sigma}$	38628	0.19	0.10	0.05	1.72
implied asset volatility $\widehat{\sigma}$	18137	0.19	0.07	0.03	0.72
residual asset volatility $\mathring{\sigma}$	42222	0.00	0.37	-1.39	1.72
market leverage $[MA/ME]$	42222	2.36	2.28	1.06	173.37
credit spreads cs	42747	300.75	245.91	8.73	1912.25
fair value spreads $\widehat{cs}$	30873	151.99	249.22	9.15	1822.99
distance-to-default DD	36875	5.98	3.29	-2.01	23.29
return on equity	42282	0.15	0.51	-0.98	16.73
tangibility ratio	34042	0.71	0.42	0.01	3.98
sales ratio	42141	1.47	2.91	0.02	52.73
income ratio	39968	0.20	0.32	-3.27	7.89
Tobin's q	32304	2.44	3.73	-2.52	62.52

Table 2.2: Relationship between investment, equity volatility, and credit spread

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t-1} + \beta_2 \log cs_{i,t-1} + \beta_3 \log \sigma^e_{i,t-1} \times \log cs_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all	(8) all
$\log \sigma^e_{i,t-1}$	-0.150***		-0.057***	0.048**	-0.031	-0.107***	0.712***	0.537***
	(-8.30)		(-3.67)	(2.55)	(-1.38)	(-3.90)	(8.48)	(6.11)
$\log cs_{i,t-1}$		-0.269***	-0.253***	-0.119***	-0.231***	-0.445***	-0.435***	-0.294***
		(-13.37)	(-13.07)	(-3.24)	(-5.22)	(-10.31)	(-16.37)	(-10.30)
$\log \sigma^e_{i,t-1} \times \log cs_{i,t-1}$							-0.137***	-0.100***
							(-9.13)	(-6.20)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls								✓
Observations	42694	42694	42694	13659	13613	12987	42694	28475
R-squared	0.114	0.137	0.138	0.164	0.142	0.129	0.145	0.220

Notes: This table documents the relationship between investment, equity volatility, and credit spread at the firm-quarter level from 1983 to 2018. Columns 4-6 use subsamples sorted by terciles every quarter on credit spread. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.3: Relationship between investment, equity volatility, and fair value spread

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t-1} + \beta_2 \log \widehat{cs}_{i,t-1} + \beta_3 \log \sigma^e_{i,t-1} \times \log \widehat{cs}_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low $\widehat{cs}$	(5) mid $\widehat{cs}$	(6) high $\widehat{cs}$	(7) all	(8) all
$\log \sigma^e_{i,t-1}$	-0.139***		0.006	0.060***	0.013	-0.050*	0.272***	0.231***
	(-6.91)		(0.37)	(3.34)	(0.56)	(-1.86)	(6.34)	(5.14)
$\log \widehat{cs}_{i,t-1}$		-0.150***	-0.151***	-0.088***	-0.097***	-0.195***	-0.228***	-0.150***
		(-13.59)	(-14.06)	(-3.29)	(-3.99)	(-10.17)	(-14.44)	(-8.17)
$\log \sigma^e_{i,t-1} \times \log \widehat{cs}_{i,t-1}$							-0.061***	-0.052***
							(-6.42)	(-4.92)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls								✓
Observations	30729	30729	30729	9835	9850	9605	30729	20570
R-squared	0.120	0.157	0.157	0.180	0.152	0.151	0.161	0.222

Notes: This table documents the relationship between investment, equity volatility, and fair value spread at the firm-quarter level from 1983 to 2018. Columns 4-6 use subsamples sorted by terciles every quarter on credit spread. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.4: Relationship between investment, asset volatility, and credit spread

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t-1} + \beta_2 \log cs_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma_{i,t-1}$	0.221***		0.193***	0.153***	0.160***	0.196***	0.161***
	(13.91)		(12.30)	(8.37)	(7.33)	(7.61)	(10.35)
$\log cs_{i,t-1}$		-0.269***	-0.248***	-0.107***	-0.226***	-0.436***	-0.153***
		(-13.35)	(-12.42)	(-2.88)	(-5.08)	(-10.15)	(-7.65)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Controls							✓
Observations	42299	42299	42299	13488	13496	12910	28278
R-squared	0.126	0.137	0.152	0.174	0.151	0.140	0.226

Notes: This table documents the relationship between investment, asset volatility, and credit spread at the firm-quarter level from 1983 to 2018. Columns 4-6 use subsamples sorted by terciles every quarter on credit spread. Control variables include quarterly return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.5: Relationship between investment, implied asset volatility, and credit spread

$$\log[I/K]_{i,t} = \beta_1 \log \widehat{\sigma}_{i,t-1} + \beta_2 \log cs_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \widehat{\sigma}_{i,t-1}$	0.283***		0.258***	0.233***	0.248***	0.220***	0.198***
	(8.62)		(7.96)	(5.58)	(4.28)	(3.88)	(5.60)
$\log cs_{i,t-1}$		-0.329***	-0.316***	-0.139***	-0.286***	-0.663***	-0.193***
		(-9.86)	(-9.41)	(-2.66)	(-4.25)	(-8.04)	(-6.05)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Controls							✓
Observations	18388	18388	18388	7388	6060	4127	12825
R-squared	0.132	0.154	0.168	0.179	0.156	0.180	0.247

Notes: This table documents the relationship between investment, implied asset volatility, and credit spreads at the firm-quarter level from 1983 to 2018. Columns 4-6 use subsamples sorted by terciles every quarter on credit spread. Control variables include quarterly return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.6: Relationship between investment, asset volatility, market leverage, and credit spread

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t-1} + \beta_2 \log[MA/ME]_{i,t-1} + +\beta_3 \log cs_{i,t-1} + \beta_4 \log \sigma_{i,t-1} \times \log cs_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all	(8) all
$\log \sigma_{i,t-1}$	0.221***		0.020	0.068***	0.015	0.013	0.471***	0.303***
	(13.91)		(1.22)	(3.74)	(0.67)	(0.48)	(4.74)	(3.19)
$\log[MA/ME]_{i,t-1}$		-0.476***	-0.464***	-0.405***	-0.490***	-0.440***	-0.406***	-0.368***
		(-18.75)	(-17.04)	(-6.70)	(-11.00)	(-12.17)	(-15.07)	(-10.46)
$\log cs_{i,t-1}$							-0.287***	-0.169***
							(-6.94)	(-3.84)
$\log \sigma_{i,t-1} \times \log cs_{i,t-1}$							-0.080***	-0.045***
							(-4.45)	(-2.58)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls								✓
Observations	42299	41832	41832	13266	13432	12751	41832	28019
R-squared	0.126	0.171	0.171	0.187	0.176	0.155	0.177	0.240

Notes: This table documents the relationship between investment, asset volatility, market leverage, and credit spreads at the firm-quarter level from 1983 to 2018. Columns 4-6 use subsamples sorted by terciles on credit spread. Control variables include quarterly return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.7: Loadings of equity volatility and credit spread on asset volatility and market leverage

$$\log y_{i,t} = \beta_1 \log \widetilde{\sigma}_{i,t} + \beta_2 \log[MA/ME]_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

Panel A: Levels	$\log \sigma^e_{i,t}$	$\log cs_{i,t}$	Panel B: Changes	$\Delta \log \sigma^e_{i,t}$	$\Delta \log cs_{i,t}$
$\log \widetilde{\sigma}_{i,t}$	0.777***	0.137***	$\Delta \log \widetilde{\sigma}_{i,t}$	0.748***	0.016***
	(71.53)	(12.32)		(69.78)	(4.83)
$\log[MA/ME]_{i,t}$	0.424***	0.574***	$\Delta \log[MA/ME]_{i,t}$	0.213***	0.231***
	(32.81)	(26.11)		(13.76)	(22.13)
Firm FE	✓	✓		✓	✓
Time FE	✓	✓		✓	✓
Observations	38225	38225	Observations	37462	37462
R-squared	0.837	0.580	R-squared	0.739	0.338

Notes: This table presents the loadings of equity volatility and credit spread on asset volatility and market leverage at the firm-quarter level from 1983 to 2018. We report results for estimations in levels in Panel A and results for estimations in first differences in Panel B. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.8: Relationship between investment, residual asset volatility, and credit spread

$$\log[I/K]_{i,t} = \beta_1 \log \mathring{\sigma}_{i,t-1} + \beta_2 \log cs_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \mathring{\sigma}_{i,t-1}$	-0.002		0.057***	0.093***	0.054**	0.031	0.058***
	(-0.09)		(3.59)	(4.81)	(2.38)	(1.12)	(3.61)
$\log cs_{i,t-1}$		-0.267***	-0.276***	-0.120***	-0.242***	-0.492***	-0.166***
		(-13.36)	(-14.00)	(-3.25)	(-5.45)	(-11.31)	(-8.09)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Controls							✓
Observations	42227	42227	42227	13437	13549	12828	28216
R-squared	0.106	0.137	0.137	0.166	0.144	0.125	0.217

Notes: This table documents the relationship between investment, residual asset volatility, and credit spreads at the firm-quarter level from 1983 to 2018. Residual asset volatility $\mathring{\sigma}_{i,t}$ is the residual of a panel regression of the log of idiosyncratic equity volatility on the log of market leverage with time and firm fixed effects: $\log \sigma_{i,t} = \beta \log[MA/ME]_{i,t} + \eta_i + \lambda_t + \log \mathring{\sigma}_{i,t}$. Columns 4-6 use subsamples sorted by terciles every quarter on credit spread. Control variables include quarterly return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.9: Relationship between investment, asset volatility derived from Merton's model, and credit spread

$$\log[I/K]_{i,t} = \beta_1 \log \widetilde{\sigma}_{i,t-1} + \beta_2 \log cs_{i,t-1} + \boldsymbol{\gamma} \mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \widetilde{\sigma}_{i,t-1}$	0.060***		0.086***	0.098***	0.080***	0.104***	0.073***
	(3.80)		(5.77)	(5.13)	(3.89)	(4.06)	(4.75)
$\log cs_{i,t-1}$		-0.258***	-0.265***	-0.094**	-0.263***	-0.487***	-0.155***
		(-12.61)	(-12.95)	(-2.54)	(-5.74)	(-10.50)	(-7.16)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Controls							✓
Observations	38968	38968	38968	12741	12422	11791	26116
R-squared	0.111	0.138	0.140	0.173	0.144	0.131	0.219

Notes: This table documents the relationship between investment, asset volatility derived from Merton's model, and credit spread at the firm-quarter level from 1983 to 2018. Columns 4-6 use subsamples sorted by terciles every quarter on credit spread. Control variables include quarterly return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.10: Relationship between investment, asset volatility, credit spread, and Tobin's q

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t-1} + \beta_2 \log cs_{i,t-1} + \beta_3 \log q_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$	(3) $\log[I/K]_{i,t}$	(4) $\log[I/K]_{i,t}$	(5) $\log[I/K]_{i,t}$	(6) $\log[I/K]_{i,t}$	(7) $\log[I/K]_{i,t}$	(8) $\log[I/K]_{i,t}$	(9) $\log[I/K]_{i,t}$	(10) $\log[I/K]_{i,t}$
$\log \sigma_{i,t-1}$	0.221***				0.165***	0.192***				0.152***
	(13.85)				(9.14)	(11.84)				(9.57)
$\log cs_{i,t-1}$		-0.269***		-0.191***	-0.184***		-0.188***		-0.157***	-0.155***
		(-13.37)		(-8.67)	(-8.56)		(-9.01)		(-7.57)	(-7.72)
$\log q_{i,t-1}$			0.197***	0.162***	0.137***			0.136***	0.107***	0.080***
			(12.88)	(10.79)	(8.94)			(9.03)	(7.28)	(5.51)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Controls						✓	✓	✓	✓	✓
Observations	42266	42694	31814	31814	31579	30284	30512	28475	28475	28282
R-squared	0.126	0.137	0.155	0.169	0.179	0.206	0.206	0.207	0.216	0.225

Notes: This table documents the relationship between investment, asset volatility, credit spread, and Tobin's q at the firm-quarter level from 1983 to 2018. Control variables include quarterly return on assets, log of tangibility ratio, log of sales ratio, log of income ratio (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.11: Relationship between investment, equity volatility, and asset volatility for firms without observable credit spreads

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t-1} + \beta_2 DD_{i,t-1} + \beta_3 \log \sigma^e_{i,t-1} \times DD_{i,t-1} + \boldsymbol{\gamma} \mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) no bonds	(2) no bonds	(3) zero lev.	(4) no bonds	(5) no bonds	(6) zero lev.
$\log \sigma^e_{i,t-1}$	-0.084***		-0.147***	-0.111***		-0.005
	(-3.78)		(-6.63)	(-5.80)		(-0.17)
$\log \sigma^e_{i,t-1} \times DD_{i,t-1}$	0.041***			0.038***		
	(3.95)			(10.84)		
$\log \sigma_{i,t-1}$		0.109**			0.153***	
		(2.36)			(8.15)	
$DD_{i,t-1}$	0.125***	0.060***		0.081***	0.043***	
	(8.37)	(4.76)		(10.48)	(10.86)	
Firm FE	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓
Controls				✓	✓	✓
Observations	189470	167236	47638	81596	73956	15980
R-squared	0.067	0.061	0.054	0.156	0.157	0.168

Notes: This table documents the relationship between investment, equity volatility, and asset volatility at the firm-quarter level from 1983 to 2018 for firms without observable credit spreads. Columns 1, 2, 4, and 5 use the subsample of firms without observable credit spreads but positive leverage. Columns 4 and 6 use the subsample of firms with zero leverage: the Compustat variables dlcq and dlttq are both equal to 0. Control variables include quarterly return on equity for regressions with equity volatility and return on assets for the regression with asset volatility, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.12: Relationship between investment, asset volatility, and credit spread for different lags and leads

$$\log[I/K]_{i,t} = \sum_1^4 \beta_\tau \log \sigma_{i,t-\tau} \; (\log \sigma_{i,t+\tau}) + \beta_5 \log cs_{i,t-1} + \boldsymbol{\gamma} \mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$	(3) $\log[I/K]_{i,t}$	(4) $\log[I/K]_{i,t}$
$\log cs_{i,t-1}$	-0.232*** (-11.52)	-0.250*** (-11.96)	-0.154*** (-7.70)	-0.158*** (-7.42)
$\log \sigma_{i,t-4}$	0.086*** (7.94)		0.069*** (5.62)	
$\log \sigma_{i,t-3}$	0.050*** (5.39)		0.033*** (3.13)	
$\log \sigma_{i,t-2}$	0.060*** (6.52)		0.043*** (3.95)	
$\log \sigma_{i,t-1}$	0.104*** (9.57)		0.091*** (7.56)	
$\log \sigma_{i,t+1}$		0.091*** (8.41)		0.055*** (4.45)
$\log \sigma_{i,t+2}$		0.045*** (4.97)		0.032*** (2.88)
$\log \sigma_{i,t+3}$		0.029*** (3.02)		0.021* (1.81)
$\log \sigma_{i,t+4}$		0.004 (0.46)		0.000 (0.02)
Firm FE	✓	✓	✓	✓
Time FE	✓	✓	✓	✓
Controls			✓	✓
Observations	41077	38047	27905	25536
R-squared	0.160	0.148	0.230	0.225

Notes: This table documents the relationship between investment, asset volatility, and credit spread at the firm-quarter level from 1983 to 2018 for different lags and leads. Control variables include quarterly return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.13: Relationship between investment, equity volatility, and credit spread for different levels of research and development and investment ratios

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t-1} + \beta_2 \log cs_{i,t-1} + \beta_3 \log \sigma^e_{i,t-1} \times \log cs_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) low R&D	(2) high R&D	(3) low R&D	(4) high R&D	(5) low inv.	(6) high inv.	(7) low inv.	(8) high inv.
$\log \sigma^e_{i,t-1}$	0.927***	0.671***	0.504***	0.368	0.574***	0.460***	0.467***	0.326***
	(4.83)	(3.14)	(2.68)	(1.55)	(4.69)	(4.51)	(3.42)	(2.66)
$\log cs_{i,t-1}$	-0.506***	-0.439***	-0.304***	-0.200***	-0.376***	-0.215***	-0.302***	-0.089**
	(-7.62)	(-6.89)	(-4.36)	(-2.70)	(-9.79)	(-6.51)	(-7.12)	(-2.37)
$\log \sigma^e_{i,t-1} \times \log cs_{i,t-1}$	-0.169***	-0.136***	-0.089***	-0.068	-0.113***	-0.084***	-0.094***	-0.054**
	(-4.87)	(-3.41)	(-2.60)	(-1.53)	(-5.17)	(-4.40)	(-3.80)	(-2.37)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls			✓	✓			✓	✓
Observations	6291	6235	4489	4824	13392	13218	9198	8421
R-squared	0.181	0.194	0.249	0.256	0.189	0.106	0.247	0.174

Notes: This table documents the relationship between investment, equity volatility, and credit spread at the firm-quarter level from 1983 to 2018 for different levels of research and development and investment ratios. The low (high) R&D category corresponds to firms sorted below (above) the quarterly median of R&D. The low (high) investment category corresponds to firms sorted every quarter in the first (third) tercile of the investment ratio $[I/K]_{i,t}$. Control variables include quarterly return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.14: Relationship between investment, level of implied asset volatility, shock to implied asset volatility, and credit spread

$$\log[I/K]_{i,t} = \beta_1 \log \widehat{\sigma}_{i,t-1} + \beta_2 \Delta \log \widehat{\sigma}_{i,t-1} + \beta_3 \log cs_{i,t-1} + \boldsymbol{\gamma} \mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$	(3) $\log[I/K]_{i,t}$	(4) $\log[I/K]_{i,t}$	(5) $\log[I/K]_{i,t}$
$\log \widehat{\sigma}_{i,t-1}$	0.328***	0.296***	0.258***		0.217***
	(8.76)	(8.02)	(7.96)		(5.61)
$\Delta \log \widehat{\sigma}_{i,t-1}$	-0.191***	-0.160***		-0.018	-0.110***
	(-6.58)	(-5.71)		(-0.87)	(-3.91)
$\log cs_{i,t-1}$		-0.311***	-0.316***	-0.327***	-0.190***
		(-9.25)	(-9.41)	(-9.88)	(-6.03)
Firm FE	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Controls					✓
Observations	18056	18056	18388	18056	12629
R-squared	0.135	0.169	0.168	0.153	0.251

Notes: This table documents the relationship between investment, level of implied asset volatility, shock to implied asset volatility, and credit spread at the firm-quarter level from 1983 to 2018. Control variables include quarterly return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.15: 2SLS regressions for equity and asset volatility

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t-1} + \beta_2 \log cs_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) log$[I/K]_{i,t}$	(2) log$[I/K]_{i,t}$	(3) log$[I/K]_{i,t}$	(4) log$[I/K]_{i,t}$
$\log \widetilde{\sigma}^e_{i,t-1}$	-1.382***		-0.647	
	(-3.61)		(-1.37)	
$\log \widetilde{\sigma}_{i,t-1}$		0.593***		0.735***
		(3.51)		(2.67)
$\log cs_{i,t-1}$	0.184	-0.255***	0.022	-0.128***
	(1.53)	(-5.02)	(0.18)	(-2.95)
First Moments	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓
Time FE	✓	✓	✓	✓
Controls			✓	✓
Observations	5097	4590	3936	4133
Kleibergen-Paap F	3.369	9.143	2.372	10.837
Sargan-Hansen p-val	0.882	0.316	0.764	0.266

Notes: This table documents the 2SLS regressions for equity and asset volatility at the firm-year level from 1990 to 2018 following Alfaro, Bloom, and Lin (2018). Realized annual volatility measures are instrumented with industry-level (3SIC) non-directional exposure to 10 aggregate sources of uncertainty shocks: the lagged exposure to annual changes in expected volatility of energy, currencies, and 10-year treasuries (as proxied by at-the-money forward-looking implied volatilities of oil, 7 widely traded currencies, and TYVIX) and economic policy uncertainty from Baker, Bloom, and Davis (2016). Annual realized equity volatility $\widetilde{\sigma}^e$ is the 12-month standard deviation of daily stock returns from CRSP. Annual realized asset volatility $\widetilde{\sigma}$ is the 12-month standard deviation of daily stock returns from CRSP unlevered using the daily market-to-book ratio of equity. Control variables include yearly return on equity for regressions with equity volatility and return on assets for regressions with asset volatility, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one year). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the 3-digit SIC industry.

Table 2.16: Relationship between equity volatility, credit spread, and investment for firms with different covenant tightness

$$\log[I/K]_{i,t} = \beta_1 \log \sigma^e_{i,t-1} + \beta_2 \log cs_{i,t-1} + \beta_3 \log \sigma^e_{i,t-1} \times \log cs_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) slack	(2) tight	(3) slack	(4) tight
$\log \sigma^e_{i,t-1}$	0.303**	0.480***		
	(2.07)	(2.93)		
$\log \sigma_{i,t-1}$			0.079***	0.119***
			(3.83)	(4.57)
$\log cs_{i,t-1}$	-0.190***	-0.395***	-0.111***	-0.300***
	(-4.06)	(-7.16)	(-3.50)	(-7.13)
$\log \sigma^e_{i,t-1} \times \log cs_{i,t-1}$	-0.055**	-0.083***		
	(-2.01)	(-3.01)		
Firm FE	✓	✓	✓	✓
Time FE	✓	✓	✓	✓
Controls	✓	✓	✓	✓
Observations	8834	7597	8801	7507
R-squared	0.237	0.217	0.238	0.222

Notes: This table documents the relationship between equity volatility, credit spread, and investment at the firm-year level from 1983 to 2018 for firms with different covenant tightness. The tight (slack) covenant sample includes firms with the covenant distance to threshold below (above) median. Each observation is a firm-year. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.17: Relationship between asset volatility, the predictable component of credit spread, the excess bond premium, and investment

$$\log[I/K]_{i,t} = \beta_1 \log \sigma_{i,t-1} + \beta_2[\log cs_{i,t-1} - \log ebp_{i,t-1}] + \beta_3 \log ebp_{i,t-1} + \boldsymbol{\gamma}\mathbf{X}_{i,t-1} + \eta_i + \lambda_t + \epsilon_{i,t}$$

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$	(3) $\log[I/K]_{i,t}$	(4) $\log[I/K]_{i,t}$	(5) $\log[I/K]_{i,t}$	(6) $\log[I/K]_{i,t}$
$\log \sigma_{i,t-1}$			0.109***			0.099***
			(6.30)			(5.45)
$\log cs_{i,t-1} - \log ebp_{i,t-1}$		-0.265***	-0.294***		-0.188***	-0.221***
		(-8.74)	(-9.02)		(-6.11)	(-6.48)
$\log ebp_{i,t-1}$	-0.196***	-0.228***	-0.202***	-0.075***	-0.107***	-0.090***
	(-8.91)	(-10.02)	(-8.48)	(-3.40)	(-4.63)	(-3.75)
Firm FE	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓
Controls				✓	✓	✓
Observations	37163	36656	33639	24992	24673	22773
R-squared	0.117	0.135	0.140	0.209	0.218	0.224

Notes: This table documents the relationship between asset volatility, the predictable component of credit spread, the excess bond premium, and investment at the firm-quarter level from 1983 to 2018. Following Gilchrist and Zakrajšek (2012), the excess bond premium $ebp_{i,t}$ is the quarterly average of the residual $\epsilon_{i,t}$ of a panel regression for credit spreads: $\log cs_{i,t} = \gamma' \mathbf{X}_{i,t} + \epsilon_{i,t}$. The vector of bond-specific characteristics $\mathbf{X}_{i,t}$ include the firm's distance-to-default, bond's amount outstanding, duration, coupon rate, industry fixed effects, credit rating fixed effects, an indicator variable for callable bonds, the interactions of callability with these bond characteristics as well as the level, slope, and curvature of the Treasury yield curve, and the realized monthly volatility of the daily ten-year Treasury yield. Control variables for the regression of investment include quarterly return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

2.8 Appendix: Data and Definitions

This section discusses the data sources used for the empirical analysis and the construction of variables.

Data Collection We use S&P's Compustat quarterly database from 1984:Q1 to 2018:Q4. We exclude firms in the financial sector (SIC code 6000 to 6999) and utility sector (SIC code 4900 to 4949), firms not in the panel for at least 3 years, and observations with missing investment rate, equity volatility and with negative sales. We use daily returns from the Center for Research in Security Prices (CRSP) database. Implied volatilities are from OptionMetrics data starting in 1996. Bond prices come from the Lehman/Warga (1984-2005) and ICE databases (1997-2018). This selection criterion yields 1,273 unique firms with 42,580 firm-quarter observations. To ensure that our results are not driven by extreme values, we trim every regression variables at the 1 and 99 percentiles. We provide summary statistics in Table 2.1 and describe how we construct our key variables below.

Investment and Equity Volatility We define investment rate as capital expenditures in quarter t scaled by net property, plant, and equipment in quarter $t-1$. Idiosyncratic equity volatility is constructed in two steps. For each firm-fiscal quarter, we extract daily idiosyncratic equity returns using the Carhart (1997) four-factor model. Then for each regression we calculate the standard deviation of residuals over one quarter, and obtain quarterly firm-specific idiosyncratic equity volatility. We only keep observations for quarters with more than 30 trading days. In addition to this realized equity volatility measure, we also use an implied equity volatility measure with at-the-money 30-day forward put options implied equity volatility from OptionMetrics.

Credit Spreads We follow Gilchrist and Zakrajšek (2012) to compute bond-level credit spreads. First, we construct a theoretical risk-free bond that replicates exactly the promised cash flows. The price of this risk-free bond is calculated by discounting the promised cash flows using continuously-compounded zero-coupon Treasury yields from Gürkaynak, Sack, and Wright (2007). The credit spread of an individual bond is the difference between the yield of the actual bond and the yield of the corresponding risk-free bond. We then define the credit spread of a firm as the average of the quarter-end credit spreads of all bonds issued by that firm.

Market Leverage Market leverage is defined as the ratio of market value of assets to market value of equity. The market value of assets is built as the book value of assets plus the market value of equity minus the book value of equity. Following Davies, Fama, and French (2000), the book value of equity is defined as the book value of stockholders' equity, plus balance sheet deferred taxes and investment tax credit, minus the book value of preferred stock. Depending on availability, we use the redemption, liquidation, or par value (in that order) for the book value of preferred stock. If this procedure generates missing values, we measure stockholders' equity as the book value of common equity plus the par value of preferred stock, or the book value of assets minus total liabilities.

Return on assets, Tangibility, Sales, Income, and Tobin's q Return on assets is operating income before depreciation divided by total assets. Tangibility is property, plant and equipment divided by total asset. The sales and income ratios are given by sales and operating income before depreciation divided by lagged property, plant and equipment. Following Erickson and Whited (2012), we construct the numerator of Tobin's q as book debt plus market value of equity minus book assets while the denominator is capital stock.

Asset Volatility and Distance to Default For our main measure of idiosyncratic asset volatility, we first unlever equity returns with market leverage to obtain asset returns, then we obtain idiosyncratic asset returns using the classic Carhart (1997) four-factor model and construct idiosyncratic asset volatility as the standard deviation of the idiosyncratic asset returns. We also construct a measure of firm-level idiosyncratic asset volatility based on Merton's (1974) model. Asset value V and (total) asset volatility σ_V can be obtained from a two-equation system as follows:

$$E = VN(d_1) - e^{-rT}BN(d_2)$$
$$\sigma_E = \left(\frac{V}{E}\right) N(d_1)\sigma_V$$

where

$$d_1 = \frac{\ln(V/B) + (r + 0.5\sigma_V^2)T}{\sigma_V\sqrt{T}}, \quad d_2 = d_1 - \sigma_V\sqrt{T}.$$

The inputs for the two-equation system are: (i) the market value of equity E, measured by the stock price multiplied by the number of shares outstanding; (ii) the equity volatility σ_E, measured by the annualized realized volatility of daily stock returns in each month; (iii) the face value of debt B, measured as the sum of the firm's current liabilities and one-half of its long-term liabilities; (iv) the debt maturity (forecasting horizon) $T = 1$; (v) the risk-free rate r, measured by the annualized monthly return on 90-day Treasury bills.

Instead of solving this two-equation system directly, we implement the iterative procedure proposed by Bharath and Shumway (2008).[24] We linearly interpolate the quarterly value of debt to a daily frequency and estimate asset value at a daily frequency. To construct idiosyncratic asset volatility, we use the daily asset values to generate times series of daily asset returns. With time series of daily asset returns, we calculate the idiosyncratic asset volatility using the same methodology used for idiosyncratic equity volatility. In addition to this realized asset volatility measure, we also use an implied asset volatility measure. Implied asset volatility is constructed as delevered implied equity volatility, that is, implied equity volatility times market value of equity divided by market value of assets.

[24]Gilchrist and Zakrajšek (2012) also adopt this iterative procedure.

Also, after we obtain the asset value V and total asset volatility σ_V, the distance to default (DD) can be easily computed according to the following equation.

$$DD = \frac{\ln(V/B) + (\mu - 0.5\sigma_V^2)T}{\sigma_V\sqrt{T}}.$$

Fair Value Spreads We use proprietary data set from Moody's on its Public Firm Expected Default Frequency (EDF) Metric, which is an equity-based measure of firm's probability of default. The core model used to generate the EDF metric belongs to the class of option-pricing based, structural credit risk models pioneered by Black and Scholes (1973) and Merton (1974). The Vasicek-Kealhofer (VK) model summarizes information on asset volatility, market value of assets, and the default point into one metric, the distance to default (DD), and then maps the DD to obtain the EDF metric. The DD-to-EDF mapping step utilizes the empirical distribution of DD and frequency of realized defaults. Nazeran and Dwyer (2015) provide a detailed description of their methodology. Most importantly for our purpose, the EDF credit risk measure relies only on equity market inputs and does not contain bond market information.

Using the EDF credit risk measure, we construct a cumulative EDF (CEDF) over T years by assuming a flat term structure, that is, $CEDF_T = 1 - (1 - EDF)^T$. Then, we convert our physical measure of default probabilities (CEDF) to risk-neutral default probabilities (CQDF) using the following equation:

$$CQDF_T = N\left[N^{-1}\left(CEDF_T\right) + \lambda\rho\sqrt{T}\right],$$

where N is the cumulative distribution function for the standard normal distribution, λ is the market Sharpe ratio and ρ is the correlation between the underlying asset returns and market returns. Given this risk-neutral default probability measure, the spread of a zero-coupon bond with duration T can be computed as:

$$\hat{cs} = -\frac{1}{T}\log(1 - CQDF_T \cdot LGD),$$

where LGD stands for the risk-neutral expected loss given default. We follow Moody's convention and set $T = 5$, $LGD = 60\%$, $\lambda = 0.546$, and $\rho = \sqrt{0.3}$ to build our "fair value spread" measure $\hat{s}$. We successfully match $39,925$ fair value spreads with our firm-quarter observations.

Covenant Tightness To measure the strength of creditor control rights, which is useful for providing empirical support for the debt overhang channel in our model, we use a covenant tightness measure based on a firm's outstanding loans. Data on covenant specifications and thresholds for loans is from DealScan. There are 18 types of covenants in the data. We first compute the distance between the actual financial ratio and the covenant threshold for each type of covenant, normalized by the firm-specific standard deviation of the actual financial ratios. We then use the minimum of the normalized distances to measure the overall covenant tightness for the firm in each quarter. See Kermani and Ma (2020) for more details on the covenant tightness measure.[25]

[25] We thank Yueran Ma and Amir Kermani for sharing their data with us.

2.9 Appendix: Proofs

Shareholders maximize their expected cash flow and device when to default. Thus, the value of equity is given by:

$$e = \max_{i,\underline{z}} \left\{ \mathbb{E}\Big[(iz - b)1\{z \geq \underline{z}\}\Big] - \phi(i) \right\}.$$

The first-order conditions for investment i and the default boundary $\underline{z}$ are given by

$$\begin{aligned} \int_{\underline{z}}^{\infty} z dF(z;\sigma) - \phi_i(i) &= 0, \\ -f(\underline{z};\sigma)(i\underline{z} - b) &= 0. \end{aligned}$$

The second-order conditions for investment i and the default boundary $\underline{z}$ are given by

$$\begin{aligned} -\phi_{ii}(i) &< 0, \\ -f(\underline{z};\sigma)i &< 0, \\ \phi_{ii}(i)f(\underline{z};\sigma)i + f(\underline{z};\sigma)^2\underline{z}^2 &> 0. \end{aligned} \tag{2.10}$$

Thus, $\phi_{ii}(i)i + f(\underline{z};\sigma)\underline{z}^2 > 0$.

In the following sections, we derive the partial derivatives of equity with respect to (i) credit spreads and asset volatility, (ii) leverage and asset volatility, (iii) credit spreads and equity volatility, (iv) Tobin's q and asset volatility, and (v) Tobin's q and credit spreads to rationalize our empirical results.

Assume we observe $\boldsymbol{\theta}$ and we want to derive the partial derivatives of $\mathbf{x}$ with respect to $\boldsymbol{\theta}$. Since $\mathbf{x}$ is the solution to a system of nonlinear equations $\boldsymbol{\mathcal{D}}(\mathbf{x}, \boldsymbol{\theta})$, we need to use the multivariate implicit function theorem:

$$\frac{\partial \mathbf{x}(\boldsymbol{\theta})}{\partial \theta_k} = - \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j} \right]^{-1} \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial \theta_k} \right].$$

Proof of Proposition 1

If we observe cs and σ, we get

$$\boldsymbol{\mathcal{D}}(\mathbf{x}, \boldsymbol{\theta}) = \begin{bmatrix} \int_{\underline{z}}^{\infty} z dF(z;\sigma) - \phi_i(i) \\ F(\underline{z};\sigma)/(1 - F(\underline{z};\sigma)) - cs \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} i \\ \underline{z} \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} cs & \sigma \end{bmatrix}.$$

We can derive the Jacobian matrix of $\boldsymbol{\mathcal{D}}(\mathbf{x}, \boldsymbol{\theta})$ as

$$\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j} \right] = \begin{bmatrix} -\phi_{ii}(i) & -\underline{z} f(\underline{z};\sigma) \\ 0 & f(\underline{z};\sigma)/(1 - F(\underline{z};\sigma))^2 \end{bmatrix}$$

and the partial derivatives as

$$\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial cs} \right] = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \qquad \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial \sigma} \right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} z dF_{\sigma}(z;\sigma) \\ F_{\sigma}(\underline{z};\sigma)/(1 - F(\underline{z};\sigma))^2 \end{bmatrix}.$$

To derive the comparative statics of interest, we only need few elements of $\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j} \right]^{-1}$. Thus, we get

$$\frac{\partial i}{\partial cs} = \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j} \right]^{-1}_{12} = - \frac{\underline{z}(1 - F(\underline{z};\sigma))^2}{\phi_{ii}(i)} < 0$$

and

$$\begin{aligned}
\frac{\partial i}{\partial \sigma} &= -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{11} \int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma) - \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} \frac{F_\sigma(\underline{z};\sigma)}{(1-F(\underline{z};\sigma))^2} \\
&= \frac{\int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma)}{\phi_{ii}(i)} + \frac{\underline{z}(1-F(\underline{z};\sigma))^2}{\phi_{ii}(i)} \frac{F_\sigma(\underline{z};\sigma)}{(1-F(\underline{z};\sigma))^2} \\
&= \frac{\nu(\underline{z},\sigma)}{\phi_{ii}(i)} > 0.
\end{aligned}$$

The sign of both these partial derivatives comes directly from Assumptions **??** and 4.

Proof of Proposition 2

If we observe b and σ, we get

$$\frac{\partial \mathbf{x}(\boldsymbol{\theta})}{\partial \theta_k} = -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1} \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial \theta_k}\right],$$

where

$$\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta}) = \begin{bmatrix} \int_{\underline{z}}^{\infty} z dF(z;\sigma) - \phi_i(i) \\ i\underline{z} - b \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} i \\ \underline{z} \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} b & \sigma \end{bmatrix}.$$

We can derive the Jacobian matrix of $\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta})$ as:

$$\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right] = \begin{bmatrix} -\phi_{ii}(i) & -\underline{z} f(\underline{z};\sigma) \\ \underline{z} & i \end{bmatrix}$$

and the partial derivatives as:

$$\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial b}\right] = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \qquad \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial \sigma}\right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma) dz \\ 0 \end{bmatrix}.$$

To derive the comparative statics of interest, we only need few elements of $\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}$. Thus, we can directly derive:

$$\frac{\partial i}{\partial b} = \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} = -\frac{\underline{z} f(\underline{z};\sigma)}{\phi_{ii}(i)i - \underline{z}^2 f(\underline{z};\sigma)} < 0,$$

$$\begin{aligned}
\frac{\partial i}{\partial \sigma} &= -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{11} \int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma) dz \\
&= \frac{i \int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma) dz}{\phi_{ii}(i)i - \underline{z}^2 f(\underline{z};\sigma)} \\
&= \frac{i(\nu(\underline{z},\sigma) - \underline{z} F_\sigma(\underline{z};\sigma))}{\phi_{ii}(i)i - \underline{z}^2 f(\underline{z};\sigma)}.
\end{aligned}$$

Proof of Proposition 2

If we observe cs and σ^e, we get

$$\frac{\partial \mathbf{x}(\boldsymbol{\theta})}{\partial \theta_k} = -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1} \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial \theta_k}\right],$$

where

$$\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta}) = \begin{bmatrix} \int_{\underline{z}}^{\infty} z dF(z;\sigma) - \phi_i(i) \\ F(\underline{z};\sigma)/(1-F(\underline{z};\sigma)) - cs \\ \frac{\sigma}{\mathbb{E}[(z-\underline{z})^+]} - \sigma^e \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} i \\ \underline{z} \\ \sigma \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} cs & \sigma^e \end{bmatrix}.$$

We can derive the Jacobian matrix of $\boldsymbol{\mathcal{D}}(\mathbf{x},\boldsymbol{\theta})$ as

$$\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right] = \begin{bmatrix} -\phi_{ii}(i) & -\underline{z} f(\underline{z};\sigma) & \int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma) \\ 0 & f(\underline{z};\sigma)/(1-F(\underline{z};\sigma))^2 & F_\sigma(\underline{z};\sigma)/(1-F(\underline{z};\sigma))^2 \\ 0 & \sigma^e_{\underline{z}} & \sigma^e_\sigma \end{bmatrix}$$

where

$$\sigma^e_{\underline{z}} = -\frac{\sigma \mu_{\underline{z}}(\underline{z},\sigma)}{\mu(\underline{z},\sigma)^2} = \frac{\sigma\left(1-F(\underline{z};\sigma)\right)}{\mu(\underline{z},\sigma)^2},$$
$$\sigma^e_{\sigma} = \frac{\mu(\underline{z},\sigma)-\sigma\nu(\underline{z},\sigma)}{\mu(\underline{z},\sigma)^2}$$
$$\mu(\underline{z},\sigma) = \mathbb{E}\left[(z-\underline{z})^+\right],$$

and the partial derivatives as

$$\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial cs}\right] = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \qquad \left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial \sigma^e}\right] = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}.$$

To derive the comparative statics of interest, we only need two elements of $\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}$. Thus, we can directly derive:

$$\frac{\partial i}{\partial cs} = \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} = \frac{(1-F(\underline{z};\sigma))^2}{\phi_{ii}(i)} \frac{\int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma) + \underline{z} f(\underline{z};\sigma)\sigma^e_{\sigma}(\underline{z},\sigma)}{F_\sigma(\underline{z};\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma) - f(\underline{z};\sigma)\sigma^e_{\sigma}(\underline{z},\sigma)},$$
$$= -\frac{\underline{z}(1-F(\underline{z};\sigma))^2}{\phi_{ii}(i)} \frac{\int_{\underline{z}}^{\infty} z/\underline{z} dF_\sigma(z;\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma) + f(\underline{z};\sigma)\sigma^e_{\sigma}(\underline{z},\sigma)}{f(\underline{z};\sigma)\sigma^e_{\sigma}(\underline{z},\sigma) - F_\sigma(\underline{z};\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma)}$$

and

$$\frac{\partial i}{\partial \sigma^e} = \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{13} = -\frac{1}{\phi_{ii}(i)} \frac{\int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma) f(\underline{z};\sigma) + \underline{z} f(\underline{z};\sigma) F_\sigma(\underline{z};\sigma)}{F_\sigma(\underline{z};\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma) - f(\underline{z};\sigma)\sigma^e_{\sigma}(\underline{z},\sigma)}$$
$$= \frac{\nu(\underline{z},\sigma)}{\phi_{ii}(i)} \frac{f(\underline{z};\sigma)}{f(\underline{z};\sigma)\sigma^e_{\sigma}(\underline{z},\sigma) - F_\sigma(\underline{z};\sigma)\sigma^e_{\underline{z}}(\underline{z},\sigma)}.$$

Positive Liquidation Value Given that the price of debt with positive liquidation value α is given by

$$D = (1-F(\underline{z};\sigma))B + i\alpha \int_0^{\underline{z}} zK dF(z;\sigma),$$

we define the credit spreads with positive liquidation value as

$$\widetilde{cs} = \frac{F(\underline{z};\sigma) - \alpha/bi \int_0^{\underline{z}} z dF(z;\sigma)}{1 - F(\underline{z};\sigma) + \alpha/bi \int_0^{\underline{z}} z dF(z;\sigma)}.$$

where $1-\alpha$ represents bankruptcy costs. For readability, we define

$$\widetilde{F}(i, \underline{z}, \sigma) = F(\underline{z};\sigma) - \alpha/bi \int_0^{\underline{z}} z dF(z;\sigma).$$

Thus, we can write:

$$\boldsymbol{\mathcal{D}}(\mathbf{x}, \boldsymbol{\theta}) = \begin{bmatrix} \int_{\underline{z}}^{\infty} z dF(z;\sigma) - \phi_i(i) \\ \frac{\widetilde{F}(i,\underline{z},\sigma)}{1-\widetilde{F}(i,\underline{z},\sigma)} - cs \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} i \\ \underline{z} \end{bmatrix}, \quad \boldsymbol{\theta} = \begin{bmatrix} cs & \sigma \end{bmatrix}.$$

We can derive the Jacobian matrix of $\boldsymbol{\mathcal{D}}(\mathbf{x}, \boldsymbol{\theta})$ as

$$\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right] = \begin{bmatrix} -\phi_{ii}(i) & -\underline{z} f(\underline{z};\sigma) \\ -\alpha/b \int_0^{\underline{z}} z dF(z;\sigma)/(1-\widetilde{F}(i,\underline{z},\sigma))^2 & f(\underline{z};\sigma)(1-\alpha)/(1-\widetilde{F}(i,\underline{z},\sigma))^2 \end{bmatrix}$$

and the partial derivatives as

$$\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial cs}\right] = \begin{bmatrix} 0 \\ -1 \end{bmatrix},$$

$$\left[\frac{\partial \boldsymbol{\mathcal{D}}(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial \sigma}\right] = \begin{bmatrix} \int_{\underline{z}}^{\infty} z dF_{\sigma}(z;\sigma) \\ \widetilde{F}_{\sigma}(i,\underline{z},\sigma)/(1-\widetilde{F}(i,\underline{z},\sigma))^2 \end{bmatrix}.$$

To derive the comparative statics of interest, we only need few elements of $\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}$. Thus, we get

$$\frac{\partial i}{\partial cs} = \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}), \boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} = -\frac{\underline{z}(1-\widetilde{F}(i,\underline{z},\sigma))^2}{\alpha/i \int_0^{\underline{z}} z dF(z;\sigma) + \phi_{ii}(i)(1-\alpha)} \leq 0$$

and

$$
\begin{aligned}
\frac{\partial i}{\partial \sigma} &= -\left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{11} \int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma) - \left[\frac{\partial \mathcal{D}_i(\mathbf{x}(\boldsymbol{\theta}),\boldsymbol{\theta})}{\partial x_j}\right]^{-1}_{12} \frac{\widetilde{F}_\sigma(i,\underline{z},\sigma)}{(1-\widetilde{F}(i,\underline{z},\sigma))^2} \\
&= \frac{(1-\alpha)\int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma)}{\alpha/i\int_0^{\underline{z}} z dF(z;\sigma) + \phi_{ii}(i)(1-\alpha)} + \frac{\underline{z}(1-\widetilde{F}(i,\underline{z},\sigma))^2}{\alpha/i\int_0^{\underline{z}} z dF(z;\sigma) + \phi_{ii}(i)(1-\alpha)} \frac{\widetilde{F}_\sigma(i,\underline{z},\sigma)}{(1-\widetilde{F}(i,\underline{z},\sigma))^2} \\
&= \frac{(1-\alpha)\int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma) + \underline{z}\widetilde{F}_\sigma(i,\underline{z},\sigma)}{\alpha/i\int_0^{\underline{z}} z dF(z;\sigma) + \phi_{ii}(i)(1-\alpha)} \\
&= \frac{(1-\alpha)\int_{\underline{z}}^{\infty} z dF_\sigma(z;\sigma) + \underline{z}F_\sigma(\underline{z};\sigma) - \alpha\int_0^{\underline{z}} z dF_\sigma(z;\sigma)}{\alpha/i\int_0^{\underline{z}} z dF(z;\sigma) + \phi_{ii}(i)(1-\alpha)} \\
&= \frac{\nu(\underline{z},\sigma)}{\alpha/i\int_0^{\underline{z}} z dF(z;\sigma) + \phi_{ii}(i)(1-\alpha)} \geq 0.
\end{aligned}
$$

2.10 Appendix: Robustness Checks

In this appendix, we provide several robustness checks for the results discussed above, and show that they yield similar results. in Table 2.18, we replicate Table 2.2 using total equity volatility instead of idiosyncratic equity volatility. In Tables 2.19, 2.20, and 2.21, we replicate Tables 2.2, 2.3, and 2.6 with implied volatility measures instead of realized volatility measures. In Table 2.22, we show the results of Table 2.4 by restricting the regressions to the set of firms with an observable implied asset volatility. In Table 2.23, we examine the loadings of firm's credit spreads and equity volatility on the average asset volatility and leverage of all firms in the same industry excluding itself, instead of the firm's asset volatility and market leverage. In Table 2.24 and Table 2.25, we replicate Table 2.12 with Merton's asset volatility and implied asset volatility. In Table 2.26, we replicate Table 2.14 with a shock to asset volatility instead of implied asset volatility.

Table 2.18: Relationship between investment, total equity volatility, and credit spread

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all	(8) all
$\log \sigma^{eT}_{i,t-1}$	-0.159***		-0.056***	0.085***	-0.040	-0.110***	0.748***	0.545***
	(-7.78)		(-3.16)	(3.96)	(-1.43)	(-3.63)	(9.24)	(6.46)
$\log cs_{i,t-1}$		-0.269***	-0.254***	-0.122***	-0.229***	-0.447***	-0.412***	-0.272***
		(-13.37)	(-13.07)	(-3.32)	(-5.19)	(-10.32)	(-17.23)	(-10.46)
$\log \sigma^{eT}_{i,t-1} \times \log cs_{i,t-1}$							-0.144***	-0.100***
							(-9.80)	(-6.45)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls								✓
Observations	42567	42694	42567	13592	13588	12963	42567	28386
R-squared	0.113	0.137	0.137	0.165	0.142	0.128	0.145	0.220

Notes: This table documents the relationship between investment, total equity volatility, and credit spread at the firm-quarter level from 1983 to 2018. Columns 4-6 use subsamples sorted by terciles every quarter on credit spread. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.19: Relationship between investment, implied equity volatility, and credit spread

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all	(8) all
$\log \widehat{\sigma}^e_{i,t-1}$	-0.360***		-0.176***	0.064	-0.110*	-0.317***	1.091***	0.998***
	(-7.98)		(-4.09)	(1.16)	(-1.68)	(-4.16)	(6.73)	(5.98)
$\log cs_{i,t-1}$		-0.269***	-0.272***	-0.159***	-0.256***	-0.514***	-0.508***	-0.371***
		(-13.37)	(-8.22)	(-3.10)	(-3.81)	(-7.01)	(-13.09)	(-8.64)
$\log \widehat{\sigma}^e_{i,t-1} \times \log cs_{i,t-1}$							-0.222***	-0.194***
							(-7.88)	(-6.53)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls								✓
Observations	18481	42694	18481	7362	6107	4211	18481	12842
R-squared	0.136	0.137	0.157	0.166	0.148	0.171	0.169	0.250

Notes: This table documents the relationship between investment, implied equity volatility, and credit spread at the firm-quarter level from 1983 to 2018. Columns 4-6 use subsamples sorted by terciles every quarter on credit spread. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.20: Relationship between investment, implied equity volatility, and fair value spread

	(1) all	(2) all	(3) all	(4) low $\widehat{cs}$	(5) mid $\widehat{cs}$	(6) high $\widehat{cs}$	(7) all	(8) all
$\log \widehat{\sigma}^e_{i,t-1}$	-0.344***		-0.052	0.120**	0.029	-0.247***	0.292***	0.248***
	(-6.80)		(-1.19)	(2.26)	(0.44)	(-3.35)	(3.73)	(3.03)
$\log \widehat{cs}_{i,t-1}$		-0.150***	-0.172***	-0.143***	-0.120***	-0.165***	-0.249***	-0.164***
		(-13.59)	(-11.52)	(-4.64)	(-3.92)	(-5.99)	(-11.36)	(-6.35)
$\log \widehat{\sigma}^e_{i,t-1} \times \log \widehat{cs}_{i,t-1}$							-0.080***	-0.070***
							(-4.97)	(-3.79)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls								✓
Observations	15965	30729	15965	5842	5438	4054	15965	11025
R-squared	0.141	0.157	0.179	0.169	0.154	0.194	0.184	0.243

Notes: This table documents the relationship between investment, implied equity volatility, and fair value spread at the firm-quarter level from 1983 to 2018. Columns 4-6 use subsamples sorted by terciles every quarter on credit spread. Control variables include quarterly return on equity, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.21: Relationship between investment, implied asset volatility, market leverage, and credit spread

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all	(8) all
$\log \widehat{\sigma}_{i,t-1}$	0.283***		-0.123***	0.027	-0.049	-0.269***	0.888***	0.723***
	(8.62)		(-3.00)	(0.51)	(-0.73)	(-3.55)	(4.74)	(3.97)
$\log[MA/ME]_{i,t-1}$		-0.606***	-0.682***	-0.455***	-0.631***	-0.739***	-0.622***	-0.586***
		(-14.62)	(-12.98)	(-4.84)	(-7.88)	(-9.34)	(-11.52)	(-8.37)
$\log cs_{i,t-1}$							-0.424***	-0.313***
							(-6.51)	(-4.70)
$\log \widehat{\sigma}_{i,t-1} \times \log cs_{i,t-1}$							-0.176***	-0.138***
							(-5.27)	(-4.22)
Firm FE	✓	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓	✓
Controls								✓
Observations	18388	18234	18234	7267	6043	4120	18234	12734
R-squared	0.132	0.189	0.192	0.186	0.176	0.211	0.202	0.268

Notes: This table documents the relationship between investment, implied asset volatility, market leverage, and credit spread at the firm-quarter level from 1983 to 2018. Columns 4-6 use subsamples sorted by terciles on credit spread. Control variables include return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.22: Relationship between investment, asset volatility, and credit spread

	(1) all	(2) all	(3) all	(4) low cs	(5) mid cs	(6) high cs	(7) all
$\log \sigma_{i,t-1}$	0.145***		0.136***	0.105***	0.120***	0.114***	0.098***
	(7.04)		(6.93)	(4.52)	(3.70)	(2.97)	(4.96)
$\log cs_{i,t-1}$		-0.327***	-0.323***	-0.143***	-0.282***	-0.700***	-0.187***
		(-9.81)	(-9.68)	(-2.72)	(-4.12)	(-8.59)	(-5.89)
Firm FE	✓	✓	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓	✓	✓
Controls							✓
Observations	18333	18333	18333	7381	6035	4105	12796
R-squared	0.123	0.154	0.161	0.172	0.151	0.174	0.245

Notes: This table documents the relationship between investment, asset volatility, and credit spread at the firm-quarter level from 1983 to 2018. We restrict the sample of firms to firms with observable implied asset volatility. Columns 4-6 use subsamples sorted by terciles every quarter on credit spread. Control variables include quarterly return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.23: Loadings of credit spread on the industry average of asset volatility and market leverage

$$\log y_{i,t} = \beta_1 \log \widetilde{\sigma}_{i,t} + \beta_2 \log[MA/ME]_{i,t} + \eta_i + \lambda_t + \epsilon_{i,t}$$

Panel A: Levels	$\log \sigma^e_{i,t}$	$\log cs_{i,t}$	Panel B: Changes	$\Delta \log \sigma^e_{i,t}$	$\Delta \log cs_{i,t}$
$\frac{1}{N_k-1}\sum_{j\neq i} \log \widetilde{\sigma}_{j,t}$	0.182***	0.032	$\Delta \frac{1}{N_k-1}\sum_{j\neq i} \log \widetilde{\sigma}_{j,t}$	0.050***	-0.011
	(8.23)	(1.00)		(4.02)	(-1.08)
$\frac{1}{N_k-1}\sum_{j\neq i} \log[MA/ME]_{j,t}$	0.256***	0.495**	$\Delta \frac{1}{N_k-1}\sum_{j\neq i} \log[MA/ME]_{j,t}$	0.305***	0.341***
	(4.91)	(6.01)		(7.09)	(9.89)
Firm FE	✓	✓	Firm FE	✓	✓
Time FE	✓	✓	Time FE	✓	✓
Observations	37901	37901	Observations	37149	37149
R-squared	0.362	0.441	R-squared	0.160	0.326

Notes: This table presents the loadings of credit spread on the industry average of asset volatility and market leverage at the firm-quarter level from 1983 to 2018. For a firm i in industry k at time t, we compute the industry average of log asset volatility excluding itself as $\frac{1}{N_k-1}\sum_{j\neq i} \log \widetilde{\sigma}_{j,t}$ and the industry average of market leverage as $\frac{1}{N_k-1}\sum_{j\neq i} \log[MA/ME]_{j,t}$. We report results for estimations in levels in Panel A and results for estimations in first differences in Panel B. Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels and standard errors are clustered at the firm level.

Table 2.24: Relationship between investment, asset volatility derived from Merton's model, and credit spread

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$	(3) $\log[I/K]_{i,t}$	(4) $\log[I/K]_{i,t}$
$\log cs_{i,t-1}$	-0.267***	-0.259***	-0.159***	-0.154***
	(-12.92)	(-12.28)	(-7.33)	(-6.78)
$\log \sigma_{i,t-4}$	0.018*		0.019	
	(1.66)		(1.59)	
$\log \sigma_{i,t-3}$	0.011		0.022**	
	(1.27)		(2.02)	
$\log \sigma_{i,t-2}$	0.009**		0.006	
	(2.01)		(0.48)	
$\log \sigma_{i,t-1}$	0.083***		0.064***	
	(5.90)		(5.08)	
$\log \sigma_{i,t+1}$		0.017		0.009
		(1.63)		(1.42)
$\log \sigma_{i,t+2}$		0.011**		0.008**
		(2.20)		(2.04)
$\log \sigma_{i,t+3}$		0.007*		0.009
		(1.80)		(1.39)
$\log \sigma_{i,t+4}$		0.002		0.004
		(0.68)		(0.94)
Firm FE	✓	✓	✓	✓
Time FE	✓	✓	✓	✓
Controls			✓	✓
Observations	36863	33888	25073	22689
R-squared	0.143	0.143	0.220	0.225

Notes: This table documents the relationship between investment, asset volatility derived from Merton's model, and credit spread at the firm-quarter level from 1983 to 2018 for different lags and leads. Control variables include quarterly return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.25: Relationship between investment, implied asset volatility, and credit spread for different lags and leads

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$	(3) $\log[I/K]_{i,t}$	(4) $\log[I/K]_{i,t}$
$\log cs_{i,t-1}$	-0.311***	-0.319***	-0.194***	-0.184***
	(-9.16)	(-9.47)	(-6.08)	(-5.51)
$\log \sigma_{i,t-4}$	0.109***		0.079***	
	(4.29)		(2.69)	
$\log \sigma_{i,t-3}$	0.059**		0.031	
	(2.46)		(0.97)	
$\log \sigma_{i,t-2}$	0.062**		0.044	
	(2.47)		(1.52)	
$\log \sigma_{i,t-1}$	0.117***		0.086***	
	(4.26)		(2.63)	
$\log \sigma_{i,t+1}$		0.185***		0.111***
		(6.17)		(3.32)
$\log \sigma_{i,t+2}$		-0.021		-0.018
		(-0.87)		(-0.62)
$\log \sigma_{i,t+3}$		0.052**		0.063**
		(2.13)		(2.03)
$\log \sigma_{i,t+4}$		-0.024		-0.061**
		(-0.91)		(-1.97)
Firm FE	✓	✓	✓	✓
Time FE	✓	✓	✓	✓
Controls			✓	✓
Observations	17354	16290	12258	11305
R-squared	0.173	0.170	0.251	0.248

Notes: This table documents the relationship between investment, implied asset volatility, and credit spread at the firm-quarter level from 1983 to 2018 for different lags and leads. Control variables include quarterly return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

Table 2.26: Relationship between investment, level of asset volatility, shock to asset volatility, and credit spread

	(1) $\log[I/K]_{i,t}$	(2) $\log[I/K]_{i,t}$	(3) $\log[I/K]_{i,t}$
$\log \sigma_{i,t-1}$	0.281***	0.247***	0.193***
	(13.52)	(12.12)	(9.07)
$\Delta \log \sigma_{i,t-1}$	-0.130***	-0.111***	-0.079***
	(-10.58)	(-9.34)	(-6.09)
$\log cs_{i,t-1}$		-0.238***	-0.153***
		(-11.88)	(-7.66)
Firm FE	✓	✓	✓
Time FE	✓	✓	✓
Controls			✓
Observations	41928	41928	28157
R-squared	0.131	0.155	0.227

Notes: This table documents the relationship between investment, level of asset volatility, shock to asset volatility, and credit spread at the firm-quarter level from 1983 to 2018. Control variables include quarterly return on assets, log of tangibility ratio, log of sales ratio, log of income ratio, log of Tobin's q (all lagged by one quarter). Coefficients are reported with t-statistics in parentheses. ***, **, and * indicate significance at 1%, 5% and 10% levels. Standard errors are clustered at the firm level.

2.11 Appendix: Endogenous Leverage Dynamics

In this appendix, we extend the framework of DeMarzo and He (2020) to include an investment function. We solve numerically the Markov perfect equilibrium and confirm that our results hold in Figure 2.7. We refer to DeMarzo and He (2020) for the proofs of the existence and uniqueness of the Markov perfect equilibrium.

We assume that agents are risk neutral with an exogenous discount rate of $r > 0$. The firm's assets-in-place generate operating cash flow at the rate of Y_t, which evolves according to a geometric Brownian motion:

$$dY_t/Y_t = \mu_t dt + \sigma dZ_t$$

where Z_t is a standard Brownian motion. A firm has at its disposal an investment technology with adjustment costs, such that $\iota_t Y_t$ spent allows the firm to grow its capital stock by $\mu(\iota_t)Y_t dt$, where $\mu(\cdot)$ is increasing and concave. Denote by B the aggregate face value of outstanding debt that pays a constant coupon rate of $c > 0$. The firm pays corporate taxes equal to $\pi(Y_t - cF_t)$. We assume that debt takes the form of exponentially maturing coupon bonds with a constant amortization rate ξ. Equity holders control the outstanding debt B_t through an endogenous issuance/repurchase policy $d\Gamma_t$ but cannot commit on a policy. Thus, the evolution of the outstanding face value of debt follows

$$dB_t = d\Gamma_t - \xi B_t dt.$$

In the unique Markov equilibrium, given the debt price $p(Y, B)$, the firm's issuance policy

$d\Gamma_t = G_t dt$ and default time τ maximize the market value of equity:

$$E(Y,B) = \max_{\tau,\iota_t,G_t} \mathbb{E}_t \bigg[\int_t^{\tau} e^{-r(s-t)} \big[(1-\iota_s)Y_s - \pi(Y_s - cB_s) - (c+\xi)B_s + G_s p_s \big] ds \bigg| Y_t = Y, B_t = B \bigg].$$

Similarly, the equilibrium market price of debt must satisfy

$$p(Y,B) = \mathbb{E}_t \left[\int_t^{\tau} e^{-(r+\xi)} (c+\xi) ds \middle| Y_t = Y, B_t = B \right].$$

The Hamilton-Jacobi-Bellman (HJB) equation for equity holders is

$$rE(Y,B) = \max_{\iota,G} \bigg[(1-\iota)Y - \pi(Y - cB) - (c+\xi)B_s + Gp(Y,B) + (G - \xi B)E_B(Y,F) + \mu(\iota) Y E_Y(Y,B) + \frac{1}{2}\sigma^2 Y^2 E_{YY}(Y,B) \bigg]. \tag{2.11}$$

Thus, in equilibrium it must be that

$$p(Y,B) = -E_B(Y,B).$$

The first-order condition for the investment rate is given by

$$1 = \mu_\iota(\iota) E_Y(Y,B).$$

In the following, we define $\{\iota(Y,B), G(Y,B)\}$ as

$$\{\iota(Y,B), G(Y,B)\} = \arg\max_{\iota,G}\Big[(1-\iota)Y - \pi(Y-cB) - (c+\xi)B_s + Gp(Y,B) + (G-\xi B)E_B(Y,F) + \mu(\iota)YE_Y(Y,B) + \frac{1}{2}\sigma^2 Y^2 E_{YY}(Y,B)\Big].$$

In this setting with scale-invariance, the relevant measure of leverage is given by

$$y_t \equiv Y_t/B_t,$$

and the equity value function $E(Y,B)$ and debt price $p(Y,B)$ satisfy

$$E(Y,B) = E\left(Y/B,1\right) \equiv e(y)B \quad \text{and} \quad p(Y,B) = p(Y/B,1) \equiv p(y).$$

We also define the following:

$$\iota(Y,B) \equiv \iota(y) \quad \text{and} \quad G(Y,B) \equiv g(y)B.$$

Thus, we can rewrite (2.11) as follows

$$(r+\xi)e(y) = \max_{\iota}\Big[(1-\iota)y - \pi(y-c) - (c+\xi) + (\mu(\iota)+\xi)ye'(y) + \frac{1}{2}\sigma^2 y^2 e''(y)\Big]. \quad (2.12)$$

The optimal default boundary is such that

$$e'(y_b) = 0.$$

The higher bound is such that

$$e'(y) = \phi y - \rho,$$

which corresponds to the value of equity without a default option. We can solve for ϕ and ρ with

$$(r+\xi)(\phi y - \rho) = \max_{\iota}\Big[(1-\iota)y - \pi(y-c) - (c+\xi) + (\mu(\iota)+\xi)\phi y\Big].$$

Thus,

$$\rho = \frac{(1-\tau)c+\xi}{r+\xi},$$
$$\phi = \frac{1-\iota^\star-\pi}{r-\mu(\iota^\star)},$$
$$1 = \mu'(\iota^\star)\phi.$$

The HJB for $p(Y,B)$ is given by

$$rp(Y,B) = c + \xi(1-p(Y,B)) + (G-\xi B)p_B(Y,B) + \mu(Y,B)Yp_Y(Y,B) + \frac{1}{2}\sigma^2 Y^2 p_{YY}(Y,B).$$

where we define $\mu(Y,B) \equiv \mu(\iota(Y,B)) \equiv \mu(y)$.

Thus, we can write the HJB for $p(y)$ as

$$rp(y) = c + \xi(1-p(y)) - (g(y)-\xi)p'(y)y + \mu(y)yp'(y) + \frac{1}{2}\sigma^2 y^2 p''(y). \tag{2.13}$$

where $g(y) = G(Y,B)/B$. We need $g(y)$ to be such that $p(y) = e'(y)y - e(y)$. From (2.12), we get

$$
\begin{aligned}
(r+\xi)e'(y)y = {} & (1-\iota(y))y - \pi y - \iota'(y)y^2 + (\mu(y)+\xi)y^2e''(y) + (\mu(y)+\xi)ye'(y) + \mu'(y)y^2e'(y) \\
& + \frac{1}{2}\sigma^2y^3e'''(y)) + \sigma^2y^2e''(y).
\end{aligned}
$$

Thus,

$$
\begin{aligned}
(r+\xi)(e'(y)y - e(y)) = {} & (1-\pi)c + \xi - \iota'(y)y^2 + (\mu(y)+\xi)ye''(y) + \mu'(y)y^2e'(y) \\
& + \frac{1}{2}\sigma^2y^2e'''(y) + \frac{1}{2}\sigma^2y^2e''(y).
\end{aligned}
$$

Thus, $g(y)$ is such that

$$
\begin{aligned}
& c + \xi - (g(y)-\xi)p'(y)y + \mu(y)yp'(y) + \frac{1}{2}\sigma^2y^2p''(y) \\
& = (1-\pi)c + \xi - \iota'(y)y^2 + (\mu(y)+\xi)y^2e''(y) + \mu'(y)y^2e'(y) \\
& + \frac{1}{2}\sigma^2y^3e'''(y)) + \frac{1}{2}\sigma^2y^2e''(y).
\end{aligned}
$$

With further algebra, we get

$$
-gp'(y)y = -\pi c - \iota'(y)y^2 + \mu'(y)y^2e'(y).
$$

Since $\mu'(\iota)e'(y) = 1$ and $\mu'(y) = \mu'(\iota)\iota'(y)$, we get

$$
g(y) = \frac{\pi c}{p'(y)y}.
$$

Plugging the solution for $g(y)$ in (2.13) yields

$$
(r+\xi)p(y) = (1-\pi)c + \xi + (\mu(y)+\xi)yp'(y) + \frac{1}{2}\sigma^2y^2p''(y).
$$

We solve numerically for the solution using ODE45 in Matlab. We use the following pseudo-algorithm.

1. Start with $y_L = 0$ and $y_H = H$, where H is a sufficiently large number.

2. Given $y_b = 1/2(y_L + y_H)$, $e(y_b) = 0$, and $e'(y_b) = 0$, we solve for $e(y)$ on $[y_b, y_B]$ where Y_B is a large number.

3. Check if $|e(Y_B) - (\phi y_B - \rho)| \leq \varepsilon$, where $\varepsilon > 0$ is a small number. If $e(Y_B) - (\phi y_B - \rho) > \varepsilon$, set $y_L = y_b$ and repeat 2-3. If $e(y_B) - (\phi y_B - \rho) < -\varepsilon$, set $y_H = y_b$ and repeat 2-3. Otherwise move to 4.

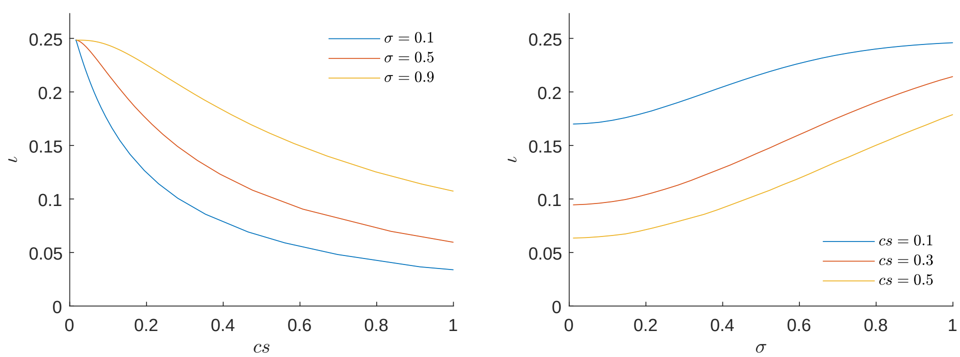
4. Start with $pp_L = 0$ and $pp_H = H$, where H is a sufficiently large number.

5. Given $pp_b = 1/2(pp_L + pp_H)$, $p(y_b) = 0$, $p'(y_b) = pp_b$ we solve for $p(y)$ on $[y_b, y_B]$.

6. Check if $|p(y_B) - \rho| \leq \varepsilon$ If $p(Y_B) - \rho > \varepsilon$, set $p_H = p_b$ and repeat 2-3. If $p(y_B) - \rho < -\varepsilon$, set $pp_L = pp_b$ and repeat 4-5. Otherwise move to 7.

7. Check if $|p'(y_b) - e''(y_b)y_b| \leq \varepsilon$. If not, increase the precision of the ODE45 solver and restart from 1.

Figure 2.7: Optimal investment in dynamic setting



Notes: This figure presents optimal investment in dynamic setting with $\mu(\iota) = \frac{\log(1+\kappa\iota)}{\kappa}$, $\kappa = 100$, $r = 0.05$, $\xi = 1/8$, $c = 0.05$, $\pi = 0.3$.

CHAPTER 3

Central Bank Digital Currency and Bank Disintermediation in a Portfolio Choice Model

with Lucyna Gornicka, Federico Grinberg, and Marcello Miccoli

3.1 Introduction

Recent years have seen a surge in the use of new kinds of privately issued digital money.[1] In response, more and more central banks have initiated work on exploring the issuance of their own digital money, denominated central bank digital currency (CBDC).[2] While different central banks pursue different objectives with CBDC, it is often hoped that its introduction can provide a more efficient, secure, and modern central bank money available to everyone, and that it can also increase resilience, availability, efficiency and contestability of retail payments, as well as broaden financial inclusion.

However, one of policymakers' key concerns is whether the introduction of CBDC could lead to bank disintermediation (Bindseil, 2020; Mancini-Griffoli et al., 2018) as a result of CBDC potentially crowding out commercial bank deposits. Deposits are a cheap and stable source of funding for banks, so if CBDC becomes successful in substituting bank deposits as a payment instrument, this could have a negative impact on banks' overall funding, and

[1] For example, payment systems provided by mobile network operators, new payment system providers, and stablecoins. See Adrian and Mancini-Griffoli (2019) for a detailed discussion. [IMPROVE THIS FN]

[2] For example, China, Canada, Sweden, The Bahamas, and European Union. See Soderberg et al. (2022) for a discussion on these central banks' policy objectives for considering a CBDC.

thus, their ability to lend.

In this paper we present a standard portfolio choice model with banks, in the spirit of Monti (1972); Klein (1971) and Drechsler et al. (2017), to analyze whether CBDC can actually generate bank disintermediation and, if so, how big this effect may be. In the model, households choose how to allocate their wealth between an illiquid asset and three imperfectly substitutable liquid assets: cash, bank deposits and CBDC. Households' utility depends on their final wealth and on services provided by the liquid assets. Deposits are offered by banks who then can invest funds in bonds that pay a fixed return or lend to firms. Banks have market power in deposits, which allows them to charge a positive spread between the return on bonds and the deposit rate.

We measure the impact of CBDC on bank intermediation by comparing the size of the bank deposit base and the size of bank lending before and after the introduction of CBDC. In the model, CBDC is simply a new, imperfect substitute for the other two liquid assets: deposits and cash. In the baseline case, when it is costless to hold the liquid assets, households would like to use all three of them, as households derive utility from variety. We show analytically that in this case the introduction of CBDC does not lead to bank disintermediation. The reason is that CBDC reduces banks' market power, to which banks optimally respond by increasing the rate of return on deposits. Thus, households choose to hold even more of bank deposits and the aggregate deposit base increases. We call this effect the *intensive margin* of CBDC introduction, and it is always positive. At the same time, due to the lost market power the net effect of CBDC introduction on bank profits is negative.[3]

But in reality there can be many barriers to access financial assets. While getting cash

[3]In a related model of Andolfatto (2021) the introduction of CBDC also has a non-negative impact on aggregate bank deposits. Similarly to our setup, this happens because competition from CBDC makes banks offer higher rates of return on deposits. However, in Andolfatto (2021) agents cannot hold positive amounts of cash, CBDC and deposits at the same time, but strategically choose only one means of payment. As a result, also the extensive margin (see below) has always positive impact on the size of bank deposit base, which is not the case in our model. Allowing deposits, cash and CBDC to be imperfect substitutes allows us to study the consequences of CBDC introduction for a more general set of household preferences.

usually has no costs for retail users[4], gaining access to deposits can be cumbersome (for example, in many countries banks require customers to provide a proof of residence and employment in order to open an account) and costly (for example, some banks charge fixed fees for setting up an account or for executing transfers). In the extended model, when the fixed cost of holding bank deposits is much higher than the cost of holding CBDC, and when we allow households to differ in their initial wealth, the introduction of CBDC can generate bank disintermediation. This happens when the high cost of access to bank deposits leads poorer households to abandon deposits and to use CBDC and cash only. We call this the *extensive margin* of CBDC introduction. If big enough, the extensive margin can more than offset the *intensive margin*. However, when we parametrize and solve the enriched model numerically, we find that aggregate bank deposits fall following CBDC introduction only under a special condition: when the wealth distribution is fairly unequal. In this case banks do not aggressively increase deposit rates to prevent the outflow of customers due to the relatively small wealth held by the poor households. This leads to an aggregate decrease in bank deposits.

Regarding the impact that CBDC may have on lending, we find it to be quantitatively small under the conditions that make aggregate deposits fall. The access to other forms of funding, such as wholesale or central bank financing, allows banks to compensate the decline in deposits without having to reduce lending too much. On the one hand, when these alternative funding sources are relatively cheap, it is easy for banks to substitute away from deposits.[5] On the other hand, when alternative forms of funding are expensive, banks fight for deposits more aggressively, further increasing deposit rates, and thus reducing their

[4]Although this is most certainly the case for retail users and small amounts of cash, storage costs can be non-negligible when they involve larger amounts. Cash also pays a lower real return compared to bank deposits (as long as interest rates on deposit are positive). which we also consider in the model.

[5]The model ignores the effect that the changes in the funding structure may have on regulatory ratios or, more generally, on financial stability.

loss of deposits.[6]

Overall, our results show the importance of taking into account the market structure and banks' strategic responses when assessing the impact of CBDC on the banking system. Policymakers aiming to examine resiliency of bank lending to the introduction of potentially very attractive means of payment should take into account the mechanisms that we unveil. Our findings point also to the need for quality data on households' preferences over means of payment to estimate the demand for liquid assets.

Related Literature This paper contributes to a growing literature on CBDC. In line with majority of past work, we consider CBDC to be means of payment that i) can pay interest, ii) is directly accessible to a broad public, and iii) is not held on an account with a commercial bank.

Our main focus is on the effects that CBDC introduction can have on the deposit base of commercial banks and on bank lending. Other papers that have also considered this question include Andolfatto (2021), Chiu et al. (2019) and Agur et al. (2022). In Andolfatto (2021), banking sector is also monopolistic, but CBDC is a perfect substitute for currency and bank deposits. Thus, agents choose to hold only one means of payment and banks always match the rate paid on deposits with the return on CBDC. Additionally, costs of accessing bank deposits and CBDC are the same. As a result, while there is an extensive margin of CBDC introduction similar to our setting, it has always a *positive* impact on bank deposits. In comparison, we model deposits, cash and CBDC as imperfect substitutes, which allows us to study implications of CBDC introduction for a broader set of household preferences.

Chiu et al. (2019) consider a model where cash and deposits serve different transactions, and where CBDC is a perfect substitute for bank deposits only. The implications of CBDC introduction depend on whether it earns an interest rate and whether banks have market

[6]An important caveat is that our model is static, so the compression of bank profits and capital erosion does not affect lending. This is clearly a channel that can have an impact on lending in a dynamic setting. See, for instance, (Van den Heuvel et al., 2002).

power in the deposit market. When banking sector is imperfectly competitive, CBDC introduction expands bank deposit base and lending if its interest rate lies in an intermediate range and it causes disintermediation only if the interest rate is set too high relative to the rate that can be offered on bank deposits without making banks non-profitable.

In contrast to these two papers, we model CBDC as an imperfect substitute for both cash and bank deposits in a simple portfolio choice model. Although the way CBDC can increase bank deposit base in our model is also by reducing the market power of banks, it is important to note that both in Chiu et al. (2019) and in Andolfatto (2021) CBDC generates these effects although it has a zero market share: just by serving as an outside option to depositors and setting the interest rate on deposits. While in our model the impact of CBDC on bank deposits and lending works through an intensive and an extensive margin as in Andolfatto (2021), we show that the two margins might actually work in the opposite directions under special circumstances: when cost of setting a CBDC account is low relative to bank deposits and when wealth distribution among households is fairly unequal.

Agur et al. (2022) consider a setup where households choose the means of payments depending on their preferences over the level of anonymity and security of transactions. While cash offers most anonymity, bank deposits provide most security. Similarly to our model, variety in payment instruments increases welfare, but this is because of the heterogeneity in household preferences. Contrary to our setup, Agur et al. (2022) do not consider the role of the market power: banks are modeled as price-takers in both deposit and loan markets. The implications of CBDC introduction crucially depend on how close it resembles cash or deposits: a cash-like CBDC can reduce the demand for cash beyond the point where network effects cause the disappearance of cash, while a deposit-like CBDC can cause an increase in deposit and loan rates, and a contraction in bank lending to firms. The optimal design of CBDC involves a trade-off between loss of utility from variety when CBDC crowds out cash and loss of bank intermediation in the presence of severe lending frictions.

Other related papers on macroeconomic implications of CBDC, include Barrdear and

Kumhof (2016), Keister and Sanches (2019), Brunnermeier and Niepelt (2019), Williamson (2019), Piazzesi and Schneider (2020), Garratt et al. (2021), Wang and Hu (2022). In particular, Keister and Sanches (2019) show that by choosing a proper interest on CBDC, policymakers can ensure that CBDC introduction never decreases welfare. Barrdear and Kumhof (2016) introduce CBDC in a DSGE model with competitive but regulated banking sector. They find that CBDC always spurs economic activity, lowers the policy and deposit rates and increases bank lending. Garratt et al. (2021) consider a model with banks that have heterogenous market shares, and analyze how an interest-bearing CBDC can affect concentration in the banking system. In sum, the impact crucially depends on the design of CBDC. Finally, Wang and Hu (2022) study the link between CBDC and financial development. They argue that in less financially developed economies, retail CBDCs can be useful for promoting financial inclusion, while in countries with high levels of financial development, CBDC can enhance financial stability by substituting out more risky non-bank e-money.

Our paper also belongs to the vast literature studying implications of imperfect competition in banking system (e.g. Drechsler et al., 2017, Repullo et al., 2020). In particular, we build on at model developed Drechsler et al. (2017) to study the deposit channel of monetary policy. The model contains two features that make it suitable for our purposes: i) imperfect substitution between liquid assets as a means of payment, and ii) imperfectly competitive banking system . Finally, our work builds on models that distinguish between the extensive and intensive margins of adjustment as in Hopenhayn (1992) and Melitz (2003).

The rest of the paper is organized as follows. Section 3.2 introduces the baseline model with homogeneous households and no fixed costs for holding bank deposits and CBDC. Section 3.3 discusses the enriched model with heterogeneous households and fixed costs of holding deposits and CBDC. Section 3.4 adds lending and wholesale funding for banks to the model. Section 3.5 concludes.

3.2 Baseline Model

This section introduces the baseline model with homogeneous households and provides the analytical solution. We show that when households are homongenous in wealth, the introduction of CBDC will always lead to an increase in total bank deposits in the model.

3.2.1 Setup

We consider a portfolio choice model with an imperfectly competitive banking sector. There are three types of agents in the model: households, banks, and a central bank.

Households Households are homogenous and have initial wealth of W_0, which they allocate among four types of assets: (i) notes (cash), denoted by N, earns no return; (ii) CBDC, denoted by C, earns return $r_C \geq 0$; (iii) deposits, denoted by D, earn r_D; and (iv) bonds, earn a non-negative rate f. The bonds are risk-free, and f is the risk-free rate set by the central bank. Bonds are also "illiquid" as they are not useful as means of payment. Cash, CBDC, and deposits can instead be used for payments, creating liquidity services value in households' utility function.

Households' utility is a function of final wealth W and liquidity services L:

$$U(W_0) = \max \left(W^{\frac{\rho-1}{\rho}} + \lambda L^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \tag{3.1}$$

where wealth and liquidity are complements, with the elasticity of substitution $\rho < 1$. Liquidity services arising from holding cash, CBDC and deposits are defined as:

$$L(N, C, D) = (N^{\frac{\epsilon-1}{\epsilon}} + \delta_C C^{\frac{\epsilon-1}{\epsilon}} + \delta_D D^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}}. \tag{3.2}$$

The three liquid assets are imperfect substitutes for households, hence the elasticity of substitution is greater than one, $\epsilon > 1$. δ_C and δ_D represent the relative usefulness of CBDC and deposits as means of payments compared to cash.

Households face the following budget constraint:

$$W = W_0(1 + f) - Nf - C(f - r_C) - D(f - r_D), \tag{3.3}$$

rearranged to highlight the opportunity costs of holding the liquid assets with respect to bonds. As cash earns no return, households face an opportunity cost of f, the return on bonds, when holding cash. The opportunity costs of holding CBDC and deposits are lower than for cash, as they guarantee positive returns r_C and r_D, respectively.

Banks Deposits are a composite good produced by a set of J banks, indexed by $j \in \{1, 2..., J\}$:

$$D = \left(\frac{1}{J} \sum_{j=1}^{J} D_j^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \tag{3.4}$$

where $\eta > 1$ is the elasticity of substitution between deposits of different banks. It is greater than one, reflecting imperfect substitutability between bank deposits.

To focus on the effect that CBDC has on the deposit market, for now we assume that banks are fully funded by deposits and can only invest in bonds. These assumptions are relaxed in Section 3.4. As deposits are imperfect substitutes, banks have market power and set the return on deposits $r_{D,j}$ with the objective of maximizing their profits, $(f - r_{D,j})D_j$, subject to deposits demand. The return on aggregate deposits is defined by the weighted average of each bank's rate of return, i.e. $r_D = \frac{1}{J} \sum_{j=1}^{J} \frac{D_j}{D} r_{D,j}$.

Central bank The central bank chooses the risk-free rate f, i.e. remuneration on bonds, and the interest rate on CBDC, r_C. It also supplies bonds and CBDC with an infinite elasticity.

3.2.2 Equilibrium

The behaviour of households is characterized by four first-order conditions. First, households choose between liquid assets and bonds according to:

$$\frac{L}{W} = \lambda^{\rho} s_L^{-\rho}, \tag{3.5}$$

where $s_L \equiv (f^{1-\epsilon} + \delta_D^{\epsilon}(s^*)^{1-\epsilon} + \delta_C^{\epsilon}(f - r_C)^{1-\epsilon})^{\frac{1}{1-\epsilon}}$ is the foregone interest of holding liquid assets. A higher forgone interest decreases the share of wealth kept in liquid assets.[7]

Second, households choose between liquid assets according to the two first-order conditions

$$\frac{C}{N} = {\delta_C}^{\epsilon} \left(\frac{f - r_C}{f} \right)^{-\epsilon}, \tag{3.6}$$

$$\frac{C}{D} = \left(\frac{\delta_C}{\delta_D} \right)^{\epsilon} \left(\frac{f - r_C}{f - r_D} \right)^{-\epsilon}. \tag{3.7}$$

It follows that households will want to hold more CBDC if it is more useful as means of payments relative to other liquid assets, and if it earns a higher return.

Third, households choose between deposits of different banks according to:

$$\frac{D_j}{D} = \left(\frac{f - r_{D,j}}{f - r_D} \right)^{-\eta}. \tag{3.8}$$

Thanks to the market power they enjoy, banks can remunerate deposits below the central bank's risk-free rate: the spread with respect to the rate f, $f - r_{D,j}$, is positive. The first-order condition for banks is given by:

$$\frac{\partial D_j}{\partial (f - r_{D,j})} \frac{(f - r_{D,j})}{D_j} = -1. \tag{3.9}$$

[7]Note that we can equivalently rewrite the households' budget constraint as $W = W_0(1 + f) - L s_L$.

Following Drechsler et al. (2017), we focus on the symmetric equilibrium with $D_j = D$. In this case it can be shown that the elasticity of aggregate deposit demand with respect to the spread $(f - r_D)$ is equal to:

$$-\frac{\partial D}{\partial (f - r_D)} \frac{(f - r_D)}{D} = 1 - (\eta - 1)(J - 1) = \mathcal{M}. \tag{3.10}$$

The elasticity of demand with respect to the spread, $\mathcal{M}$, decreases in the level of competition in the deposit market. In turn, the competitiveness of the deposit market increases with the number banks J, and with higher substitutability of deposits across banks, η.

A closed-form solution to the model can be obtained for the limit case in which $\lambda \to 0$. In this case, following proposition 1 in Drechsler et al. (2017), if $\epsilon > \mathcal{M} > \rho$, the deposit remuneration and aggregate deposits are given by:

$$f - {r_D}^* = \delta_D^{\frac{\epsilon}{\epsilon - 1}} \left[\frac{\mathcal{M} - \rho}{\epsilon - \mathcal{M}}\right]^{\frac{1}{\epsilon - 1}} \left[f^{1-\epsilon} + \delta_C^{\epsilon}(f - r_C)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} > 0, \tag{3.11}$$

and

$$D^* = \delta_D^{\frac{\epsilon(1-\rho)}{1-\epsilon}} (f - {r_D}^*)^{-\rho} \left[1 + \delta_D^{-\epsilon} \left(\frac{f - {r_D}^*}{f}\right)^{\epsilon - 1} + \left(\frac{\delta_D}{\delta_C}\right)^{-\epsilon} \left(\frac{f - {r_D}^*}{f - r_C}\right)^{\epsilon - 1}\right]^{\frac{\rho - \epsilon}{\epsilon - 1}}. \tag{3.12}$$

If $\mathcal{M} < \rho$, then ${r_D}^* = f$. Throughout the analysis we focus on the case when the return on deposits is strictly less than the policy rate f, hence we impose that $\epsilon > \mathcal{M} > \rho$.

For completeness, it is worth noting that just like in Drechsler et al. (2017) the equilibrium spread $s^* \equiv f - {r_D}^*$ is *non-decreasing* and the amount of deposits D^* is non-increasing in the policy rate f, giving a rise to a "bank deposit channel":

$$\frac{\partial s^*}{\partial f} \geq 0 \tag{3.13}$$

$$\frac{\partial D^*}{\partial f} \leq 0. \tag{3.14}$$

In equilibrium, a higher rate f increases the opportunity cost of using cash or CBDC instead of deposits to service liquidity needs, allowing banks to increase the rate paid on bank deposits, but by not as much as f, hence the spread s^* increases. In response to the higher opportunity cost of holding deposits, the total supply of deposits by households declines because it is now more profitable to save through bonds than through deposits.

3.2.3 Impact of CBDC introduction

In this section we analyze the impact of the introduction of CBDC on the equilibrium deposit return and the amount of deposits. For simplicity, we will present the results in terms of the deposit spread, i.e. the spread between the policy rate and the bank deposit rate, $s^* \equiv f - r_D{}^*$.

Equilibrium deposit interest rate and deposit base with and without CBDC. In the absence of CBDC, proxied by setting $\delta_C = 0$, the equilibrium deposit spread and aggregate amount of deposits simplify to:

$$\tilde{s}^* = \delta_D^{\frac{\epsilon}{\epsilon-1}} \left[\frac{\mathcal{M} - \rho}{\epsilon - \mathcal{M}}\right]^{\frac{1}{\epsilon-1}} \times f, \tag{3.15}$$

and

$$\tilde{D}^* = \delta_D^{\frac{\epsilon(1-\rho)}{1-\epsilon}} (\tilde{s}^*)^{-\rho} \left[1 + \delta_D^{-\epsilon} \left(\frac{\tilde{s}^*}{f}\right)^{\epsilon-1}\right]^{\frac{\rho-\epsilon}{\epsilon-1}}. \tag{3.16}$$

Comparing equations (3.11)-(3.12) and (3.15)-(3.16), it is clear that introduction of CBDC will have an instant impact on the deposit rate of return and on aggregate deposits. Two opposing effects will come to play. First, as long as CBDC is not a perfect substitute for deposits and cash ($\epsilon > 1$), its introduction will induce households to diversify their liquidity basket, reducing but not fully eliminating the demand for cash and deposits. At the same time, however, it will also force banks to reduce the deposit spread in order to keep households from substituting deposits with CBDC. This will increase households' demand for deposits.

Overall, it can be shown that the deposit spread will always *decline* following introduction of CBDC, or equivalently that the return on deposits will increase, and that the aggregate deposits will always increase in equilibrium.

Lemma 1 *The equilibrium spread on deposits always declines when CBDC, with* $\delta_C > 0$*, is introduced, and the equilibrium level of deposits always increases:*

$$s^* - \tilde{s}^* = \Delta s \leq 0. \tag{3.17}$$

$$D^* - \tilde{D}^* = \Delta D \geq 0. \tag{3.18}$$

The intuition for the preceding lemma is following. CBDC is a substitute for deposits, hence banks decrease the deposit spread in order to fight the competition from CBDC. Deposits in equilibrium increase for a more subtle reason. One can rewrite aggregate deposits in the limit case of $\lambda \to 0$ as: $D^* = \delta_D^\epsilon \left(\frac{s^*}{s_l}\right)^{-\epsilon} s_l^{-\rho}$, where $s_l \equiv (f^{1-\epsilon} + \delta_D^\epsilon (s^*)^{1-\epsilon} + \delta_C^\epsilon (f - r_C)^{1-\epsilon})^{\frac{1}{1-\epsilon}}$ represents the opportunity cost of holding liquid assets. Thus, the ratio $\left(\frac{s^*}{s_l}\right)^{-\epsilon}$ represents the substitution forces between the cost of deposits and overall liquidity, while $s_l^{-\rho}$ represents the demand for liquidity, which is decreasing in the opportunity cost of liquidity. When CBDC is introduced, the opportunity cost of liquidity, s_l, decreases. Directly, because there is a new liquid asset (*love for variety* effect), and indirectly because CBDC competition induces banks to decrease the deposits spread, s^*. But banks adjust the spread so that households are exactly indifferent between holding one more unit of CBDC and one more unit of deposits and do not substitute away from deposits (equivalently, $\frac{s^*}{s_l(s^*)} = \frac{\tilde{s}^*}{s_l(\tilde{s}^*)}$)[8]. As a result of these two effects, the increase in liquid asset demand increases the demand for

[8]Note that this is not true when $\lambda \nrightarrow 0$. However the change in the ratio is very small.

deposits by the households and the aggregate amount of bank deposits in equilibirum.

3.2.3.1 Comparative statics: policy rate, CBDC remuneration, and bank market power.

Quantitatively, implications of CBDC introduction for bank deposits and the deposit spread will depend on the level of the policy rate f, remuneration offered by the CBDC and the level of market power in the banking sector. In what follows, we explain how these factors will matter more in detail.

In line with intuition, the decline in the deposit spread Δs is larger, and the increase in the aggregate deposits ΔD is larger, the higher the rate on CBDC is:

$$\frac{\partial|\Delta s|}{\partial r_E} > 0, \tag{3.19}$$

$$\frac{\partial \Delta D}{\partial r_E} > 0. \tag{3.20}$$

A higher rate of return on CBDC implies that banks will need to compensate households by paying a higher deposit rate in order to prevent them from switching to CBDC. Thus a higher r_E pushes the deposit spread further down and results in a larger increase in the amount of liquidity held in bank deposits.

For the policy rate, which is also the rate of return on bonds in which banks invest, we can show that the decline in the deposit spread Δs is larger, and the increase in the aggregate deposits ΔD is smaller, the higher the policy rate is, i.e.:

$$\frac{\partial|\Delta s|}{\partial f} > 0, \tag{3.21}$$

$$\frac{\partial \Delta D}{\partial f} < 0 \tag{3.22}$$

The result that a higher policy rate implies a higher decline in the deposit spread follows from the comparison of equations (3.11) and (3.15). In the absence of CBDC, the deposit spread $\tilde{s}^*$ is increasing by a fixed proportion, $\delta_D^{\frac{\epsilon}{\epsilon-1}}\left[\frac{\mathcal{M}-\rho}{\epsilon-\mathcal{M}}\right]^{\frac{1}{\epsilon-1}}$, each time f is raised by one. This elasticity declines once CBDC is introduced. Intuitively, although a higher policy rate still allows banks to raise the spread, they are more constrained in the ability to raise it due to the competition from CBDC.

The result that the increase in the aggregate deposits is highest for low levels of the policy rate f might seem counter-intuitive, given that the decline in the deposit spread is the largest when f is high. To understand this result, it is again useful to express aggregate deposits as a function of the deposit spread and the overall cost of liquidity: $D^* = \delta_D^{\epsilon}\left(\frac{s^*}{s_l}\right)^{-\epsilon} s_l^{-\rho}$ and $\tilde{D}^* = \delta_D^{\epsilon}\left(\frac{\tilde{s}^*}{\tilde{s}_l}\right)^{-\epsilon} \tilde{s}_l^{-\rho}$, where $\tilde{s}_l \equiv (f^{1-\epsilon} + \delta_D^{\epsilon}(\tilde{s}^*)^{1-\epsilon})^{\frac{1}{1-\epsilon}}$. As discussed before, following the introduction of CBDC, the deposit spread adjusts so that the ratios $\frac{s^*}{s_l}$ and $\frac{\tilde{s}^*}{\tilde{s}_l}$ are equal. Thus, a larger decline in s^* relative to $\tilde{s}^*$ when f goes up is simply necessary in order to keep the relative share of deposits in the households' liquidity basket from falling. It follows that the difference in the response of D^* and $\tilde{D}^*$ to changes in f is solely due to differences in how the overall demand for liquidity, $\tilde{s}_l^{-\rho}$ and $s_l^{-\rho}$, changes with the policy rate when there is no CBDC and when CBDC is present.

Finally, when we consider different levels of competition in the banking sector, we find that the decline in the deposit spread Δs is smaller, and the increase in the aggregate deposits ΔD is larger, the higher the elasticity of substitution among deposits (η) or the number of banks (J) is. Formally:

$$\frac{\partial|\Delta s|}{\partial \eta} < 0, \quad \frac{\partial|\Delta s|}{\partial J} < 0 \tag{3.23}$$

$$\frac{\partial \Delta D}{\partial \eta} > 0, \quad \frac{\partial \Delta D}{\partial J} > 0 \tag{3.24}$$

Higher competition has two implications. First, the deposit rate is higher, as banks have less market power. Second, the aggregate elasticity of deposits with respect to deposit rate (the negative of elasticity with respect to the deposit spread) increases as either there are more banks the household can substitute to (higher J), or there is higher elasticity of substitution across deposits. Hence, when CBDC is introduced, banks have less capacity to increase deposits rates compared to a deposit market with less competition and, even with a smaller increase in the deposit rate, the higher elasticity of aggregate deposits will imply a larger increase in deposit holdings by the households.

3.3 Extensive Margin in Liquid Asset Holdings

So far, we have assumed that households do not differ in the amount of initial wealth they hold. As a result, the only way through which the introduction of CBDC was altering households' portfolio allocation decisions was through higher or lower holdings of different assets by the representative household.

However, many view CBDC as a means to bolster financial inclusion, particularly in countries where banking penetration is low and where cash no longer offers a viable alternative. Importantly, if CBDC introduction increased financial inclusion—understood as access to and use of formal financial services—its effect on bank intermediation through this channel would be ambiguous. On the one hand, if banks increase the return offered on deposits in response to competition from CBDC as shown in Section 3.2), some previously unbanked households could decide to open bank accounts, pushing the total amount of deposits further up. On the other hand, if setting up a CBDC account is considerably cheaper than opening a bank account, this could encourage poorer households to switch from deposits to CBDC entirely. Thus, to enrich our analysis and to capture these potentially important effects, in this section we introduce two additional features to the model: i) heterogeneity in the initial household wealth, and ii) fixed costs of holding both CBDC and deposits.

The solution of the model will be now characterized by equilibrium wealth thresholds under which households will not hold CBDC and/or deposits. Thus, changes in aggregate deposit holdings will be driven by changes in how much deposits households hold conditional on having deposits at all (*intensive margin*) and how many households hold deposits (*extensive margin*).

3.3.1 Model setup

We assume that households' initial wealth W_0 has now a Pareto (Type I) distribution with the shape parameter α. The probability density function is given by $f(W_0) = \frac{\alpha \underline{W_0}^{\alpha}}{W_0^{\alpha+1}}$, where $\underline{W_0}$ is the lowest possible wealth level. For simplicity and without loss of generality we set $\underline{W_0} = 1$.

Households also need to pay a fixed cost (ϕ^D) to hold deposits and a fixed cost (ϕ^C) to hold CBDC. These costs are measured in terms of utility to simplify the model solution. The introduction of $\phi^D > 0$ and $\phi^C > 0$ allows us to capture pecuniary and non-pecuniary frictions that households face when accessing payment instruments. In addition, we assume that the cost of holding deposits is higher than the cost of holding CBDC: ($\phi^D > \phi^C$). We think it is a reasonable assumption because introduction of CBDC—a policy intervention—would likely be aimed at increasing access to payment instruments and/or increasing financial inclusion.

Under these two new assumptions, households' utility can be written as

$$u(W_0) = \max \left[(W^{\frac{\rho-1}{\rho}} + \lambda L^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}} - 1(\phi) \right], \tag{3.25}$$

where

$$1(\phi) \equiv \begin{cases} \phi^C & \text{if } C > 0 \text{ and } D = 0 \\ \phi^D & \text{if } D > 0 \text{ and } C = 0 \\ \phi^C + \phi^D & \text{if } C > 0 \text{ and } D > 0 \end{cases}$$

3.3.2 Solution characterization

With the fixed costs of setting up a CBDC or a bank deposit account, households may choose to hold neither CBDC nor deposit (N), only hold CBDC (C), only hold bank deposit (D), or hold both (B). We show that this choice depends on the individual household's initial wealth.

We first show that the indirect utility is linear in initial wealth W_0. The indirect utility $u(W_0)$ is defined as the maximized utility with optimal choice of assets allocation $\{N, C, D, B\}$, and is defined for each discrete choice over whether to set up an account for deposit or CBDC. Let $t(s_l) \equiv (1+\lambda^\rho s_l^{1-\rho})^{\frac{1}{\rho-1}}$, the indirect utility can be expressed as follows:

$$U(W_0) = W_0(1+f)t\left(1(s_l)\right) - 1(\phi), \tag{3.26}$$

where

$$1(s_l) \equiv \begin{cases} \delta_N^{\frac{\epsilon}{1-\epsilon}} f & \text{if } C = 0 \text{ and } D = 0 \\ (\delta_N^\epsilon f^{1-\epsilon} + \delta_C^\epsilon (f - r_C)^{1-\epsilon})^{\frac{1}{1-\epsilon}} & \text{if } C > 0 \text{ and } D = 0 \\ (\delta_N^\epsilon f^{1-\epsilon} + \delta_D^\epsilon s^{1-\epsilon})^{\frac{1}{1-\epsilon}} & \text{if } D > 0 \text{ and } C = 0 \\ (\delta_N^\epsilon f^{1-\epsilon} + \delta_D^\epsilon s^{1-\epsilon} + \delta_C^\epsilon (f - r_C)^{1-\epsilon})^{\frac{1}{1-\epsilon}} & \text{if } C > 0 \text{ and } D > 0 \end{cases}$$

The benefit of having a deposit or a CBDC account is equal to the lower cost of liquidity service, reflected in different $1(s_l)$ associated with different choices. The cost is the fixed cost ϕ^C, ϕ^D.

We can compute the indirect utility for each of the four choices and compare them, in order to find the cutoff wealth values for preferred choice within each pair. Since $\phi^D > \phi^C$, there are four possible scenarios: (i) households with low initial wealth use only cash, and households with high initial wealth use cash, CBDC, and deposits; (ii) households with low initial wealth use only cash, households with medium initial wealth use both cash and CBDC, and households with high initial wealth use cash, CBDC, and deposits; (iii) households with low initial wealth use only cash, households with medium initial wealth use both cash and deposits, and households with high initial wealth use cash, CBDC, and deposits; (iv) households with low initial wealth use only cash, households with medium low initial wealth use both cash and CBDC, households with medium high initial wealth use both cash and deposits, and households with high initial wealth use cash, CBDC, and deposits. For a given parametrization, only one scenario exists.

We will focus on the second scenario in the main text, as it is the one that arises under our preferred parametrization, and relegate the discussion on other scenarios to Appendix 3.7.2. In the Appendix, we show how the realization of different scenarios depends on the parameter values.

In scenario number 2, households with initial wealth below $\hat{W}_1 = \frac{\phi^C}{(1+f)\left(t\left(s_l^C\right)-t(s_l^N)\right)}$ would choose to hold cash only, and those with initial wealth above $\hat{W}_1$ and below $\hat{W}_2 = \frac{\phi^D}{(1+f)\left(t(s_l^B)-t(s_l^C)\right)}$ would choose to hold both cash and CBDC, while those with initial wealth above $\hat{W}_2$ would hold hold cash, CBDC, and deposits. These expressions show that the cutoff wealth levels increase with the cost of switching from/to an asset (the numerator) and fall with the benefit of switching (the denominator).

Aggregate bank deposits are now given by

$$D = \delta_D^\epsilon \left(\frac{s_l^B}{s}\right)^\epsilon \frac{\lambda^\rho \left(s_l^B\right)^{-\rho}}{1+\lambda^\rho \left(s_l^B\right)^{1-\rho}}(1+f) \times \int_{\hat{W}_2} W_0 dF(W_0), \tag{3.27}$$

which shows that aggregate deposits can change along both an intensive and an extensive margin.

Similar to Section 3.2, we compare equilibria without CBDC ($\delta_C = 0$) and when CBDC is introduced ($\delta_C > 0$). Different from the basic model presented in Section 3.2, we calibrate the model and rely on numerical methods to solve it.

3.3.3 Calibration

The baseline parameter values used for numerical solutions are summarized in Table 3.1.

We set the interest rate on bonds to 3 percent. We choose λ to generate a share of liquid assets to total household wealth of 5 percent (as observed in Advanced Economies). [9] Given these parameters, we choose ρ and ϵ so that the condition required for the existence of equilibrium holds ($\epsilon > \mathcal{M} > \rho$) [10]. The parameter governing the liquidity services of cash (δ_N) is normalized to one, so we choose δ_D and δ_C to reflect the assumed rank in terms of how good these assets are in providing liquidity services: CBDC provides more liquidity services than deposits, and these provide more liquidity services than cash.[11] Three parameters are chosen such that the model generates disintermediation when CBDC is introduced: α is such that the distribution of initial wealth is relatively unequal and the fixed costs of holding CBDC and deposits is such that its CBDC is sufficiently less expensive than deposits. In the subsection below, we discuss how these key parameter choices drive the results and the mechanisms behind. Finally, the parameters η and J correspond to the monopoly power of banks.

3.3.4 Results

Figure 3.2 summarizes the change in the intensive margin and the extensive margin following the introduction of CBDC. The x-axis correspond to households with different initial wealth levels and the y-axis correspond to the share of wealth. The blue lines plot the share of wealth that a household will allocate to deposits (D/W) for an initial level of wealth W_0.

[9] In US, the share of wealth in checking accounts as a fraction of household net worth (excluding equity in own home) is $2000/41200 \approx 0.05$ in 2019. See https://www.census.gov/data/tables/2019/demo/wealth/wealth-asset-ownership.html.

[10] See discussions on this constraint following equation (3.12) in Section 3.2.2.

[11] See Agur et al. (2022) for a discussion on users' preferences over anonymity and security when choosing payment instruments.

First, consider the dashed line, which represents the allocations before CBDC introduction ($\delta^C = 0$). In this case, there is a cutoff value of wealth W_A. Households with initial wealth lower than W_A do not have a bank account and thus $D/W = 0$. Households with initial wealth higher than W_A hold both cash and deposits. All households choose the same fraction of wealth to be allocated to bonds. The solid blue line represents the allocations when CBDC is introduced. Now only households with wealth higher than W_B choose to open a bank account. For households with initial wealth lower than W_A, they all choose to set up a CBDC account: the red line showing the share C/W is above zero.

Comparing the solid blue line to the dashed line, $W_B > W_A$ means that a smaller fraction of households chooses to open a bank account when CBDC is present. Some of the poor households which would otherwise hold deposits, now choose to hold CBDC instead. This corresponds to a decrease in bank deposits via the extensive margin. On the other hand, D/W is higher for households that choose to hold deposits when CBDC is present (intensive margin). In other words, those households that still choose to set up a bank account, now hold more deposits. The reason is that banks raise the interest rate on deposits when they face the competition from CBDC. The extensive margin and the intensive margin work in different directions, and the net effect depends on the assumed parameter values.

The red line represents the share of wealth allocated to CBDC, C/W. First, notice that all households choose to hold CBDC with our baseline choices of parameters. This is because we assume that CBDC is very easy to access by choosing a low value of ϕ^C. In this case, the introduction of CBDC improves financial inclusion since the poorest households ($W_0 < W_A$) who would hold only cash, will now hold CBDC as well. The fraction of wealth allocated to CBDC is much lower for richer households. This is not surprising as these households have access to deposits that pay interest and are thus a less costly liquidity instruments.

Figure 3.3 shows the change in aggregate deposits when CBDC is introduced by plotting $Df(W)$ on the y-axis, while the x axis again plots households according to their initial wealth level. The blue area reflects aggregate deposits lost through the extensive margin and the red area reflects the increase in aggregate deposits through the intensive margin. With our baseline choice of parameters, the aggregate deposits fall by 4.5% following the introduction of CBDC.

As mentioned above, the parameters crucial for the relative strength of the extensive and intensive margins are the fixed costs of setting up a CBDC or a bank account, distribution of initial wealth, and the risk-free rate f. In particular, we find that the extensive margin dominates the intensive margin and the aggregate deposits decline in equilibrium when (i) access to CBDC is much cheaper than bank deposits $\phi^C << \phi^D$. (ii) wealth distribution is unequal (large α); (iii) relatively low policy rate. Given the baseline calibration of other parameters including the policy rate ($f = 0.3$), we perform our quantitative experiments using different combinations of α and ϕ^C/ϕ^D, and summarize the results in Figure 3.4. The objective is to give a sense of the parameter space that could generate disintermediation following the introduction of CBDC.

We now discuss the intuition behind this results. The introduction of CBDC increase the competition faced by banks to attract deposits, and banks will increase the interest rate on deposits (decrease deposit spread) in response. With higher interest rate, households increase their deposit holding and aggregate deposit increases. This is what happened in our basic model when households are homogeneous.

In the enriched model, there is the extensive margin that works in a different direction because poor households will switch from deposits to CBDC. However, for the impacts on the extensive margin to be stronger enough to outweigh the impact on the intensive margin, we need banks to have less than enough incentives to further increase deposits rates to go after households who chose not to hold deposits accounts.

Easy access to CBDC means that many households with low wealth will choose to set up a CBDC account instead of a bank deposit account so that the extensive margin is large enough. At the same time, unequal wealth distribution guarantees that low wealth households own only a small fraction of wealth so that they are not important enough for banks' profits. Finally, the fall in deposits happens for low policy rate because low policy rate reduces the return from holding deposits and thus their advantage over CBDC.

Therefore, only under special circumstances detailed above, we will observe that the introduction of CBDC leads to a reduction in the amount of bank deposits. Introducing CBDC will lead to an increase in aggregate deposits otherwise. Figure 3.5 is one example. When we set $\alpha = 1.16$, which is below our baseline value 1.3 and corresponds to a more equal wealth distribution, the result changes and the aggregate deposit increases after introducing CBDC. In the Appendix, we describe the parameter space and show how combinations of low ϕ^C/ϕ^D and high α correspond to results of disintermediation.

Negative return on CBDC Our results still hold when the return on CBDC is negative. Quantitatively, when the return on CBDC is higher, the effect of introducing is stronger/ Figure 3.6 presents the percentage drop in aggregate deposit when we impose different interest rates on CBDC. As can be seen from the figure, the curve is continuous around $r_C = 0$ and our model works well with negative returns on CBDC. The reason is that we assume liquidity is a CES composite of cash, CBDC and deposits, so the households still want to hold some CBDC even if the opportunity costs of holding CBDC is higher than for cash ($r_C < 0$).

Comparative statics on δ The parameters $\{\delta_N, \delta_C, \delta_D\}$ govern how good cash, CBDC, and deposits are in terms of providing liquidity services. We normalize $\delta_N = 1$ and assume $\delta_C = 1.5$ and $\delta_D = 1.3$ in our baseline calibration. Here we perform comparative statics with respect to these δ's. Figure 3.7 shows that the better CBDC is in providing liquidity (i.e., higher δ_C), the larger the drop in aggregate deposit following the the introduction of CBDC. This is intuitive because CBDC is a better substitute for bank deposits and thus cause stronger bank disintermediation when δ_C is larger.

Figure 3.8 shows that the percentage drop in the aggregate deposit doesn't change with the the choice of δ_D. When δ_D is larger, the introduction of CBDC will cause a smaller drop in the aggregate deposit as δ_C will be relatively smaller so that CBDC is less of a threat to bank deposits. At the same time the initial deposit is smaller when δ_D is larger since the households need less deposits to obtain the same level of liquidity service. Thus the percentage change in the aggregate deposit is the same as the numerator and the denominator move together.

3.4 Extended Model with Lending

In this section, we introduce lending and wholesale funding to bank's problem. We solve the model numerically using the same values for main parameters and carefully picked values for new lending parameters. We find that our main results still hold qualitatively, that is the introduction of CBDC leads to a reduction in lending under the special circumstances, while the drop in lending is quantitatively very small and it is hard to make it large.

3.4.1 Model setup and solution characterization

Everything on the consumer side is unchanged and only the banking problem is now different. Banks can now fund both with deposits (D_i) and wholesale funding (H_i). They lend L_i , which is "unproductive" and given to firms outside of the economy.

The problem of the bank is:

$$\max_{D_i,H_i} \left(f + l_0 - \frac{l_1}{2}L_i\right) L_i - \left(f + \frac{h}{2}H_i\right) H_i - (f - s_i)D_i \tag{3.28}$$
$$\text{s.t. } L_i = H_i + D_i$$

Quote from Drechsler et al. (2017): "$l_0, l_1, h > 0$ are parameters that control the bank's lending opportunities and wholesale funding costs. The bank earns a profit from lending (first term), pays a cost for wholesale funding (second term), and earns profits from its deposit franchise (third term). If the bank has more deposits than profitable lending opportunities, we assume it simply buys securities that pay the competitive rate f. The case $l_1 > 0$ captures the idea that the bank has a limited pool of profitable lending opportunities. Similarly, $h > 0$ captures a limited pool of wholesale funding, which makes the cost of wholesale funding increasing in the amount borrowed."

The first order conditions for D_i, H_i are

$$[D_i] :(f + l_0) - l_1 L_i - (f - s_i) + \frac{\partial s_i}{\partial D_i} D_i = 0 \tag{3.29}$$

$$[H_i] :(f + l_0) - l_1 L_i - f - hH_i = 0 \tag{3.30}$$

From (3.30) one can derive

$$H_i = \frac{l_0}{l_1 + h} - \frac{l_1}{l_1 + h} D_i \quad \Longrightarrow \quad L_i = \frac{l_0}{l_1 + h} + \frac{h}{l_1 + h} D_i$$

and so lending co-moves with deposits. Using the previous result in (3.29), after some algebraic manipulation:

$$\frac{h}{l_1 + h}(l_0 - l_1 D_i) + s_i \left(1 + \frac{\partial s_i}{\partial D_i} \frac{D_i}{s_i}\right) = 0$$

which define the individual bank demand of deposits. Quote from Drechsler et al. (2017): "The first term is the marginal lending profit the bank earns from raising another dollar of deposits. The second term is the marginal profit on the bank's deposit franchise from raising this dollar. In the baseline model ($l_0 = l_1 = 0$), the deposit franchise is the bank's only source of profits, so the bank increases deposits until this marginal profit is zero. With profitable lending opportunities the bank goes further and continues raising deposits until the marginal loss of deposit rents offsets the marginal profit from lending. The bank thus gives up some of its deposit rents in order to fund a large balance sheet and take advantage of profitable lending opportunities."

To derive aggregate deposits demand, remember that in a symmetric equilibrium $s_i = s, D_i = D$ and that $\frac{\partial D_i}{\partial s_i}\frac{s_i}{D_i} = \frac{1}{N}\frac{\partial D}{\partial s}\frac{s}{D} - \eta\left(1 - \frac{1}{N}\right)$. Substituting in before and rearranging gives that equation for aggregate deposits demand:

$$-\frac{\partial D}{\partial s}\frac{s}{D} = [1 - (N-1)(\eta - 1)] - N\left(\frac{\frac{h}{l_1+h}(l_0 - l_1 D)}{\frac{h}{l_1+h}(l_0 - l_1 D) + s}\right) \tag{3.31}$$

The first term on the right hand side was already in the baseline model, while the second captures the new setup. Equalizing this to the elasticity from household side gives the equilibrium equation to solve for s.

3.4.2 Quantitative results

As above, we resort to solving the model numerically. We use the same values for parameters summarized in Table 3.1 and pick values for new lending parameters. We find that our main results on deposits are extended qualitatively to lending: the introduction of CBDC leads to a reduction in lending under the special circumstances and leads to an increase in lending otherwise. In the cases that lending does contract, its fall is quantitatively small and it is hard to make it large. In our baseline calibration, we set $l_0 = 0.001, l_1 = 0.001, h = 0.0001$, and find that the introduction of CBDC generates a 1.9% drop in deposits and only a 0.14% drop in lending. By contrast, the drop in deposits and lending is 4.5% in the enriched model in Section 3.3.

The intuition for these results are as follows. First consider when $l_1 = 0$, the solution for H and D are independent. $H \equiv \frac{l_0}{h}$ before and after the introduction of CBDC. Then when l_0 is very small, the solution for s and D are similar to the case without lending. As l_0 increases, the introduction of CBDC is more likely to increase lending because lending is more profitable, so banks will respond more actively to the introduction of CBDC, which means the equilibrium interest rate on deposit is higher and thus the aggregate deposit is higher. One might think that as a larger l_0 or a smaller l_1 makes lending more profitable so that the banks fight harder and the drop in lending is smaller, we could assuming a smaller l_0 or a larger l_1 so that the drop in lending is larger. However, there seems to be a limit on the ratio of l_1/l_0. Numerical experiments indicate that the ratio needs to be roughly smaller than 1 and otherwise $H < 0$. The idea is that the profits on lending is so long such that the banks choose to invest, rather than, borrow from the wholesale funding channel.

Next, consider the intuition behind parameter h. When $h \to \infty$, there is no wholesale funding. When $h = 0$, lending is constant $L = \frac{l_0}{l_1}$ regardless of the introduction of CBDC. The idea is that When h is small, wholesale funding is cheaper. Banks care less about deposits, so the drop in deposits can be large, but the drop in lending is small. When h is large, banks care more about deposits, so the drop in deposits is small, and the drop in lending is also small.

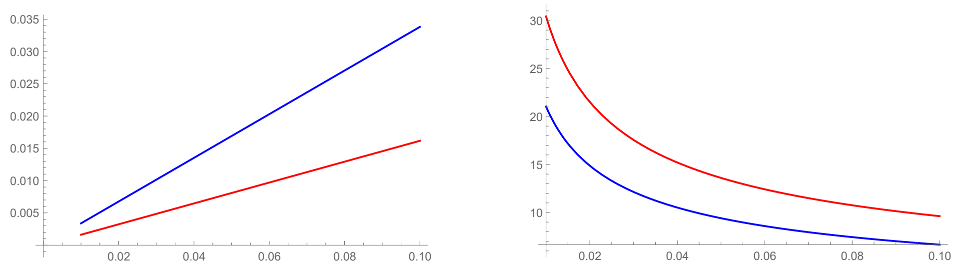
Therefore, it is hard to make the drop in lending large. So the introduction of CBDC may lead to a drop in bank deposits under some circumstances, but even when this happens, the drop in bank lending is very small when they have access to wholesale funding. We could interpret wholesale funding H as a use of a central bank lending facility, then this means the central bank can help avoid bank disintermediation following the introduction of CBDC through lending facilities available to commercial banks.

3.5 Conclusion

In this paper, we set up a portfolio choice model as a laboratory to investigate the effects of the introduction of CBDC on bank deposits and lending. We find that only in special cases introducing CBDC reduces bank credit and when it does, the effect is small. This is a fairly standard model and many important features on the banking sector or CBDC design can be added and discussed in the framework. The parameters can be calibrated more carefully to match the economy. CBDC can be designed with easy or hard access, and can feature time-varying remuneration, which will capture what different countries or analysis might have in mind. Also, the model can be extended so that banks also lend to firms and fund productive projects, or a different demand system can be assumed. We leave the explorations of these questions for future work.

3.6 Appendix: Figures and Tables

Figure 3.1: Impacts of introducing CBDC as a function of policy rate



(a) deposit spread (b) aggregate deposits

Notes: This figure presents the impact of the introduction of CBDC (red lines) on deposit spread (left panel) and on deposits (right panel) as a function of the policy rate f.

Figure 3.2: Portfolio adjustment when CBDC is introduced

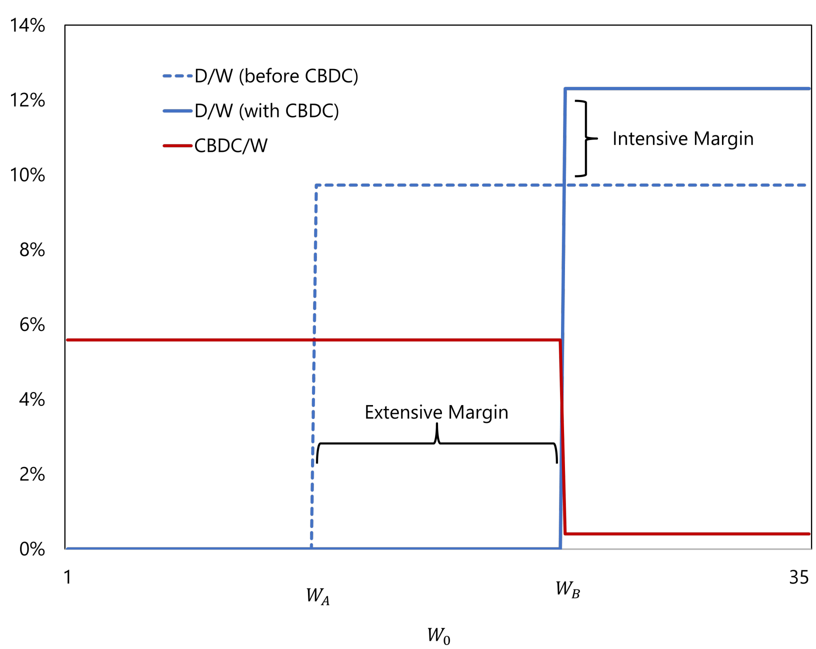


Figure 3.3: Aggregate deposits when CBDC is introduced

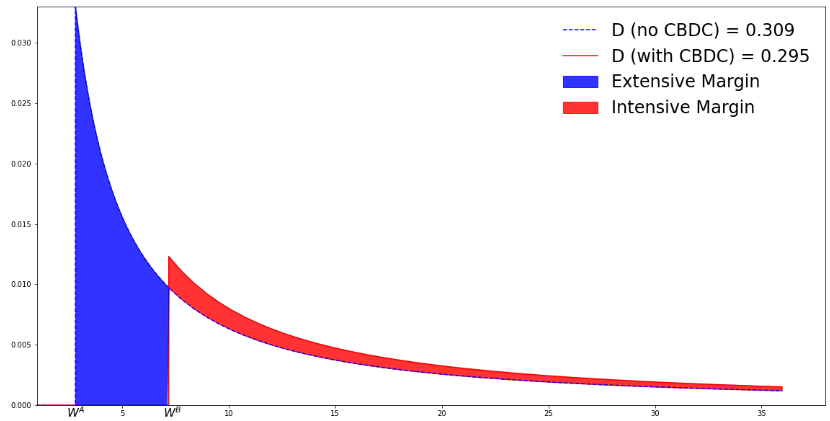


Figure 3.4: Parameter space for the disintermediation result

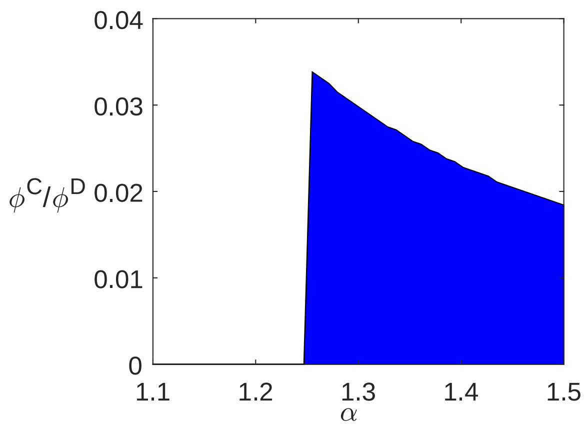


Figure 3.5: Aggregate deposit when CBDC is introduced (low α)

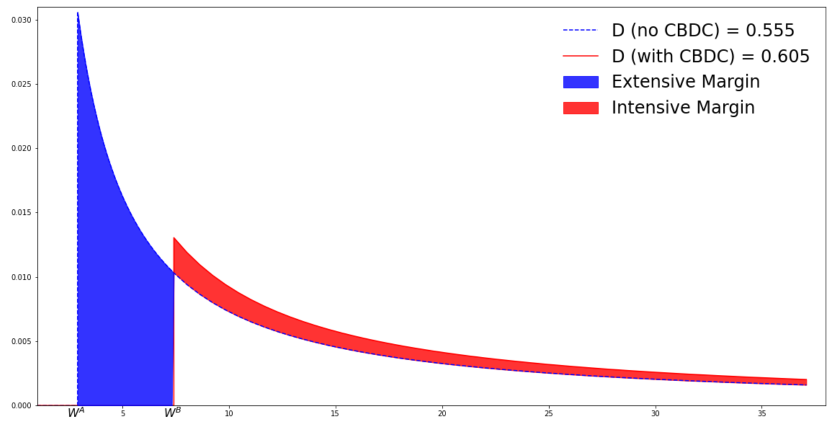


Figure 3.6: Percentage drop in aggregate deposit for different interest rate on CBDC

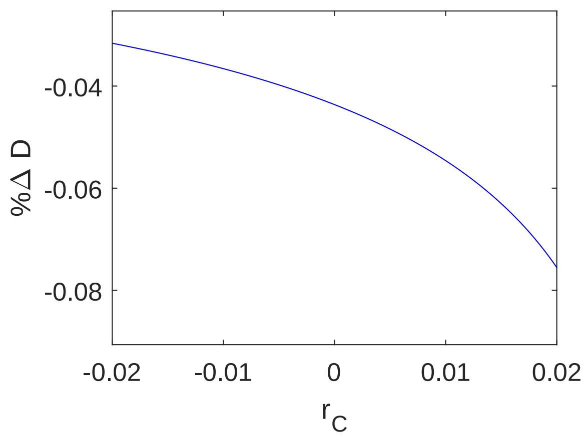


Figure 3.7: Percentage drop in aggregate deposit for different δ_C

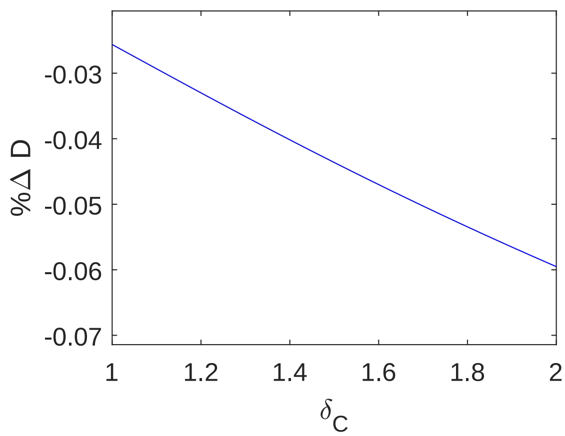


Figure 3.8: Percentage drop in aggregate deposit for different δ_D

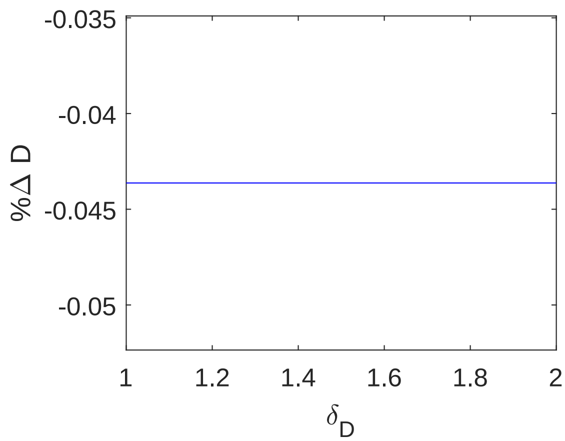


Table 3.1: Parameter values

Parameter	Definition	Value
λ	Share of liquidity assets	0.001
ρ	Complementarity b/w wealth & liquidity	0.2
ϵ	Substitutability b/w different liquidity assets	2
η	Substitutability b/w deposits at different banks	1.1
J	Number of banks	4
δ_D	Share of deposits	1.3
δ_C	Share of CBDC	1.5
f	Risk-free rate	0.03
r_C	Return on CBDC	0
ϕ^D	Fixed cost of accessing deposits	$0.15 \times \lambda^\rho$
ϕ^C	Fixed cost of accessing CBDC	$0.001 \times \lambda^\rho$
$\underline{W}$	Normalized lowest wealth	1
α	Inequality of wealth distribution	1.3

3.7 Appendix: Derivations

3.7.1 Equilibrium in model with household heterogeneity

The deposit spread s is determined in the equilibrium by the following equation:

$$
\begin{aligned}
\mathcal{M} = \epsilon & \left(\frac{\delta_N^\epsilon f^{1-\epsilon} + \delta_C^\epsilon (f - r_C)^{1-\epsilon}}{\delta_N^\epsilon f^{1-\epsilon} + \delta_D^\epsilon s^{1-\epsilon} + \delta_C^\epsilon (f - r_C)^{1-\epsilon}} \right) \\
& + \rho \left(\frac{\delta_D^\epsilon s^{1-\epsilon}}{\delta_N^\epsilon f^{1-\epsilon} + \delta_D^\epsilon s^{1-\epsilon} + \delta_C^\epsilon (f - r_C)^{1-\epsilon}} \right) \\
& + \frac{(1-\rho)\left(s_l^B\right)^{1-\rho}}{\lambda^{-\rho} + \left(s_l^B\right)^{1-\rho}} \left(\frac{\delta_D^\epsilon s^{1-\epsilon}}{\delta_N^\epsilon f^{1-\epsilon} + \delta_D^\epsilon s^{1-\epsilon} + \delta_C^\epsilon (f - r_C)^{1-\epsilon}} \right) \\
& + (\alpha - 1) \frac{\lambda^\rho \left(t(s_l^B)\right)^{2-\rho} \left(s_l^B\right)^{1-\rho}}{t(s_l^B) - t(s_l^C)} \left(\frac{\delta_D^\epsilon s^{1-\epsilon}}{\delta_N^\epsilon f^{1-\epsilon} + \delta_D^\epsilon s^{1-\epsilon} + \delta_C^\epsilon (f - r_C)^{1-\epsilon}} \right) \times \mathcal{I}_{\hat{W}_1 > \underline{W}}
\end{aligned}
\tag{3.32}
$$

3.7.2 Extensive margin

As can be seen from Equation (3.26), the indirect utility of a household is a linear function of the wealth level W_0, with different slopes and intercepts given different choices over the extensive margin, $\{N, C, D, B\}$. Notice that the slopes are such that $B > C, D > N$, and the intercepts are such that $B < D < C < N$, so the poorest households always choose N, and the richest households always choose B, and the households in the middle might choose C or D.

Denote the cutoff wealth levels to for a household choose C, D, and B over N as $\hat{W}_{11}, \hat{W}_{12}$ and $\hat{W}_{13}$, respectively. If $\hat{W}_{13} \leq \hat{W}_{11}$ and $\hat{W}_{13} \leq \hat{W}_{12}$, the scenario would be that poor households with $W \leq \hat{W}_{13}$ choose N and other households choose B (Scenario 1). The expressions for the cutoff wealth levels are given by the following equations:

$$
\hat{W}_{11} = \frac{\phi^C}{(1+f)\left(t(s_l^C) - t(s_l^N)\right)}, \quad \hat{W}_{12} = \frac{\phi^D}{(1+f)\left(t(s_l^D) - t(s_l^N)\right)}, \quad \hat{W}_{13} = \frac{\phi^D + \phi^C}{(1+f)\left(t(s_l^B) - t(s_l^N)\right)},
$$

Similarly, let $\hat{W}_{21}$ and $\hat{W}_{22}$ denote cutoff wealth levels for a household to choose D and B over C, respectively:

$$\hat{W}_{21} = \frac{\phi^D - \phi^C}{(1+f)\,(t(s_l^D) - t(s_l^C))}, \quad \hat{W}_{22} = \frac{\phi^D}{(1+f)\,(t(s_l^B) - t(s_l^C))}.$$

Following similar reasoning, we can show that there are four possible scenarios in the distribution of households over these four choices, summarized as follows.

1. N→ B

   Conditions: $\hat{W}_{13} \leq \hat{W}_{11}$, $\hat{W}_{13} \leq \hat{W}_{12}$

2. N→ C→ B

   Conditions: $\hat{W}_{11} < \hat{W}_{13}$, $\hat{W}_{11} < \hat{W}_{12}$, $\hat{W}_{22} \leq \hat{W}_{21}$ or $t(s_l^D) \leq t(s_l^C)$

3. N→ D→ B

   Conditions: $\hat{W}_{12} < \hat{W}_{13}$, $\hat{W}_{12} \leq \hat{W}_{11}$, $t(s_l^D) > t(s_l^C)$

4. N→ C→ D→ B

   Conditions: $\hat{W}_{11} < \hat{W}_{13}$, $\hat{W}_{11} < \hat{W}_{12}$, $\hat{W}_{21} < \hat{W}_{22}$, $t(s_l^D) > t(s_l^C)$

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