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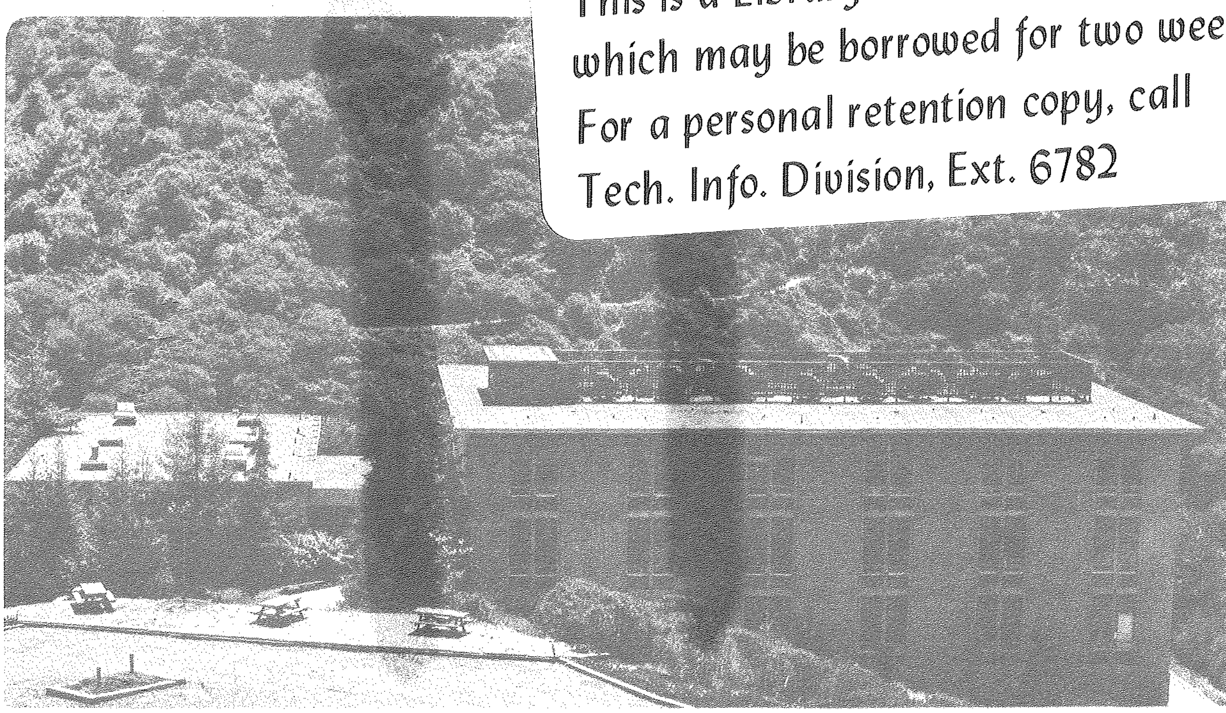

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BOUNDARY LAYER CONTROL IN PIPES THROUGH STRONG INJECTION

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ABSTRACT (ZUSAMMENFASSUNG)

In coal gasification, oxidation and sulfidization cause serious pipe corrosion. This paper attempts to determine the feasibility of reducing such corrosion by injecting steam at pipe entry to modify the boundary-layer gas composition along pipe walls. The injection will form a thin layer on the inner wall, preventing, for a time, contact with the corrosive gases. (Turbulence will eventually force diffusion through the protective layer.) The gas products are assumed to be hydrogen sulfide and steam. The Method of Integral Relations is used to obtain the numerical solutions to the governing equations. With several different injectant lengths and velocities, the concentration of H_2S along the pipe wall is calculated and is found low enough to prevent corrosion.

Section I: Introduction

In the production of synthetic natural gas (SNG) from coal, the environment in a gasifier contains hydrogen, water, carbon dioxide, carbon monoxide, methane and hydrogen sulfide (see Table 1). The operating temperature is about 1000°C in all three typical coal gasification processes. The material for all the metallic internal components of the gasifier must be corrosion resistant at temperatures ranging from 700°C to 1000°C. We shall take the pipe material to be 310 stainless steel, because this steel is used in existing coal gasifiers. Since hydrogen sulfide is the most corrosive gas and steam is the dominant gas in the pipe mixture, these two gases are considered to be the only fluid components in the mixture.

There are many ways to inject a protective layer of water along the inner side of pipes, but only two are considered here: (1) water is initially injected at the leading edge in a direction parallel to the axis of the pipe; (2) water is injected at the leading edge for a finite length L in a direction normal to the axis of the pipe. In both cases, the amount of injectant can be small enough to ensure that the contents of the mixture will not be changed greatly.

The effect of strong (i.e., non-viscous) injection on boundary layer characteristics has previously been investigated in [1] and [2]. These theories are adapted to the present problem, to determine the shape of the interface between the pipe flow and the injectant. Viscous diffusion about this interface is then calculated by extending the viscous mixing theory of Chapman [3].

Section 2: Parallel Injection

In this method, water is injected parallel to the axis of a straight circular pipe. To simulate the problem, a two-dimensional free shear model has been used (see Fig. 1). In the upper half-plane, the velocity of the gas mixture is 1524 cm/sec. In the lower half-plane, the velocity of the injectant is taken to be 75 cm/sec and the temperature is taken to be 300°C, which is the saturated water temperature at 68 atm pressure.

The following assumptions are made:

1) The pipe wall in the lower half-plane does not affect the boundary layer growth.
2) The flow is laminar and steady.
3) Pr = 1, where Pr = ν/α, α is the heat diffusivity, and ν is the kinematic viscosity of the gas mixture.
4) Le = 1, where Le = $\bar{D}/\alpha$ and $\bar{D}$ is the mass diffusion coefficient.
5) $\frac{\bar{\mu}}{\bar{\mu}_\infty} = C(\bar{T}/\bar{T}_\infty)^W$, where C = 1 and W = 1 in the subsonic case.

   The viscosity of the gas mixture behaves as $\bar{\mu}/\bar{\mu} \;\; = \bar{T}/\bar{T}_\infty$, where $\bar{\mu}$ is the dynamic viscosity and $\bar{T}$ is the temperature of the gas mixture, and the subscript   denotes the free stream value.
6) The specific heat coefficient, c_p, is constant.
7) The ideal gas law holds.

Governing Equations

Since the Reynolds number is high, the boundary layer equations can be applied:

$$\frac{\partial \bar{\rho}\ \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{\rho}\bar{v}}{\partial \bar{y}} = 0 \ , \qquad (2.1)$$

$$\bar{\rho}\bar{u}\ \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{\rho}\bar{v}\ \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}}\left(\bar{\mu}\ \frac{\partial \bar{u}}{\partial \bar{y}}\right) \ , \qquad (2.2)$$

$$\bar{\rho}\bar{u}\ \frac{\partial (c_p\bar{T})}{\partial \bar{x}} + \bar{\rho}\bar{v}\ \frac{\partial (c_p\bar{T})}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}}\ k\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right) \ ; \qquad (2.3)$$

$$\bar{\rho}\bar{u}\ \frac{\partial \bar{m}}{\partial \bar{x}} + \bar{\rho}\bar{v}\ \frac{\partial \bar{m}}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}}\left(\bar{\rho}\bar{D}\ \frac{\partial \bar{m}}{\partial \bar{y}}\right) \ . \qquad (2.4)$$

In the above, $\bar{u}$ and $\bar{v}$ are the velocity components in the $\bar{x}$ and $\bar{y}$ directions, respectively, $\bar{\rho}$ is the density, $\bar{m}$ is the mass concentration of each species, and k is the thermal conductivity. With the assumptions that Pr = Le = 1 and c_p = constant, the energy equation and species equation can be rewritten as:

$$\bar{\rho}\ \bar{u}\ \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{\rho}\bar{v}\ \frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}}\left(\bar{\mu}\ \frac{\partial \bar{T}}{\partial \bar{y}}\right) \ , \qquad (2.5a)$$

$$\bar{\rho}\bar{u}\ \frac{\partial \bar{m}}{\partial \bar{x}} + \bar{\rho}\bar{v}\ \frac{\partial \bar{m}}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}}\left(\bar{\mu}\ \frac{\partial \bar{m}}{\partial \bar{y}}\right) \ . \qquad (2.5.b)$$

Introduce the non-dimensional quantities:

$$u = \frac{\bar{u}}{\bar{U}_\infty} \ , \quad T = \frac{\bar{T}}{\bar{T}_\infty} \ , \quad x = \frac{\bar{x}}{\bar{L}} \ ,$$

$$\rho = \bar{\rho}/\bar{\rho}_\infty \ , \quad \mu = \bar{\mu}/\bar{\mu}_\infty \ , \quad m = \bar{m} \ , \qquad (2.6)$$

$$y = \frac{\bar{y}}{L}\left(\frac{\bar{U}_\infty L}{\nu_\infty}\right)^{1/2} \quad , \quad v = \frac{\bar{v}}{\bar{U}}\left(\frac{\bar{U}_\infty L}{\nu_\infty}\right)^{1/2} \quad ,$$

where L is a reference length. The governing equations can be written as:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \quad , \tag{2.7}$$

$$\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \quad , \tag{2.8}$$

$$\rho u\frac{\partial T}{\partial x} + \rho v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\mu\frac{\partial T}{\partial y}\right) \quad , \tag{2.9}$$

$$\rho u\frac{\partial m}{\partial x} + \rho v\frac{\partial m}{\partial y} = \frac{\partial}{\partial y}\left(\mu\frac{\partial m}{\partial y}\right) \quad . \tag{2.10}$$

The boundary conditions are:

$$\begin{aligned} &\text{at } y = \infty, \quad u = 1 \quad , \quad T = 1 \quad , \quad m = 0.02; \\ &\text{at } y = -\infty \quad , \quad u = 0.05 \quad , \quad T = 0.45 \quad , \quad m = 0 \quad . \end{aligned} \tag{2.11}$$

Under the assumption Pr = 1, when applied to the boundary layer equations (as pointed out in [4]), the energy equation can be decoupled from the other equations. Using Crocco's relation, T is a function of u only, and $T(u) = Au + B$, where A and B are constants determined from the boundary conditions. Similarly, the species equation can also be decoupled by the assumption Le = 1 and can be written as $m(u) = A'u + B'$, where A' and B' are constants determined by the boundary conditions.

Applying the boundary conditions (2.11), we get

$$T\ (u) = 0.5684\ u + 0.4316 \quad , \tag{2.12a}$$

$$m\ (u) = 0.0211\ u - 0.00105 \quad . \tag{2.12b}$$

Once the velocity profile has been determined, the temperature profile and mass concentration profile can also be solved.

To obtain the velocity profile, the continuity equation and momentum equation were solved by the Method of Integral Relations [5]. At first the equations were transformed into incompressible form, by the Stewartson transformation:

$$\begin{aligned} &\xi = cx, \quad c = 1 \quad \text{for the subsonic case;} \\ &\eta = \int_0^y \rho dy \quad , U = u \quad , \mu = \mu \quad , \\ &\rho = \rho \quad , \rho V = Vc - U(\partial\eta/\partial x)_y \quad . \end{aligned} \tag{2.13}$$

Since $\mu = T$, the ideal gas law gives $\rho T = 1$. The continuity equation and momentum equation become:

$$\frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial \eta} = 0 \quad , \tag{2.14}$$

$$U\frac{\partial U}{\partial \xi} + V\frac{\partial U}{\partial \eta} = \frac{\partial^2 U}{\partial \eta^2} \quad . \tag{2.15}$$

Let f(U) be a weighting function. Mutiply equation (2.14) by f(U) and equation (2.15) by f'(U), and add the result. We obtain:

$$\frac{\partial}{\partial \xi}(Uf) + \frac{\partial}{\partial \eta}(Vf) = f'\frac{\partial^2 U}{\partial \eta^2} \quad .$$

The equation is now solved separately in the two regions: one is in the upper half-plane, and the other in the lower half-plane.

For the upper half plane, we integrate with respect to η from 0 to ∞. We obtain:

$$\frac{\partial}{\partial \xi}\int_0^\infty fU d\eta + fV \Big|_{\eta=0}^{\eta=\infty} = \int_0^\infty f'(U)\,\frac{\partial^2 U}{\partial \eta^2}\,d\eta \quad . \tag{2.16}$$

Since $V = 0$ at $\eta = 0$, $U = 1$ at $\eta = \infty$, and $f(1) = 0$, the second term on the left-hand side of the above equation becomes zero. We change the variable of integration from η to U and integrate the term on the right-hand side of the equation by parts. We obtain:

$$\frac{\partial}{\partial \xi}\int_{U_0}^{1} \theta U f(U) dU = \frac{f'(U)}{\theta}\Big|_{U_0}^{1} - \int_{U_0}^{1} \frac{f''(U)}{\theta}\,dU \quad , \tag{2.17}$$

where

$$\theta = \frac{1}{\partial U/\partial \eta} \quad , \quad U_0 = U(\eta=0) \quad .$$

Since at $\eta=\infty$, $U=1$, $\frac{\partial U}{\partial \eta} = 0$, $\theta=\infty$, therefore

$$\theta \sim \frac{1}{1-U} \quad .$$

To avoid a singularity in the equation, we may choose

$$f(U) \sim (1-U) \quad , \quad (1-U)^2 \quad , \quad \ldots\ldots \quad , \quad (1-U)^n \quad , \quad \ldots\ldots$$

For the first approximation, we take $f(U) = 1 - U$ and $\theta = \frac{1-U_0}{1-U}\theta_0(\xi)$

where

$$\theta_0(\xi) = \theta(\xi,\eta)|_{\eta=0} \quad .$$

Substitution in equation (2.17) gives

$$\frac{\partial}{\partial\xi}\int_{U_0}^{1} (1-U_0)\theta_0\; UdU = \frac{f'(U)}{\theta}\Bigg|_{U_0}^{1} = \frac{1}{\theta_0} \quad .$$

Since the problem is similar, U_0 = constant, and

$$(1-U_0)\;(1-U_0^2)\;\frac{d\theta_0}{d\xi} = \frac{2}{\theta_0} \quad . \tag{2.18}$$

For the lower half-plane, we use coordinates ξ, $\hat{\eta}$, where $\hat{\eta} = -\eta$, and put a circumflex (^) above each variable. For the first approximation, we take:

$$\hat{f}(\hat{U}) = \hat{U} - 0.05 \quad , \quad \hat{\theta} = \frac{\hat{U}_0-0.05}{\hat{U}-0.05}\;\hat{\theta}_0(\xi) \quad .$$

Similar to the procedures for the upper half-plane, we obtain:

$$\frac{\partial}{\partial\xi}\int_{\hat{U}_0}^{0.05} (U_0-0.05)\hat{\theta}_0\hat{U}d\hat{U} = \frac{\hat{f}'(\hat{U})}{\hat{\theta}_0}\Bigg|_{\hat{U}_0}^{0.05} = \frac{-1}{\theta_0} \quad ,$$

$$(\hat{U}_0-0.05)(0.05^2-\hat{U}_0^2)\;\frac{d\hat{\theta}_0}{d\xi} = \frac{-2}{\hat{\theta}_0} \quad . \tag{2.19}$$

Applying the matching conditions at $\eta = \hat{\eta} = 0$, we obtain:

$$\theta_0 = -\hat{\theta}_0 \quad , \quad U_0 = \hat{U}_0 \quad , \quad \frac{d\theta_0}{d\xi} = -\frac{d\hat{\theta}_0}{d\xi}$$

$$(1-U_0)(1-U_0^2) = -(U_0-0.05)(0.05^2-U_0^2) \tag{2.20}$$

Solving numerically for U_0, we find $U_0 = 0.6274$. From similarity we have:

$$\theta_0 = \beta_0 \xi^{1/2} \quad ,$$

$$\hat{\theta}_0 = \hat{\beta}_0 \xi^{1/2} \quad ,$$

where β_0 and $\hat{\beta}_0$ are constants. Equation (2.18) then becomes:

$$(1-U_0)(1-U_0^2)\,\frac{1}{2}\,\beta_0 \xi^{-1/2} = \frac{2}{\beta_0 \xi^{1/2}} \quad ,$$

or

$$\beta_0 = 2/[(1-\bar{U}_0)\ (1-\bar{U}_0^2)]^{1/2}$$

Recalling

$$\eta = \int_0^y \rho dy \ , \quad \rho T = 1 \text{ and } U = u \quad ,$$

we have:

$$y = \int_0^y \frac{d\eta}{\rho} = \int_{U_0}^{U} \theta T dU \quad ,$$

$$y = \beta_0(1-U_0)\,\xi^{1/2} \int_{U_0}^{U} \frac{AU+B}{1-U}\, dU \quad ,$$

$$\frac{y}{\xi^{1/2}} = \bar{y}\left(\frac{\bar{U}_\infty}{\nu_\infty \bar{X}}\right)^{1/2} = 1.5678\ [0.5684\ U - \ell n(1-u) - 1.344] \quad .$$

Similarly, equation (2.20) becomes:

$$\hat{y} = \xi^{1/2}\ \hat{\beta}_0(U_0-0.05)\int_{U_0}^{U} \frac{AU+B}{U-0.05}\, dU \qquad (2.21)$$

or

$$\bar{y}\left(\frac{\bar{U}_\infty}{\nu_\infty \bar{X}}\right)^{1/2} = 2.4299\ [0.5684u + 0.45\ell n(u-0.05)-0.1095]\ . \qquad (2.22)$$

With the velocity profile known, the H_2S concentration profile can be found from equations (2.11) and (2.12). Same calculations were performed with injectant velocity equal to 500 cm/sec. Results are plotted in Figure 2 and Figure 3.

Define the hydrodynamic boundary layer thickness of the lower half-plane as that where the mass concentration of H_2S is 0.0002, which is 99% smaller than that in the coal gasification mixture. Outside the boundary layer, in the lower half-plane, the H_2S concentration is even smaller, and is considered to be low enough to prevent corrosion of the pipes. Then we can say that the pipes are free from corrosion from the entry to the point where the boundary layer thickness meets the wall.

For a large pipe diameter, for example, 90 cm, we can inject large amounts of steam. For a width of injection equal to 1 cm and an injection velocity equal to 75 cm/sec, we find that the boundary layer hits the wall at X = 19.4 meters. Therefore, the pipe is protected from corrosion for the initial length of 19.4 m. For an injection velocity equal to 500 cm/sec and a width of injection equal to 1 cm, we find that the initial 31.9 m of the pipe is protected from corrosion. For 90-cm-diameter pipes, Re is high and the flow is turbulent. The turbulent model will reduce the protected length considerably.

The usual diameter of pipes having serious corrosion in the coal gasification process is 5 cm. Since we want to inject less than 1% of volume of steam into the pipes, we choose an injection velocity of 500 cm/sec and width of injection of 0.3 cm, resulting in the initial 28.7 cm of the pipe being protected from corrosion.

Section III: NORMAL INJECTION

In this section, we discuss injecting water normal to the axis of a straight circular pipe. A uniform strong injection is applied over a finite entry length of the pipe. To simplify the problem, the circumferential effect is neglected. A two-dimensional model is used to simulate the problem (see Figure 4). It has been pointed out in [2] that, for strong injection, $V_0/u_\infty >> Re^{1/2}$, and the injectant layer is inviscid. The viscous effect is confined to a thin layer along the dividing streamline of the injectant and the coal gasification gas mixture. In the first part of this section, we find the dividing streamline, using an inviscid model. In the second part, we find the mass diffusion of H_2S across the dividing streamline by means of the Method of Integral Relations.

Inviscid Analysis

To find the dividing streamline, a model based on uniform flow over a line source is used to analyze the problem (see Figure 5). A line source of strength σ is located between $s = 0$ and $s = L$, where L is the injection length and s is measured along the streamline. The amount of water injected must be equal to the amount of water coming out from one side of the line source. So $\sigma = 2V_0$, where V_0 is injection velocity. By means of the complex variable $z = s + it$, where t is the transverse coordinate, the complex velocity is given by:

$$u(z) - iv(z) = U_\infty + \int_0^L \frac{\sigma}{2\pi} \frac{ds'}{(z-s')} \quad , \tag{3.1}$$

$$u(z) = \frac{\sigma}{2\pi}\int_0^L \frac{s-s'}{(s-s')^2+t^2}\, ds' + U_\infty \quad , \tag{3.2}$$

$$v(z) = \frac{\sigma}{2\pi}\int_0^L \frac{t}{(s-s')^2+t^2}\, ds' \quad . \tag{3.3}$$

The velocity components of the gas mixture on the dividing streamline are:

$$u(s,\delta) = \frac{V_0}{2\pi}\,\ell n\left[\frac{s^2+\delta^2}{(s-L)^2+\delta^2}\right] + U_\infty \quad , \tag{3.4}$$

$$v(s,\delta) = \frac{V_0}{\pi}\left(\tan^{-1}\frac{s}{\delta} - \tan^{-1}\frac{s-L}{\delta}\right) \quad , \tag{3.5}$$

where δ is the height of the dividing streamline.

Since $\frac{d\delta}{dS} = \frac{V(s,\delta)}{U(s,\delta)}$, we have:

$$\frac{d\delta}{ds} = \frac{\frac{V_0}{\pi}\left(\tan^{-1}\frac{S}{\delta} - \tan^{-1}\frac{S-L}{\delta}\right)}{\frac{V_0}{\pi}\,\ell n\left[\frac{S^2+\delta^2}{(S-L)^2+\delta^2}\right] + u_\infty} \quad . \tag{3.6}$$

We know that far downstream, the velocity is uniform and equal to U_∞. Hence by conservation of mass, we have $\delta = V_0 L/U_\infty$. With this as the initial condition, the dividing streamline, $\delta(s)$, can be found by integrating equation (3.6) backward along s. The velocity of the gas mixture U_1 along the dividing streamline is given by

$$U_1 = \left[u(s,\delta)^2 + v(s,\delta)^2\right]^{1/2} .$$

Across the dividing streamline, the pressure is continuous, but the velocity and the H_2S concentration are discontinuous. Applying Bernoulli's equation on the dividing streamline, we find the velocity of the injectant U_2 is:

$$U_2 = \left[V_0^2 + (\rho_\infty/\hat{\rho}_\infty)\, U_1^2\right]^{1/2} . \tag{3.7}$$

<u>Mixing Layer Analysis</u>

To solve for the mass diffusion across the dividing streamline, the following assumptions have been made:

1) Laminar flow.
2) Steady state.
3) Le = 1.
4) Pr = 1.
5) Since the concentration of H_2S is small in the mixture, we can assume the gas constant R and the specific heats c_p and c_v are all constant in the mixing layer.

Use Cartesian coordinates x and y; x is the distance along the dividing streamline and y is the distance normal to it. The basic governing boundary layer equations are as follows:

$$\frac{\partial}{\partial x}(\bar{\rho}\bar{u}) + \frac{\partial}{\partial y}(\bar{\delta}\bar{v}) = 0 \quad , \tag{3.8}$$

$$\bar{\rho}\bar{u}\,\frac{\partial \bar{u}}{\partial x} + \bar{\rho}\bar{n}\,\frac{\partial \bar{u}}{\partial y} = -\frac{dp_e}{dx} + \frac{\partial}{\partial y}\left(\bar{\mu}\,\frac{\partial \bar{u}}{\partial y}\right) \quad , \tag{3.9}$$

$$\bar{\rho}\bar{u}\,\frac{\partial}{\partial x}\left(c_pT + \frac{1}{2}\bar{u}^2\right) + \bar{\rho}\bar{v}\,\frac{\partial}{\partial y}\left(c_pT + \frac{1}{2}\bar{u}^2\right)$$

$$= \frac{\partial}{\partial y}\left[\bar{\mu}\,\frac{\partial}{\partial y}\left(c_pT + \frac{1}{2}\bar{u}^2\right)\right] \quad , \tag{3.10}$$

$$\bar{\rho}\bar{u}\,\frac{\partial \bar{m}}{\partial x} + \bar{\rho}\bar{v}\,\frac{\partial \bar{m}}{\partial y} = \frac{\partial}{\partial y}\left(\bar{\rho}D\,\frac{\partial \bar{m}}{\partial y}\right) \quad , \tag{3.11}$$

where

$$Pe = \bar{\rho}_\infty R\bar{T} \quad .$$

The boundary conditions are:

$$\bar{u} \longrightarrow U_1 \text{ as } y \longrightarrow \infty \quad ,$$

$$\bar{u} \longrightarrow U_2 \text{ as } y \longrightarrow -\infty \quad .$$

We solve the above equations by the Method of Integral Relations. We solve separately the region above the dividing streamline and the region below it, using the matching conditions along it as boundary conditions.

For the upper region, apply the Dorodnitsyn transformation:

$$\xi = \frac{1}{V_m L}\int_0^x \frac{P_e(x)}{P_S}\,U_1(x)dx \quad ,$$

$$\eta = \frac{U_1(x)}{\left(\nu_S V_m L\right)^{1/2}} \int_0^y \frac{\bar{\rho}}{\bar{\rho}_s} \, dy \quad ,$$

$$v = \frac{V_m L}{U_1} \frac{u \frac{\partial \eta}{\partial x}}{P_e/P_s} + \frac{\left(V_m L\right)^{1/2}}{U_1 \left(\nu_s\right)^{1/2}} \bar{v} \frac{\bar{T}_s}{T} \quad ,$$

$$u = \frac{\bar{u}}{U_1} \quad , \quad 1-h = \frac{c_p \bar{T} + \frac{1}{2} \bar{u}^2}{c_p \bar{T}_s} \quad , \quad m = \bar{m} - \bar{m}_\infty \ . \qquad (3.12)$$

Here subscript "s" represents quantities at the stagnation point and $\frac{1}{2} V^2_m = c_p \bar{T}_s$, while L is some reference length.

The resulting governing equations become:

$$\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \eta} = 0 \quad , \qquad (3.13)$$

$$u \frac{\partial u}{\partial \xi} + v \frac{\partial u}{\partial \eta} = \beta(1-h-u^2) + \frac{\partial^2 u}{\partial \eta^2} \quad , \qquad (3.14)$$

$$u \frac{\partial h}{\partial \xi} + v \frac{\partial h}{\partial \eta} = \frac{\partial^2 h}{\partial \eta^2} \quad , \qquad (3.15)$$

$$u \frac{\partial m}{\partial \xi} + v \frac{\partial m}{\partial \eta} = \frac{\partial^2 m}{\partial \eta^2} \quad , \qquad (3.16)$$

where

$$\beta = \frac{\alpha_e}{\alpha_e(1-\alpha_e)} \quad , \quad \alpha_e = \frac{U_1(x)}{V_m} \ .$$

Let f(u) and g(u) be weighting functions. Multiply equation (3.13) by f(u) and equation (3.14) by f'(u), and add the result. We obtain:

$$\frac{\partial}{\partial\xi}(uf) + \frac{\partial}{\partial\eta}(fv) = \beta f'(1-h-u^2) + f' \frac{\partial^2 u}{\partial\eta^2} \; . \qquad (3.17)$$

Multiply equation (3.13) by hg(u), equation (3.14) by hg'(u), equation (3.15) by g(u), and add the result. We obtain:

$$\frac{\partial}{\partial\xi}(hug) + \frac{\partial}{\partial\eta}(hvg) = \beta(1-h-u^2)hg'(u) + \frac{\partial^2 u}{\partial\eta^2} hg'(u) + g \frac{\partial^2 h}{\partial\eta^2} \; . \qquad (3.18)$$

Integrate the above three equations with respect to η from 0 to ∞ and then change the variable of integration from η to u. We get:

$$\frac{\partial}{\partial\xi}\int_{u_0}^{1} \theta u f(u)du = \beta\int_{u_0}^{1} f'(u)(1-u^2)\theta du - \beta\int_{u_0}^{1} f'(u)\theta h du + \left.\frac{f'(u)}{\theta}\right|_{u_0}^{1}$$
$$-\int_{u_0}^{1} \frac{f''(u)}{\theta} du - fv\Big|_{\eta=0}^{\eta=\infty} \; , \qquad (3.20)$$

$$\frac{\partial}{\partial\xi}\int_{u_0}^{1} hug\theta du = \beta\int_{u_0}^{1} (1-u^2)hg'\theta du - \beta\int_{u_0}^{1} h^2 g'\theta du + \left.\frac{1}{\theta}hg'\right|_{u_0}^{1}$$
$$-\int_{u_0}^{1}\frac{1}{\theta}\left(g''h+g'\frac{\partial h}{\partial u}\right)du + \left.\frac{g}{\theta}\frac{\partial h}{\partial u}\right|_{u_0}^{1} - \int_{u_0}^{1} g'\frac{\partial h}{\partial u}\frac{1}{\theta}du - hvg\Big|_{\eta=0}^{\eta=\infty} \; , \qquad (3.21)$$

$$\frac{\partial}{\partial\xi}\int_{u_0}^{1} mug\theta du = \beta\int_{u_0}^{1} (1-u^2)mg'\theta du - \beta\int_{u_0}^{1} mhg'\theta du + \left.\frac{1}{\theta} mg'\right|_{u_0}^{1} \qquad (3.22)$$
$$-\int_{u_0}^{1}\frac{1}{\theta}\left(g''m+g'\frac{\partial m}{\partial u}\right)du + \left.\frac{g}{\theta}\frac{\partial m}{\partial u}\right|_{u_0}^{1} - \int_{u_0}^{1} g'\frac{\partial m}{\partial u}\frac{1}{\theta}du - mvg\Big|_{\eta=0}^{\eta=\infty} \; ,$$

where

$$\Theta = \frac{1}{\partial u/\partial \eta} \quad , \qquad u_0 = u(\xi,\eta)_{\eta=0} \quad .$$

For the first approximation, we choose:

$$f(u) = 1-u \quad ,$$

$$g(u) = 1 \quad ,$$

$$\Theta = \frac{1-u_0}{1-u}\,\Theta_0(\xi) \quad ,$$

$$K = K_0\ (\xi) = h\Theta \quad ,$$

$$J = J_0\ (\xi) = m\Theta \quad .$$

Then equations (3.20), (3.21) and (3.22) respectively become:

$$\frac{\partial}{\partial \xi}\left[\frac{\Theta_0}{2}(1-u_0)(1-u_0^2)\right] = \beta(1-u_0)\Theta_0(u_0 + \frac{u_0^2}{2} - \frac{3}{2}) + \beta K_0\ (1-u_0) + \frac{1}{\Theta_0} \quad , \tag{3.23}$$

$$\frac{\partial}{\partial \xi}\left[J_0(1-u_0^2)\right] = \frac{2J_0}{\Theta_0^2(1-u_0)} \quad , \tag{3.24}$$

$$\frac{\partial}{\partial \xi}\left[K_0(1-u_0^2)\right] = \frac{2K_0}{\Theta_0^2(1-u_0)} \quad , \tag{3.25}$$

For the lower region, we put circumflex (^) above all variables as before, and take the Dorodnitsyn transformation:

$$\hat{\xi} = \frac{1}{\hat{V}_m L}\int_0^x \frac{P_e(x)}{\hat{P}_s}\,U_2(x)dx \quad ; \quad \hat{P}_e(x) = P_e(x) \quad ,$$

$$\hat{n} = \frac{U_2(x)}{\left(\nu_s \hat{V}_m L\right)^{1/2}} \int_0^y \frac{\bar{\rho}}{\bar{\rho}_s}\, dy \quad ,$$

$$\hat{v} = \frac{\hat{V}_m L \; U_2 \frac{\partial \hat{n}}{\partial x}}{U_2 P_e / P_s} + \frac{\left(V_m L\right)^{1/2}}{U_2 \left(\nu_s\right)^{1/2}} \hat{\bar{v}} \frac{\hat{\bar{T}}_s}{\bar{T}} \quad ,$$

$$\hat{u} = \frac{\bar{u}}{U_2} \quad ,$$

$$\hat{m} = \hat{\bar{m}} \quad ,$$

Using the same procedures that lead to equation (3.23), we get the following integral relation for the lower region:

$$\frac{\partial}{\partial \hat{\xi}} \left[\frac{\hat{\theta}_0}{2} (1-\hat{u}_0)(1-\hat{u}_0^2) \right] = \hat{\beta}(1-\hat{u}_0)\hat{\theta}_0 \left(\hat{u}_0 + \frac{u_0^2}{2} - \frac{3}{2} \right) + \hat{\beta}\hat{K}_0 (1-\hat{u}_0) + \frac{1}{\hat{\theta}_0} \quad . \tag{3.26}$$

Since u, θ, T and m are continuous across the dividing streamline, we have four matching conditions. Together with the four integral relations (3.23), (3.24), (3.25), and (3.26), we have eight equations to solve for eight unknowns.

The four matching conditions are:

$$\hat{u}_0 = u_0 U_1 / U_2 \quad , \tag{3.27}$$

$$\hat{\theta}_0 = - \theta_0 \left(\frac{V_m}{\hat{V}_m} \right)^{1/2} \left(\frac{U_2}{U_1} \right)^2 \quad , \tag{3.28}$$

$$K_0 = - \theta_0 \left(\frac{V_m}{\hat{V}_m} \right)^{1/2} \left(\frac{U_2}{U_1} \right)^2 \frac{\hat{T}_s - T_s + K_0 T_s / \theta_0}{\hat{T}_s} \quad , \tag{3.29}$$

$$m = \left(\bar{m}_\infty - \frac{J_0}{\theta_0}\right)\left(\frac{1-\hat{u}}{1-\hat{u}_0}\right) \quad . \tag{3.30}$$

Substituting these matching conditions into the equations (3.23), (3.24), (3.25), and (3.26) to eliminate the unknowns $\hat{\theta}_0, \hat{U}_0, \hat{K}_0$, we obtain the following:

$$\begin{bmatrix} A_{11} & A_{12} & 0 & 0 \\ A_{21} & A_{22} & 0 & 0 \\ 0 & A_{32} & A_{33} & 0 \\ 0 & A_{42} & 0 & A_{44} \end{bmatrix} \begin{bmatrix} \dot{\theta}_0 \\ \dot{U}_0 \\ \dot{J}_0 \\ \dot{K}_0 \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} \quad , \tag{3.31}$$

where

$$A_{11} = (1-u_0)(1-u_0^2) \quad ,$$

$$A_{12} = (3u_0+1)(u_0-1) \quad ,$$

$$A_{21} = (1-\alpha u_0)(1-\alpha^2 u_0^2)/\alpha^2 \quad ,$$

$$A_{22} = (-\frac{1}{\alpha} - 2u_0 + 3\alpha u_0^2)/\theta_0 \quad ,$$

$$A_{32} = -2u_0 J_0 \quad ,$$

$$A_{33} = 1 - u_0^2 \quad ,$$

$$A_{42} = -2u_0 K_0 \quad ,$$

$$A_{44} = 1 - u_0^2 \quad ,$$

$$B_1 = \beta(1-u_0)(u_0+3)(u_0-1)\theta_0 + 2\beta K_0(1-u_0) + \frac{2}{\theta_0} \quad ,$$

$$B_2 = \frac{2J_0}{\theta_0^2(1-u_0)} \quad ,$$

$$B_3 = \frac{2K_0}{\theta_0^2(1-u_0)} \quad ,$$

$$B_4 = -\theta_0\left(-\frac{2}{\alpha^3} + \frac{u_0}{\alpha^2} + u_0^3\right)\frac{d\alpha}{d\xi} + \frac{2\alpha}{\theta_0}$$

$$+ \frac{2\theta_0 V_m}{\alpha^2 \hat{V}_m}\hat{\beta}\,(1-\alpha u_0)(\alpha u_0 - \frac{\alpha^2 u_0^2}{2} - \frac{1}{2} - \frac{T_s}{\hat{T}_s} - \frac{K_0 T_s}{\theta_0 \hat{T}_s} \quad ;$$

$$\alpha = U_1/U_2 \quad ,$$

$$\hat{\beta} = \frac{1}{\hat{\alpha}_e(1-\hat{\alpha}_e)}\frac{d\hat{\alpha}_e}{d\hat{\xi}} = \frac{\hat{V}_m}{V_m}\frac{U_1}{U_2}\frac{\hat{p}_s}{p_s}\frac{1}{\alpha_e(1-\hat{\alpha}_e)}\frac{d\hat{\alpha}_e}{d\xi} \quad ,$$

$$\hat{\alpha}_e = u_2/\hat{V}_m \quad .$$

Initial conditions are needed to solve the above system of differential equations (3.31). Since at $\xi = 0, \theta_0 = 0$, we have a singularity at $\xi = 0$. In order to avoid the singularity point, we integrate the equations starting at $\xi = \varepsilon$, where ε is a small positive number. From the inviscid model, $U_1(0)$ and $U_2(0)$ are found. Assuming that the pressure gradient has no effect in the vicinity of the leading edge, then the technique discussed in Section 2 can be applied to solve for θ_0, U_0, J_0 and K_0 at $\xi = \varepsilon$. We take these as the initial conditions and solve the system of differential equations numerically. With θ_0, U_0, J_0 and K_0 determined for general ξ, the velocity, temperature and H_2S concentration can be found.

Define the boundary layer thickness of the lower half-plane as the normal distance where the H_2S concentration is 0.0002, which is 1% of that in the gas mixture. For comparison with the parallel injection model, we use the same pipe diameter of 5 cm and the same amount of steam, for an injection length of 5 cm and injection velocity of 30 cm/sec. We find that the boundary layer hits the wall at a position 17 cm from the leading edge. The concentration of H_2S at the wall is calculated and plotted in Figure 6.

Section IV. Conclusion

The Method of Integral Relations has been applied successfully to solve the present mixing problem. The accuracy of the solution increases with succeeding orders of approximation, but the calculations are increasingly complicated. In this paper only the first order of approximation is used.

For large pipe diameter, say 1 meter, we can inject a layer 1 cm thick. From the parallel injection analysis, an initial pipe length of approximately 32 meters is found to be protected from corrosion. For small pipe diameter, say 5 cm, the usual pipe diameter in present coal gasification processes, only a thin layer can be injected. From the calculations in Section III, the concentration of H_2S at the wall of the pipes was found to be reduced approximately 70 percent for an initial length of 6 m.

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<u>List of Symbols</u>

A_{ij}	matrix elements
c_p, c_v	specific heats
$\bar{D}$	molecular diffusion coefficient
D	diffusivity of H_2S
f, g	weighting functions
h	dimensionless enthalpy
J	jθ
k	thermal conductivity
K	mθ
L	characteristic length
Le	Lewis number
$\bar{m}, m$	species mass concentration
p	pressure
Pr	Prandtl number
R	gas constant
Re	Reynolds numbers
t	transverse coordinate
$\bar{T}, T$	temperatures
$\bar{u}, \bar{v}$	velocity components (dimensional)
u, v	velocity components (dimensionless)
$\bar{U}_{\infty}$	free stream velocity
U_1	velocity along dividing streamline
U, V	transformed velocity components

V_0	injection velocity
$\bar{x},\bar{y}$	Cartesian coordinates (dimensional)
x,y	Cartesian coordinates (dimensionless)
z	$s+it$, complex variable in inviscid flow
α,β	functions of velocity at outer edge of mixing layer
η	$\int_0^y \rho\,dy$
θ	$1/(\partial u/\partial \eta)$
$\bar{\mu},\mu$	coefficients of viscosity
$\bar{\nu},\nu$	coefficients of kinematic viscosity
ν_∞	free stream value
ξ	cx, coordinate
$\bar{\rho},\rho$	densities
σ	line source strength

Diacritical Symbols

–	dimensional quantities
^	variables in lower half of mixing region

Subscripts

e	value at outer edge of mixing layer
0	quantities at $\eta = 0$, mixing layer interface
s	quantities at stagnation point
∞	free stream value

Table 1: Coal Gasifier Operating Environment

Item	Hygas	Synthane	Consol
Gas Composition: In Mole %			
H_2O	24	37	17
CO	7	11	14
CO_2	4	18	6
H_2S	0.8	0.3	0.03
H_2	24	18	45
CH_4	13.3	15	17
N_2	0.3	0.5	0.2
Gas Pressure			
Psig	1100	1000	300
Gas Temperature °C	1000	1000	875

Figure Captions

Figure 1. Coordinate system for parallel injection.

Figure 2. Numerical solutions of velocity profile for parallel-injection model.

Figure 3. Numerical solutions of H_2S concentration profile for parallel-injection model.

Figure 4. Coordinate system for normal injection.

Figure 5. Diagram for inviscid analysis; uniform flow over a line source.

Figure 6. Numerical solutions of H_2S concentration at the wall of pipe for normal-injection model.

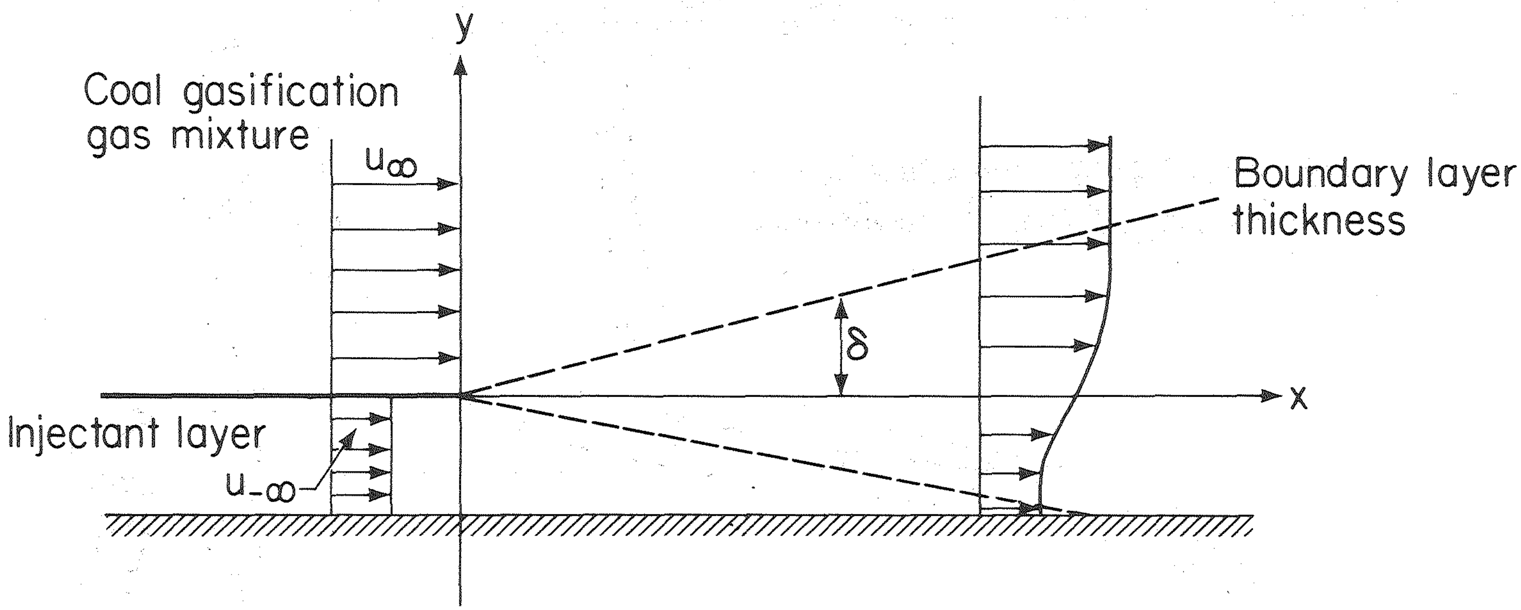

Figure 1

XBL 813-457

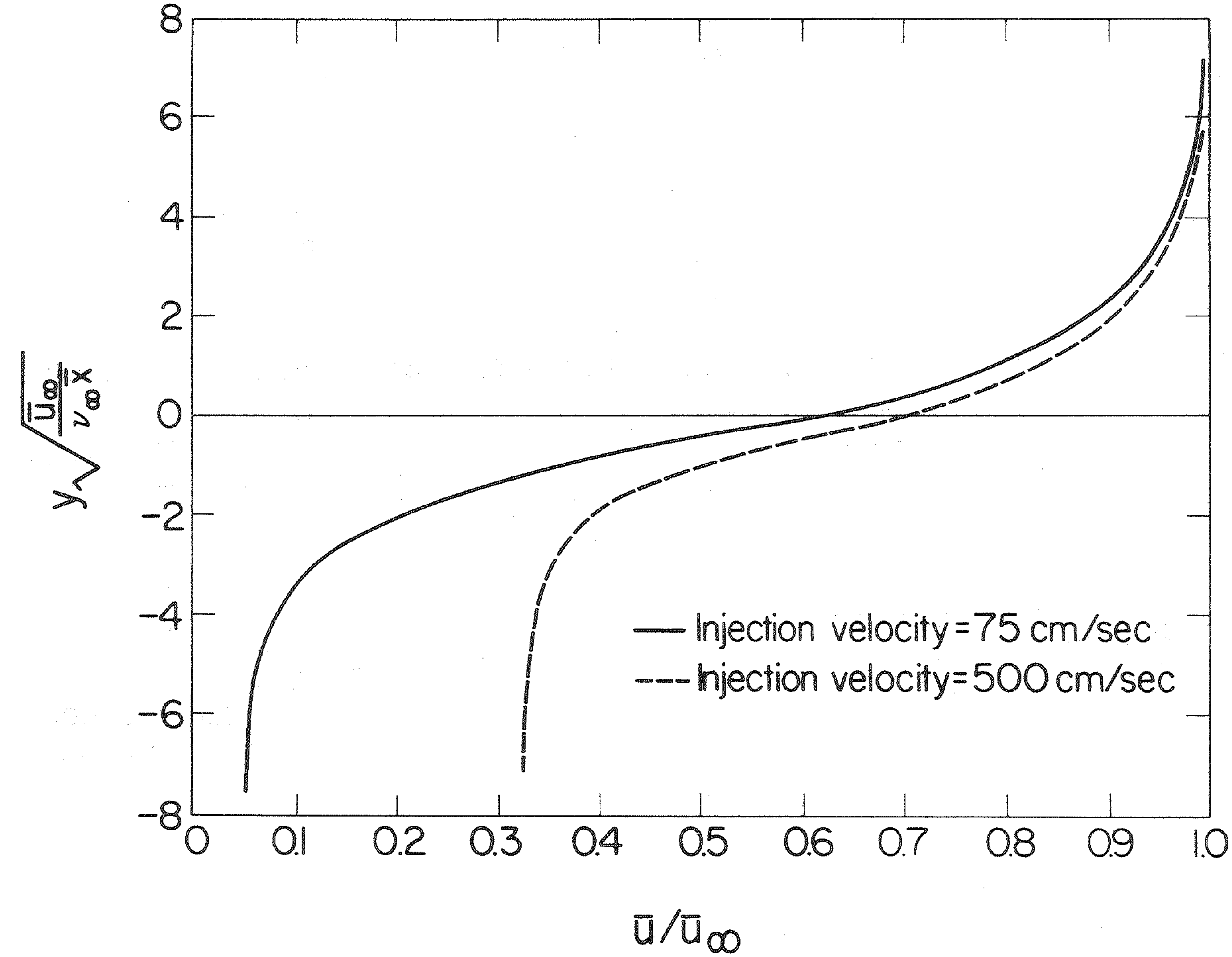

Figure 2

XBL 813-458

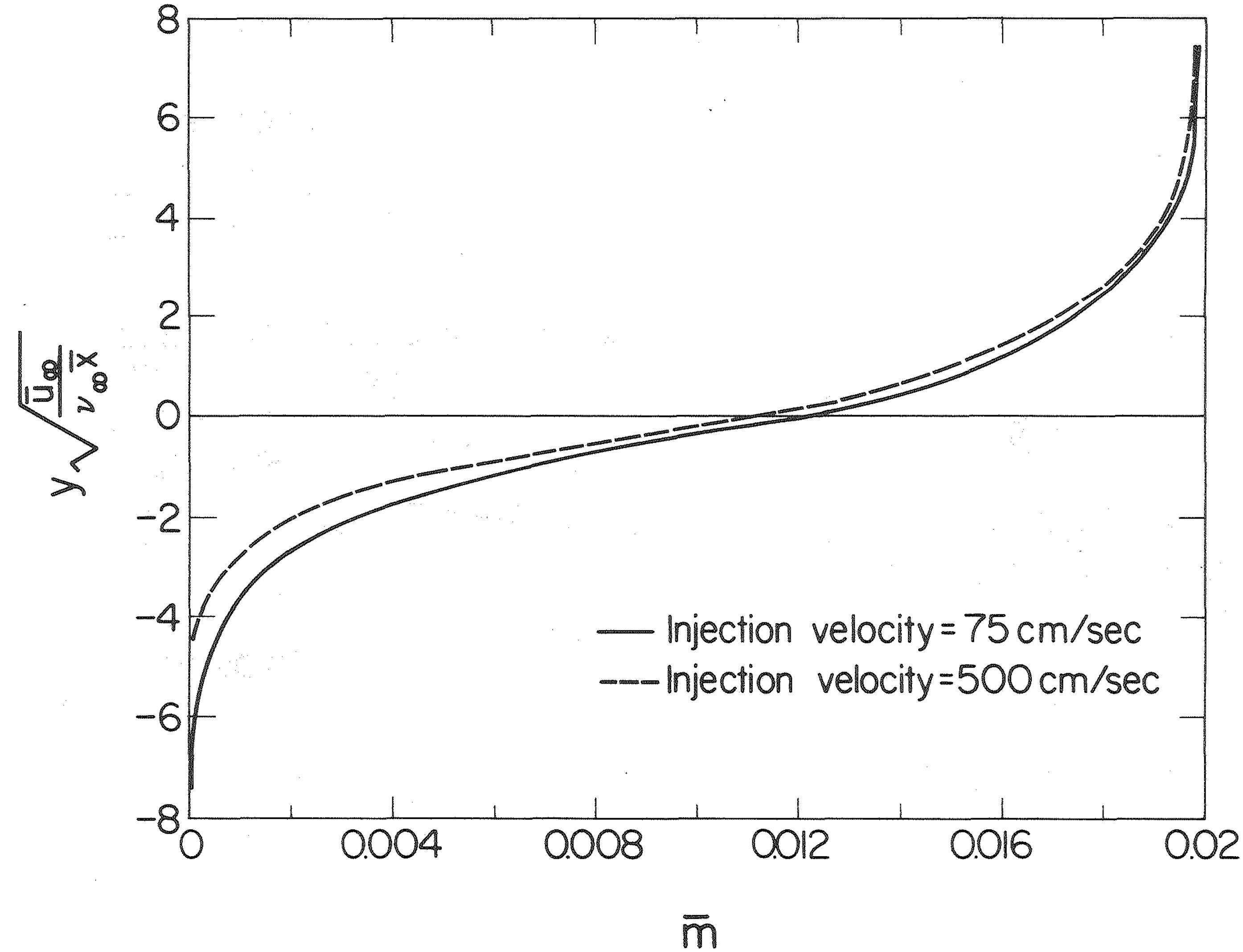

Figure 3

XBL 813-459

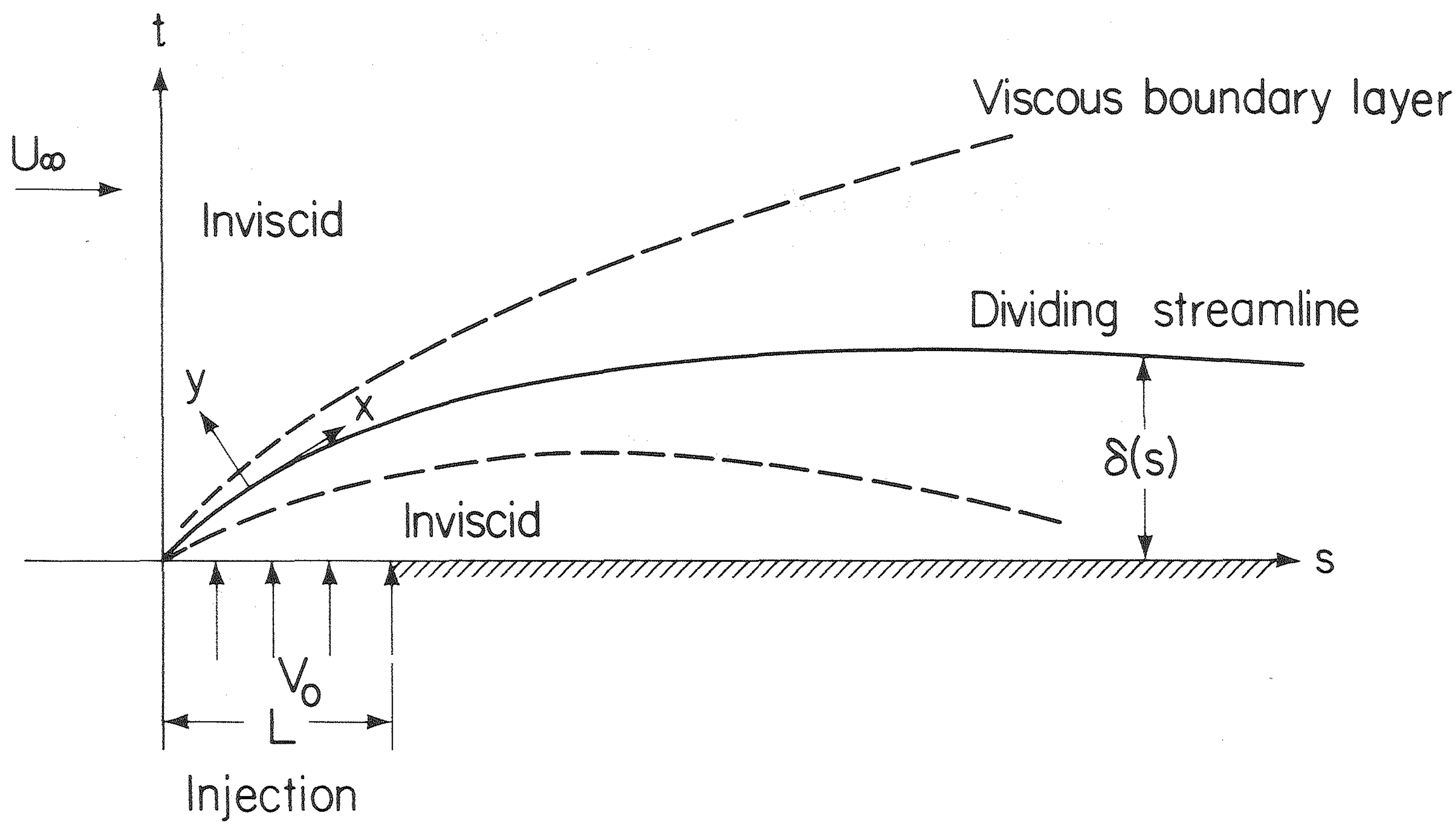

Figure 4

XBL 813-460

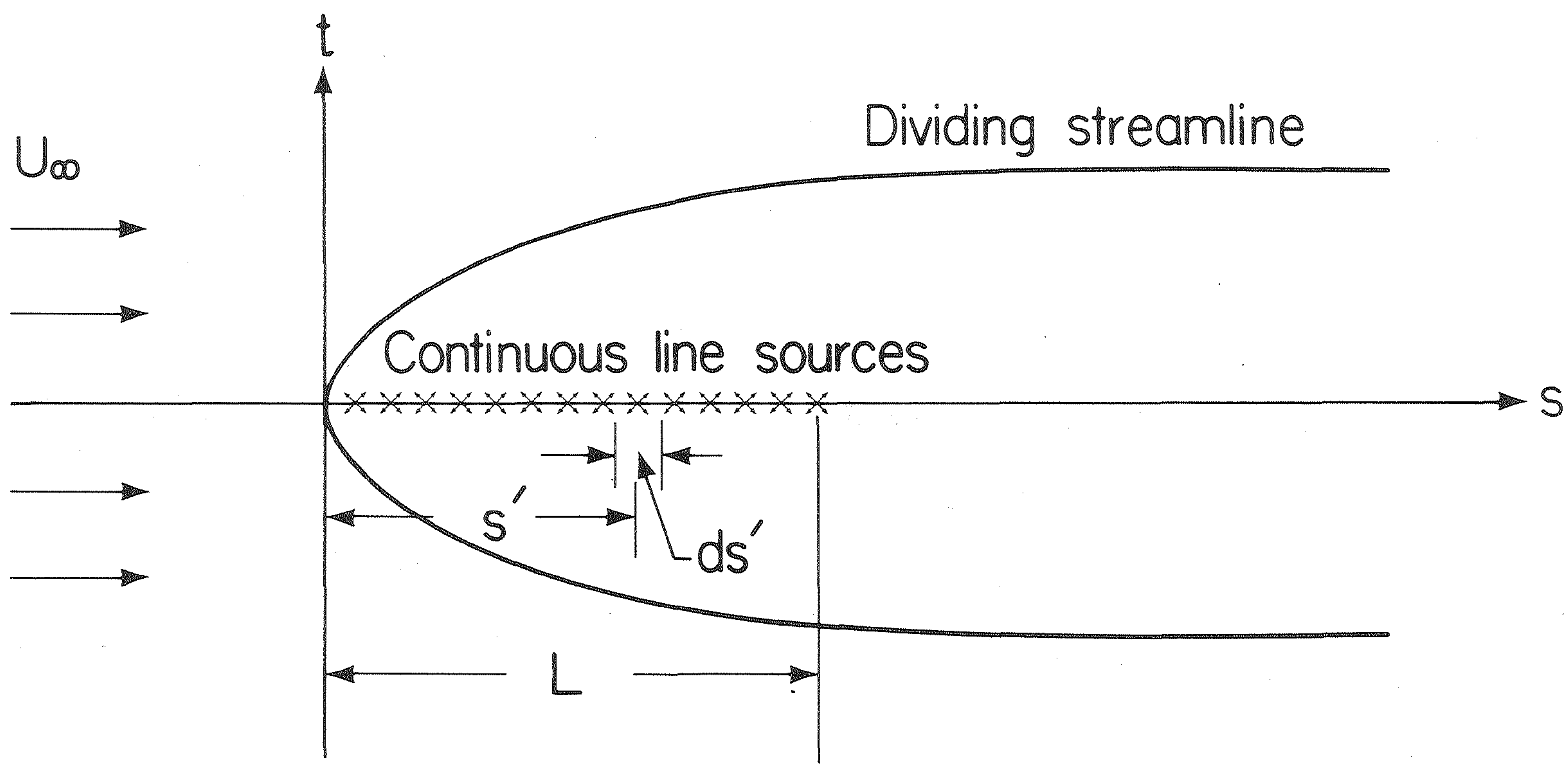

Figure 5

XBL 813-461

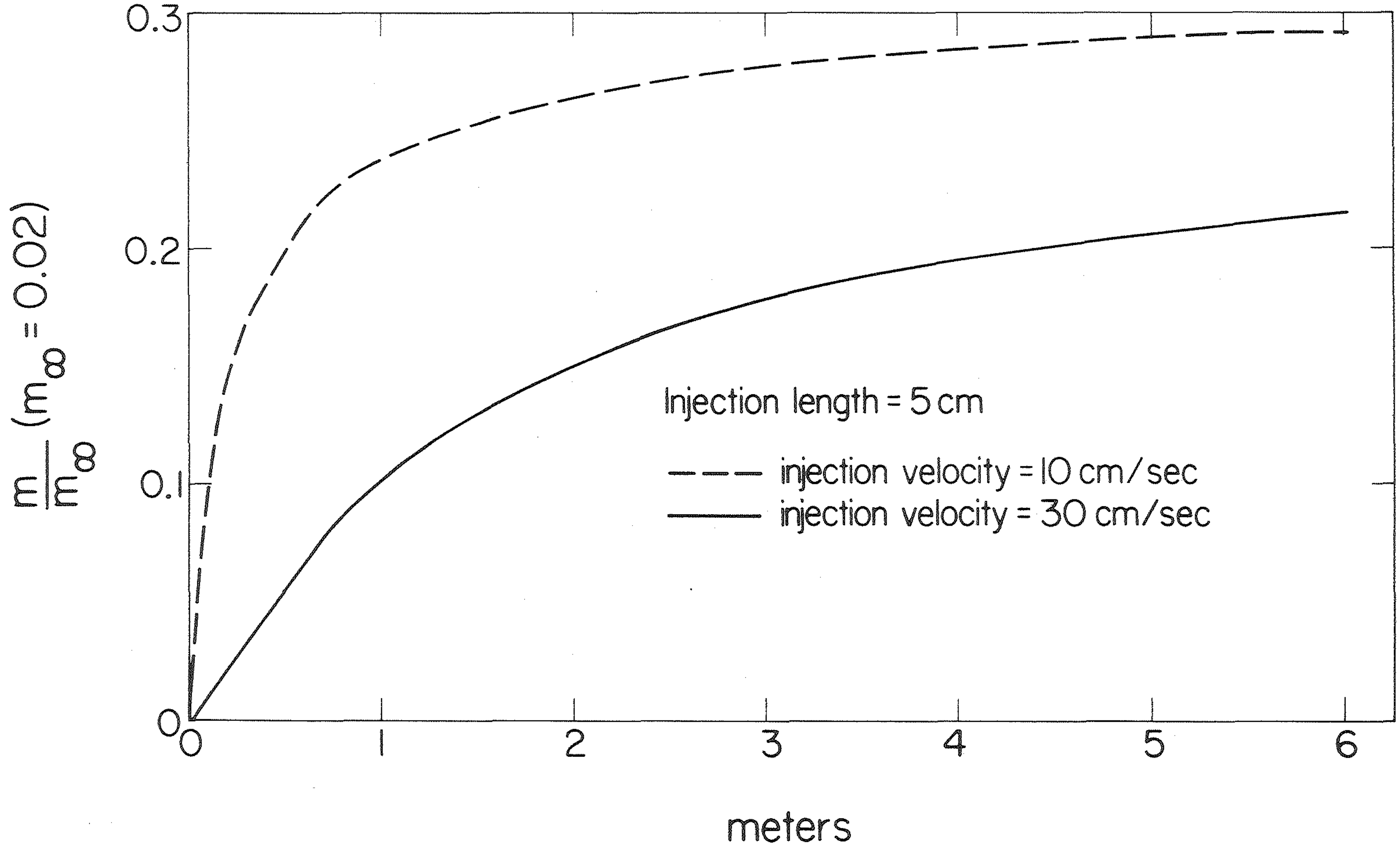

Figure 6

XBL 813-462