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On the Hall Effect in Ferromagnetics

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With precise measurements at magnetic fields well above those required for saturation, it is shown that the Hall electric field per unit current density consists of two distinct parts. Its value averaged between poles is given by R_0H+R_1M, where H and M are the magnetizing force and average intensity of magnetization, respectively, in the sample.

The value $R_0=-0.611\times10^{-12}$ volt-cm/amp.-oersted indicates 1.16 s-band electrons with no d-band conduction, 0.6 s-band electrons with 30 percent d-band conduction, or something intermediate between these two.

The value $R_1=-74.9\times10^{-12}$ volt-cm/amp.-oersted suggests the existence of a very large average magnetic field acting on the conduction electrons. An explanation for this is suggested.

I. INTRODUCTION

It has long been known[1] that the Hall electromotive force in practically all of the ferromagnetic materials is directly proportional, at low magnetic fields, to the intensity of magnetization, M. The accuracy with which the Hall e.m.f. follows this rule is very high; for example, when measurements are made while the M of the material is carried around a hysteresis loop,[2] the measured e.m.f.'s follow curved loops when plotted against either H or B, but follow precise straight lines when plotted against M.

Following a suggestion of Smith and Sears[3] one of us proposed[4] that for ferromagnetics the Hall relation should be written as:

$$e_H=-E_H/ib=R_0H+R_1M, \tag{1}$$

where E_H = the Hall e.m.f., e_H = Hall electric field per unit current density, i = the current density in the ferromagnetic, b = the width of the plate between the Hall probes, and H = the magnetic field which in a thin plate perpendicular to the field is given by $H=B-4\pi M$, B being the magnetic induction.

R_0 and R_1 are Hall coefficients and $|R_1|\gg|R_0|$ in most ferromagnetic materials. R_0 is too small to be easily detectable in bars of ferromagnetics shaped to facilitate simultaneous magnetic and Hall e.m.f. measurements.

Using the more conventional plate-shaped samples of nickel, which are more suitable for high field measurements, it has been shown that the Hall e.m.f. does increase in fields beyond those required for saturation and is linear with H in this region. Furthermore, the slope of this straight line of E_H *versus* H is close to the slope predicted by theory. Figure 1 shows a plot of E_H *versus* H measured in an annealed plate of 99.6 percent pure nickel at 28.5, 20.3, and 14.3°C. The experimental points taken at fields well above those required to saturate the nickel fall quite accurately on a straight line whose slope determines the value of R_0. The intercept of this line with the $H=0$ axis, together with the separately determined saturation magnetization of the nickel, determines the value of R_1. At 20.3°C these values, when corrected for the Ettingshausen effect, are $R_0=-0.611\times10^{-12}$ volt-cm/amp.-oersted and $R_1=-74.9\times10^{-12}$ volt-cm/amp.-oersted. In Fig. 2 values of R_1 are plotted as a function of temperature over the small range of temperatures for which they have been determined. Over the observed range of temperatures there is no significant variation of R_0 with temperature, while R_1 appears to increase rather rapidly with increasing temperature.

In 1910 Smith[5] published a rather complete study of the Hall effect in nickel over a wide range of temperatures, which agrees as well as could be expected with the results reported here. Since he did not measure the saturation magnetization of his samples as a function of temperature and since he did not employ very high fields, it is impossible to calculate reliable values of R_0 and R_1 from his data. This is especially true in the neighborhood of the Curie temperature where the saturation magnetization needs to be known with great accuracy. Nevertheless, R_0 and R_1 have been calculated from Smith's data by using accepted values for the magnetic properties of nickel and these are plotted in Fig. 3. If the behavior in the immediate neighborhood of the Curie temperature is ignored, R_0 remains constant while R_1 increases with temperature. It is particularly noteworthy that R_0 is the same above the Curie temperature as below. This is just what should be expected, since R_0 should be the ordinary Hall coefficient depending chiefly upon the number of conduction carriers (electrons and holes), which should not change with temperature. Experimentally, then, the Hall effect in ferromagnetics separates into two distinct phenomena which can be analyzed separately. The two effects will be called ordinary (proportional to H) and extraordinary (proportional to M) to distinguish them from the terms "normal" and "abnormal," which have so often been used to designate negative and positive Hall coefficients, respectively.

[1] A. Kundt, Wied. Ann. **49**, 257 (1893).
[2] E. M. Pugh and T. W. Lippert, Phys. Rev. **42**, 709 (1932).
[3] A. W. Smith and R. W. Sears, Phys. Rev. **34**, 1466 (1929).
[4] E. M. Pugh, Phys. Rev. **36**, 1503 (1930).
[5] A. W. Smith, Phys. Rev. **30**, 1 (1910).

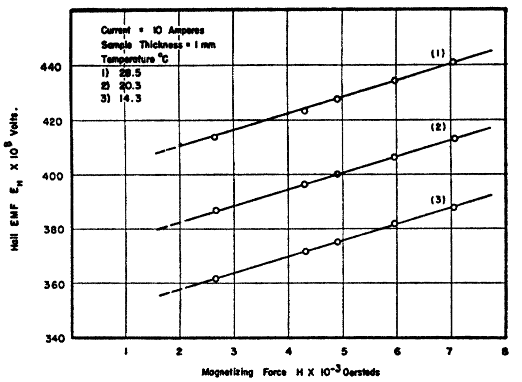

FIG. 1. Hall electromotive force in a magnetically saturated sample of nickel.

II. EXTRAORDINARY EFFECT

The extraordinary Hall e.m.f. is the one which is usually measured, since it is obtainable at low fields with relatively insensitive instruments. The coefficient R_1 is far too large to agree with generally accepted theories. To obtain a better understanding of this effect one must take the modern domain theory of magnetization into consideration. According to this theory, a current passing through a ferromagnetic material is passing through domains that are magnetized to saturation at all times. When the measured value of magnetization is less than the saturation value, the domains are oriented in different directions. Under these circumstances the Hall electric field per unit current in each domain is given by an expression of the form:

$$\mathbf{e_H}=R_1\mathbf{m}\times\mathbf{i}, \tag{2}$$

where $\mathbf{m}$ = vector intensity of magnetization in the domain, R_1 = a constant which is the same for each domain, and $\mathbf{i}$ = vector current density assumed to be of unit magnitude and parallel to the x-axis. A rectangular parallelopiped as shown in Fig. 4 is employed and E_H is measured between probes located at $(x, 0, z)$ and (x, b, z). The current density $\mathbf{i}$ is assumed to be uniform and parallel to the x-axis. The macroscopically

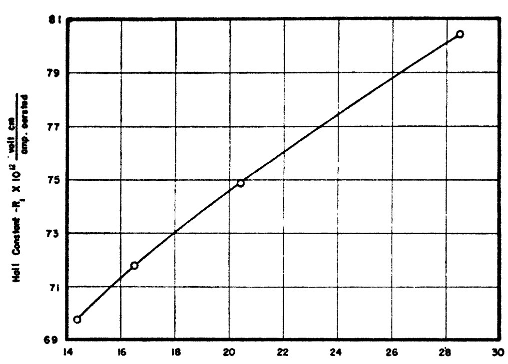

FIG. 2. Temperature dependence of extraordinary Hall constant R_1 in nickel.

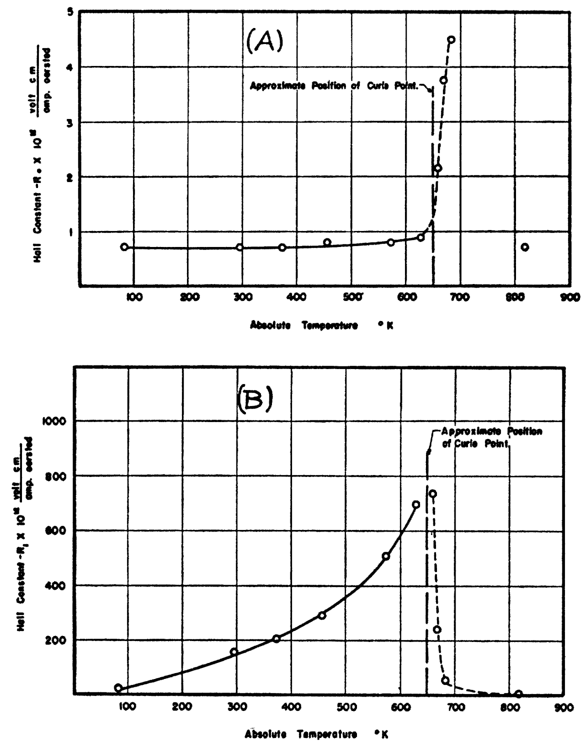

FIG. 3. (A) Temperature dependence of ordinary Hall constant R_0 in nickel (calculated from data of A. W. Smith). (B) Temperature dependence of extraordinary Hall constant R_1 in nickel (calculated from data of A. W. Smith).

measured M is parallel to the z-axis and is the average of the z-component of $\mathbf{m}$. For convenience this average may be taken along a line parallel to the y-axis; thus

$$M=(1/b)\int_0^b(\mathbf{m}\cdot\mathbf{k})dy.$$

M is assumed to be uniform throughout the specimen in the sense that it has the same value when averaged over any volume whose dimensions are small compared with the dimensions of the block, but large compared with the dimensions of the domains.

The average Hall electric field e_H is given by

$$e_H=(1/b)\int_0^b\mathbf{e_H}\cdot\mathbf{j}dy=(R_1/b)\int_0^b(\mathbf{m}\times\mathbf{i})\cdot\mathbf{j}dy,$$

and since $(\mathbf{m}\times\mathbf{i})\cdot\mathbf{j}=(\mathbf{i}\times\mathbf{j})\cdot\mathbf{m}=\mathbf{k}\cdot\mathbf{m}$, it may be expressed in terms of M as

$$e_H=-E_H/b=R_1M. \tag{3}$$

Although the Hall effect is of different direction and magnitude in each domain, the resultant macroscopic effect is proportional to the intensity of magnetization, and the result would be the same if the magnetization were regarded as a simple uniform field in the z direction. Furthermore the macroscopic Hall constant is the same as the Hall constant for the domains.

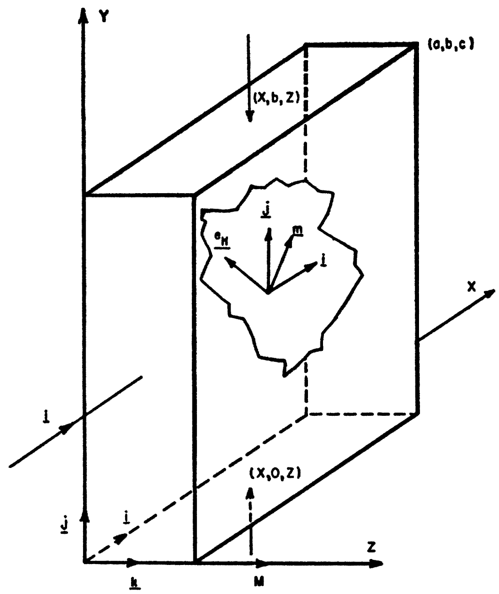

FIG. 4. Domain structure in Hall effect sample (relative size of domain is greatly exaggerated).

The complete expression for $\mathbf{e_H}$ in the domain is

$$\mathbf{e}_{\mathrm{H}}=(R_0\mathbf{H}+R_1\mathbf{m})\times\mathbf{i}. \quad (2.1)$$

This leads to the following expression for the observable Hall e.m.f.:

$$E_H=-ibR_0[H+4\pi\alpha M], \quad \alpha=R_1/4\pi R_0. \quad (4)$$

Equation (2) for the Hall effect in domains is what would be expected on the basis of Lorentz forces acting on conduction electrons; and, since this leads to the correct empirical formula (1) for the observed Hall effect with the same macroscopic Hall constants, it is probable that conduction electrons in domains are subjected to a Lorentz magnetic force-field $\mathbf{H}+4\pi\alpha\mathbf{m}$. The effective uniform field acting on conduction electrons would be

$$h=H+4\pi\alpha M. \quad (5)$$

The most recent work on effective Lorentz force-fields acting on charged particles has been carried out by Wannier[6] in connection with the deflection of beams of cosmic-ray particles by a ferromagnetic medium whose magnetism is due to electron spins. Wannier finds that the effective uniform Lorentz field for a beam of high velocity charged particles is

$$h=H+2\pi(p+1)M, \quad (6)$$

where p is the probability of coincidence of beam particles and "magnetic" electrons compared to the random probability of such coincidences. (By random probability is meant the coincidence probability of beam particles and magnetic electrons in the absence of any interaction between them.) Wannier considered in detail the case of Coulomb interaction and found that $p>1$ for attractive interaction and $p<1$ for repulsive interaction.

Webster[7] first applied Wannier's ideas to conduction electrons and arrived at the conclusion that p should be very nearly zero due to repulsive Coulomb interaction between conduction electrons and "magnetic" electrons. There is no assurance, however, that Webster's speculations concerning conduction electrons are correct, since many of the approximations employed by Wannier are doubtful for particles of energy as low as the energy of conduction electrons. This fact has been pointed out by Wannier.

The experimental evidence from the Hall effect in nickel indicates that $p=2\alpha-1$ must be considerably greater than unity. Values of the field parameter α from the extraordinary Hall effect are shown in Fig. 5. It is difficult to understand how α can attain such large values on the basis of Wannier's analysis for beams of negative particles or Webster's considerations of conduction electrons. However, from the work of previous investigators it may be concluded that for p to be greater than unity there must be a correlation in position between the conduction electrons and the locations of strong magnetic fields that is more than random. It is likely that there is such a correlation between the conduction electrons and "magnetic" holes characteristic of ferromagnetics. This type of process appears to offer the only possibility of explaining the unusually large value of the field parameter α within existing framework of concepts.

No attempt will be made in this paper to explain quantitatively the large value of α. However, experimental and theoretical researches are being carried on which will appear in forthcoming papers.

III. ORDINARY EFFECT

The ordinary Hall e.m.f. is measured at magnetic fields well above those required to saturate the material, and is expressed by the first term of Eqs. (1) or (4). It is generally supposed[8] that in nickel the $3d$-band does not contribute significantly to electrical conduction. With this assumption the value of R_0 in nickel should be a direct measure of the number of electrons per unit volume N_s in the $4s$-band; i.e.,

$$N_s=|1/R_0ec|,$$

where e = charge of the electron (e.s.u.), c = velocity of light (cm/sec.), and R_0 = ordinary Hall constant (statvolt-cm/stat-volt-oersted). From the experimental value of R_0, the number of electrons n_s per atom in the $4s$-band is 1.16. However, $n_s=0.6$ seems to be pretty well established.[9] Its most direct measure is obtained from the value of the saturation magnetization, which yields

[6] G. H. Wannier, Phys. Rev. **72**, 304 (1947). (Wannier uses the symbol b in place of h.)

[7] D. L. Webster, Am. J. Phys. **14**, 360 (1946).

[8] N. F. Mott, Proc. Roy. Soc. **A153**, 699 (1935–6).

[9] N. F. Mott, Proc. Phys. Soc. London **47**, 571 (1935).

0.6 Bohr magnetons per atom of nickel. Ferromagnetism results from exchange forces which align as many as possible of the spins of the 3*d* electrons parallel to the field, the balance being antiparallel. If n_p and n_a are the number per atom of 3*d* electrons whose spins are aligned with the field parallel and antiparallel, respectively, then $n_p - n_a = 0.6$. Then, since $n_s + n_p + n_a = 10$ for nickel and since it is assumed that $n_p = 5$, $n_s = 0.6$. The assumption that $n_p = 5$ is made because calculations[9] indicate that the parallel half of the 3*d* should be full. However, if this assumption is eliminated and instead the Hall effect measurements are taken at face value, then there are three equations for the three unknowns n_s, n_p, and n_a. For nickel these are

$$n_s + n_p + n_a = 10, \quad n_p - n_a = 0.6, \quad n_s = 1.16. \tag{7}$$

The solution of these equations gives $n_p = 4.72$ and $n_a = 4.12$. However, there is another possible explanation of the experimental results obtained here.

The above explanation assumed that the 3*d*-band does not contribute significantly to the electrical conduction. Let us suppose that the 3*d*-band does contribute to the conduction and that $n_p - n_a = 0.6 = n_s$. The total conductivity $\sigma = \sigma_d + \sigma_s$, where σ_d and σ_s are the conductivities due to the 3*d*- and 4*s*-bands, respectively. Since the *d*-band is nearly full, its conductivity is caused by holes and should contribute a positive Hall e.m.f., whereas the *s*-band should contribute a negative Hall e.m.f. This could account for the small observed value of R_0. Under these circumstances its value is given by:

$$R_0 = -\frac{n_d \sigma_s^2 - n_s \sigma_d^2}{n_d n_s \sigma^2} \frac{1}{Nec} = \frac{-1}{n_s' Nec}. \tag{8}$$

(N = the number of atoms per unit volume),

where n_d is the number of holes per atom in the 3*d*-band and n_s' (=1.16 in nickel) is the apparent number of electrons in the conduction band as measured by the Hall effect. Solving (8) gives

$$n_s = n_s'[(\sigma_s^2/\sigma^2) - (n_s \sigma_d^2 / n_d \sigma^2)] \text{ or, since } n_s = n_d,$$
$$n_s = n_s'(\sigma_s - \sigma_d)/\sigma.$$

Putting $n_s = 0.6$ and $n_s' = 1.16$ gives $\sigma_d/\sigma_s = 0.3$.

IV. CONCLUSIONS

(1). The Hall effect in nickel has been separated into two terms, the "ordinary" Hall effect caused by a uniform field (the magnetizing force) and the "extraordinary" Hall effect due to magnetization. The empirical expression (Eq. 1) for the Hall effect satisfies the requirements of the domain theory of ferromagnetism if the Hall effect is due to Lorentz type magnetic forces where the average Lorentz magnetic force-field is $h = H + 4\pi\alpha M$. This appears to be the only possible explanation of the experimental facts at present although it is conceivable that other types of forces may also contribute to the Hall effect in nickel. The surprisingly large value of the field parameter α is interpreted as being due to strong correlation between the positions of conduction electrons and the locations of very high magnetic fields identified with the *d*-band "magnetic" holes. The reason for this strong correlation is the effectively attractive interaction between *s*-band electrons and *d*-band holes.

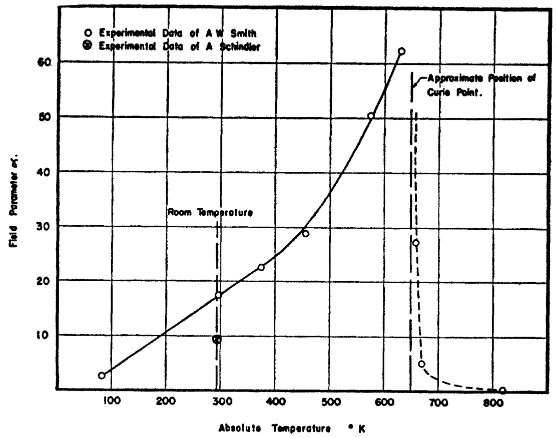

FIG. 5. Temperature dependence of field parameter α in nickel.

(2). The "ordinary" Hall effect involves a Hall constant R_0 which corresponds closely to the Hall constant of the conventional theory of Hall effect due to a uniform magnetic field. As such it should be a measure of the number of conduction particles. There is a relatively small but real discrepancy between the number of *s*-band electrons predicted by the measured value of R_0 and the generally accepted value. Two distinct interpretations have been suggested.

(a) One interpretation assumes that the conduction by holes is negligible. The numbers of electrons in the *s*- and *d*-bands are then calculated by means of Eq. (7). From the experimental value of R_0 we obtain $n_s = 1.16$ for the *s*-band, and $n_p = 4.72$, $n_a = 4.12$ for the *d*-band. Since the Hall effect measurements were made at room temperature and the magnetization at room temperature differs from that at 0°K only by a factor of the order of 0.96, this calculation would be approximately correct for room temperature.

The parallel *d*-band in nickel is generally regarded as being filled at low temperatures. It follows that there must be 0.6 electrons per atom in the *s*-band. Although there must be a change in the distribution of parallel and antiparallel spin electrons in the *d*-band at higher temperatures, it seems highly improbable that there would be as large a change in the numbers of electrons in the *s*- and *d*-bands as indicated by the above calculations at the temperatures and field strengths associated with these Hall effect measurements. It is thus clear that the above interpretation of the ordinary Hall constant R_0 is at variance with established concepts.

(b) The other interpretation assumes that the parallel

d-band is full but the hole conduction cannot be neglected. The experimental value of R_0 then requires that $\sigma_d/\sigma_s=0.3$. According to existing theory of conductivity[8] for temperatures above the characteristic temperature of the lattice

$$\frac{\sigma_d}{\sigma_s}=\frac{m_s\tau_d}{m_d\tau_s}, \tag{9}$$

where m_s = effective mass of s-band electrons, m_d = effective mass of d-band electrons (absolute value), τ_s = relaxation time of s-band electrons, and τ_d = relaxation time of d-band electrons. The specific heat measurements of Keesom and Clark[10] indicate that $m_s/m_d \cong 1/28$, which would give $\tau_d/\tau_s \cong 8$. No physically significant calculations have yet been given for τ_d/τ_s, but it is plausible that $\tau_d > \tau_s$ because of the low velocity of d-band electrons relative to the velocity of s-band electrons.

[10] W. H. Keesom and C. W. Clark, Physica 2, 513 (1935).

Of the two possible interpretations of the experimental value of R_0, the latter, in which the negative Hall effect from s-band conduction is counteracted by the positive Hall effect due to hole conduction, appears to be the most likely alternative. The result $\sigma_d/\sigma_s=0.3$ does not appear to contradict any existing experimental evidence. It is believed that this is the only quantitative evaluation of the conductivity ratio that has been attempted. Furthermore, it is necessary that $\sigma_d/\sigma_s>1$ for Co and Fe to produce the observed positive Hall effects, unless some agency entirely different from hole conduction is responsible for these positive Hall effects.

It is expected that a clearer understanding of the hole conduction will be achieved with the completion of measurements of R_0 and R_1 that are now being carried out on the Ni–Cu and the Ni–Co series of alloys.

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Magneto-Hydrodynamic Shocks*

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A mathematical treatment of the coupled motion of hydrodynamic flow and electromagnetic fields is given. Two simplifying assumptions are introduced: first, the conductivity of the medium is infinite, and second, the motion is described by a plane shock wave. Various orientations of the plane of the shock and the magnetic field are discussed separately, and the extreme relativistic and unrelativistic behavior is examined. Special consideration is given to the behavior of weak shocks, that is, of sound waves. It is interesting to note that the waves degenerate into common sound waves and into common electromagnetic waves in the extreme cases of very weak and very strong magnetic fields.

I. INTRODUCTION

It has been shown recently that the interaction between hydrodynamic motion and magnetic fields in a conducting liquid is of importance in problems of astrophysics, geophysics, and the behavior of interstellar gas masses.[1] The non-linear character of the hydrodynamic equations raises difficulties in the treatment of these problems. So far only the linear problem of sound propagation has been treated[2] and this one only for a transverse wave propagating along the lines of force. It is the purpose of the following investigation to clarify the behavior of plane waves in magneto-hydrodynamics. The non-linear case of shock waves will be of primary interest and the simpler behavior of sound waves will be obtained by considering shocks of small amplitude. Two special cases of magneto-hydrodynamic waves are well known in physics. One is the hydrodynamic shock and the other the pure electromagnetic wave. It will be clear from the formalism which we are going to develop that these can be obtained from the magneto-hydrodynamic shock as limiting cases. In order to permit a treatment of waves which are similar to electromagnetic waves we shall need to discuss shock velocities which are close to the velocity of light. We therefore must include a relativistic treatment.

In order to limit ourselves to the simplest possible case we shall make the assumption that the conductivity is infinite. One consequence of this assumption is that the self-induction will prevent a change in the magnetic fields if the substance carrying the magnetic field is at rest. Actually the conductivity is finite. However, the

* This document is based on work performed at Los Alamos Scientific Laboratory of the University of California under the auspices of the AEC.

[1] H. Alfvén, Arkiv. f. Mat. Astron. Fysik 29A, 12 (1943). E. C. Bullard, Proc. Roy. Soc. A 197, 433 (1949)—See this paper for additional bibliography. E. Fermi, Phys. Rev. 75, 1169 (1949). C. Walen, Arkiv. f. Mat. Astron. Fysik 30A, 15 (1944).

[2] H. Alfvén, Arkiv. f. Mat. Astron. Fysik 29B, 2 (1943).