Title

Optimal Reservation Deposit Policies in the Presence of Rational Customers and Retail Competition

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Publication Date

2010-03-01

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March 6, 2010

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Abstract

We study two reservation deposit policies for a service firm to increase its revenue through higher capacity utilization. First, under the "no deposit" policy, the firm requires no reservation deposit and imposes no "no show" penalty. Anticipating potential "no shows," a firm may overbook; hence, there is no guarantee that the reserved service will be provided under the no deposit policy. On the contrary, under the "guarantee deposit" policy, a guarantee deposit is required for each customer to make a non-cancelable reservation. To honor the reserved service under the guarantee deposit policy, the firm will not overbook. We analyze each deposit policy as a Stackelberg game in which the firm acts as the leader who selects the booking capacity under the no deposit policy (or the required deposit under the guarantee deposit policy), and each customer acts as the follower who decides whether to reserve or not. Our model incorporates rational customer behavior so that each customer will take other customers' behavior into consideration. Using the firm's optimal booking capacity (optimal required deposit) in equilibrium under the no deposit policy (the guarantee deposit policy), we compare the firm's expected profits under these two policies in a monopolistic environment. Our results suggest that the firm should charge a higher optimal retail price under the no deposit policy, and adopt the no deposit policy when the demand rate is below a certain threshold. By analyzing a game of duopolistic competition between two firms, we develop the conditions under which the firms will adopt a particular pair of deposit policies in equilibrium, and we show this game can lead to a Prisoner's Dilemma. Moreover, when both firms charge the same retail price, we show the existence of an equilibrium in which both firms adopt the no deposit policy.

Keywords: Reservations, Deposit Policies, Revenue Management, Rational Customer Behavior, Retail Competition.

1 Introduction

An important aspect of revenue management is to develop mechanisms that would enable service firms with limited capacity to improve their revenues. With limited capacity, capacity utilization is a key performance measure of a service firm. That is why hotels measure occupancy rates and car rental companies track the number of idle cars in their parking lots. In view of limited capacity, customers often reserve in advance but they may not show up for their reserved service due to a variety of reasons which include change of plans. "No shows" can be a significant problem in the service industry: 10-15% of passengers do not show up to claim their reserved seats in the airline industry, and 25% of guests do not show up for their reserved rooms in the hotel industry (Rothstein (1974), Rothstein (1985) , and USA Today (1998)). When competition is fierce and profit margins are slim, "no-shows" can have detrimental effects on a service firm's survival.

In this paper we examine the use of two different reservation deposit policies that are intended to reduce the risk of unused capacity and improve the revenue of a service firm. Two basic deposit policies observed in practice are the "no deposit" policy N , and the "guarantee deposit" policy D. Under the no deposit policy N, there is no required reservation deposit, and there is no penalty for customers not showing up for the reserved service. Anticipating potential "no shows," a firm may "overbook" by accepting more reservations than his capacity; hence, there is no guarantee that the reserved service will be provided. However, if a customer shows up and discovers that her reserved service is denied due to overbooking, then she will receive a compensation for the inconvenience that the firm has caused. For example, in the airline industry, the US Department of Transportation (DOT) issues guidelines on the compensation schemes for passengers who are denied boarding involuntarily due to overbooking. The reader is referred to http://airconsumer.ost.dot.gov/reports/index.htm for details. Also, in the hotel industry, the US law courts have ruled that hotels are obligated to compensate guests who are denied service involuntarily due to overbooking (Rothstein (1974)). Under the guarantee deposit policy D, a guarantee deposit is required for a customer to make a non-cancelable reservation.[1] To honor the reserved service under the guarantee deposit policy, the firm will not overbook.

In the United States, the guarantee deposit policy is becoming more common in the hotel industry even though some hotels do not require deposits. However, the no deposit policy is commonly observed in car rental companies and restaurants.[2] While both policies are commonly observed, it is unclear which reservation policy (N or D) is more effective for a firm to improve his revenue. To select an effective reservation policy, a firm needs to examine the following tradeoffs: customers are eager to reserve under policy N, but their commitment to show up can be low because there is no penalty for not showing up for the reserved service. On the other hand, customers are more reluctant to reserve when non-refundable

[1] Without dealing with the issue of service guarantee, Xie and Gerstner (2007) argue that a firm can obtain a higher expected profit by offering cancelable reservations with refund. Their argument is based on the assumption that cancelable reservations encourage more customers to reserve.

[2] Some highly acclaimed restaurants in the US such as Chez Panisse in California (www.chezpanisse.com) and Rainbow Room in New York (www.rainbowroom.com) require customers to pay non-refundable deposits to guarantee their reservations.

deposits are required under policy D; however, the non-refundable deposits provide strong incentives for customers to show up for their reserve service.

Even though both reservation polices N and D are common in practice, there are no formal analytical models for analyzing the case when: (1) customer demand is uncertain; (2) customers are rational in the sense that they take other customers' behavior into consideration when deciding whether to reserve; and (3) market competition is present. As an initial attempt, we first examine the monopolistic case in which a single firm with a fixed capacity $m(\geq 1)$ who needs to decide on his deposit policy (N or D). We then extend our model by analyzing a game of duopolistic competition between two identical firms. Our model captures the key tradeoffs associated with each deposit policy. We consider the case in which an uncertain number of customers, with valuation v and show up probability ψ (private information), who need to decide whether to make a reservation. First, suppose the firm adopts policy N and charges a retail price r_N (a decision variable).[3] Then each rational customer will take the "deny probability" $(1-\beta)$ into consideration when deciding on whether to reserve or not, where this deny probability depends on the rational behavior of other customers in the system. Anticipating customers' reservation behavior, the firm can estimate the number of customers who would attempt to reserve under policy N. However, due to potential "no shows," the firm needs to decide on the booking capacity n ($n \geq m$); i.e., the maximum number of reservations to accept. When a customer shows up, she is obligated to pay r_N for her reserved service. However, in the event when the firm overbooks and denies a customer who shows up for her reserved service, the firm will offer her a compensation c for the inconvenience. The tradeoff under policy N is captured by the booking capacity n: (a) when n is too large, the risk of compensating too many denied customers increases; and (b) when n is too small, then the risk of unused capacity increases.

Next, suppose the firm opts for policy D and charges a retail price r_D (a decision variable). Then the firm needs to decide on the non-refundable deposit $d > \theta > 0$ (where θ is the minimum deposit) that each customer is required to pay upfront to ensure her reservation is guaranteed. In this case, the customer is obligated to pay the remaining amount $(r_D - d)$ when she shows up to redeem her reserved service. However, for any given deposit d, each rational customer will take her show up probability ψ into consideration when deciding on whether to reserve or not. The customer reservation behavior will enable the firm to determine the number of customers who would attempt to reserve under policy D. Anticipating customers' reservation behavior, the firm can determine the number of customers who would attempt to reserve under policy D. Effectively, the required deposit d captures a key trade-off: (a) when d is too small, customers have a lower incentive to show up and the risk of unused capacity increases; and (b) when d is large, customers are reluctant to reserve and the risk of unused capacity increases.

To analyze the tradeoffs associated with each deposit policy, we model the dynamics between the firm and its customers as a Stackelberg game in which the firm acts as the leader who selects the booking capacity under the no deposit policy N (or the required deposit under the guarantee deposit policy D), and each rational customer acts as the follower who decides whether to reserve or not. To deal with the

[3] To enable us to focus on the issue of reservation policies in the presence of rational customers and competition, we shall treat the retail price as given first and then determine the optimal retail price numerically in Section 3.5.2.

issue of market competition, we extend our model to capture duopolistic competition between two firms by analyzing a non-cooperative game between two firms on top of the aforementioned Stackelberg game between each firm and the customers. Our analysis enables us to answer the following questions:

- For any given retail price, what is the customer's optimal reservation decision under each deposit policy?
- For any given retail price, what is the firm's optimal booking capacity n^* under policy N? What is the firm's optimal guarantee deposit d^* under policy D? Also, what is the firm's optimal retail price under each policy?
- What conditions render deposit policy N more profitable for the firm?
- In a duopolistic environment, which deposit policy will each firm adopt in equilibrium?

In a monopolistic environment, we show that the no deposit policy N dominates the guarantee deposit policy D when the demand rate is below a certain threshold or when the customer valuation is below a certain threshold. Hence, market condition is a key driver when choosing a deposit policy. Also, our results suggest that the firm should always charge a higher optimal retail price under the no deposit policy N. This result is consistent with a common practice in the hotel industry: the daily rate of a room with guarantee deposit requirement tends to be lower.

When we examine the issue of market competition, we analyze a game of duopolistic competition between two firms. We develop the conditions under which the firms will adopt a particular pair of deposit policies in equilibrium, and we show this game can lead to a Prisoner's Dilemma. In addition, we show that the results obtained in the monopolistic case do not necessarily carry over to the duopolistic case. Moreover, when both firms charge the same retail price, we show that there exists an equilibrium in which both firms adopt policy N without overbooking. Finally, we show the existence of an asymmetric equilibrium in which one firm adopts policy N and the other adopts policy D.

The primary contributions of this paper are four-fold. First, our paper is the first to examine the trade-off between the no deposit policy N and the guarantee deposit policy D in the presence of rational customers and market competition. Second, by exploring the underlying mathematical structure, we obtain insights regarding the optimal booking capacity n^* under policy N, the optimal guarantee deposit d^* under policy D, and the optimal retail price a firm should charge under each policy. Third, we derive conditions under which one reservation policy dominates the other. Fourth, we extend our analysis to the case in which two competing firms need to determine their reservation policies in equilibrium. We determine the equilibrium of each subgame analytically and the meta-game equilibrium numerically.

This paper is organized as follows. Section 2 provides a brief review of related literature. Section 3 presents the base model in a monopolistic setting. We determine the optimal booking capacity under policy N and the optimal deposit under policy D. In section 4, we extend our analysis to the duopolistic case in which the industry is comprised of two competing firms. Here, we examine the deposit policy that

each firm will adopt in equilibrium. We conclude in Section 5 with a brief summary of our results, and a discussion of the limitations of our model and potential future research topics. In order to streamline our presentation, all proofs are given in Appendix 1.

2 Literature Review

Recognizing the fact that unused capacity has no salvage value in the service industry, we witness an increasing research interest in revenue management recently. Many researchers have examined different pricing mechanisms to segment the market so that a firm can extract the surplus from different customer segments (Talluri and Van Ryzin (2004)). Besides pricing mechanisms, various operations management researchers have explored other mechanisms that involve opaque selling (Jerath et. al. (2009)), strategic stockout (Liu and Van Ryzin (2008)), partial inventory information (Yin et. al. (2009)), and reservations (Alexandrov and Lariviere (2006), Elmaghraby et. al. (2009) and Png (1989)). The reader is referred toBitran and Caldentey (2003), Netessine and Tang (2009), Philips (2005), Talluri and Van Ryzin (2004), and Weatherford and Bodily (1992) for different comprehensive reviews on this important research area.

Our paper is related to a research stream in revenue management that deals with the issues of reservations and overbooking. In the reservation literature, Png (1989) is one of the first papers that examines a monopolistic airline which, in order to increase capacity utilization, takes customer reservations. While customers with reservations are not obligated to claim their reserved seats, customers with high valuation will exercise the purchase option (reserve and show up). He shows that overbooking is an effective strategy to reduce the risk of unused capacity. By consider the duopoly case, Lim (2009) is the first paper to analyze the effect of overbooking in a competitive environment. She shows that overbooking is a dominant strategy for both firms in equilibrium. Recently, Alexandrov and Lariviere (2006) study the role of reservations in the context of restaurants. They show that a restaurant should never offer reservations when there is no demand uncertainty and when customers have identical valuation. However, when competition is present, it is beneficial for restaurants to take reservations especially when the number of interested customers or the number of restaurants in the market is sufficiently large. Instead of examining the value of reservations and the value of overbooking, we analyze two reservation deposit policies N and D. By comparing the firm's expected revenues under these two policies, we establish the conditions under which one policy dominates the other in a monopolistic environment. Also, we extend our model to deal with market competition in a duopolistic environment.

Because there is no penalty for "no shows", it is beneficial for a firm to overbook (Png (1989) and Alexandrov and Lariviere (2006)). Instead of imposing penalty for "no shows", Biyalogorsky et. al. (1999) examine a situation in which the market is comprised of two types of customers with low and high valuations. To reduce the risk of unused capacity, a firm accepts customer reservations by offering a lower price to the low valuation customers (leisure travelers) who arrive in the first period. However, the firm reserves the rights to recall (i.e., cancel) these reservations so that it can sell the recalled units at a higher price to the high valuation customers (business travelers) in the second period. They show how "callable" reservations can enable the firm to reduce the risk of unused capacity and to obtain a

higher expected revenue. By considering more general assumptions, Gallego et. al. (2008) study the issue of callable products and obtain similar results. In addition, they determine the optimal number of callable reservations to be allocated for sales in the first period. Instead of restricting the case that only callable units are available for sale in the first period and non-callable units are available for sale in the second period, Elmaghraby et. al. (2009) examine a situation in which the firm offers both callable and non-callable units at different prices at any point in time. By considering the case when customers with different valuations arrive at the firm according to a Poisson process, they show that a firm can obtain an even higher profit by offering customers both options. Their result is due to the fact that, when both options are available at any point in time, rational customers would feel the competitive pressure among customers to purchase the non-callable units at a higher price. Our model differs from these models in the following manner. First, instead of focusing on a particular policy, we are interested in comparing the performance of two common deposit policies (N and D) observed in practice. Second, in addition to the monopolistic setting, we examine the impact of retail competition on these two reservation policies. Third, while the customer valuation is assumed to follow a two-point distribution in the aforementioned models, we assume that the customer valuation follows a uniform distribution. With the exception of Elmaghraby et. al. (2009), these models assume that low valuation customers will only arrive early and high valuation customers will only arrive late. Instead, we allow customers with different valuations to be present in the system simultaneously.

While overbooking has been shown to be an effective strategy for a firm to reduce the risk of unused capacity, it can cause customer disloyalty due to customer dissatisfaction. To mitigate the negative effect of overbooking, some firms may offer guaranteed service to their loyal customers at a higher retail price. For example, Continental Airlines offer their loyal customers with Gold or Platinum status guaranteed seats for their reservations if they purchase their tickets at a higher price (e.g. the unrestricted Y class ticket) at least 48 hours before departure. To guarantee these loyal customers' reservations, the airlines would need to either reduce its booking capacity or to increase its compensation to customers for being denied (Biyalogorsky et. al. (2000)). Instead of charging a higher price to ensure that a reservation will be honored, we consider the case when the firm charges an upfront guarantee deposit d under policy D. We show that the firm should always offer a lower retail price under the guarantee deposit reservation policy D. Hence, instead of charging a higher price to guarantee a reservation (Biyalogorsky et. al. (2000)), a lower retail price is more likely to be welcome by most customers even when a non-refundable deposit is required.

3 Base Model: The Monopolistic Case

Consider a firm with a fixed capacity of $m \geq 1$ units who needs to specify his reservation policy (N or D) before customers are present in the system. Let us describe the firm's decision under each deposit policy. If he chooses the no deposit policy N, then he needs to decide on his retail price r_N and his booking capacity n (two decision variables), where $n \geq m$, so that he will accept no more than n reservations. Once the firm accepts a reservation from a customer without a deposit, there are three possible outcomes: (a) the customer does not show up for her reserved service; (b) the customer shows up and the service is

available; and (c) the customer shows up and the service is not available (due to overbooking). The firm earns nothing when outcome (a) occurs, earns r_N when (b) occurs, and pays a penalty c to the customer whose reserved service is being denied.

If the firm chooses the guarantee deposit policy D, then he needs to determine his retail price r_D and his guarantee deposit $d \geq \theta > 0$, where r_N, d are decision variables and θ is an exogenously given minimum deposit. (Because the firm will not overbook under policy D, it is reasonable to expect the firm to charge a minimum deposit to θ to defray some operating cost.) To guarantee that each reserved service will be honored, the firm will not overbook so that no more than m reservations will be accepted. Once the firm accepts a reservation from a customer who pays an upfront non-refundable deposit d, there are two possible outcomes: (a) the customer does not show up for her reserved deposit; and (b) the customer shows up for the reserved service. In the former case, the customer's deposit d is forfeited, and the firm earns d. In the latter case, the customer pays the remaining portion $(r_D - d)$, and the firm earns the entire retail price r_D in total.

To model market uncertainty, let A be the number of "potential" customers who are present in the system after the firm announces the booking capacity n if policy N is chosen (or the required guarantee deposit d if policy D is selected), where A is assumed to be a Poisson random variable with rate λ.[4] For each of these A potential customers, her "net valuation" of the service is equal to $\psi \cdot v$, where v is her gross valuation and ψ is her "show up" probability for the reserved service. To capture market heterogeneity and obtain tractable results, we assume that the gross valuation v is fixed, and ψ is uniformly distributed over $[0, 1]$. Thus, the net valuation of the service $\psi \cdot v \sim U[0, v]$. To eliminate trivial cases, let us assume that $x_D \triangleq v - r_D > 0$ so that every customer is a potential customer when the firm chooses policy D. Similarly, to eliminate potential arbitrage opportunities, we assume that $x_N \triangleq v - r_N > c$. This assumption is reasonable when reservations are non-transferable or when the number of "speculators" in the market is negligible. Here, the term speculators is referred to those "phantom" customers who do not care for the service but they have a strong desire to get the compensation c for being denied service. The reader is referred to Su (2008) for an interesting study that shows speculators can increase a firm's expected profits.

In this paper, we assume that the firm and all customers are endowed with the following knowledge: the market size is a Poisson random variable with rate λ; the customer's gross valuation is v; and the customer's show up probability ψ is uniformly distributed over $[0, 1]$. In this case, the sequence of events can be described as follows. The firm first decides and announces his retail price r_N and his booking capacity n if policy N is chosen (or retail price r_D and guarantee deposit d if policy D is selected). Then the number of customers in the system is realized. Each customer needs to decide whether to reserve with the firm, and the firm can only accept reservations up to his booking capacity (i.e., n under policy N, or m under policy D). After that, each customer with a reservation decides to show up or not according to

[4] Besides the fact that the Poisson assumption enables us to obtain tractable results, Lariviere and Van Mieghem (2004) argue that Poisson demand is an acceptable assumption for modeling the number of rational customers in a sufficiently large market.

her private show up probability ψ, and the firm determines his revenue based on the number of accepted reservations and the number of customers who show up for their reserved service.

3.1 Customer Surplus and Reservations

We now examine customers' rational behavior under policies N and D. We first conduct our analysis for any given retail price. Then we determine the optimal retail price numerically in Section 3.5.2.

3.1.1 *No Deposit Policy* N

Consider a customer who makes a reservation with the firm who adopts policy N. Given her private show up probability ψ, she will obtain an expected surplus $\psi \cdot \beta \cdot x_N$ if her reserved service is honored when she shows up, and she will obtain an expected surplus $\psi \cdot (1-\beta) \cdot c$ if her reserved service is denied when she shows up, where $(1-\beta)$ is the "deny" probability that her reserved service will be not be honored due to overbooking. Because each customer will take this deny probability $(1-\beta)$ into consideration when deciding whether to reserve or not, the deny probability $(1-\beta)$ needs to be determined endogenously.[5] Because $(1-\beta) \geq 0$ and $\psi \geq 0$, it is rational for each customer to reserve because her expected surplus $\pi_N \geq 0$, where

$$\pi_N = \psi \cdot [\beta \cdot x_N + (1-\beta) \cdot c]. \tag{1}$$

Knowing that all customers are rational and hence they will attempt to reserve, the firm can estimate the number of customers who would attempt to reserve under policy N is a random variable $A_N \sim Poi(\gamma_N \lambda)$, where $\gamma_N = 1$ is the probability that a customer in the system will attempt to reserve with the firm who adopts policy N.[6]

Recognizing the fact that $\gamma_N = 1$ is a common knowledge, we now discuss how customers can infer β correctly. Suppose the firm announces his booking capacity n. Then each customer knows that the firm will accept R_N reservations, where $R_N = \min\{n, A_N\}$. For any number of accepted reservations $R_N = j$ and for any show up probability ψ, the number of customers who show up for their reserved service can be denoted by S_N, where $(S_N \,|\, R_N, \psi)$ is a binomial random variable so that $\Pr\{S_N = k \,|\, R_N = j, \psi\} = \binom{j}{k} \psi^k (1-\psi)^{j-k}$. Because $\psi \sim U[0,1]$, the conditional probability of $S_N = k$ given $R_N = j$ satisfies

$$\Pr\{k \,|\, j\} \equiv \Pr\{S_N = k \,|\, R_N = j\} = \int_0^1 \binom{j}{k} \psi^k (1-\psi)^{j-k} \, d\psi = \frac{1}{j+1}, \text{ for } 0 \leq k \leq j. \tag{2}$$

[5] To obtain tractable results, some researchers assume that this kind of information is given exogenously (e.g. Liu and Van Ryzin (2008) and Cachon and Swinney (2009)). In some cases, the deny probability $(1-\beta)$ can be deduced from historical data. For example, in the airline industry, the likelihood of being denied for service is published in the public domain. The US department of transportation provides detailed statistics about the deny probability of different airlines on a bi-monthly basis. See: http://airconsumer.ost.dot.gov/reports/index.htm. In this paper, we show that the deny probability can be determined endogenously with some efforts.

[6] It is important for a firm to take this rational customer behavior into consideration when selecting its booking capacity. Otherwise, the firm will not be able to set the right booking capacity, which will result in lower expected revenue. In a different context, Yin et. al. (2009) show that a firm's expected revenue can suffer significantly from making decisions without taking rational customer behavior into consideration.

Therefore, when a firm with capacity m announces a booking capacity n, each customer can use the probability distributions of R_N and S_N to infer $\beta(m, n)$ correctly, where

$$\beta(m, n) = E_{R_N} E_{S_N|R_N} \left[\min \left\{ 1, \frac{m}{S_N} \right\} \right]. \tag{3}$$

By noting that $(S_N|R_N)$ is stochastically increasing in any realization of R_N, which is stochastically increasing in γ_N, it is easy to check from (3), that $\beta(m, n)$ is stochastically decreasing in γ_N. This property will enable us to determine the deny probability for the duopolistic case.

3.1.2 *Guarantee Deposit Policy D*

Consider the case when the firm chooses the guarantee deposit policy D. Because the firm will not overbook, the "deny" probability $1 - \beta = 1$ (i.e. $\beta = 1$). Also because the guarantee deposit d will be forfeited for not showing up with probability $(1 - \psi)$, each customer will obtain an expected surplus π_D, where

$$\pi_D = (1 - \psi)(-d) + \psi \cdot (v - r_D) = -d + \psi(x_D + d) \tag{4}$$

Hence, a customer will attempt to reserve if $\pi_D \geq 0$ or equivalently, if $\psi \geq \frac{d}{x_D + d}$. By noting that $\psi \sim U[0, 1]$, the probability that a customer will attempt to reserve, denoted by γ_D, satisfies

$$\gamma_D = \Pr \left\{ \psi \geq \frac{d}{x_D + d} \right\} = \frac{x_D}{x_D + d}. \tag{5}$$

To guarantee that each reserved service will be honored, the firm will not overbook. Consequently, the firm will accept R_D reservations, where $R_D = \min\{m, A_D\}$, $A_D \sim Poi(\gamma_D \lambda)$, and γ_D is given in (5). For any number of accepted reservations $R_D = j$ and for any show up probability ψ, the number of customers who show up for their reserved service can be denoted by S_D, where $(S_D \,|\, R_D, \psi)$ is a binomial random variable so that $\Pr\{S_D = k \,|\, R_D = j, \psi\} = \binom{j}{k} \psi^k (1 - \psi)^{j-k}$. By using the fact the show up probability of each customer who reserves is $\psi \sim U\left[\frac{d}{x_D + d}, 1\right]$, we can use the same approach as presented in Section 3.1.1 to determine the conditional probability of $S_D = k$ conditional on $R_D = j$, where

$$\Pr\{k \,|\, j\} = \int_{\frac{d}{x_D + d}}^{1} \binom{j}{k} \psi^k (1 - \psi)^{j-k} \cdot \frac{x_D + d}{x_D} d\psi. \tag{6}$$

3.2 Optimal Booking Capacity n^* under Policy N

We now determine the optimal booking capacity n^* for a firm with capacity m who adopts policy N. Recall from Section 3.1.1 that S_N customers will show up for the reserved service after accepting $R_N = min\{A_n, n\}$ reservations. Specifically, for any realization $S_N = k \geq 1$, the firm's revenue can be expressed as: $\Pi_N(m\,;\, n|\, k) \triangleq z(k, m) = r_N \cdot \min\{k, m\} - c \cdot \max\{k - m, 0\}$. By considering the probability distributions of R_N and $(S_N|R_N)$ as discussed in Section 3.1.1, one can show that the firm's expected

revenue under policy N satisfies:

$$\Pi_N(m,n) = E_{R_N}\left\{E_{S_N|R_N}\left\{\Pi_N\left(m,n\,|\,S_N=k\right)\right\}\right\} = \sum_{j=1}^{n-1}\sum_{k=1}^{j} z\left(k,m\right)\Pr\left\{k|j\right\}p_{j|\lambda} + \sum_{k=1}^{n} z\left(k,m\right)\Pr\left\{k|j\right\}p_{j\geq n|\lambda} \tag{7}$$

where $\Pr\{k\,|\,j\}$ is given in (2), and $p_{j\geq n|\lambda} = \left(1-\sum_{j=1}^{n-1} p_{j|\lambda}\right)$.[7] Using (2), the expected revenue function for firm N can be simplified further as:

$$\Pi_N\left(m,n\right) = \sum_{j=1}^{n-1}\left[\frac{p_{j|\lambda}}{j+1}\cdot Z\left(j,m\right)\right] + \left[\frac{p_{j\geq n|\lambda}}{n+1}\cdot Z\left(n,m\right)\right], \tag{8}$$

where $Z\left(j,m\right) \triangleq \sum_{k=1}^{j} z\left(k,m\right) = \begin{cases} r_N\cdot\frac{j(j+1)}{2} & \\ r_N\cdot\left[\frac{m(m+1)}{2}+(j-m)\,m\right] - c\cdot\frac{(j-m)(j-m+1)}{2} & \text{if} \quad j>m \end{cases}$

By examining the marginal gain as we increase the booking capacity from n to $n+1$, we obtain the following result:

Proposition 1. *Under policy N, it is optimal for a firm with capacity m to set his booking capacity to $n^*(m)$ that satisfies*

$$n^*\left(m\right) = \begin{cases} \left\lfloor\sqrt{\left(\frac{r_N}{c}+1\right)m\left(m+1\right)}-1\right\rfloor & \text{if} \quad f\left(\left\lfloor\sqrt{\left(\frac{r_N}{c}+1\right)m\left(m+1\right)}-1\right\rfloor\right) > f\left(\lceil\cdot\rceil\right) \\ \left\lceil\sqrt{\left(\frac{r_N}{c}+1\right)m\left(m+1\right)}-1\right\rceil & \text{if} \quad f\left(\left\lfloor\sqrt{\left(\frac{r_N}{c}+1\right)m\left(m+1\right)}-1\right\rfloor\right) < f\left(\lceil\cdot\rceil\right) \end{cases} \tag{9}$$

where $f\left(n\right) = m\left(\frac{r_N}{c}+1\right)\left(\frac{2n+1-m}{n+1}\right) - n$ is a concave function in n and $\lfloor\bullet\rfloor$ and $\lceil\bullet\rceil$ are the "round-down" and "round-up" functions, respectively.

Observe from (9) that the optimal booking capacity $n^*\left(m\right)$ is an increasing function in $\frac{r_N}{c}$. This result is intuitive because the firm can afford to increase his booking capacity when the retail price r_N is high or when the penalty for overbooking c is low. Also, it is easy to see that it is optimal for the firm to overbook so that $n^*\left(m\right) \geq m$ when $r_N > c$.

Notice that the firm's optimal booking capacity $n^*\left(m\right)$ is the "ideal" booking capacity that is based on its capacity m and the tradeoff between r_N and c, but it is independent of the actual number of customers A_N who attempt to reserve; i.e., it is independent of the demand rate λ and the reserve probability γ_N. To elaborate, consider the case when the firm sets a different booking capacity n, where $n < n^*\left(m\right)$. Suppose the number of customers who would like to reserve is $A_N \leq n$. Then the firm will accept the same number of reservations and obtain the same expected revenue regardless whether the booking capacity is equal to n or $n^*\left(m\right)$. Next, suppose $n < A_N < n^*\left(m\right)$. Then the firm will accept a larger number of reservations for the case when the booking capacity is $n^*\left(m\right)$. Because the firm's expected revenue $\Pi_N\left(m,n\right)$ given in

[7] For notational convenience, let $p_{j|\xi} = \Pr\left\{X=j\,|\,\xi\right\}$ for any random variable $X\sim Poi\left(\xi\right)$. For example, when the firm offers policy N, the number of customers who would attempt to reserve is a Poisson random variable $A_N \sim Poi\left(\gamma_N\lambda\right)$. Hence, $p_{j|\gamma_N\lambda} = \Pr\left\{A_N=j\right\} = \frac{e^{\gamma_N\lambda}\left(\gamma_N\lambda\right)^j}{j!}$.

(8) is concave in n, the firm will earn a higher expected revenue for the case when the booking capacity is $n^*(m)$. Finally, suppose $A_N > n^*(m) > n$. Then we can use the same argument to show that the firm will earn a higher expected revenue for the case when the booking capacity is $n^*(m)$. Hence, the firm is worse off by setting his booking capacity $n < n^*(m)$. We can use the same approach to argue that the firm will be worse off if he sets a different booking capacity $n > n^*(m)$. This explains why the optimal booking capacity $n^*(m)$ is independent of the demand rate λ.

We now examine the impact of the capacity m and the ratio $\frac{r_N}{c}$ on the optimal booking capacity $n^*(m)$ given in (9). Observe from (9) that $n^*(m)$ is increasing and concave in m so that it exhibits the "pooling" effect of having multiple units of capacity. To examine the pooling effect further, we compare the optimal booking capacity for a single firm with capacity of m units (i.e. $n^*(m)$) and the total optimal booking capacity for m "independent" firms, each of which has 1 unit of capacity (i.e., $m \cdot n^*(1)$). By considering (9), we establish the following lemma.

Corollary 2. *Under policy N, $n^*(m) > m \cdot n^*(1)$ if and only if*

$$m < \frac{1}{\frac{r_N}{c} + 2 + 2\sqrt{2\left(\frac{r_N}{c} + 1\right)}} \tag{10}$$

Moreover, this threshold is decreasing in the ratio $\frac{r_N}{c}$.

The above Corollary suggests that the pooling effect is more prominent (i.e., $n^*(m) > m \cdot n^*(1)$) when capacity m is below a certain threshold and that threshold is decreasing in the ratio $\frac{r_N}{c}$. This result is intuitive, because when the ratio $\frac{r_N}{c}$ is sufficiently small, each of those m independent firms with single unit capacity will have to be more cautious when setting their booking capacity in order to avoid the risk of having to compensate too many customers, whereas when the ratio $\frac{r_N}{c}$ is large, then they can be more aggressive when setting their booking capacity.

Next, let us examine the impact of the demand rate λ on the firm's optimal expected profit $\Pi_N(m, n^*(m))$. In view of the expressions given in (8), the expression for $\Pi_N(m, n^*(m))$ is complex. However, we are able to obtain the following result:

Corollary 3. *For any given demand rate $\lambda > 0$, the firm's optimal expected revenue $\Pi_N(m, n^*(m))$ under policy N is bounded above by $\frac{\lambda r_N}{2}$. In addition, $\lim_{m\to\infty} \Pi_N(m, n^*(m)) = \frac{\lambda r_N}{2}$.*

The result stated in Corollary 3 is intuitive for the following reasons. For a firm with capacity m, the firm cannot earn more than he would have earned under the "best case scenario" in which he accepts all reservations without paying any penalty to those denied customers. By noting that the firm will earn $r_N \cdot E(A) \cdot E(\psi) = \frac{\lambda r_N}{2}$ in the best case scenario, we obtain an upper bound on $\Pi_N(m, n^*(m))$. Also, Corollary 3 asserts that this bound is tight when the firm's capacity $m \to \infty$.
Because the expression for $\Pi_N(m, n^*(m))$ is complex, analytical comparison between the firm's optimal revenue under policy N and under policy D is intractable. For this reason, we shall focus our comparison for the case when $m = 1$ in Section 3.4. In preparation, we establish the following Corollary that is intended to examine the property of the optimal booking capacity $n^*(1)$ for the case when $m = 1$.

Corollary 4. *When $m = 1$, the optimal booking capacity $n^*(1)$ for different ranges of the ratio $\frac{r_N}{c}$ can be described in the following table:*

$\frac{r_N}{c} \in$	$(0, 2]$	$[2, 5]$	$[5, 9]$	$[9, 14]$	$[14, 20]$
$n^*(1) =$	1	2	3	4	5

Corollary 4 suggests that when $m = 1$, the firm should set his optimal booking capacity $n^*(1)$ at a lower level so as to avoid paying too much penalty. This observation motivates us to compare the firm's expected revenue under policy N (for the case when the capacity $m = 1$ and the booking capacity n is small) and the firm's expected revenue under policy D (for any guarantee deposit d) in Section 3.4. This analytical comparison will enable us to establish a conjecture to verify numerically in Section 3.5.

3.3 Optimal Deposit d^* under Policy D

We now determine the optimal deposit d^* that maximizes the firm's expected revenue under policy D. Recall from Section 3.1.2 that the firm will not overbook; hence, the firm will accept R_D reservations under policy D, where $R_D = \min\{m, A_D\}$, $A_D \sim Poi(\gamma_D \lambda)$, and the reserve probability γ_D is given in (5). For any number of accepted reservations R_D, the number of customers who show up is denoted by S_D. By noting that the firm receives d for accepting a reservation and receives $(r_D - d)$ for honoring each customer who shows up for her reserved service, the firm's revenue satisfies $\Pi_D(m, d) = d \cdot R_D + (r_D - d) \cdot S_D$ for any realization of S_D and R_D. Also, due to the fact that $\psi \sim U\left[\frac{d}{x_D + d}, 1\right]$, the conditional probability of $S_D = k$ given $R_D = j$ is given by (6). These two observations enable us to determine the firm's expected revenue for any deposit d, where

$$\Pi_D(m, d) = d\left(\sum_{j=1}^{m-1} j p_{j|\lambda\gamma_D} + m p_{\geq m|\lambda\gamma_D}\right) + (r_D - d)\left[\sum_{j=1}^{m-1}\sum_{k=1}^{j} k \Pr\{k \mid j\} p_{j|\lambda\gamma_D} + m p_{j\geq m|\lambda\gamma_D} \sum_{k=1}^{m} k \Pr\{k \mid j\}\right] \quad (11)$$

By noting the fact that

$$\sum_{k=1}^{j} k \Pr\{k \mid j\} = \sum_{k=1}^{j} k \int_{\frac{d}{x_D+d}}^{1} \binom{j}{k} \psi^k (1-\psi)^{j-k} \cdot \frac{1}{1 - \frac{d}{x_D+d}} d\psi = \frac{j}{2} \cdot \frac{x_D + 2d}{x_D + d},$$

$\Pi_D(m, d)$ can be simplified as:

$$\Pi_D(m, d) = \left[\frac{r_D x_D + d(r_D + v)}{2(x_D + d)}\right]\left[\sum_{j=1}^{m-1} j \cdot p_{j|\lambda\cdot\gamma_D} + m \cdot p_{j\geq m|\lambda\cdot\gamma_D}\right] \quad (12)$$

By differentiating (12) with respect to d and by considering the first-order condition, we establish the following result for the case when $\theta = 0$. When $\theta > 0$, we can impose this bound via truncation as illustrated in Proposition 7.

Proposition 5. *Under policy D, it is optimal for a firm with capacity m to charge a guarantee deposit*

$d^*(m)$ *that satisfies:*

$$[(r_D+d)(x_D+d)+(r_D-d)d]\left[(x_D+d)P_2-((\lambda+m)x_D+md)P_1+\lambda x_D m P_0\right] \\ = vx_D(x_D+d)(m+P_1-mP_0), \quad (13)$$

where $P_k \triangleq \sum_{j=0}^{m-1} j^k \cdot p_{j|\lambda\gamma_D}$.

Although there is no explicit closed form expression for the optimal guarantee deposit $d^*(m)$ and the corresponding optimal revenue $\Pi_D(m, d^*(m))$, we can develop the following characteristics. First, by using the implicit function theorem, one can show that $d^*(m)$ is decreasing in m. Also, it can be seen from (12) that the firm's expected profit is increasing in its capacity m. In addition, when the capacity m is sufficiently large, we obtain the following result:

Proposition 6. *Under policy D, the optimal deposit* $d^*(m)$ *is bounded below so that* $d^*(m) \geq \frac{(v-r_D)^2}{v+r_D} > 0$. *Also, the firm's optimal expected revenue* $\Pi_D(m, d^*(m))$ *is bounded above so that* $\Pi_D(m, d^*(m)) \leq \frac{\lambda(v+r_D)^2}{8v}$. *Moreover, these bounds are tight when* $m \to \infty$ *so that*

$$\lim_{m\to\infty} d^*(m) = \frac{(v-r_D)^2}{v+r_D} > 0 \quad \textit{and} \quad \lim_{m\to\infty} \Pi_D(m, d^*(m)) = \frac{\lambda(v+r_D)^2}{8v} \quad (14)$$

The above Proposition asserts that, regardless of the value of the minimum deposit θ, the firm should always charge a positive amount of deposit $d^*(m) > 0$ under policy D.

Although we show that the optimal deposit $d^*(m)$ satisfies (13) in Proposition 5, we were unable to show its uniqueness. To investigate this matter further, let us consider the special case when $m = 1$. It follows from (12) that $\Pi_D(1, d)$ takes the following simplified form when $m = 1$, where

$$\Pi_D(1,d) = \left[\frac{r_D x_D + d(r_D+v)}{2(x_D+d)}\right]\left[1-e^{-\frac{\lambda\cdot x_D}{x_D+d}}\right]. \quad (15)$$

By considering the first-order condition associated with (15), we establish the following result:

Proposition 7. *Under policy D, it is optimal for a firm with capacity* $m = 1$ *to charge a guarantee deposit* $d^*(1) > 0$ *that maximizes the firm's expected profit* $\Pi_D(1, d)$, *where* $d^*(1) = \max\left\{\theta, \min\left\{r_D, d'(1)\right\}\right\}$ *and* $d'(1)$ *is the unique solution to the following equation:*

$$\lambda \cdot e^{-\frac{\lambda\cdot x_D}{x_D+d}} \cdot \left[\frac{r_D\cdot x_D + d\cdot(r_D+v)}{x_D+d}\right] = \left(1-e^{-\frac{\lambda\cdot x_D}{x_D+d}}\right)v \quad (16)$$

Also, $d^*(1)$ *is increasing in the rate* λ *and* $d^*(1) \to r_D$ *as* $\lambda \to \infty$. *Moreover, it is optimal for the firm to charge a "partial" deposit (i.e.* $d^*(1) < r_D$ *) if and only if* $1 + 2\lambda\left(\frac{r_D}{v}\right) > e^{\lambda\left(1-\frac{r_D}{v}\right)}$.

Proposition 7 can be interpreted as follows. First, when the demand rate λ is small, the firm needs to charge a small deposit d to increase the reserve probability γ_D given in (5) so that the number of

reservations $R_D = \min\{1,\, A_D\}$ is sufficient. On the contrary, when the demand rate λ is large, the firm can afford to charge a higher deposit d by taking reservations from those customers with high show up probability ψ that is uniformly distributed over $\left[\frac{d}{x_D+d},\, 1\right]$. This explains why $d^*(1)$ is increasing in the rate λ and $d^*(1,d) \to r_D$ as $\lambda \to \infty$.

Next, observe that $1 + 2\lambda\left(\frac{r_D}{v}\right)$ is a strictly increasing function in $\left(\frac{r_D}{v}\right)$, that $e^{\lambda\left(1-\frac{r_D}{v}\right)}$ is a decreasing function in $\left(\frac{r_D}{v}\right)$, that $1 + 2\lambda\left(\frac{r_D}{v}\right) < e^{\lambda\left(1-\frac{r_D}{v}\right)}$ when $\left(\frac{r_D}{v}\right) = 0$, and that $1 + 2\lambda\left(\frac{r_D}{v}\right) > e^{\lambda(1-\frac{r_D}{v})}$ when $\frac{r_D}{v} = 1$, we can conclude that $1 + 2\lambda\left(\frac{r_D}{v}\right) > e^{\lambda\left(1-\frac{r_D}{v}\right)}$ when $\left(\frac{r_D}{v}\right)$ is sufficiently close to 1 so that $d^*(1) < r_D$. Intuitively speaking, when$\left(\frac{r_D}{v}\right)$ is sufficiently close to 1, customers are more likely to show up for their service because their show up probability ψ under policy D is uniformly distributed over $\left[\frac{d}{x_D+d},\, 1\right] = \left[\frac{d}{(v-r_D)+d},\, 1\right]$. Hence, the firm can afford to offer partial deposit $d^*(1) < r_D$ without the fear of losing the the remaining portion of the revenue $(r_D - d^*(1))$ due to no-shows. On the contrary, when the the price r_D is relatively low in comparison to the customer valuation v, the customer may not show up for her reserved service. As a way to reduce the potential loss of the remaining portion of the revenue $(r_D - d^*(1))$ due to no shows, the firm should request full deposit so that $d^*(1) = r_D$.

3.4 Choosing the Reservation Policy: N versus D.

Because analytical comparison of $\Pi_N(m,\, n^*(m))$ and $\Pi_D(m,\, d^*(m))$ is intractable, we first examine the case when the capacity m is very large (as $m \to \infty$) and then study the case when m is small, say, $m = 1$. The results associated with these two special cases will enable us to develop a conjecture for any general value of $m > 1$, which we will examine numerically in Section 3.5. To begin, let us consider the case when m is very large. By using the results stated in Lemmas 3 and 6, we obtain the following result:

Corollary 8. *When the capacity m is sufficiently large, say, $m \to \infty$, policy N dominates policy D if and only if:*

$$r_N > r_D + \frac{(v - r_D)^2}{4v}.$$

This corollary suggests that, when capacity is abundant, say $m \to \infty$, policy N is preferred if and only if the firm can charge a higher retail price r_N under policy N.

Next, let us consider the case when $m = 1$. We now compare $\Pi_N(1,n)$, the firm's expected revenue for any given booking capacity n under policy N, and $\Pi_D(1,d)$, the firm's expected profit for any guarantee deposit d under policy D. Because $\Pi_N(1,n)$ given in (8) is a complex function, we shall compare these two profit functions for small values of n analytically. (We shall compare the optimal expected revenues $\Pi_N(m,\, n^*(m))$ and $\Pi_D(m,\, d^*(m))$ numerically in Section 3.5.) In view of Corollary 4, it is reasonable to focus on small values of n because the optimal booking capacity n^* is likely to be small. In preparation, apply (7) and (2) to show that:

$$\Pi_N(1,1) = r_N \cdot \Pr\{1 \mid 1\} \cdot \left(1 - p_{0|\lambda}\right) = \frac{r_N}{2} \cdot \left(1 - e^{-\lambda}\right) \quad \text{, and} \tag{17}$$

$$\Pi_N(1,2) = r_N \cdot \Pr\{1|1\} \cdot p_{1|\lambda} + [r_N \cdot \Pr\{1|2\} + (r_N - c) \cdot \Pr\{2\,|\,2\}] \cdot \left(1 - p_{0|\lambda} - p_{1|\lambda}\right)$$
$$= \frac{r_N}{2}\lambda e^{-\lambda} + \frac{2r_N - c}{3}\left(1 - e^{-\lambda} - \lambda e^{-\lambda}\right). \quad (18)$$

By considering $\Pi_D(1,d)$ given in (15), we establish the following result:

Proposition 9. *For any given $d > 0$, there exist unique thresholds $\tau_1, \tau_2 > 0$ such that $\Pi_N(1,1) > \Pi_D(1,d)$ and $\Pi_N(1,2) > \Pi_D(1,d)$ if and only if the demand rate $\lambda < \tau_1$ and $\lambda < \tau_2$, respectively. Moreover, $\tau_1 = \infty$ and $\tau_2 = \infty$ if and only if $r_N \geq r_D + \frac{dv}{v - r_d + d}$ and $2r_N \geq \frac{3}{2}\left[\frac{r_D(v - r_D) + d(r_D + v)}{v - r_D + d}\right] + c$, respectively.*

Proposition 9 can be interpreted as follows. When the retail price r_N is sufficiently large in relation to r_D, the firm can obtain a higher expected revenue under policy N for any demand rate λ. However, when r_N is below a certain threshold, policy N dominates policy D if and only if the demand rate λ is sufficiently low. This is because, when the number of customers in the system is low, the firm can use the no deposit policy N to entice more customers to reserve so as to obtain a higher expected revenue. However, due to the required deposit d under policy D, the firm is unable to receive enough reservations from customers who are willing to pay the upfront deposit d and the remaining amount $(r_D - d)$ later. Therefore, deposit policy N dominates the guarantee deposit policy D when the demand rate λ is small.

By considering the conditions as stated in Corollary 8 (for the case when $m \to \infty$) and Proposition 9 (for the case when $m = 1$) , we develop the following conjecture that speculates the conditions under which deposit policy N dominates the guarantee deposit policy D for any general value of $m \geq 1$. We shall examine this conjecture numerically in the next section.

Conjecture 1. *For any capacity m, policy N dominates policy D (i.e., $\Pi_N(m, n^*) > \Pi_D(m, d^*)$ when (a) the demand rate λ is sufficiently low; (b) the retail price r_N is sufficiently large in relation to r_D; (c) the penalty c is sufficiently low; and (d) the customer valuation v is sufficiently low.*

3.5 Numerical Analysis

In this section, we first develop numerical experiments to test Conjecture 1 established in the last section. Then we examine the characteristics of the optimal retail prices r_N^* and r_D^*.

3.5.1 The Dominance of Policy N for any capacity $m \geq 1$

We examine the conditions under which policy N dominates policy D for any general capacity $m \geq 1$, we construct our numerical experiments as follows. In each of the experiments, we set $m = 5$, $r_D = 80$, and we vary the demand rate λ from 0.1 to 20. First, we investigate the effect of the price r_N on the dominance of policy N. To do so, we vary r_N from 80 to 130, but we fix the value of v and c so that $v = 150$ and $c = 20$. Figure 1 (a) (below) reports the region in which policy N dominates policy D so that $\Pi_N(1, n^*) > \Pi_D(1, d^*)$. Specifically, when r_N is high in relation to r_D, policy N dominates policy

D regardless of the demand rate λ. This result is consistent with the first statement of Proposition 9. Also, when r_N is in the medium range, policy N dominates policy D if and only if the demand rate λ is below a certain threshold. The result is consistent with the second statement of Proposition 9. Overall, our results support statements (a) and (b) of Conjecture 1.

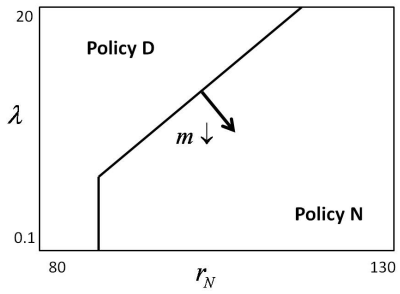

Figure 1 (a)

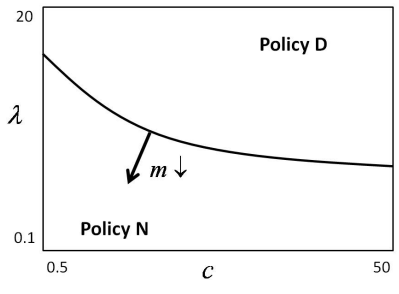

Figure 1 (b)

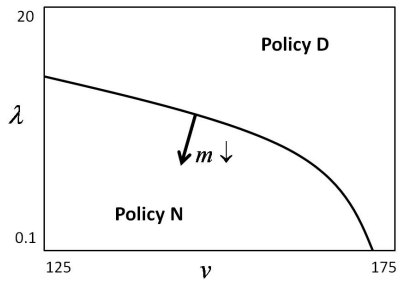

Figure 1 (c)

Figure 1. The dominance of Policy N

Second, to examine the effect of the penalty c on the dominance of policy N, we vary c from 0.5 to 50, but we fix the value of r_N and v so that $r_N = 80$, and $v = 150$. Figure 1b reports the region in which policy N dominates policy D so that $\Pi_N(1, n^*) > \Pi_D(1, d^*)$. Observe from Figure 1 (b) that, for any penalty c, policy N dominates policy D when the demand rate λ is below a certain threshold, and this threshold increases as c decreases. This result is intuitive because, as the penalty c decreases, the firm can afford to overbook more. This observation is based on the fact that the optimal booking capacity $n^*(m)$ given in Proposition 1 is decreasing in c. Hence, as the penalty c decreases, policy N becomes more attractive. Therefore our numerical result supports statement (c) of Conjecture 1.

Next, to investigate the effect of the valuation v on the dominance of policy N, we vary v from 125 to 175, but we fix the value of r_N and c so that $r_N = 80$, and $c = 20$. Figure 1 (c) reports the region in which policy N dominates policy D so that $\Pi_N(1, n^*) > \Pi_D(1, d^*)$. As shown in Figure 1 (c), for any demand rate λ, policy N dominates policy D if and only if the valuation v is sufficiently low. This result can be explained as follows. When the valuation v is sufficiently high, customers are more willing to pay a guarantee deposit (even if d^* is high) and more likely to show up for the reserved service under policy D. Because the firm can obtain a higher expected revenue under policy D, policy D dominates when customer valuation v is sufficiently high. This result supports statement (d) of Conjecture 1.

Finally, to investigate the impact of capacity m on the dominant policy, we conduct the same set of experiments as described above by varying m from 1 to 10, and we obtain similar results as before except that the threshold curve decreases as m decreases. The pattern of this effect is indicated by the arrow associated with $m \downarrow$(decreasing in m) as shown in Figures 1 (a), 1 (b) and 1 (c) (To reduce repetition, the detailed figures are omitted). Hence, Conjecture 1 continues to hold for different values of m. In addition, as capacity m decreases, policy N becomes less desirable. This result is intuitive because, as capacity m decreases, the "effective" demand rate per unit of capacity $\lambda' = \left(\frac{\lambda}{m}\right)$ increases. Hence, as the "effective"

demand rate increases, statement (a) of Conjecture 1 hinted that policy D will become more desirable because there will be more customers with high show up probability ψ who are willing to pay a guarantee deposit and show up for the reserved service under policy D. Hence, policy D becomes more desirable as capacity m decreases.

3.5.2 Optimal Retail Price

We have conducted our analysis for the case when the retail prices r_N and r_D are given. We now determine the optimal retail price under each deposit policy. First, let us analyze the optimal retail price r_N^* that maximizes the firm's expected revenue under policy N. Recall from Section 3.1.1 that all customers will attempt to reserve as long as the retail price $r_N \leq v$. Therefore, it is always optimal for the firm to set his optimal retail price $r_N^* = v$ under policy N so that the firm can extract the entire surplus from the customers.

We now analyze the optimal retail price r_D^* that maximizes the firm's expected revenue under policy D. As one can observe from Propositions 5 and 7, it is extremely difficult to analyze r_D^* analytically. For this reason, we shall compute the optimal price r_D^*, the optimal deposit d^*, and the firm's optimal revenue $\Pi_D(m, d^*)$ numerically. To examine the effect of capacity, demand, and minimum deposit, we determine r_D^*, d^*, and $\Pi_D(m, d^*)$ for different combinations of m, λ and θ in a succinct manner. By considering the case when customer valuation $v = 150$, we obtain our numerical results as summarized in Table 1 below. Observe from Table 1 that, in all cases, the optimal retail price $r_D^* < 150 = v = r_N^*$. This result suggests that, relative to policy N, a firm should charge a lower retail price r_D^* under policy D. This result is consistent with common practice: most hotels offer rooms with non-refundable deposits at lower daily rates.

	$m = 1$		$m = 5$	
	$\lambda = 2$	$\lambda = 8$	$\lambda = 2$	$\lambda = 8$
$\theta = 5$	$r_D^* = 142.75$ $d^* = 5$ $\Pi_D^* = 70.76$	$\begin{pmatrix} 147.6 \\ 5 \\ 115.18 \end{pmatrix}$	$\begin{pmatrix} 135.2 \\ 8.62 \\ 128.9 \end{pmatrix}$	$\begin{pmatrix} 142.2 \\ 15.88 \\ 408.26 \end{pmatrix}$
$\theta = 10$	$r_D^* = 137.55$ $d^* = 10$ $\Pi_D^* = 68.48$	$\begin{pmatrix} 145.35 \\ 10 \\ 114.09 \end{pmatrix}$	$\begin{pmatrix} 128.1 \\ 13.34 \\ 119.91 \end{pmatrix}$	$\begin{pmatrix} 136.35 \\ 28.12 \\ 393.65 \end{pmatrix}$

Table 1. Optimal retail price, optimal deposit, and optimal expected revenue under policy D.

Let us examine the results reported in Table 1. First, by comparing the results reported in columns 1 and 2 (and columns 3 and 4), we can examine the impact of demand rate λ for any given capacity m and minimum deposit θ. Observe that the optimal price r_D^*, the optimal deposit d^*, and the firm's optimal expected revenue are non-decreasing in the demand rate λ. This result is intuitive because, as more customers are present in the system, the firm can afford to charge a higher retail price and a higher deposit without the fear of not getting enough reservations.

Next, by comparing the results reported in columns 1 and 3 (and columns 2 and 4), we can evaluate the effect of capacity m. Clearly, the firm can always obtain a higher expected revenue as the capacity increases. However, the impact of capacity m on the optimal price r_D^* and the optimal deposit d^* is unclear. Our comparisons suggest that, as capacity m increases, it is optimal for the firm to reduce his optimal price r_D^* and to increase his optimal deposit d^*. Intuitively speaking, when the firm has more capacity, he is more concerned about the reserve probability γ_D given in (5), where $\gamma_D = \frac{v-r_D}{d+v-r_D}$. By noting that γ_D is decreasing in the deposit d and the retail price r_D, the firm can increase the reserve probability γ_D by reducing the deposit and/or the retail price. However, to mitigate the potential loss of revenue due to "no shows," our numerical examples suggest that it is advantageous for the firm to reduce the retail price and to increase the deposit as capacity becomes more abundant. This result is consistent with the way policy D is implemented in practice: during the low season (i.e., when the capacity is large in relation to customer demand), most firms tend to reduce the retail price but they often command a guarantee deposit.

Finally, by comparing the results reported in rows 1 and 2, we can examine the impact of the minimum deposit requirement θ. In this case, it is quite clear that the optimal deposit d^* is increasing in θ. However, to ensure that the reserve probability γ_D given in (5) does not decrease too much, the firm needs to compensate this increase in the minimum deposit by offering a lower retail price.

In summary, our numerical results support Conjecture 1 established in Section 3.4 that policy N dominates policy D (i.e., $\Pi_N(m, n^*) > \Pi_D(m, d^*)$) when (a) the "effective" demand rate $\frac{\lambda}{m}$ is sufficiently low; (b) the retail price r_N is sufficiently large in relation to r_D; (c) the customer valuation v is sufficiently low; and (d) the penalty c is sufficiently low. Also, our numerical analysis enables us to gain a better understanding about how capacity m, customer demand rate λ, and the minimum deposit θ affect the optimal retail price r_D^* and optimal deposit d^* under policy D. Specifically, our numerical analysis suggests that it is optimal for the firm to set a higher price under policy N so that $r_N^* > r_D^*$. In view of this result, we shall examine the duopolistic case in the next section by focusing our attention on the case when $r_N > r_D$.

4 Duopolistic Case

We now extend our analysis for the monopolistic case presented in Section 3 to the duopolistic case in which 2 identical firms compete in the same market with customer demand $A \sim Poi(\lambda)$. We consider the case when the customer will behave in the following manner: (a) each customer will attempt to reserve with the firm that yields the higher expected surplus that is non-negative; (b) each customer will leave the system if the higher expected surplus is negative; and (c) each customer will leave the system if her attempt to reserve with the chosen firm is unsuccessful. The sequence of events is the same as described in the monopolistic case. Specifically, prior to the presence of customers in the system, both firms will announce their deposit policies simultaneously; i.e., a firm will announce his booking capacity n if policy N is chosen and the required deposit d if policy D is selected. Then, for each customer who is present in the system, she would infer the deny probability $(1-\beta)$ when she evaluates the expected surplus for

reserving with a firm who adopts policy N. Figure 2 depicts the sequence of events, where $d = 0$ when a firm adopts policy N and $\beta = 1$ when a firm adopts policy D.

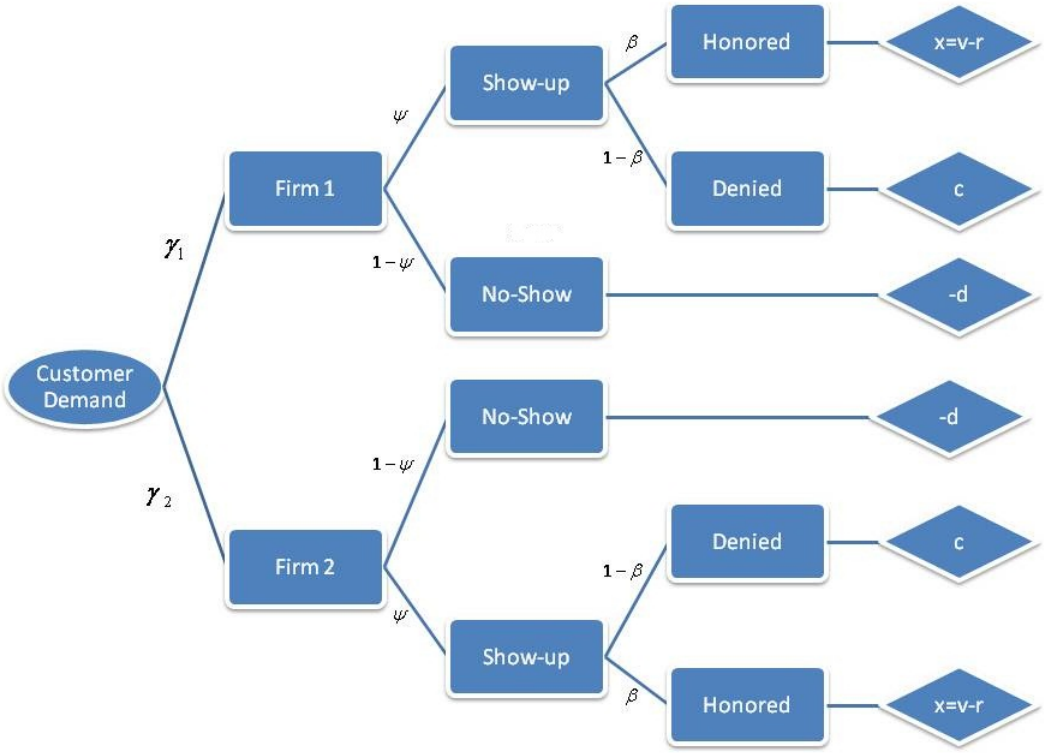

Figure 2. The sequence of events.

This section is organized as follows. In Sections 4.1-4.3 we examine the equilibria associated with the following three subgames: (1) (N, N): both firms adopt the no deposit policy N; (2) (D, D): both firms adopt the guarantee deposit policy D; and (3) (N, D): one firm adopts policy N, while the other adopts policy D. By using the firm's expected revenue obtained in equilibrium for different subgames, we characterize the Nash equilibrium of the meta-game in Section 4.4. To identify the conditions under which a particular pair of deposit policies (N, N), (D, D), (N, D) or (D, N) will constitute an equilibrium in the meta-game, we report our extensive numerical analysis in Section 4.5. We show that, in most cases, both firms will choose the same deposit policy in equilibrium (i.e., either (N, N) or (D, D) in equilibrium). In addition, we show the Prisoner's Dilemma can occur in this meta-game. Also, when both firms charge the same retail price, we show the existence of an equilibrium in which both firms adopt the no deposit policy. Because each subgame involves the analysis of a competitive game between two firms within which a separate Stackelberg game is played between each firm and the customers, the analysis of each subgame is non-trivial and the analysis of the meta-game is highly complex. To obtain tractable results, we shall limit our analysis to the case when the capacity of each firm $m = 1$ and when the retail price is policy-dependent (but firm-independent) so that $r_N > r_D$.

4.1 Subgame 1: Both Firms Adopt Policy N

Consider the case when both firms adopt policy N and charge the same retail price r_N. If a customer makes a reservation with firm i, $i = 1, 2$, then we can apply (1) to show that her expected surplus is equal to: $\pi_{Ni} = \psi\left[\beta_i x_N + (1-\beta_i)\, c\right]$, where $i = 1, 2$. We now establish the subgame equilibrium. In preparation, let us make the following observations: (1) the expected surplus π_{Ni} is increasing in β_i because of our "no arbitrage" assumption $x_N = v - r_N > c$; (2) all customers will attempt to reserve with the firm that has a lower deny probability $(1-\beta_i)$; and (3) each firm i can always guarantee that the reserved service will be honored (i.e. $\beta = 1$) by setting his booking capacity to $n_i = 1$. By using these three observations, it is easy to check that there exists a Nash equilibrium in which neither firm will overbook (i.e. $(n_1, n_2) = (1, 1)$ is an equilibrium). To identify other equilibria, let us establish the following Lemma:

Lemma 10. *Consider the case when both firms adopt policy N. Then the service probability of firm 1 will be equal to the service probability of firm 2 (i.e. $\beta_1 = \beta_2$) and each firm will have the same reserve probability $\gamma_{N1} = \gamma_{N2} = \frac{1}{2}$ in equilibrium.*

Because Lemma 10 suggests that the subgame (N, N) may have multiple equilibria, which are symmetric, let us consider the case when $n_1 = n_2 = n$. By using the fact that the show up probability $\psi \sim U[0, 1]$, we can apply (8) to show that each firm's expected revenue can be expressed as[8]:

$$\Pi_{N,N}(n\,;\, n) = \sum_{j=1}^{n-1} \frac{p_{j|\gamma_{Ni}\lambda}}{j+1}\left[j \cdot r_N - \frac{j\,(j-1)}{2} \cdot c\right] + \frac{p_{j\geq n|\gamma_{Ni}\lambda}}{n+1}\left[n \cdot r_N - \frac{n\,(n-1)}{2} \cdot c\right]. \tag{19}$$

The following result establishes the Nash equilibria for this subgame and the conditions under which one equilibrium dominates the other:

Proposition 11. *Suppose both firms adopt policy N. Then (i) there exists an equilibrium in which both firms will set their booking capacity to $n_{N,N} = 1$ and (ii) if $\Pi_N\left(n^*; \frac{\lambda}{2}\right) > \frac{r_N}{2}\left(1 - e^{-\lambda}\right)$, then there exists a payoff-dominant equilibrium in which both firms set their booking capacity to $n_{N,N} = n^*$, where n^* and $\Pi_N(\cdot)$ are given by (9) and (8), respectively.*

The result stated in Proposition 11 complements the result presented in Lim (2009). Specifically, when low valued customers only arrive early and high valuation customers only arrive late, Lim (2009) shows that it is a dominant equilibrium policy for both firms to overbook so that $n_{N,N} > 1$. Our results differs from her's slightly, because allows customers with different valuations to be present in the system simultaneously. Therefore, Proposition 11 identifies the condition (i.e., $\Pi_N\left(n^*; \frac{\lambda}{2}\right) > \frac{r_N}{2}\left(1 - e^{-\lambda}\right)$) under which both firms should overbook in equilibrium.

[8] In the duopolistic case, we use the notation $\Pi_{X,Y,}(x,\, y)$ to denote a firm's expected revenue when he adopts policy X with decision x given that his competitor adopts policy Y with decision y. For example, $\Pi_{D,N}(d,\, n)$ represents a firm's expected revenue when he adopts policy D with a required deposit d given that the other firm adopts policy N with a booking capacity n.

To investigate the condition under which one equilibrium dominates the other, let us consider the case when $n^* = 2$. This case is interesting because it captures the case when $2 \leq \frac{r_N}{c} \leq 5$ as reported in Corollary 4 in Section 3.2.

Corollary 12. *Suppose* $2 \leq \frac{r_N}{c} \leq 5$ *so that* $n^* = 2$. *Then there exists a threshold* λ_{crit} *so that the equilibrium* $(n^*, n^*) = (2, 2)$ *dominates the equilibrium* $(1, 1)$ *when demand rate* $\lambda > \lambda_{crit}$.

In view of Corollary 12, we establish the following Conjecture, which we shall examine numerically in Section 4.6.

Conjecture 2. *When both firms adopt policy N, the equilibrium* (n^*, n^*) *dominates* $(1, 1)$ *if and only if the demand rate* λ *is sufficiently large.*

4.2 Subgame 2: Both Firms Adopt Policy D

We now consider the subgame in which both firms adopt policy D so that firm i requires a non-refundable deposit $d_i \geq \theta$ for $i = 1, 2$. Without loss of generality, consider the case when $d_2 > d_1$. For any customer with show-up probability ψ, it is easy to check from (4) that $\pi_D(d_1) = -d_1 + \psi \cdot (x_D + d_1) > -d_2 + \psi \cdot (x_D + d_2) = \pi_D(d_2)$. Combine this observation with the requirement that $\pi_D(d_1) \geq 0$, we can conclude that every customer with $\psi \geq \frac{d_1}{x_D + d_1}$ will attempt to reserve with firm 1. Also, every customer with $\psi < \frac{d_1}{x_D + d_1}$ will leave the system because her expected surplus is negative. Hence, to compete for customer reservations under policy D, both firms will undercut each other's required deposit. Consequently, in equilibrium, both firms will set their deposits at the minimum value $d_1 = d_2 = \theta$, because no firm can obtain a higher expected revenue by setting his deposit above θ. Moreover, it is easy to check that this equilibrium is unique. This proves the following Proposition:

Proposition 13. *When both firms adopt the guarantee deposit policy D, both firms will require the same deposit* $d^*_{D,D} \equiv \theta$ *in equilibrium.*

Because both firms require deposit θ in equilibrium and because both firms are identical, customers will attempt to reserve with each firm with the same probability γ_D, where

$$\gamma_D = \frac{1}{2} \cdot \Pr\{\pi_D(\theta) \geq 0\} = \frac{1}{2} \cdot \Pr\left\{\psi \geq \frac{\theta}{x_D + \theta}\right\} = \frac{x_D}{2 \cdot (x_D + \theta)} \tag{20}$$

Observe that the reserve probability $\gamma_D < \frac{1}{2}$ for any $\theta > 0$. Because the show up probability ψ of each customer who reserves with either firm satisfies $\psi \sim U\left(\frac{\theta}{x_D + \theta}, 1\right)$, one can apply (12) to show that each firm's expected revenue is equal to:

$$\Pi_{D,D}(\theta, \theta) = \frac{r_D x_D + \theta(r_D + v)}{2(x_D + \theta)} \cdot \left[1 - e^{-\frac{\lambda x_D}{2(x_D + \theta)}}\right] \tag{21}$$

By using the fact that $\Pi_{D,D}(\theta, \theta) \to 0$ as $\lambda \to 0$, that $\Pi_{D,D}(\theta, \theta)$ is strictly increasing λ, and that $\Pi_{D,D}(\theta, \theta) \to \frac{r_D \cdot x_D + \theta \cdot (r_D + v)}{2 \cdot (x_D + \theta)}$ when $\lambda \to \infty$, the expected profit of each firm in equilibrium $\Pi_{D,D}(\theta, \theta)$

is bounded above by $\frac{r_D \cdot x_D + \theta \cdot (r_D + v)}{2 \cdot (x_D + \theta)}$; (i.e. $\Pi_{D,D}(\theta, \theta) \leq \frac{r_D \cdot x_D + \theta \cdot (r_D + v)}{2 \cdot (x_D + \theta)}$) and this bound is tight when the demand rate $\lambda \to \infty$.

4.3 Subgame 3: One firm adopts Policy N, while the other firm adopts Policy D.

We now examine the subgame in which one firm adopts policy N, while the other adopts policy D. (For ease of exposition, we shall refer to the firm who adopts policy N as firm N and the other firm as firm D.) In this case, each customer has to decide whether to reserve and which firm to reserve with. First, recall from (1) that each customer can obtain an expected surplus $\pi_N = \psi \cdot [\beta \cdot x_N + (1 - \beta) \cdot c] \geq 0$ by reserve with firm N. As a result, we can infer that all customers will always attempt to reserve with firm N unless they can obtain a higher surplus by reserving with firm D. Also, recall from (15) that each customer can obtain an expected surplus $\pi_D = -d + \psi \cdot (x_D + d)$ by reserving with firm D. Hence, we can conclude that each customer will attempt to reserve with firm D if $\pi_D > \pi_N \geq 0$ and reserve with firm N, otherwise. By comparing π_D with π_N, it is easy to check that a customer will attempt to reserve with firm D if her show-up probability $\psi > \frac{d}{d + (r_N - r_D) + (x_N - c) \cdot (1 - \beta)}$ and will attempt to reserve with firm N, otherwise. Because the show-up probability $\psi \sim U[0, 1]$, the probability that a customer attempts to reserve with firm D is $\gamma_D(d)$, where

$$\gamma_D(d) = \Pr\left\{\psi > \frac{d}{d + (r_N - r_D) + (x_N - c) \cdot (1 - \beta)}\right\} = \frac{(r_N - r_D) + (x_N - c) \cdot (1 - \beta)}{d + (r_N - r_D) + (x_N - c) \cdot (1 - \beta)} > 0. \quad (22)$$

Conversely, the probability that a customer will attempt to reserve with firm N is equal to $\alpha \equiv \gamma_N = 1 - \gamma_D(d)$, where

$$\alpha = \Pr\left\{\psi < \frac{d}{d + (r_N - r_D) + (x_N - c) \cdot (1 - \beta)}\right\} = \frac{d}{d + (r_N - r_D) + (x_N - c) \cdot (1 - \beta)} > 0. \quad (23)$$

In this case, we can interpret the probability α and $\gamma_D(d)$ as the "market share" of firm N and firm D, respectively. Thus, the number of customers who will attempt to reserve with firm N and firm D are Poisson random variables with rates $\lambda\alpha$ and $\lambda\gamma_D(d)$, respectively.

Observe from (23) that the reserve probability $\alpha \equiv \gamma_N = 1 - \gamma_D(d)$ depends on the probability β given in (3), where β depends on the following elements: (i) the booking capacity n selected by firm N; and (ii) $\alpha = 1 - \gamma_D(d)$, which depends on the the deposit d that chosen by firm D. Therefore, we need to use these circular relationships to estimate $\alpha = 1 - \gamma_D(d)$ in equilibrium. In preparation, we first analyze firm D's expected revenue for any given β. Then we analyze firm N's expected revenue for any given α.

First, let us consider the firm that adopts policy D. We now determine $\Pi_{D,N}(d, n|\beta)$, the expected revenue for firm D who imposes a deposit d given any β (when firm N sets his booking capacity to n). Recall from (22) that each customer will attempt to reserve with firm D if her show-up probability $\psi > \frac{d}{(x_N - c) \cdot (1 - \beta) + (r_N - r_D) + d}$. Therefore, the "effective" show-up probability of those who reserve with firm

D is uniformly distributed over $\left[\frac{d}{(x_N-c)\cdot(1-\beta)+(r_N-r_D)+d},\,1\right]$. Because $m=1$, one can apply (15) to show that:

$$\Pi_{D,N}\left(d,n|\beta\right)=\left[1-e^{-\lambda\frac{(r_N-r_D)+(x_N-c)\cdot(1-\beta)}{d+(r_N-r_D)+(x_N-c)\cdot(1-\beta)}}\right]\cdot\left[d+\left(1+\frac{d}{d+(r_N-r_D)+(x_N-c)\cdot(1-\beta)}\right)\cdot\frac{r_D-d}{2}\right] \tag{24}$$

By considering the first-order condition associated with the expected revenue $\Pi_{D,N}\left(d\,,n|\,\beta\right)$ given in (24), we establish the following result:

Lemma 14. *For any given $\beta\leq 1$, there exists an optimal deposit $d_{D,N}\left(\beta\right)$ that maximizes $\Pi_{D,N}\left(d\,,n|\,\beta\right)$, where $d_{D,N}\left(\beta\right)=\max\left\{\theta\,,\,\min\left\{d^{'}\left(\beta\right)\,,\,r_D\right\}\right\}$ and $d^{'}\left(\beta\right)$ satisfies:*

$$\left(e^{\lambda\frac{(r_N-r_D)+(x_N-c)(1-\beta)}{d+(r_N-r_D)+(x_N-c)(1-\beta)}}-1\right)\left[r_N+\left(x_N-c\right)\left(1-\beta\right)\right]=\lambda\left[2d+\left(1+\frac{d}{d+(r_N-r_D)+(x_N-c)\cdot(1-\beta)}\right)\left(r_D-d\right)\right] \tag{25}$$

Furthermore, $d_{D,N}\left(\beta\right)$ is a decreasing, continuous function of β.

Next, let us consider the firm that adopts policy N. We now determine $\Pi_{N,D}\left(n\,,d|\,\alpha\right)$; i.e. the expected revenue for firm N for any given reserve probability $\gamma_N=\alpha$. Recall from (23) that each customer will attempt to reserve with firm N if her show-up probability $\psi<\alpha$. Therefore, the "effective" show up probability of those who reserve with firm N is uniformly distributed over $[0,\,\alpha]$. When one firm adopts policy N and the other firm adopts policy D, one can use (7) and (2) to show that $\Pi_{N,D}\left(n\,,d|\,\alpha\right)$ satisfies

$$\Pi_{N,D}(n\,,d|\,\alpha)=\sum_{j=1}^{n-1}\left[\left(r_N+c\right)\left(\frac{j+(1-\alpha)^{j+1}}{\alpha\left(j+1\right)}\right)-\frac{cj\alpha}{2}\right]\cdot p_{j|\lambda\alpha}+\left[\left(r_N+c\right)\left(\frac{n+(1-\alpha)^{n+1}}{\alpha\left(n+1\right)}\right)-\frac{cn\alpha}{2}\right]\cdot p_{j\geq n|\lambda\alpha} \tag{26}$$

By using the same approach as in Proposition 1, we can obtain the following result:

Lemma 15. *For any given reserve probability $\alpha\in[0,1]$, there exists a unique booking capacity $n_{N,D}$ for firm N that maximizes $\Pi_{N,D}\left(n\,,d|\,\alpha\right)$, where $n_{N,D}\left(\alpha\right)$ satisfies:*

$$n_{N,D}\left(\alpha\right)=\arg\max_{n\in\mathbb{N}}\hat{f}\left(n;\alpha\right), \tag{27}$$

where $\hat{f}\left(n;\alpha\right)=\left(\frac{r_N}{c}+1\right)\cdot\frac{n+(1-\alpha)^{n+1}}{\alpha(n+1)}-\frac{n\alpha}{2}$ is a quasi-concave function of n.

Using the results stated in Lemmas 14 and 15, we can compute the optimal booking capacity $n_{N,D}(\alpha)$ and the optimal deposit $d_{N,D}\left(\beta\right)$ for any given values of β and α. It remains to show how to compute β

and α. To do so, let us review the aforementioned circular relationships among these quantities. First, recall from (3) that β is a function of $\gamma_N = \alpha$ and $n_{N,D}$, where $n_{N,D}$ can be expressed as a function of α by using Lemma 15. Given $\beta(\alpha, n_{N,D})$, we can express $d_{D,N}(\beta)$ as a function of α and $n_{N,D}$ by applying Lemma 14. It follows immediately from (23) that the reserve probability α must satisfy:

$$\alpha = \frac{d_{D,N}(\beta(\alpha, n_{N,D}(\alpha)))}{d_{D,N}(\beta(\alpha, n_{N,D}(\alpha))) + (r_N - r_D) + (x_N - c)[1 - \beta(\alpha, n_{N,D}(\alpha))]}. \tag{28}$$

Hence, by solving the fixed point (i.e. the reserve probability $\alpha^* \in (0, 1]$) that satisfies (28), we can retrieve other quantities as follows: we first compute the booking capacity $n^*_{N,D} = n_{N,D}(\alpha^*)$ for firm N by using Lemma 15, then compute $\beta^* = \beta\left(\alpha^*, n^*_{N,D}\right)$ using (3), and finally compute the guarantee deposit $d^*_{D,N} = d_{D,N}(\beta^*)$ for firm D by using Lemma 14.

The following proposition establishes existence of a Nash equilibrium in this subgame:

Proposition 16. *There exists a Nash equilibrium in this subgame where firm N sets his booking capacity to $n^*_{N,D}$ and firm D requires deposit $d^*_{D,N}$, where $\left(n^*_{N,D}, d^*_{D,N}\right)$ and the associated (α^*, β^*) satisfy (28).*

4.4 Analysis of Equilibria in the Meta-Game

By using the equilibrium outcomes (i.e. the booking capacity selected by firm N and the guarantee deposit chosen by firm D) as stated in Propositions 11, 13, and 16, associated with subgames (N, N), (D, D) and (N, D); respectively, we can determine the payoff of each firm in each of the three subgames. Table 2 provides a summary of the payoff function associated with each subgame, which constitutes the payoff function in the meta-game.

		Firm 2	
		Policy N	Policy D
Firm 1	Policy N	$(\Pi_{N,N}(n_{N,N}, n_{N,N})\ ,\ \Pi_{N,N}(n_{N,N}, n_{N,N}))$	$\left(\Pi_{N,D}\left(n^*_{N,D}\right)\ ,\ \Pi_{D,N}\left(d^*_{N,D}\right)\right)$
	Policy D	$\left(\Pi_{D,N}\left(d^*_{N,D}\right)\ ,\ \Pi_{N,D}\left(n^*_{N,D}\right)\right)$	$(\Pi_{D,D}(\theta, \theta)\ ,\ \Pi_{D,D}(\theta, \theta))$

Table 2. Payoff Function in the Meta-Game.

By examining the payoffs associated with the different subgames, we can establish the necessary and sufficient conditions for a particular pair of policy (i.e. $\{N, N\}$, $\{D, D\}$ or $\{N, D\}$) to be the equilibrium policy that the firms will adopt in the meta-game.

4.4.1 Equilibrium Policy $\{N, N\}$ in the Meta-Game

We now establish the necessary and sufficient condition for $\{N, N\}$ to be the equilibrium policy in the meta-game. First, let us examine the subgame $\{N, N\}$ in which both firms will set their equilibrium booking capacity in accord to $n_{N,N}$ stated in Proposition 11. By applying (19), we can compute each firm's expected revenue $\Pi_{N,N}(n_{N,N}, n_{N,N})$. Thus, policy $\{N, N\}$ will be the equilibrium policy in the meta-game if and only if neither firm can improve his expected revenue from a unilateral move by deviating

from the adopted policy N with booking capacity $n_{N,N}$. By symmetry, it suffices to analyze this condition associated with one firm who makes a unilateral move (while the other firm's policy is fixed at policy N with booking capacity $n_{N,N}$). First, observing that $n_{N,N}$ is the equilibrium booking capacity in subgame $\{N, N\}$, it is clear that this firm cannot improve his expected revenue if he makes a unilateral move by changing his booking capacity. Second, suppose this firm changes his policy from N to D. Then we can utilize the same approach as presented in Section 4.3 for the subgame $\{N, D\}$ to determine this firm's "best response" under policy D (denoted by guarantee deposit $\tilde{d}$), given that the other firm's policy is fixed at policy N with booking capacity $n_{N,N}$. By making this unilateral move, this firm's expected revenue is equal to $\Pi_{D,N}\left(\tilde{d};\, n_{N,N}\right)$. Hence, we can conclude that this firm cannot improve his expected revenue via a unilateral move if and only if $\Pi_{N,N}\left(n_{N,N}, n_{N,N}\right) \geq \Pi_{D,N}\left(\tilde{d};\, n_{N,N}\right)$. More formally, we have:

Proposition 17. *Suppose both firms adopt policy N and set the same booking capacity $n_{N,N}$. Then $\{N, N\}$ will be an equilibrium policy in the meta-game if and only if $\Pi_{N,N}\left(n_{N,N}, n_{N,N}\right) \geq \Pi_{D,N}\left(\tilde{d};\, n_{N,N}\right)$, where $\tilde{d} = d_{D,N}\left(\beta\left(\tilde{\alpha}\right)\right)$ and $\tilde{\alpha}$ is the solution to (28).*

4.4.2 Equilibrium Policy $\{D, D\}$ in the Meta-Game

We now establish the necessary and sufficient condition for $\{D, D\}$ to be the equilibrium policy in the meta-game. First, let us examine the subgame $\{D, D\}$ in which both firms will set their equilibrium deposit to θ as stated in Proposition 13 so that each firm's expected revenue is equal to $\Pi_{D,D}\left(\theta, \theta\right)$ as given in (21). Thus, policy $\{D, D\}$ will be the equilibrium policy in the meta-game if and only if neither firm can improve his expected revenue from a unilateral move by deviating from the adopted policy D with a required deposit θ. By symmetry, it suffices to analyze this condition associated with one firm who makes a unilateral move (while the other firm's policy is fixed at policy D with a required deposit θ). First, observing that θ is the equilibrium deposit in subgame $\{D, D\}$, it is clear that this firm cannot improve his expected revenue if he makes a unilateral move from changing his required deposit. Second, suppose this firm changes his policy from D to N. Then we can utilize the same approach as presented in Section 4.3 for the subgame $\{N, D\}$ to determine this firm's "best response" under policy N (denoted by booking capacity $\hat{n}$), given that the other firm's policy is fixed at policy D with a required deposit θ. By making this unilateral move, this firm's expected revenue is equal to $\Pi_{N,D}\left(\hat{n}\,;\,\theta\right)$. Hence, we can conclude that this firm cannot improve his expected revenue via a unilateral move if and only if $\Pi_{D,D}\left(\theta, \theta\right) \geq \Pi_{N,D}\left(\hat{n}\,;\,\theta\right)$. More formally, we have:

Proposition 18. *Suppose both firms adopt policy D and charge the same guarantee deposit θ. Then $\{D, D\}$ will be an equilibrium policy in the meta-game if and only if $\Pi_{D,D}\left(\theta, \theta\right) \geq \Pi_{N,D}\left(\hat{n}\,;\,\theta\right)$, where $\hat{n} = n_{N,D}\left(\hat{\alpha}\right)$ and $\hat{\alpha}$ is the solution to (28).*

4.4.3 Equilibrium Policy $\{N, D\}$ in the Meta-Game

We now establish the necessary and sufficient conditions for $\{N, D\}$ to be the equilibrium policy in the meta-game. First, let us examine the subgame (N, D) in which firm N will set his booking capacity to $n^*_{N,D}$ and firm D requires deposit $d^*_{N,D}$ as stated in Proposition 16. By applying (26) and (24), one can determine firm N's expected revenue is equal to $\Pi_{N,D}\left(n^*_{N,D},\, d^*_{N,D}\right)$ and firm D's expected revenue is

equal to $\Pi_{D,N}\left(d^*_{N,D},\, n^*_{N,D}\right)$. Thus, policy $\{N,\, D\}$ will be the equilibrium policy in the meta-game if and only if the following conditions hold: (1) firm N cannot improve his expected revenue from a unilateral move by deviating from his booking capacity at $n^*_{N,D}$; and (2) firm D cannot improve his expected revenue from a unilateral move by deviating from his require deposit $d^*_{N,D}$. By using the same approach as presented above, we can establish these two conditions formally in the following Proposition:

Proposition 19. *Suppose the two firms adopt different policies. Then* $\{N\,,\, D\}$ *will be an equilibrium policy in the meta-game if and only if (i)* $\Pi_{N,D}\left(n^*_{N,D},\, d^*_{N,D}\right) \geq \Pi_{Dev,D}$ *; and (ii)* $\Pi_{D,N}\left(d^*_{N,D},\, n^*_{N,D}\right) \geq \Pi_{Dev,N}$ *, where:*

$$\Pi_{Dev,D} = \begin{cases} \Pi_D\left(\hat{d};\, \lambda\right) & if \quad d^*_{N,D} > \theta \\ \Pi_D\left(\hat{d};\, \frac{\lambda}{2}\right) & otherwise \end{cases} \quad and \quad \Pi_{Dev,N} = \begin{cases} \Pi_N\left(\bar{n}\,;\, \lambda\right) & if \;\; n^*_{N,D} \geq 2 \;\; and \;\; \bar{n} = 1 \\ \Pi_N\left(\bar{n}\,;\, \frac{\lambda}{2}\right) & otherwise. \end{cases} \tag{29}$$

$$And\ the\ term\ \hat{d} = \begin{cases} \min\left\{d^*\,, d^*_{N,D} - \epsilon\right\} & if \;\; d^*_{N,D} > \theta \\ \theta & otherwise \end{cases} \quad and \;\; \bar{n} = \begin{cases} n_{N,N} \;\; if \;\; n^*_{N,D} \geq 2 \\ 1 \quad otherwise \end{cases}$$

where $\epsilon > 0$ *is an infinitesimally small number. Also, the functions* $\Pi_N(\bullet)$ *and* $\Pi_D(\bullet)$ *are given in (8) and (15); and* $d^*\,, n_{N,N}\,, d^*_{D,N}$ *and* $n^*_{N,D}$ *are stated in Propositions 7, 11, 16 and 16, respectively.*

By using the results stated in Propositions 17, 18, and 19, we develop Table 3 that summarizes the necessary and sufficient condition(s) for a particular pair of policies (i.e. $\{N,\, N\}$,$\{D,\, D\}$ or $\{N,\, D\}$) to be the equilibrium policy that the firms will adopt in the meta-game.

		Firm 2	
		Policy N	Policy D
Firm 1	Policy N	$\Pi_{N,N}\left(n_{N,N}, n_{N,N}\right) \geq \Pi_{D,N}\left(\tilde{d}\middle| n_{N,N}\right)$	$\Pi_{N,D}\left(n^*_{N,D}\right) \geq \Pi_{Dev,D}$ $\Pi_{D,N}\left(d^*_{D,N}\right) \geq \Pi_{Dev,N}$
	Policy D	$\Pi_{N,D}\left(n^*_{N,D}\right) \geq \Pi_{Dev,D}$ $\Pi_{D,N}\left(d^*_{D,N}\right) \geq \Pi_{Dev,N}$	$\Pi_{D,D}\left(\theta, \theta\right) \geq \Pi_{N,D}\left(\hat{n}, \theta\right)$

Table 3. Necessary and Sufficient Condition(s) for a pair of policy to be an equilibrium in the meta-game.

4.5 A Special Case: Policy-Independent Retail Price $r_N = r_D$

We now examine a special case in which both firms charge the same retail price regardless of the deposit policy so that $r_N = r_D$. By using the fact that a firm who adopts policy N can set his service probability $\beta = 1$ by setting his booking capacity to $n = 1$ (i.e. no overbooking), (1) asserts that all customers will attempt to reserve with this firm unless the other firm follows suit and also adopts policy N and sets his booking capacity to $n = 1$. This observation enables us to establish the following proposition:

Proposition 20. *Suppose the retail price is both policy- and firm-independent so that $r_N = r_D = r$. Then there exists an equilibrium in the meta-game in which both firms adopt policy N and set their booking capacity $n_{N,N} = 1$.*

Although Proposition 20 suggests that there exists an equilibrium in the meta-game in which both firms adopt policy N and set their booking capacity $n = 1$, we learn from Propositions 11 and 17 that it is possible for another, payoff-dominant equilibrium to exist. This observation motivates us to develop the following conjecture.

Conjecture 3. *A Prisoner's Dilemma can occur when the retail prices are policy-independent; i.e. when $r_N = r_D$.*

4.6 Numerical Examples

To determine the equilibrium deposit policies associated with the meta-game that captures duopolistic competition and to examine Conjectures 2 and 3 as established in Section 4.4 and 4.5, we conduct numerical experiments by fixing the value $r_D = 80$, $\theta = 10$ and $v = 150$. In each experiment, we vary the demand rate λ from 0.1 to 20 and vary the retail price r_N from 80 to 130. Figures 3 (a), 3 (b) and 3 (c) report the regions in which $\{N\,,\,N\}$ or $\{D\,,\,D\}$ is the unique pure-strategy equilibrium and the regions in which both $\{N\,,\,N\}$ are $\{D\,,\,D\}$ are pure-strategy equilibria when $c = 5$, $c = 10$ and $c = 20$, respectively.[9] (The region highlighted in bold corresponds to the region in which $\{N\,,\,N\}$ is the equilibrium when both firms set their booking capacity to $n_{N,N} = 1$. By noting the fact that this occurs when $r_N = r_D = 80$, we verify Proposition 20. Also, the region with the "no equilibrium" label represents the region in which no pure-strategy equilibrium exists). As shown in Figures 3 (a), 3 (b) and 3 (c), $\{D\,,\,D\}$ appears to be the common equilibrium policies that both firms will adopt, followed by the equilibrium policies $\{N\,,\,N\}$. With this set of parameter values, we noticed that policy $\{N\,,\,D\}$ is never an equilibrium. However, we discover, in some rare instances, that policy $\{N\,,\,D\}$ is a Nash equilibrium. For ease of exposition, we shall discuss these rare instances in Appendix 2.

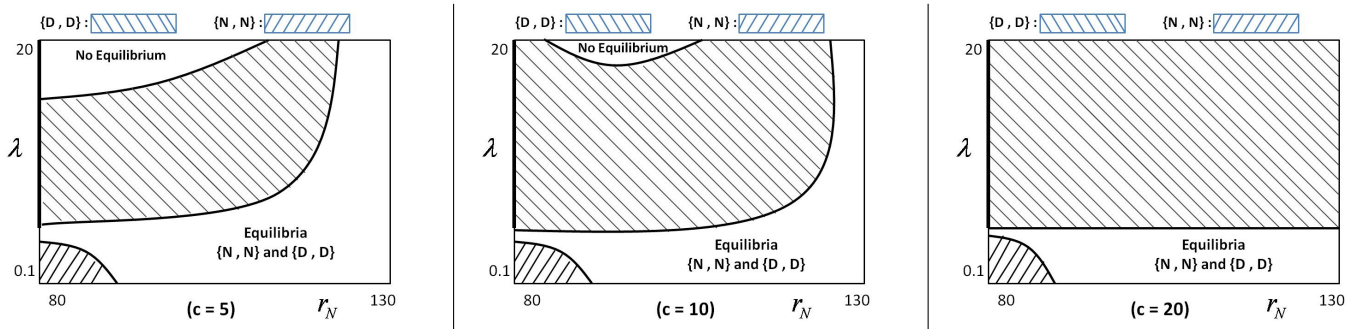

Figure 3 (a) Figure 3 (b) Figure 3 (c)

Figure 3. Equilibrium Deposit Policies for the Meta-Game.

[9] We have also conducted numerical experiments to examine the effect of the minimum deposit θ on the equilibrium policies when $\theta = 1$, $\theta = 10$ and $\theta = 20$, respectively. By setting the penalty $c = 10$, we obtain similar results as reported in Figures 3 (a), 3 (b) and 3 (c). To reduce repetition, we omit the details.

Although we have shown analytically in Section 3.4 and numerically in Section 3.5 (Figure 1 (a)) that policy N dominates policy D when the retail price r_N is sufficiently larger than r_D in the monopoly case, when the firms enter a duopolistic competition using the same set of parameter values, we observe in Figure 3 that, when $r_N \gg r_D$, this results holds only when the penalty c and the demand rate λ are sufficiently small. However, in contrast to the monopolistic case, we find that $\{N\,,\,N\}$ is the unique equilibrium policy only when the retail price r_N is sufficiently close to r_D (and when the demand rate λ is sufficiently low). To understand why the equilibrium policy in the duopoly case differs from the monopoly case in this occasion, let us examine the case when r_N is much larger than r_D. In this case, Proposition 9 and Figure 1 (a) suggest that policy N dominates policy D in the monopolistic environment. However, in the duopolistic environment, we need to account for the interplays between the firms. To examine the dynamics of each firm, let us suppose first that both firms adopt policy N and set their booking capacity to $n_{N,N}$ as given in Proposition 11. Because $r_N \gg r_D$, one can observe from (22) that: (1) a firm can obtain a high market share of customers by switching to policy D unilaterally; and (2) overbooking is less beneficial when the compensation c is large. Hence, policy D is more attractive. As one firm switches to policy D and obtains a high market share, the other firm may have to follow suit to ensure sufficient market share (if the penalty c or the demand rate λ are relatively large). Therefore, even when $r_N \gg r_D$, both firms may adopt policy D in the meta-game as shown in Figure 3. These firms dynamics explain why the results obtained in the duopoly case can be opposite from the results obtained in the monopoly case.

Next, let us examine the case when no pure-strategy equilibrium exists as shown in the "no equilibrium" region in Figure 3 (b). Let us consider a specific instance in which $r_D = 80$, $r_N = 90$, $v = 150$, $c = 10$, $\theta = 10$ and demand rate $\lambda = 20$. First, suppose both firms adopt policy N in equilibrium. Then one can apply Proposition 11 to show that each firm will set his booking capacity $n_{N,N} = 4$ and enjoy an expected revenue $\Pi_{N,N}(4,4) = 59.98$. Because the demand rate is large and the retail price r_N is not much larger than r_D, one can check from (24) that a firm can obtain a higher expected revenue (i.e. $\Pi_{D,N}(d = 80\,,\,n = 4) > \Pi_{N,N}(4,4)$) if he unilaterally switches from policy N to policy D and charges a deposit $\tilde{d} = 80 = r_D$. Consequently, $\{N\,,\,N\}$ is not a Nash equilibrium. Second, suppose both firms adopt policy D in equilibrium. Then one can apply Proposition 13 and (21) to show that both firms will require the same deposit $d^*_{D,D} = \theta = 10$ and enjoy an expected revenue $\Pi_{D,D}(10,10) = 49.37$. In this case, one can check from (26) that one of the firms can switch unilaterally from policy D to policy N by setting his booking capacity to $\hat{n} = 12$ and enjoy a higher expected revenue (i.e. $\Pi_{N,D}(\hat{n} = 12\,,\,d = 10) = 49.47 > 49.37 = \Pi_{D,D}(10,10)$. Therefore, policy $\{D\,,\,D\}$ is not a Nash equilibrium either. It remains to check to see if policy $\{N\,,\,D\}$ is a Nash equilibrium. By applying (26) and (24), we can determine the expected revenues for firms N and D as $\Pi_{N,D}\left(n^*_{N,D} = 5,\, d^*_{N,D} = 80\right)$ and $\Pi_{D,N}\left(d^*_{N,D} = 80,\, n^*_{N,D} = 5\right)$, respectively. In this case, one can also check that policy $\{N\,,\,D\}$ cannot be an equilibrium, because firm N can make a unilateral switch to policy D and obtain a higher expected revenue by undercutting the other firm D's required deposit. Based on this argument, we can conclude that there are instances in which no pure-strategy equilibrium policy exists.

Finally, let us examine Conjectures 2 and 3 as established in Sections 4.2 and 4.5, respectively. Specifically,

we consider the following instance: $r_N = r_D = 100$, $c = 20$, $v = 150$ and $\theta = 10$. First, our numerical analysis shows that in the subgame (N, N), both firms will set their booking capacity to $n_{N,N} > 1$ if and only if the demand rate $\lambda > 3$. Therefore, our result supports Conjecture 2. Also, our numerical result suggests that in this instance, there exists a unique Nash equilibrium in the meta-game in which both firms adopt policy N and set up their booking capacity to $n_{N,N} = 1$ when the demand rate $\lambda \leq 3$. This result verifies Proposition 20.

We now examine Conjecture 3 that speculates the existence of a Prisoner's Dilemma situation when $r_N = r_D$. Consider the following instance: $r_N = r_D = 100$, $c = 20$, $v = 150$, $\theta = 10$, and $\lambda = 3$. For this particular instance, it can be shown that the set Pareto efficient actions is for each firm to adopt policy D, require deposit $d^* = 29.79$, and enjoy expected revenue $\Pi_{D,D}(d^*, d^*) = 47.53$. Because of the undercutting dynamics between both firms as exhibited in the subgame (D, D), Proposition 13 states that both firms will set their required deposits at $\theta = 10$ and enjoy expected revenue $\Pi_{D,D}(\theta, \theta) = 44.59$. However, a firm can increase his expected revenue by making a unilateral switch from policy D to policy N. By setting his booking capacity to $\hat{n} = 4$, this firm can obtain a higher expected revenue: $\Pi_{N,D}(\hat{n}\,, \theta) = 45.19 > 44.59 = \Pi_{D,D}(\theta, \theta)$. As one firm switches to policy N, the other firm would follow suit. As both firms adopt policy N, one can check from Proposition 11 that both firms will set their booking capacity to $n_{N,N} = 1$ in equilibrium and both firms will obtain an expected revenue equal to $\Pi_{N,N}(1, 1) = 38.84$. It is interesting to note that, had each firm sets its booking capacity to $n^* = 3$, they would have obtained a higher expected revenue equal to $\Pi_{N,N}(n^*, n^*) = 43.26 > 38.84 = \Pi_{N,N}(1, 1)$. Thus, we can conclude that a Prisoner's Dilemma occurs in this instance, which supports Conjecture 3 as established in Section 4.5.

5 Discussion

We have examined how two common deposit policies (i.e. the no deposit policy N and the guarantee deposit policy D) affect a rational customer's reservation decision and a firm's optimal expected revenue. In a monopolistic environment, we have analyzed each deposit policy as a Stackelberg game in which the firm acts as the leader who selects the booking capacity n under the no deposit policy N (or the required deposit d under the guarantee deposit policy D) and each customer acts as the follower who decides whether to reserve or not. By solving these two Stackelberg games, we have determined the optimal booking capacity n^* under the no deposit policy N and the optimal guarantee deposit d^* under the guarantee deposit policy D. In addition, we have shown that policy N dominates policy D when when (a) the "effective" demand rate $\frac{\lambda}{m}$ is sufficiently low; (b) the retail price r_N is sufficiently large in relation to r_D; (c) the customer valuation v is sufficiently low; and (d) the penalty c is sufficiently low. Also, our numerical analysis enabled us to gain additional insights about the impact of the capacity m, the customer demand rate λ, and the minimum deposit θ on the optimal retail price r_D^*, and optimal deposit d^* under policy D. More importantly, our numerical analysis suggested that it is optimal for the firm to charge a higher retail price under policy N so that $r_N^* > r_D^*$. This result may have helped us to explain formally why it is commonly observed in practice that firms tend to charge lower retail prices when guarantee deposits are required.

To understand how market competition affects the way a firm selects his deposit policy, we have analyzed a game of duopolistic competition between two firms. For any given pair of policies adopted by the firms (i.e. (N, N), (D, D), (N, D) and(D, N)), we have examined each subgame between two firms by incorporating the underlying Stackelberg game that is played between each firm and its customers. By analyzing a non-cooperative game with an embedded Stackelberg game, we have highlighted the interplays between the two firms and developed the subgame equilibrium that specifies each firm's decision for any given pair of policies. By comparing the payoffs associated with different pairs of policies, we have developed conditions under which a particular pair of policies constitutes the equilibrium policy to be adopted by both firms in the meta-game. Our numerical analysis enabled us to obtain the following insights: (1) policy $\{D\,,\,D\}$ is the most common equilibrium policy that both firms will adopt in the meta-game; (2) policy $\{N\,,\,N\}$ is a unique equilibrium policy for the meta-game when the demand rate λ and the retail price r_N is sufficiently small; (3) policy $\{N\,,\,D\}$ can be the equilibrium policy in rare occasions; (4) equilibrium policy are not necessarily unique; (5) a pure-strategy equilibrium may not exist in some cases; and (6) the Prisoner's Dilemma can certainly occur in the meta-game. Finally, when both firms charge the same retail price, we have shown that there exists an equilibrium in which both firms adopt the no deposit policy N.

There are various research opportunities for addressing the limitations of the model presented in this paper. First, it would be of interest to examine other deposit policies including cancelable reservations with partial refunds. Second, we have assumed that all customers are present simultaneously in the system. It would be of interest to analyze the case when customers arrive dynamically over time according to a certain stochastic process and when the firm can adjust its retail price dynamically over time. Third, our model assumes that all parties are risk-neutral. It would be of interest to examine the case when firm and customers are risk-averse. Fourth, our model assumes that each customer will leave the system if her attempt to reserve with a firm fails. It would be of interest to extend our model to the case when each customer would consider reserving with the other firm after a failed attempt before she leaves the system. Fifth, our model does not incorporate the existence of speculators in the system who do not care for the service but they have a strong desire to get the compensation c for being denied. For instance, in the airline industry, there are passengers who are eager to give up their seats voluntarily in order to receive compensations. Sixth, our model assumes common knowledge. It would be of interest to examine a situation in which customers do not know the firm's booking capacity under policy N. Finally, another potentially interesting extension could be the extension of the duopolistic model to a K-firm oligopolistic model.

Acknowledgments: This research is supported by Edward Carter Research fund.

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Appendix 1: Proofs

Proposition 1:

Proof. Let $\Delta(n) \triangleq (\Pi_N(m, n+1) - \Pi_N(m, n))$ be the marginal gain for increasing the booking capacity from n to $(n+1)$. By using (7) and by rearranging various terms, one can show that $\Delta(n)$ can be re-expressed as: $\Delta(n) = (\Pi_N(m, n+1) - \Pi_N(m, n)) = [f(n+1) - f(n)]\,\frac{c}{2} \cdot p_{\geq n+1|\lambda}$, where

$$f(n) = 2\left(\frac{r_N}{c} + 1\right)\sum_{k=1}^{m} k \Pr\{k|n\} + 2m\left(\frac{r_N}{c} + 1\right)\sum_{k=m+1}^{n} \Pr\{k|n\} - 2\sum_{k=1}^{n} k \Pr\{k|n\} \tag{30}$$

By using the fact that $p_{\geq n+1|\lambda} = \Pr\{A \geq n+1\} > 0$, we can conclude that the optimal n^* that maximizes the firm's expected revenue $\Pi_N(n)$ is equal to the optimal n^* that maximizes the function $f(n)$ given in (30). Hence, it remains to determine the optimal n^* that maximizes the function $f(n)$. By using (2), the function $f(n)$ can be simplified as: $f(n) = m\left(\frac{r_N}{c} + 1\right)\left(\frac{2n+1-m}{n+1}\right) - n$. Since the function $f(n)$ is concave in n, we can use the difference equation defined by $\Delta f(n) \triangleq f(n+1) - f(n) = \left(\frac{r_N}{c} + 1\right)\frac{m(m+1)}{(n+1)(n+2)} - 1$, to show that $n^* \in \left\{\left\lfloor \sqrt{\left(\frac{r_N}{c} + 1\right) m(m+1)} - 1 \right\rfloor, \lceil \bullet \rceil\right\}$. This completes our proof. □

Corollary 2:

Proof. Let $\xi \triangleq \frac{r_N}{c} + 1$. Observe from (9) that $n^*(m) > m \cdot n^*(1)$ if and only if $\sqrt{\xi m(m+1)} > \left(\sqrt{2\xi} - 1\right)m + 1$. Squaring both sides, re-arranging terms, this condition can be simplified as: $m < \frac{1}{(\xi - 2\sqrt{2\xi} + 1)}$. Our result follows immediately by using the fact that $\xi = \frac{r}{c} + 1$. □

Corollary 3:

Proof. Observe from (8) that $\Pi_N(m, n^*(m))$ is increasing in the capacity m. To establish an upper bound on $\Pi_N(m, n^*(m))$, it suffices to examine the case when $m \to \infty$. As $m \to \infty$, (9) suggests that $n^*(m) \to \infty$. This implies that the firm has enough capacity to accept and to serve all customers who attempt to reserve so that the firm will not deny service to any customer. Hence, (7) suggests that the firm's expected revenue can be re-written as: $\lim_{m\to\infty} \Pi_N(m, n^*(m)) = \sum_{j=1}^{\infty}\sum_{k=1}^{j} k r_N \Pr\{k \mid j\}\, p_{j|\lambda} = \sum_{j=1}^{\infty} \frac{r_N}{j+1} p_{j|\lambda} \sum_{k=1}^{j} k = \frac{\lambda r_N}{2}$. This completes the proof. □

Corollary 4:

Proof. Observe that the difference equation $\Delta f(n) \triangleq f(n+1) - f(n) = \left(\frac{r_N}{c} + 1\right)\frac{m(m+1)}{(n+1)(n+2)} - 1$, where the function $f(n)$ is given in Proposition 1. When $m = 1$, the difference equation reduces to $\Delta f(n) = \left(\frac{r_N}{c} + 1\right)\frac{2}{(n+1)(n+2)} - 1$. Since $f(n)$ is concave, it follows that $n^* = k$ if and only if $\Delta f(k-1) \geq 0$ and $\Delta f(k) \leq 0$. The result follows immediately by substituting $k = 1, ..., 5$ into $\Delta f(k)$. □

Proposition 5:

Proof. First differentiate (12) with respect to d and solve for $\frac{\partial}{\partial d}\Pi_D(m, d) = 0$. By re-arranging the terms, defining $P_k \triangleq \sum_{j=0}^{m-1} j^k p_{j|\lambda\gamma_D}$, it is easy to show that the first order condition satisfies (13). □

Proposition 6:

Proof. Because $d^*(m)$ is decreasing in m and $\Pi_D(m, d^*(m))$ is increasing in m, it suffices to analyze the case when $m \to \infty$. Using the results in corollary 5, observe that as $m \to \infty$, $P_0 \to 1$, $P_1 \to \frac{\lambda x_D}{x_D+d}$ and $P_2 \to \frac{\lambda x_D}{x_D+d}\left(\frac{\lambda x_D}{x_D+d}+1\right)$. Substituting into (13) the first order condition reduces to $\frac{\lambda x_D}{2}\left[\frac{x_D^2-(x_D+2r_D)d^\infty}{(x_D+d^\infty)^3}\right]=0$, which implies that $d^*(m)=\frac{x_D^2}{x_D+2r_D}$ as $m \to \infty$. Noting that (11) can be rewritten as $\Pi_D(m,d)=\left[\frac{(r_D+d)(x_D+d)+d(r_D-d)}{2(x_D+d)}\right](m+P_1-mP_0)$, substituting $d^*(m)=\frac{x_D^2}{x_D+2r_D}$ we obtain $\lim_{m\to\infty}\Pi_D(m,d^*(m))=\frac{\lambda(x_D+2r_D)^2}{8(x_D+r_D)}$. Finally, substituting $v=x_D+r_D$ we obtain the desired result. □

Proposition 7:

Proof. First, we differentiate (15) with respect to d, getting $\frac{\partial}{\partial d}\Pi_D(1,d)=-\frac{1}{2}\frac{\lambda x_D}{(x_D+d)^2}e^{-\frac{\lambda\cdot x_D}{x_D+d}}\cdot\left[\frac{r_D\cdot x+d(r_D+v)}{x_D+d}\right]+\frac{1}{2}\left(1-e^{-\frac{\lambda x_D}{x_D+d}}\right)\frac{x_D v}{(x_D+d)^2}$. Then solving for $\frac{\partial}{\partial d}\Pi_D(1,d)=0$ we obtain the first-order condition in (16). By noting that $\frac{\partial}{\partial d}\left[e^{-\frac{\lambda\cdot x_D}{x_D+d}}\right]>0$ and $\frac{\partial}{\partial d}\left[\frac{r_D x_D+d(r_D+v)}{x_D+d}\right]>0$, we can conclude that the LHS of (16) is strictly increasing in d the the RHS is strictly decreasing in d. By observing that $\lambda\cdot e^{-\lambda}\cdot r<\left(1-e^{-\lambda}\right)v$, it is easy to check from (16) that the LHS is strictly less than the RHS when $d=0$. Combine this observation with the fact that the LHS is strictly increasing and the RHS is strictly decreasing in d, we can conclude that the first-order condition has a unique solution $d'(1)>0$. Because $\Pi_D(1,d)$ is concave in d, the optimal deposit satisfies $d^*(1)=\max\left\{\theta,\min\left\{r_D, d'(1)\right\}\right\}$. Next, by considering $\frac{\partial}{\partial d}\Pi_D(1,d)$ given above, one can check that $\frac{\partial}{\partial d}\Pi_D(1,d)|_{d=r_D}<0$ if and only if $1+2\lambda\left(\frac{r_D}{v}\right)>e^{\frac{\lambda x_D}{v}}$. Hence, we can conclude that $d^*<r_D$ if and only if $1+2\lambda\left(\frac{r_D}{v}\right)>e^{\lambda\left(1-\frac{r_D}{v}\right)}$. To complete the proof, differentiate (16) with respect to λ, apply the implicit function theorem to show that $\frac{\partial d'(1)}{\partial\lambda}>0$ and it follows by monotonicity that $d^*(1)$ is increasing in λ. □

Corollary 8:

Proof. The result follows immediately from comparing the firm's expected revenue reported in Lemma 3 and from Lemma 6. We omit the details. □

Proposition 9:

Proof. First, let us examine the case when $n=1$. Define $h_1(\lambda)\triangleq 2\left[\Pi_D(1,d)-\Pi_N(1,1)\right]=\left[\frac{r_D x_D+d(r_D+v)}{x_D+d}\right]\cdot\left(1-e^{-\frac{\lambda\cdot x_D}{x_D+d}}\right)-r_N\left(1-e^{-\lambda}\right)$. It is easy to check that $h_1(0)=0$ and $h_1(\lambda)\to r_D+\frac{dv}{v-r_d+d}-r_N$ as $\lambda\to\infty$. Differentiating h_1 we obtain $\frac{d}{d\lambda}h_1(\lambda)=e^{-\lambda}\left[\frac{r_D\cdot x_D+d\cdot(r_D+v)}{(x_D+d)^2}x_D\cdot e^{\frac{\lambda d}{x_D+d}}-r_N\right]$. First note that $e^{-\lambda}>0\ \forall\lambda$ and the term inside the bracket is increasing in λ. Then for any given d, it is easy to check from $\frac{d}{d\lambda}h_1(\lambda)$ that there exists some threshold κ such that the function $h_1(\lambda)$ first decreases and then increases if and only if $\lambda>\kappa$. It is easy to check that there exists a threshold τ_1 such that $h_1(\lambda)<0$ if and only if $\lambda<\tau_1$. Moreover, one can show that $\tau_1=\infty$ if and only if $r_D+\frac{dv}{v-r_d+d}<r_N$. Thus, we can conclude that such that $\Pi_N(1,1)<\Pi_D(1,d)$ if and only if $\lambda<\tau_1$. This proves the statement for the

case when $n = 1$. By using the same approach, we can complete the proof for the case when $n = 2$. We omit the details. □

Lemma 10:

Proof. We have already established the equilibrium $n_1 = n_2 = 1$ in which case $\beta_1 = \beta_2 = 1$ holds. This implies that $\pi_{N1} = \pi_{N2}$ and therefore customers are indifferent as to which firm to reserve with. Therefore, each firm enjoys the same reserve probability $\gamma_N = \frac{1}{2}$ in equilibrium. Since $n_1 = n_2 = 1$ is an equilibrium, it suffices to check β_1, β_2 for n_1, $n_2 \geq 2$. We aim for a contradiction. Without loss of generality, suppose that $\beta_1 > \beta_2$. Then our observation (2) asserts that all customers will attempt to reserve with firm 1, while no customer will attempt to reserve with firm 2. Consequently, $\beta_1 < 1 = \beta_2$, contradicting the assumption. Hence we must have $\beta_1 = \beta_2$ in equilibrium. Combining this observation with the fact that the customer's expected surplus $\pi_{Ni} = \psi\left[\beta_i x_N + (1 - \beta_i)\, c\right] \geq 0$, we can conclude that all customers will attempt to reserve and all customers are indifferent about these two firms. Consequently, both firms have the same reserve probability so that $\gamma_{N1} = \gamma_{N2} = \frac{1}{2}$. □

Proposition 11:

Proof. First, we have already shown that $n_{N,N} = 1$ is a Nash equilibrium. Next, it follows from Lemma 10 that $\gamma_{N1} = \gamma_{N2} = \frac{1}{2}$ and from Proposition 1 that the optimal booking capacity n^* given in (19) is independent of its demand rate, so we can conclude that n^* is the same as the optimal booking capacity given in (9). To proceed, consider the case when both firms set $n_1 = n_2 = n^* \geq 2$ (the case when $n^* = 1$ is trivial), so that each firm has the same reserve probability. Observe that a firm cannot increase his reserve probability (and thus his expected revenue) unless he unilaterally sets $n_i = 1$. By noting that the expected revenue is equal to $\Pi_N(1;\lambda) = \frac{r_N}{2}\left(1 - e^{-\lambda}\right)$, ($n^*$, n^*) is an equilibrium if and only if $\Pi_N\left(n^*; \frac{\lambda}{2}\right) \geq \frac{r_N}{2}\left(1 - e^{-\lambda}\right)$. Noting that each firm will have the same reserve probability in equilibrium, it follows from Proposition 1 that $\Pi_N\left(n^*; \frac{\lambda}{2}\right) > \Pi_N\left(1; \frac{\lambda}{2}\right)$ $\forall \lambda > 0$ and as a result if (n^*, n^*) is an equilibrium, then it payoff-dominates the equilibrium (1, 1). This completes our proof. □

Corollary 12:

Proof. First note that $\Pi_N(1;\lambda) = \frac{r_N}{2}\left(1 - e^{-\lambda}\right)$ and $\frac{r_N}{c} \geq 2$. Next define $h\left(\frac{r_N}{c}, \lambda\right) \triangleq \frac{6}{c}\left[\Pi_N\left(n^*; \frac{\lambda}{2}\right) - \frac{r_N}{2}\left(1 - e^{-\lambda}\right)\right]$. It is easy to check from (18) that: $h\left(\frac{r_N}{c}, \lambda\right) = \left(4\frac{r_N}{c} - 2\right)\left(1 - e^{-\frac{\lambda}{2}}\right) - \left(\frac{r_N}{c} - 2\right)\frac{\lambda}{2}e^{-\frac{\lambda}{2}} - 3\frac{r_N}{c}\left(1 - e^{-\lambda}\right)$ and differentiating with respect to λ we obtain $\frac{\partial h\left(\frac{r_N}{c}, \lambda\right)}{\partial \lambda} = e^{-\frac{\lambda}{2}}\left[\left(\frac{r_N}{c} + 1\right) - 3\frac{r_N}{c}e^{-\frac{\lambda}{2}} + \frac{\lambda}{4}\left(\frac{r_N}{c} - 2\right)\right]$. To proceed, note that $e^{-\frac{\lambda}{2}} > 0$ and let us define $\hat{h}\left(\frac{r_N}{c}, \lambda\right) \triangleq e^{\frac{\lambda}{2}}\frac{\partial h\left(\frac{r_N}{c}, \lambda\right)}{\partial \lambda} = \left(\frac{r_N}{c} + 1\right) - 3\frac{r_N}{c}e^{-\frac{\lambda}{2}} + \frac{\lambda}{4}\left(\frac{r_N}{c} - 2\right)$. Noting that (i) $\hat{h}\left(\frac{r_N}{c}, \lambda = 0\right) = -\left(2\frac{r_N}{c} - 1\right) < 0$, (ii) $\hat{h}\left(\frac{r_N}{c}, \lambda \to \infty\right) > 0$, (iii) $\hat{h}\left(\frac{r_N}{c}, \lambda\right)$ is increasing in λ and (iv) $h\left(\frac{r_N}{c}, \lambda \to \infty\right) = \frac{r_N}{c} - 2 \geq 0$, we can conclude by the mean-value theorem that there exists some λ_{crit} such that $\Pi_N\left(n^*; \frac{\lambda}{2}\right) \geq \frac{r_N}{2}\left(1 - e^{-\lambda}\right)$ $\forall \lambda > \lambda_{crit}$. This completes the proof. □

Proposition 13:

Proof. Omitted. □

Lemma 14:

Proof. Showing that equation (25) satisfies the first order conditions for $\Pi_{D,N}(d;\beta)$ involves a simple differentiation of (24) with respect to d and the proof is omitted. To simplify the analysis, we define $y(d,\beta) \triangleq LHS - RHS$ of (25). Letting $d = 0$, it is easy to check that for fixed β, $y(d=0,\beta) = \left(e^{\lambda}-1\right)[r_N + (x_N - c)(1-\beta)] - \lambda r_D > 0$ as long as $r_N - r_D \geq 0$. Letting $d \to \infty$, it is easy to check that $y(d \to \infty, \beta) = -\infty$. Moreover, $y(d,\beta)$ is strictly decreasing and continuous in d. We can therefore conclude by the mean value theorem that given fixed β, there exists some unique $d'(\beta)$ that maximizes $\Pi_{D,N}(d;\beta)$. Because $\Pi_{D,N}(d;\beta)$ is concave in β, it follows that $d_{D,N}(\beta) = \max\left\{\theta, \min\left\{d'(\beta), r_D\right\}\right\}$ maximizes $\Pi_{D,N}(d;\beta)$ on $[\theta, r_D]$.

Now we "unfix" β. It is easy to check that for any fixed d, $y(d,\beta)$ is strictly decreasing in β, since the LHS is strictly decreasing in β, while the RHS is strictly increasing in β. Therefore $\forall \beta_1 > \beta_2$ it follows that for any fixed $d \geq 0$, $y(d,\beta_1) < y(d,\beta_2)$.

We can thus conclude that $\exists d_1, d_2$ that satisfy $y(d_1,\beta_1) = y(d_2,\beta_2) = 0$ and $d_1 < d_2$. Therefore $d'(\beta)$ is strictly decreasing in β. Since $y(d,\beta)$ is continuously differentiable in both d,β, we can also conclude that $d'(\beta)$ is continuous in β.

By monotonicity it follows that $d_{D,N}(\beta)$ is a decreasing, continuous function of β and this completes the proof. □

Lemma 15:

Proof. Let $\Delta_{N,D}(n) \triangleq \Pi_{N,D}(n+1;\alpha) - \Pi_{N,D}(n;\alpha)$ be the marginal gain for increasing the booking capacity from n to $(n+1)$. By using (26) and rearranging various terms, one can show that $\Delta_{N,D}(n)$ can be re-expressed as: $\Delta_{N,D}(n) = \left[\hat{f}(n+1;\alpha) - \hat{f}(n;\alpha)\right] \cdot c \cdot p_{j \geq n+1|\lambda\alpha}$, where $\hat{f}(n;\alpha) = \left(\frac{r_N}{c}+1\right) \cdot \frac{n+(1-\alpha)^{n+1}}{\alpha(n+1)} - \frac{n\alpha}{2}$. By noting that $p_{j \geq n+1|\lambda\alpha} > 0$, we can conclude that the optimal $n^*_{N,D}$ that maximizes firm N's expected revenue $\Pi_{N,D}(n;\alpha)$ is equal to the n^* that maximizes $\hat{f}(n;\alpha)$.

As a result, it suffices to show that $\hat{f}(n;\alpha)$ is a quasi-concave function of n. Observe that $\frac{\partial}{\partial n}\left(-\frac{n\alpha}{2}\right) = -\frac{\alpha}{2} < 0$ and $\frac{\partial}{\partial n}\left(\frac{n+(1-\alpha)^{n+1}}{\alpha(n+1)}\right) = \frac{1+\left[\ln(1-\alpha)^{n+1}-1\right](1-\alpha)^{n+1}}{\alpha^2(n+1)^2} \geq 0$. To prove the last inequality, observe that it suffices to show that: $1 + \left[\ln(1-\alpha)^{n+1} - 1\right](1-\alpha)^{n+1} \geq 0$. Rearranging the terms in the above inequality, it is easy to check that it reduces to: $(1-\alpha)^{n+1}\ln(1-\alpha)^{n+1} \geq (1-\alpha)^{n+1} - 1$. By properties of the natural logarithm, the above inequality holds for all $(1-\alpha)^{n+1} > 0$. Furthermore: $\lim_{n\to\infty}\frac{\partial}{\partial n}\left(\frac{n+(1-\alpha)^{n+1}}{\alpha(n+1)}\right) = 0$ and $\lim_{n\to\infty}\hat{f}(n;\alpha) = -\infty$. This implies that $\frac{n+(1-\alpha)^{n+1}}{n+1}$ is increasing with a slope that diminishes to 0 as n grows large, while $-\frac{n\alpha}{2}$ is decreasing with a constant slope. Therefore $\hat{f}(n;\alpha)$ is a quasi-concave function of n and this completes our proof. □

Proposition 16:

Proof. For fixed $n \in \mathbb{N}$ define the function $q(\alpha_n) \triangleq \frac{d_{D,N}(\beta(\alpha_n,n))}{d_{D,N}(\beta(\alpha_n,n))+(r_N-r_D)+(x_N-c)[1-\beta(\alpha_n,n)]}$. Note that $\beta(\alpha_n,n)$ is a continuous function conditional on n being fixed and by lemma 14, $d_{D,N}(\bullet)$ is also a continuous function, so $q(\alpha_n)$ is a continuous function. Noting that $q : [0,1] \to [0,1]$, where $[0,1]$ is a compact set, it follows by Brouwer's fixed point theorem that for every $n \in \mathbb{N}$, the exists some $\alpha_n \in [0,1]$ that satisfies $\alpha_n = q(\alpha_n)$. So far, we have established that $\forall n \in \mathbb{N}$, there exists some α_n that satisfies

$q(\alpha_n) = \alpha_n$. Since by assumption there exists some upper bound $\bar{N} \in \mathbb{N}$, the sequence $\{\alpha_n\}_{n\in\{1,...,\bar{N}\}}$ is finite and as a result, there exists some $n^*_{N,D}$ that satisfies $n^*_{N,D} = \arg\max_{n\in\{1,...,\bar{N}\}} \hat{f}(n;\alpha_n)$, where $\hat{f}(n;\alpha_n)$ was given in (27). Finally, note that such $n^*_{N,D}$ trivially satisfies $n^*_{N,D} = n_{N,D}\left(\alpha_{n^*_{N,D}}\right)$ and this completes the proof. □

Proposition 17:

Proof. Recall from Lemma 14 that, in the subgame (N, D), it is optimal for firm D to charge a deposit $d_{D,N}(\beta)$ in equilibrium. Also, recall from 3.1.1 that β can be expressed as a function of the reserve probability $\gamma_N = \alpha$ and the booking capacity n. Therefore, firm D's best response is to set his guarantee deposit $\tilde{d} = d_{D,N}(\beta(\tilde{\alpha}))$, where $\tilde{\alpha}$ satisfies (28) for the case when firm N sets his booking capacity to $n_{N,N}$, getting $\tilde{\alpha} = \frac{\tilde{d}}{\tilde{d}+(r_N-r_D)+(x_N-c)\left(1-\beta\left(\tilde{\alpha},n_{N,N}\right)\right)}$. Similarly to above, existence of such $\tilde{\alpha}$ follows from the first part of the proof of Proposition 16 and is omitted here. Therefore, policy $\{N, N\}$ will be an equilibrium when a unilateral move is undesirable; i.e., when $\Pi_{D,N}\left(\tilde{d};\, n_{N,N}\right) \leq \Pi_{N,N}(n_{N,N}, n_{N,N})$. This completes the proof. □

Proposition 18:

Proof. Recall from Lemma 15 that, in the subgame (N, D), it is optimal for firm N to set his booking capacity $n_{N,D}(\alpha)$ in equilibrium. Also, recall from 3.1.1, β can be expressed as a function of the reserve probability $\gamma_N = \alpha$ and the booking capacity n. Therefore, firm N's best response is to set its booking capacity to $n_{N,D}(\hat{\alpha})$, where $\hat{\alpha}$ satisfies the reserve probability $\hat{\alpha} = \frac{\theta}{\theta+(r_N-r_D)+(x_N-c)(1-\beta(\hat{\alpha},\hat{n}))}$ as given in (28) for the case when $d = \theta$. Existence of such $\hat{\alpha}$ and $\hat{n}$ follows by the same arguments as in the proof for Proposition 16 and are therefore omitted here. Trivially, the associated expected revenue for the firm deviating to policy N will be equal to $\Pi_{N,D}(\hat{n};\, \theta)$. Therefore $\{D, D\}$ is an equilibrium policy if and only if such optimal deviation is not desirable ; i.e. when $\Pi_{D,D}(\theta, \theta) \geq \Pi_{N,D}(\hat{n};\, \theta)$. This completes the proof. □

Proposition 19:

Proof. First, let us examine the optimal deviation strategy for firm N. It follows from Proposition 16 that a firm cannot increase his expected revenue by requiring a deposit different than $d^*_{D,N}$. As a result, it suffices to check only strategies in which the firm switches to policy D. By the same argument used for Proposition 13, the firm cannot capture any market share if he requires deposit $\hat{d} > d^*_{D,N}$. To proceed, first suppose that $d^*_{D,N} > \theta$. Then the firm can enjoy market share $\gamma_D = \frac{x_D}{2\cdot\left(x_D+d^*_{D,N}\right)}$ by setting $\hat{d} = d^*_{D,N}$ or $\gamma_D = \frac{x_D}{x_D+\hat{d}}$ by setting $\hat{d} < d^*_{D,N}$. Noting that there exists some $d^* \geq \theta$ that maximizes the firm's expected revenue when it operates in a monopoly, it is easy to check that the firm's optimal strategy is to set $\hat{d} = \min\left\{d^*,\, d^*_{D,N} - \epsilon\right\}$, where $\epsilon > 0$ is an infinitesimally small number [10]. Now suppose that

[10] Technically, if $d^* > d^*_{N,D}$ then a best response does not exist, because any response that yields a positive expected revenue must lie in interval $\left[d^*_{N,D} - \epsilon,\, d^*_{N,D}\right)$ and this interval is not compact. As a result, for every response, a better response exists and at the limit $\hat{d} \to d^*_{N,D}$, which is clearly not a best response. This issue can be resolved easily by letting the deposit d only take values on a discrete grid (i.e. $d \in \{\theta, \theta+\epsilon, ..., r_D-\epsilon, r_D\}$, where $\epsilon > 0$ is fixed). Because this technicality has negligible effect for ϵ sufficiently small, we choose to omit it for tractability.

$d^*_{D,N} = \theta$ and noting that the firm cannot capture any market share by requiring deposit $\hat{d} > d^*_{D,N}$, we can conclude that the optimal strategy is to set $\hat{d} = d^*_{D,N} = \theta$.
Now let us examine the optimal deviation strategy for firm D. As before, it follows by Proposition 16 that a firm cannot increase his expected revenue by setting his booking capacity to any value other than $n^*_{N,D}$. As a result, it suffices to check only strategies in which the firm switches to policy N. First, suppose that $n^*_{N,D} \geq 2$. Then it follows by lemma 10 that the firm can capture $\frac{1}{2}$ of the market share if it sets its booking capacity to $\bar{n} \geq 2$ or the entire market share if it sets his booking capacity to $\bar{n} = 1$. It follows by Proposition 11 that the optimal strategy is to set $\bar{n} = n_{N,N}$, where $n_{N,N} = n^*$ if $\Pi_N\left(n^*; \frac{\lambda}{2}\right) > \frac{1}{2}\left(1 - e^{-\lambda}\right)$ and $n_{N,N} = 1$ otherwise. Now suppose that $n^*_{N,D} = 1$ and note that the associated service probability $\beta = 1$ as a result. Because the firm cannot capture any market share unless it also sets $\bar{n} = 1$, we conclude that the optimal strategy is to set his booking capacity to $\bar{n} = 1$.
Finally, using (8) and (15) it is easy to check that the expected revenue associated with the optimal deviation strategies satisfy (29). This completes our proof. □

Proposition 20:

Proof. Suppose both firms adopt policy N and set their booking capacity to $n_{N,N} = 1$. It follows from 4.1 that $(1, 1)$ is an equilibrium for the subgame $(N\,, N)$. Using (23) it follows from (i) the fact that $n_{N,N} = 1 \Rightarrow \beta = 1$ and (ii) the assumption that $r_N = r_D$, that the reservation probability for firm N and for firm D is $\gamma_N = 1$ and $\gamma_D = 0\ \forall d > 0$, respectively. As a result a firm cannot capture any market share by deviating to policy D and we can thus conclude that there exists an equilibrium in the meta-game in which both firms adopt policy N and set their booking capacity to $n_{N,N} = 1$. □

APPENDIX 2: The Existence of Asymmetric Equilibrium Policy $\{N\,, D\}$

As reported in Section 4.6, $\{D\,, D\}$ appears to be the most common equilibrium policies that both firms will adopt, followed by the equilibrium policies $\{N\,, N\}$. Because both firms are identical, one would expect all equilibria for the meta-game to be symmetric. As it turns out, there are rare instances in which policy $\{N\,, D\}$ is the unique Nash equilibrium for the meta-game. In this appendix, we first establish the existence of this asymmetric equilibrium policy $\{N\,, D\}$ using a specific numerical example. Then we provide some basic intuition to explain why such an asymmetric equilibrium exists.

Consider the case when $r_N = r_D = 100$, $v = 150$, $c = 5$, $\theta = 20$, and $\lambda = 3.5$. First, in the subgame $(N,\, N)$, one can check from Proposition 11 that both firms will set their booking capacity to $n_{N,N} = 6$ in equilibrium so that each firm can obtain an expected revenue $\Pi_{N,N}\left(n_{N,N}, n_{N,N}\right) = 51.05$. In this case, one can check from Proposition 17 that $\Pi_{N,N}\left(n_{N,N}, n_{N,N}\right) = 51.05 < 52.86 = \Pi_{D,N}\left(\tilde{d} = \theta = 20;\, n_{N,N}\right)$, where $\tilde{d}$ is the best response in the event when one of the firms makes a unilateral move to adopt policy D. As the condition for $\{N\,, N\}$ to be an equilibrium policy in the meta-game as stated in Proposition 17 is violated, we can conclude that $\{N\,, N\}$ is not an equilibrium for the meta-game in this specific instance.

Next, us examine the (D, D) subgame. Proposition 13 claims that both firms will require deposit $d^*_{D,D} = \theta = 20$ in the subgame equilibrium, and each firm can obtain an expected revenue $\Pi_{D,D}(\theta, \theta) = 50.96$. It follows from Proposition 18 that $\Pi_{D,D}(\theta, \theta) = 50.96 < 53.89 = \Pi_{N,D}(\hat{n} = 8\,;\, \theta)$, where $\hat{n}$ is the best response in the event when one of the firms makes a unilateral move to adopt policy N. As the condition for $\{D\,,\, D\}$ to be an equilibrium policy in the meta-game as stated in Proposition 18 is violated, we can conclude that $\{D\,,\, D\}$ is not an equilibrium for the meta-game in this specific instance.

It remains to examine the (N, D) subgame as discussed in Section 4.3. One can check from Proposition 16 that, in the subgame equilibrium, firm N will set $n^*_{N,D} = 8$ and firm D will require deposit $d^*_{D,N} = 20$. As a result, firm N and D will obtain an expected revenue $\Pi_{N,D}\left(n^*_{N,D} = 8,\, d^*_{D,N} = 20\right) = 53.89$ and $\Pi_{D,N}\left(d^*_{D,N} = 20,\, n^*_{N,D} = 8,\right) = 52.95$, respectively. To check if $\{N\,,\, D\}$ is an equilibrium policy in the meta-game, it suffices to check if there exists a profitable unilateral move for each firm. In this case, we can apply Proposition 19 to show that both conditions (i) and (ii) hold; hence we can conclude that $\{N\,,\, D\}$ is the unique Nash equilibrium in the meta-game in this instance.

After establishing the existence of an asymmetric equilibrium policy$\{N\,,\, D\}$ as the unique Nash equilibrium in the meta-game, we now provide the basic intuition to explain why such an symmetric equilibrium policy$\{N\,,\, D\}$ exists. When both firms adopt the same policy (i.e. under (N, N) or (D, D)), they compete for customers in the same segment (with the same show up probability distribution) by undercutting each other's booking capacity under policy(N, N) (or each other's required deposit under policy (D, D)). Consequently, one can check from the equilibrium outcomes from Propositions 11 and 13 that a Prisoner's Dilemma situation can occur under policy (N, N) or (D, D). On the contrary, under policy (N, D), one can observe from (21) and (22) that firm N and firm D compete in different customer segments: firm N focuses on customers whose show up probability $\psi < \frac{d}{d+(r_N - r_D)+(x_N - c)\cdot(1-\beta)}$, firm D focuses on customers whose show up probability $\psi > \frac{d}{d+(r_N - r_D)+(x_N - c)\cdot(1-\beta)}$, and both firms capture the entire market. Hence, policy (N, D) appears to be more efficient in terms of market segmentation. However, our numerical experiments reveal that policy (N, D) serves as the unique Nash equilibrium in the meta-game on rare occasions, which tend to occur when the demand rate is medium because policy (N, N) (policy (D, D)) tends to be the Nash equilibrium when the demand rate λ is low (high) as observed in Figure 3 and speculated in Conjecture 1.