Title
MASS TRANSFER TO A ROTATING SPHERE AT HIGH SCHMIDT NUMBERS

Author
Newman, John.

Publication Date
1971-06-01

Submitted to Journal of
Electrochemical Society

UCRL-20599
Preprint c.2

MASS TRANSFER TO A ROTATING SPHERE AT HIGH SCHMIDT NUMBERS

John Newman

June 1971

AEC Contract No. W-7405-eng-48

LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

Mass Transfer to a Rotating Sphere at High Schmidt Numbers

John Newman

Inorganic Materials Research Division,
Lawrence Radiation Laboratory, and
Department of Chemical Engineering,
University of California, Berkeley

June, 1971

Abstract

The Lighthill transformation is used to calculate limiting rates of mass transfer to a rotating sphere on the basis of shear-stress calculations found in the literature.

Key words: current distribution, limiting current

Chin has recently[1] calculated rates of mass transfer to a rotating sphere on the basis of calculated velocity profiles from the literature. Near the poles of the sphere, the situation is essentially identical to a rotating disk. Chin has calculated also the next term in an expansion for small values of θ, the angle from the pole. He takes the Schmidt number to be large, a good approximation for electrolytic solutions.

This is an excellent situation in which to use the Lighthill transformation[2-4] to calculate the mass-transfer rate at angles θ which need not be small. For high Schmidt numbers, the diffusion layer is much thinner than the hydrodynamic boundary layer, and the shear stress or the velocity derivative at the wall

$$\beta = \left.\frac{\partial v_\theta}{\partial r}\right|_{r=r_o} \tag{1}$$

provides sufficient information about the velocity profiles to permit the calculation of a first approximation to the mass-transfer rate. For axisymmetric bodies, this result takes the form[4]

$$i_n = \frac{nFD_i c_\infty \sqrt{\mathcal{R}\beta}}{s_i \Gamma(4/3)} \Bigg/ \left[9D_i \int_0^x \mathcal{R}\sqrt{\mathcal{R}\beta}\, dx \right]^{1/3} , \tag{2}$$

where x is the distance along the electrode from its upstream end, $\mathcal{R}(x)$ is the normal distance of the surface from the axis of symmetry, and s_i is the stoichiometric coefficient of the limiting reactant in the electrode reaction

$$\sum_i s_i M_i^{z_i} \rightarrow ne^- \qquad . \tag{3}$$

This result applies at high Schmidt numbers to the limiting current density i_n when the effect of ionic migration on the limiting reactant can be ignored. Thus, the concentration of the limiting reactant is zero at the electrode surface and equal to c_∞ in the bulk solution outside the diffusion layer.

For a sphere,

$$\mathcal{R} = r_o \sin\theta \qquad \text{and} \qquad x = r_o\theta \qquad . \tag{4}$$

If we define a dimensionless shear-stress distribution $B(\theta)$ according to

$$B(\theta) = \beta\nu^{1/2}/r_o\Omega^{3/2} \qquad , \tag{5}$$

then equation 2 can be written as

$$\mathrm{Nu}(\theta) = \frac{2r_o s_i i_n}{nFD_i c_\infty} = \frac{2\sqrt{B\sin\theta}\ \mathrm{Re}^{1/2}\mathrm{Sc}^{1/3}}{\Gamma(\frac{4}{3})\left[9\int_0^\theta \sqrt{B\sin\theta}\,\sin\theta\,d\theta\right]^{1/3}} \qquad , \tag{6}$$

where $\mathrm{Nu}(\theta)$ is the local Nusselt number, $\mathrm{Re} = r_o^2\,\Omega/\nu$ is the Reynolds number, and $\mathrm{Sc} = \nu/D_i$ is the Schmidt number. For an electrode extending from the pole to the angle θ, we can define the average Nusselt number Nu_{avg} analogously in terms of the average current density. Appropriate integration of equation 6 yields

$$Nu_{avg} = \frac{3^{1/3} Re^{1/2} Sc^{1/3}}{\Gamma(4/3)(1-\cos\theta)} \left[\int_0^\theta \sqrt{B\sin\theta}\,\sin\theta\, d\theta\right]^{2/3} \quad . \tag{7}$$

Banks[5] has solved the boundary-layer equations for a rotating sphere and expresses the shear-stress distribution in an expansion for small θ:

$$B(\theta) = 0.51023\,\theta - 0.22129\,\theta^3 + 0.02071\,\theta^5 - 0.00189\,\theta^7 \;. \tag{8}$$

Manohar[6] has solved the boundary-layer equations numerically and gives a graph of the shear-stress distribution. His results should be better than those of Banks when the angle θ is not small.

On the basis of these results, we have expressed B as

$$B(\theta) = 0.51023\,\theta - 0.1808819\,\theta^3 - 0.040408\,\sin^3\theta \quad , \tag{9}$$

as shown in figure 1. For small values of θ, this reduces to

$$B(\theta) = 0.51023\,\theta - 0.22129\,\theta^3 + 0.020204\,\theta^5 - 0.00438\,\theta^7 + O(\theta^9) \tag{10}$$

and approximates well Banks's expression 8 for the first three terms. Figure 1 can be compared with the graph given by Manohar.[6]

The local and average Nusselt numbers are now obtained from equations 6 and 7 by carrying out the integration with $B(\theta)$ given by equation 9. The results are shown in figure 2 along with Chin's result for the local Nusselt number. Chin obtained correctly the first two terms for the local Nusselt number in an expansion for small values of θ. However, he predicts too high a mass-transfer rate near the equator of the sphere. Thus, his

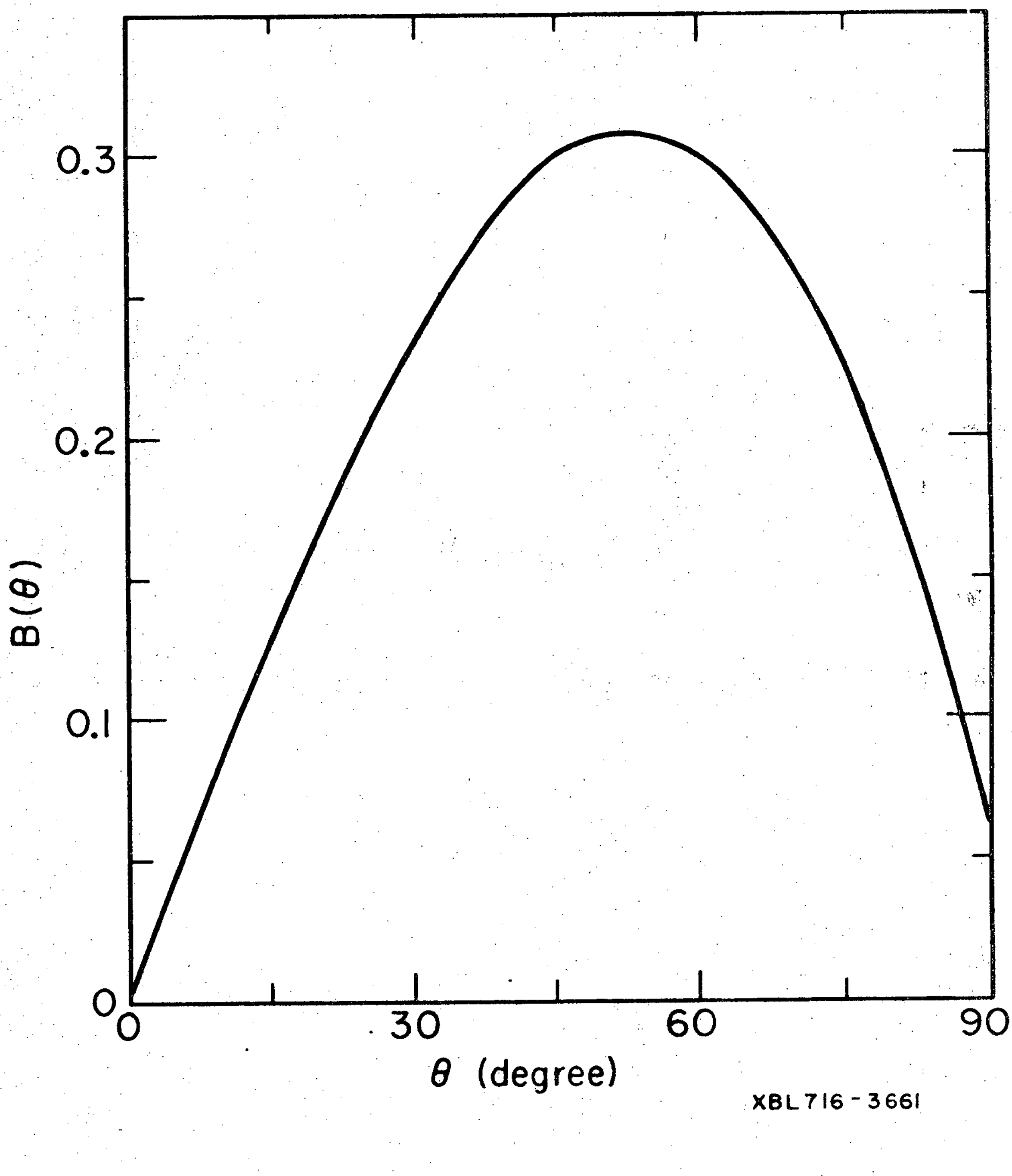

Figure 1. Dimensionless velocity derivative on the surface of a rotating sphere.

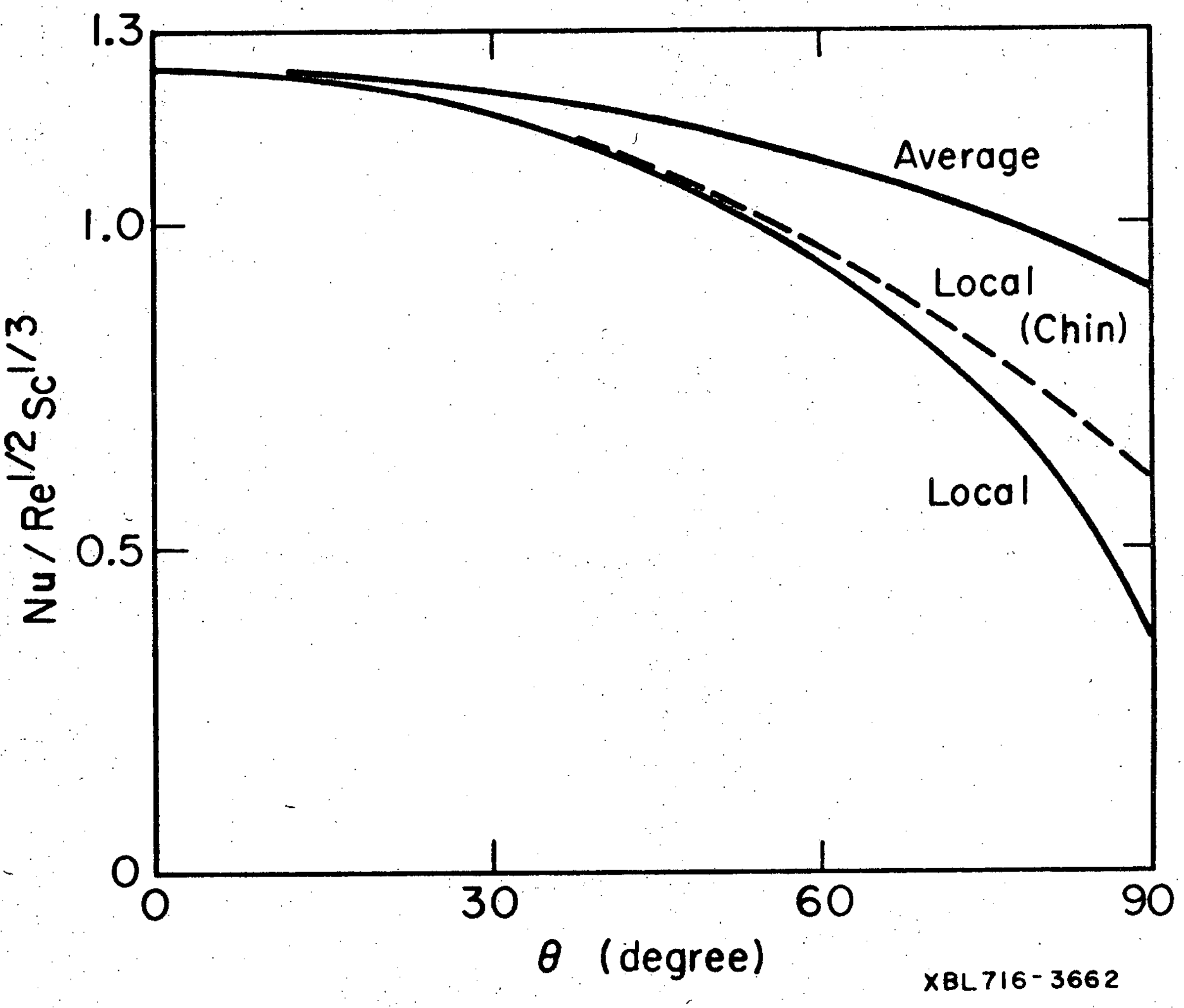

Figure 2. Local and average Nusselt numbers for a rotating sphere at high Schmidt numbers, calculated from the shear stress results of Manohar. The local Nusselt numbers obtained from Chin's analysis are shown for comparison.

value of 0.474 for $Nu_{avg}/2Re^{1/2}Sc^{1/3}$ for a sphere or a hemisphere should be replaced by the value 0.4508.

The boundary-layer results do not predict that the shear stress goes to zero at the equator, as it must. Thus, there must be a small region near the equator where B goes from the value predicted by boundary-layer theory to zero at the equator itself. This will lower the mass-transfer rates predicted for this region.

These results can be extended to metal deposition on a rotating sphere at the limiting current from a solution of a single electrolyte by redefining the local Nusselt number as

$$Nu(\theta) = \frac{(1-t_+)2r_o i_n}{z_+ F D c_\infty} \tag{11}$$

and replacing the Schmidt number by $Sc = \nu/D$, where D is the diffusion coefficient of the electrolyte and z_+ and t_+ are the charge number and transference number of the reacting cation.

Acknowledgment

This work was supported by the United States Atomic Energy Commission.

Nomenclature

B	dimensionless velocity derivative at the surface
c_∞	bulk concentration of the limiting reactant, mole/cm^3
D	diffusion coefficient of single electrolyte, cm^2/sec
D_i	diffusion coefficient of limiting reactant, cm^2/sec

e^-	symbol for the electron
F	Faraday's constant, 96,487 C/equiv
i_n	limiting current density, A/cm^2
M_i	symbol for the chemical formula of species i
n	number of electrons transferred in the electrode reaction
$Nu(\theta)$	local Nusselt number
Nu_{avg}	average Nusselt number
r	radial distance, cm
r_o	radius of sphere, cm
Re	Reynolds number
$\mathcal{R}$	normal distance of surface from axis of symmetry, cm
s_i	stoichiometric coefficient of species i in the electrode reaction
Sc	Schmidt number
t_+	cation transference number
v_θ	velocity in the θ-direction, cm/sec
x	distance along electrode from its upstream end, cm
z_i	charge number of species i
β	velocity derivative at the surface, sec^{-1}
$\Gamma(4/3)$	0.89298, the gamma function of 4/3
θ	angle from the pole of the sphere
ν	kinematic viscosity of the fluid, cm^2/sec
Ω	rotation speed of sphere, radian/sec

References

1. Der-Tau Chin. "Convective Diffusion on a Rotating Spherical Electrode." Journal of the Electrochemical Society, to be published.

2. M. J. Lighthill. "Contributions to the theory of heat transfer through a laminar boundary layer." Proceedings of the Royal Society, A202, 359-377 (1950).

3. Andreas Acrivos. "Solution of the Laminar Boundary Layer Energy Equation at High Prandtl Numbers." The Physics of Fluids, 3, 657-658 (1960).

4. John Newman. "Engineering Design of Electrochemical Systems." Industrial and Engineering Chemistry, 60, no. 4, 12-27 (April, 1968).

5. W. H. H. Banks. "The Boundary Layer on a Rotating Sphere." The Quarterly Journal of Mechanics and Applied Mathematics, 18, 443-454 (1965).

6. R. Manohar. "The Boundary Layer on a Rotating Sphere." Zeitschrift für angewandte Mathematik und Physik, 18, 320-330 (1967).

TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720