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HORIZONTAL MERGERS:  
AN EQUILIBRIUM ANALYSIS

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Abstract

We analyze mergers as concentration-increasing transfers of industry-specific capital among oligopolists. Such capital transfers affect industry structure, and induce changes in the subsequent oligopolistic Cournot equilibrium. We provide general conditions under which they raise the market price. We also examine the social and private incentives to merge, accounting for the fact that mergers alter the distribution of outputs across firms as well as the aggregate output level. We find that small firms typically have insufficient incentives to merge, while large firms have excessive incentives to do so. In one reasonable special case, a merger is socially more attractive the more concentrated is production among the non-participant firms.



# Horizontal Mergers: An Equilibrium Analysis

Joseph Farrell and Carl Shapiro

## 1. Introduction

In the United States, horizontal merger policy is administered by the Department of Justice and the Federal Trade Commission, who apply rules summarized in the Department of Justice's Merger Guidelines (1984). These Guidelines reflect the view that concentration can diminish industry performance, but are not based on explicit analysis of how a corporate transaction is likely to affect industry equilibrium.

In this paper we use oligopoly theory to examine the trade-off between the evils of concentration and the possible benefits of mergers.<sup>1</sup> We view a merger as transfer of industry-specific assets, and trace its effects on prices, output, profits, and industry performance. At a practical, policy level, we seek to provide some guidance for using information about market structure to assess the likely desirability of a proposed merger. Although theories of oligopoly behavior underlie many economists' thinking about the impact of industry structure on market behavior and performance, oligopoly theory has not been systematically used to analyze the effect of a corporate transaction in a concentrated market. In particular, the responses of large non-participant firms have not been adequately included in previous analyses of horizontal mergers.

The need for explicit oligopoly theory in judging mergers is perhaps seen most clearly in the use of the Herfindahl index,  $H$ , to guide merger analysis. Although  $H$  has some attractive welfare properties (see, *e.g.*, Dansby and Willig (1979)), these properties relate to the social benefits of increasing production, not to the costs or benefits of redeploying

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<sup>1</sup> Williamson (1968) examined the tradeoff between exogenous cost savings and price increases, but did not study how the two are linked by the nature of a merger, as we do.

industry assets. The Merger Guidelines, while surely more sophisticated than what they replaced, are *ad hoc* in the way they use the level and predicted change in the concentration index to judge the likely anticompetitive effects of a merger.

In particular, the rule used to compute a merger's effect on  $H$  is logically flawed. This rule takes the initial market shares of the merging firms,  $s_1$  and  $s_2$ , and assumes that the new entity's market share will be  $s_1 + s_2$ , so that  $H$  will rise by  $(s_1 + s_2)^2 - (s_1^2 + s_2^2) = 2s_1s_2$ . But if indeed all firms maintain their outputs at the pre-merger level, then the merger will have no effect on either consumers or non-participant firms, so it will be socially desirable if and only if it is privately profitable. If, as is more likely, there are output responses to the merger, then the  $2s_1s_2$  formula is presumably invalid.

An even more perverse consequence of naively using the Herfindahl index emerges when the market leader proposes to build a new plant that will increase its output. Such an expansion expands industry output but raises  $H$ ; thus, reliance on  $H$  to measure the extent of oligopolistic output restriction is misleading.

We believe that correct use of concentration measures requires distinguishing more clearly between a merger's (or an investment's) effect on industry *structure* — the distribution of assets across firms — and its effect on *conduct* — firms' chosen outputs. We study the effects of horizontal mergers, as structural changes, on conduct and hence on performance, using an explicit theory of oligopoly. In this approach, firms foresee the effects of asset reconfiguration on output and pricing competition. In game-theoretic terms, we evaluate analyze mergers and capital transfers (as well as investments) by examining their effects on production decisions.

Any theory of horizontal mergers intended to guide merger policy must have the following features. First, it must recognize that mergers may have anticompetitive effects: otherwise, mergers should always be approved. Second, it must recognize that mergers may enhance efficiency, for instance by reducing costs: otherwise, mergers should never be allowed. Finally, it must yield predictions of these effects in terms of observable variables in a reasonably general model.

An ideal theory would begin with a coherent, reasonably general oligopoly theory, in which, realistically, some mergers are privately profitable and others are not. It would

develop formulae for the net increase in overall economic welfare,  $\Delta W$ , as a function of market observables and (perhaps) of information privy to the participant firms, and for the net private profitability  $\Delta\pi \equiv \Delta\pi_1 + \Delta\pi_2$  of the merger between firms 1 and 2. It would then develop a “rule” that specifies for what values of the market observables the expected value of  $\Delta W$  is positive, conditional on the privately observed value of  $\Delta\pi$  being positive (since mergers are voluntary).

Our theory is a step in this direction. We develop, in particular, a relatively general and concise description of the factors determining the sign of the net “wedge”  $\Delta W - \Delta\pi$  resulting from a merger. Any privately profitable merger for which this wedge is positive should be approved. If the wedge is negative, then, the more negative it is, the heavier should be the burden of proof (presumably, on the participants) that the “unobservable” variables are favorable. Our model does not formally contain unobservable variables, so we interpret a negative value of the wedge to indicate that only mergers that are *sufficiently* profitable for the participants should be permitted.

We incorporate anticompetitive effects by using standard Cournot oligopoly theory to determine prices and outputs, given capital stocks. We permit efficiency benefits of mergers by considering the prospect that assets may be better employed as a result of the merger. We include the possibility that the acquiring firm is simply more efficient than the target firm, that there are direct synergies between merging firms, and that the assets of non-participant firms may be used more effectively in response to the merger.

There is a small theoretical literature on horizontal mergers that we take as our starting point. First, Salant, Switzer and Reynolds (1983) considered mergers in the simplest model of Cournot oligopoly: constant average costs and linear demand. They found that the joint profits of merging firms typically *decline*; of course, in this model all mergers lower welfare, since they yield no efficiency gains and reduce competition. As we argue above, merger analysis should try to predict the welfare consequences of mergers, *given* that they are privately profitable. For this program, the Salant *et al.* model is unhelpful, both because it omits the efficiency factors that can make mergers desirable, and because it predicts that no mergers would ever be proposed.

Nonetheless, their analysis is useful for understanding horizontal mergers. The reason that mergers are privately unprofitable is that rival firms expand output in response to

the contraction of the merging parties. We believe that the possibility of such responses is important in merger analysis, and we emphasize it in our treatment.

A second problem with the Salant *et al.* assumption of constant marginal costs is that each firm has no assets, other than its identity as an active oligopolist, so the combined firm is not intrinsically “larger” than the constituent firms. That is, a “merger” in their model simply means one firm committing itself to shutting down. In fact, mergers typically involve transfers of physical or organizational capital. Recognizing that, Perry and Porter (1985) examined the incentives for competitive “fringe” firms to combine their capital stocks and form a new oligopolist instead of behaving competitively. Working with linear demand and quadratic cost functions, they found conditions under which such consolidation is profitable for the merging parties. But they did not examine the welfare effects of such mergers, or consider the merger of two firms that are already “large.”

Like Perry and Porter, we model a merger as a consolidation of capital among existing rivals.<sup>2</sup> Unlike them, we study the incentives for two large firms to combine, and we focus on the welfare analysis. We also consider a much more general set of cost conditions than do Perry and Porter.

## 2. Cournot Oligopoly with Assets

Our analysis uses the traditional model of Cournot oligopoly with homogeneous goods.<sup>3</sup> We begin by describing the Cournot equilibrium for a given allocation of capital across firms. Demand is given by  $p(X)$ , where  $p$  is price,  $X$  is industry output, and  $p'(X) < 0$ . The corresponding total benefit function is  $B(X) \equiv \int_0^X p(z) dz$ .

The number of firms,  $n$ , is fixed, reflecting some important barriers to entry.<sup>4</sup> We model a merger as the transfer of all or some of one firm's capital stock to another firm.

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<sup>2</sup> While writing up our results, we learned of the related work of McAfee and Williams (1988). They study by simulation the linear-quadratic model that Perry and Porter used and that we develop as an example.

<sup>3</sup> For the drawbacks of the Cournot model, see Shapiro (1988).

<sup>4</sup> Our analysis can easily accommodate entry by, or the existence of, price-taking fringe firms, if we reinterpret the demand curve  $p(X)$  as the residual demand curve facing the oligopolists that we model. What we are ruling out is entry by, or the presence of, additional large firms who behave oligopolistically.

We denote firm  $i$ 's capital stock by  $k_i$ ; the industrywide vector of capital stocks is  $\mathbf{k} \equiv (k_1, \dots, k_n)$ . Firm  $i$ 's variable cost of producing output  $x$  is  $c^i(x, k_i)$ .<sup>5</sup> For notational ease, we write  $c^i \equiv c^i(x_i, k_i)$  for firm  $i$ 's total cost, and similarly for partial derivatives such as  $c_x^i \equiv \partial c^i(x_i, k_i)/\partial x_i$  and  $c_k^i \equiv \partial c^i(x_i, k_i)/\partial k_i$ . We assume throughout that  $c_{xk}^i < 0$ : additional capital lowers the marginal cost curve.

In the Cournot equilibrium, each firm  $i$  picks its output  $x_i$  to maximize its profits,  $\pi_i \equiv p(X)x_i - c^i(x_i, k_i)$ , given its rivals' outputs. Firm  $i$ 's first-order condition,  $\partial \pi_i / \partial x_i = 0$ , is

$$p(X) + x_i p'(X) - c_x^i = 0, \quad i = 1, \dots, n. \quad (1)$$

A Cournot equilibrium, given the capital stocks  $\mathbf{k}$ , is a vector  $\mathbf{x} \equiv (x_1, \dots, x_n)$  such that equation (1) holds for all  $n$  firms. We assume throughout the paper that the firms' reactions curves slope downward, and that the equilibrium is unique and "stable."<sup>6</sup> We denote firm  $i$ 's market share by  $s_i \equiv x_i/X$ .

Comparing two firms  $i$  and  $j$ , equation (1) tells us that  $x_i > x_j$  if and only if  $c_x^i < c_x^j$ : in equilibrium, larger firms have lower marginal costs. In any Cournot equilibrium in which different firms produce different quantities, marginal costs differ across firms, so that costs are not minimized given the aggregate output level. This observation will be important below, since mergers generally affect the distribution of outputs across firms. Like many of our qualitative results, this should extend to oligopoly behavior other than Cournot, so long as larger firms have lower marginal costs in equilibrium, as they will if they are exploiting market power more than their smaller rivals.

Our measure of industry performance or welfare is simply the sum of consumer and producer surplus. This in turn equals consumers' gross benefits less production costs:

$$W \equiv B(X) - \sum_{i=1}^n c^i(x_i, k_i).$$

<sup>5</sup> We permit the firms to have access to different technologies, or, equivalently, to have firm-specific, non-transferable assets. At times below, however, we shall assume that all firms have the same technology, although they will typically differ in their capital stocks.

<sup>6</sup> See Dixit (1986) or Shapiro (1988) for uniqueness and stability conditions in Cournot oligopoly.



### *Traditional Physical Capital*

For illustrative purposes and concreteness, we develop two special cases of our general analysis. The first involves linear demand and quadratic costs: Demand is given by  $p(X) = A - X$ , and costs are given by  $c(x, k) = \frac{1}{2}x^2/k$  for all  $i$ .<sup>7</sup> Notice that this cost function exhibits constant returns to scale (it is homogeneous of degree one in output and capital). We interpret this example as applying to industries where “capital” is traditional physical capital.<sup>8</sup> Marginal costs are  $c_x = x/k$ . In the short run, firms’ marginal costs rise linearly with output. Clearly, additional capital lowers marginal costs,  $c_{xk} = -x/k^2 < 0$ .

Firm  $i$ ’s reaction schedule is given by  $A - X - x_i - x_i/k_i = 0$ . Defining firm  $i$ ’s *adjusted capital stock* as  $\kappa_i \equiv k_i/(k_i + 1)$ , the reaction schedule becomes  $x_i = (A - X)\kappa_i = p\kappa_i$ ; the firms’ outputs are proportional to their adjusted capital stocks. Firm  $i$ ’s marginal cost is  $x_i/k_i = p/(1 + k_i)$ . In equilibrium, therefore, firms with more capital produce more output and have lower marginal costs. Writing  $\kappa \equiv \sum_{i=1}^n \kappa_i$ , the equilibrium outputs are  $x_i = A\kappa_i/(1 + \kappa)$ . Aggregate output is  $X = A\kappa/(1 + \kappa)$ , and price is  $p = A/(1 + \kappa)$ .

### *Patents, Know-How or Managerial Talent*

Our second example is the case where a firm’s unit cost depends on its capital stock but not on its output:<sup>9</sup> thus,  $c(x, k) = x\phi(k)$ , where  $\phi'(k) < 0$ . This represents capital such as know-how or managerial talent that is not subject to congestion: patents, know-how, and (perhaps) good managers. With such capital, a natural economy of scale arises, since it can be applied to any scale of output at no additional cost. As we shall see, the analysis of mergers in this case depends critically on whether different units of know-how are substitutes (less valuable when combined with others) or complements (more valuable in combination) in unit-cost-reduction, *i.e.*, on the convexity or concavity of  $\phi(\cdot)$ .

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<sup>7</sup> These are the cost and demand functions used by Perry and Porter (1985) and by McAfee and Williams (1988). The analysis would be identical in a model with  $p = A - bX$  and  $c(x, k) = cx + \frac{e}{2}x^2/k$ , but the parameters  $b, c$ , and  $e$  would clutter our formulae.

<sup>8</sup> This cost function corresponds to the Cobb-Douglas production function  $z = \sqrt{LK}$ .

<sup>9</sup> Clarke (1987) studies mergers between firms with different constant marginal costs, but does not consider the possibility that asset transfers affect marginal costs.

If  $\phi''(k) > 0$ , there are diminishing returns to the concentration of capital, in terms of its effect on unit cost reduction.

Firm  $i$ 's reaction curve is  $p(X) + x_i p'(X) = \phi(k_i)$ . Adding across firms we get  $np(X) + Xp'(X) = \sum_{i=1}^n \phi(k_i) \equiv \Phi(\mathbf{k})$ . Total output depends in a simple way on the distribution of capital across firms.

### 3. Changes in Capital Stocks

We begin with a Cournot equilibrium, as described by equation (1), and consider an arbitrary small change in industry structure (*i.e.*, in the firms' capital stocks),  $d\mathbf{k} \equiv (dk_1, \dots, dk_n)$ . To determine the effect of this structural change on industry conduct and performance, we must trace the effect on all firms' outputs,  $d\mathbf{x} \equiv (dx_1, \dots, dx_n)$ .

In mathematical terms, we perform comparative statics on the set of equations (1). In economic terms, this approach assumes that output competition takes place in the short run, while capital allocation occurs more slowly, so we can treat the allocation of capital as prior to the determination of outputs.

#### *Profit and Welfare Effects*

We develop general formulae measuring the changes in profits and welfare resulting from the shift  $d\mathbf{k}$  in the firms' capital stocks. The change in firm  $i$ 's profits is

$$d\pi_i = (p - c_x^i)dx_i + x_i p'(X)dX - c_k^i dk_i. \quad (2)$$

Firm  $i$  benefits from increased production proportionately to its markup, is affected by price changes proportionately to its output, and of course its costs will be affected by any change in its own capital stock.

The welfare effect is given by

$$dW = \sum_{i=1}^n (p - c_x^i)dx_i - \sum_{i=1}^n c_k^i dk_i. \quad (3)$$

The first term is the net welfare effect of the changes in output: recall that marginal benefits  $B'(X)$  are measured by price. The second term is the direct effect of the capital shift on costs, given outputs. These results apply for any oligopoly theory.

For the Cournot case, substituting from (1) into (3) we have

$$dW = -p'(X) \sum_{i=1}^n x_i dx_i - \sum_{i=1}^n c_k^i dk_i. \quad (4)$$

We can relate this to the familiar Herfindahl Index,  $H \equiv \sum_{i=1}^n s_i^2 = \sum_{i=1}^n (\frac{x_i}{X})^2$ , using the fact that  $\sum_{i=1}^n x_i dx_i = \frac{1}{2} \sum_{i=1}^n d[x_i^2] = \frac{1}{2} d[X^2 H] = HX dX + \frac{1}{2} X^2 dH$ . So, the welfare expression (4) can be rewritten as

$$dW = -p'(X)(HX dX + \frac{1}{2} X^2 dH) - \sum_{i=1}^n c_k^i dk_i. \quad (5)$$

Equation (5) decomposes welfare changes into three parts, one related to aggregate output,  $X$ , one related to the distribution of output,  $H$ , and one related to direct cost effects. Increases in output raise welfare, since price exceeds marginal cost in equilibrium. More surprisingly, so do increases in  $H$ . For any given change in  $X$ , welfare rises by *more*, the larger is the increase in  $H$ : large firms have the lowest marginal costs, so expansion by them is particularly valuable socially. Consequently, the Merger Guidelines, or indeed any analyses suggesting that increases in  $H$  are bad, are misleading if Cournot competition applies. For example, given the firms' capital stocks and a level of aggregate output, redistributions of output that increase  $H$  increase welfare.

### *Induced Output Effects*

We now determine how the  $x_i$ 's change in response to the shift in the firms' capital stocks. Differentiating totally firm  $i$ 's first-order condition (1), we get

$$[p'(X) + x_i p''(X)] dX + p'(X) dx_i - c_{xx}^i dx_i - c_{xk}^i dk_i = 0.$$

Writing  $\lambda_i \equiv \frac{-p'(X) - x_i p''(X)}{c_{xx}^i - p'(X)}$ , and  $\delta_i \equiv \frac{-c_{xk}^i}{c_{xx}^i - p'(X)}$ , output changes are

$$dx_i = -\lambda_i dX + \delta_i dk_i. \quad (6)$$

With downward-sloping reaction schedules and a stable equilibrium,  $\lambda_i$  and  $\delta_i$  are positive for all  $i$ .

The  $\lambda_i$ 's are critical in what follows, so we pause here to interpret them.<sup>10</sup> For any firm  $i$  whose capital stock is not changing, *i.e.*,  $dk_i = 0$ , we have  $dx_i = -\lambda_i dX$ , so these firms' output "responses" to aggregate output changes are proportional to their  $\lambda_i$ 's. So,  $\lambda_i$  measures firm  $i$ 's response to price changes.<sup>11</sup> Large firms respond more than small firms to any given price increase if and only if  $\lambda_i$  increases with  $x_i$ .

To find the change in total output, we add up equation (6) across firms. Writing  $\Lambda \equiv \sum_{i=1}^n \lambda_i$ , we get

$$dX = (1 + \Lambda)^{-1} \sum_i \delta_i dk_i. \quad (7)$$

#### 4. Investment in Oligopoly

Our main goal is to examine mergers, which we model as transfers of capital from one oligopolist to another. But we begin with another question that is of independent economic interest and serves as a tool for our study of mergers below: What is the welfare effect of investment by an oligopolist? In particular, what external effects on consumers and other firms result from an oligopolist's decision to buy capital?

In our model, investment amounts to a change in  $\mathbf{k}$  of the form  $d\mathbf{k} = (dk_1, 0, \dots, 0)$ . From (7), the effect of this change on aggregate output is  $dX/dk_1 = \delta_1/(1 + \Lambda)$ . Hence from (6), for  $i \neq 1$ ,  $dx_i/dk_1 = -\delta_1 \lambda_i/(1 + \Lambda)$ , and  $dx_1/dk_1 = \delta_1(1 - \frac{\lambda_1}{1 + \Lambda})$ . As firm 1 invests, it expands its output. In response, its rivals contract, but overall, output rises.

Using these expressions, equation (2) gives

$$\frac{d\pi_1}{dk_1} = -p'(X)\delta_1(x_1 \frac{\Lambda - \lambda_1}{1 + \Lambda}) - c_k^1$$

and equation (4) becomes

$$\frac{dW}{dk_1} = -p'(X)\delta_1(x_1 - \frac{1}{1 + \Lambda} \sum_{i=1}^n \lambda_i x_i) - c_k^1. \quad (8)$$

<sup>10</sup> See also Gaudet and Salant (1988), who write  $\alpha$  for what we call  $\lambda$ , and discuss its importance in coordinated output reduction.

<sup>11</sup> Since price and aggregate output are endogenous, these statements are not totally precise. Strictly, firm  $i$ 's response to the output changes of all other firms combined,  $dx_i/dX_{-i}$ , is  $-\lambda_i/(1 + \lambda_i)$ .

We wish to compare  $d\pi_1/dk_1$  and  $dW/dk_1$ . The difference is the net effect of firm 1's investment on consumers and on other firms.<sup>12</sup> From equations (2) and (3),

$$d\pi_1 - dW = x_1 p'(X) dX - \sum_{i=2}^n (p - c_x^i) dx_i. \quad (9)$$

With output rising,  $dX > 0$ , the first term in (9) reflects a private cost to firm 1 that is not a social cost: firm 1 loses revenues on each unit it sells as the price falls, but this private loss is just a transfer to consumers. The second term in (9) reflects a social cost not borne by firm 1: other firms, whose marginal costs are less than the price, reduce their outputs. These observations, including equations (8) and (9), apply to any oligopoly theory.

In the Cournot case, equation (1) gives  $p - c_x^i = -p'(X)x_i$  and equation (6) gives  $dx_i = -\lambda_i dX$  for  $i \neq 1$ , so that equation (9) becomes

$$\frac{d\pi_1}{dk_1} - \frac{dW}{dk_1} = -p'(X) \frac{dX}{dk_1} (-x_1 + \sum_{i=2}^n \lambda_i x_i). \quad (10)$$

In equation (10), firm  $i$ 's equilibrium output response to firm 1's investment is proportional to  $\lambda_i$ , while its markup is proportional to  $x_i$ . Hence the social cost of the output-reduction at firms 2 through  $n$  is proportional to  $\sum_{i=2}^n \lambda_i x_i$ : in fact, it equals  $(-dp/dk_1) \sum_{i=2}^n \lambda_i x_i$ . This social cost is compared with the transfer losses borne by firm 1, which equal  $(dp/dk_1)x_1$ .

Converting (10) into market shares, we have

**Proposition 1.** *Firm 1 has excessive incentives to invest, i.e.,  $d\pi_1/dk_1 > dW/dk_1$ , if and only if*

$$s_1 < \sum_{i=2}^n \lambda_i s_i. \quad (11)$$

Clearly, if firm 1 is sufficiently small, it has excessive incentives to invest. Although investment by a small firm lowers price and benefits consumers, it harms rival firms by more.<sup>13</sup> The marginal investment by a small firm worsens industry performance.

<sup>12</sup> See Katz and Shapiro (1988) for a general treatment of these "two wedges."

<sup>13</sup> Of course, this overinvestment cannot occur in a competitive market. As the Cournot equilibrium becomes more competitive,  $dX/dk_1$  approaches zero, and the overinvestment incentive vanishes.

To see more generally which firms have excessive incentives to invest, we rewrite equation (11) as  $s_1(1 + \lambda_1) < \sum_{i=2}^n \lambda_i s_i$ . Firms for which  $s_i(1 + \lambda_i)$  is small are the most likely to have excessive investment incentives. If larger firms respond more to rivals' output changes, then  $\lambda_i$  increases with  $s_i$  and we can conclude that it is smaller firms (if any) who have excessive incentives to invest, while larger firms have insufficient incentives to do so. In this case, Proposition 1 tells us that especially efficient firms (or those whose capital stocks are already large), who produce more output in equilibrium, tend to underinvest, while inefficient, small firms overinvest.

Intuitively, investment by firm 1 induces other firms to reduce their outputs. Firm 1's investment incentives are most likely to be excessive if its rivals have large market shares and therefore high price-cost margins, and if their equilibrium outputs are sensitive to expansion by firm 1. In such a market, firm 1 "steals" a great deal of valuable "business" from its rivals when it invests. This negative externality imposed on rivals must be compared against the transfer to consumers of the price change on firm 1's output (the price effect on other firms' outputs is a mere pecuniary effect).

### *Quadratic Costs*

With linear demand and quadratic costs,  $p'(X) = -1$ ,  $p''(X) = 0$ , and  $c_{zx}^i = 1/k_i$ , so  $\lambda_i = k_i/(k_i + 1)$ ;  $\lambda_i$  is simply equal to firm  $i$ 's adjusted capital stock,  $\kappa_i$  defined earlier. Since  $\lambda_i = x_i/p$ ,  $\sum_{i=2}^n \lambda_i x_i = \sum_{i=2}^n x_i^2/p$ , and inequality (11) becomes

$$s_1 < \frac{1}{\epsilon^D} \sum_{i=2}^n s_i^2,$$

where  $\epsilon^D$  is the elasticity of demand,  $-p(X)/Xp'(X)$ . Firm 1 is most likely to have excessive incentives to invest if its market share is small, if its rivals are concentrated, and if market demand is inelastic.

### *Constant Marginal Costs*

With constant marginal costs,  $c_{zx} = 0$ , so  $\lambda_i = (p' + x_i p'')/p' = 1 + x_i p''/p'$ . Defining  $E \equiv -Xp''(X)/p'(X)$  as the elasticity of the slope of the inverse demand curve,<sup>14</sup> we have

<sup>14</sup> See Dixit (1986) for a discussion of the role played by  $E$  in oligopoly comparative statics.

$\lambda_i = 1 - s_i E$ , and condition (11) becomes

$$s_1 < \frac{1}{2} - \frac{E}{2} \sum_{i=2}^n s_i^2.$$

In the plausible case where  $p'' > 0$ ,  $E > 0$  as well, and we see that an increase in concentration among other firms makes it *less* likely that firm 1 will overinvest. With  $E > 0$ , larger firms respond less to firm 1's investment than do small firms, so the detrimental indirect output effects of 1's investment are smaller, the more concentrated are its rivals.

Formally, an increase in concentration among firm 1's rivals can be seen as a mean-preserving spread of the  $x_i$ . Such a spread raises the sum  $\sum_{i=2}^n \lambda_i x_i$  if and only if  $\lambda_i$  is larger at larger firms. In the quadratic case,  $\lambda_i = x_i/p$  increases with  $x_i$  and concentration increases the magnitude of  $dX_{-i}/dk_i$ , while in the constant marginal costs case with  $E > 0$ ,  $\lambda_i$  decreases with  $x_i$ , so concentration reduces that magnitude.

Proposition 1 concerns an "external" market for capital, *i.e.*, bringing new capital into the industry. If it is impossible, difficult or slow to buy capital externally, then a firm can expand more readily using the "internal" capital market — that is, buying existing capital from a rival. We now consider such internal capital markets.

## 5. Mergers as Transfers of Capital

From now on we will take the industrywide capital stock as fixed, and consider only transfers of capital from one firm to another. We suppose (without loss of generality) that a small amount of capital is transferred from firm 2 to firm 1: formally we take  $dk = (dk, -dk, 0, \dots, 0)$ , where  $dk > 0$ ; *i.e.*,  $dk_1 = dk = -dk_2$ . We consider transfers of capital that "increase the concentration of capital," *i.e.*, where the firm buying capital is larger than that selling it;  $k_1 > k_2$ . Transfers that decrease the concentration of capital, including divestitures, are simply the inverse of those studied here, and have precisely the opposite effects.

By considering small transfers of capital, we can measure the "local" effects of increasing the concentration of capital. We can also examine the *extent* of a merger, rather than treating it as a discrete event. "Discrete" or "complete" mergers between two firms can be studied as the sum of small transfers of capital from the smaller firm to the larger: see Section 8 below.

## 6. The Effect of Mergers On Price

In this section we provide two general results giving conditions under which increasing the concentration of capital reduces output and raises price in Cournot equilibrium.<sup>15</sup> First, from equation (7),

$$\frac{dX}{dk} = \frac{\delta_1 - \delta_2}{1 + \Lambda}, \quad (12)$$

so we have

**Proposition 2.** *A small transfer of capital from firm 2 to firm 1 reduces output and raises price if and only if  $\delta_1 < \delta_2$ , i.e., if and only if the expression  $\frac{-c_{xk}}{c_{xx} - p'(X)}$  is smaller at firm 1 than at firm 2.*

From equation (6),  $\delta_i$  measures the direct effect of an increase in  $k_i$  on firm  $i$ 's output. Proposition 2 tells us that a comparison of these direct effects between firms 1 and 2 determines the direction of the equilibrium change in aggregate output when firm 2 transfers capital to firm 1.

### Examples

In the example with quadratic costs, direct computations show that  $\delta_i = p/(1 + k_i)^2$ . Since  $\delta_i$  is smaller for larger firms, all mergers raise price.

In the example with the cost function  $c = x\phi(k)$ ,  $\delta_i = \phi'(k_i)/p'$ , which decreases with  $k_i$  if and only if  $\phi'' > 0$ . More directly, price is an increasing function of the sum of all the firms' marginal costs,  $\Phi(\mathbf{k})$ , and  $d\Phi/dk = \phi'(k_1) - \phi'(k_2)$ , so price increases with a merger if and only if  $\phi'' > 0$ . Capital transfers raise price if and only if different pieces of know-how are substitutes rather than complements, i.e., if and only if adding capital lowers a firm's unit cost at a diminishing rate.

In principle, Proposition 2 allows one to predict the price effect of a merger on the basis of simple pre-merger variables. But these variables, the  $\delta_i$ , may be hard to measure in practice. We therefore seek general conditions on the cost function under which  $\delta_i$  is

<sup>15</sup> In the Conclusion we discuss how our results would differ if mergers take us away from Cournot equilibrium by making collusion more likely.



a decreasing function of  $k_i$  in cross-section — that is, looking across firms of different sizes within a single equilibrium — so that a concentration-increasing transfer is certain to reduce output.

**Proposition 3.** *Suppose that all firms have access to the same technology, represented by the cost function  $c(x, k)$ . A concentration-increasing merger necessarily reduces output if marginal cost is a convex function of  $x$  and  $k$ .*

*Proof.* Equation (1) relates firm  $i$ 's output  $x_i$  to aggregate output,  $X$ , and firm  $i$ 's capital stock,  $k_i$ . Call this relationship  $x_i = \psi(X, k_i)$ . Adding this up across firms gives

$$np(X) + Xp'(X) = \sum_{i=1}^n c_x(\psi(X, k_i), k_i). \quad (13)$$

Denote the right-hand side of (13) by  $\sum_{i=1}^n h(X, k_i)$ . Since we assume downward-sloping industry marginal revenue, the left hand side of (13) is decreasing in  $X$ , while the right-hand side is increasing in  $X$ . Therefore aggregate output  $X$  decreases with a change  $d\mathbf{k}$  if and only if the right-hand side of (13) increases for the equilibrium value of  $X$ .

Now an increase in the concentration of capital is a mean-preserving spread in  $\mathbf{k}$ , which increases the average value of  $h$  for a given  $X$  if  $h$  is convex in  $k$ . But  $h_k = c_{xx}\psi_k$ , so

$$h_{kk} = c_{xxx}(\psi_k)^2 + 2c_{xxk}\psi_k + c_{xkk},$$

and this is always positive if  $c_x$  is jointly convex in  $(x, k)$ .<sup>16</sup> ■

For the Cobb-Douglas production function  $x = k^\alpha v^{1-\alpha}$ , where  $k$  is capital and  $v$  is a variable input,  $c_x$  is convex in  $(x, k)$  if and only if  $\alpha < \frac{1}{3}$ . Thus Proposition 3 does not apply to our quadratic-cost example ( $\alpha = \frac{1}{2}$ ), but does apply when  $\alpha \approx .25$ , the value suggested by national-income statistics (recall that  $\alpha$  is capital's share of income).<sup>17</sup> As we saw above, although Proposition 3 does not cover the case  $\alpha = \frac{1}{2}$ , mergers do nevertheless

<sup>16</sup> This expression has the form  $\gamma D\gamma'$ , where  $\gamma = (\psi_k, 1)$  and  $D$  is the matrix of second derivatives of  $c_x$ .

<sup>17</sup> Caution is due here, however. "Capital" in our analysis means factors fixed in the short run, which might include management and other skilled labor, and exclude certain factors classified as capital in national-income accounts.

reduce output in that case (at least when demand is linear); the sufficient condition of Proposition 3 is not necessary.

If the conditions of Proposition 3 hold, then any merger — small or large — that increases the concentration of capital necessarily raises price. Propositions 2 and 3 give sufficient conditions for consumers to be harmed by a merger. Equivalently, they give conditions for rival firms to benefit from a merger.

We pause to offer a comment on antitrust enforcement in merger policy. If merger policy is meant simply to protect consumers, and to ban all price-increasing mergers, then Propositions 2 and 3 give conditions under which mergers should be blocked. But rival firms benefit from a merger if and only if the equilibrium price rises, so their interests are diametrically opposed to consumers', and they should not have standing to sue to block proposed mergers.<sup>18</sup> Indeed, if, as often happens, rival firms contest a proposed merger, arguing (ostensibly with commendable unselfishness) that the proposed merger would reduce output and should be forbidden, we should perhaps infer that they believe the opposite, and that the merger would probably benefit consumers!

If merger policy is instead meant to promote overall efficiency, then a merger's effects on price do not determine its desirability; one must carefully examine its effect on overall welfare. We turn to this next.

## 7. The Effects of Mergers on Profits and Welfare

In this section we calculate the effects of a small capital transfer on welfare and on the participants' profits. Our principal result is a formula for the difference between these two, the net externality of the transfer. Since a small capital transfer from firm 2 to firm 1 is equivalent to a small investment by firm 1 and an equal (small) disinvestment by firm 2, the analysis follows easily from that in Section 4 above.

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<sup>18</sup> Sometimes rivals argue a theory of "incipient predation," arguing that the merged entity will indeed increase output, but only as a form of predation against them. This too is unconvincing since predation can be directly fought under the antitrust laws.

### Effect of a Merger on Overall Welfare

Using our earlier result, equation (8), for the welfare consequences of firm 1's investment, we have the welfare effect of a small capital transfer as

$$\frac{dW}{dk} = \frac{dW}{dk_1} - \frac{dW}{dk_2} = -p'(X)(\delta_1 x_1 - \delta_2 x_2 - \frac{\delta_1 - \delta_2}{1 + \Lambda} \sum_{i=1}^n \lambda_i x_i) - c_k^1 + c_k^2. \quad (14)$$

The most straightforward component of this formula is the direct cost effect,  $-c_k^1 + c_k^2$ . One might expect this to be positive because capital lowers costs by more at larger firms, since  $c_{zk} < 0$ . But larger firms already have more capital, and (plausibly)  $c_{kk} > 0$ , which suggests the opposite. In fact, for cost-functions with constant returns to scale,  $c_k$  is increasing in cross-section, so that the direct cost effect is unfavorable.<sup>19</sup> That is, with constant returns to scale, the direct effect of transferring a little capital from firm 2 to firm 1 is to *increase* total production costs.

But of course outputs are not constant (if they were, then mergers would pose no policy problem). The capital transfer also affects welfare through the induced output effects given

<sup>19</sup> With constant returns, we can use the Euler equation  $zc_z + kc_k = c$ , which implies that  $zc_{zz} + kc_{zk} = 0$  and  $zc_{zk} + kc_{kk} = 0$ , whence  $c_{kk} - c_{zk}^2/c_{zz} = 0$ . Also, since  $c_{zk} < 0$ , we know that  $c_{zz} > 0$ . Now we introduce a technique that we call *cross-sectional differentiation*. This technique determines whether a function  $f(k, z)$  increases or decreases with  $k$ , looking across firms in a given equilibrium, and accounting for the cross-sectional variation in  $z$  that accompanies the variation in  $k$ . In cross-section,  $z$  varies with  $k$  in such a way that (1) holds for all firms, for the equilibrium values of  $p$  and  $p'$ . Thus, in cross-section,

$$\frac{d^{cs}}{dk} [c_z - p'(X)z] = 0,$$

which implies

$$\frac{d^{cs}}{dk} z = \frac{-c_{zk}}{c_{zz} - p'} \equiv \delta(z, k).$$

Now we must totally differentiate  $\delta$  with respect to  $k$ , remembering that industrywide variables, such as  $p'(X)$  and  $X$  itself, are constants in this differentiation. Since

$$\frac{d^{cs}}{dk} \delta = \frac{\partial \delta}{\partial k} + \frac{\partial \delta}{\partial z} \frac{d^{cs}}{dk} z,$$

we obtain

$$\frac{d^{cs}}{dk} \delta = \frac{\partial \delta}{\partial k} + \delta \frac{\partial \delta}{\partial z}.$$

Now by definition,  $\frac{d^{cs}}{dk} c_k = c_{kk} + \delta c_{zk} = c_{kk} - c_{zk}^2 / (c_{zz} - p')$ , which can be written as  $c_{kk} - (c_{zk}^2 / c_{zz}) \theta$ , where  $\theta \equiv c_{zz} / (c_{zz} - p') \in (0, 1)$ . Since  $c_{zz} > 0$ ,  $\frac{d^{cs}}{dk} c_k > c_{kk} - c_{zk}^2 / c_{zz}$  which was just shown to equal zero.

by the quantity in parentheses in (14). Since  $\delta_1$  is the direct or “first-round” change in firm 1’s output  $x_1$ , and since  $-p'(X)x_1$  is firm 1’s markup (the social value of increases in its output), the  $\delta_1 x_1$  term captures the welfare benefit of firm 1’s first-round output increase due to its new capital; this of course must be balanced against the  $-\delta_2 x_2$  term. These are the merely the first-round output effects. The term involving  $\sum_{i=1}^n \lambda_i x_i$  measures the induced output effects as the system restores equilibrium: firm  $i$ ’s equilibrating output change is  $-\lambda_i(\delta_1 - \delta_2)/(1 + \Lambda)$ , and its margin is  $-p'(X)x_i$ . If the merger reduces output, then  $\delta_1 < \delta_2$ , so this term’s contribution is positive.

Conventional merger analysis emphasizes one harmful output effect: total output often falls. Our analysis identifies another important output effect: output is redistributed across firms, as firm 1 increases output, firm 2 reduces output by more, and other firms increase output in response to the net output reduction. Because marginal costs differ among firms, this redistribution of output has welfare consequences, which can be favorable. That is, the term in parentheses in equation (14) can be positive. Recalling equation (5), this term is equal to  $HX(dX/dk) + \frac{1}{2}X^2 dH/dk$ . Even if  $dX/dk < 0$ , welfare increases if  $H$  rises by enough. Although initially large values of  $H$  make output-reducing mergers less desirable (since the  $dX/dk$  term is multiplied by  $H$ ), large *increases* in  $H$  make mergers *more* desirable.<sup>20</sup> This is potentially important for merger policy: traditional policy, including the Merger Guidelines, sees increases in  $H$  as bad, especially if  $H$  is already high. Our model suggests the opposite: while a high value of  $H$  may be bad, increases in  $H$  may be associated with increased performance.

The ambiguity of some of these considerations suggests that increasing the concentration of capital can raise welfare, even if it raises price. For an example, consider any capital transfer that has a favorable direct cost effect and raises price. Then equation (14) shows that this capital transfer raises welfare if  $x_1 \delta_1 > x_2 \delta_2$ . Since this condition is met if firm 2 is sufficiently small, we find that any acquisition with favorable direct cost effects involving a sufficiently small target firm raises welfare if it raises price!

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<sup>20</sup> Of course, the two may go together.

### *Effect of a Merger on Participants' Profits*

The capital transfer's effect on the merging firms' joint profits  $\pi \equiv \pi_1 + \pi_2$  is

$$\frac{d\pi}{dk} = -p'(X)(\delta_1 x_1 - \delta_2 x_2 - \frac{\delta_1 - \delta_2}{1 + \Lambda}((1 + \lambda_1)x_1 + (1 + \lambda_2)x_2)) - c_k^1 + c_k^2. \quad (15)$$

Again, the  $\delta_i x_i$  terms reflect the direct output effects of the altered capital stocks, the  $\delta_1 - \delta_2$  term captures the induced output effects after accounting for all firms' output adjustments, and the final term measures the direct cost effects.

Our focus here is on capital transfers that increase concentration, so we do not explore all possible transfers or ask what configuration of capital will be an equilibrium in the sense that no further transfers are profitable.<sup>21</sup> We restrict attention instead to transfers of capital from a smaller firm to a larger, that raise their joint profits.<sup>22</sup>

### *Net Externality from a Capital Transfer*

As we suggest in the Introduction, it is valuable to measure the net externality associated with a merger, since merger policy should look for conditions under which a merger is socially desirable conditional on its private profitability. For example, it seems plausible that certain kinds of "synergies" internal to the merging firms may be both important and hard for an antitrust agency to observe, while the external effects due to price changes are more likely to be predictable and measurable. Suppose that, in addition to the known variable costs  $c^i(x_i, k_i)$ , there are fixed costs  $F_i(k_i)$  that the antitrust agency cannot observe. Then our formulae for the change in the merging firms' joint profits, and for the change in welfare, are incomplete, since each lacks the term  $\Delta F \equiv F_1'(k_1) - F_2'(k_2)$ , but our expression for the net externality is correct and is the proper basis for policy.

Combining (14) and (15) and using (12), we get an expression for the net externality, or wedge, from a small transfer of capital from firm 2 to the larger firm 1, as

$$\frac{dW}{dk} - \frac{d\pi}{dk} = -\frac{dp}{dk}(x_1 + x_2 - \sum_{i=3}^n \lambda_i x_i). \quad (16)$$

<sup>21</sup> Absent diseconomies of scale or antitrust enforcement, the accumulation of all capital by a single firm would clearly be the only such configuration.

<sup>22</sup> If decreasing the concentration of capital raises the firms' joint profits, we might see divestitures rather than mergers.

In the “normal” case where the capital transfer raises price (see Propositions 2 and 3), the net externality has the sign of

$$\eta \equiv \sum_{i=3}^n \lambda_i x_i - (x_1 + x_2). \quad (17)$$

**Proposition 4.** *Suppose that a small transfer of capital from firm 2 to firm 1 would raise the market price. Then firms 1 and 2 have excessive incentives to transfer the capital if and only if*

$$s_1 + s_2 > \sum_{i=3}^n \lambda_i s_i.$$

Proposition 4 tells us that capital transfers have positive external effects when the participants’ market shares are small compared to a weighted sum of other firms’ market shares.<sup>23</sup> In (17), the  $\sum_{i=3}^n \lambda_i x_i$  term reflects the welfare benefits of rivals’ increased production in response to the merging firms’ contraction, and the  $x_1 + x_2$  term reflects the participants’ transfer gains as the price rises. Intuitively, the participants’ anticompetitive incentive to raise price is related to their combined size, while the socially beneficial effect (apart from direct cost effects, which are internalized) is related to their rivals’ increases in output and the corresponding markups. This “induced output effect” is too often ignored.

To see what kinds of firms are most likely to have excessive incentives to merge, we re-write (17) as

$$\eta = \sum_{i=1}^n \lambda_i x_i - (1 + \lambda_1)x_1 - (1 + \lambda_2)x_2.$$

From this expression we see that it is large firms that are most likely to have excessive incentives to merge, provided that  $x(1 + \lambda)$  is increasing in cross-section.

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<sup>23</sup> Of course, all the comparisons go the other way when a merger *lowers* price, which (as we note above) is perfectly possible. However, we think of price increases as the normal case.

### Quadratic Costs

We showed above that with quadratic costs mergers always reduce output. Despite this, and although there are no economies of scale, mergers can indeed raise welfare by causing a favorable redistribution of output across firms.

In the quadratic case, substituting for  $\lambda_i$  and  $\delta_i$  into the general expression for  $dW/dk$ , equation (14) gives

$$\frac{1}{p^2} \frac{dW}{dk} = \left( \frac{1}{(k_2 + 1)^2} - \frac{1}{(k_1 + 1)^2} \right) \left( \frac{H\Lambda^2}{1 + \Lambda} - \frac{1}{2} \right) + \frac{k_1}{(k_1 + 1)^3} - \frac{k_2}{(k_2 + 1)^3} \quad (18)$$

It is easy to construct numerical examples where (18) is positive: that is, where mergers increase welfare. McAfee and Williams (1988) report numerical simulations (for this example) showing that welfare rises if the participants' market shares are small.

Increases in the *industrywide* variables,  $H$  and  $\Lambda$ , increase (18). Thus, we have

**Proposition 5.** *Consider the welfare effect of increasing capital concentration in Cournot oligopoly with linear demand and quadratic cost functions. Given the sizes of the participant firms,  $k_1$  and  $k_2$ , the capital transfer is more likely to increase welfare the greater is the total adjusted industry capital stock,  $\Lambda$ , and the greater is the industry's concentration, as measured by the Herfindahl Index,  $H$ .*

While a higher value of  $H$  may be bad in itself, it offers the greatest chance of a positive  $dW/dk$ , since a concentrated market means that the welfare benefits of expansion by the large non-participant firms are greatest. Again, this casts doubt on the wisdom of the Merger Guidelines' assumption that mergers in already-concentrated industries are the most likely to be harmful.

Now we ask how industrywide conditions affect the profitability of a merger.<sup>24</sup> Calculations using equation (15) give

$$\frac{1}{p^2} \frac{d\pi}{dk} = \left( \frac{1}{(k_2 + 1)^2} - \frac{1}{(k_1 + 1)^2} \right) \left( \frac{\left( \frac{k_1}{k_1 + 1} \right)^2 + \left( \frac{k_2}{k_2 + 1} \right)^2}{1 + \Lambda} - \frac{1}{2} \right) + \frac{k_1}{(k_1 + 1)^3} - \frac{k_2}{(k_2 + 1)^3}.$$

<sup>24</sup> See McAfee and Williams (1988) for simulation results indicating which mergers are profitable.

**Proposition 6.** *With quadratic costs and linear demand, a merger by one pair of firms increases the profitability of merger for any other pair of firms.*

*Proof.* Industrywide conditions affect  $d\pi/dk$  only through  $p$  and  $1 + \Lambda$ . A merger between two other firms, say 3 and 4, raises price and reduces  $\Lambda$ , both of which increase  $d\pi/dk$ . ■

Proposition 6 shows the possibility of bandwagons or merger waves in an industry. If one pair of firms merges, that may make merger become profitable for other pairs as well.<sup>25</sup>

Finally, consider the net externality imposed by merger. Since  $\lambda_i$  is proportional to  $x_i$ , we know that  $\lambda$  increases in the cross section, and that large firms are the ones most likely to have excessive incentives to merge. Substituting for  $\lambda_i$  into expression (17), we have

**Proposition 7.** *In the case of quadratic costs and linear demand, firms 1 and 2 have excessive incentives to increase the concentration of capital if and only if*

$$s_1 + s_2 > \frac{1}{\epsilon^D} \sum_{i=3}^n s_i^2. \quad (19)$$

Two factors thus determine whether firms' incentives to transfer capital are excessive or insufficient: the degree of concentration of the *rest of the industry*, and the combined market shares of the participant firms. Propositions 5 & 7 tell us that *given the participants' sizes, the more concentrated is the rest of the industry, the more likely is the capital transfer to raise welfare and to create net positive effects on consumers and rivals*, because of the induced output effect. If demand is very elastic, however, then markups are small and these induced output effects contribute relatively little to welfare.

### *Constant Marginal Costs*

In our example with constant marginal costs,  $\delta_i = \phi'(k_i)/p'$  and  $\lambda_i = 1 - s_i E$ . Now  $x_i(1 + \lambda_i) = Xs_i(2 - Es_i)$  increases in cross-section with  $x_i$ , for  $s_i \leq 1/E$ . Thus if  $\phi'' > 0$ , large firms are again the ones most likely to have excessive incentives to merge.<sup>26</sup>

<sup>25</sup> The first pair might account for this effect, and that would increase the first pair's incentive to merge.

<sup>26</sup> If  $\phi'' < 0$ , however, it is small firms who are too eager to merge, for they cause output to be shifted away from their more efficient rivals.



Substituting for  $\lambda_i$  and  $\delta_i$  into equation (14), we find that a small transfer of capital from firm 2 to firm 1 raises welfare if and only if

$$(-\phi'(k_1))(s_1 - \sigma) > (-\phi'(k_2))(s_2 - \sigma),$$

where  $\sigma = \frac{1 - EH}{2(1 + n - E)}$ .

Suppose that  $s_1 > \sigma$ .<sup>27</sup> Then the capital transfer increases welfare if and only if

$$\frac{-\phi'(k_1)}{-\phi'(k_2)} > \frac{s_2 - \sigma}{s_1 - \sigma}.$$

This condition is satisfied if  $\phi'' < 0$ : if different pieces of know-how are complements, then the efficiency gains from increasing the concentration of capital overwhelm any anticompetitive losses. If  $\phi'' > 0$ , we must compare the marginal cost effects at the two firms to their market shares. If firm 1 is much larger, or if its unit costs fall nearly as much as firm 2's rise, then the capital transfer raises welfare.

Finally, substituting into (16), we find that

$$\frac{dW}{dk} - \frac{d\pi}{dk} = X \frac{dp}{dk} (1 - 2(s_1 + s_2) - E \sum_{i=3}^n s_i^2).$$

In the natural case where  $\phi'' > 0$  and  $E > 0$ , increasing the concentration at nonparticipating firms makes it less likely that the capital transfer will generate net positive externalities.

## 8. Small Capital Transfers and Large Mergers

We have analyzed the effects of a small capital transfer in order to understand the effects on conduct and performance as capital becomes more concentrated. We now show how our analysis can be used to study large mergers. As above, we focus on the net externality associated with the merger.

We begin with the case where the smaller firm remains active until its last unit of capital is transferred. A sufficient condition for this is that  $c_x(0, k) = 0$  for all  $k > 0$ .<sup>28</sup> The effect

<sup>27</sup> If  $E > 0$ , then  $\sigma < 1/(2n)$  follows from the fact that  $H \geq 1/n$ . In this case,  $s_1 > \sigma$  is clearly the case of interest. And note that if demand is isoelastic,  $E = 1 + 1/\epsilon^D$ , so  $E$  in fact exceeds unity.

<sup>28</sup> This condition is met by the quadratic cost function introduced earlier.

of a large merger is found by integrating our expressions for small capital transfers as the two firms' capital stocks go from  $(k_1, k_2)$  to  $(k_1 + k_2, 0)$ . We are interested in what we can say about this integral in terms of observable variables.

The expression  $\eta$  measuring the sign of the net externality, given by (17), changes as firm 2 transfers capital to firm 1. We argue that, plausibly,  $\eta$  *increases* as the participants' capital becomes more concentrated. If so, this implies that if  $\eta \geq 0$  before a merger, then *any* privately profitable merger (partial or total) between firms 1 and 2 should be approved, indeed encouraged.<sup>29</sup>

When  $dk$  is transferred from firm 2 to firm 1, we have shown (Proposition 3) that industry output generally falls, and that this consists of a fall in firms 1 and 2's joint output and an increase in other firms'. We could infer that  $\eta$  increases as capital is shifted to the larger firm, except for the fact that the  $\lambda_i$ , ( $i > 2$ ) may change.<sup>30</sup> If the  $\lambda_i$  increase, this only strengthens the result. So the only possibility that we must rule out is that the  $\lambda_i$  fall enough to overcome the increase in the  $x_i$  and the fall in  $x_1 + x_2$ .

**Proposition 8.** *Provided that  $d[\lambda_i x_i]/dk \geq 0$  for  $i > 2$ , any privately profitable merger that raises price also raises welfare if  $\eta \geq 0$  at the pre-merger equilibrium.*

More formally, we can calculate the effect of a capital transfer on  $\eta$ :

$$\begin{aligned} \frac{d\eta}{dk} &= \sum_{i=3}^n \lambda_i \frac{dx_i}{dk} + \sum_{i=3}^n x_i \frac{d\lambda_i}{dk} - \frac{d(x_1 + x_2)}{dk} \\ &= \left(-\frac{dX}{dk}\right) \left(1 + \sum_{i=3}^n \lambda_i (\lambda_i + 1)\right) + \sum_{i=3}^n x_i \frac{d\lambda_i}{dk}. \end{aligned}$$

If the merger reduces output, so  $dX/dk < 0$ , then  $d\eta/dk > 0$ , provided that the  $d\lambda_i/dk$  are not too negative. That is, so long as the nonparticipant firms do not become much less sensitive to price as they expand, the net external benefits generated by capital transfers only become more likely to be positive as capital becomes more concentrated.

<sup>29</sup> The net externality from the complete merger is the integral of (16), which is positive if  $\eta$  starts out positive and grows.

<sup>30</sup> As we shall see below, in the quadratic case the  $\lambda_i$  are unaffected by the merger.

What if, in the pre-merger equilibrium,  $\eta < 0$ ? Then a small capital transfer imposes net costs on rivals and consumers collectively, but still may raise welfare. One response is to require to prove that their gain from the capital transfer exceeds the net loss imposed on others. But the net externality from the *complete merger* may be positive, even if  $\eta < 0$  for the first unit of capital transferred. If the condition in Proposition 8 is met, then  $\eta$  increases as capital is transferred, so it may begin negative but become positive after some capital is transferred. Thus if  $\eta$  is negative but small, and especially if the merger is a large one, the complete merger will have net positive effects on rivals and consumers, and is therefore socially desirable if it is privately profitable.

Finally, consider the case where a firm with a small amount of capital would close down. Then we must view the capital transfer beyond that point as investment (see Section 4). We can still integrate (16) from  $(k_1, k_2)$  to  $(k_1 + k_2, 0)$ , but we must recognize that  $dp/dk > 0$  (typically) until the point where firm 2 closes down, after which  $dp/dk < 0$ : capital transfers raise price, but investment lowers price. Even if  $\eta$  is positive until firm 2 closes down, the merging parties may have excessive incentives to merge if firm 1 then has excessive incentives to invest (Proposition 1 gives conditions for this).

### *Quadratic Costs*

In the quadratic case,  $\lambda_i = x_i/p$ , so  $\lambda_i$  is increasing in the cross-section. However, since  $\lambda_i = k_i/(1 + k_i)$ ,  $\lambda_i$  ( $i > 2$ ) is unaffected by merger of firms 1 and 2. Hence  $\eta$  can only increase during the course of a merger, and so any privately profitable merger such that  $\eta \geq 0$  at the pre-merger equilibrium raises welfare. That is, any proposed merger for which (19) is violated in the pre-merger equilibrium should be approved.

Our policy conclusions are less clear when (19) holds ( $\eta < 0$ ) in the pre-merger equilibrium. But if (19) holds even at the post-merger equilibrium, then the net externality from the merger is certainly negative.

### Constant Marginal Costs With $\phi'' > 0$

For illustration, suppose that  $E$  is a positive constant.<sup>31</sup> As a merger proceeds, the change in  $\eta$  is given by

$$\frac{d\eta}{dk} = \left(-\frac{dX}{dk}\right) \left(1 + \sum_{i=3}^n (1 - Es_i)(2 - Es_i)\right) + \sum_{i=3}^n x_i \left(1 - E \frac{ds_i}{dk}\right)$$

and calculations show that this is positive at least for values of  $E$  between 1 and 2, corresponding to constant demand elasticities greater than 1. In these cases, we again have a one-way test: any privately profitable merger such that  $\eta \geq 0$  in the pre-merger equilibrium raises welfare.

## 9. Conclusion

We have studied the welfare and profit effects of horizontal mergers, viewing these corporate transactions as affecting industry structure by increasing the concentration of industry-specific capital. We learned a number of lessons regarding merger policy:

First, an important welfare effect of a horizontal merger comes from the "induced output effect:" the response of *non-participant* firms to any output reduction by the merging parties. If non-participant firms with large markups expand their outputs noticeably in response to the merger, these responses can provide a significant and favorable welfare effect. In Cournot oligopoly, markups are proportional to market shares, so the responses by large non-participant firms are especially important. These responses appear to have been neglected in previous analysis of horizontal mergers.

In particular, conventional merger policy, based on the desire not to increase measures of concentration such as the Herfindahl index when they are already high, is often misleading. In fact, for any given change in output, welfare rises by more, the larger the increase in  $H$ , properly computed. The general point is that the distribution of outputs across firms, as well as the aggregate output level, is an important aspect of industry performance.

If larger firms have lower marginal costs, as in Cournot oligopoly, then shifting output towards them improves performance. Mergers may raise welfare through such output-shifting effects, even if they lower aggregate output, raise the Herfindahl index, and afford

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<sup>31</sup> This includes linear and constant-elasticity demand functions.

no direct cost savings! The output-shifting effect is larger if nonparticipant firms are larger and if they are more responsive to price increases.

On the other hand, if non-participant firms *reduce* their outputs along with the merging parties, then the merger will surely generate negative net external effects, and may well lower welfare even though it is profitable. This case is most relevant if the merger makes collusion more likely, or if the oligopolists compete in price among differentiated products.<sup>32</sup> Our most robust point, then, is that it is crucial for merger analysis to consider the likely responses of rival firms.

Although most of our analysis assumed Cournot behavior, the welfare importance of large non-participant firms' responses to a merger is far more general. Under any theory of oligopoly behavior in which larger firms exert more monopoly power and hence have lower marginal revenue, these firms will have lower marginal costs in equilibrium, and their responses will be most pivotal in the welfare analysis.

For the Cournot case, we developed concise and easily-interpreted formulae for the net externality, *i.e.*, the net effect on rivals and on consumers, caused by additions to the capital stock (investment) or by movements of capital from one firm to another (mergers). When mergers reduce output, a merger is more likely to generate negative externalities, the larger is the combined market share of the participants, and the smaller is the weighted sum of the nonparticipant firms' market shares, where the weights (the  $\lambda_i$ ) measure how responsive the firms are to price increases.

We also examined firms' incentives to invest unilaterally in industry-specific capital. We found that small, inefficient firms typically have excessive incentives to invest, while large firms have insufficient incentives to do so. These results suggest, then, if the Cournot model is believed, that large firms should generally be free to invest in new capital but that their merger plans should be subject to careful scrutiny; small firms should generally be allowed to merge, but their new-investment plans should be scrutinized!

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<sup>32</sup> Deneckere and Davidson (1985) showed that rivals behave *less* competitively after a merger in industries where firms sell differentiated products and set prices. In such a setting, when the merging parties raise their prices, other firms respond in kind, since price-reaction schedules slope upward. These induced price increases redound to the further benefit of the participant firms, so mergers are always profitable in this context, but are never socially desirable.

We believe our approach offers a clear improvement in providing a coherent application of oligopoly theory to the study of horizontal mergers. Mergers must be studied as the transfer of capital, recognizing that such structural changes will affect the outputs of participating firms and of their rivals.

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