Title
Stochastic Simulation of Progressive Fiber Breaking in Longitudinally Fiber-Reinforced Composites

Author
Wu, Yi

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Stochastic Simulation of Progressive Fiber Breaking in Longitudinally Fiber-Reinforced Composites (FRC)

A thesis submitted in partial satisfaction

of the requirement for the degree Master of Science

in Statistics

by

Yi Wu

2012

ABSTRACT OF THE THESIS

Stochastic Simulation of Progressive Fiber Breaking in Longitudinally

Fiber-Reinforced Composites (FRC)

by

Yi Wu

Master of Science in Statistics

University of California, Los Angeles, 2012

Professor Yingnian Wu, Chair

Statistics has a wide application in science and engineering fields. This research work is aim to study the progressive fiber breaking evolution in the longitudinally reinforced composites from a statistical perspective. First of all, the fiber breaking evolution in a single fiber composite is studied. The Kolmogorov-Smirnov goodness-of-fit test is performed on the experimental data to characterize the damage pattern of the fiber in a single fiber composite. The results indicate that the fragmentation evolution of single fiber composites follows the Weibull statistic. Further investigation is focused on the damage initiation sequence in the multi-fiber composites. Four stochastic competing mechanisms are proposed to address the local load redistribution from the

broken fibers to the intact fibers. These mechanisms are based on the dominating weakness selection, random walk selection, the self-avoiding walk (SAW) select and the all surrounding neighborhood fiber selection. Finally, the evolutionary fiber breaking process in the multi-fiber composites is demonstrated by using the Weibull statistic to govern the fiber breakings in the longitudinal direction, and the competing models to describe the sequence of damage initiation.

The thesis of Yi Wu is approved.

Qing Zhou

Jiann-Wen Ju

Yingnian Wu, Chair

University of California, Los Angeles

2012

DEDICATION

To my mom and dad

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

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VITA OF YI WU

2003-2007	B.S., Water Resource and Ocean Engineering, Zhejiang University, Hangzhou, China
2007-2008	M.S., Civil and Environmental Engineering, University of California, Los Angeles, US
2010	Candidate in Philosophy Degree, University of California, Los Angeles, US

CHAPTER 1 INTRODUCTION

1.1 Introduction

Statistics has a wide application in science and engineering fields. In mechanics, various stochastic models are proposed in the hope of capturing the randomness of material behavior from the statistical perspective. Henstenburg and Phoenix [16] performed the Monte Carlo simulation to interpret experimental data from the single-filament-composite test. The simulation was based on the Poisson/Weibull probability model for failure that characterizes the strength in terms of the random flaws distributed along the fiber [1]. Milani and Nazari [15] applied the gene expression programming to predict ductile-to-brittle transition temperature of functionally graded steels in both crack divider and crack attester configurations. Meyer and his coworker [14] proposed a stochastic simulation model for microcrack initiation in martensitic steels. The probability that a microcrack of a specific length and orientation initiated in a grain was determined and inserted in a stochastic simulation model based on a random cell structure. Considerable applications reveal that appropriate statistical techniques are powerful of interpreting the randomness in different damage scenarios. Motivated by these innovative applications, this research work is aim to study the progressive multiple fiber breakings in the longitudinally reinforced composites.

1.2 Motivation

Longitudinally reinforced composites have been receiving the increasing attractiveness due to their highly specific mechanical properties and potential applications in various industries. Many experimental evidences reveal that fiber fragmentation dominates the catastrophic failure of longitudinally reinforced composites under tensile loading. Theoretically, if i) the failure strain of the embedded fibers is much lower than that of the matrix, ii) the bonding between the matrix and the fiber is perfect, fiber breaks occur prior to other damage modes. Therefore, we confine our interest to composites with brittle fibers, relatively ductile matrix, and perfectly bonded interface.

The fiber fragmentation evolution in multi-fiber composites is characterized as a stochastic process for several reasons. First of all, the fiber strength is not uniform due to the manufacturing defects. Second, the local microstructure is complex. Third, the bonding between the fiber and the matrix may not be uniform and perfect. Moreover, fiber damage in some local region increases the stress around that region and drives further damage locally. The mechanism of the load transferring from broken fibers to intact fibers is difficult to be determined analytically. As a result, the composite failure becomes statistical and the fiber fragmentation evolution follows some probability distribution.

Motivated by experimental observations and the statistical nature of fiber damage mechanism, the fiber breaking evolution of the longitudinally reinforced composites is investigated. The rest of the thesis is organized as follows. **Chapter 2** introduces the single fiber composite fragmentation test and its corresponding experimental data. In **Chapter 3**, the Kolmogorov-Smirnov goodness-of-fit test is performed to characterize the damage pattern of the fiber in a single fiber composite. **Chapter 4** concerns the damage initiation sequence of multi-fiber composites. Four statistical models are proposed to address the local load redistribution from the broken fibers to the intact fibers. **Chapter 5** summarizes this thesis work and proposes some feasible future works.

CHAPTER 2 FRAGMENTATION TEST OF SINGLE FIBER COMPOSITES

2.1Introduction

Single fiber composites (SFCs) have a long history of application to investigate the effects of fiber surface treatments on adhesion. Experimental observations of the number of fiber breaks versus applied stress, the spatial distribution of the breaks versus stress, and the average fragment length distribution at the end of the test can be inverted to quantity interfacial material properties. More importantly, many features of the SFC are preserved in the real multi-fiber composites of practical interest. Therefore, SFCs can give insight on some fundamental aspects of multi-fiber composites. The damage evolution of SFC can be used in the multi-fiber composite to predict the tensile strength.

2.2 Fragmentation Test of Single-Fiber Composites

A schematic picture of the apparatus of fragmentation test on a SFC is shown in Figure 2.1 [13]. Specimens are loaded between two grips and bound tightly with eight bolts. Two polarizers are used for observing fracture phenomena. A motor translates the specimen end and gives a tensile load to the specimen. The recorder shows how much load is applied.

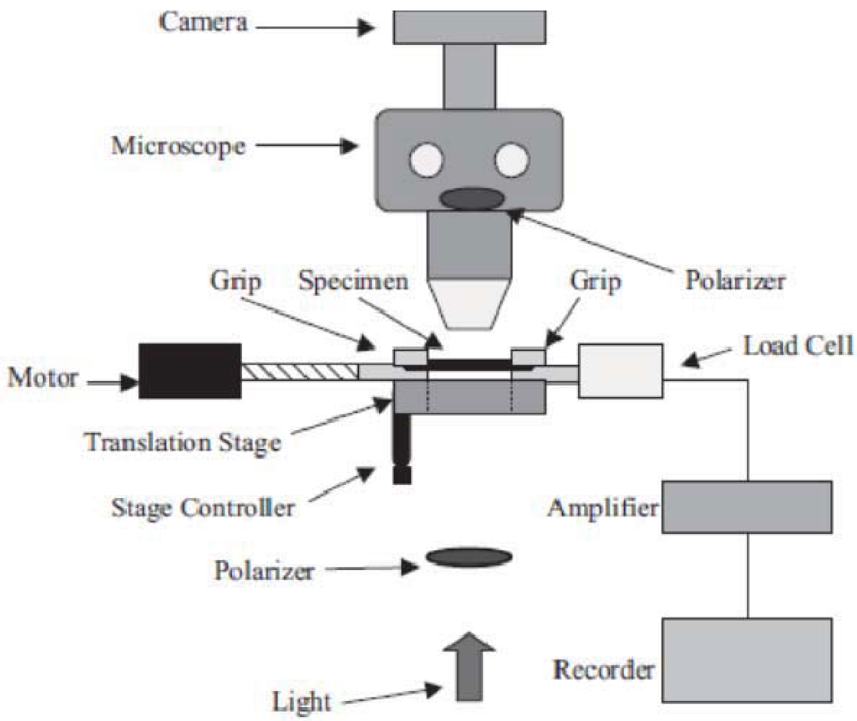


Figure 2.1 Schematic of experimental setup of the fragmentation test of single-fiber composites [13]

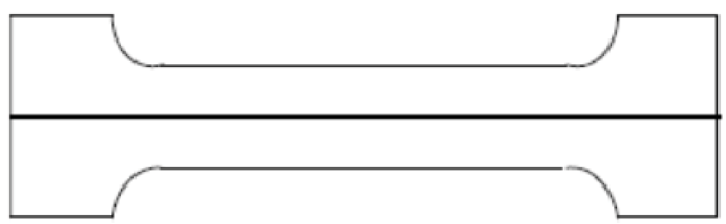


Figure 2.2 Typical dog-bone specimen for the fragmentation test of single-fiber composites [13]

The E-glass fiber/Epoxy matrix composite with different types of surface treatments are used in the fragmentation test. The specimen, as shown in Figure 2.2, is loaded gradually to reach different strain levels. The number of fiber breaks is counted until the breaks achieve saturation under each strain level. For the E-glass fiber/Epoxy matrix specimen, it takes about 10 minutes for crack number to stabilize at a constant strain level. Namely, the crack number is counted after 10 minutes when the strain arrives at a new higher level. As the loading increases, the number of fiber breaks increases and the fiber fragment length decreases. Ultimately, no new fiber breaks will occur even continuously increases the tensile loading. In other words, the crack density becomes constant and the fragment length reaches its minimum critical length. The specimen is then unloaded and the critical length of fragment is recorded

2.3 Experimental Data

The number of observed cracks at each stress level is recorded until the specimen reaches its critical fragmentation length. The experimental data of composites with the water-sized E-glass fiber, gamma-GPS treated E-glass fibers, gamma-MPS treated E-glass fibers, is tabulated in Table 2.1, Table 2.2, Table 2.3, respectively [18].

Table 2.1 Incrementally observed numbers of fiber breaks from the fragmentation test of single-fiber composites with water-sized E-glass fibers [18]

	Applied Stress Level (MPa)	Number of Observed Cracks
Data 1	648	1
Data 2	721	1
Data 3	880	2
Data 4	1028	4
Data 5	1175	7
Data 6	1323	11
Data 7	1495	17
Data 8	1607	27
Data 9	1791	29
Data 10	1952	42
Data 11	2261	56
Data 12	2544	66
Data 13	2887	69
Data 14	3316	77
Data 15	4063	80
Data 16	4552	83

Table 2.2 Incrementally observed numbers of fiber breaks from the fragmentation test of single-fiber composites with gamma-GPS treated E-glass fibers [18]

	Applied Stress Level (MPa)	Number of Observed Cracks
Data 1	1802	1
Data 2	1941	1
Data 3	2099	3
Data 4	2238	5
Data 5	2436	7
Data 6	2574	12
Data 7	2832	19
Data 8	3248	27
Data 9	3545	32
Data 10	3842	37
Data 11	4139	41
Data 12	4416	46
Data 13	4713	50
Data 14	4990	52

Table 2.3 Incrementally observed numbers of fiber breaks from the fragmentation test of single-fiber composites with gamma-MPS treated E-glass fibers [18]

	Applied Stress Level (MPa)	Number of Observed Cracks
Data 1	1449	1
Data 2	1588	2
Data 3	1727	4
Data 4	1866	4
Data 5	1985	5
Data 6	2223	9
Data 7	2521	16
Data 8	2700	22
Data 9	3017	30
Data 10	3434	42
Data 11	4288	55
Data 12	4526	59

CHAPTER 3 KOLMOGOROV-SMIRNOV GOODNESS-OF-FIT TEST FOR FIBER BREAKING BEHAVIOR OF SINGLE FIBER COMPOSITES

3.1 Introduction

The Kolmogorov-Smirnov (K-S) test is a non-parametric and distribution free test [1],[2]. It is used to decide if a sample comes from a population with a specific continuous distribution. The K-S test is based on the empirical cumulative distribution function. For a random sample from the distribution F_x, the empirical cumulative distribution function, denoted $S_n\left(x\right)$, is defined as

$$S_n\left(x\right)=\frac{\text{number of sample values} \leq x}{n} \tag{3.1}$$

The one sample K-S statistic is based on the difference between the hypothesized cumulative distribution function and the empirical distribution function of the sample. The empirical distribution function $S_n\left(x\right)$ is a consistent estimate of the population C.D.F.. As n increases, the step function $S_n\left(x\right)$, with jumps at the value of the sample order statistics $X_{(1)}, X_{(2)},\ldots, X_{(n)}$ approaches the true distribution $F_x\left(x\right)$ for all x. Therefore, the K-S one-sample statistic is defined as follows:

$$D_n=\sup_x\left|S_n\left(x\right)-F_x\left(x\right)\right| \tag{3.2}$$

3.2 Test Results

The K-S test is performed to determine whether the experimental data follows a Weibull distribution. The null and the alternative hypotheses are:

H_0 : The data follows the Weibull distribution

H_a : The data does not follow the Weibull distribution

The experimental data, mentioned in Chapter 2, are fitted by a two-parameter Weibull model and the fitting results of the Weibull parameters as well as the fiber breaking properties are summarized in Table 3.1. The comparisons of the fiber break density evolution between the experimental data and the fitting results for E-glass fiber reinforced composites with three different surface treatments are shown in Figure 3.1-Figure 3.3.Moreover, the K-S test results are shown in Table 3.2 and Figure 3.4-Figure 3.6 for E-glass fiber reinforced SFCs with the three different surface treatments. The corresponding $p-$values of water-sized fiber treatment, Gamma-GPS fiber treatment and Gamma-MPS fiber treatment are 0.3357, 0.0654 and 0.1057, respectively. Clearly, all the $p-$values are greater than 0.05, a default value of the level of significance. The difference between the experimental data for is not significantly different from the Weibull distribution. Therefore, the null hypothesis cannot be rejected at the 5% level of significance.

Table 3.1 Summary of the fitting results of Weibull parameters and fragmentation properties of the fragmentaion test of single-fiber composites

	S	M	σ_{cr} (MPa)	$n_{saturated}$
Water-sized	1475	2.026	731	81.27
Gamma-GPS	2040	1.334	1954	63.26
Gamma-MPS	2467	2.949	843	60.28

Table 3.2 Summary of the Kolmogorov-Smirnov (K-S) goodness of fit test at 5% significance level

	Accept H_0	p-value	K-S test statistics
Water-sized	Yes	0.3357	0.2259
Gamma-GPS	Yes	0.0654	0.3152
Gamma-MPS	Yes	0.1057	0.3754

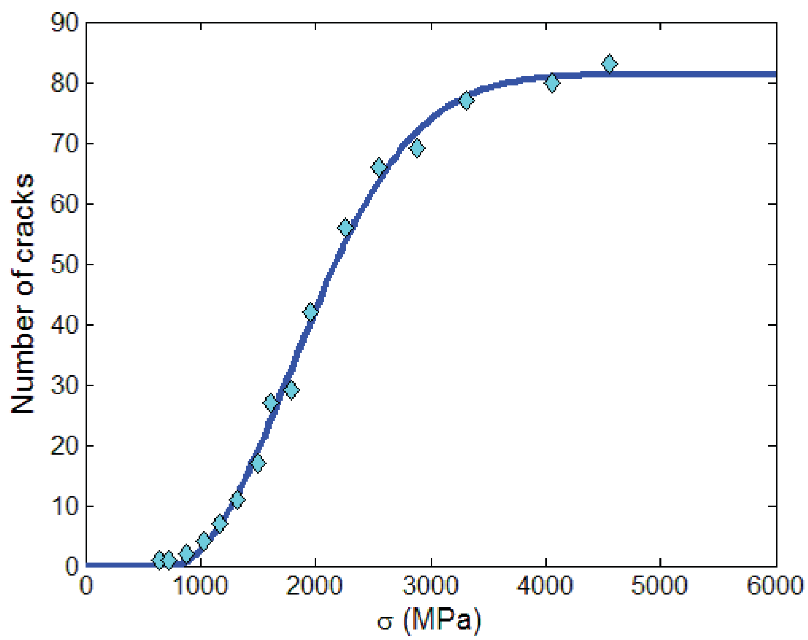


Figure 3.1 Comparison of number of observed fiber breaks versus applied macro stress between the fitted Weibull model and the experimental data (water-sized E-glass fibers) [18]

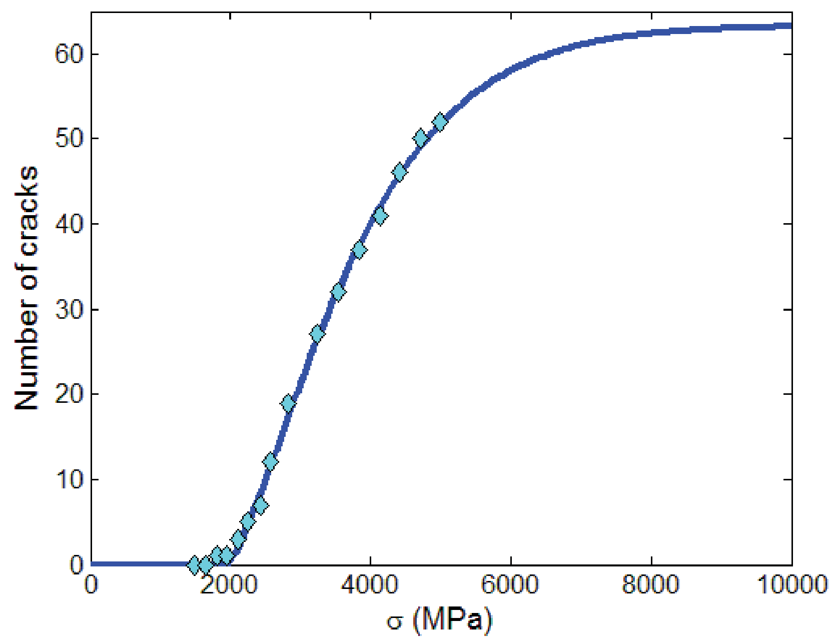


Figure 3.2 Comparison of number of observed fiber breaks versus applied macro stress between the fitted Weibull model and the experimental data (gamma-GPS treated E-glass fibers) [18]

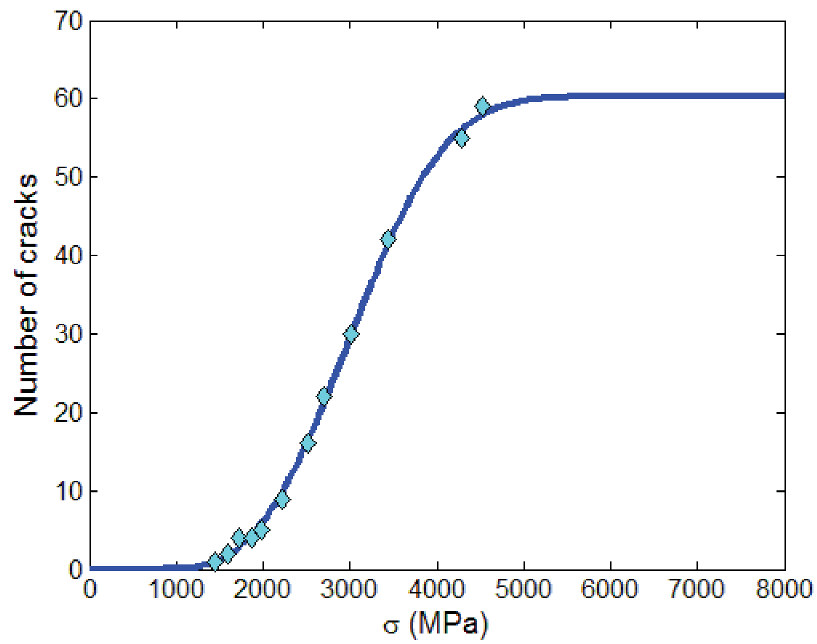


Figure 3.3 Comparison of number of observed fiber breaks versus applied macro stress between the fitted Weibull model and the experimental data (gamma-MPS treated E-glass fibers) [18]

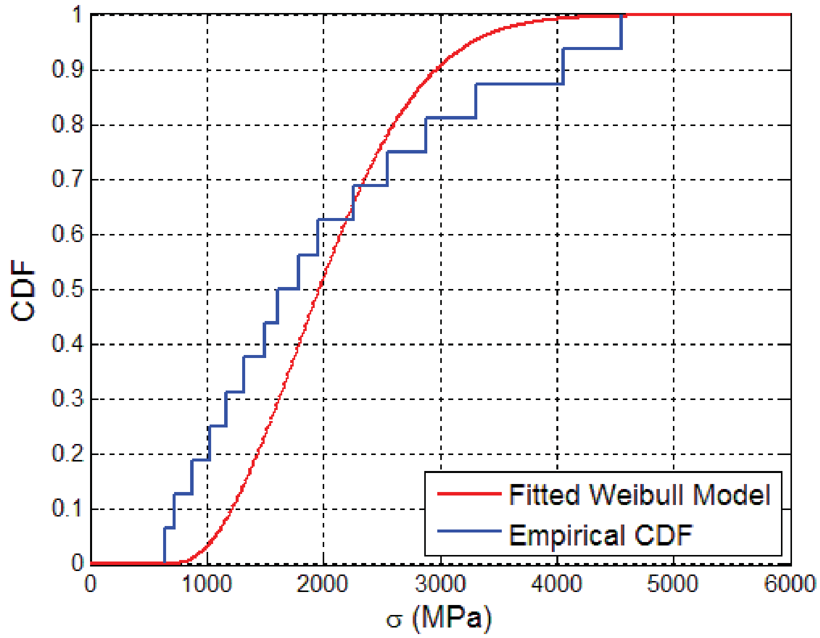


Figure 3.4 Comparison between the fitted Weibull model and the empirical CDF (water-sized E-glass fiber)

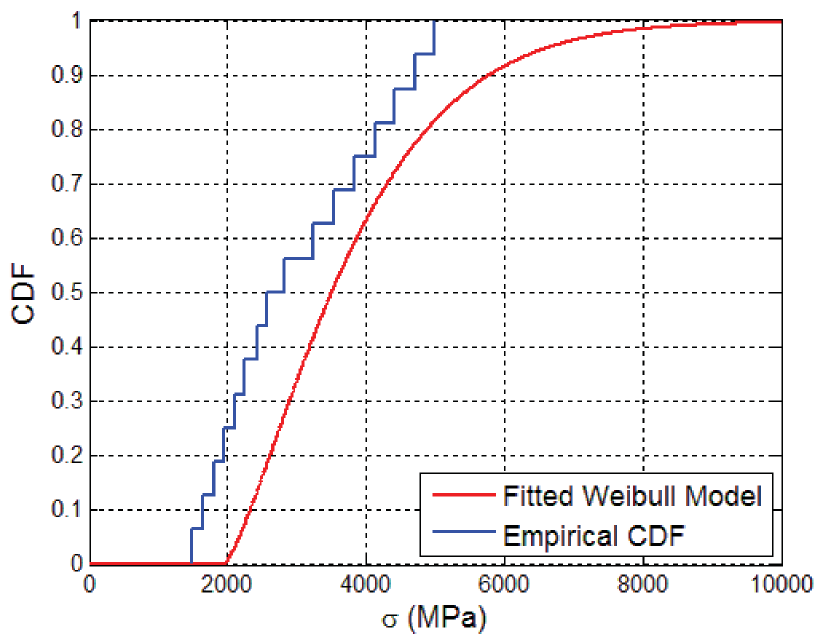


Figure 3.5 Comparison between the fitted Weibull model and the empirical CDF (gamma-GPS treated E-glass fiber)

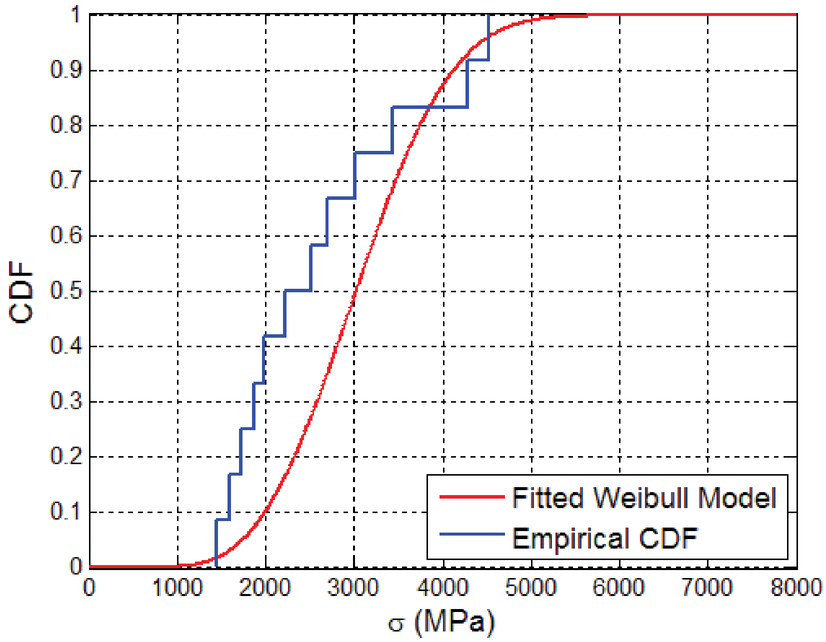


Figure 3.6 Comparison between the fitted Weibull model and the empirical CDF (gamma-MPS treated E-glass fiber)

CHAPTER 4 STOCHASTIC SIMULATION OF MULTIPLE FIBER BREAKING IN FIBER-REINFORCED COMPOSITES

4.1 Micromechanical Formulation of Progressive Fiber Breaking in Longitudinally Fiber-Reinforced Composites

Consider a far-field macro strain $\boldsymbol{\varepsilon}_0$ applied on the boundary ∂D of the representative volume element (RVE) D with an inhomogeneous inclusion Ω as shown in Figure 4.1, the total stress field $\boldsymbol{\sigma}^t$ in the composite material can be expressed as

$$\begin{cases} \boldsymbol{\sigma}^t = \mathbf{C}_1 : \left(\boldsymbol{\varepsilon}^0 + \boldsymbol{\varepsilon}' - \boldsymbol{\varepsilon}^*_{cr} \right) & \text{in } \Omega \\ \boldsymbol{\sigma}^t = \mathbf{C}_0 : \left(\boldsymbol{\varepsilon}^0 + \boldsymbol{\varepsilon}' \right) & \text{in } \mathrm{D} - \Omega \end{cases} \tag{4.1}$$

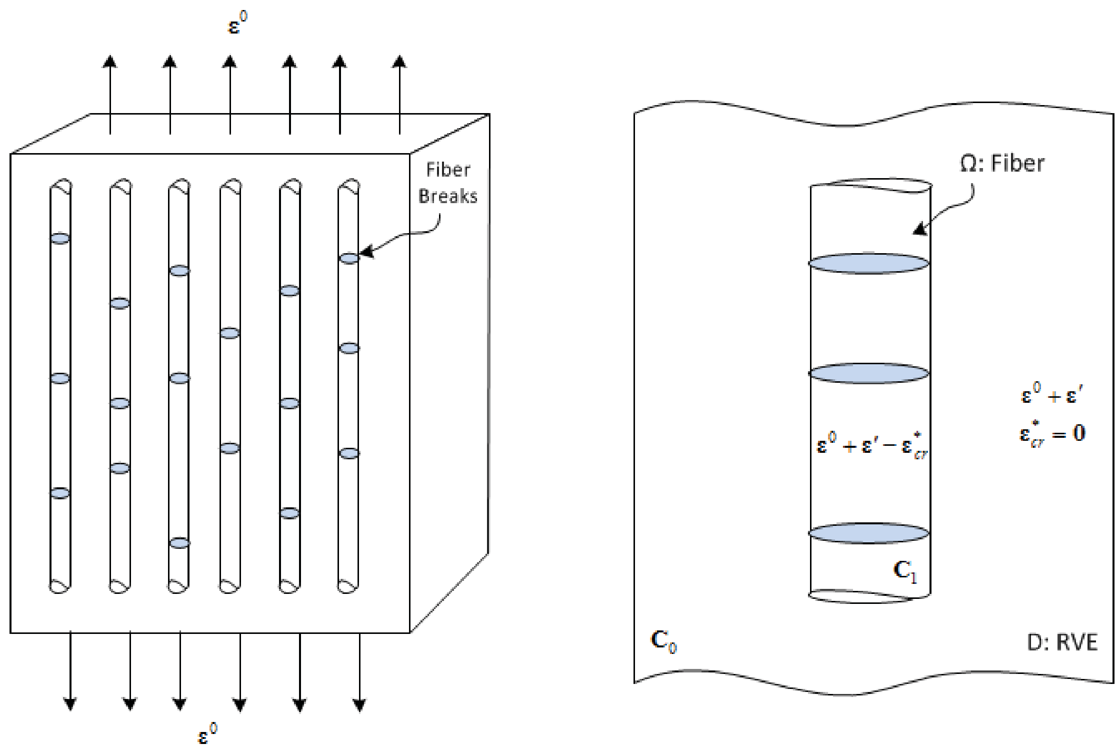
where $\mathbf{C}_1$ and $\mathbf{C}_0$ are the elasticity tensor of the inclusion (fiber) phase and matrix phase, respectively, $\boldsymbol{\varepsilon}'$ is the perturbed strain due to the inhomogeneus inclusion, and $\boldsymbol{\varepsilon}^*_{cr}$ is the eigenstrain due to fiber breaking effects.

Utilizing Eshelby's equivalent inclusion principle [4],[5] leads to

$$\mathbf{C}_1 : \left(\boldsymbol{\varepsilon}^0 + \boldsymbol{\varepsilon}' - \boldsymbol{\varepsilon}^*_{cr} \right) = \mathbf{C}_0 : \left(\boldsymbol{\varepsilon}^0 + \boldsymbol{\varepsilon}' - \boldsymbol{\varepsilon}^*_{cr} - \boldsymbol{\varepsilon}^* \right) \quad \text{in } \Omega \tag{4.2}$$

where the perturbed strain $\boldsymbol{\varepsilon}_p$ due to the inhomogeneitiy can be related to the total eigenstrain $\boldsymbol{\varepsilon}^{**}$ based on Eshelby's solution [4],[5]:

$$\boldsymbol{\varepsilon}' = \mathbf{S} : \boldsymbol{\varepsilon}^{**} = \mathbf{S} : \left(\boldsymbol{\varepsilon}^*_{cr} + \boldsymbol{\varepsilon}^* \right) \tag{4.3}$$



(a) Schematic of the RVE for Longitudinally Fiber-Reinforced Composites with Fiber Breaks

(b) Inhomogeneous inclusion (fiber phase) with the presence of fiber breaks in a homogeneous body (matrix)

Figure 4.1 Illustration of a representative volume element (RVE) in which the cylindrical fibers with fiber breaks embedded in an infinite elastic medium

and $\mathbf{S}$ is the Eshelby tensor depending on the material properties of the matrix and the geometry of the inclusion. In the present study, we consider the composites embedded with cylindrical fibers, and the detailed expression for the Eshebly tensor can be found in [9],[10]. Therefore, with the aid of Eq. (4.2), the total stress can be recast as

$$\begin{cases} \boldsymbol{\sigma}^t = \mathbf{C}_1 : \left(\boldsymbol{\varepsilon}^0 + \boldsymbol{\varepsilon}' - \boldsymbol{\varepsilon}^*_{cr} - \boldsymbol{\varepsilon}^*\right) & \text{in } \Omega \\ \boldsymbol{\sigma}^t = \mathbf{C}_0 : \left(\boldsymbol{\varepsilon}^0 + \boldsymbol{\varepsilon}'\right) & \text{in D} - \Omega \end{cases} \tag{4.4}$$

The ensemble-volume averaged strain $\langle \bar{\boldsymbol{\varepsilon}} \rangle$ can be determined by [6],[9]

$$\left\langle \bar{\boldsymbol{\varepsilon}} \right\rangle = \phi_m \left\langle \bar{\boldsymbol{\varepsilon}}_m \right\rangle + \phi_r \left\langle \bar{\boldsymbol{\varepsilon}}_r \right\rangle = \boldsymbol{\varepsilon}^0 + \mathbf{S} : \left(\sum_{r=1}^{n} \phi_r \left\langle \bar{\boldsymbol{\varepsilon}}_r^{**} \right\rangle \right) \tag{4.5}$$

where $\left\langle \bar{\bullet} \right\rangle$ denotes the ensemble-volume averaged operator, ϕ_r is the volume fraction of phase r inhomogeneity, and $\left\langle \bar{\boldsymbol{\varepsilon}}_r^{**} \right\rangle$ is the ensemble-averaged eigenstrain in phase r inclusion. Similarly, the ensemble-volume averaged stress is

$$\begin{aligned} \left\langle \bar{\boldsymbol{\sigma}} \right\rangle &= \frac{1}{V} \left[V_m \left\langle \bar{\boldsymbol{\sigma}}_m \right\rangle : + \sum_{r=1}^{n} V_r \left\langle \bar{\boldsymbol{\sigma}}_r \right\rangle \right] \\ &= \frac{1}{V} \left[V_m \mathbf{C}_0 : \left\langle \bar{\boldsymbol{\varepsilon}}_m \right\rangle + \sum_{r=1}^{n} \mathbf{C}_0 : \left(\left\langle \bar{\boldsymbol{\varepsilon}}_r \right\rangle - \left\langle \bar{\boldsymbol{\varepsilon}}_r^{**} \right\rangle \right) V_r \right] \\ &= \mathbf{C}_0 : \left[\left\langle \bar{\boldsymbol{\varepsilon}} \right\rangle - \sum_{r=1}^{n} \phi_r \left\langle \bar{\boldsymbol{\varepsilon}}_r^{**} \right\rangle \right] \end{aligned} \tag{4.6}$$

where V_m and V_r are the volume of the matrix phase and phase r inclusion, respectively, and V is the volume of the RVE. In order to relate the ensemble-volume averaged stress (Eq. (4.6)) to the ensemble-volume averaged strain (Eq. (4.5)) and to derive the homogenized overall moduli, we are aiming to express the ensemble-volume averaged eigenstrains $\left\langle \bar{\boldsymbol{\varepsilon}}_r^{**} \right\rangle$ in each inclusion phase and the far-field strain $\boldsymbol{\varepsilon}^0$ in terms of the ensemble-volume averaged strains $\left\langle \bar{\boldsymbol{\varepsilon}} \right\rangle$.

4.1.1 Eigenstrain due to Fiber Breaking in Fiber-Reinforced Composites

In the presence of fiber breaks in the inclusion phase, the corresponding ensemble averaged eigenstrains in unidirectional fiber-reinforced composites can be expressed as [6],[11]

$$\left\langle \boldsymbol{\varepsilon}_{cr}^{*} \right\rangle (\mathbf{x}) = f(\mathbf{x}) \cdot \frac{1}{2} \int_{S_i} \left\langle [\mathbf{u}] \otimes \mathbf{n} + \mathbf{n} \otimes [\mathbf{u}] \right\rangle (\mathbf{x}' | \mathbf{x}) \, dS_i \tag{4.7}$$

where $f(\mathbf{x})$ is the probability density function for a fiber break centered at $\mathbf{x}$, $\mathbf{n}=[0,0,1]^T$ is the orientation of this fiber break assumed to be aligned in the *z* direction in the following derivation, $\mathbf{x}'$ denotes a point on the crack surface S_i, and $[\mathbf{u}]$ is the microcrack opening displacement based on the theory of linear fracture mechanics which has the form [8],[12]

$$\begin{Bmatrix} [u_x] \\ [u_y] \\ [u_z] \end{Bmatrix} = \frac{8(1-\nu^2)}{\pi E(2-\nu\ )}\sqrt{c^2-r^2}\begin{Bmatrix} 2s \\ 2t \\ (2-\nu_0)p \end{Bmatrix} \tag{4.8}$$

where E and ν are the Young's modulus and Poisson's ratio of the fiber material, c is the radius of the fiber break, $r=|\mathbf{x}'-\mathbf{x}|$ is the distance of point $\mathbf{x}'$ away from the center of fiber break, and p, s, and t are the *z*-direction *normal*, *x*-direction and *y*-direction *shear* stresses projected on the fiber microcrack surface in its local coordinates. By carrying out the integration of Eq. (4.7) with respect to all the points $\mathbf{x}'$ on the crack surface S_i, we obtain

$$\langle \boldsymbol{\varepsilon}_{cr}^* \rangle (\mathbf{x}) = f(\mathbf{x}) \cdot \frac{16(1-\nu^2)}{3\pi E(2-\nu\ )} \cdot c^3 \cdot \mathbf{g} \cdot \mathbf{K}_0 : \langle \boldsymbol{\sigma}_f \rangle_r \tag{4.9}$$

where $\langle \boldsymbol{\sigma}_f \rangle_r$ is the ensemble averaged fiber stress associated with the rth phasge fiber inclusion, $\mathbf{g}$ and $\mathbf{K}_0$ are the transformation matrices and have the forms:

$$\mathbf{g}^T = \begin{bmatrix} 0 & 0 & 2-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{K}_0 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{4.10}$$

Performing the volume averaged due the existence of the fiber breaks in the rth fiber inclusion phase, we have

$$\left\langle \bar{\boldsymbol{\varepsilon}}_{cr}^{*} \right\rangle = \frac{16(1-\nu^2)}{3\pi E(2-\nu\ )} \cdot c^3 \cdot \mathbf{g} \cdot \mathbf{K}_0 : \left\langle \bar{\boldsymbol{\sigma}}_f \right\rangle_r \cdot \frac{\int_{V_f} f(\mathbf{x}) d\mathbf{x}}{V_f} = \beta \cdot n \cdot \boldsymbol{\Gamma} : \left\langle \bar{\boldsymbol{\sigma}}_f \right\rangle_r \tag{4.11}$$

where $\beta = 16(1-\nu^2)c / 3\pi E(2-\nu\ )L_f$, n denotes the number density of fiber breaks, L_f is the fiber length, and $\boldsymbol{\Gamma}$ is the 4th order tensor which has the form

$$\boldsymbol{\Gamma} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = diag\left[0,0,2-\nu,2,2,0\right] \tag{4.12}$$

It can be easily seen from Eqns. (4.11) and (4.12) that the presence of the fiber breaks only contributes to the z-direction normal, *x*-direction and *y*-direction shear strain components of the eigenstrain field. By substituting the stresses of the fiber phase defined in Eq. (4.4) into Eq. (4.11), we reach the following expression

$$\left\langle \bar{\boldsymbol{\varepsilon}}_{cr}^{*} \right\rangle = \beta \cdot n \cdot \boldsymbol{\Gamma} \bullet \left\langle \bar{\boldsymbol{\sigma}}_r \right\rangle = \beta \cdot n \cdot \boldsymbol{\Gamma} \bullet \left\{ \mathbf{C}_0 : \left[\boldsymbol{\varepsilon}^0 + \left\langle \bar{\boldsymbol{\varepsilon}}' \right\rangle - \left\langle \bar{\boldsymbol{\varepsilon}}_r^{**} \right\rangle \right] \right\} \tag{4.13}$$

In the current study, we make the assumption that the number density of fiber breaks follows a Weibull distribution which has the form

$$n = p_1 \cdot n_{saturated} \tag{4.14}$$

$$p_1 = \begin{cases} 1 - \exp\left[-\left(\dfrac{\sigma - \sigma_{cr}}{S} \right)^M \right] & \text{if } \sigma \geq \sigma_{cr} \\ 0 & \text{if } \sigma < \sigma_{cr} \end{cases} \tag{4.15}$$

where $n_{saturated}$ and σ_{cr} are the saturated fiber breaks number density and stress threshold for first initiated fiber break, respectively, which are characterized from the fragmentation test of the single-fiber composite introduced in **Section 2**, S and M are the two Weibull parameters that can be obtained by carrying out the nonlinear fitting on the experimental data as illustrated in **Section 3**. Therefore, we obtain the expression for the ensemble-volume averaged eigenstrains in the rth inclusion phase as follows:

$$\left\langle \bar{\boldsymbol{\varepsilon}}_r^{**} \right\rangle = \mathbf{T}_r : \boldsymbol{\varepsilon}^0 \tag{4.16}$$

The 4th order tensor $\mathbf{T}_r$ in Eq. (4.16) relates the ensemble-volume averaged eigenstrains in the rth inclusion phase to the far-field macro strains as follwos:

$$\mathbf{T}_r = -\left[\mathbf{C}_1 \bullet \mathbf{S} - \mathbf{C}_0 \bullet \left(\mathbf{S} - \mathbf{I}\right) - \beta \cdot n \cdot \boldsymbol{\Gamma} \bullet \mathbf{C}_0 \bullet \left(\mathbf{S} - \mathbf{I}\right)\right]^{-1} \bullet \left[\mathbf{C}_1 - \left(\mathbf{I} + \beta \cdot n \cdot \boldsymbol{\Gamma}\right) \bullet \mathbf{C}_0\right] \tag{4.17}$$

where $\mathbf{I}_{ijkl} = \delta_{ij}\delta_{kl} + \frac{1}{2}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)$ is the 4th order identity tensor. It can be easily seen that the ensemble-volume averaged eigenstrains in the inclusion phase is induced by both the mismatch of the inhomogeneity and the fiber breaking effects.

4.1.2 Overall Homogenized Elastic Moduli of Fiber-Reinforced Composites with Progressive Fiber Breaking

Substituting Eq. (4.16) into Eq. (4.5) leads to the expression of the far-field strain in terms of the ensemble-volume averaged strain as

$$\boldsymbol{\varepsilon}^0 = \mathbf{B}^{-1} : \left\langle \bar{\boldsymbol{\varepsilon}} \right\rangle \tag{4.18}$$

where 4th order tensor $\mathbf{B}$ has the form

$$\mathbf{B} = \mathbf{I} + \sum_{r=1}^{n} \phi_r \cdot \mathbf{S} \bullet \mathbf{T}_r \tag{4.19}$$

By combining Eq. (4.16) and Eq. (4.18), we can reach the expression as follows:

$$\left\langle \overline{\boldsymbol{\varepsilon}}_r^{**} \right\rangle = \left(\mathbf{B}^{-1} \bullet \mathbf{T}_r \right) : \left\langle \overline{\boldsymbol{\varepsilon}} \right\rangle \tag{4.20}$$

With the aid of Eqns. (4.6) and (4.20), the ensemble-volume averaged strain and ensemble-volume averaged stress is

$$\left\langle \overline{\boldsymbol{\sigma}} \right\rangle = \left\{ \mathbf{C}_0 \bullet \left[\mathbf{I} - \sum_{r=1}^{n} \phi_r \left(\mathbf{B}^{-1} \bullet \mathbf{T}_r \right) \right] \right\} : \left\langle \overline{\boldsymbol{\varepsilon}} \right\rangle \tag{4.21}$$

Hence, based on the theory of micromechanics and linear elastic fracture mechanics, the overall homogenized elasticity moduli for the longitudinally fiber-reinforced composites with progressive fiber breaking are derived:

$$\mathbf{C}^* = \mathbf{C}_0 \bullet \left[\mathbf{I} - \sum_{r=1}^{n} \phi_r \cdot \mathbf{B}^{-1} \bullet \mathbf{T}_\mathbf{r} \right] \tag{4.22}$$

The fiber stress in the rth phase inclusion is then obtained by using Eshelby's equivalent inclusion principle and taking the ensemble-volume averaging of the stress of the rth inclusion:

$$\begin{aligned} \left\langle \overline{\boldsymbol{\sigma}}_f \right\rangle_r &= \mathbf{C}_0 : \left[\boldsymbol{\varepsilon}^0 + \left(\mathbf{S} - \mathbf{I} \right) : \left\langle \overline{\boldsymbol{\varepsilon}}^{**} \right\rangle \right] \\ &= \mathbf{C}_0 : \left[\mathbf{I} + \left(\mathbf{S} - \mathbf{I} \right) \bullet \mathbf{T}_r \right] : \boldsymbol{\varepsilon}^0 \\ &= \left\{ \mathbf{C}_0 \bullet \left[\mathbf{I} + \left(\mathbf{S} - \mathbf{I} \right) \bullet \mathbf{T}_r \right] \bullet \mathbf{B}^{-1} \right\} : \left\langle \overline{\boldsymbol{\varepsilon}} \right\rangle \end{aligned} \tag{4.23}$$

4.1.3 Computational Algorithm for Modeling of Progressive Damage of Fiber Breaking in Longitudinally Fiber-Reinforced Composites

In the following numerical simulation of the progressive fiber breaking in the longitudinally fiber-reinforced composites, we employ a strain-driven algorithm to determine the overall stress history by the given overall strain history. Given the known state from the previous time step $t = t_\lambda$, we are seeking to determine the unknown state $\left\{ \bar{\boldsymbol{\varepsilon}}_{\lambda+1}, \bar{\boldsymbol{\varepsilon}}^*_{cr\lambda+1}, \bar{\boldsymbol{\varepsilon}}'_{\lambda+1}, n_{\lambda+1}, \bar{\boldsymbol{\sigma}}_{\lambda+1} \right\}$ at time step $t = t_{\lambda+1}$. The fiber stress and its induced fiber break number density are computed by an internal numerical iteration at the (λ+1)th time step, with the following convergence criteria:

$$\left| n^{\nu}_{\lambda+1} - n^{\nu-1}_{\lambda+1} \right| \le TOL \tag{4.24}$$

and

$$\left(\bar{\boldsymbol{\sigma}}_f \right)^{\nu}_{\lambda+1} = \left\{ \mathbf{C}_0 \bullet \left[\mathbf{I} - \sum_{r=1}^{n} \phi_r \left(\left(\mathbf{B}^{\nu}_{\lambda+1} \right)^{-1} \bullet \left(\mathbf{T}_r \right)^{\nu}_{\lambda+1} \right) \right] \right\} : \bar{\boldsymbol{\varepsilon}}_{\lambda+1} \tag{4.25}$$

$$n^{\nu}_{\lambda+1} = \left(p_1 \right)^{\nu}_{\lambda+1} \cdot n_{saturated} \tag{4.26}$$

$$\left(p_1 \right)^{\nu}_{\lambda+1} = 1 - \exp\left[-\left(\frac{ \left(\bar{\boldsymbol{\sigma}}_{zzf} \right)^{\nu}_{\lambda+1} - \sigma_{cr} }{S} \right)^{M} \right] \quad \text{if} \quad \left(\bar{\boldsymbol{\sigma}}_{zzf} \right)^{\nu}_{\lambda+1} \ge \sigma_{cr} \tag{4.27}$$

where ν is the internal iteration index. Therefore, the overall elastic moduli at the current time step can be computed by

$$\mathbf{C}^*_{\lambda+1} = \mathbf{C}_0 \bullet \left[\mathbf{I} - \sum_{r=1}^{n} \phi_r \cdot \left(\mathbf{B}_{\lambda+1} \right)^{-1} \bullet \left(\mathbf{T}_{\mathbf{r}} \right)_{\lambda+1} \right] \tag{4.28}$$

Consequently, the overall stress $\bar{\boldsymbol{\sigma}}_{\lambda+1}$ at the current time step can be updated as

$$\bar{\boldsymbol{\sigma}}_{\lambda+1} = \bar{\boldsymbol{\sigma}}_{\lambda} + \mathbf{C}^*_{\lambda+1} : \Delta\bar{\boldsymbol{\varepsilon}}_{\lambda+1} \tag{4.29}$$

For convenience, Table 4.1 summarizes the above micromechanical iterative computational algorithm for the overall elastic responses of longitudinally fiber-reinforced composites with progressive damages due to fiber breaks.

Table 4.1 Numerical algorithm for determination of overall homogenized elasticity moduli of fiber-reinforeced composites with progressive fiber breaking

Given: $\left\{\bar{\boldsymbol{\varepsilon}}_{\lambda}, \bar{\boldsymbol{\varepsilon}}^*_{cr\lambda}, \bar{\boldsymbol{\varepsilon}}'_{\lambda}, n_{\lambda}, \bar{\boldsymbol{\sigma}}_{\lambda}\right\}$ at time step λ and with the strain increment $\left\{\Delta\bar{\boldsymbol{\varepsilon}}_{\lambda+1}\right\}$

Sovling: $\left\{\bar{\boldsymbol{\varepsilon}}_{\lambda+1}, \bar{\boldsymbol{\varepsilon}}^*_{cr\lambda+1}, \bar{\boldsymbol{\varepsilon}}'_{\lambda+1}, n_{\lambda+1}, \bar{\boldsymbol{\sigma}}_{\lambda+1}\right\}$ at time step (λ+1)

(i) Initialize: $\left\{\bar{\boldsymbol{\varepsilon}}_0 = 0, \bar{\boldsymbol{\varepsilon}}^*_{cr0} = 0, \bar{\boldsymbol{\varepsilon}}'_0 = 0, n_0 = 0, \bar{\boldsymbol{\sigma}}_0 = 0\right\}$ $\left\{p_{10} = 0\right\}$

(ii) Compute: $\bar{\boldsymbol{\varepsilon}}_{\lambda+1} = \bar{\boldsymbol{\varepsilon}}_{\lambda} + \Delta\bar{\boldsymbol{\varepsilon}}_{\lambda+1}$

(iii) Compute: $\left\{\bar{\boldsymbol{\varepsilon}}^*_{cr\lambda+1}, n_{\lambda+1}, p_{1\lambda+1}\right\}$ with the internal iteration until $\left|n^{\nu}_{\lambda+1} - n^{\nu-1}_{\lambda+1}\right| \le TOL$

$$n^0_{\lambda+1} = n_{\lambda}$$

$$\left(\mathbf{T}_r\right)^{\nu}_{\lambda+1} = -\left[\mathbf{C}_1 \bullet \mathbf{S} - \mathbf{C}_0 \bullet \left(\mathbf{S} - \mathbf{I}\right) - \beta \cdot n^{\nu-1}_{\lambda+1} \cdot \boldsymbol{\Gamma} \bullet \mathbf{C}_0 \bullet \left(\mathbf{S} - \mathbf{I}\right)\right]^{-1} \bullet \left[\mathbf{C}_1 - \left(\mathbf{I} + \beta \cdot n^{\nu-1}_{\lambda+1} \cdot \boldsymbol{\Gamma}\right) \bullet \mathbf{C}_0\right]$$

$$\mathbf{B}^{\nu}_{\lambda+1} = \mathbf{I} + \sum_{r=1}^{n} \phi_r \cdot \mathbf{S} \bullet \left(\mathbf{T}_r\right)^{\nu}_{\lambda+1}$$

$$\left(\bar{\boldsymbol{\sigma}}_f\right)^{\nu}_{\lambda+1} = \left\{\mathbf{C}_0 \bullet \left[\mathbf{I} - \sum_{r=1}^{n} \phi_r \left(\left(\mathbf{B}^{\nu}_{\lambda+1}\right)^{-1} \bullet \left(\mathbf{T}_r\right)^{\nu}_{\lambda+1}\right)\right]\right\} : \bar{\boldsymbol{\varepsilon}}_{\lambda+1}$$

$$\left(\bar{\boldsymbol{\varepsilon}}^*_{cr}\right)^{\nu}_{\lambda+1} = \beta \cdot n \cdot \boldsymbol{\Gamma} \bullet \left(\bar{\boldsymbol{\sigma}}_f\right)^{\nu}_{\lambda+1}$$

$n^{\nu}_{\lambda+1} = \left(p_1\right)^{\nu}_{\lambda+1} \cdot n_{saturated}$ with the progressive evolution of fiber breaking

$$\left(p_1\right)^{\nu}_{\lambda+1} = \begin{cases} 1 - \exp\left[-\left(\dfrac{\left(\bar{\boldsymbol{\sigma}}_{zzf}\right)^{\nu}_{\lambda+1} - \sigma_{cr}}{S}\right)^M\right] & \text{if } \left(\bar{\boldsymbol{\sigma}}_{zzf}\right)^{\nu}_{\lambda+1} \ge \sigma_{cr} \\ 0 & \text{if } \left(\bar{\boldsymbol{\sigma}}_{zzf}\right)^{\nu}_{\lambda+1} < \sigma_{cr} \end{cases}$$

(iv) Compute: $\mathbf{C}^*_{\lambda+1} = \mathbf{C}_0 \bullet \left[\mathbf{I} - \sum_{r=1}^{n} \phi_r \cdot \left(\mathbf{B}_{\lambda+1}\right)^{-1} \bullet \left(\mathbf{T}_{\mathbf{r}}\right)_{\lambda+1}\right]$

(v) Calculate: $\bar{\boldsymbol{\sigma}}_{\lambda+1} = \bar{\boldsymbol{\sigma}}_{\lambda} + \mathbf{C}^*_{\lambda+1} : \Delta\bar{\boldsymbol{\varepsilon}}_{\lambda+1}$

(vi) GO TO (ii) (the next time step computation)

4.2 Stochastic Simulations of Progressive Damage Evolution due to Fiber Breaking in Fiber-Reinforced Composites

The local micro-structure of a multi-fiber composite is clearly much more complicated than that of a single fiber composite. A periodic unit cell model is widely used in the micromechanics analysis for its simplicity and well-representation. Therefore, a planar periodic unit cell structure as shown in Figure 4.2 is adopted to simulate the damage accumulation in the present work. Each fiber sub-unit is assumed to represent a single fiber embedded in the matrix material. The evolution of fiber fragmentation in multi-fiber composites during loading is, in principle, different from that in the S.F.C.. Each individual fiber experiences a non-uniform stress due to the uniform applied stress plus stresses transferred from other broken fibers in the composites. Fiber damage in some local region increases the stresses in the surrounding neighborhood and drives further damage locally. As a result, the composite failure becomes statistical, with some probability distribution. In this sense, the evolution of fiber damage thus depends crucially on the nature of the load transfer from broken or slipping fibers to intact fibers. The key to describe the fragmentation process in the multi-fiber composite is to understand how the local loading sharing influences the composite behavior. Four statistical models are proposed to determine the nature of the load sharing and selection of broken fibers in any particular system for arbitrary spatial locations of breaks.

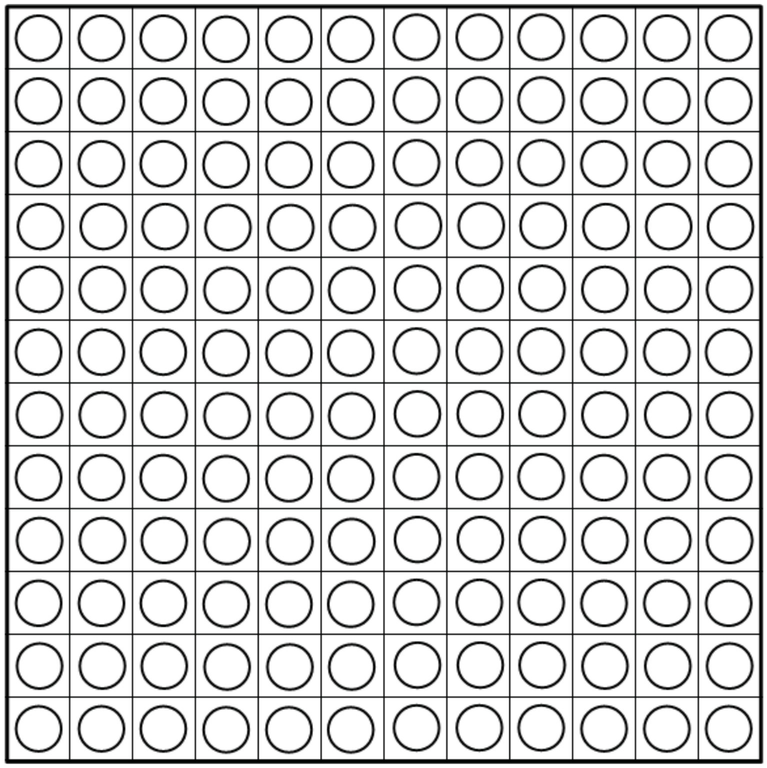


Figure 4.2 Unit cell of 12x12 fiber sub-units for stochastic simulation of multiple fiber breaking process in fiber-reinforced composites

4.2.1 Simulation Algorithms for Stochastic Modeling of Fiber Fracture

Figure 4.3 demonstrates the computational algorithm for numerical simulation of multiple fiber breakings in fiber-reinforced composites. In this study, four mechanisms are proposed to describe the local load sharing process and broken fiber selection as shown in the blue triangle, whereas the red process block represents the calculation of the homogenized macro stress-macro strain relationship as presented in **Section 4.1**. In the subsequent sections, the proposed four algorithms for describing the fiber breaking mechanisms are introduced.

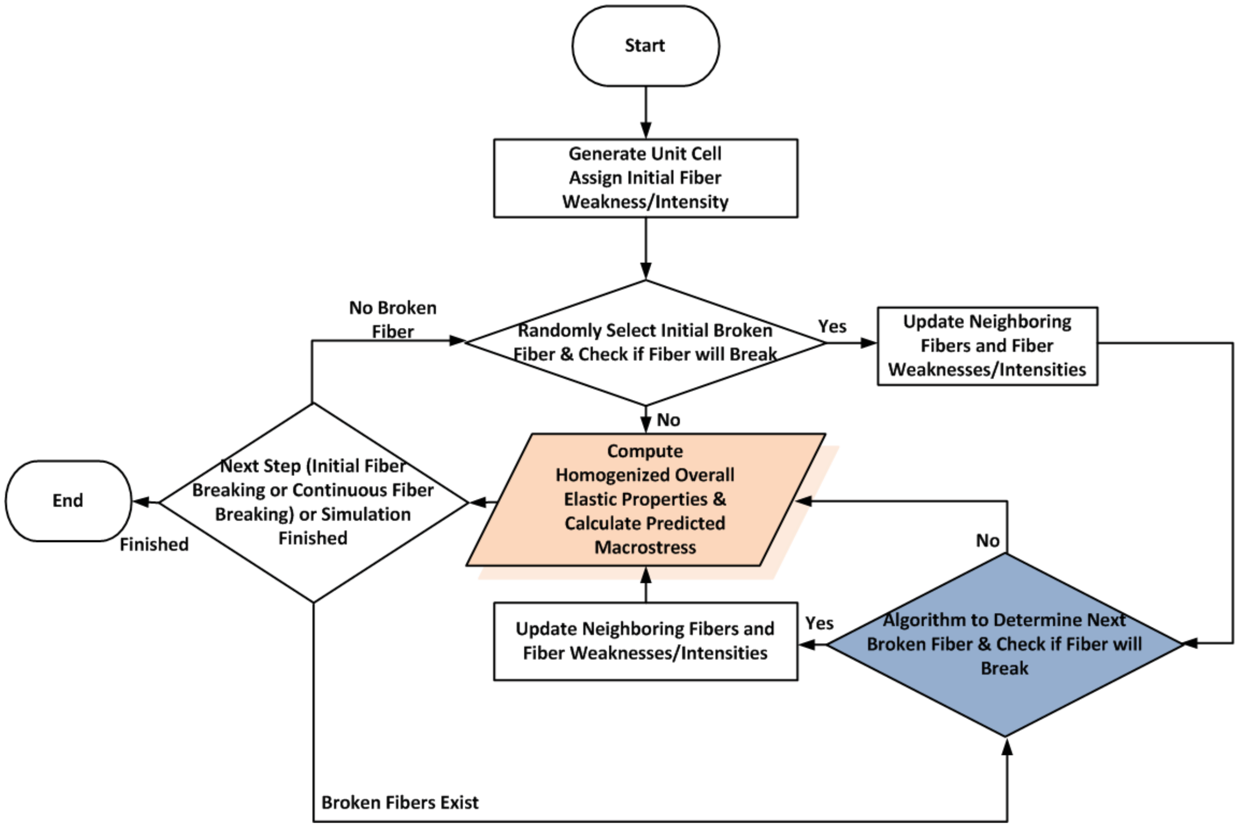


Figure 4.3 Flowchart of computational algorithm for stochastic simulation of progressive fiber breaking in fiber-reinforced composites

4.2.2 Fiber Breaking Mechanism 1 – Dominating Weakness Selection

The first fiber breaking mechanism is illustrated in Figure 4.4 and the computational procedure is described as follows:

(1) Assign weakness to the cells based on the normal distribution

(2) Randomly choose the first fiber for fragmentation

(3) Degrade the weakness of the nearest 4 fibers with a factor ρ

(4) Select the weakest fiber among the four neighboring fibers (yellow triangles in Figure 4.4) and fracture fiber j with the probability:

$$p_j = \frac{weakness_j}{\sum_{i=1}^{4} weakness_i} \tag{4.30}$$

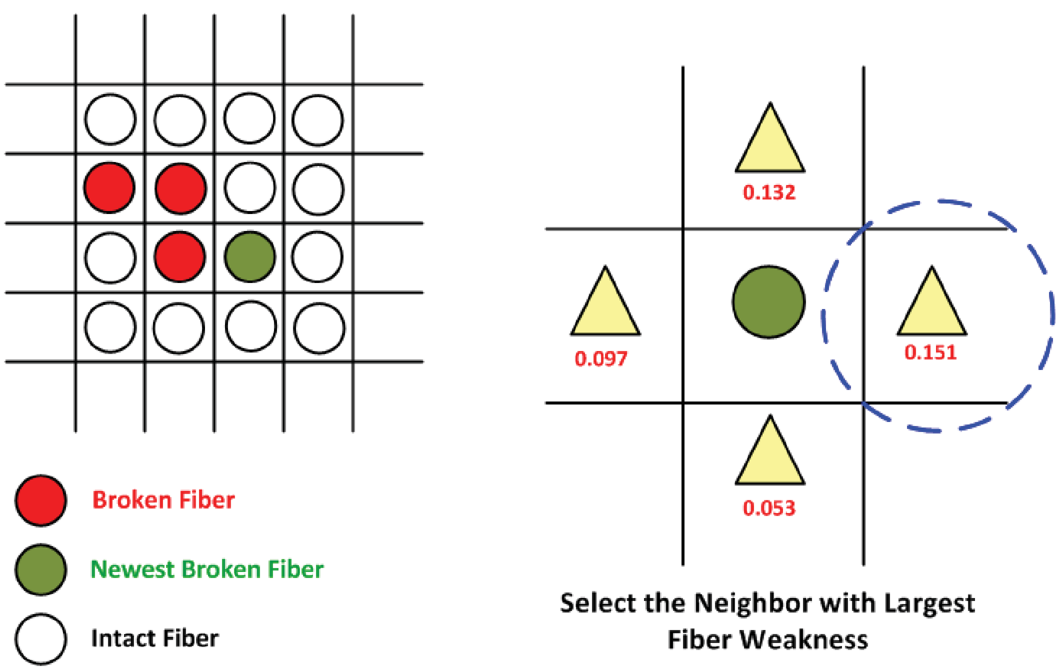


Figure 4.4 Schematic of Mechanism 1 – weakest fiber selection

4.2.3 Fiber Breaking Mechanism 2 – Random Walk Selection

Fiber breaking Mechanism 2 is illustrated in Figure 4.5 and the corresponding computational procedure is summarized as follows:

(1) Assign weakness to the cells based on the normal distribution

(2) Randomly choose the first fiber for fragmentation

(3) Degrade the weakness of the nearest 4 fibers with a factor ρ

(4) Randomly select the fiber among the four neighboring fibers (yellow triangles in Figure 4.5) and fracture fiber j with the probability:

$$p_j = \frac{weakness_j}{\sum_{i=1}^{4} weakness_i} \tag{4.31}$$

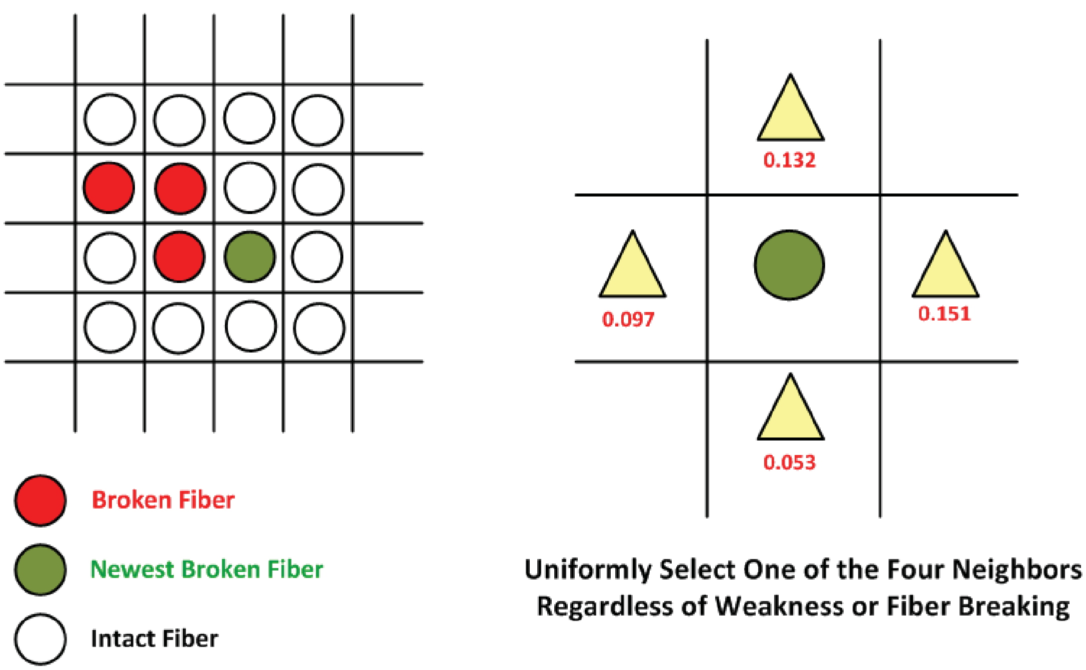


Figure 4.5 Schematic of Mechanism 2 – random walk selection

4.2.4 Fiber Breaking Mechanism 3 – Self-Avoiding Walk (SAW) Selection

The third fiber breaking mechanism is illustrated in Figure 4.6 and the computational procedure is depicted as below:

(1) Assign weakness to the cells based on the normal distribution

(2) Randomly choose the first fiber for fragmentation

(3) Degrade the weakness of the nearest 4 fibers with a factor ρ

(4) Randomly select the fiber among the "unbroken" neighboring fibers and fracture fiber j with the probability:

$$p_j = \frac{weakness_j}{\sum_{i \in S_j} weakness_i} \tag{4.32}$$

where S_j is the set of intact neighboring fibers for fiber sub-unit j (yellow triangles)

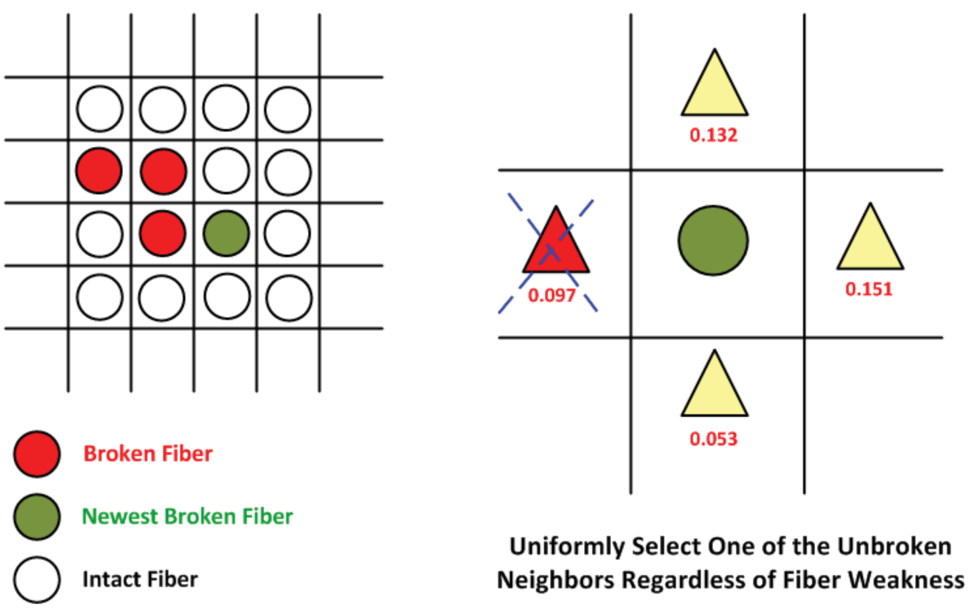


Figure 4.6 Schematic of Mechanism 3 – self-avoiding walk (SAW) selection

4.2.5 Fiber Breaking Mechanism 4 – All Surrounding Neighboring Fiber Selection

Fiber breaking Mechanism 4 is illustrated in Figure 4.7 and the computational procedure is given as follows:

(1) Assign weakness to the cells based on the normal distribution

(2) Randomly choose the first fiber for fragmentation

(3) Degrade the weakness of the nearest 4 fibers with a factor ρ

(4) Select the fiber among the all neighboring fibers with equal probability and fracture fiber j with the probability:

$$p_j = \frac{weakness_j}{\sum_{i \in V} weakness_i} \tag{4.33}$$

where set V contains all the fiber sub-units (yellow triangles in Figure 4.7) surrounding the broken fibers.

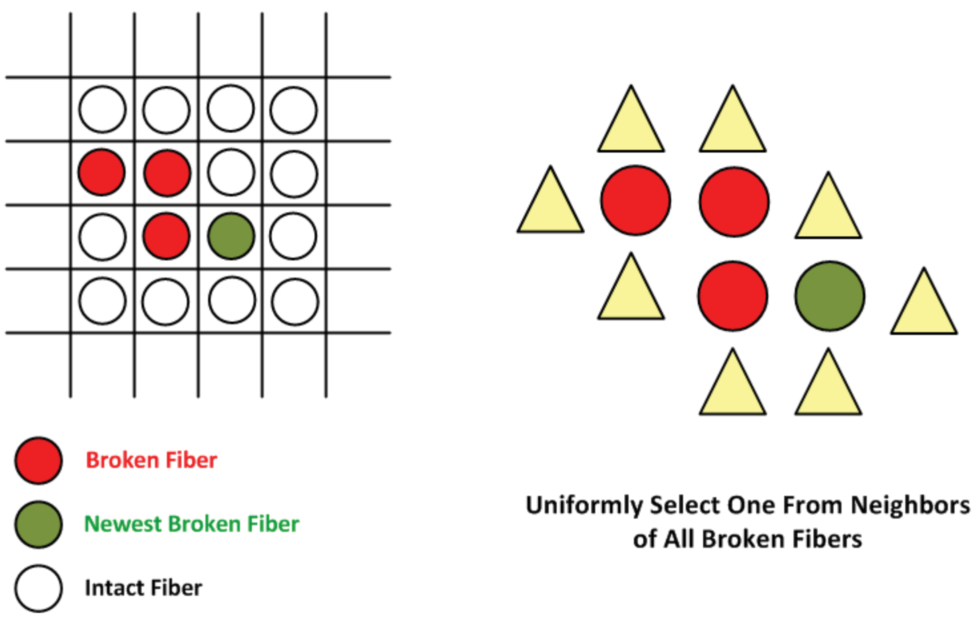


Figure 4.7 Schematic of Mechanism 4 – all neighboring fiber selection

4.2.6 Numerical Results

In the numerical studies, a unit cell with 12x12 (144) fiber sub-units is considered as shown in Figure 4.2. Initial weaknesses with a normal distribution $\mathcal{N}\left(0.05, 0.01\right)$ are assigned to each fiber sub-unit. When the fiber sub-unit is broken, a reduction coefficient ρ of 0.90 is adopted to reduce the strength of the neighboring fiber sub-units associated with this broken fiber sub-unit. The fiber stress of the broken fiber sub-unit is decreased by 10% and the fiber stress is increased by 2.5% for each of the nearest neighboring fiber sub-units. 50 simulations are performed for each proposed algorithm with an incremental strain of 0.001% up to 4% applied macro strain. The material and physical properties of the composites are as follows: $E_m = 2600$ MPa , $\nu_m = 0.34$, $E_f = 72.5$ GPa , $\nu_f = 0.22$, $L_f = 25.4$ mm , $r_f = 7$ μm , $\phi_f = 0.3$ ($\phi_{f_i} = \phi_f / n_{sub-unit}$). Weibull parameters ($S = 1475.65$ MPa and $M = 2.026$) determined in the previous chapter and fiber breaking parameters ($\sigma_{cr} = 731$ MPa and $n_{saturaed} = 4.064$ /mm) are utilized in the calculations of the homogenized stress-strain relationship as introduced in **Section 4.1**. **Appendix C** summarizes the MATLAB codes of the proposed four fiber breaking mechanisms for stochastic simulation of multiple fiber-breaking behaviors in the fiber-reinforced composites.

The predicted macro stress-macro strain relationships of 50 numerical simulations for the four mechanisms are shown in Figure 4.8, Figure 4.12, Figure 4.16, and Figure 4.20. Figure 4.9, Figure 4.13, Figure 4.17, and Figure 4.21 illustrate the mean and variation of stress-strain curves. Similarly, the total numbers of broken fibers in the composites versus the applied macro strain for the four mechanisms are given in Figure 4.10, Figure 4.14, Figure 4.18, and Figure 4.22. The corresponding mean and variation of total number of broken fibers evolutions are shown in

Figure 4.11, Figure 4.15, Figure 4.19, and Figure 4.23. The progressive damage patterns due to fiber breaking of the four mechanisms at each 0.5% strain interval are illustrated in Figure 4.24, Figure 4.25, Figure 4.26, and Figure 4.27.

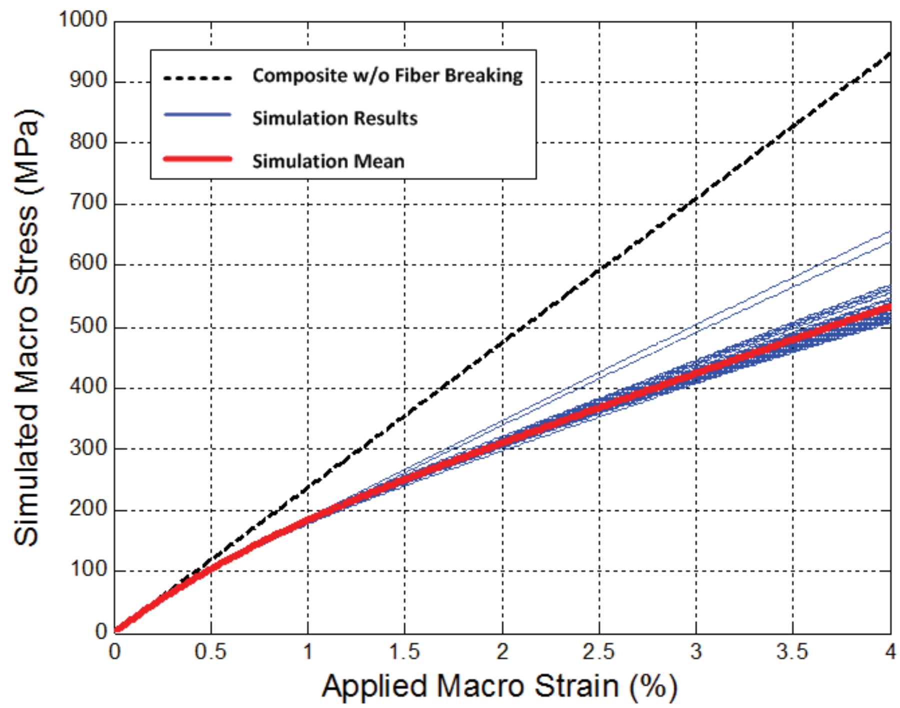


Figure 4.8 Simulation results of stress-strain relations – Mechanism 1

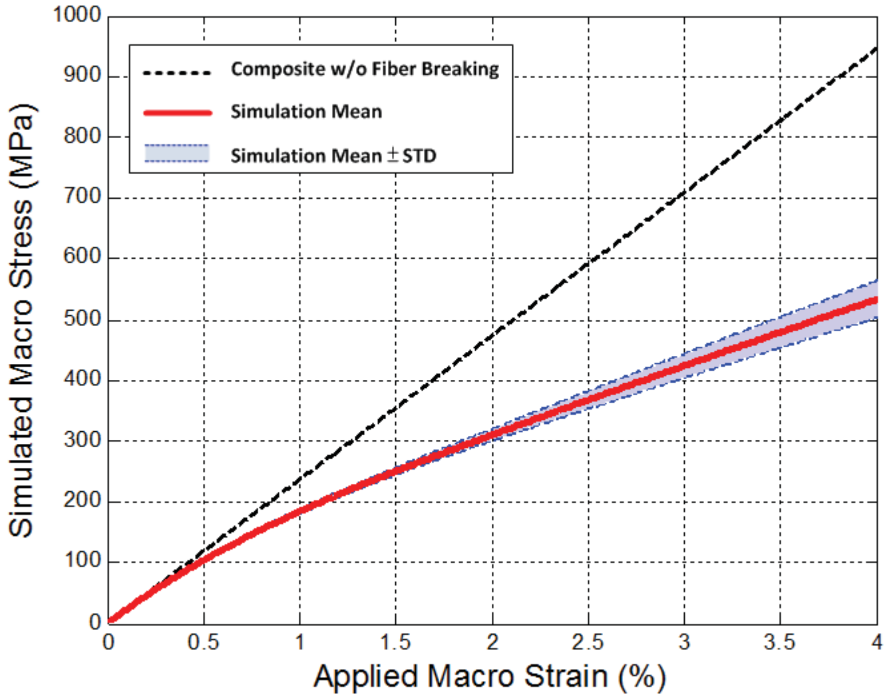


Figure 4.9 Simulated mean and variation of stress-strain relations – Mechanism 1

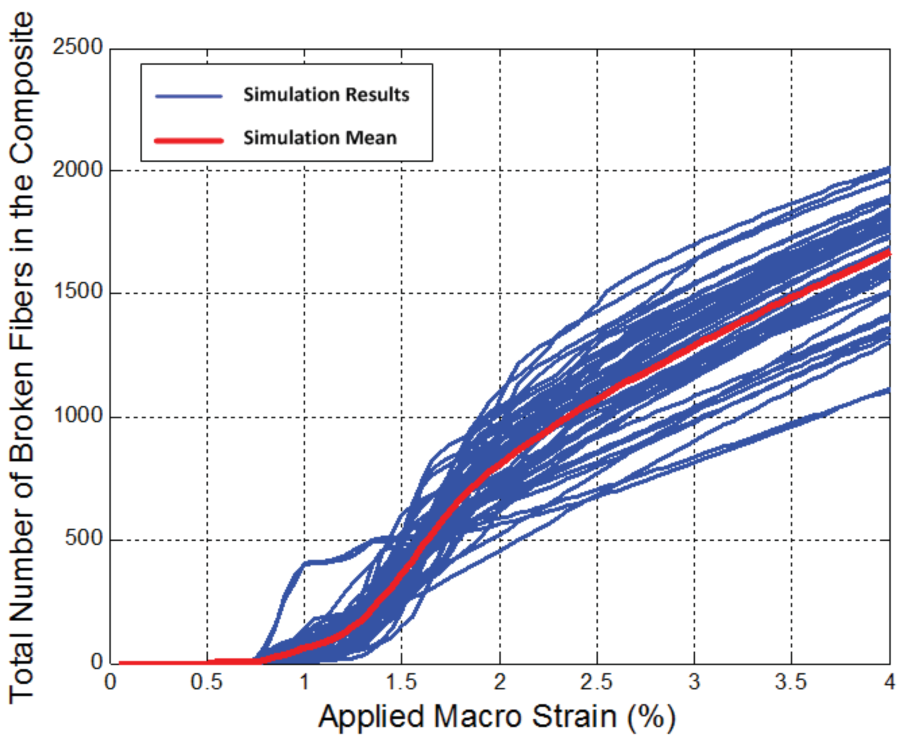


Figure 4.10 Simulation results of total number of broken fibers – Mechanism 1

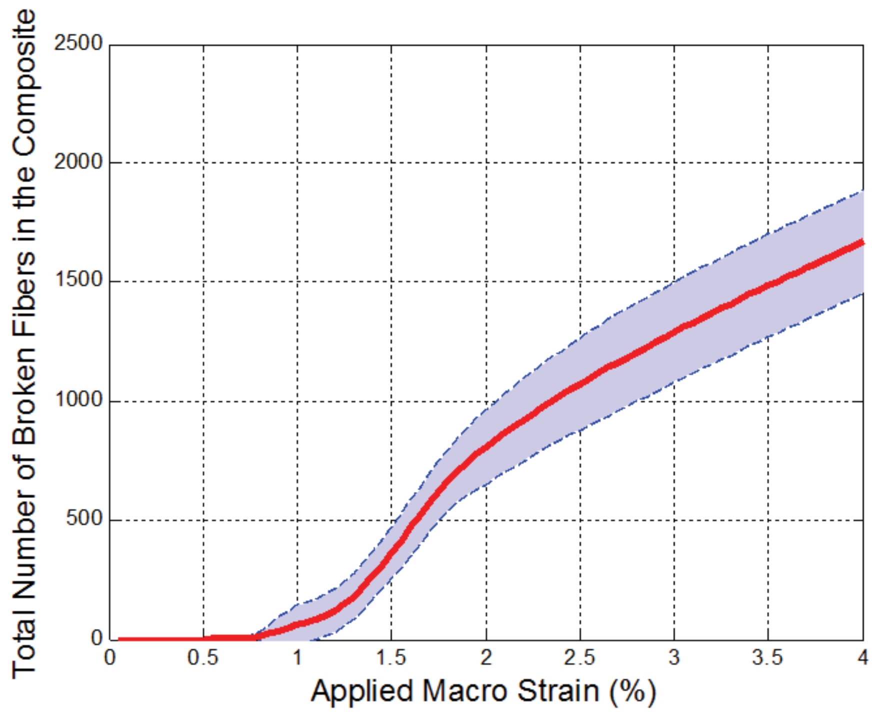


Figure 4.11 Simulated mean and variation of total number of broken fibers – Mechanism 1

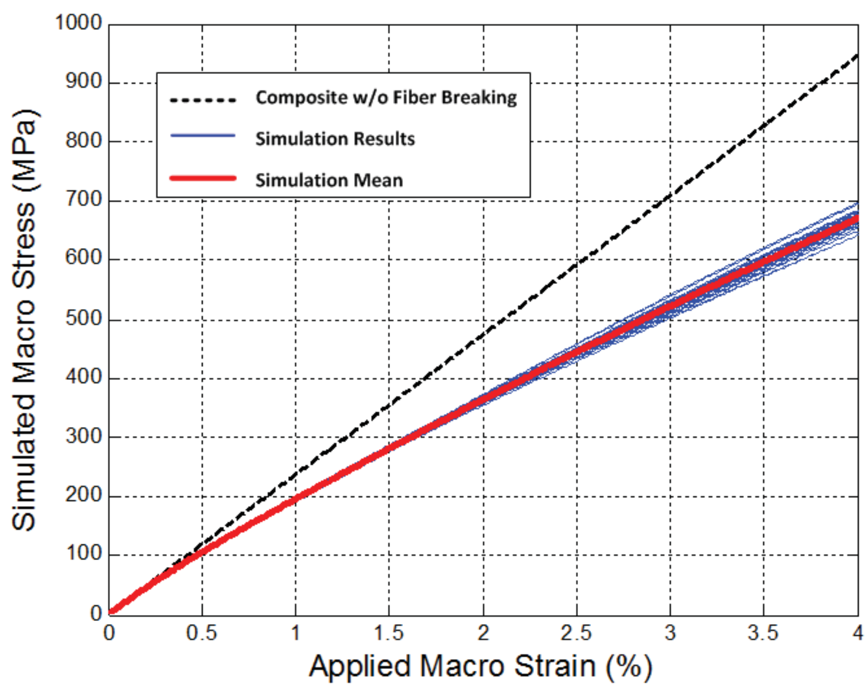


Figure 4.12 Simulation results of stress-strain relations – Mechanism 2

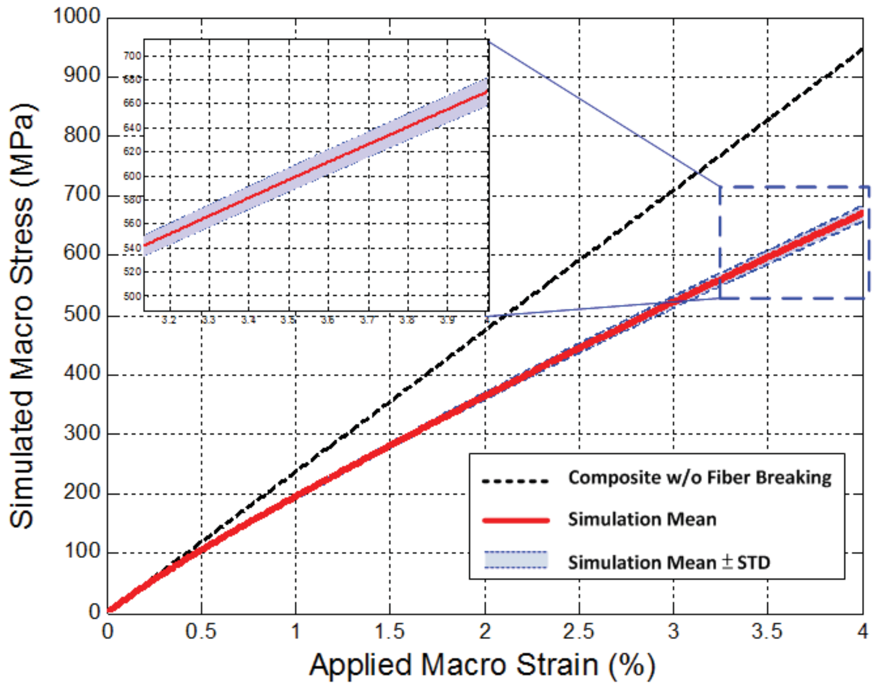


Figure 4.13 Simulated mean and variation of stress-strain relations – Mechanism 2

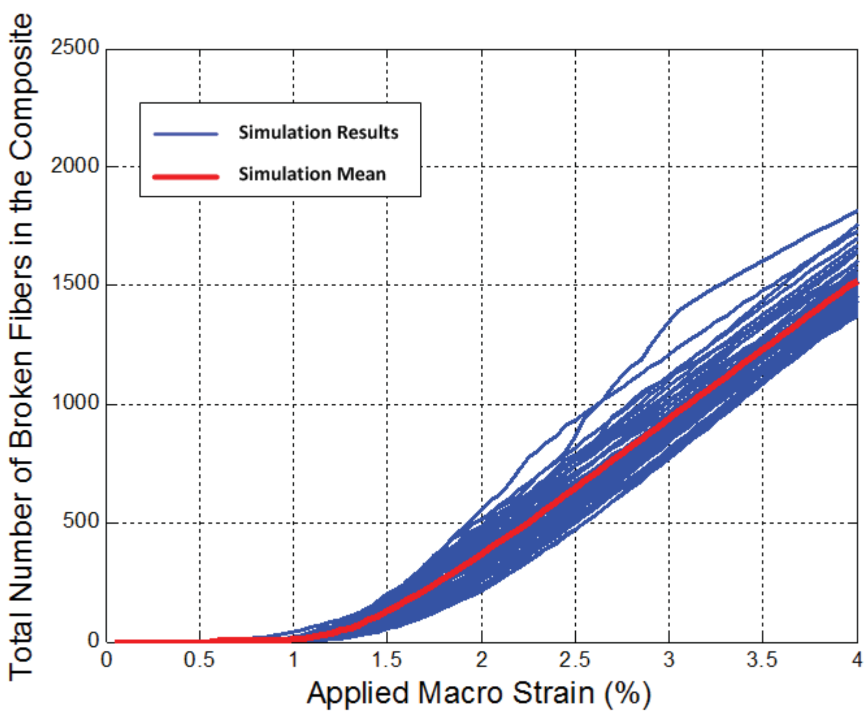


Figure 4.14 Simulation results of total number of broken fibers – Mechanism 2

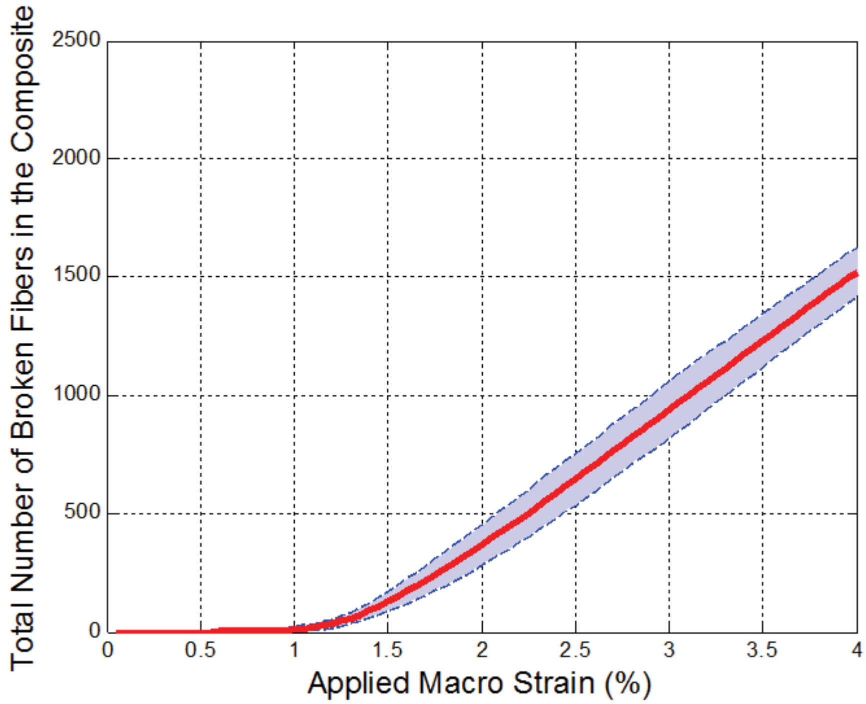


Figure 4.15 Simulated mean and variation of total number of broken fibers – Mechanism 2

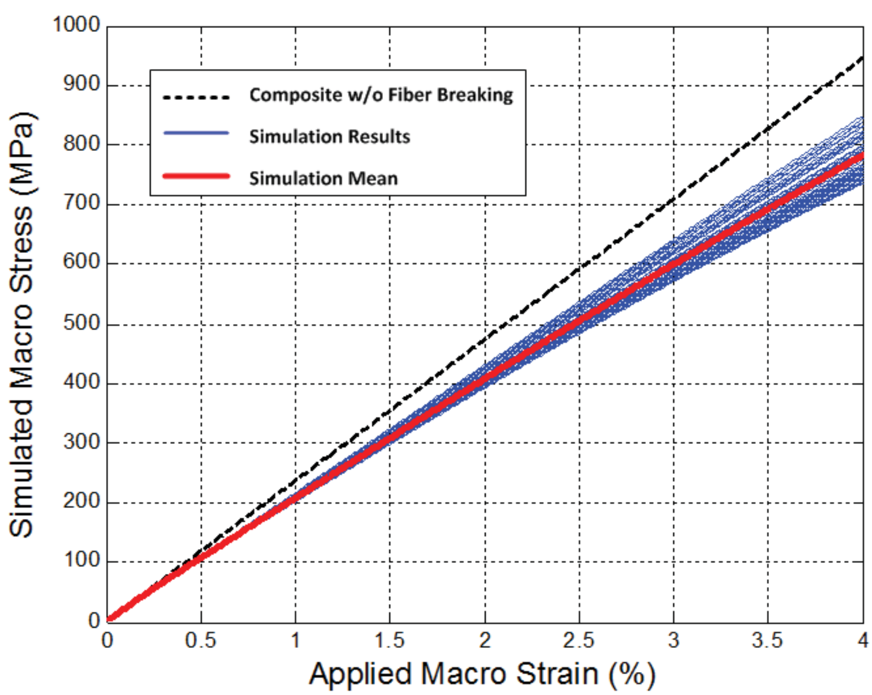


Figure 4.16 Simulation results of stress-strain relations – Mechanism 3

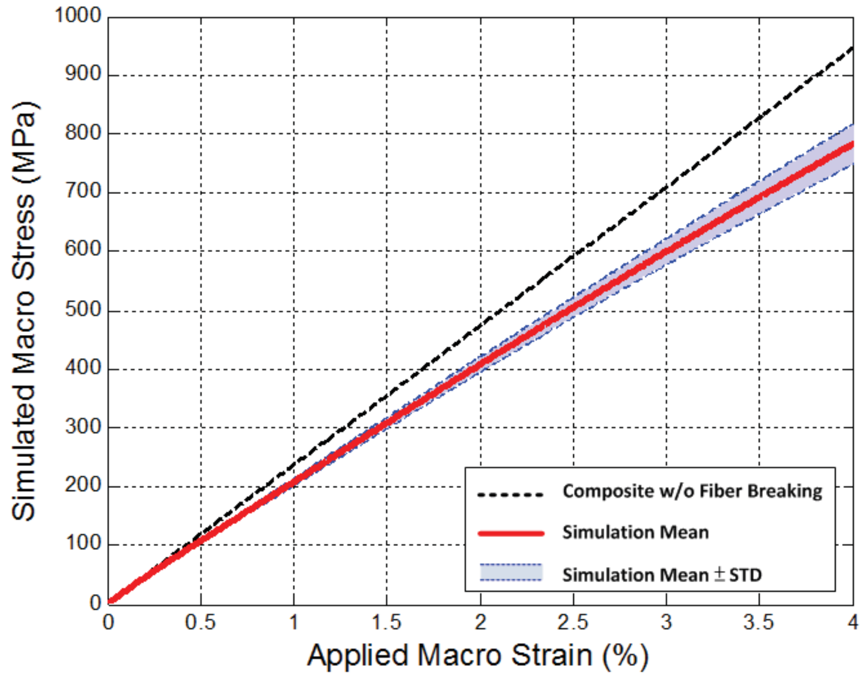


Figure 4.17 Simulated mean and variation of stress-strain relations – Mechanism 3

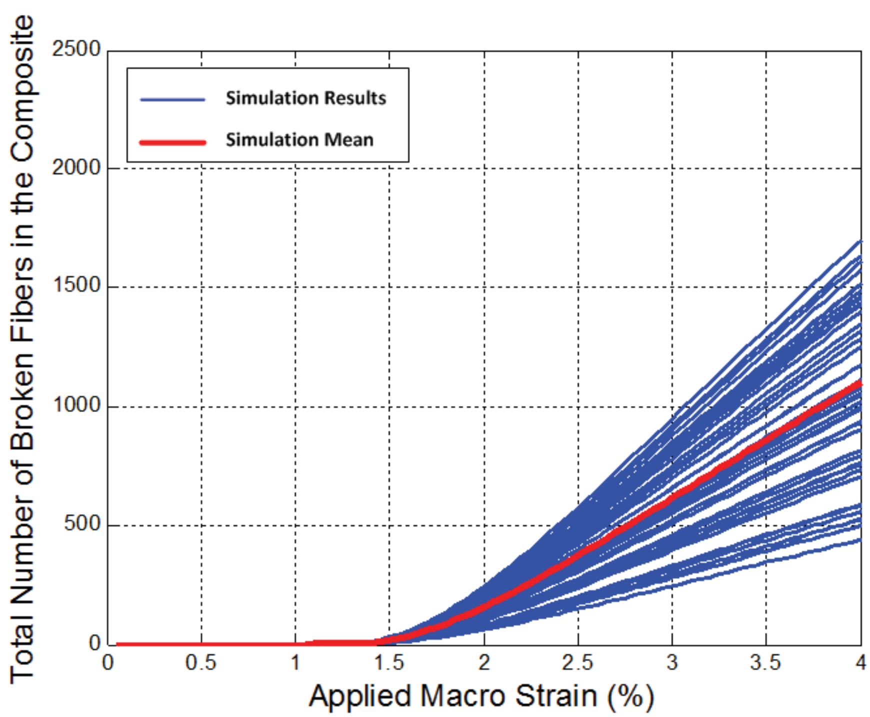


Figure 4.18 Simulation results of total number of broken fibers – Mechanism 3

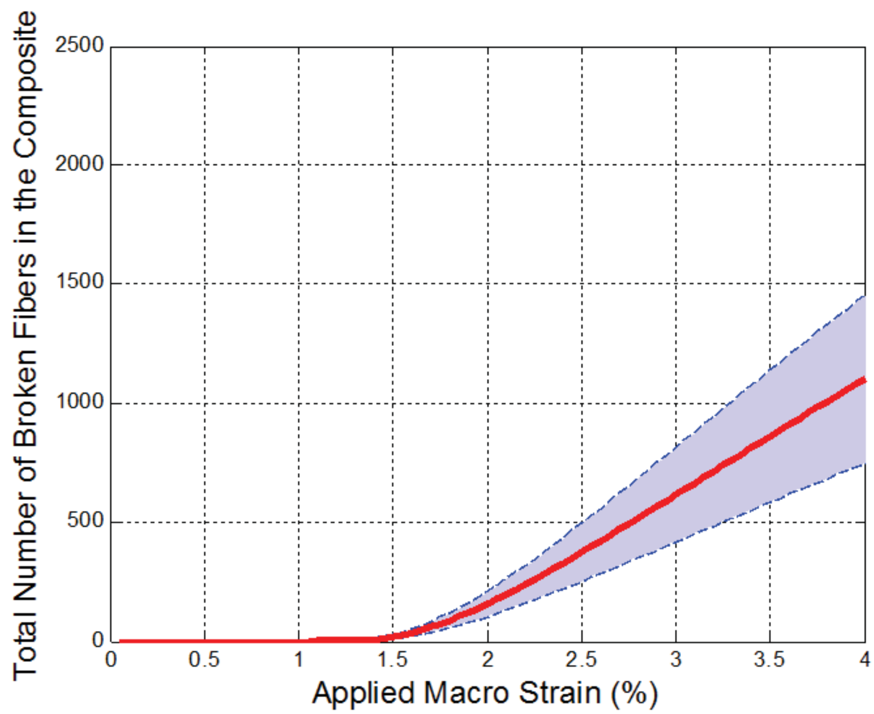


Figure 4.19 Simulated mean and variation of total number of broken fibers – Mechanism 3

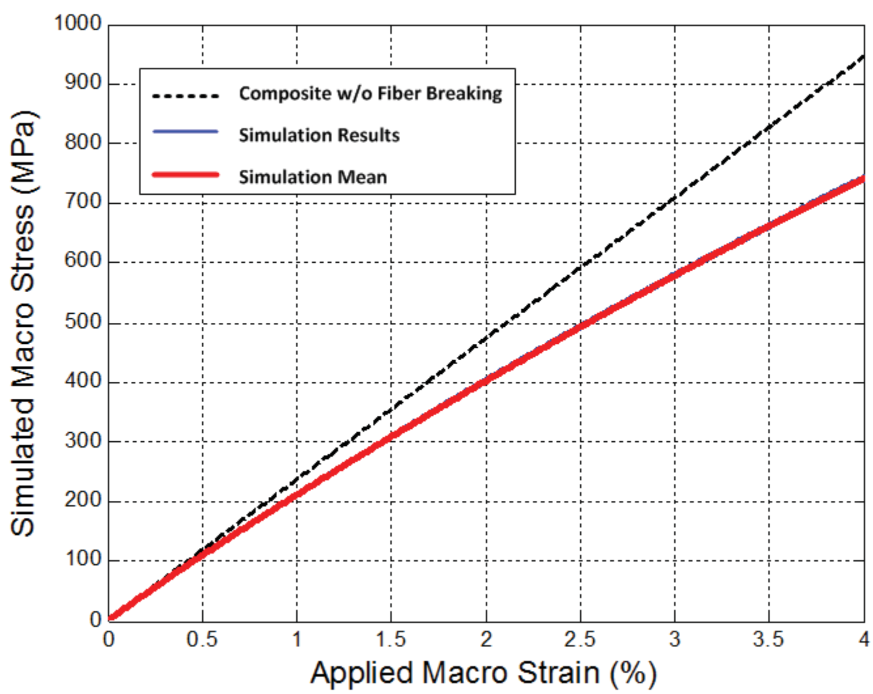


Figure 4.20 Simulation results of stress-strain relations – Mechanism 4

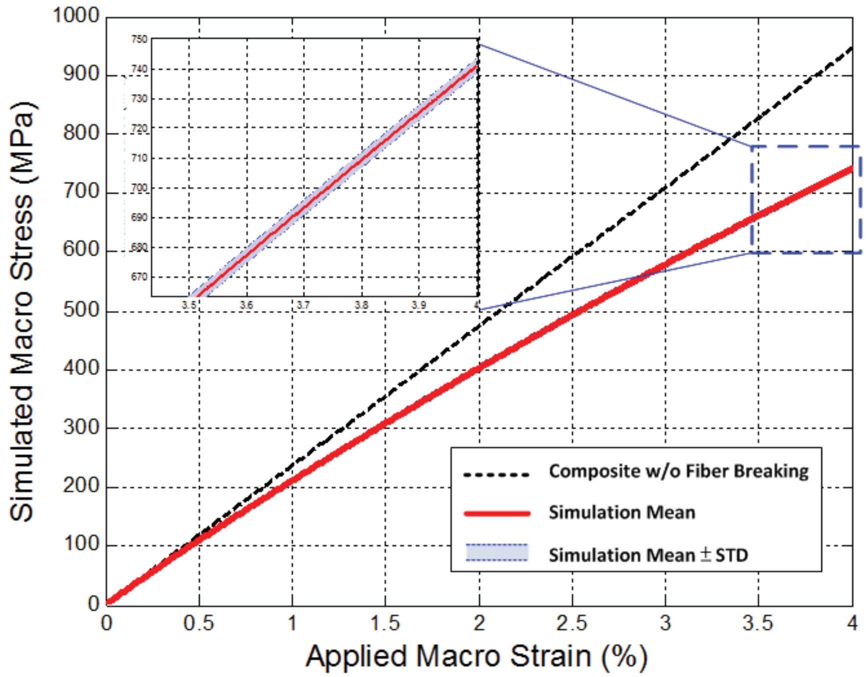


Figure 4.21 Simulated mean and variation of stress-strain relations – Mechanism 4

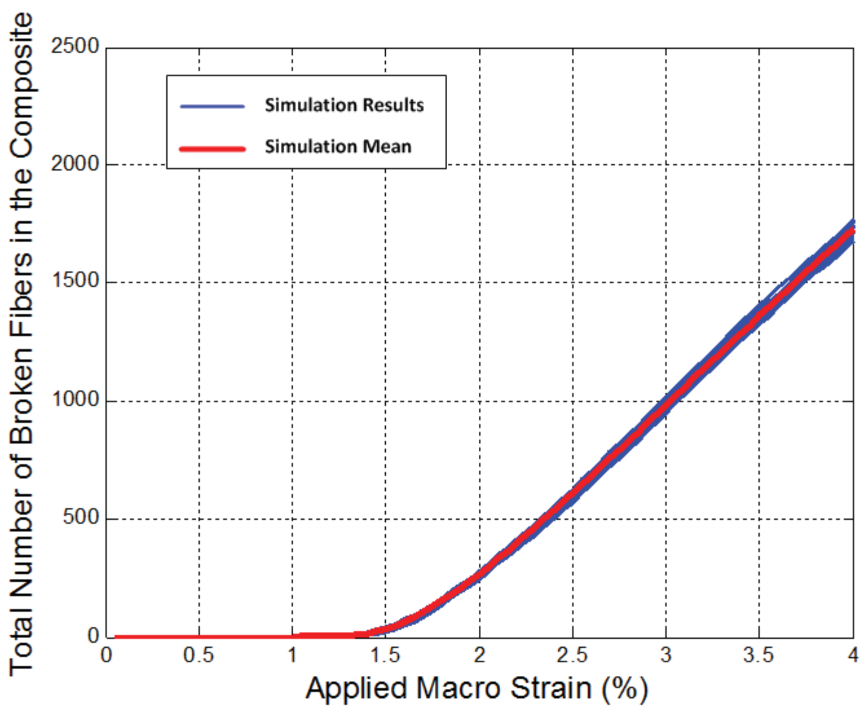


Figure 4.22 Simulation results of total number of broken fibers – Mechanism 4

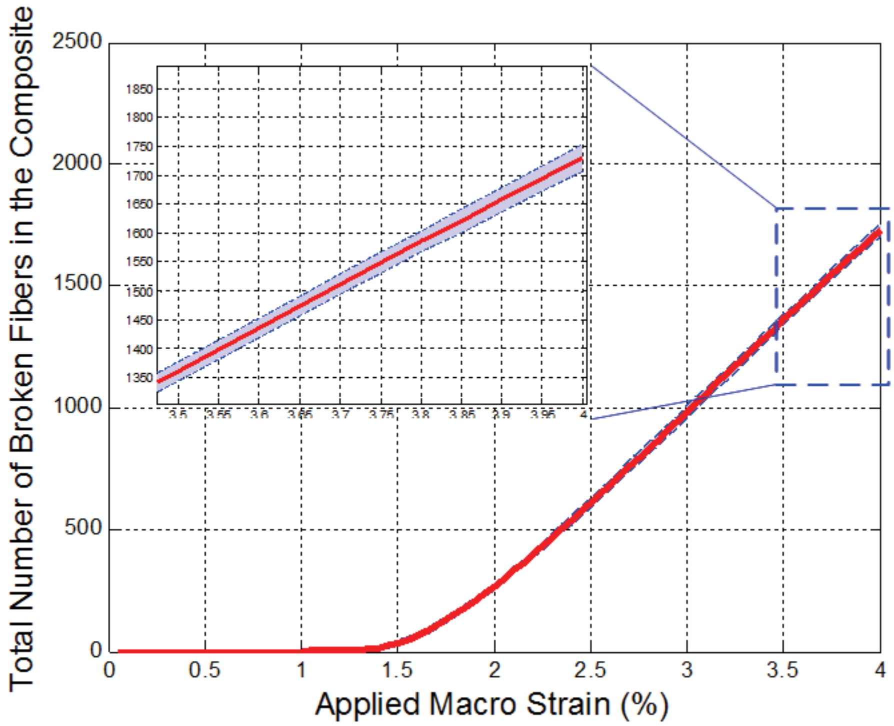


Figure 4.23 Simulated mean and variation of total number of broken fibers – Mechanism 4

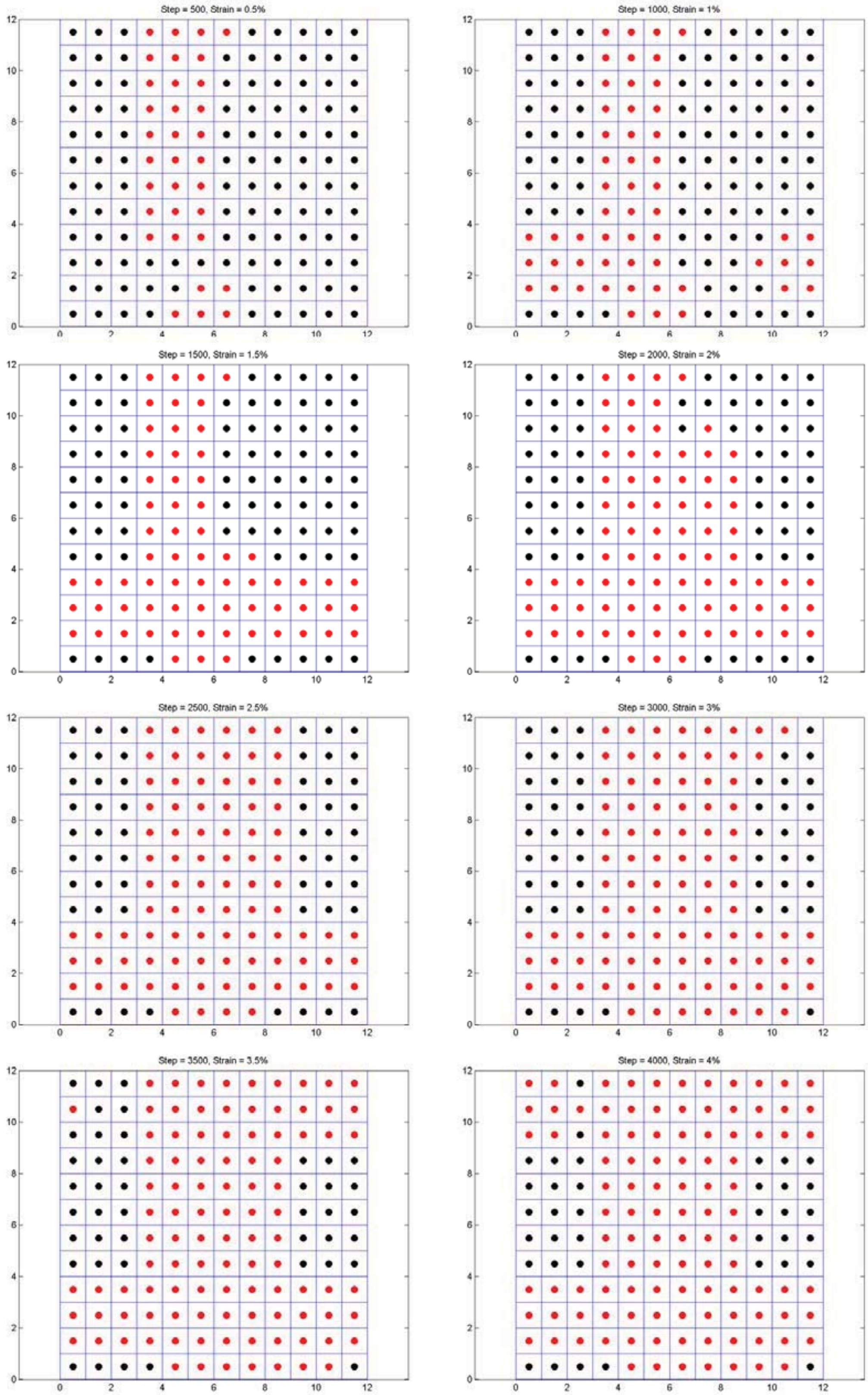


Figure 4.24 Damage evolution of broken fibers in the composite – Mechanism 1

Figure 4.25 Damage evolution of broken fibers in the composite – Mechanism 2

Figure 4.26 Damage evolution of broken fibers in the composite – Mechanism 3

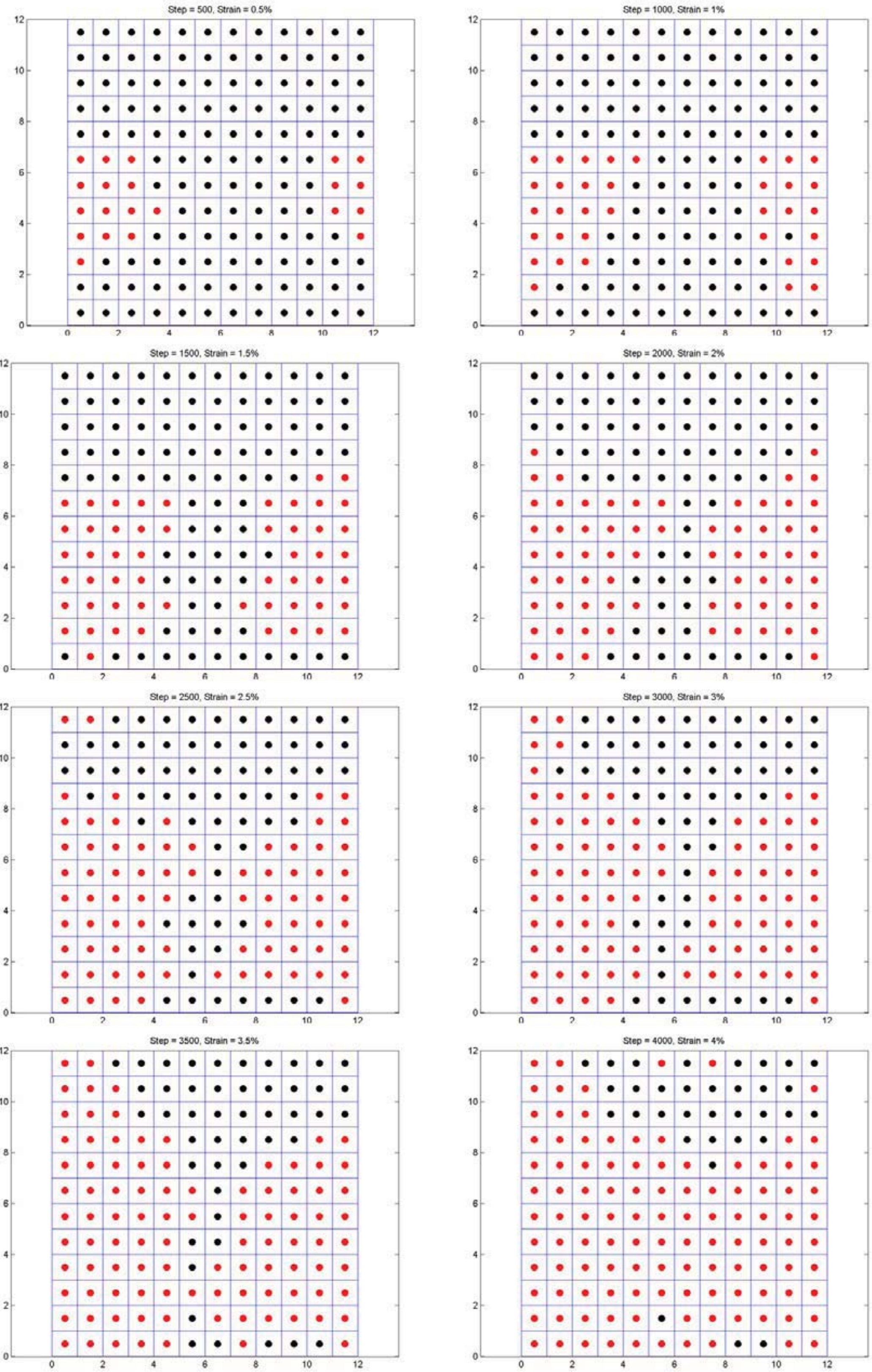


Figure 4.27 Damage evolution of broken fibers in the composite – Mechanism 4

4.2.7 Discussion

Figure 4.28 and Figure 4.29 show the comparisons of the predicted simulation-mean stress-strain curves and simulation-mean evolution curves of fiber break numbers among the four fiber-breaking mechanisms. Comparison of the typical damage patterns for the four mechanisms is made in Figure 4.30. Since the neighboring fiber with smallest weakness value is selected as the candidate broken fiber and each evolved broken fiber results in the reduction of the intensity values of the surrounding fibers, Mechanism 1 will produce more localized and I-shaped damage pattern which leads to more degradation of the overall stiffness of the composites and a softer stress-strain response. The candidate broken fiber is selected from all the neighboring fibers around the existing broken fibers in Mechanism 4, and, therefore, it produces more spread-out damage pattern among these four mechanisms. Moreover, the selection of the candidate broken fiber will be terminated once all the four neighboring fibers are already broken due to the SAW algorithm in Mechanism 3, which leads to larger variation in the fiber break numbers. On the other hand, the random walk algorithm in Mechanism 2 provides a better and more realistic way of detecting the candidate broken fiber. Observed from the numerical simulation results, Mechanism 2 (random walk selection) and Mechanism 4 (all neighboring fiber selection) are recommended for the stochastic modeling of multiple fiber breaking in the fiber-reinforced composites.

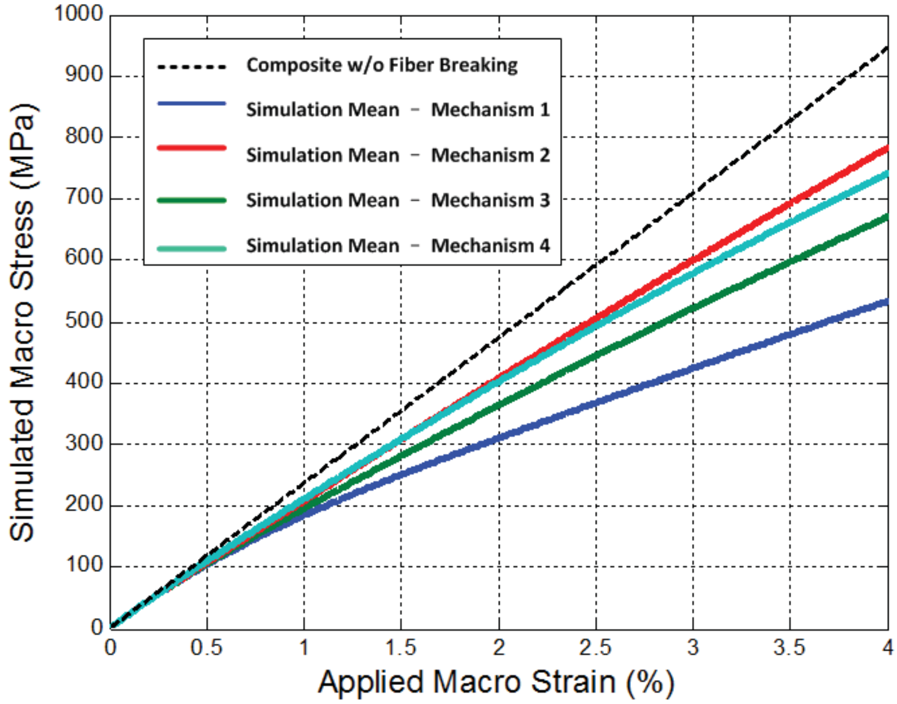


Figure 4.28 Comparisons of stress-strain relations among four Mechanisms

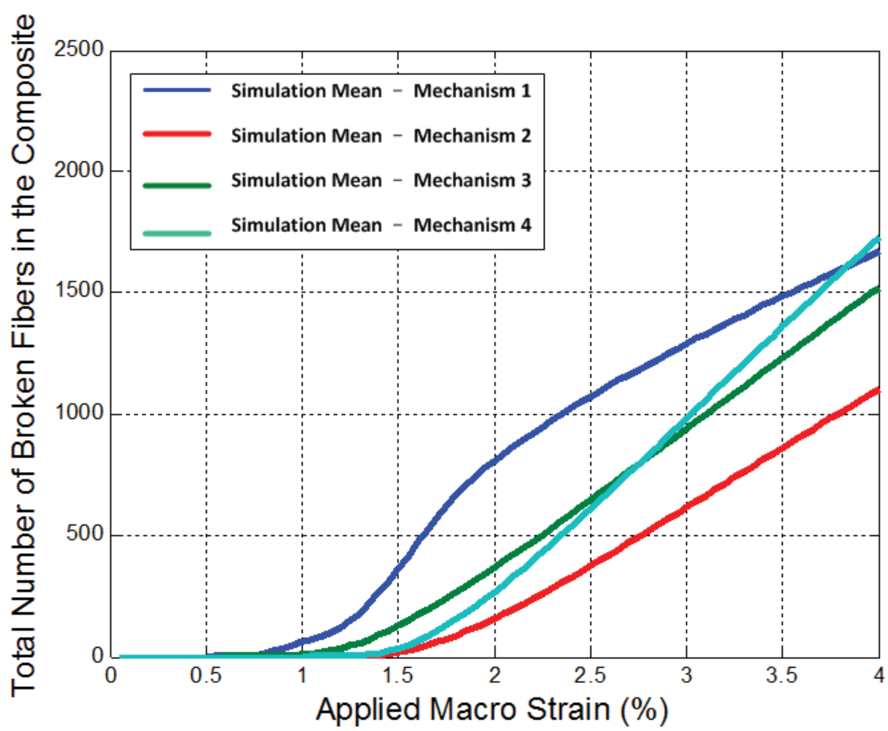


Figure 4.29 Comparisons of total numbers of broken fibers among four mechanisms

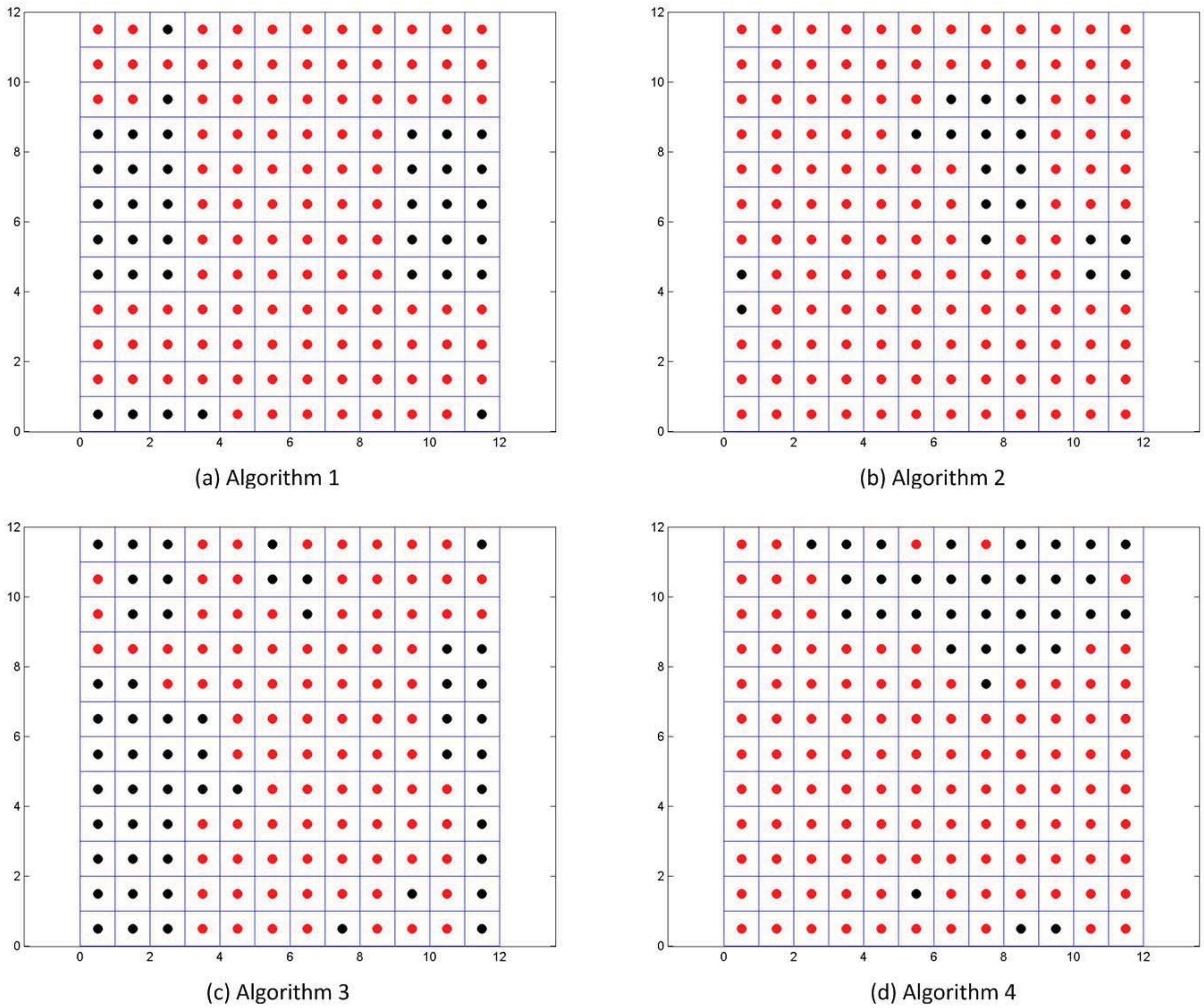


Figure 4.30 Comparison of damage patterns of broken fibers in the composites among four mechanisms

CHAPTER 5 CONCLUSION AND FUTURE WORK

5.1 Conclusion

The progressive fiber breaking of the longitudinally reinforced composites is investigated from the statistical perspective. The Kolmogorov-Smirnov goodness-of-fit test is performed to characterize the damage pattern of the fiber in a single fiber composite. The results indicate that the fragmentation evolution of single fiber composites follows the Weibull statistic. Moreover, four stochastic competing models are proposed to address the local load redistribution from the broken fibers to the intact fibers. These models provide possible explanations of the damage initiation sequence in the multi-fiber composites. Finally, the evolutionary fiber breaking process in the multi-fiber composites is demonstrated by using the Weibull statistic to govern the fiber breakings in each fiber along the longitudinal direction, and the stochastic competing models to describe the sequence of damage initiation.

5.2 Future work

1) *Coupling of fiber breaking with fiber/matrix interfacial debonding*

Many experimental studies [2] reveal that debonding between the fiber and the matrix frequently occurs along with fiber fracture. The stress distribution near the crack tip will be significantly affected by the existence of the fiber and matrix debonding [17]. Therefore, it is crucial to take account of the fiber breaking and the fiber-matrix debonding simultaneously. Theoretically, if a fiber is completely debonded from the matrix, it loses the shear transferring capability. Thus, it will hardly break. A competing model of the two damage scenarios can be developed to study their coupling influence on the material behavior.

2) *Probabilistic and sensitivity analyses of global system*

In dealing with real-world problems, the presence of uncertainties is unavoidable. For engineers, it is important to recognize the major sources of uncertainties in real applications. Sensitivity analyses will be performed on the statistical models that have been developed. Failure criteria will be proposed by using a set of limit-state functions with random variables. The limit state function of each damage mode can be combined in a general system, further to study the failure probability of composites with the existence of different damage scenarios.

APPENDIX A NONLINEAR CURVE FITTING (MATLAB CODE)

(1) Main Program

```
global itype

itype = 1; % water-sized
% itype = 2; % gamma_GPS
% itype = 3; % gamma_MPS

           % sigma_cr    S0    M   n_sat
beta_pre = [0           1500  4.5  50.0]';

beta_lb = [0 0 1 20];
beta_ub = [2000 5000 25 100];

temp1 = {@crack_density_sigma};

options = optimset('TolFun', 1e-5, 'TolX', 1e-6,
'FinDiffType','central','MaxIter',1000, 'MaxFunEvals', 50000);

[beta_now,resnorm,residual,exitflag,output] = lsqnonlin(temp1, beta_pre,
beta_lb, beta_ub, options);

if itype == 1
    load('exp_data.mat', 'water_sized_1', 'water_sized_2');
    data = [water_sized_1; water_sized_2];
    n1 = length(water_sized_1);
    n2 = length(water_sized_2)+1;
elseif itype == 2
    load('exp_data.mat', 'gamma_GPS_1', 'gamma_GPS_2');
    data = [gamma_GPS_1; gamma_GPS_2];
    n1 = length(gamma_GPS_1);
    n2 = length(gamma_GPS_2)+1;
elseif itype == 3
    load('exp_data.mat', 'gamma_MPS_1', 'gamma_MPS_2');
    data = [gamma_MPS_1; gamma_MPS_2];
    n1 = length(gamma_MPS_1);
    n2 = length(gamma_MPS_2)+1;
end
data(:,2) = round(data(:,2));

if itype == 2
    xx = 0:50:10000;
elseif itype == 3
    xx = 0:50:8000;
else
    xx = 0:50:6000;
end
Fval = zeros(length(xx),1);
Fval_d = zeros(length(xx),1);
for ii = 1:length(xx)
    if xx(ii) >= beta_now(1)
```

```
        Fval(ii) = beta_now(4) * (1 - exp(-((xx(ii)-
beta_now(1))/beta_now(2)).^beta_now(3)));

        Fval_d(ii) = beta_now(3)/beta_now(2)*((xx(ii)-
beta_now(1))/beta_now(2))^(beta_now(3)-1) * exp(-((xx(ii)-
beta_now(1))/beta_now(2)).^beta_now(3));
    end
end

Fval_d = Fval_d / sum(Fval_d);

figure(1)
plot(xx, Fval, 'b-','LineWidth',3)
hold on
plot(data(1:n1,1), data(1:n1,2), 'kd','MarkerFaceColor', 'c','MarkerSize',8)
hold off
if itype == 2
    ylim([0 65])
end
xlabel('\sigma (MPa)','FontSize',14)
ylabel('Number of cracks','FontSize',14)
```

(2) Function Evaluation for Nonlinear Curve Fitting

```
function Fval = crack_density_sigma(x)

global itype

if itype == 1
    load('exp_data.mat', 'water_sized_1', 'water_sized_2');
    data = [water_sized_1];
elseif itype == 2
    load('exp_data.mat', 'gamma_GPS_1', 'gamma_GPS_2');
    data = [gamma_GPS_1];
elseif itype == 3
    load('exp_data.mat', 'gamma_MPS_1', 'gamma_MPS_2');
    data = [gamma_MPS_1];
end
data(:,2) = round(data(:,2));
ndata = length(data(:,1));
Fval = zeros(ndata,1);

for ii = 1:ndata
    if data(ii,1) >= x(1)
        num_val = (1 - exp(-((data(ii,1)-x(1))/x(2))^x(3)));
        Fval(ii) = (x(4) * num_val - data(ii,2))^2;
    end
end
return
```

APPENDIX B ONE-SAMPLE KOLMOGOROV-SMIRNOV TEST (MATLAB CODE)

```
load('exp_data.mat')

exp_x = [water_sized_1(1:end,1)];

n_saturated = 81.27681;
sigma_cr = 730.9965;
S = 1475.6508;
M = 2.02637;

xxx = 0:10:6000;
fff = 1-exp(-((xxx-sigma_cr)/S).^M);
for ii = 1:length(xxx)
    if xxx(ii) <= sigma_cr
        fff(ii) = 0;
    end
end
CCDF = [xxx',fff'];
[h,p,k,cv] = kstest(exp_x,CCDF,0.05)

figure(1)
plot(xxx,fff,'r-','LineWidth',2)
hold on
F1 = cdfplot(exp_x);
set(F1,'LineWidth',2,'Color','b');
xlabel('\sigma (MPa)','FontSize',14)
ylabel('CDF','FontSize',14)
set(gca,'FontSize',11)
hh = legend('Fitted Weibull Model','Empirical CDF',4);
set(hh,'FontSize',13);
title('')
ylim([-0.0025 1.005])
hold off
```

APPENDIX C SIMULATION OF MULTIPLE FIBER BREAKING IN METAL MATRIX COMPOSITES (MATLAB CODE)

(1) Mechanism 1 – Weakest Fiber Selection

```
ncell = 15;

rho = 0.9;
beta1 = 1.025;
beta2 = 0.9;
ibreak = 0;

nidx = floor(ncell/2);
ninc = floor(ncell/2);

eps_cr = 0.00125;
eps_total = 0.04;
nstep = 2000;

cell_conn = cell(ncell,ncell);
for ii = 1:ncell
  for jj = 1:ncell
    cell_conn{ii,jj} = [ii-1 jj-1
                        ii   jj-1
                        ii   jj
                        ii-1 jj
                        ii-1 jj-1];
  end
  end

cell_weak = zeros(ncell,ncell);
cell_intensity = zeros(ncell,ncell);
cell_fiber_share = zeros(ncell,ncell)+1;
cell_fiber_check = zeros(ncell,ncell);
cell_fiber_check_hist = cell(nstep/50+1,1);
cell_n_L_num = zeros(nstep+1,ncell,ncell);
rand_weak = randn(ncell*ncell)*0.0085 + 0.05;
ic = 1;
for ii = 1:ncell
  for jj = 1:ncell
    cell_weak(ii,jj) = rand_weak(ic);
    cell_intensity(ii,jj) = 1.0 - rand_weak(ic);
    ic = ic + 1;
  end
end

%%% Epoxy Matrix
Em = 2600;
vm = 0.3402;
[C0] = Elasticity(Em,vm);

%%% E-Glass Fiber
Ef = 72500;
vf = 0.22;
```

```
[C1] = Elasticity(Ef,vf);

L0 = 25.40;
cc = 7e-3;
a11 = 25.40;
a31 = 7e-3;
alpha = a11/a31;
phi_f = 0.3;
phi_f_cell = phi_f/ncell^2;

Scr = 1475.65;
M = 2.026;
sigma_cr = 731;
n_L_0 = 81.2768/20;

[S10]= Eshelby(a11,a31,vm);
[S1] = change_matrix_dir(1,3,S10);

%%% 4th Order Identity Tensor
I = eye(6);
I(4,4) = 0.5;
I(5,5) = 0.5;
I(6,6) = 0.5;

eps_inc = eps_total/nstep;
deps = [0;0;eps_inc;0;0;0];
eps_macro = zeros(6, nstep+1);
sig_macro = zeros(6, nstep+1);
sig_initial = zeros(6, nstep+1);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
beta = 16*(1-vm^2)/3/Em/(2-vm)/pi * 30000;
gamma_crack = diag([0,0,2-vm,2,2,0]);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

idx_curr = 0;
jdx_curr = 0;
neighbor_curr = [];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
TTr = cell(ncell,ncell);
AAr = cell(ncell,ncell);
CCr = cell(ncell,ncell);
BB = I;
for mm = 1:ncell
  for nn = 1:ncell
    coeff_temp = beta * cc * cell_n_L_num(ii,mm,nn);
    CCr{mm,nn} = C1;
    AAr{mm,nn} = multiplication(CCr{mm,nn},S1) - multiplication(C0,S1-I) -
                 coeff_temp * multiplication(gamma_crack, multiplication(C0,
                 S1-I));
    TTr{mm,nn} = -multiplication(inverse_4th(AAr{mm,nn}), CCr{mm,nn} -
                  multiplication(I + coeff_temp * gamma_crack, C0));
    BB = BB + phi_f_cell * multiplication(S1, TTr{mm,nn});
  end
end
C_eff_initial = C0;
```

```
for mm = 1:ncell
  for nn = 1:ncell
    C_eff_initial = C_eff_initial - phi_f_cell * multiplication(C0,
                    multiplication(inverse_4th(BB), TTr{mm,nn}));
  end
end
[C_eff_initial] = get_symm(C_eff_initial);

crack_list = [];
for ii = 1:nstep
  % step 1 %
  eps_macro(:, ii+1) = eps_macro(:, ii) + deps;
  if ibreak == 0 && eps_macro(3, ii+1) >= eps_cr %%% initial break !!!
    iselect = floor(ncell * ncell * rand(1) + 1);
    idx_curr = rem(iselect,ncell);
    if idx_curr == 0
      idx_curr = ncell;
    end
    jdx_curr = (iselect - rem(iselect,ncell))/ncell+1;
    if jdx_curr > ncell
      jdx_curr = jdx_curr - ncell;
    end
    if idx_curr >= min(1,nidx - ninc) && idx_curr <= max(ncell,nidx+ + ninc)
       &&jdx_curr >= min(1,nidx - ninc) && jdx_curr <= max(ncell,nidx+ + ninc)
      [neighbor_curr] = find_neighbor(idx_curr,jdx_curr,ncell);
      cell_weak_temp = [cell_weak(neighbor_curr(1,1),neighbor_curr(1,2))
                        cell_weak(neighbor_curr(2,1),neighbor_curr(2,2))
                        cell_weak(neighbor_curr(3,1),neighbor_curr(3,2))
                        cell_weak(neighbor_curr(4,1),neighbor_curr(4,2))
                        cell_weak(idx_curr,jdx_curr)];
      if rand(1,1) <= cell_weak(idx_curr,jdx_curr) / sum(cell_weak_temp)
        for i = 1:4
          iid = neighbor_curr(i,1);
          jid = neighbor_curr(i,2);
          if cell_fiber_check(iid,jid) ~= 0
            cell_fiber_share(iid,jid) = cell_fiber_share(iid,jid) * beta1;
          else
            cell_fiber_share(iid,jid) = cell_fiber_share(iid,jid) * beta1;
            cell_intensity(iid,jid) = cell_intensity(iid,jid) * rho;
            cell_weak(iid,jid) = 1.0 - cell_intensity(iid,jid);
          end
      end
      cell_fiber_check(idx_curr,jdx_curr) =
      cell_fiber_check(idx_curr,jdx_curr)+1;
      cell_fiber_share(idx_curr,jdx_curr) =
      cell_fiber_share(idx_curr,jdx_curr) * beta2;
      ibreak = 1;
      crack_list = [crack_list; idx_curr, jdx_curr];
    end
  end
 elseif ibreak == 1 && length(find(cell_fiber_check ~= 0)) < ncell^2
 weak_check = [];
 num_neighbor_curr = length(neighbor_curr(:,1));
 for kk = 1:num_neighbor_curr
    weak_check = [weak_check; neighbor_curr(kk,1), neighbor_curr(kk,2),
                  cell_weak(neighbor_curr(kk,1),neighbor_curr(kk,2))];
  end
```

```
  [temp1 temp2] = sort(weak_check);
  iidd2 = temp2(end);
  if rand(1) <= weak_check(iidd2,3) / sum(weak_check(:,3))
    idx_curr = weak_check(iidd2,1);
    jdx_curr = weak_check(iidd2,2);
    cell_fiber_check(idx_curr,jdx_curr) =
    cell_fiber_check(idx_curr,jdx_curr) + 1;
    cell_fiber_share(idx_curr,jdx_curr) =
    cell_fiber_share(idx_curr,jdx_curr) * beta2;
    crack_list = [crack_list; idx_curr, jdx_curr];
    [neighbor_now] = find_neighbor(idx_curr,jdx_curr,ncell);
  for i = 1:4
    iid = neighbor_now(i,1);
    jid = neighbor_now(i,2);
    if cell_fiber_check(iid,jid) ~= 0
        cell_fiber_share(iid,jid) = cell_fiber_share(iid,jid) * beta1;
    else
        cell_fiber_share(iid,jid) = cell_fiber_share(iid,jid) * beta1;
        cell_intensity(iid,jid) = cell_intensity(iid,jid) * rho;
        cell_weak(iid,jid) = 1.0 - cell_intensity(iid,jid);
    end
  end
  neighbor_curr = [];
  neighbor_curr = [neighbor_curr; neighbor_now];
  idlist = [];
  for kk = 1:length(neighbor_curr)
    if cell_fiber_check(neighbor_curr(kk,1),neighbor_curr(kk,2)) ~= 0
        idlist = [idlist; kk];
    end
  end
 end
end

% step 2 %
TTr = cell(ncell,ncell);
AAr = cell(ncell,ncell);
CCr = cell(ncell,ncell);
BB = I;
for mm = 1:ncell
  for nn = 1:ncell
    coeff_temp = beta * cc * cell_n_L_num(ii,mm,nn);
    CCr{mm,nn} = C1 * cell_intensity(mm,nn);
    AAr{mm,nn} = multiplication(CCr{mm,nn},S1) - multiplication(C0,S1-I) -
       coeff_temp * multiplication(gamma_crack, multiplication(C0, S1-I));
    TTr{mm,nn} = -multiplication(inverse_4th(AAr{mm,nn}), CCr{mm,nn} -
                  multiplication(I + coeff_temp * gamma_crack, C0));
    BB = BB + phi_f_cell * multiplication(S1, TTr{mm,nn});
  end
end

for mm = 1:ncell
  for nn = 1:ncell
    Cf_r = multiplication(C0, multiplication(I +
           multiplication(S1-I, TTr{mm,nn}), inverse_4th(BB)));
    sigma_f = Cf_r * eps_macro(:, ii+1);
    sig_f = sigma_f(3) * cell_fiber_share(mm,nn);
    if sig_f <= sigma_cr || cell_fiber_check(mm,nn) == 0
```

```
      cell_n_L_num(ii+1,mm,nn) = 0;
    else
      P1temp = 1 - exp(-((sig_f - sigma_cr)/Scr)^M);
      cell_n_L_num(ii+1,mm,nn) = P1temp * n_L_0;
    end
    coeff_temp = beta * cc * cell_n_L_num(ii+1,mm,nn);
    AAr{mm,nn} = multiplication(CCr{mm,nn},S1) - multiplication(C0,S1-I) -
      coeff_temp * multiplication(gamma_crack, multiplication(C0, S1-I));
    TTr_temp = TTr{mm,nn};
    TTr{mm,nn} = -multiplication(inverse_4th(AAr{mm,nn}), CCr{mm,nn} -
                  multiplication(I + coeff_temp * gamma_crack, C0));
    BB = BB + phi_f_cell * multiplication(S1, TTr{mm,nn}) - phi_f_cell *
                           multiplication(S1, TTr_temp);
  end
end

% step 3 %
C_eff = C0;
for mm = 1:ncell
  for nn = 1:ncell
    C_eff = C_eff - phi_f_cell * multiplication(C0,
                                 multiplication(inverse_4th(BB),  TTr{mm,nn}));
  end
end
[C_eff] = get_symm(C_eff);

% step 4 %
dsig = C_eff * deps;
sig_macro(:, ii+1) = sig_macro(:, ii) + dsig;
sig_initial(:, ii+1) = sig_initial(:, ii) + C_eff_initial * deps;

set(gcf,'Color','w')
for mm = 1:ncell
    for nn = 1:ncell
        temp = cell_conn{mm,nn};
        plot(temp(:,1), temp(:,2), 'b-')
        if mm == 1 && nn == 1
            hold on
        end
        cent2 = [sum(temp(1:4,1))/4-0.1 sum(temp(1:4,2))/4-0.35];

        if cell_fiber_check(mm,nn) == 0
           plot(sum(temp(1:4,1))/4,sum(temp(1:4,2))/4,
                  'ko','MarkerFaceColor','k','MarkerSize',8)
        else
           plot(sum(temp(1:4,1))/4,sum(temp(1:4,2))/4,
                  'ro','MarkerFaceColor','r','MarkerSize',8)
        end
    end
  end
  title(['Step = ' num2str(ii)])
  hold off
  axis equal
 end
end

end
```

```
eps_macro = eps_macro * 100;

figure(1)
set(gcf,'color','w');
plot(eps_macro(3,:), sig_macro(3,:), 'b-', 'LineWidth', 3)
hold on
plot(eps_macro(3,:), sig_initial(3,:), 'k--', 'LineWidth', 2)
hold off
xlim([0 5])
ylim([0 2000])
xlabel('Applied Macrostrain (%)','FontSize',13)
ylabel('Predicted Macrostress (MPa)','FontSize',13)
grid on
```

(2) Mechanism 2 – Random Walk Selection

```
if ibreak == 0 && eps_macro(3, ii+1) >= eps_cr %%% initial break !!!
  iselect = floor(ncell * ncell * rand(1) + 1);
  idx_curr = rem(iselect,ncell);
  if idx_curr == 0
    idx_curr = ncell;
  end
  jdx_curr = (iselect - rem(iselect,ncell))/ncell+1;
  if jdx_curr > ncell
    jdx_curr = jdx_curr - ncell;
  end

  if idx_curr >= min(1,nidx - ninc) && idx_curr <= max(ncell,nidx+ + ninc) &&
     jdx_curr >= min(1,nidx - ninc) && jdx_curr <= max(ncell,nidx+ + ninc)
   [neighbor_curr] = find_neighbor(idx_curr,jdx_curr,ncell);
   cell_weak_temp = [cell_weak(neighbor_curr(1,1),neighbor_curr(1,2))
                     cell_weak(neighbor_curr(2,1),neighbor_curr(2,2))
                     cell_weak(neighbor_curr(3,1),neighbor_curr(3,2))
                     cell_weak(neighbor_curr(4,1),neighbor_curr(4,2))
                     cell_weak(idx_curr,jdx_curr)];
  if rand(1,1) <= cell_weak(idx_curr,jdx_curr) / sum(cell_weak_temp) %%%
    for i = 1:4
      iid = neighbor_curr(i,1);
      jid = neighbor_curr(i,2);
      if cell_fiber_check(iid,jid) ~= 0
        cell_fiber_share(iid,jid) = cell_fiber_share(iid,jid) * beta1;
      else
        cell_fiber_share(iid,jid) = cell_fiber_share(iid,jid) * beta1;
        cell_intensity(iid,jid) = cell_intensity(iid,jid) * rho;
        cell_weak(iid,jid) = 1.0 - cell_intensity(iid,jid);
      end
    end
      cell_fiber_check(idx_curr,jdx_curr) =
        cell_fiber_check(idx_curr,jdx_curr) + 1;
      cell_fiber_share(idx_curr,jdx_curr) =
        cell_fiber_share(idx_curr,jdx_curr) * beta2;
      ibreak = 1;
      crack_list = [crack_list; idx_curr, jdx_curr];
    end
  end
```

```
elseif ibreak == 1 && length(find(cell_fiber_check ~= 0)) < ncell^2
  weak_check = [];
  num_neighbor_curr = length(neighbor_curr(:,1));
  for kk = 1:num_neighbor_curr
    weak_check = [weak_check; neighbor_curr(kk,1), neighbor_curr(kk,2),
                  cell_weak(neighbor_curr(kk,1),neighbor_curr(kk,2))];
  end
  iidd2 = floor(rand(1,1)*num_neighbor_curr+1);
  if rand(1) <= weak_check(iidd2,3) / sum(weak_check(:,3))
    idx_curr = weak_check(iidd2,1);
    jdx_curr = weak_check(iidd2,2);
    cell_fiber_check(idx_curr,jdx_curr) =
      cell_fiber_check(idx_curr,jdx_curr) + 1;
    cell_fiber_share(idx_curr,jdx_curr) =
      cell_fiber_share(idx_curr,jdx_curr) * beta2;
    crack_list = [crack_list; idx_curr, jdx_curr];
    [neighbor_now] = find_neighbor(idx_curr,jdx_curr,ncell);
    for i = 1:4
      iid = neighbor_now(i,1);
      jid = neighbor_now(i,2);
      if cell_fiber_check(iid,jid) ~= 0
        cell_fiber_share(iid,jid) = cell_fiber_share(iid,jid) * beta1;
      else
        cell_fiber_share(iid,jid) = cell_fiber_share(iid,jid) * beta1;
        cell_intensity(iid,jid) = cell_intensity(iid,jid) * rho;
        cell_weak(iid,jid) = 1.0 - cell_intensity(iid,jid);
      end
    end
   neighbor_curr = [];
   neighbor_curr = [neighbor_curr; neighbor_now];
   idlist = [];
   for kk = 1:length(neighbor_curr)
     if cell_fiber_check(neighbor_curr(kk,1),neighbor_curr(kk,2)) ~= 0
       idlist = [idlist; kk];
     end
   end
  end
end
```

(3) Mechanism 3 – Self-Avoiding Walk (SAW) Selection

```
if ibreak == 0 && eps_macro(3, ii+1) >= eps_cr %%% initial break !!!
  iselect = floor(ncell * ncell * rand(1) + 1);
  idx_curr = rem(iselect,ncell);
  if idx_curr == 0
    idx_curr = ncell;
  end
  jdx_curr = (iselect - rem(iselect,ncell))/ncell+1;
  if jdx_curr > ncell
    jdx_curr = jdx_curr - ncell;
  end
  if idx_curr >= min(1,nidx - ninc) && idx_curr <= max(ncell,nidx+ + ninc) &&
     jdx_curr >= min(1,nidx - ninc) && jdx_curr <= max(ncell,nidx+ + ninc)
    [neighbor_curr] = find_neighbor(idx_curr,jdx_curr,ncell);
    cell_weak_temp = [cell_weak(neighbor_curr(1,1),neighbor_curr(1,2))
                      cell_weak(neighbor_curr(2,1),neighbor_curr(2,2))
```

```
                    cell_weak(neighbor_curr(3,1),neighbor_curr(3,2))
                    cell_weak(neighbor_curr(4,1),neighbor_curr(4,2))
                    cell_weak(idx_curr,jdx_curr)];
        if rand(1,1) <= cell_weak(idx_curr,jdx_curr) / sum(cell_weak_temp) %%%
          for i = 1:4
            iid = neighbor_curr(i,1);
            jid = neighbor_curr(i,2);
            if cell_fiber_check(iid,jid) ~= 0
              cell_fiber_share(iid,jid) = cell_fiber_share(iid,jid) * beta1;
            else
              cell_fiber_share(iid,jid) = cell_fiber_share(iid,jid) * beta1;
              cell_intensity(iid,jid) = cell_intensity(iid,jid) * rho;
              cell_weak(iid,jid) = 1.0 - cell_intensity(iid,jid);
            end
          end
          cell_fiber_check(idx_curr,jdx_curr) =
            cell_fiber_check(idx_curr,jdx_curr) + 1;
          cell_fiber_share(idx_curr,jdx_curr) =
            cell_fiber_share(idx_curr,jdx_curr) * beta2;
          ibreak = 1;
          crack_list = [crack_list; idx_curr, jdx_curr];
        end
      end
    elseif ibreak == 1 && length(find(cell_fiber_check ~= 0)) < ncell^2
      weak_check = [];
      num_neighbor_curr = length(neighbor_curr(:,1));
      for kk = 1:num_neighbor_curr
        weak_check = [weak_check; neighbor_curr(kk,1), neighbor_curr(kk,2),
                      cell_weak(neighbor_curr(kk,1),neighbor_curr(kk,2))];
      end
      iidd2 = floor(rand(1,1)*num_neighbor_curr+1);
      if rand(1) <= weak_check(iidd2,3) / sum(weak_check(:,3)) &&
         cell_fiber_check(weak_check(iidd2,1),weak_check(iidd2,2)) == 0
        idx_curr = weak_check(iidd2,1);
        jdx_curr = weak_check(iidd2,2);
        cell_fiber_check(idx_curr,jdx_curr) =
          cell_fiber_check(idx_curr,jdx_curr) + 1;
        cell_fiber_share(idx_curr,jdx_curr) =
          cell_fiber_share(idx_curr,jdx_curr) * beta2;
        crack_list = [crack_list; idx_curr, jdx_curr];
        [neighbor_now] = find_neighbor(idx_curr,jdx_curr,ncell);
        for i = 1:4
          iid = neighbor_now(i,1);
          jid = neighbor_now(i,2);
          if cell_fiber_check(iid,jid) ~= 0
            cell_fiber_share(iid,jid) = cell_fiber_share(iid,jid) * beta1;
          else
            cell_fiber_share(iid,jid) = cell_fiber_share(iid,jid) * beta1;
            cell_intensity(iid,jid) = cell_intensity(iid,jid) * rho;
            cell_weak(iid,jid) = 1.0 - cell_intensity(iid,jid);
          end
        end
        neighbor_curr = [];
        neighbor_curr = [neighbor_curr; neighbor_now];
        idlist = [];
        for kk = 1:length(neighbor_curr)
          if cell_fiber_check(neighbor_curr(kk,1),neighbor_curr(kk,2)) ~= 0
```

```
        idlist = [idlist; kk];
      end
    end
  end
end
```

(4) Mechanism 4 – All Neighboring Fiber Selection

```
if ibreak == 0 && eps_macro(3, ii+1) >= eps_cr %%% initial break !!!
  iselect = floor(ncell * ncell * rand(1) + 1);
  idx_curr = rem(iselect,ncell);
  if idx_curr == 0
    idx_curr = ncell;
  end
  jdx_curr = (iselect - rem(iselect,ncell))/ncell+1;
  if jdx_curr > ncell
    jdx_curr = jdx_curr - ncell;
  end
  if idx_curr >= min(1,nidx - ninc) && idx_curr <= max(ncell,nidx+ + ninc) &&
     jdx_curr >= min(1,nidx - ninc) && jdx_curr <= max(ncell,nidx+ + ninc)
    [neighbor_curr] = find_neighbor(idx_curr,jdx_curr,ncell);
    cell_weak_temp = [cell_weak(neighbor_curr(1,1),neighbor_curr(1,2))
                      cell_weak(neighbor_curr(2,1),neighbor_curr(2,2))
                      cell_weak(neighbor_curr(3,1),neighbor_curr(3,2))
                      cell_weak(neighbor_curr(4,1),neighbor_curr(4,2))
                      cell_weak(idx_curr,jdx_curr)];
    if rand(1,1) <= cell_weak(idx_curr,jdx_curr) / sum(cell_weak_temp) %%%
      for i = 1:4
        iid = neighbor_curr(i,1);
        jid = neighbor_curr(i,2);
        if cell_fiber_check(iid,jid) ~= 0
          cell_fiber_share(iid,jid) = cell_fiber_share(iid,jid) * beta1;
        else
          cell_fiber_share(iid,jid) = cell_fiber_share(iid,jid) * beta1;
          cell_intensity(iid,jid) = cell_intensity(iid,jid) * rho;
          cell_weak(iid,jid) = 1.0 - cell_intensity(iid,jid);
        end
      end
      cell_fiber_check(idx_curr,jdx_curr) =
        cell_fiber_check(idx_curr,jdx_curr) + 1;
      cell_fiber_share(idx_curr,jdx_curr) =
        cell_fiber_share(idx_curr,jdx_curr) * beta2;
      ibreak = 1;
      crack_list = [crack_list; idx_curr, jdx_curr];
      end
    end
elseif ibreak == 1 && length(find(cell_fiber_check ~= 0)) < ncell^2
  weak_check = [];
  num_neighbor_curr = length(neighbor_curr(:,1));
  for kk = 1:num_neighbor_curr
    weak_check = [weak_check; neighbor_curr(kk,1), neighbor_curr(kk,2),
                  cell_weak(neighbor_curr(kk,1),neighbor_curr(kk,2))];
  end
```

```
  iidd2 = floor(rand(1,1)*num_neighbor_curr+1);
  if rand(1) <= weak_check(iidd2,3) / sum(weak_check(:,3))
    idx_curr = weak_check(iidd2,1);
    jdx_curr = weak_check(iidd2,2);
    cell_fiber_check(idx_curr,jdx_curr) =
      cell_fiber_check(idx_curr,jdx_curr) + 1;
    cell_fiber_share(idx_curr,jdx_curr) =
      cell_fiber_share(idx_curr,jdx_curr) * beta2;
    crack_list = [crack_list; idx_curr, jdx_curr];
    [neighbor_now] = find_neighbor(idx_curr,jdx_curr,ncell);
    for i = 1:4
      iid = neighbor_now(i,1);
      jid = neighbor_now(i,2);
      if cell_fiber_check(iid,jid) ~= 0
        cell_fiber_share(iid,jid) = cell_fiber_share(iid,jid) * beta1;
      else
        cell_fiber_share(iid,jid) = cell_fiber_share(iid,jid) * beta1;
        cell_intensity(iid,jid) = cell_intensity(iid,jid) * rho;
        cell_weak(iid,jid) = 1.0 - cell_intensity(iid,jid);
      end
    end
    neighbor_curr = [];
    for kk = 1:length(crack_list)-1
      [neighbor_temp] =
        find_neighbor(crack_list(kk,1),crack_list(kk,2),ncell);
      neighbor_curr = [neighbor_curr; neighbor_temp];
    end
    neighbor_curr = [neighbor_curr; neighbor_now];
    idlist = [];
    for kk = 1:length(neighbor_curr)
      if cell_fiber_check(neighbor_curr(kk,1),neighbor_curr(kk,2)) ~= 0
        idlist = [idlist; kk];
      end
    end
    neighbor_curr(idlist,:) = [];
  end
end
```

(5) Function of Changing Fiber Direction of 4th Order Tensor

```
function [DD_new] = change_matrix_dir(idx1,idx2,DD)
DDtemp = zeros(3,3,3,3);
for ii = 1:3
    for jj = 1:3
        for kk = 1:3
            for ll = 1:3
                if ii == idx2
                    itemp = idx1;
                elseif ii == idx1
                    itemp = idx2;
                else
                    itemp = ii;
                end
```

```
                if jj == idx2
                    jtemp = idx1;
                elseif jj == idx1
                    jtemp = idx2;
                else
                    jtemp = jj;
                end
                if kk == idx2
                    ktemp = idx1;
                elseif kk == idx1
                    ktemp = idx2;
                else
                    ktemp = kk;
                end
                if ll == idx2
                    ltemp = idx1;
                elseif ll == idx1
                    ltemp = idx2;
                else
                    ltemp = ll;
                end
                [qq] = mapping2(itemp,jtemp);
                [rr] = mapping2(ktemp,ltemp);
                DDtemp(ii,jj,kk,ll) = DD(qq,rr);
            end
        end
    end
end
DD_new = zeros(6);
for mm = 1:6
    [ii jj] = mapping(mm);
    for nn = 1:6
        [kk ll] = mapping(nn);
        DD_new(mm,nn) = DDtemp(ii,jj,kk,ll);
    end
end
return
```

(6) Function of Computing 4th Order Elasticity Tensor

```
function [C] = Elasticity(E,mu)
lame1 = E*mu/(1+mu)/(1-2*mu);
lame2 = E/2/(1+mu);
C = zeros(6,6);
for mm = 1:6
    [ii jj] = mapping(mm);
    for nn = 1:6
        [kk ll] = mapping(nn);
        if (ii == jj && kk == ll)
            C(mm,nn) = C(mm,nn)+lame1;
        end
        if ((ii == kk && jj == ll))
            C(mm,nn) = C(mm,nn) + lame2;
        end
        if ((ii == ll && jj == kk) )
            C(mm,nn) = C(mm,nn) + lame2;
```

```
        end
    end
end
```

(7) Function of Computing 4th Order Eshelby Tensor

```
function [S]= Eshelby(a1,a3,mu)
a = a1^2-a3^2;
I1 = -4*pi*a3^2/a-2*pi*a1*a3^2/a^1.5*(log((a1-sqrt(a))/(a1+sqrt(a))));
I2 = 0.5*(4*pi-I1);
I11 = -4*pi*a3^2/(3*a*a1^2)+I1/a;
I12 = 2*pi/a-3*I1/2/a;
I22 = pi/a3^2-1/4*I12;
I = [I1;I2;I2];
II = [I11,I12,I12;I12,I22,I22;I12,I22,I22];
aI = [a1;a3;a3];
S = zeros(6,6);
for mm = 1:6
  [ii jj] = mapping(mm);
  for nn = 1:6
    [kk ll] = mapping(nn);
      if(ii == jj && kk == ll)
        S(mm,nn) = S(mm,nn) + 2*mu*I(ii)-I(kk)+aI(ii)^2*II(kk,ii);
      end
      if((ii == kk && jj == ll))
        S(mm,nn) = S(mm,nn) + aI(ii)^2*II(ii,jj) - I(jj) +
                  (1-mu)*(I(kk)+I(ll));
      end
      if((jj == kk && ii == ll))
        S(mm,nn) = S(mm,nn) + aI(ii)^2*II(ii,jj) - I(jj) +
                  (1-mu)*(I(kk)+I(ll));
      end
  end
end
S = S / (8*pi*(1-mu));
return
```

(8) Function of Finding Nearest Neighboring Fibers

```
function [neighbor_now] = find_neighbor(idx,jdx,ncell)
neighbor_now  = [idx-1 jdx
                 idx+1 jdx
                 idx   jdx-1
                 idx   jdx+1];
for i = 1:4
    for j = 1:2
        if neighbor_now(i,j) > ncell
            neighbor_now(i,j) = neighbor_now(i,j) - ncell;
        elseif neighbor_now(i,j) <= 0
            neighbor_now(i,j) = neighbor_now(i,j) + ncell;
        end
    end
end
return
```

(9) Function of Finding Symmetric Matrix

```
function [C_sym] = get_symm(C_eff)
C_sym = zeros(6);
for ii = 1:6
    for jj = 1:6
        C_sym(ii,jj) = max(C_eff(ii,jj),C_eff(jj,ii));
    end
end
return
```

(10) Function of Fining Inverse of 4th Order Tensor

```
function [B]= inverse_4th(A)
I = eye(6);
I(4,4) = 0.5;
I(5,5) = 0.5;
I(6,6) = 0.5;
A_bar = A;
for ii = 1:6
    for jj = 4:6
        A_bar(ii,jj) = 2*A(ii,jj);
    end
end
B = inv(A_bar)*I;
Return
```

(11) Functions of Finding Index Mapping

```
function [ii jj] = mapping(mm)
if (mm == 1)
    ii = 1;
    jj = 1;
elseif (mm == 2)
    ii = 2;
    jj = 2;
elseif (mm == 3)
    ii = 3;
    jj = 3;
elseif (mm == 4)
    ii = 2;
    jj = 3;
elseif (mm == 5)
    ii = 1;
    jj = 3;
elseif (mm == 6)
    ii = 1;
    jj = 2;
end
return

function [mm] = mapping2(ii,jj)
if (ii == 1 && jj == 1)
    mm = 1;
```

```
elseif (ii == 2 && jj == 2)
    mm = 2;
elseif (ii == 3 && jj == 3)
    mm = 3;
elseif ((ii == 2 && jj == 3) || (ii == 3 && jj == 2))
    mm = 4;
elseif ((ii == 1 && jj == 3) || (ii == 3 && jj == 1))
    mm = 5;
elseif ((ii == 1 && jj == 2) || (ii == 2 && jj == 1))
    mm = 6;
end
return
```

(12) Function of Computing 4th Order Tensor Multiplication

```
function [C] = multiplication(A,B)
Ctemp = zeros(3,3,3,3);
for ii = 1:3
    for jj = 1:3
        for kk = 1:3
            for ll = 1:3
                for mm = 1:3
                    for nn = 1:3
                        [pp] = mapping2(ii,jj);
                        [qq] = mapping2(mm,nn);
                        [rr] = mapping2(kk,ll);
                        Ctemp(ii,jj,kk,ll) = Ctemp(ii,jj,kk,ll) +
A(pp,qq)*B(qq,rr);
                    end
                end
            end
        end
    end
end
C = zeros(6);
for mm = 1:6
    [ii jj] = mapping(mm);
    for nn = 1:6
        [kk ll] = mapping(nn);
        C(mm,nn) = Ctemp(ii,jj,kk,ll);
    end
end
return
```

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