Title
A Super-Covariant Derivative Expansion of the Effective Action

Authors
Jain, V.
Kotcheff, A.

Publication Date
1990-07-01

LBL-27944
LAPP-TH-285/90
July 1st, 1990

A Super–Covariant Derivative Expansion of the Effective Action

Vidyut Jain*

L.A.P.P., B.P. 110, 74941 Annecy-le-Vieux Cedex, France

Aaron Kotcheff

Lawrence Berkeley Laboratory
1 Cyclotron Road, Berkeley, California 94720

Abstract

We present a superspace formulation of covariant derivative expansion techniques. As an example we compute the leading one–loop corrections to the effective action for a supersymmetric nonlinear σ–model in four space–time dimensions. We briefly discuss applications to effective low energy models suggested by superstrings.

*Bitnet address: JAIN@frlapp51.

L.A.P.P. B.P. 110 F-74941 ANNECY-LE-VIEUX CEDEX ○ TÉLÉPHONE 50.23.32.45 + ○ TÉLEX 385180 F ○ TÉLÉCOPIE 50 27 94 95

DISCLAIMER

1. Introduction and motivation.

Nonlinear σ–models play an important role in physics, from the two–dimensional field theory that lives on the worldsheet that a string maps out as it evolves in space–time, to the effective theory describing a one–dimensional lattice of spin one–half states [1] or the effective low energy theory of the large Higgs mass limit of the standard model [2] to effective low energy theories suggested by superstring theory [3,4]. In all such cases, it is important to study the full quantum theory. For example, in the case of the field theory on the world sheet, conformal invariance of the full quantum theory gives restrictions on the background (massless) matter fields [5].

There is a class of $N = 1$ four–dimensional supergravity models, of the "no–scale" type, that are suggested by the low energy limit of string theory [3]. These models are highly degenerate at tree level, even after the supersymmetry is spontaneously broken by nonperturbative effects [6]. In particular, the scalar potential has flat directions in field space, so that the gravitino mass is undetermined at tree level. In addition, in these models, in which the Kähler potential is invariant [7] under a group of global nonlinear transformations among the scalar fields (in the limit of vanishing gauge and Yukawa couplings) the gauge nonsinglets remain massless at tree level. To make contact with physics at observable energies it is necessary to determine the mass scales in these models. To do this, one must determine radiative corrections to the tree level supergravity action. This is a complex undertaking, and much work has already been done to this end [8,9]. However, there are still some corrections that need to be determined, and before the results can be used they must be made consistent with the symmetries of the effective low energy theory [10].

The classical action has a number of invariances. We expect radiative corrections to respect the underlying supersymmetry, and also the Yang–Mills gauge invariance and the invariance under reparamterizations of the scalar fields. We therefore seek a formalism in which the perturbative expansion manifestly displays these invariances. In particular, we are interested here in determining the one–loop corrections. The background field method, together with a normal–coordinate expansion of the scalar fields [11,12] and (covariant) derivative expansion techniques [13,8] yield one–loop results that display manifest background scalar field reparametrization invariance. These techniques have been further developed to include background gauge invariance [14] and background general coordinate invariance [15].

Here we present a superspace formulation of the derivative expansion techniques, which we believe is useful for calculating the leading radiative corrections for theories with broken supersymmetry. For theories with explicitly broken supersymmetry this is acheived as usual at the superfield level by use of a spurion field. However, our main goal is the calculation of corrections in a theory with spontaneosly broken supersymmetry, as is the case in the above mentioned "no-scale" models, keeping manifest also the other invariances of the theory. The main result of this paper is to present a manifestly supersymmetric and reparameterization invariant method to compute the radiative corrections for a spontaneously broken supersymmetric nonlinear σ-models, as the beginning step for the more complicated case of a general supergravity + Yang–Mills + matter model. We believe a generalization to a gauged nonlinear σ-model should not be too difficult.

Although the explicit corrections we compute have existed in the literature for some time [16] a completely covariant superfield approach has not. Furthermore, the covariant derivative techniques have proved to be a very compact and useful procedure [8] for the type of applications we have in mind and provide an elegant alternative to Feynman diagrams.

In section two we show how to modify the derivative expansion technique for a simple chiral Lagrangian in four dimensions. As a primer for the nonlinear σ-model we compute the quadratically divergent one–loop corrections for a simple model where supersymmetry is explicitly broken by a 'spurion' field. Our results agree with previous diagrammatic calculations and component calculations [16].

In section three we further extend the techniques to the supersymmetric nonlinear σ-model in four dimensions. We present a modification of the usual reparametrization covariant background field expansion [12] which is appropriate for chiral superfields and solve the chirality constraints by introducing unconstrained fields in terms of which the action has a gauge invariance. Although the two expansions agree at the one–loop level, the chirality constraints beyond one–loop are not easy to solve if one uses the expansion of Ref. [12].

In section four we consider the gauge fixing of this action which, as in our spurion example of section 2, yields an infinite tower of ghosts. A covariant approach requires that the ghost determinants have background field dependence and so do not decouple. However, it is still possible to perform finite explicit calculations which give the required results.

Finally, in section five we conclude by discussing the application of our results to effective low–energy models suggested by string theory, and comment on regularization procedures that may be necessary in order to extract supersymmetric results.

2. Derivative expansion in Superspace.

Here we develop superspace functional methods to calculate the leading one–loop corrections. These methods have already been developed in the component field approach [13,8]. In general, to functionally evaluate the one–loop effective action we need methods for calculating determinants in superspace. Equivalently, we are interested in calculating $Tr\ ln\ \mathcal{O}$ for some operator $\mathcal{O}$ which is a function of superspace coordinates and their derivatives.* In our applications $\mathcal{O}$ will be the second functional derivative of the superspace action functional with respect to the superfields, i.e. the inverse superspace propogator. It is thought of as an infinite dimensional matrix with two (continious) labels. We first develop the formalism and then use it for the simple case of a model with explicitly broken supersymmetry.

Following [8,15] we assume that a function $F(\mathcal{O})$, of an operator $\mathcal{O}$, has an expansion

$$Tr\ F(\mathcal{O}) = \sum c_l\, Tr\ \mathcal{O}^l. \tag{2.1}$$

When $\mathcal{O}$ is of the form $\mathcal{O} = \mathcal{O}(\partial_Y, Y)\delta(X-Y)$ we have

$$Tr\mathcal{O}^l = \int \prod_{k=1}^{l} dX_k\ \mathcal{O}(\partial_{X_{k+1}}, X_{k+1})\delta(X_k - X_{k+1}), \tag{2.2}$$

where $X_{l+1} = X_1$, and the measure dX_k stands for $dX_k \equiv d^4x_k d^2\theta_k d^2\bar{\theta}_k$. To perform the integrals in this expression we need to remove the derivatives that act on the δ–functions. We do this by using a Fourier transform in superspace.† Schematically, we write

$$\delta(X-Y) = \int dP\ e^{-(X-Y)\cdot P}, \tag{2.3}$$

where $P_A = (ip_{\alpha\dot{\alpha}}, \psi_\alpha, \bar{\psi}_{\dot{\alpha}})$ stands for the "supermomentum", with $P_A^\dagger = (-ip_{\alpha\dot{\alpha}}, \bar{\psi}_{\dot{\alpha}}, \psi_\alpha)$. The measure dP stands for $dP \equiv \frac{d^4p}{(2\pi)^4} d^2\psi d^2\bar{\psi}$. We also have the identities

$$\begin{aligned} X^A e^{X\cdot P} &= -\partial_P^A e^{X\cdot P}, \\ \partial_{X^A} e^{X\cdot P} &= P_A e^{X\cdot P}, \end{aligned} \tag{2.4}$$

where $\partial_P^A = (i\frac{\partial}{\partial p_{\alpha\dot{\alpha}}}, \frac{\partial}{\partial\psi_\alpha}, \frac{\partial}{\partial\bar{\psi}_{\dot{\alpha}}})$ and $\partial_P^{A\dagger} = (i\frac{\partial}{\partial p_{\alpha\dot{\alpha}}}, \frac{\partial}{\partial\bar{\psi}_{\dot{\alpha}}}, \frac{\partial}{\partial\psi_\alpha})$. We obtain,

$$\mathcal{O}(\partial_{X_i}, X_i)\delta(X_{i-1} - X_i) \equiv \int dP_i\ e^{-X_{i-1}\cdot P_i}\mathcal{O}(P_i, -\partial_{P_i})e^{X_i\cdot P_i}. \tag{2.5}$$

*Our notation is that of Ref. 16. In the following an upper case X (for example) stands for both ordinary space–time x and the anticommuting variables $\theta, \bar{\theta}$. In addition, our Tr generally means a supertrace, also including a trace over any indicies that label an internal symmetry that the action might posses.

†It is not necessary to Fourier transform the anticommuting coordinates as their δ–function is known explicitly, but it appears simpler to Fourier transform all the variables.

Use of the inverse Fourier transform,

$$\int dX \; e^{X\cdot(P_2-P_1)} = \delta(P_2 - P_1), \tag{2.6}$$

lets us write

$$Tr\mathcal{O}^l = \int dX \int dP \; e^{-X\cdot P}\mathcal{O}(P, -\partial_P)^l e^{X\cdot P}, \tag{2.7}$$

from which we get

$$Tr \; ln \; \mathcal{O} = \int dX \int dP \; Tr \; ln\left\{e^{-X\cdot P}\mathcal{O}(P, -\partial_P)e^{X\cdot P}\right\}. \tag{2.8}$$

Finally, the translation operator appearing in (2.8) has the following properties:

$$\begin{aligned} e^{-X\cdot P}P_A e^{X\cdot P} &= P_A, \\ e^{-X\cdot P}\partial_P^A e^{X\cdot P} &= \partial_P^A - X^A, \end{aligned} \tag{2.9}$$

so that

$$Tr \; ln \; \mathcal{O} = \int dX \int dP \; Tr \; ln\mathcal{O}(P, -\partial_P + X). \tag{2.10}$$

We can use the additional identities

$$\begin{aligned} e^{-\partial_P\cdot\partial_X}X^A e^{\partial_P\cdot\partial_X} &= X^A - \partial_P^A, \\ e^{-\partial_P\cdot\partial_X}(P_A - \partial_{X^A})e^{\partial_P\cdot\partial_X} &= P_A, \end{aligned} \tag{2.11}$$

to write

$$Tr \; ln \; \mathcal{O} = \int dX \int dP \; Tr \; ln\left\{e^{-\partial_P\cdot\partial_X}\mathcal{O}(P - \partial_X, X)e^{\partial_P\cdot\partial_X}\right\}. \tag{2.12}$$

An immediate consequence is that if $\mathcal{O}(\partial_X, X)$ contains no anticommuting variables then $Tr \; ln \; \mathcal{O} = 0$. This is because in this case the argument of the ln contains no anticommuting variables, so that under the Berezin integral it vanishes. A simple example is the noninteracting massless chiral Lagrangian which leads to a propogator that depends only on the ordinary space–time d'Alembertian.

The expression (2.10) or (2.12) does not have manifest $SUSY$ invariance. To see this, consider a superfield $\Phi(X)$. Then

$$\begin{aligned} e^{-\partial_P\cdot\partial_X}\Phi(X)e^{\partial_P\cdot\partial_X} &= e^{-i\partial_p\cdot\partial_x}\left\{\Phi(X) + \left[e^{-\partial_\psi\cdot\partial_\theta - \bar\partial_{\bar\psi}\cdot\bar\partial_{\bar\theta}}, \Phi(X)\right]e^{\partial_\psi\cdot\partial_\theta + \bar\partial_{\bar\psi}\cdot\bar\partial_{\bar\theta}}\right\}e^{+i\partial_p\cdot\partial_x} \\ &= e^{-i\partial_p\cdot\partial_x}\left\{\Phi(X) - \left(\partial_\psi\cdot\partial_\theta\Phi(X) + \bar\partial_{\bar\psi}\cdot\bar\partial_{\bar\theta}\Phi(X)\right) + \cdots\right\}e^{+i\partial_p\cdot\partial_x}, \end{aligned} \tag{2.13}$$

which is an expansion of $\Phi(X)$ involving ordinary derivatives w.r.t. θ and $\bar\theta$ and not $SUSY$ covariant derivatives D_α and $\bar D_{\dot\alpha}$. In addition to this, under the series of operations ending with (2.12) the supercovariant derivative D_α becomes:

$$D_\alpha(\partial_X, X) = \frac{\partial}{\partial\theta^\alpha} + \frac{1}{2}i\bar\theta^{\dot\alpha}\partial_{\alpha\dot\alpha} \rightarrow D_\alpha(P - \partial_X, X) = \psi_\alpha - \frac{1}{2}\bar\theta^{\dot\alpha}p_{\alpha\dot\alpha} - D_\alpha(\partial_X, X), \tag{2.14}$$

with a similar expression for $\bar{D}_{\dot{\alpha}}$. The appearance of the "$\bar{\theta}^{\dot{\alpha}} p_{\alpha\dot{\alpha}}$" term means that we are going to have problems with manifest $SUSY$ invariance if $\mathcal{O}$ contains supercovariant derivatives. To cast the expansions in a manifestly invariant form note that inside the $d^2\psi d^2\bar{\psi}$ integral we have:

$$\int d^2\psi d^2\bar{\psi}\; e^{-(\partial_\psi\cdot\Theta+\bar{\partial}_{\bar{\psi}}\cdot\bar{\Theta})}\; F\; e^{\partial_\psi\cdot\Theta+\bar{\partial}_{\bar{\psi}}\cdot\bar{\Theta}} = \int d^2\psi d^2\bar{\psi}\; F, \tag{2.15}$$

for any function F and any anticommuting function Θ not containing any ψ or $\bar{\psi}$ dependence. In fact, we first write

$$\begin{aligned}&\int dP\; e^{-\partial_P\cdot\partial_X} F(P-\partial_X, X) e^{\partial_P\cdot\partial_X}\\ &\quad = \int \frac{d^4p}{(2\pi)^4} e^{-i\partial_p\cdot\partial_x}\left\{\int d^2\psi d^2\bar{\psi}\; F(ip-\partial_x,\psi_\alpha,\bar{\psi}_{\dot{\alpha}};x,\theta^\alpha-\partial_\psi^\alpha,\bar{\theta}^{\dot{\alpha}}-\bar{\partial}_{\bar{\psi}}^{\dot{\alpha}})\right\} e^{i\partial_p\cdot\partial_x},\end{aligned} \tag{2.16}$$

and then use

$$\begin{aligned}&\int d^2\psi d^2\bar{\psi}\; F(ip-\partial_x,\psi_\alpha,\bar{\psi}_{\dot{\alpha}};x,\theta^\alpha-\partial_\psi^\alpha,\bar{\theta}^{\dot{\alpha}}-\bar{\partial}_{\bar{\psi}}^{\dot{\alpha}})\\ &\quad = \int d^2\psi d^2\bar{\psi}\; G\; F(ip-\partial_x,\psi_\alpha,\bar{\psi}_{\dot{\alpha}};x,\theta^\alpha-\partial_\psi^\alpha,\bar{\theta}^{\dot{\alpha}}-\bar{\partial}_{\bar{\psi}}^{\dot{\alpha}})\; G^{-1},\end{aligned} \tag{2.17}$$

where

$$G = e^{-\frac{1}{2}(\partial_\psi^\alpha\bar{\theta}^{\dot{\alpha}}p_{\alpha\dot{\alpha}}+\partial_{\bar{\psi}}^{\dot{\alpha}}\theta^\alpha p_{\alpha\dot{\alpha}})} e^{-(\partial_\psi^\alpha D_{\theta\alpha}+\partial_{\bar{\psi}}^{\dot{\alpha}}\bar{D}_{\bar{\theta}\dot{\alpha}})} e^{+(\partial_\psi^\alpha\partial_{\theta\alpha}+\partial_{\bar{\psi}}^{\dot{\alpha}}\bar{\partial}_{\bar{\theta}\dot{\alpha}})}. \tag{2.18}$$

In particular, for a superfield $\Phi(X)$ we have:

$$\begin{aligned}&G\;\Phi(x,\theta^\alpha-\partial_\psi^\alpha,\bar{\theta}^{\dot{\alpha}}-\bar{\partial}_{\bar{\psi}}^{\dot{\alpha}})\;G^{-1}\\ &\qquad = e^{-(\partial_\psi^\alpha D_{\theta\alpha}+\partial_{\bar{\psi}}^{\dot{\alpha}}\bar{D}_{\bar{\theta}\dot{\alpha}})}\Phi(x,\theta^\alpha,\bar{\theta}^{\dot{\alpha}})e^{+(\partial_\psi^\alpha D_{\theta\alpha}+\partial_{\bar{\psi}}^{\dot{\alpha}}\bar{D}_{\bar{\theta}\dot{\alpha}})}\\ &\qquad = \Phi(X)-\big(\partial_\psi\cdot D_\theta\Phi(X)+\bar{\partial}_{\bar{\psi}}\cdot\bar{D}_{\bar{\theta}}\Phi(X)\big)+\cdots,\end{aligned} \tag{2.19}$$

and for the supercovariant derivatives we get

$$G\; D_\alpha(ip-\partial_x,\psi_\alpha,\bar{\psi}_{\dot{\alpha}};x,\theta^\alpha-\partial_\psi^\alpha,\bar{\theta}^{\dot{\alpha}}-\partial_{\bar{\psi}}^{\dot{\alpha}})\;G^{-1} = \psi_\alpha+\frac{1}{2}\bar{\partial}_{\bar{\psi}}^{\dot{\alpha}}(p_{\alpha\dot{\alpha}}+i\partial_{\alpha\dot{\alpha}}). \tag{2.20}$$

The net result of all of this is easily seen by considering $\mathcal{O}(\partial_X,X)=\mathcal{O}(D_\alpha,\bar{D}_{\dot{\alpha}};\Phi(X))$ a function of supercovariant derivatives and superfields. Then

$$Tr\; ln\;\mathcal{O} = \int dX\int dP\; Tr\; ln\mathcal{O}\left(D_\alpha^P,\bar{D}_{\dot{\alpha}}^P;\Phi^P(X)\right), \tag{2.21}$$

where

$$\begin{aligned}D_\alpha^P &= \psi_\alpha+\frac{1}{2}\bar{\partial}_{\bar{\psi}}^{\dot{\alpha}}p_{\alpha\dot{\alpha}},\\ \Phi^P(X) &= e^{-(i\partial_p\cdot\partial_x+\partial_\psi^\alpha D_{\theta\alpha}+\partial_{\bar{\psi}}^{\dot{\alpha}}\bar{D}_{\bar{\theta}\dot{\alpha}})}\Phi(X)e^{+(i\partial_p\cdot\partial_x+\partial_\psi^\alpha D_{\theta\alpha}+\partial_{\bar{\psi}}^{\dot{\alpha}}\bar{D}_{\bar{\theta}\dot{\alpha}})}\\ &= e^{-\partial_P\cdot D_X}\Phi(X)e^{+\partial_P\cdot D_X},\end{aligned} \tag{2.22}$$

and $D^A_X = (\partial_x, D_{\theta^\alpha}, \bar{D}_{\bar{\theta}^{\dot{\alpha}}})$.

As a simple application of (2.21), consider the action given by

$$S = \int d^4x\, d^2\theta\, d^2\bar{\theta}\, D^2\bar{\chi}(1+U)\bar{D}^2\chi, \tag{2.23}$$

where U is the "spurion", a *fixed* superfield, which explicitly breaks supersymmetry. As is well known [16], this action has a gauge invariance because the fields χ and $\bar{\chi}$ are unconstrained:

$$\begin{aligned} \chi &\to \chi + \bar{D}^{\dot{\alpha}} w_{\dot{\alpha}} \\ \bar{\chi} &\to \bar{\chi} + D^{\alpha}\bar{w}_{\alpha}. \end{aligned} \tag{2.24}$$

After gauge fixing in the standard way [16] the action becomes:

$$S = \int d^4x\, d^2\theta\, d^2\bar{\theta}\, \bar{\chi}(\Box + D^2 U\bar{D}^2)\chi, \tag{2.25}$$

with $\Box = \frac{1}{2}\partial_{\alpha\dot{\alpha}}\partial^{\alpha\dot{\alpha}}$. There are also ghosts (or more precisely, there is an infinite tower of ghosts) which arise from the gauge fixing procedure, but these decouple since their determinants have no spurion field dependence. The effective action is a function of the spurion field and is given by the super–determinant of the propogator in (2.25), or:

$$\begin{aligned} \Gamma &= -2\times\frac{1}{2}Tr\, ln(\Box + D^2U\bar{D}^2) \\ &= -Tr\, ln(1+\Box^{-1}D^2U\bar{D}^2) + \ldots \\ &= -Tr\sum_n \frac{-1}{n}(-\Box^{-1}D^2U\bar{D}^2)^n + \ldots, \end{aligned} \tag{2.26}$$

where the factor 2 arises as we have complex fields, the "$+\ldots$" represents the Hausdorff expansion, and we have used $Tr\, ln\, \Box = 0$. Since $D^2\bar{D}^2D^2 = \Box D^2$, the quadratically divergent terms will come from the leading term in

$$(-\Box^{-1}D^2U\bar{D}^2)^n = (-U)^n\Box^{-1}D^2\bar{D}^2 + \cdots. \tag{2.27}$$

From the results of this section, eq. (2.21), we obtain:

$$\Gamma_{quad} = -\int dX\int\frac{d^4p}{(2\pi)^4}\int d^2\psi d^2\bar{\psi}\,\sum_n\frac{-1}{n}(-U)^n(-p^{-2})D_P^2\bar{D}_P^2. \tag{2.28}$$

Carrying out the $d^2\psi d^2\bar{\psi}$ integral, and resumming the resultant series yields, for the quadratically divergent Lagrangian:

$$\mathcal{L}_{quad} = \int d^2\theta d^2\bar{\theta}\, ln(1+U)\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2}. \tag{2.29}$$

The $d^2\theta d^2\bar{\theta}$ integral can be performed once U is given. For example, if $U = \mu_0^2\theta^2\bar{\theta}^2$ we get

$$\mathcal{L}_{quad} = \mu_0^2 \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2}. \tag{2.30}$$

This choice of U explicitly breaks supersymmetry by giving the scalars a different mass than the fermions and our result agrees with the expected $STrM^2$ expression found by superspace Feynman diagrams [16] or component calculations.

3. Non-Linear Sigma Model

As is well known the supersymmetric non-linear sigma model in four dimensions is described by a single real function of the chiral and anti-chiral superfields, the Kähler potential $K(\Phi^i, \bar{\Phi}^{\bar{j}})$. The action is given by

$$S = \int d^4x\ d^2\theta\ d^2\bar{\theta}\ K(\Phi^i, \bar{\Phi}^{\bar{j}}), \tag{3.1}$$

where the chiral superfields $\Phi^i, \bar{\Phi}^{\bar{j}}$, are subject to the constraints

$$D^\alpha\bar{\Phi} = \overline{D}^{\dot{\alpha}}\Phi = 0. \tag{3.2}$$

The action is invariant both under holomorphic coordinate transformations:

$$\begin{aligned} \Phi^i &\to \tilde{\Phi}^i = f(\Phi^i) \\ \bar{\Phi}^{\bar{j}} &\to \tilde{\bar{\Phi}}^{\bar{j}} = f(\bar{\Phi}^{\bar{j}}) \end{aligned} \tag{3.3}$$

and Kähler transformations $K(\Phi, \bar{\Phi}) \to K(\Phi, \bar{\Phi}) + F(\Phi) + \bar{F}(\bar{\Phi})$.

In order to compute radiative corrections to the tree level theory, including derivative corrections, we will use the backround field method. We wish to expand the action (3.1) around a backround superfield configuration in such a way as to manifestly retain invariance under holomorphic coordinate transformations. The formalism for obtaining such a covariant expansion of the action is well known, and has been given for non-supersymmetric sigma models and supersymmetric sigma models in component formalism [12], with particular emphasis on sigma–models in two space–time dimensions. We briefly review these methods here, and modify them for the case of chiral superfields in four space-time dimensions. We also give the background field expansion upto forth order in the quantum fields which, although unnecessary for the explicit one-loop calculations we will later perform, should facilitate covariant computations of two–loop and higher corrections either by functional methods or superspace Feynman diagrams.

The coordinate field Φ^I of a sigma model on an arbitary Riemannian manifold M is split into the backround classical and quantum parts:

$$\Phi^I = \phi^I + \pi^I \tag{3.4}$$

The quantum field π^I represents the difference between coordinate values on the manifold, and does not transform as a vector under coordinate transformations. Hence the expansion of some object on the manifold as a power series in π will not be covariant at each order. Instead a vector $\xi^I(\pi)$ is choosen to play the role of the quantum field. ξ^I is defined as the tangent to the geodesic at the point ϕ^I that passes through the point $\phi^I + \pi^I$ at unit affine parameter. Solving the geodesic equation gives

$$\xi^I(\pi) = \pi^I + \tfrac{1}{2}\Gamma^I{}_{JK}(\phi)\pi^J\pi^K + \tfrac{1}{6}\left(\Gamma^I{}_{JK,L}(\phi) + \Gamma^I{}_{MJ}\Gamma^M{}_{KL}(\phi)\right)\pi^J\pi^K\pi^L + \cdots \tag{3.5}$$

where $\Gamma^I{}_{JK}(\phi)$ is the connection on M evaluated at the point ϕ^I. ξ^I is just the Riemann normal coordinate of the point $\phi^I+\pi^I$, and (3.5) defines a coordinate transformation from an arbitary coordinate system $\{\pi^I\}$ to Riemann normal coordinates. Hence in normal coordinates the relation between ξ^I and π^I is simply $\xi^I = \pi^I$ and the expansion of an arbitary tensor T is

$$T(\phi+\pi) = \sum_n \frac{1}{n!}\partial_{I_1}\cdots\partial_{I_n}T(\phi)\xi^{I_1}\cdots\xi^{I_n}. \tag{3.6}$$

To obtain the explicitly covariant form of this expansion the following proceedure is adopted [12]. The derivatives in (3.6) are replaced with covariant derivatives. In normal coordinates the resulting expression differs from (3.6) by derivatives of the connection which can then be reexpressed in terms of the curvature tensor using the identities true in normal coordinates:

$$\Gamma^I{}_{JK}(\phi) = \Gamma^I_{\{JK,L_1\cdots L_m\}}(\phi) = 0, \tag{3.7}$$

where $\{\}$ indicates symmetrisation.

Unfortunately this method is inappropriate on a Kähler manifold, $\mathcal{K}$. This can be seen by considering the transformation to normal coordinates. Starting in an arbitary hermitean coordinate system $\{\pi^i, \bar{\pi}^{\bar{j}}\}$, the normal coordinates are defined by

$$\begin{aligned}\xi^i =& \pi^i + \tfrac{1}{2}\Gamma^i{}_{jk}(\phi,\bar{\phi})\pi^j\pi^k + \tfrac{1}{6}\left(\Gamma^i{}_{jk,l}(\phi,\bar{\phi}) + \Gamma^i{}_{mj}\Gamma^m_{kl}(\phi,\bar{\phi})\right)\pi^j\pi^k\pi^l \\ &- \tfrac{1}{6}R^i{}_{jk\bar{l}}(\phi,\bar{\phi})\pi^j\pi^k\bar{\pi}^{\bar{l}} + \cdots\end{aligned} \tag{3.8}$$

and similarly for $\bar{\xi}^{\bar{j}}$. This expansion transforms by construction as a vector under (3.3). However, on a Kähler manifold the curvature tensor is given by

$$\begin{aligned}\Gamma^i{}_{jk,\bar{l}} &= -R^i{}_{jk\bar{l}} \\ \Gamma^{\bar{i}}{}_{\bar{j}\bar{k},l} &= -R^{\bar{i}}{}_{\bar{j}\bar{k}l}.\end{aligned} \tag{3.9}$$

Since in a normal coordinate system (3.7) is true, unless the curvature (and it's higher derivatives) vanishes at the origin, normal coordinates are incompatible with the hermitean structure of a Kähler manifold. Furthermore we would not expect the action (3.1) to be invariant under such a transformation. For the purposes of evaluating the Feynmann path integral it would be possible to proceed in this manner by simply treating $\mathcal{K}$ as a real manifold M. However, it appears that the constraints (3.2) become intractible in this approach because the ξ^i are not chiral, as one may readily verify by examining the expression for $\partial\xi^i$ given in Ref. [12].

The Riemann normal coordinate proceedure can, however, be straightforwardly adapted to the case of a Kähler manifold. We expand the chiral superfields $\Phi, \bar{\Phi}$ in terms of backround and quantum superfield parts:

$$\begin{aligned} \Phi^i &= \phi^i + \pi^i \\ \bar{\Phi}^{\bar{j}} &= \bar{\phi}^{\bar{j}} + \bar{\pi}^{\bar{j}}. \end{aligned} \tag{3.10}$$

We wish to define chiral and antichiral functions ξ^i and $\bar{\xi}^{\bar{j}}$, respectively, that transform under (3.3) as holomorphic and anti-holomorphic vectors respectively. The use of Riemann normal coordinates is too restrictive since, in general, the set of holomorphic coordinate transformations on $\mathcal{K}$ is smaller than the full set of coordinate transformations with $\mathcal{K}$ regarded as a real manifold. In analogy with (3.5) it is clear that the following *holomorphic* functions of $\pi, \bar{\pi}$ transform in the required way,

$$\begin{aligned} \xi^i(\pi) &= \pi^i + \tfrac{1}{2}\Gamma^i{}_{jk}(\phi,\bar{\phi})\pi^j\pi^k + \tfrac{1}{6}\left(\Gamma^i{}_{jk,l}(\phi,\bar{\phi}) + \Gamma^i{}_{mj}\Gamma^m_{kl}(\phi,\bar{\phi})\right)\pi^j\pi^k\pi^l + \cdots \\ \bar{\xi}^{\bar{i}}(\bar{\pi}) &= \bar{\pi}^{\bar{i}} + \tfrac{1}{2}\Gamma^{\bar{i}}{}_{\bar{j}\bar{k}}(\phi,\bar{\phi})\bar{\pi}^{\bar{j}}\bar{\pi}^{\bar{k}} + \tfrac{1}{6}\left(\Gamma^{\bar{i}}{}_{\bar{j}\bar{k},\bar{l}}(\phi,\bar{\phi}) + \Gamma^{\bar{i}}{}_{\bar{m}\bar{j}}\Gamma^{\bar{m}}_{\bar{k}\bar{l}}(\phi,\bar{\phi})\right)\bar{\pi}^{\bar{j}}\bar{\pi}^{\bar{k}}\bar{\pi}^{\bar{l}} + \cdots. \end{aligned} \tag{3.11}$$

This expression is similar to (3.8) but with all the terms involving $\bar{\pi}$ (π) in the expansion of ξ^i ($\bar{\xi}^{\bar{i}}$) missing. Since holomorphic (anti–holomorphic) transformations mix only πs with πs ($\bar{\pi}$s with $\bar{\pi}$s) these expressions transform as vectors under (3.3). Equation (3.11) defines a transformation to holomorphic normal coordinates, which is always possible on a Kähler manifold. In a holomorphic normal coordinate system we have $\xi^i = \pi^i, \bar{\xi}^{\bar{i}} = \bar{\pi}^{\bar{i}}$ and hence the following identities,

$$\begin{aligned} \Gamma^i{}_{jk}(\phi,\bar{\phi}) &= \Gamma^i{}_{jk,l_1\cdots l_m}(\phi,\bar{\phi}) = 0 \\ \Gamma^{\bar{i}}{}_{\bar{j}\bar{k}}(\phi,\bar{\phi}) &= \Gamma^{\bar{i}}{}_{\bar{j}\bar{k},\bar{l}_1\cdots\bar{l}_m}(\phi,\bar{\phi}) = 0. \end{aligned} \tag{3.12}$$

In exact analogy to the expansion of a tensor in Riemann normal coordinates the backround field expansion of the Kähler potential in a holomorphic normal coordinate system is simply,

$$K(\phi+\pi,\bar{\phi}+\bar{\pi}) = \sum_{n,m}\frac{1}{n!}\frac{1}{m!}\partial_{i_1\cdots i_n}\partial_{\bar{j}_1\cdots\bar{j}_m}K(\phi,\bar{\phi})\xi^{i_1}\cdots\xi^{i_n}\bar{\xi}^{\bar{j}_1}\cdots\bar{\xi}^{\bar{j}_m}. \tag{3.13}$$

To obtain the explicitly covariant expansion we follow a similar proceedure to the non-Kähler case. The derivatives in (3.13) are replaced by covariant derivatives. The holomorphic and anti-holomorphic covariant derivatives, ∇_i and $\bar{\nabla}_{\bar{j}}$, do not commute and we have to choose in what order they act on $K(\phi,\bar{\phi})$. However the explicitly covariant expression cannot be affected by the ordering chosen so it is best to order in a way which makes the calculation the easiest. Once this is done the resulting expression is considered in holomorphic normal coordinates. It will differ from (3.13) by derivatives of the connection, which are either zero by the identities (3.12), or can be re-written using the expression for the curvature tensor on a Kähler manifold, eq. (3.9).

This method can now be used to derive the fully covariant expansion of the Kähler potential to any order. We give here the expansion up to fourth order,

$$\begin{aligned}
K(\phi+\pi,\bar{\phi}+\bar{\pi}) = {} & K(\phi,\bar{\phi}) + \nabla_i K\,\xi^i + \bar{\nabla}_{\bar{i}} K\,\bar{\xi}^{\bar{i}} + \tfrac{1}{2}\left(\nabla_i\nabla_j K\,\xi^i\xi^j + \text{c.c.}\right) + \nabla_i\bar{\nabla}_{\bar{j}} K\,\xi^i\bar{\xi}^{\bar{j}} \\
& + \tfrac{1}{6}\left(\nabla_i\nabla_j\nabla_k K\,\xi^i\xi^j\xi^k + \text{c.c.}\right) + \tfrac{1}{2}\left[\left(\nabla_{\{i}\nabla_j\bar{\nabla}_{\bar{k}\}} K - \tfrac{1}{3}R^l{}_{ij\bar{k}}\nabla_l K\right)\xi^i\xi^j\bar{\xi}^{\bar{k}} + \text{c.c.}\right] \\
& + \tfrac{1}{24}\left(\nabla_i\nabla_j\nabla_k\nabla_l K\,\xi^i\xi^j\xi^k\xi^l + \text{c.c.}\right) \\
& + \tfrac{1}{6}\left[\left(\nabla_{\{i}\nabla_j\nabla_k\bar{\nabla}_{\bar{l}\}}\,K - \tfrac{1}{2}\nabla_i R^m_{jk\bar{l}}\nabla_m K - R^m_{jk\bar{l}}\nabla_i\nabla_m K\right)\xi^i\xi^j\xi^k\bar{\xi}^{\bar{l}} + \text{c.c.}\right] \\
& + \tfrac{1}{4}\left[\nabla_{\{i}\nabla_j\bar{\nabla}_{\bar{k}}\bar{\nabla}_{\bar{l}\}} K - \tfrac{1}{6}\left(\bar{\nabla}_{\bar{l}} R^m_{ij\bar{k}}\nabla_m K + 4R^m_{ij\bar{k}}\bar{\nabla}_{\bar{l}}\nabla_m K\right) + \text{c.c.}\right]\xi^i\xi^j\bar{\xi}^{\bar{k}}\bar{\xi}^{\bar{l}} \\
& + \cdots
\end{aligned} \tag{3.14}$$

where, for generality, we have let ∇_i and $\bar{\nabla}_{\bar{j}}$ act on $K(\phi,\bar{\phi})$ symmetrically.

The backround fields $\phi,\bar{\phi}$ satisfy the classical equations of motion and the constraints (3.2). Since the new fields $\xi,\bar{\xi}$ are holomorphic functions of the $\pi,\bar{\pi}$ respectively, the constraints on the quantum fields are unchanged to all orders:

$$D^\alpha\bar{\xi} = \overline{D}^{\dot{\alpha}}\xi = 0. \tag{3.15}$$

In order to solve these constraints it will be useful to introduce the reparamaterisation covariant superderivatives $\mathcal{D}^\alpha, \overline{\mathcal{D}}^{\dot{\alpha}}$ whose action, on arbitary vectors $W^i, \bar{W}^{\bar{j}}$ is

$$\begin{aligned}
\mathcal{D}^\alpha W^i &= D^\alpha W^i + \Gamma^i{}_{jk}(D^\alpha\phi^j)W^k \\
\mathcal{D}^\alpha \bar{W}^{\bar{j}} &= D^\alpha \bar{W}^{\bar{j}} \\
\overline{\mathcal{D}}^{\dot{\alpha}} W^i &= \overline{D}^{\dot{\alpha}} W^i \\
\overline{\mathcal{D}}^{\dot{\alpha}} \bar{W}^{\bar{j}} &= \overline{D}^{\dot{\alpha}} \bar{W}^{\bar{j}} + \Gamma^{\bar{j}}_{\bar{k}\bar{l}}(\overline{D}^{\dot{\alpha}}\bar{\phi}^{\bar{k}})\bar{W}^{\bar{l}}.
\end{aligned} \tag{3.16}$$

These derivatives satisfy the following (anti)commutation relations:

$$
\begin{aligned}
\{\mathcal{D}^\alpha, \mathcal{D}_\beta\} &= \{\overline{\mathcal{D}}^{\dot\alpha}, \overline{\mathcal{D}}^{\dot\beta}\} = 0 \\
\{\mathcal{D}^\alpha, \overline{\mathcal{D}}^{\dot\alpha}\} W^i &= \left(i\mathcal{D}^{\alpha\dot\alpha}\delta^i_j + R^{\alpha\dot\alpha\,i}{}_j\right) W^j \\
\left[\mathcal{D}_\alpha, \{\mathcal{D}^\alpha, \overline{\mathcal{D}}^{\dot\alpha}\}\right] W^i &= \left(\mathcal{D}_\alpha R^{\alpha\dot\alpha\,i}{}_j\right) W^j + R^{\dot\alpha\,i}{}_j W^j \\
\left[\overline{\mathcal{D}}_{\dot\alpha}, \{\mathcal{D}^\alpha, \overline{\mathcal{D}}^{\dot\alpha}\}\right] W^i &= \left(\overline{\mathcal{D}}_{\dot\alpha} R^{\alpha\dot\alpha\,i}{}_j\right) W^j - R^{\alpha\,i}{}_j W^j,
\end{aligned}
\tag{3.17}
$$

where $\mathcal{D}_{\alpha\dot\alpha}$ is the covariant derivative corresponding to $\partial_{\alpha\dot\alpha}$ and

$$
\begin{aligned}
R^{\alpha\dot\alpha\,i}{}_j &= R^i{}_{jk\bar l}(D^\alpha\phi^k)(\overline{D}^{\dot\alpha}\bar\phi^{\bar l}) \\
R^{\dot\alpha\,i}{}_j &= R^i{}_{jk\bar l}(D_\alpha\phi^k)(i\partial^{\alpha\dot\alpha}\bar\phi^{\bar l}) \\
R^{\alpha\,i}{}_j &= R^i{}_{jk\bar l}(\overline{D}_{\dot\alpha}\bar\phi^{\bar l})(i\partial^{\alpha\dot\alpha}\phi^k).
\end{aligned}
\tag{3.18}
$$

We can now solve the constraints (3.15) in a completely covariant manner by introducing the unconstrained superfields $\chi^i, \bar\chi^{\bar j}$ transforming as holomorphic and anti-holomorphic vectors respectively, and then writing,

$$
\begin{aligned}
\xi^i &\equiv \overline{\mathcal{D}}^2\chi^i = \overline{D}^2\chi^i \\
\bar\xi^{\bar i} &\equiv \mathcal{D}^2\bar\chi^{\bar i} = D^2\bar\chi^{\bar i}.
\end{aligned}
\tag{3.19}
$$

Notice that in this change of coordinates the Jacobian can be ignored since it contains no background field dependence. As for the spurion example of section two, this proceedure introduces the abelian gauge invariance,

$$
\begin{aligned}
\chi^i &\rightarrow \chi^i + \overline{D}^{\dot\alpha}\omega^i_{\dot\alpha} \\
\bar\chi^{\bar i} &\rightarrow \bar\chi^{\bar i} + D^\alpha\bar\omega^{\bar i}_\alpha
\end{aligned}
\tag{3.20}
$$

where $\bar\omega, \omega$ are arbitary anticommuting superfields.

4. Covariant gauge–fixing and one–loop results.

In this section we find the leading one–loop corrections for the nonlinear σ–model. In order to obtain an invertable propogator we must first gauge fix the invariance (3.20). A straightforward application of the Fadeev-Popov procedure, however, quickly leads to problems. This is because in order to keep manifest reparameterization covariance the gauge fixing functions must be constructed out of covariant derivatives which have background field dependence. Then gauge fixing leads to infinite ghosts all with background field dependence. In the spurion example of section two because the gauge fixing functions were field independent the ghosts decoupled and could be ignored. In the nonlinear σ–model case we must determine the contribution of the ghosts in order to obtain the complete background field dependent effective action. This requires calculating an infinite number of functional determinants. Fortunately, by a clever choice of the gauge–fixing functions, and making use of the fact that physical results must be independent of this choice, we are able to extract the required results by evaluating only one functional determinant. We present this in detail in section 4.2. First, however, we describe why the straightforward approach fails us. The reader who is not interested in this should skip to section 4.2.

4.1 Ghosts with K dependence.

Following the Fadeev-Popov procedure, let us choose the backround covariant gauge fixing functions:

$$\begin{aligned} F^{i\alpha} &= \mathcal{D}^{\alpha}\chi^{i} \\ \bar{F}^{\bar{i}\dot{\alpha}} &= \overline{\mathcal{D}}^{\dot{\alpha}}\bar{\chi}^{\bar{i}}. \end{aligned} \tag{4.1}$$

Since physical results cannot depend on the anticommuting fields F and $\bar{F}$ we integerate over them in the path integral with the gauge fixing term

$$S_{GF} = \int d^4x\, d^2\theta\, d^2\bar{\theta}\; K_{\bar{i}j}\, \bar{F}^{\bar{i}\dot{\alpha}} M_{\alpha\dot{\alpha}} F^{j\alpha}, \tag{4.2}$$

added to the action, where $M_{\alpha\dot{\alpha}} = -\mathcal{D}_{\alpha}\overline{\mathcal{D}}_{\dot{\alpha}} - \frac{3}{4}\overline{\mathcal{D}}_{\dot{\alpha}}\mathcal{D}_{\alpha}$. Due to the nontrivial factor $M_{\alpha\dot{\alpha}}$ in (4.2) we must normalize our path integral by compensating with Nielsen-Kallosh ghosts, so that the integral over the Nielsen–Kallosh ghosts and the F and $\bar{F}$ fields yields 1 ignoring all else. There is a subtlety here which introduces additional ghosts. Note the smearing fields F and $\bar{F}$ themselves satisfy a constraint. For example, taking the first of the gauge fixing expressions (4.1) and applying $\mathcal{D}^{\beta}$ one easily obtains

$$\mathcal{D}^{\beta}F^{i\alpha} + \mathcal{D}^{\alpha}F^{i\beta} = 0. \tag{4.3}$$

This means that the simple approach of integrating over all unconstrained fields F and $\bar{F}$, and over all unconstrained Nielson–Kallosh ghosts, is incorrect, and in general will yield physical answers that depend on the gauge–fixing functions. This problem can be rectified by using so–called hidden ghosts [17], but these are not important to what we wish to illustrate here.

We also have Fadeev-Popov ghosts whose action can be found from a gauge variation on the gauge fixing functions (4.1). Unlike the spurion case, the Fadeev-Povov and

Nielsen-Kallosh determinants cannot be ignored since they involve the backround fields through the backround covariant superderivatives. We have,

$$Z[\phi,\bar{\phi}]_{FP} = \int \mathcal{D}\omega\mathcal{D}\bar{\omega}e^{-S_{FP}} \quad Z[\phi,\bar{\phi}]_{NK} = \int \mathcal{D}b\mathcal{D}\bar{b}e^{-S_{NK}} \tag{4.4}$$

with

$$\begin{aligned} S_{FP} &= \int d^4x\, d^2\theta\, d^2\bar{\theta}\; K_{\bar{i}j}\,\bar{\omega}^{\bar{i}}_{\alpha}\mathcal{D}^{\alpha}\overline{\mathcal{D}}^{\dot{\alpha}}\omega^{j}_{\dot{\alpha}} \\ S_{NK} &= \int d^4x\, d^2\theta\, d^2\bar{\theta}\; K_{\bar{i}j}\,\bar{b}^{\bar{i}}_{\dot{\alpha}}M^{\alpha\dot{\alpha}}b^{j}_{\alpha} \end{aligned} \tag{4.5}$$

where the $\bar{\omega}^{\bar{i}}_{\alpha}, \omega^{i}_{\dot{\alpha}}$ are anticommuting ghost fields and the $\bar{b}^{\bar{i}}_{\dot{\alpha}}, b^{i}_{\alpha}$ are commuting ghost fields.

To see that we have ghosts of ghosts notice that S_{FP} is itself invariant under the gauge transformations

$$\begin{aligned} \omega^{i}_{\dot{\alpha}} &\to \omega^{i}_{\dot{\alpha}} + \overline{\mathcal{D}}^{\dot{\beta}} L^{i}_{\{\dot{\alpha}\dot{\beta}\}} \\ \bar{\omega}^{\bar{i}}_{\alpha} &\to \bar{\omega}^{\bar{i}}_{\alpha} + \mathcal{D}^{\beta}\bar{L}^{\bar{i}}_{\{\alpha\beta\}}. \end{aligned} \tag{4.6}$$

Gauge fixing this invariance leads to a second generation of ghosts etc. and, as in superfield approaches to Super-Yang-Mills theory [18], we obtain an infinite tower of ghosts all coupling to the backround field.

The important question at this stage is: can we explicitly calculate the effective action? To investigate this consider the first level Fadeev-Popov ghost term. After gauge fixing we get

$$S_{FP} = \int d^4x\, d^2\theta\, d^2\bar{\theta}\; K_{\bar{i}j}\,\bar{\omega}^{\bar{i}}_{\alpha}\left(\mathcal{D}^{\alpha}\overline{\mathcal{D}}^{\dot{\alpha}} + \alpha\overline{\mathcal{D}}^{\dot{\alpha}}\mathcal{D}^{\alpha}\right)\omega^{j}_{\dot{\alpha}}, \tag{4.7}$$

with α the gauge fixing parameter. Examining the ghost terms that arise from the gauge invariance, eq. (4.6), it is easy to see that all the higher generation Fadeev-Popov terms are of the same form as S_{FP} but with ghosts of alternating statistics. No further Nielsen-Kallosh determinants arise. Choosing $\alpha = 1$, eq. (4.7) becomes

$$S_{FP} = \int d^4x\, d^2\theta\, d^2\bar{\theta}\; K_{\bar{i}j}\,\bar{\omega}^{\bar{i}}_{\alpha}\left(i\mathcal{D}^{\alpha\dot{\alpha}}\delta^{j}{}_{k} + R^{\alpha\dot{\alpha}}{}^{j}{}_{k}\right)\omega^{k}_{\dot{\alpha}}, \tag{4.8}$$

and hence by the results of section 2[‡] we have $Z_{FP} = 1$, since it is the superdeterminant of an operator containing no supercovariant derivatives. If we attempt to evaluate the second level of Fadeev-Popov ghosts in this manner it is clear they will also give 1, and so on for the third, etc. level.

With this gauge–fixing procedure, the fully backround covariant unconstrained action relevant for calculating quantum corrections can now be obtained by taking the expansion

‡ See the comment after equation (2.12).

(3.14), setting the pieces linear in $\xi, \bar{\xi}$ to zero, making the substitution (3.19), and adding the gauge fixing term (4.2). For briefness of presentation we do not give the relevant terms here. We find that these terms do not yield any quadratically divergent corrections. This is because equations (2.21) and (2.22) imply that we need two Ds and two $\bar{D}$s in $ln\mathcal{O}$ in order to get a nonvanishing contribution to $Tr\ ln\ \mathcal{O}$. For a quadratic divergence we also need a term proportional to $\Box^{-1}$ in $ln\mathcal{O}$. We find no such terms in the part of the action quadratic in $\chi, \bar{\chi}$. A similar analysis can be used to note that the contribution from the Nielson–Kallosh sector also does not yield any quadratically divergent piece.

In fact, this simple procedure simply keeps pushing the quadratic divergence to the next level of ghosts.[§] Apart from the already mentioned problems in Super-Yang-Mills [18], covariant approaches to quantization of the superparticle [19] and the Green–Scwarz superstring [20] also lead to infinite towers of ghosts. We have not suceeding in finding a covariant gauge fixing procedure which has a finite number of ghosts, and as for the superparticle or the superstring, perhaps there is none. In fact, for the superstring it appears that an infintite tower of ghosts is needed because there is no number of finite functional integrals over covariant quantities that can yield the desired result [21]. In our case, an inspection of the leading divergent contribution that we find in section 4.2 suggests a similar problem. However, it is not the aim of this paper to consider such questions. We will merely assume that a consistent covariant gauge fixing procedure exists.

4.2 Ghosts without K dependence.

Given that we have an infinite tower of ghosts, can we perform finite explicit calculations to obtain the effective action? In fact, it is possible to extract the required physical results by evaluating only one determinant. The crucial observation is that the physical results are independent of the gauge fixing functions. Hence, by carefully choosing the gauge fixing functions so that the ghost determinants are all independent of the Kähler potential, we are able to have all the K dependence in a single determinant. Of course, we can use the noncovariant background field independent gauge fixing procedure as for the spurion case of section two. Then, although the explicit determinant will not have manifest reparameterization covariance, physical results will be reparameterization invariant since they cannot depend on the choice of gauge fixing. However, we chose to keep manifest covariance. Assuming there is a consistent way of gauge fixing and given an arbitrary background field dependent covariant tensor $V_{i\bar{j}}$ we again follow the gauge–

[§]If we had chosen a noncovariant bacground field independent gauge fixing for the ghost invariance (4.6) we would have obtained a quadratic contribution from the ghost determinant.

fixing procedure given in section 4.1. However, instead of (4.1) we pick the following gauge fixing conditions:

$$
\begin{aligned}
F_{\bar{i}\alpha} &= D_\alpha V_{j\bar{i}}\chi^j \\
\bar{F}_{i\dot{\alpha}} &= \bar{D}_{\dot{\alpha}} V_{i\bar{j}}\bar{\chi}^{\bar{j}}.
\end{aligned}
\tag{4.9}
$$

These are covariant by construction (because of the transformation properties of $V_{i\bar{j}}$) but have no explicit dependence on the Kähler potential. We integrate over the fields F and $\bar{F}$ with a gauge fixing term

$$
S_{GF} = \int d^4x\, d^2\theta\, d^2\bar{\theta}\; V^{l\bar{i}}\, F_{\bar{i}\alpha}(M^{\alpha\dot{\alpha}})_l^i F_{i\dot{\alpha}}, \tag{4.10}
$$

where $V^{i\bar{j}}$ is the inverse matrix of $V_{i\bar{j}}$, and $(M^{\alpha\dot{\alpha}})_k^i = -D^\alpha V_{i\bar{j}}\bar{D}^{\dot{\alpha}}V^{\bar{j}k} - \frac{3}{4}V_{i\bar{j}}\bar{D}^{\dot{\alpha}}V^{\bar{j}k}D^\alpha$. We need Fadeev–Popov, Nielson–Kallosh and "hidden" ghosts to take care of all the constraints properly. All these ghost determinants will be manifestly covariant but have no *explicit* dependence on the Kähler potential. The gauged fixed action of $O(\chi,\bar{\chi})$ will have both K and V dependence. However, by gauge invariance, the total effective action must be independent of V[¶], or:

$$
S_{eff}[K] = S_\chi[K,V] + S_g[V], \tag{4.11}
$$

where S_χ comes from the functional integral over the $O(\chi,\bar{\chi})$ terms and S_g comes from the integral over the ghost sectors (i.e. the rest). This means that S_χ must factorize into a part that depends only on K and a part that depends only on V, and the V independent terms then yield the required effective action. Hence if we compute only S_χ and drop all the V dependent terms[‖] then we will obtain the required effective action, S_{eff}.

Using this methodolgy, we find, with some minor discomfort, for the $O(\chi,\bar{\chi})$ gauged fixed action (after integration over the fields F and $\bar{F}$) the expression

$$
S_{\text{quad}}[\phi,\bar{\phi},\chi,\bar{\chi}] = \frac{1}{2}\int d^4x\, d^2\theta\, d^2\bar{\theta}\; \left(\bar{\chi}^{\bar{i}},\chi^i\right)\begin{pmatrix} \mathcal{O}_{\bar{i}j} & \mathcal{O}_{\bar{i}\bar{j}} \\ \mathcal{O}_{ij} & \mathcal{O}_{i\bar{j}} \end{pmatrix}\begin{pmatrix} \chi^j \\ \bar{\chi}^{\bar{j}} \end{pmatrix} \tag{4.12}
$$

with

$$
\mathcal{O}_{ij} = \mathcal{O}^\dagger_{\bar{i}\bar{j}} = (\overline{\mathcal{D}}^2\nabla_i K_j)\overline{\mathcal{D}}^2, \tag{4.13}
$$

and

$$
\mathcal{O}_{\bar{i}k} = \mathcal{O}^\dagger_{i\bar{k}} = V_{\bar{i}j}\left[(\Box_V)^j_k + V^{j\bar{l}}D^2(K_{\bar{l}k} - V_{\bar{l}k})\overline{\mathcal{D}}^2\right], \tag{4.14}
$$

¶In the background field formalism, when properly used, the effective action is manifestly invariant under background gauge transformations. The effective action may not be completely independent of V but any background gauge invariant terms in the effective action that depend on V must vanish by the background equations of motion so do not present a problem. In fact, the leading divergent correction we calculate has no dependence on V.

‖Actually, we can only require such a factorization up to background field Kähler transformations.

and where $\Box_V$ contains only explicit V dependence:

$$
\begin{aligned}
(\Box_V)^j_k = & \tilde{\Box}\delta^j_k - i\tilde{R}^{\alpha\dot{\alpha}\,j}{}_k\tilde{\mathcal{D}}_{\alpha\dot{\alpha}} \\
& + \frac{1}{2}(\bar{\tilde{\mathcal{D}}}_{\dot{\alpha}}\tilde{R}^{\dot{\alpha}\,j}{}_k) - \frac{1}{2}(\tilde{\mathcal{D}}_\alpha\tilde{R}^{\alpha\,j}{}_k) - \tfrac{1}{4}(\tilde{\mathcal{D}}_\alpha\bar{\tilde{\mathcal{D}}}_{\dot{\alpha}}\tilde{R}^{\alpha\dot{\alpha}\,j}{}_k) \\
& - \frac{1}{2}(\bar{\tilde{\mathcal{D}}}_{\dot{\alpha}}\tilde{R}^{\alpha\dot{\alpha}\,j}{}_k)\tilde{\mathcal{D}}_\alpha - \frac{1}{2}(\tilde{\mathcal{D}}_\alpha\tilde{R}^{\alpha\dot{\alpha}\,j}{}_k)\bar{\tilde{\mathcal{D}}}_{\dot{\alpha}} + \frac{1}{2}\tilde{R}^{\alpha\,j}{}_k\tilde{\mathcal{D}}_\alpha - \frac{1}{2}\tilde{R}^{\dot{\alpha}\,j}{}_k\bar{\tilde{\mathcal{D}}}_{\dot{\alpha}}.
\end{aligned}
\tag{4.15}
$$

Here $\tilde{\Box} = \frac{1}{2}\tilde{\mathcal{D}}_{\alpha\dot{\alpha}}\tilde{\mathcal{D}}^{\alpha\dot{\alpha}}$, and derivative terms $\tilde{\mathcal{D}}_\alpha$, $\bar{\tilde{\mathcal{D}}}_{\dot{\alpha}}$, $\tilde{\mathcal{D}}_{\alpha\dot{\alpha}}$ and curvature terms $\tilde{R}$ are as in (3.16) – (3.18) but with all explicit and implicit K dependence replaced by corresponding V dependence.

The effective action is proportional to the log–determinant arising from the χ fields that we must calculate

$$
ln\ det \begin{pmatrix} \mathcal{O}^i_j & \mathcal{O}^i_{\bar{j}} \\ \mathcal{O}^{\bar{i}}_j & \mathcal{O}^{\bar{i}}_{\bar{j}} \end{pmatrix}, \tag{4.16}
$$

where $\mathcal{O}^i_j = V^{ik}\mathcal{O}_{j\bar{k}}$, and so on. Perhaps it is worth restating that the superdeterminant of a super–covariant derivative independent quantity is 1, which is what allows us to drop the *ln det V* implied by (4.16).

For the quadratically divergent corrections we can immediately apply the results of the spurion field calculation of section 2. The off–diagonal terms in the matrix of (4.16) do not contribute quadratically divergent corrections. Also, in this case none of the background field dependent terms in $\Box_V$ are important. Comparing (2.25) with (4.12) and the propogator term (4.14) we see that the quadratically divergent corrections come from the last term in (4.14). More precisely, the role of U is played by $V^{i\bar{k}}K_{j\bar{k}} - \delta^i_j$ and its complex conjugate here. Then, adapting the result (2.29) we find for the quadratically divergent correction:

$$
\mathcal{L}_{quad} = \int d^2\theta d^2\bar{\theta}\ tr\ ln(V^{i\bar{k}}K_{j\bar{k}}) \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2}. \tag{4.17}
$$

The promised factorization occurs because under the integral over the anticommuting coordinates we can use $ln(V^{-1}K) = lnV^{-1} + lnK$. Hence, in *component* form we are able to drop all V dependence. We obtain the same answer as previous diagrammatic and component calculations [16].

In fact, given the answer we are able to make a comment about our problems with infinite ghosts. It should be stressed that although what we are interested in is a supersymmetric *action*, what we actually calculate is a reparameterization invariant quantity which is a function of (background) superfields, that is *without* the superspace integral.

There is no way to form such a quantity from the Kähler metric $K_{i\bar{j}}$ that gives the required result. This is why we needed another tensorial object ($V_{i\bar{j}}$), i.e. so that we can form an invariant object. Having introduced V we would like its contributions to cancel in the total gauge fixing independent calculation. However, again there is no invariant term we can construct which is a function of $V_{i\bar{j}}$ only that is needed to cancel the lnV^{-1} that we obtained from the calculation above.

Finally in this section we briefly comment on modifications that are necessary to the procedure of section 2 in order to go beyond the leading correction we have found. In general, the procedure of section two will yield supersymmetric results but not holomorphic reparameterization invariant results. This is because the derivative expansion in eq. (2.22) is in terms of ordinary super-covariant derivatives, not reparameterization covariant ones. However, the method for regaining manifest covariance is just the same as in the case of the purely bosonic nonlinear σ–model. Then, as specified in Ref. [8], one multiples the integrand of eq. (2.21) on the left and on the right by suitably chosen functions of the derivatives and background fields so as to "shift" the ordinary derivatives to covariant ones. Our only difference here is that we deal with superfields and we have three different derivatives (D^A_X). In any case, these considerations do not affect the leading correction.

5. Conclusion.

We have presented a procedure for calculating radiative corrections in theories with broken supersymmetry which we believe is especially useful in the case of spontaneous breaking since our method manifestly displays the background field supersymmetry invariance. We further extended this to the case of the nonlinear σ–model in four dimensions and presented a method for retaining manifest invariance under the holomorphic (chiral superfields) and antiholomorphic (antichiral superfield) reparameterizations.

The physical motivation for this work is as follows. Although the nonlinear σ–model is not renormalizable, with suitable interpretation it can be used to describe physics as an effective low energy theory valid below some energy scale. Loop corrections will yield terms not in the original Lagrangian and that have a dependence on the scale μ used to regulate the divergent integrals. If we interpret the nonlinear σ–model as some appropriate limit of some underlying finite or renormalizable theory then we must take the scale μ to be of order the scale at which the physics of the full theory would enter to dampen the otherwise divergent integrals. In other words, if we interpret the scale μ

properly then loop corrections computed with the nonlinear σ–model should correspond to a suitable low energy limit of corrections computed with the full theory. Of course, we only expect this correspondence to hold for the μ^2 and $ln\mu^2$ corrections since the nonlinear σ model is insensitive to the details of the underlying physics. It should be noted that this correspondence has been made exact at the one loop level in the case of the large λ (coupling constant) limit of the standard model (see Ref. [2] for a review).

The philosophy in the case of string theory is the same, although the application tends to be a little more involved. Assuming supersymmetry survives compactification its generaly believed that the effective low energy theory should be of the "no–scale" type [4]. The tree–level model posseses a high amount of degeneracy and particles that we would like to have mass remain massless at this level. Therefore it is important to study radiative corrections. Ideally, one would want to compute the full effective action in the context of string (field?) theory and then take an appropriate low energy limit (i.e. remove the tower of massive modes). Another approach involves calculating loop corrections to the field theory that lives on the worldsheet of a string in the presence of a background of the massless modes [5]. Then, requirement of conformal invariance at the quantum level gives conditions on the background fields which can be interpreted as the equations of motion arising from some low energy effective action for the massless modes [22]. Our methodology is that by first taking the point field limit of string theory and then calculating loop corrections, the two approaches should agree in the $O(\mu^2)$ and $O(ln\mu^2)$ corrections (at least at one–loop) with a suitable interpretation for μ, and when all the symmetries of the theory and threshold effects are taken into account [10]. The correct identification for μ in this case is the compactification scale, since this sets the scale of the infinite tower of Kaluza–Klien modes that help to make the underlying theory more finite. Many of calculations required to find the one–loop corrections have been determined in component formalism (Burton *et. al.* in [9]) but require "supersymmetric completion" before they can be used.

Another complication is that the compactification scale is a field dependent quantity (i.e. μ is a field, not a constant). For example, in Witten's simple reduction [23] the compactification scale is

$$\mu = \frac{M_{Pl}}{(x\rho)^{\frac{1}{2}}}, \tag{5.1}$$

where M_{Pl} is as always the Planck mass and the quantities x and ρ are related to his gauge singlet fields S and T and the nonsinglet fields ϕ by

$$x = \text{Re } S, \quad \rho = \text{Re } T - \frac{1}{2}|\phi|^2. \tag{5.2}$$

The field dependency of μ means that extra care must be taken to ensure a supersymmetric answer [10] for the one–loop corrections. Since a major motivation for this work was to find a manifestly supersymmetric answer the problem of supersymmetric regularization in the case of a field dependent μ must also be addressed before the results of this work can be directly applied to these models.

It is clear that cutting of the momentum integral in (4.17) at $|p^2| = \mu^2$ will not yield a supersymmetric answer when μ is a field. However, it is equally clear that cutting off the momentum integral at $|p^2| = \Lambda^2$, where Λ is a *superfield* whose scalar component is μ will yield a supersymmetric result inside the Berezin integral. However, if Λ itself transforms nontrivially under the action of some symmetry S which we wish to keep then a simple cut–off prescription is not sufficient. In this case a more general and correct procedure in studying loop corrections is to work with a regulated Lagrangian in which fields of mass $O(\mu^2)$ act as the regulators [10], much as in Pauli–Villars. The couplings of these fields to the light fields must be invariant under S, and the expansion techniques used in this paper must be modified so that all derivatives are covariant with respect to S, thus ensuring the final answer will be manifestly invariant under S. Since supersymmetric Pauli–Villars regularization is well known [16] we do not forsee any major obstacle in implementing this strategy at the superfield level as long as one is careful in only using he "mass" superfield Λ and not μ which is not a superfield.

We stress that it should not be necessary to reevaluate all the corrections to supersymmetrize previous component results, although this will certainly provide a useful check on these difficult calculations. All one needs are the tree + one–loop supersymmetry transformations. In a formulation without auxiliary fields these are different than the tree level supersymmetry transformations. One way to see this is to calculate the corresponding corrections in a superfield approach keeping all the auxiliary fields, in which case the tree + one–loop transformations are necessarily the same as the tree level transformations. However, the tree + one–loop equations of motion for the auxiliary fields will now depend on the one–loop corrections, i.e. on the regulating scale. Hence, when the auxiliary fields are eliminated at the one–loop level they will induce corrections to the supersymmetry transformations of the other fields that depend on the regulating scale. It was a lack of knowledge of these corrections to the tree level transformations that prevented, for example, Burton *et. al.* [9] from supersymmetrizing their results for the scalar and gauge effective action. Our belief is that by using a manifestly supersymmetric superfield approach such as ours it should be possible to determine the one–loop corrections to the supersymmetry transformations by evaluating only a few carefully chosen terms in the effective action, not the entire effective action. Once found these can be used to supersymmetrize previous component results. It should be noted that even when in principle both the bosonic and fermionic parts of the effective action have been determined via component calculations it is not straightforward to supersymmetrize the results. For example, Ref. [10] achieves this for the quadratically divergent corrections that scale with the number of chiral supermultiplets (N) in a no-scale type model. An extension of the approach of Ref. [10] to terms subleading in N is not easy.

Of course the techniques developed here must be extended to Yang–Mills and supergravity before we can use them for the string inspired low energy models. In addition, one would want to compute the full $O(\mu^2)$ and order $O(ln\mu^2)$ to the full tree–level Lagrangian in these "no–scale" models. However, we feel this is best left to a future presentation.

Acknowledgements.

We wish to thank Massimo Porrati for various engaging conversations, many related to this work. Vidyut Jain would also like to thank Mary K. Gaillard for encouragement and useful comments.

This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY–85–15857. In addition, V.J. acknowledges partial support by the National Science Foundation under grant INT–87–15131, and A.K. thanks the Science and Engineering Research Council of Great Britain for a N.A.T.O. fellowship.

References.

[1] R. Shankar, NSF–ITP–89–116 (1989); *Phys. Rev. Lett.* **63:** 203 (1989).

[2] See, for example, M. K. Gaillard, "Effective Nonrenormalizable Theories at One Loop", in *Particle Physics, Cargèse 1987*, ed. M. Lévy, *et. al.*, Plenum Press, New York (1988), p. 189.

[3] D. Gross, J. Harvey, E. Martinec and R. Rohm, *Phys. Rev. Lett.* **54**, 502 (1985), *Nucl. Phys.* **B256**, 253 (1985) and *Nucl. Phys.* **B267**, 75 (1986).

[4] N. Chang, S. Ouvry and X. Wu, *Phys. Rev. Lett.* **51:** 327 (1981); E. Cremmer, S. Ferrara, C. Kounnas, and D.V. Nanopoulos, *Phys. Lett.* **133B:** 61 (1983); J. Ellis, C. Kounnas, and D.V. Nanopoulos, *Nucl. Phys.* **B241:** 406 (1984); *Nucl. Phys.* **B247:** 373 (1984); N. Dragon, U. Ellwanger and M.G. Schmidt, *Phys. Lett.* **145B:** 192 (1984).

[5] Green, Schwarz, and Witten, "Superstring theory," volumes 1 and 2, Cambridge monographs on mathematical physics (1987), and references therein.

[6] M. Dine, R. Rohm, N. Seiberg and E. Witten, *Phys. Lett.* **156B:** 55 (1985).

[7] P. Binétruy and M.K.Gaillard, *Phys. Lett.* **195B:** 382 (1987);U. Ellwanger and M. Schmidt, *Nucl. Phys.* **B294:** 445 (1987).

[8] M.K.Gaillard, *Nucl. Phys.* **B268:** 669 (1986).

[9] J.D. Breit, B.A. Ovrut and G. Segré, *Phys. Lett.* **162B:** 303 (1985); P. Binétruy and M.K. Gaillard, *Phys. Lett.* **168B:** 347 (1986); P. Binétruy, S.Dawson, M.K. Gaillard and I. Hinchliffe, *Phys. Rev.* **D37:** 2633 (1988); J. Burton, M.K.Gaillard, and V. Jain, *Phys. Rev.* **D41:** 3118 (1990); and C. Chiou-Lahanas *et. al.*, Athens U. preprint UA/NPPS-5-89.

[10] P. Binétruy and M.K.Gaillard, *Phys. Lett.* **B220:** 68 (1989).

[11] S.Honerkamp, *Nucl. Phys.* **36**, 130 (1973).

[12] L. Alvarez–Gaumé, D. Z. Freedman and S. Mukhi, *Annals of Physics* **134:** 85 (1981); S. Mukhi, *Nucl. Phys.* **B264:** 640 (1986).

[13] L.-H. Chan, *Phys. Rev. Lett.* **57:** 1199 (1986), *Phys.Rev.* **D36:** 3755 (1987); R.D. Ball, *Phys. Rept.* **182:** 1 (1989) .

[14] O. Cheyette, *Nucl. Phys.* **B297:** 183 (1988); M. K. Gaillard and V. Jain, *Phys. Rev.* **D39:** 3755 (1989).

[15] P. Binétruy and M.K.Gaillard, *Nucl. Phys.* **B312:** 341 (1989).

[16] S.J.Gates, M.T. Grisaru, M. Rocek, W. Siegel, "Superspace *or* One Thousand and One Lessons in Supersymmetry," Frontiers in Physics 58 (1983).

[17] W. Siegel, *Phys. Lett.* **93B:** 170 (1980).

[18] P.K. Townsend in "Supersymmetry and Supergravity '84," p. 408.

[19] U. Lindström, M. Roček, W. Siegel, P. van Nieuwenhuizen, and A. E. van de Ven, *Phys. lett.* **224B:** 285 (1989).

[20] R. Kallosh, *Phys. lett.* **195B:** 369 (1987).

[21] L. Brink, "Covariant Superstrings," GÖTEBORG, ITP preprint GÖTEBORG-90-7.

[22] C. M. Hull and P. K. Townsend, *Nucl. Phys.* **B301:** 197 (1988).

[23] E. Witten, *Phys. Lett.* **155B:** 51 (1985).