ESTIMATION OF INTERREGIONAL IN- AND OUT-MIGRATION FLOWS FROM PLACE-OF-BIRTH-BY-RESIDENCE DATA

by

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ERRATA

Eq. (27):

$$D_u^{(t)} = \left[\begin{array}{cc} {}_A D_A^{(t)} & 0 \\ 0 & {}_B D_A^{(t)} \\ \hdashline {}_A D_B^{(t)} & 0 \\ 0 & {}_B D_B^{(t)} \end{array}\right] \left[\begin{array}{c:c} {}_A d_A^{(t,0)} \; \cdots & {}_B d_A^{(t,0)} \; \ddots \\ \hdashline {}_A d_B^{(t,0)} \; \cdots & {}_B d_B^{(t,0)} \; \cdots \end{array}\right] \qquad (27)$$

Eq. (56):

$$\underline{v}^{(t+1)} = R\underline{v}^{(t)} = RD_V^{(t)} \, \underline{w}^{(t)} \qquad (56)$$

Eq. (57):

$$\underline{v}^{(t+1)} = D_V^{(t+1)} \, \underline{w}^{(t+1)} = D_V^{(t+1)} \, G \, \underline{w}^{(t)} \quad . \qquad (57)$$

Eq. (58):

$$RD_V^{(t)} = D_V^{(t+1)} \, G \quad , \qquad (58)$$

NOTATION

PARAMETERS

${}_k g_{ij}$ = the proportion of women, present in region i in age group k at time t, who will be present in region j in age group k + 1 at time t + 1;

${}_k b_{ij}$ = the expected number of female children, born alive per woman present in region i in age group k at time t, who will be alive and present in region j in the first age group at time t + 1.

DATA

${}_h w_{ij}^{(t,k)}$ = the number of women, born in region h, present in region i in age group k at time t and alive and present in region j in age group k + 1 at time t + 1;

${}_h w_{i}^{(t,k)}$ = the number of women, born in region h, present in region i in age group k at time t;

$w_{ij}^{(t,k)}$ = the number of women present in region i in age group k at time t and alive and present in region j in age group k + 1 at time t + 1;

$w_{i}^{(t,k)}$ = the number of women present in region i in age group k at time t;

${}_h w^{(t,k)}$ = the number of women, born in region h, alive in age group k at time t.

ESTIMATION OF INTERREGIONAL IN- AND OUT-MIGRATION FLOWS FROM PLACE-OF-BIRTH-BY-RESIDENCE DATA

Several recent studies have used place-of-birth-by-residence data, tabulated in two successive censuses, to estimate intercensal net migration [Eldridge and Kim (1968), George (1970), and Lee, et al. (1957)]. The purpose of this paper is to describe a method for using these same data to estimate in- and out-migration. We begin by defining the multiregional matrix model of population growth and the estimation problem. Then we develop the estimation method and illustrate its application to U.S. place-of-birth-by-residence data. Finally, we discuss some of the problems that are encountered in applying the estimation method to empirical data and compare our estimates of net migration with those obtained by Eldridge and Kim's DOB method. A brief consideration of the stable growth properties of the place-of-birth vector concludes the paper.

1. The Multiregional Matrix Model and the Estimation Problem

Imagine an interregional population system that contains only two regions, regions A and B say, and, to further simplify the exposition, let us only consider the female populations of these two regions.[1] Moreover, assume that this interregional population system is closed to emigration and immigration. We then may express the growth and distribution of this system by means of the following matrix model [Keyfitz (1968), p. 321]:

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[1] No generality is lost by adopting these assumptions, and it easily can be established that our results are equally valid for any finite number of regions and for both sexes.

$$\underline{w}^{(t+1)} = G\underline{w}^{(t)} \quad (1)$$

$$= \left[\begin{array}{c|c} G_{AA} & G_{BA} \\ \hline G_{AB} & G_{BB} \end{array}\right] \left[\begin{array}{c} \underline{w}_A^{(t)} \\ \hline \underline{w}_B^{(t)} \end{array}\right] \quad (2)$$

$$= \left[\begin{array}{cccccc|cccccc} 0 & 0 & {}_{10}b_{AA} & {}_{15}b_{AA} & \cdots & 0 & 0 & 0 & {}_{10}b_{BA} & {}_{15}b_{BA} & \cdots & 0 \\ {}_0g_{AA} & 0 & 0 & 0 & \cdots & 0 & {}_0g_{BA} & 0 & 0 & 0 & \cdots & 0 \\ 0 & {}_5g_{AA} & 0 & \cdots & \cdots & 0 & 0 & {}_5g_{BA} & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots & \vdots & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 \\ \hline 0 & 0 & {}_{10}b_{AB} & {}_{15}b_{AB} & \cdots & 0 & 0 & 0 & {}_{10}b_{BB} & {}_{15}b_{BB} & \cdots & 0 \\ {}_0g_{AB} & 0 & 0 & 0 & \cdots & 0 & {}_0g_{BB} & 0 & 0 & 0 & \cdots & 0 \\ 0 & {}_5g_{AB} & 0 & \cdots & \cdots & 0 & 0 & {}_5g_{BB} & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots & \vdots & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 \end{array}\right] \left[\begin{array}{c} w_A^{(t,0)} \\ w_A^{(t,5)} \\ \vdots \\ \hline w_B^{(t,0)} \\ w_B^{(t,5)} \\ \vdots \end{array}\right] \quad (3)$$

where

$_kg_{ij}$ = the proportion of women, present in region i in age group k at time t, who will be present in region j in age group k+1 at time t+1 ;

$_kb_{ij}$ = the expected number of female children, born alive per woman present in region i in age group k at time t, who will be alive and present in region j in the first age group at time t + 1;

$w_i^{(t,k)}$ = the number of women present in region i in age group k at time t.[2]

[2] By age group k we mean the group aged k at last birthday. Thus, in our example, k is incremented by units of five, that is, k=0,5,10,....

At any time t, the female population in each of our two regions is made up of: (1) women present and born in the region, and (2) women present in the region but born in the other region. Let

${}_h w_i^{(t,k)}$ = the number of women, born in region h, present in region i in age group k at time t.

Then

$$\underline{w}_A^{(t)} = {}_A\underline{w}_A^{(t)} + {}_B\underline{w}_A^{(t)} , \qquad (4)$$

and

$$\underline{w}_B^{(t)} = {}_A\underline{w}_B^{(t)} + {}_B\underline{w}_B^{(t)} , \qquad (5)$$

where

$$\underline{w}_i^{(t)} = \begin{bmatrix} w_i^{(t,0)} \\ w_i^{(t,5)} \\ \vdots \end{bmatrix} \qquad {}_h\underline{w}_i^{(t)} = \begin{bmatrix} {}_h w_i^{(t,0)} \\ {}_h w_i^{(t,5)} \\ \vdots \end{bmatrix} ,$$

and our particular estimation problem may be expressed as follows: Given the vectors ${}_A\underline{w}_A^{(t)}$, ${}_A\underline{w}_B^{(t)}$, ${}_B\underline{w}_A^{(t)}$, and ${}_B\underline{w}_B^{(t)}$ for two successive points in time, t and t+1 say, find the subdiagonal elements of the four submatrices G_{AA}, G_{AB}, G_{BA}, and G_{BB}, defined in (3).

2. Additional Definitions and Notation

Before proceeding with the development of our estimation method, we shall find it useful to introduce some additional definitions and notation. First, recalling the definitions set out in (2) and (3), we have that

$$\underline{w}_A^{(t+1)} = G_{AA}\underline{w}_A^{(t)} + G_{BA}\underline{w}_B^{(t)} \quad (6)$$

and

$$\underline{w}_B^{(t+1)} = G_{AB}\underline{w}_A^{(t)} + G_{BB}\underline{w}_B^{(t)} \quad . \quad (7)$$

Next, recalling (4) and (5), we substitute into (6) and (7) to find

$$\underline{w}_A^{(t+1)} = {}_A\underline{w}_A^{(t+1)} + {}_B\underline{w}_A^{(t+1)} = G_{AA}\left[{}_A\underline{w}_A^{(t)} + {}_B\underline{w}_A^{(t)}\right] + G_{BA}\left[{}_A\underline{w}_B^{(t)} + {}_B\underline{w}_B^{(t)}\right] \quad (8)$$

and

$$\underline{w}_B^{(t+1)} = {}_A\underline{w}_B^{(t+1)} + {}_B\underline{w}_B^{(t+1)} = G_{AB}\left[{}_A\underline{w}_A^{(t)} + {}_B\underline{w}_A^{(t)}\right] + G_{BB}\left[{}_A\underline{w}_B^{(t)} + {}_B\underline{w}_B^{(t)}\right] . \quad (9)$$

Since women born outside of region h can never become members of ${}_h w_i^{(t)}$, we may break up (8) and (9) into the following four equations:

$$ {}_A\underline{w}_A^{(t+1)} = G_{AA}\ {}_A\underline{w}_A^{(t)} + G_{BA}\ {}_A\underline{w}_B^{(t)} \quad (10)$$

$$ {}_B\underline{w}_A^{(t+1)} = G_{AA}\ {}_B\underline{w}_A^{(t)} + G_{BA}\ {}_B\underline{w}_B^{(t)} \quad (11)$$

$$ {}_A\underline{w}_B^{(t+1)} = G_{AB}\ {}_A\underline{w}_A^{(t)} + G_{BB}\ {}_A\underline{w}_B^{(t)} \quad (12)$$

$$ {}_B\underline{w}_B^{(t+1)} = G_{AB}\ {}_B\underline{w}_A^{(t)} + G_{BB}\ {}_B\underline{w}_B^{(t)} \quad , \quad (13)$$

or, more compactly,

$$\underline{u}^{(t+1)} = \left[\begin{array}{c} \underline{u}_A^{(t+1)} \\ \hline \underline{u}_B^{(t+1)} \end{array}\right] = \left[\begin{array}{c} {}_A\underline{w}_A^{(t+1)} \\ {}_B\underline{w}_A^{(t+1)} \\ \hline {}_A\underline{w}_B^{(t+1)} \\ {}_B\underline{w}_B^{(t+1)} \end{array}\right] = \left[\begin{array}{cc|cc} {}_AG_{AA} & 0 & {}_AG_{BA} & 0 \\ 0 & {}_BG_{AA} & 0 & {}_BG_{BA} \\ \hline {}_AG_{AB} & 0 & {}_AG_{BB} & 0 \\ 0 & {}_BG_{AB} & 0 & {}_BG_{BB} \end{array}\right] \left[\begin{array}{c} {}_A\underline{w}_A^{(t)} \\ {}_B\underline{w}_A^{(t)} \\ \hline {}_A\underline{w}_B^{(t)} \\ {}_B\underline{w}_B^{(t)} \end{array}\right] = Q\underline{u}^{(t)} \text{ say,} \quad (14)$$

where we now have introduced an additional subscript to the regional growth matrices to indicate the region of birth of the population to which they are applied. Thus ${}_{A}G_{ij}$ defines the growth regime of women born in region A, while ${}_{B}G_{ij}$ defines the corresponding growth schedule of women born in region B. Note that we may recombine the four equations (10)-(13) in the following alternative way:

$$\underline{v}^{(t+1)} = \left[\begin{array}{c} \underline{v}_A^{(t+1)} \\ \hline \underline{v}_B^{(t+1)} \end{array}\right] = \left[\begin{array}{c} {}_{A}\underline{w}_A^{(t+1)} \\ {}_{A}\underline{w}_B^{(t+1)} \\ \hline {}_{B}\underline{w}_A^{(t+1)} \\ {}_{B}\underline{w}_B^{(t+1)} \end{array}\right] = \left[\begin{array}{cc|cc} {}_{A}G_{AA} & {}_{A}G_{BA} & 0 & 0 \\ {}_{A}G_{AB} & {}_{A}G_{BB} & 0 & 0 \\ \hline 0 & 0 & {}_{B}G_{AA} & {}_{B}G_{BA} \\ 0 & 0 & {}_{B}G_{AB} & {}_{B}G_{BB} \end{array}\right] \left[\begin{array}{c} {}_{A}\underline{w}_A^{(t)} \\ {}_{A}\underline{w}_B^{(t)} \\ \hline {}_{B}\underline{w}_A^{(t)} \\ {}_{B}\underline{w}_B^{(t)} \end{array}\right] = R\underline{v}^{(t)} \text{ say.} \quad (15)$$

We shall call $\underline{u}^{(t)}$ the place-of-residence-by-place-of-birth vector and $\underline{v}^{(t)}$ the place-of-birth-by-place-of-residence vector.

Note that Q and R are orthogonally similar, since

$$Q = P^{-1}RP = P'RP = PRP \qquad (16)$$

and

$$R = PQP^{-1} = PQP' = PQP \quad , \qquad (17)$$

where

$$P = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix},$$

and I is an identity matrix of the appropriate order.

Finally, let

${}_{h}w^{(t,k)}$ = the number of women, born in region h, alive in age group k at time t;

and

$$h^{\underline{w}(t)} = \begin{bmatrix} h^{w(t,0)} \\ h^{w(t,5)} \\ \vdots \end{bmatrix} .$$

Then

$${}_A\underline{w}^{(t)} = {}_A\underline{w}_A^{(t)} + {}_A\underline{w}_B^{(t)} \tag{18}$$

and

$${}_B\underline{w}^{(t)} = {}_B\underline{w}_A^{(t)} + {}_B\underline{w}_B^{(t)} . \tag{19}$$

Thus if we let

$$\underline{\tilde{w}}^{(t)} = \left[\begin{array}{c} {}_A\underline{w}^{(t)} \\ \hdashline {}_B\underline{w}^{(t)} \end{array}\right] \quad \text{and} \quad C = \left[\begin{array}{c:c} I & I \end{array}\right] , \tag{20}$$

where I is an identity matrix of the appropriate order, then

$$\underline{\tilde{w}}^{(t)} = \left[\begin{array}{c} {}_A\underline{w}^{(t)} \\ \hdashline {}_B\underline{w}^{(t)} \end{array}\right] = \left[\begin{array}{c:c} I & I \end{array}\right] \left[\begin{array}{c} {}_A\underline{w}_A^{(t)} \\ {}_B\underline{w}_A^{(t)} \\ \hdashline {}_A\underline{w}_B^{(t)} \\ {}_B\underline{w}_B^{(t)} \end{array}\right] = \left[\begin{array}{c} {}_A\underline{w}_A^{(t)} + {}_A\underline{w}_B^{(t)} \\ \hdashline {}_B\underline{w}_A^{(t)} + {}_B\underline{w}_B^{(t)} \end{array}\right] = C\underline{u}^{(t)} \tag{21}$$

and, similarly,

$$\underline{w}^{(t)} = \left[\begin{array}{c} \underline{w}_A^{(t)} \\ \hdashline \underline{w}_B^{(t)} \end{array}\right] = \left[\begin{array}{c:c} I & I \end{array}\right] \left[\begin{array}{c} {}_A\underline{w}_A^{(t)} \\ {}_A\underline{w}_B^{(t)} \\ \hdashline {}_B\underline{w}_A^{(t)} \\ {}_B\underline{w}_B^{(t)} \end{array}\right] = \left[\begin{array}{c} {}_A\underline{w}_A^{(t)} + {}_B\underline{w}_A^{(t)} \\ \hdashline {}_A\underline{w}_B^{(t)} + {}_B\underline{w}_B^{(t)} \end{array}\right] = C\underline{v}^{(t)} . \tag{22}$$

Note that

$$\underline{w}^{(t+1)} = G\underline{w}^{(t)} = GC\underline{v}^{(t)} , \tag{23}$$

by virtue of (1) and (22), and if we define

$$\tilde{\underline{w}}^{(t+1)} = \check{G}\tilde{\underline{w}}^{(t)} ,$$

where $\tilde{G}$ is as yet unspecified, then, recalling (21), we have that

$$\tilde{\underline{w}}^{(t+1)} = \tilde{G}\tilde{\underline{w}}^{(t)} = \tilde{G}C\underline{u}^{(t)} . \tag{24}$$

But

$$\tilde{\underline{w}}^{(t+1)} = C\underline{u}^{(t+1)} \quad \text{, by (21),}$$

and

$$\underline{u}^{(t+1)} = Q\underline{u}^{(t)} \quad \text{, by (14) .}$$

Thus if the matrix $D_u^{(t)}$ is defined in a manner such that

$$\underline{u}^{(t)} = D_u^{(t)}\tilde{\underline{w}}^{(t)}, \tag{25}$$

then

$$\tilde{\underline{w}}^{(t+1)} = C\underline{u}^{(t+1)} = CQ\underline{u}^{(t)} = CQD_u^{(t)}\tilde{\underline{w}}^{(t)} = \tilde{G}\tilde{\underline{w}}^{(t)}, \tag{26}$$

where $\tilde{G} = CQD_u^{(t)}$. It can easily be demonstrated that the matrix $D_u^{(t)}$ is equal to C', with proportions taking the place of unities [Rogers(1969)]:

$$D_u^{(t)} = \left[\begin{array}{c} {}_A D_A^{(t)} \\ \hdashline {}_A D_B^{(t)} \end{array}\right] = \left[\begin{array}{ccc} {}_A d_A^{(t,0)} & & \\ & {}_A d_A^{(t,5)} & \\ & & \ddots \\ \hdashline {}_A d_B^{(t,0)} & & \\ & {}_A d_B^{(t,5)} & \\ & & \ddots \end{array}\right], \tag{27}$$

where

$$ {}_A d_A^{(t,k)} = \frac{{}_A w_A^{(t,k)}}{{}_A w_A^{(t,k)} + {}_A w_B^{(t,k)}}$$

and ${}_A d_B^{(t,k)} = 1 - {}_A d_A^{(t,k)}$, for $k = 0, 5, \ldots$.

With an analogous argument, we have that

$$\underline{w}^{(t+1)} = C\underline{v}^{(t+1)} = CR\underline{v}^{(t)} = CRD_v^{(t)}\underline{w}^{(t)} = G\underline{w}^{(t)}, \tag{28}$$

where $G = CRD_v^{(t)}$ and $D_v^{(t)}$ is defined in a manner analogous to $D_u^{(t)}$ in (25).

3. Estimation of the Age-Specific In- and Out-Migration Probabilities

Consider the definition of an arbitrary element of $\underline{w}_A^{(t)}$ and $\underline{w}_B^{(t)}$ in (4) and (5), respectively:

$$w_A^{(t,k)} = {}_A w_A^{(t,k)} + {}_B w_A^{(t,k)} \tag{29}$$

$$w_B^{(t,k)} = {}_A w_B^{(t,k)} + {}_B w_B^{(t,k)} \quad . \tag{30}$$

From (3), we have that

$$w_A^{(t+1,k+5)} = {}_kg_{AA} w_A^{(t,k)} + {}_kg_{BA} w_B^{(t,k)} \quad (31)$$

and

$$w_B^{(t+1,k+5)} = {}_kg_{AB} w_A^{(t,k)} + {}_kg_{BB} w_B^{(t,k)} \ . \quad (32)$$

Thus substituting (29) and (30) into (31) and (32), we find

$$w_A^{(t+1,k+5)} = {}_A w_A^{(t+1,k+5)} + {}_B w_A^{(t+1,k+5)} = {}_kg_{AA}\left[{}_A w_A^{(t,k)} + {}_B w_A^{(t,k)}\right] + {}_kg_{BA}\left[{}_A w_B^{(t,k)} + {}_B w_B^{(t,k)}\right] \quad (33)$$

and

$$w_B^{(t+1,k+5)} = {}_A w_B^{(t+1,k+5)} + {}_B w_B^{(t+1,k+5)} = {}_kg_{AB}\left[{}_A w_A^{(t,k)} + {}_B w_A^{(t,k)}\right] + {}_kg_{BB}\left[{}_A w_B^{(t,k)} + {}_B w_B^{(t,k)}\right] . \quad (34)$$

And, since women born outside of region h can never become members of ${}_h w_i^{(t,k)}$, we may break up (33) and (34) into the following four equations:

$${}_A w_A^{(t+1,k+5)} = {}_kg_{AA}\ {}_A w_A^{(t,k)} + {}_kg_{BA}\ {}_A w_B^{(t,k)} \quad (35)$$

$${}_B w_A^{(t+1,k+5)} = {}_kg_{AA}\ {}_B w_A^{(t,k)} + {}_kg_{BA}\ {}_B w_B^{(t,k)} \quad (36)$$

$${}_A w_B^{(t+1,k+5)} = {}_kg_{AB}\ {}_A w_A^{(t,k)} + {}_kg_{BB}\ {}_A w_B^{(t,k)} \quad (37)$$

$${}_B w_B^{(t+1,k+5)} = {}_kg_{AB}\ {}_B w_A^{(t,k)} + {}_kg_{BB}\ {}_B w_B^{(t,k)} \ . \quad (38)$$

Note that (4)-(13) are the matrix equivalents of (29)-(38).

Equations (35)-(38) may be expressed as

$$\begin{bmatrix} {}_A w_A^{(t+1,k+5)} & {}_B w_A^{(t+1,k+5)} \\ {}_A w_B^{(t+1,k+5)} & {}_B w_B^{(t+1,k+5)} \end{bmatrix} = \begin{bmatrix} {}_kg_{AA} & {}_kg_{BA} \\ {}_kg_{AB} & {}_kg_{BB} \end{bmatrix} \begin{bmatrix} {}_A w_A^{(t,k)} & {}_B w_A^{(t,k)} \\ {}_A w_B^{(t,k)} & {}_B w_B^{(t,k)} \end{bmatrix} \quad (39)$$

or, more compactly,

$$W^{(t+1)} = [{}_k g]\, W^{(t)} \;; \tag{40}$$

whence

$$[{}_k g] = W^{(t+1)} \left\{ W^{(t)} \right\}^{-1} . \tag{41}$$

An alternative expression of (35)-(38) is

$$\begin{bmatrix} {}_A w_A^{(t+1,k+5)} \\ {}_B w_A^{(t+1,k+5)} \\ \hline {}_A w_B^{(t+1,k+5)} \\ {}_B w_B^{(t+1,k+5)} \end{bmatrix} = \left[\begin{array}{cc|cc} {}_A w_A^{(t,k)} & {}_A w_B^{(t,k)} & 0 & 0 \\ {}_B w_A^{(t,k)} & {}_B w_B^{(t,k)} & 0 & 0 \\ \hline 0 & 0 & {}_A w_A^{(t,k)} & {}_A w_B^{(t,k)} \\ 0 & 0 & {}_B w_A^{(t,k)} & {}_B w_B^{(t,k)} \end{array}\right] \begin{bmatrix} {}_k g_{AA} \\ {}_k g_{BA} \\ \hline {}_k g_{AB} \\ {}_k g_{BB} \end{bmatrix} \tag{42}$$

or, more compactly,

$$\underline{y}^{(t+1,k+5)} = \begin{bmatrix} \underline{y}_A^{(t+1,k+5)} \\ \hline \underline{y}_B^{(t+1,k+5)} \end{bmatrix} = \left[\begin{array}{c|c} W^{(t)\prime} & 0 \\ \hline 0 & W^{(t)\prime} \end{array}\right] \begin{bmatrix} {}_k \underline{g}_A \\ \hline {}_k \underline{g}_B \end{bmatrix} = X_k^{(t)} \underline{g} \;; \tag{43}$$

whence

$${}_k \underline{g} = \left\{ X^{(t)} \right\}^{-1} \underline{y}^{(t+1,k+5)} , \tag{44}$$

or, equivalently,

$${}_k \underline{g}_A = \left\{ W^{(t)\prime} \right\}^{-1} \underline{y}_A^{(t+1,k+5)} \tag{45}$$

$${}_k \underline{g}_B = \left\{ W^{(t)\prime} \right\}^{-1} \underline{y}_B^{(t+1,k+5)} . \tag{46}$$

Therefore

$$\begin{bmatrix} {}_k\underline{g}_A & {}_k\underline{g}_B \end{bmatrix} = \left\{ W^{(t)'} \right\}^{-1} \begin{bmatrix} \underline{y}_A^{(t+1,k+5)} & \underline{y}_B^{(t+1,k+5)} \end{bmatrix} ,$$

or

$$[{}_k g]' = \left\{ W^{(t)'} \right\}^{-1} W^{(t+1)'} , \tag{47}$$

from which we may obtain (41) by taking transposes of both sides of the equation.

Equations (41) and (44) provide us with two alternative formulations of the estimation method. The former is somewhat simpler to grasp and follows more logically from our previous arguments. The latter, however, can be more readily extended to include the case of estimation on the basis of several successive censuses and permits the incorporation of side constraints that restrict the ranges of feasible values that the g's can take on in such instances [Rogers (1967) and (1968), pp. 35-45].

4. Some Empirical Results Using U.S. Place-of-Birth-by-Residence Data: 1950-1960

Table 1 sets out place-of-birth-by-residence data for white females in the two-region system of: East North Central Division and the Rest of the United States in 1950 and 1960.[3] The application of our estimation method to these data produces the estimates set out in Figure 1. Applying this growth matrix to the 1950 data, we find that it projects exactly the 1960 observed data (Table 2). Table 3 presents the estimated in-, out- and net migration flows for the decade.

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[3] Since the model assumes an interregional system that is closed to the rest of the world, adjustments were made to take out the impact of emigration and immigration, following the procedure described in Eldridge (1968).

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.9440	0	0	0	0	0	0	0	0.0153	0	0	0	0	0	0	0
0	0.8850	0	0	0	0	0	0	0	0.0345	0	0	0	0	0	0
0	0	0.9367	0	0	0	0	0	0	0	0.0148	0	0	0	0	0
0	0	0	0.9373	0	0	0	0	0	0	0	0.0066	0	0	0	0
0	0	0	0	0.9271	0	0	0	0	0	0	0	0.0027	0	0	0
0	0	0	0	0	0.8758	0	0	0	0	0	0	0	-0.0014	0	0
0	0	0	0	0	0	0.5699	0	0	0	0	0	0	0	0.0003	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.0589	0	0	0	0	0	0	0	0.9885	0	0	0	0	0	0	0
0	0.1068	0	0	0	0	0	0	0	0.9476	0	0	0	0	0	0
0	0	0.0684	0	0	0	0	0	0	0	0.9885	0	0	0	0	0
0	0	0	0.0396	0	0	0	0	0	0	0	0.9718	0	0	0	0
0	0	0	0	0.0320	0	0	0	0	0	0	0	0.9619	0	0	0
0	0	0	0	0	0.0354	0	0	0	0	0	0	0	0.9108	0	0
0	0	0	0	0	0	-0.0013	0	0	0	0	0	0	0	0.6062	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Source: Calculated using the data in Table 1.

FIG. 1--ESTIMATED SURVIVORSHIP AND MIGRATION GROWTH OPERATOR FOR WHITE FEMALES: EAST NORTH CENTRAL DIVISION AND THE REST OF THE UNITED STATES, 1950-1960

TABLE 1 -- FEMALE WHITE POPULATION BORN IN CONTERMINOUS UNITED STATES ON OR BEFORE APRIL 1, 1950, AND LIVING IN CONTERMINOUS UNITED STATES AT THE CENSUS DATES BY AGE AND REGION OF BIRTH AND RESIDENCE, 1950 AND 1960

Age Group (in 1960)	1950				1960			
	Born in East North Central Division		Born in Rest of the United States		Born in East North Central Division		Born in Rest of the United States	
	Residing in East North Central Division	Residing in Rest of the United States	Residing in East North Central Division	Residing in Rest of the United States	Residing in East North Central Division	Residing in Rest of the United States	Residing in East North Central Division	Residing in Rest of the United States
10-19	2,483,718	147,851	155,931	9,801,590	2,346,848	292,484	297,340	9,698,355
20-29	1,756,766	143,998	160,716	7,273,751	1,559,717	324,010	393,398	6,909,767
30-39	1,825,245	294,625	376,258	7,894,963	1,713,947	416,038	469,151	7,829,598
40-49	1,704,201	318,673	401,588	7,448,813	1,599,400	377,140	425,969	7,254,629
50-59	1,326,921	297,031	328,358	5,807,603	1,231,041	328,126	320,060	5,596,623
60-69	1,053,967	281,921	225,756	4,306,859	922,727	294,143	191,736	3,930,805
70 and over	1,291,692	494,198	233,102	4,927,849	736,294	297,936	134,183	2,987,094

Source: U.S. Census of Population: 1950 and 1960, State of Birth. Adjusted.

TABLE 2 -- OBSERVED AND PROJECTED WHITE FEMALE AGE DISTRIBUTIONS: EAST NORTH CENTRAL DIVISION AND THE REST OF THE UNITED STATES, 1950-1960

Region	Age Group in 1960	Female Population		
		Observed		Projected
		1950	1960	1960
East North Central Division	10-19	2,639,649	2,644,188	2,644,188
	20-29	1,917,482	1,953,115	1,953,115
	30-39	2,201,503	2,183,098	2,183,098
	40-49	2,105,789	2,025,369	2,025,369
	50-59	1,655,279	1,551,101	1,551,101
	60-69	1,279,723	1,114,463	1,114,463
	70 and over	1,524,794	870,476	870,476
Rest of the United States	10-19	9,949,441	9,990,839	9,990,839
	20-29	7,417,749	7,233,777	7,233,777
	30-39	8,189,588	8,245,636	8,245,636
	40-49	7,767,486	7,631,769	7,631,769
	50-59	6,104,634	5,924,749	5,924,749
	60-69	4,588,780	4,224,948	4,224,948
	70 and over	5,422,047	3,285,030	3,285,030

Source: Calculated using the data in Table 1 and Figure 1.

TABLE 3 -- IN-MIGRATION, OUT MIGRATION, AND NET MIGRATION OF WHITE FEMALES: EAST NORTH CENTRAL DIVISION, 1950-1960

Age Group (in 1960)	In-Migration	Out-Migration	Net-Migration
10 - 19	152,409	155,516	-3,107
20 - 29	256,136	204,716	51,420
30 - 39	121,090	150,541	-29,451
40 - 49	51,698	83,350	-31,652
50 - 59	16,425	52,920	-36,495
60 - 69	-6,385	45,365	-51,750
70 and over	1,468	-1,960	3,427
Total	592,841	690,448	-97,607

Source: Calculated using the data in Table 1 and Figure 1.

Without accurate data on migration flows during the 1950-1960 decade, it is difficult to evaluate the accuracy of the estimated volumes of flow. We can, however, (1) immediately identify certain obvious inaccuracies, such as the presence of negative elements in the estimated population growth matrix; (2) examine closely the principal assumption that is implicit in our estimation method; and (3) contrast the results produced by our estimation method with those generated an alternative estimation procedure.

5. Conditions Which Produce Negative Estimates of Migration Probabilities

Clearly, the two negative migration probabilities in Figure 1 are in error, since by definition these probabilities must be nonnegative.[4] Several factors could be acting to produce this result, of which the most likely ones are (1) errors in age reporting; (2) errors in enumeration; (3) errors in reporting the place of birth; and (4) errors in the adjustment of the data for international migration. To identify the circumstances under which our estimation method will produce negative migration estimates, we recall (41) and proceed to compute the elements of $[{}_k g]$, using Cramer's Rule. Consider, for example, the element ${}_k g_{AB}$:

$$ {}_k g_{AB} = \frac{{}_B w_B^{(t,k)} \; {}_A w_B^{(t+1,k+5)} - {}_A w_B^{(t,k)} \; {}_B w_B^{(t+1,k+5)}}{{}_A w_A^{(t,k)} \; {}_B w_B^{(t,k)} - {}_A w_B^{(t,k)} \; {}_B w_A^{(t,k)}} \; . \qquad (48) $$

For ${}_k g_{AB}$ to be negative either, but not both, of the following two conditions must be satisfied:

---

[4] Note the similarity of this problem with the one reported in Rogers (1967).

$$ {}_B w_B^{(t,k)} \; {}_A w_B^{(t+1,k+5)} < {}_A w_B^{(t,k)} \; {}_B w_B^{(t+1,k+5)} \tag{49} $$

or

$$ {}_A w_A^{(t,k)} \; {}_B w_B^{(t,k)} < {}_A w_B^{(t,k)} \; {}_B w_A^{(t,k)} \quad . \tag{50} $$

Let us dispense with the second condition first, for it is unlikely to be realized. Rewriting (50) as follows:

$$ \frac{{}_A w_A^{(t,k)}}{{}_A w_B^{(t,k)}} < \frac{{}_B w_A^{(t,k)}}{{}_B w_B^{(t,k)}} \quad , \tag{51} $$

and noting that typically

$$ {}_A w_B^{(t,k)} < {}_A w_A^{(t,k)} $$

and

$$ {}_B w_A^{(t,k)} < {}_B w_B^{(t,k)} \quad , $$

we conclude that (50) is most unlikely to be satisfied in practical situations.

Returning to the first condition in (49), we rewrite it as follows:

$$ \frac{{}_B w_B^{(t,k)}}{{}_A w_B^{(t,k)}} < \frac{{}_B w_B^{(t+1,k+5)}}{{}_A w_B^{(t+1,k+5)}} \quad , \tag{52} $$

and conclude that ${}_k g_{AB}$ will be negative (provided the second condition is not satisfied) if a region's ratio of in-born to out-born increases with time and age. This explains, perhaps, why our negative migration probabilities occurred only in the higher age brackets. An examination of the negative $_k g_{AB}$ for the seventh age group in Figure 1 shows that (52) is satisfied, while (51) is not.

If the survivorship probabilities exceed unity or the migration probabilities are negative, we may be certain that our estimated growth matrix is not equal to the "true" or "observed" growth matrix that would have been obtained from direct observations. However, even if neither of these two potential errors occurs, there is still no assurance that the estimated growth matrix is equal to the observed growth matrix. For this to be true, certain other conditions also have to be met. These pertain to the principal assumption that is implicit in our estimation method.

6. The Principal Assumption Underlying the Estimation Method

Our estimation method is founded on the equations set out in (35)-(38) and their matrix equivalents presented in (10)-(13). Thus, the principal assumption that underlies our estimation procedure is that $_A G_{ij} = {}_B G_{ij}$. This can be readily seen by recalling (15) and observing that

$$\underline{w}^{(t+1)} = C\underline{v}^{(t+1)} = CR\underline{v}^{(t)}, \text{ by (22) and (15)}, \tag{53}$$

and

$$\underline{w}^{(t+1)} = G\underline{w}^{(t)} = GC\underline{v}^{(t)}, \text{ by (1) and (22)} . \tag{54}$$

Equation (53) is the errorless projection because it arises from the unconsolidated growth model which differentiates between $_A G_{ij}$ and $_B G_{ij}$. Equation (54), however, generates a potentially erroneous projection since it applies the same growth matrix to the A-born and B-born. Hence <u>perfect aggregation</u> will occur if

$$CR = GC \quad , \tag{55}$$

which is merely another way of expressing that ${}_{A}G_{ij} = {}_{B}G_{ij}$ [Rogers (1969)].[5]

The above argument is useful in that it may be applied to show under what other circumstance we may still obtain perfect aggregation even if ${}_{A}G_{ij} \neq {}_{B}G_{ij}$. To establish this alternative condition for perfect aggregation, we observe that

$$\underline{w}^{(t+1)} = G\underline{w}^{(t)} = GD_{v}^{(t)}\ \underline{v}^{(t)} \tag{56}$$

and

$$\underline{w}^{(t+1)} = D_{v}^{(t+1)}\underline{v}^{(t+1)} = D_{v}^{(t+1)}R\underline{v}^{(t)}\ . \tag{57}$$

Once again, the first equation is the errorless projection and the second is the potentially erroneous one. Thus perfect aggregation will occur if

$$GD_{v}^{(t)} = D_{v}^{(t+1)}R\quad , \tag{58}$$

which expresses the condition that the growth matrices for the A-born and B-born female populations may be perfectly aggregated if their population distributions retain a constant proportional relationship to one another. This consolidation rule is met only if the interregional population system is stable.

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[5] By perfect aggregation we mean a consolidation scheme that produces results which would be obtained in the absence of consolidation.

7. Comparison of the Estimation Method with an Alternative Estimation Procedure

To our knowledge, no other method has been suggested for estimating in- and out-migration flows from place-of-birth-by-place-of-residence data. However, Eldridge and Kim (1968) describe a method, which they call the DOB (Division-of-Birth) method, for estimating net migration from these data and argue that their estimates of the net migration of the in-born and out-born populations of a region yield acceptable measures of primary, but not total, interregional migration flows.

Eldridge and Kim's DOB method proceeds as follows:

(1) Division-of-birth-specific survival rates are estimated and applied to the respective components of the observed 1950 population of each residence division to yield the expected 1960 population, by place of residence, that would have resulted if net migration were zero for each division-of-residence-by-division-of-birth component.

(2) The differences between the observed and the expected 1960 population components are estimates of the net migration for each place of residence, cross-classified by division of birth, over the ten-year period preceeding 1960.

In order to compare the DOB method with our estimation method, henceforth referred to as the PRPB (Place-of-Residence-by-Place-of-Birth) method, we shall need the following additional definitions and notation.

Recall the growth matrix R, defined in (15). If all elements of this matrix were known, we could obtain survival probabilities classified by place of birth and place of residence, simply by aggregating the migration and survivorship proportions along the columns of R to define

$$P = \left[\begin{array}{cc|cc} {}_A S_A & 0 & 0 & 0 \\ 0 & {}_A S_B & 0 & 0 \\ \hline 0 & 0 & {}_B S_A & 0 \\ 0 & 0 & 0 & {}_B S_B \end{array}\right], \tag{59}$$

where

$$\begin{aligned} {}_A S_A &= {}_A G^*_{AA} + {}_A G^*_{AB} \\ {}_A S_B &= {}_A G^*_{BA} + {}_A G^*_{BB} \\ {}_B S_A &= {}_B G^*_{AA} + {}_B G^*_{AB} \\ {}_B S_B &= {}_B G^*_{BA} + {}_B G^*_{BB} \end{aligned} ,$$

where ${}_h G^*_{ij}$ is simply ${}_h G_{ij}$ with all of the fertility elements set equal to zero. Then

$$\hat{\underline{v}}^{(t+1)} = P\underline{v}^{(t)} . \tag{60}$$

where $\hat{\underline{v}}^{(t+1)}$ is the population vector that would be expected if no inter-regional migration were to take place. Henceforth, the first age group will always be equal to zero since we have eliminated fertility from the system.

Consolidating the place-of-residence-by-place-of-birth survivorship matrix P to obtain S and $\tilde{S}$, the place-of-residence and the place-of-birth survivorship matrices, respectively, we have:

$$S = \left[\begin{array}{c|c} S_A & 0 \\ \hline 0 & S_B \end{array}\right] = \left[\begin{array}{cc|cc} I & 0 & I & 0 \\ 0 & I & 0 & I \end{array}\right] \left[\begin{array}{cc|cc} {}_AS_A & 0 & 0 & 0 \\ 0 & {}_AS_B & 0 & 0 \\ \hline 0 & 0 & {}_BS_A & 0 \\ 0 & 0 & 0 & {}_BS_B \end{array}\right] \left[\begin{array}{cc} {}_AD_A^{(t)} & 0 \\ 0 & {}_BD_A^{(t)} \\ \hline {}_AD_B^{(t)} & 0 \\ 0 & {}_BD_B^{(t)} \end{array}\right] \tag{61}$$

and

$$\tilde{S} = \left[\begin{array}{c|c} {}_AS & 0 \\ \hline 0 & {}_BS \end{array}\right] = \left[\begin{array}{cc|cc} I & I & 0 & 0 \\ 0 & 0 & I & I \end{array}\right] \left[\begin{array}{cc|cc} {}_AS_A & 0 & 0 & 0 \\ 0 & {}_AS_B & 0 & 0 \\ \hline 0 & 0 & {}_BS_A & 0 \\ 0 & 0 & 0 & {}_BS_B \end{array}\right] \left[\begin{array}{cc} {}_AD_A^{(t)} & 0 \\ {}_AD_B^{(t)} & 0 \\ \hline 0 & {}_BD_A^{(t)} \\ 0 & {}_BD_B^{(t)} \end{array}\right], \tag{62}$$

where the ${}_iD_j^{(t)}$ matrices are defined as in (27), and where

$$\hat{\underline{w}}^{(t+1)} = S\underline{w}^{(t)} \tag{63}$$

and

$$\hat{\tilde{\underline{w}}}^{(t+1)} = \tilde{S}\tilde{\underline{w}}^{(t)} . \tag{64}$$

Note that the survivorship matrices S and $\tilde{S}$ do not project the population by place of residence ($\underline{w}^{(t)}$) and the population by place of birth ($\tilde{\underline{w}}^{(t)}$), respectively, and observe that according to (60), to project the place-of-birth-by-place-of-residence vector $\hat{\underline{v}}^{(t)}$ in a population system with no migration we need the survivorship matrix P defined in (59). Since the data to estimate R or P are not available, Eldridge and Kim compute age-specific survival ratios, by division of birth, by dividing each element of $\underline{w}^{(t+1)}$ by the appropriate element in $\underline{w}^{(t)}$:

$$ {}_h s^k = \frac{{}_h w^{(t+1,k+5)}}{{}_h w^{(t,k)}} \qquad (k=0,5,10,\ldots). \tag{65} $$

Collecting these elements they form the matrix $\tilde{\bar{S}}$, and, to add the place-of-residence dimension, assume that place-of-birth-specific survival ratios do not vary with the place of residence, and that therefore the ${}_h S_i$ in P may be replaced by the ${}_h S$ in $\tilde{S}$, for all i, to obtain:

$$ \bar{\hat{\underline{v}}}^{(t+1)} = \begin{bmatrix} \bar{\hat{\underline{v}}}_A^{(t+1)} \\ \hline \bar{\hat{\underline{v}}}_B^{(t+1)} \end{bmatrix} = \begin{bmatrix} {}_A\bar{\hat{\underline{w}}}_A^{(t+1)} \\ {}_A\bar{\hat{\underline{w}}}_B^{(t+1)} \\ \hline {}_B\bar{\hat{\underline{w}}}_A^{(t+1)} \\ {}_B\bar{\hat{\underline{w}}}_B^{(t+1)} \end{bmatrix} = \begin{bmatrix} {}_A\bar{S} & 0 & 0 & 0 \\ 0 & {}_A\bar{S} & 0 & 0 \\ \hline 0 & 0 & {}_B\bar{S} & 0 \\ 0 & 0 & 0 & {}_B\bar{S} \end{bmatrix} \begin{bmatrix} {}_A\underline{w}_A^{(t)} \\ {}_A\underline{w}_B^{(t)} \\ \hline {}_B\underline{w}_A^{(t)} \\ {}_B\underline{w}_B^{(t)} \end{bmatrix} = \bar{P}\underline{v}^{(t)} \text{ say.} \tag{66} $$

Note the differences between (15), (60), and (66), and observe that $\underline{v}^{(t+1)}$ is the "correct" vector in the presence of interregional migration, $\hat{\underline{v}}^{(t+1)}$ is the "correct" vector in the absence of interregional migration, and $\bar{\hat{\underline{v}}}^{(t+1)}$ is Eldridge and Kim's estimate of $\hat{\underline{v}}^{(t+1)}$.

From $\bar{P}$ we may obtain place-of-residence-specific survival ratios by weighting place-of-birth-specific survival ratios for each place-of-residence by the contribution of their population components to the total regional population:

$$\bar{S} = \begin{bmatrix} \bar{S}_A & 0 \\ 0 & \bar{S}_B \end{bmatrix} = \begin{bmatrix} I & 0 & I & 0 \\ 0 & I & 0 & I \end{bmatrix} \begin{bmatrix} {}_A S & 0 & 0 & 0 \\ 0 & {}_A S & 0 & 0 \\ 0 & 0 & {}_B S & 0 \\ 0 & 0 & 0 & {}_B S \end{bmatrix} \begin{bmatrix} {}_A D_A^{(t)} & 0 \\ 0 & {}_A D_B^{(t)} \\ {}_B D_A & 0 \\ 0 & {}_B D_B \end{bmatrix} \quad (67)$$

where ${}_h D_i^{(t)}$ are defined as in (27) but with the modification:

$${}_h d_A^{(t,k)} = \frac{{}_h w_A^{(t,k)}}{{}_h w_A^{(t,k)} + {}_B w_h^{(t,k)}} .$$

It may easily be verified that the regional population totals projected by $\bar{S}$ are the same as those projected by $\bar{P}$ if the place-of-birth components for each region are added together.

Having compared Eldridge and Kim's estimation method with our own, let us now contrast the results that each method produces when applied to the same data base. First, consider the two-region results set out in Figure 1 and Tables 2 and 3. We may compare them with the results that Eldridge and Kim's DOB method would have produced by comparing the survival and net migration rates that are generated by each method. These data are set out in Figure 2 and Tables 4 and 5. Note that in Table 4, the two sets of DOB survival rates are from $\tilde{S}$ and $\bar{S}$, respectively, and observe that, for purposes of comparability, the in- and out-migration rates in Table 5 have been given a common denominator and therefore may be added to yield the net migration rate,as is the case in Eldridge and Kim (1968). The survival

FIGURE 2 - RATES OF IN-, OUT-, AND NET MIGRATION FOR NATIVE WHITE FEMALES, 10 YEARS OLD AND OVER

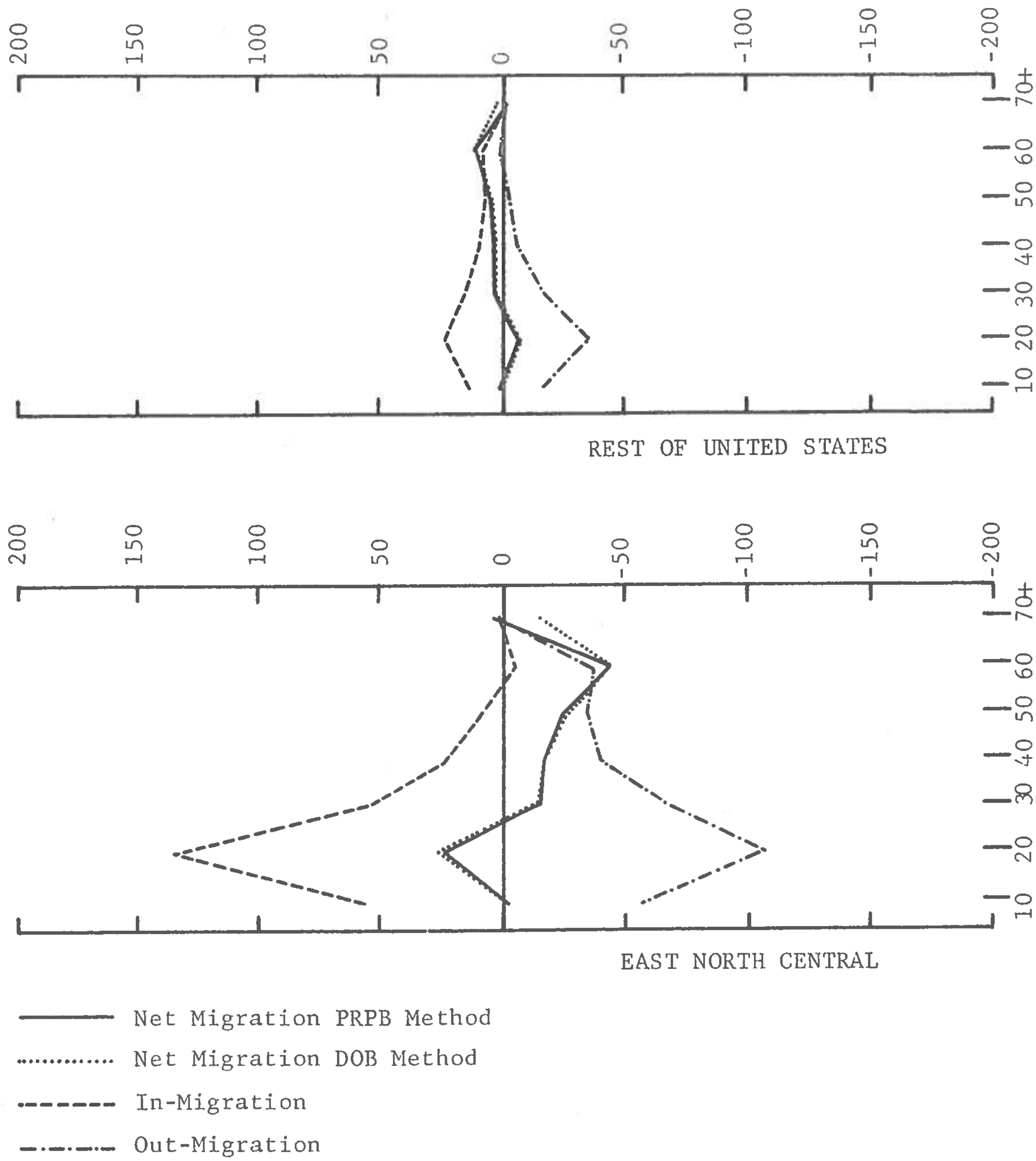

TABLE 4

DIVISION-OF-BIRTH (DOB) SURVIVAL RATES, DIVISION-OF-RESIDENCE SURVIVAL RATES, AND PRPB SURVIVAL RATES FROM GROWTH MATRIX, FOR NATIVE WHITE FEMALE POPULATION, 10 YEARS OLD AND OVER, BY AGE AND TWO-WAY DIVISION OF CONTERMINOUS UNITED STATES, 1950-1960

Division of Birth/ Residence and Age in 1960	Survival Rates by Division of		
	Birth DOB	Residence DOB	PRPB
East North Central			
10 - 19	1.002939	1.002991	1.002884
20 - 29	.991023	.990293	.991755
30 - 39	1.004760	1.004511	1.005008
40 - 49	.977082	.977323	.976831
50 - 59	.960090	.960911	.959098
60 - 69	.910887	.910639	.911288
70+	.579100	.583022	.568630
Rest of United States			
10 - 19	1.003834	1.003821	1.003849
20 - 29	.982312	.982481	.982104
30 - 39	1.003304	1.003357	1.003223
40 - 49	.978344	.978293	.978426
50 - 59	.964230	.964029	.964520
60 - 69	.909481	.909568	.909387
70+	.604757	.602419	.606466

TABLE 5

RATES OF IN-, OUT-, AND NET MIGRATION OF NATIVE WHITE FEMALES, 10 YEARS OLD AND OVER, BY AGE, AS DERIVED BY DOB AND PRPB METHODS, FOR TWO-REGIONAL DIVISION OF UNITED STATES, 1950-1960

Residence and Age in 1960	Rates per 1,000 Average Population			
	In-Mig. PRPB	Out-Mig. PRPB	Net Migration PRPB	Net Migration DOB
East North Central				
10-19	57.7	58.9	- 1.2	- 1.3
20-29	132.3	105.8	26.5	28.0
30-39	55.2	68.7	-13.4	-12.9
40-49	25.0	40.3	-15.3	-15.8
50-59	10.2	33.0	-22.8	-24.6
60-69	-5.3	37.9	-43.2	-42.5
70+	1.2	-1.6	2.9	-15.5
Total, 10+	46.2	53.8	-7.6	-9.3
Rest of United States				
10-19	15.6	15.3	0.3	0.3
20-29	27.9	35.0	-7.0	-7.4
30-39	18.3	14.7	3.6	3.4
40-49	10.8	6.7	4.1	4.2
50-59	8.8	2.7	6.1	6.6
60-69	10.3	1.4	11.7	11.6
70+	-0.4	0.3	-0.8	4.3
Total, 10+	14.4	12.3	2.0	2.5

rates from the PRPB method are obtained simply by aggregating along the columns of the estimated growth matrix in Figure 1 (omitting the fertility elements, of course).

A quick glance at the three columns in Table 4 reveals that the three sets of survival rates are remarkably similar. Table 5 and Figure 2 show that the same may be said about the net migration rates that are produced by each method. Thus it appears that the PRPB and the DOB methods produce almost identical estimates of net migration. However, the PRPB method, in addition, provides estimates of in- and out-migration.

A two-region population system does not permit a convincing comparison of the PRPB and DOB methods because of insufficient data observations. Hence, in order to carry out a more extensive comparison of the two estimation methods, we now expand our two-region system to Eldridge and Kim's nine-region division of the conterminous United States.[6]

Figure 3 and Tables 6 and 7 summarize the results for the nine-region population system and correspond directly to the results for the two-region system that appear in Figure 2 and Tables 4 and 5, respectively. Once again the principal finding is that both methods generate almost the same survival and net migration rates in all age groups except for the last.

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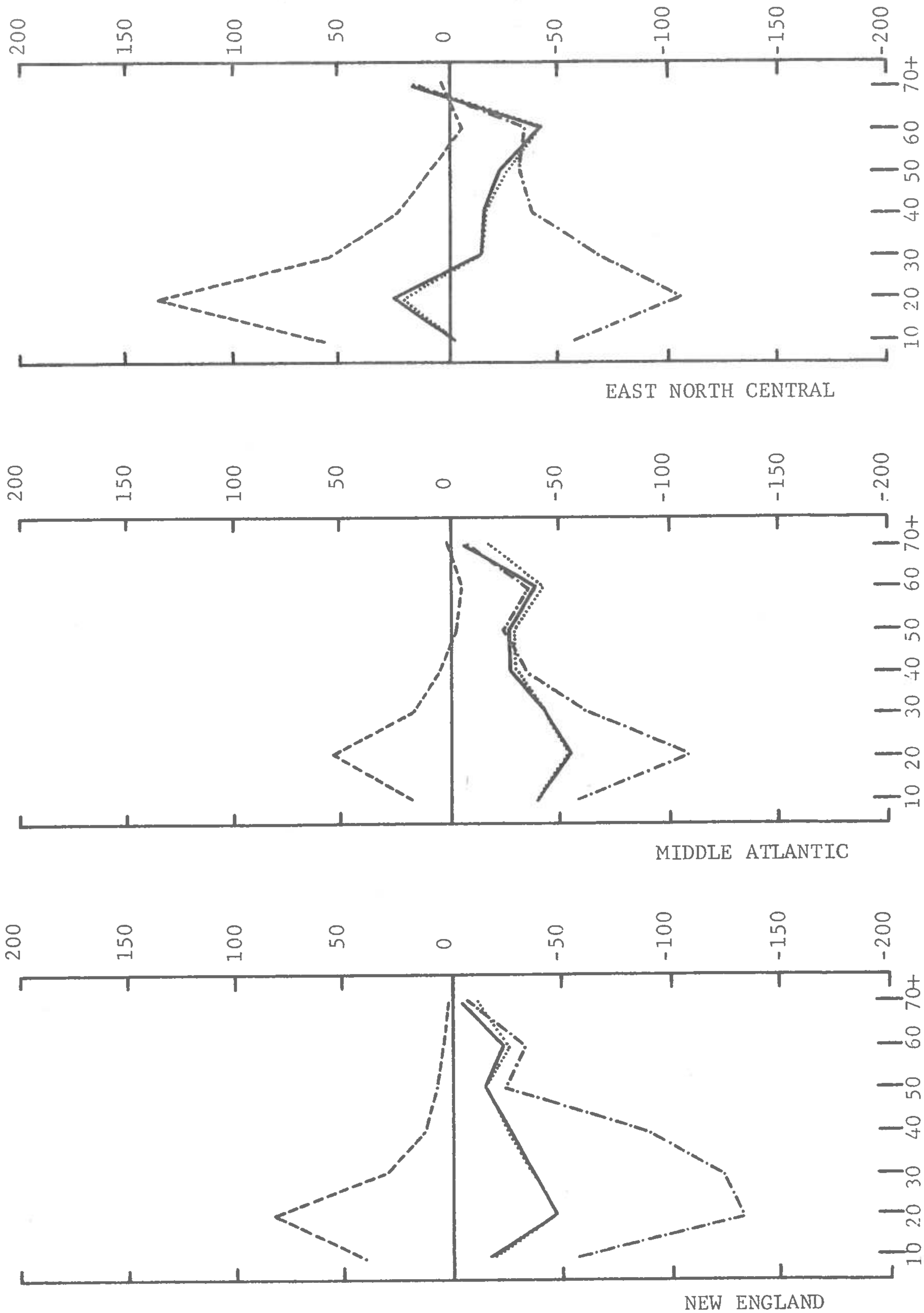
[6] For purposes of comparison, we have adjusted the published Census data in precisely the way described by Eldridge and Kim. However, for some unexplainable reason, we have been unable to eliminate small discrepancies between our data base and theirs. Therefore, we present their estimates and the ones we obtained by applying their method on our data base.

FIGURE 3 - RATES OF IN-, OUT-, AND NET MIGRATION FOR NATIVE WHITE FEMALES, 10 YEARS OLD AND OVER



Net Migration PRPB Method

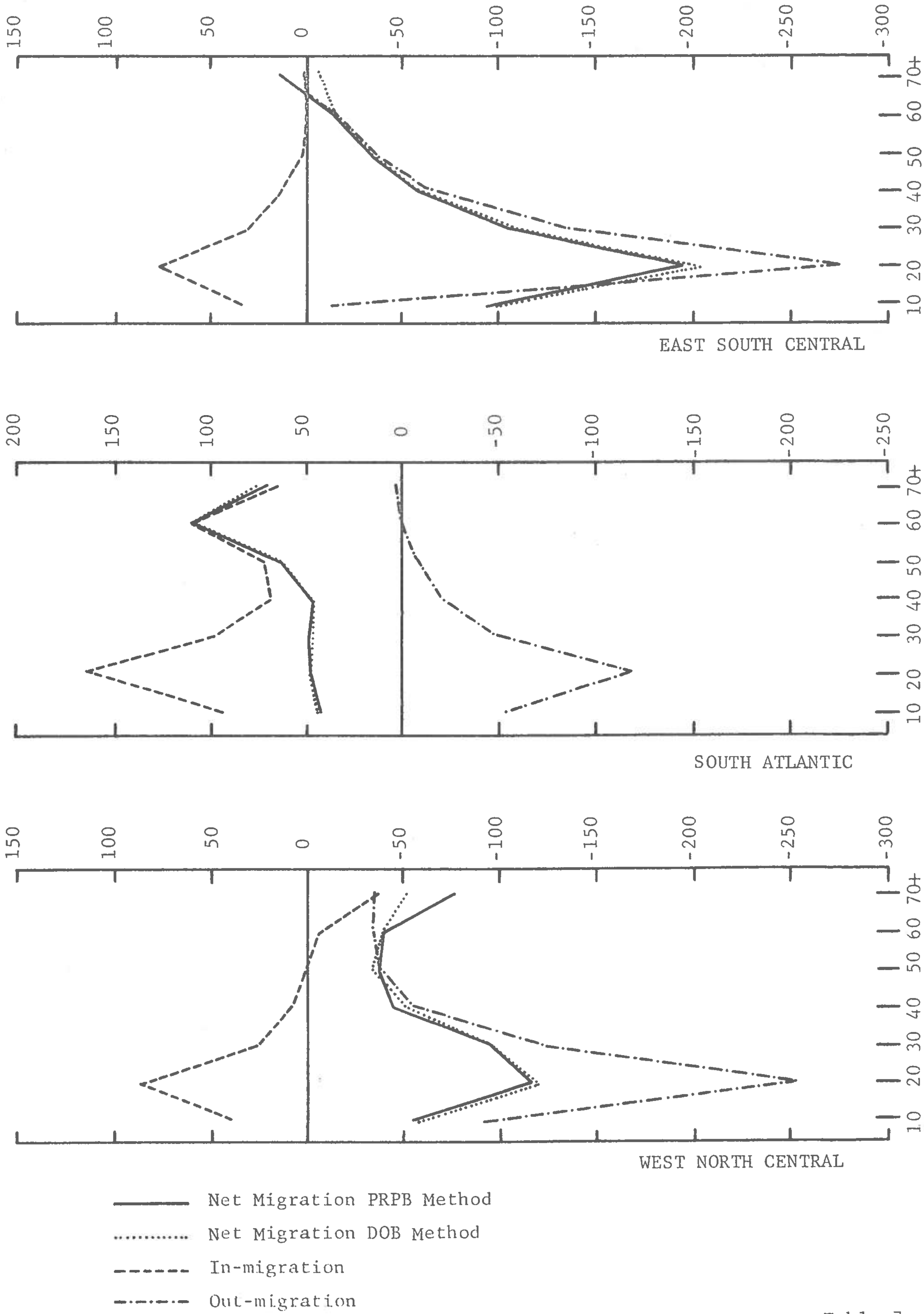
Net Migration DOB Method

In-migration

Out-migration

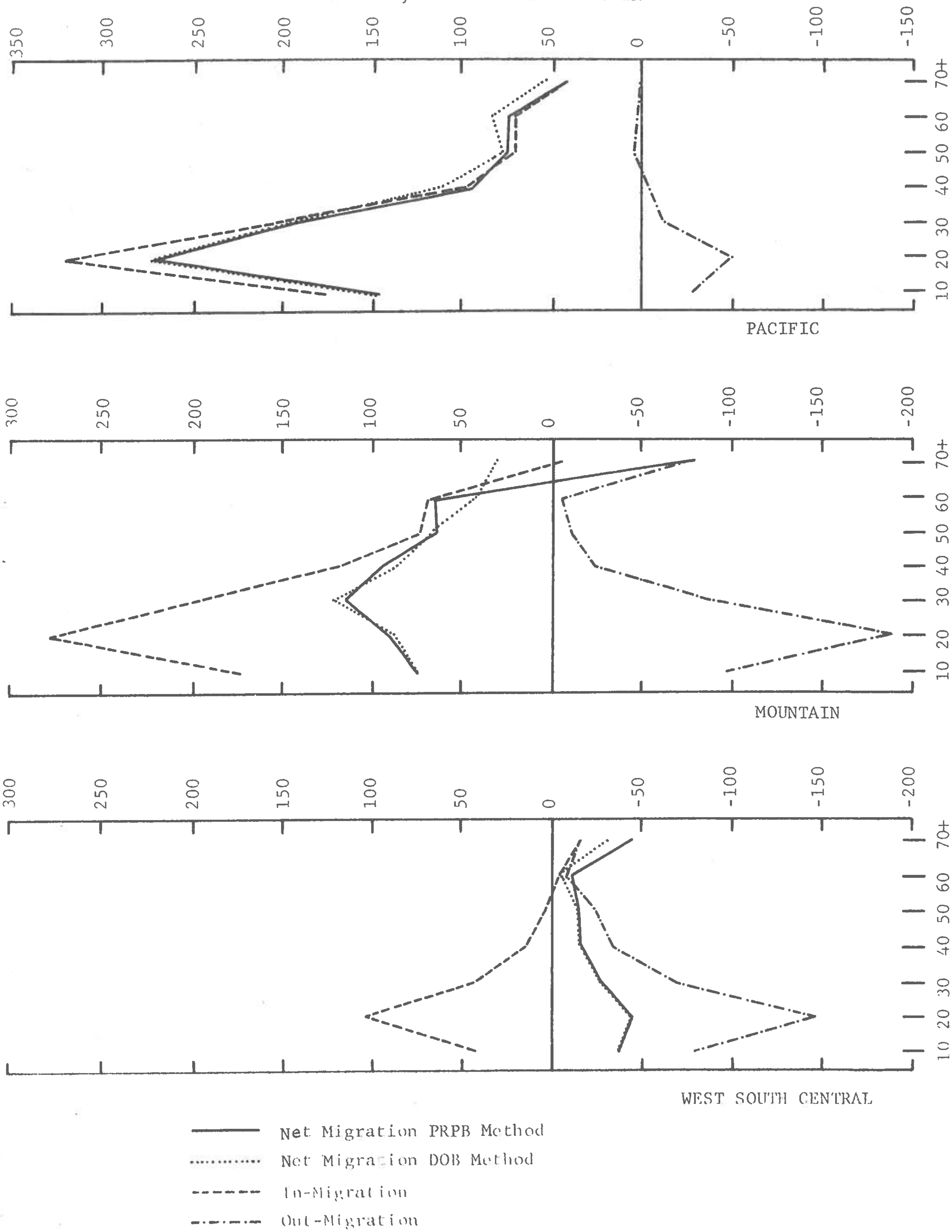
Source: Table 7

FIGURE 3 - RATES OF IN-, OUT-, AND NET MIGRATION FOR NATIVE WHITE FEMALES, 10 YEARS OLD AND OVER



Source: Table 7

FIGURE 3 - RATES OF IN-, OUT-, AND NET MIGRATION FOR NATIVE WHITE FEMALES, 10 YEARS OLD AND OVER



Source: Table 7

TABLE 6

DIVISION-OF-BIRTH (DOB) SURVIVAL RATES, DIVISION-OF-RESIDENCE SURVIVAL RATES AND PRPB SURVIVAL RATES FROM GROWTH MATRIX, FOR NATIVE WHITE FEMALE POPULATION, 10 YEARS OLD AND OVER, BY AGE AND GEOGRAPHIC DIVISIONS OF CONTERMINOUS UNITED STATES, 1950-1960

Division of Birth/ Residence and Age in 1960		Survival Rates by Division of			
		Birth		Residence	
		DOB*[1]	DOB[2]	DOB[3]	PRPB
New England					
10 - 19	10-14 15-19	1.01157 0.99253	1.003302	1.003410	1.003162
20 - 29		0.98627	.986712	.986690	.986697
30 - 39		0.99941	.999919	1.000372	.999140
40 - 49		0.98311	.983648	.982633	.984615
50 - 59		0.96187	.962677	.962201	.963097
60 - 69		0.90220	.903066	.903171	.902236
70+		0.57718	.577848	.578114	.574649
Middle Atlantic					
10 - 19	10-14 15-19	1.01601 0.99675	1.007478	1.007337	1.007646
20 - 29		0.98623	.986426	.986399	.986382
30 - 39		1.00480	1.005135	1.004846	1.005483
40 - 49		0.97162	.971976	.972646	.970205
50 - 59		0.95503	.955505	.956342	.953816
60 - 69		0.89989	.900631	.901342	.898812
70+		0.57022	.570634	.572629	.564371

[1] From Eldridge and Kim (1968), Table A-2

[2] Obtained using our data base .

[3] Using the rates in Column 2.

Division of Birth/ Residence and Age in 1960		Survival Rates by Division of Birth		Survival Rates by Division of Residence	
		DOB*[1]	DOB[2]	DOB[3]	PRPB
East North Central					
10 - 19	10-14	1.00919	1.002939	1.002860	1.002932
	15-19	0.99463			
20 - 29		0.99060	.991023	.989870	.991648
30 - 39		1.00427	1.004760	1.004152	1.004670
40 - 49		0.97656	.977082	.976954	.975465
50 - 59		0.95944	.960090	.960765	.958059
60 - 69		0.91015	.910887	.910612	.910001
70+		0.57864	.579100	.52785	.558653
West North Central					
10 - 19	10-14	1.00133	.995025	.995778	.993938
	15-19	0.98734			
20 - 29		0.97668	.976557	.977275	.974119
30 - 39		1.00670	1.006694	1.006505	1.005927
40 - 49		0.97919	.979200	.979325	.972290
50 - 59		0.97170	.971967	.970729	.973790
60 - 69		0.91455	.914689	.914311	.913902
70+		0.64067	.640838	.628026	.646076
South Atlantic					
10 - 19	10-14	1.01584	1.004712	1.004565	1.004852
	15-19	0.99318			
20 - 29		0.98496	.983690	.983425	.983781
30 - 39		.99849	.997221	.998083	.996269
40 - 49		.97574	.974018	.974420	.973387
50 - 59		.96845	.965653	.964573	.966613
60 - 69		.90893	.905194	.905819	.904342
70+		.59213	.589672	.589332	.587546

Division of Birth/ Residence and Age in 1960		Survival Rates by Division of Birth		Survival Rates by Division of Residence	
		DOB*1	DOB2	DOB3	PRPB
East South Central					
10 - 19	10-14	1.00783	.995374	.995977	.994365
	15-19	.98096			
20 - 29		.96592	.966032	.967186	.962492
30 - 39		.99268	.991737	.993087	.987193
40 - 49		.97237	.972622	.973223	.970113
50 - 59		.95843	.958786	.959543	.955851
60 - 69		.91148	.911773	.911696	.910501
70+		.59439	.594629	.595289	.578816
West South Central					
10 - 19	10-14	1.02225	1.007271	1.006864	1.007658
	15-19	.98976			
20 - 29		.98400	.984353	.984015	.983990
30 - 39		1.00675	1.007172	1.006501	1.007065
40 - 49		.98306	.983363	.982501	.980859
50 - 59		.97062	.970825	.969761	.971043
60 - 69		.92615	.926274	.922781	.929699
70+		.66282	.662468	.642268	.668790
Mountain					
10 - 19	10-14	1.01491	1.006265	1.005835	1.006450
	15-19	.99665			
20 - 29		.98212	.981272	.981845	.978633
30 - 39		1.01586	1.015403	1.011558	1.019996
40 - 49		.98892	.987911	.984617	.977976
50 - 59		.97477	.973438	.970437	.975231
60 - 69		.90432	.903576	.910461	.889152
70+		.98754	.687357	.636445	.729101
Pacific					
10 - 19	10-14	1.01033	1.007654	1.006999	1.008093
	15-19	1.00437			
20 - 29		.99395	.993541	.989484	.994423
30 - 39		1.00987	1.009723	1.008113	1.009949
40 - 49		1.00731	1.006052	.989114	1.009330
50 - 59		.97394	.972689	.968940	.973236
60 - 69		.92457	.923720	.915076	.925450
70+		.62677	.626643	.615898	.625773

TABLE 7

RATES OF IN-, OUT-, AND NET MIGRATION OF NATIVE WHITE FEMALES, 10 YEARS OLD AND OVER, BY AGE, AS DERIVED BY DOB AND PRPB METHODS, AND GEOGRAPHIC DIVISIONS OF CONTERMINOUS UNITED STATES, 1950-1960

Rates per 1,000 Average Population

Residence and Age in 1960		In-Mig. PRPB	Out-Mig. PRPB	Net Migration		
				PRPB	DOB[2]	DOB*[1]
New England						
10-19	10-14	40.9	58.4	-17.5	-17.8	-26.
	15-19					- 7.
20-29		82.8	131.8	-49.0	-49.0	-49.
30-39		32.9	72.1	-39.2	-40.5	-40.
40-49		13.2	39.9	-26.7	-24.7	-24.
50-59		7.6	21.8	-14.2	-13.3	-13.
60-69		4.5	26.3	-21.8	-22.8	-22.
70+		2.6	8.4	- 5.9	-10.3	- 9.
Total 10+		30.3	56.8	-26.5	-26.8	-26.
Middle Atlantic						
10-19	10-14	19.0	58.9	-39.9	-39.6	-38.
	15-19					-42.
20-29		52.8	107.5	-54.7	-54.8	-54.
30-39		18.7	61.7	-43.0	-42.4	-42.
40-49		5.3	34.4	-29.0	-31.5	-31.
50-59		-0.7	25.9	-26.6	-29.2	-29.
60-69		-4.0	34.7	-38.7	-41.4	-41.
70+		1.0	8.0	- 6.9	-17.5	-17.
Total 10+		15.3	51.3	-36.1	-37.8	-37.

[1] Source: DOB* estimates from Eldridge and Kim (1968) Table 2.

[2] DOB estimates from our data base.

Rates per 1,000 Average Population

Residence and Age in 1960		In-Mig. PRPB	Out-Mig. PRPB	Net Migration PRPB	DOB[2]	DOB*[1]
East North Central						
10-19	10-14 15-19	57.7	58.9	-1.2	-1.1	- 4. 3.
20-29		132.0	105.3	26.7	28.4	29.
30-39		54.8	67.9	-13.1	-12.6	-12.
40-49		24.6	38.5	-13.9	-15.4	-15.
50-59		9.9	31.6	-21.7	-24.5	-24.
60-69		-5.3	36.6	-41.9	-42.5	-42.
70+		1.5	-14.0	15.5	-15.2	-15.
Total, 10+		46.0	51.9	- 5.9	- 9.0	- 9.
West North Central						
10-19	10-14 15-19	38.7	94.3	-55.6	-57.5	-63. -51.
20-29		84.3	201.3	-117.0	-120.4	-121.
30-39		28.1	124.2	-96.2	-96.8	-97.
40-49		9.3	55.5	-46.2	-53.5	-53.
50-59		0.2	39.2	-39.0	-35.8	-36.
60-69		-6.0	33.9	-39.9	-40.4	-40.
70+		-37.8	36.8	-74.7	-51.9	-52.
Total, 10+		21.3	89.3	-68.0	-67.2	-67.
South Atlantic						
10-19	10-14 15-19	92.2	51.4	40.8	41.1	38. 44.
20-29		165.3	117.9	47.5	47.8	47.
30-39		97.8	48.3	49.5	47.7	47.
40-49		68.0	19.6	48.3	47.3	46.
50-59		71.2	6.7	64.5	66.5	64.
60-69		110.2	-0.3	110.5	109.1	106.
70+		65.3	-3.3	68.6	66.4	64.
Total, 10+		97.6	41.5	56.1	55.7	54.

Rates per 1,000 Average Population

Residence and Age in 1960		In-Mig. PRPB	Out-Mig. PRPB	Net Migration PRPB	Net Migration DOB[2]	Net Migration DOB*[1]
East South Central						
10-19	10-14 15-19	31.9	124.5	-92.6	-94.3	-96. -109.
20-29		76.7	272.5	-195.8	-201.1	-220.
30-39		30.0	133.4	-103.4	-109.6	-121.
40-49		11.8	69.0	-57.2	-60.4	-66.
50-59		1.2	33.8	-32.6	-36.4	-40.
60-69		0.3	15.1	-14.8	-16.1	-14.
70+		0.7	-17.7	12.5	-8.3	-12.
Total, 10+		26.6	108.8	-82.3	-87.5	-96.
West South Central						
10-19	10-14 15-19	42.6	80.2	-37.7	-36.9	-34. -41.
20-29		102.0	147.0	-45.0	-45.0	-45.
30-39		42.2	69.9	-27.7	-27.1	-27.
40-49		16.4	32.4	-16.0	-17.7	-17.
50-59		3.4	17.3	-13.9	-12.6	-12.
60-69		-1.2	8.4	-9.6	-2.4	-2.
70+		-30.8	14.9	-45.8	-13.2	-14.
Total, 10+		32.8	61.9	-29.1	-25.4	-25.
Mountain						
10-19	10-14 15-19	171.3	98.5	72.8	73.4	80. 65.
20-29		278.2	188.2	90.0	86.9	86.
30-39		201.7	89.0	112.7	120.6	120.
40-49		117.5	23.2	94.2	87.8	87.
50-59		72.9	9.4	63.5	68.2	68.
60-69		70.1	3.7	66.4	44.6	44.
70+		-6.1	71.4	-77.5	33.8	34.
Total, 10+		153.7	80.7	73.0	80.1	80.

Rates per 1,000 Average Population

Residence and Age in 1960		In-Mig. PRPB	Out-Mig. PRPB	Net Migration PRPB	Net Migration DOB[2]	Net Migration DOB*[1]
Pacific						
10-19	10-14	176.2	28.9	147.3	148.3	153.
	15-19					143.
20-29		319.7	49.4	270.3	274.6	274.
30-39		204.4	11.2	193.2	194.9	195.
40-49		97.6	2.8	94.8	113.9	114.
50-59		71.1	-4.0	75.1	79.2	79.
60-69		71.9	-3.5	75.4	85.7	86.
70+		41.4	-0.6	42.0	53.9	54.
Total, 10+		153.8	14.7	139.1	146.0	146.

The estimated nine-region survivorship and migration growth matrix is too large to be included in this paper. However, we may illustrate its utility by returning to our results for the East North Central Division, reported in Table 3, and using the nine-region growth matrix to disaggregate, by division of origin and destination, respectively, the in- and out-migration flows that are presented there. These details are set out in Table 8. Note that by adding the row elements of the three matrices presented there we may obtain, except for errors introduced by rounding, the totals that are listed in the three columns in Table 3.

TABLE 8 -- INTERREGIONAL IN-, OUT-, AND NET MIGRATION OF WHITE FEMALES INTO AND OUT OF THE EAST NORTH CENTRAL DIVISION, 1950-1960, BY AGE, PLACE OF ORIGIN, AND PLACE OF DESTINATION

A. In-Migration

Age Group (in 1960)	Division of Origin							
	NE	MA	WNC	SA	ESC	WSC	MT	PAC
10-19	3,908	24,724	17,977	31,949	55,106	12,629	2,435	3,787
20-29	8,533	39,290	38,691	53,071	86,822	17,710	4,409	6,941
30-39	3,533	21,354	19,103	26,848	43,046	6,803	- 195	- 353
40-49	2,233	10,084	4,319	12,643	20,373	1,639	240	- 685
50-59	13	- 11	3,588	4,268	8,888	1,065	- 454	-1,404
60-69	- 573	-3,174	-2,797	- 339	1,053	- 601	-54	153
70 and over	- 721	-3,189	2,634	679	1,934	365	289	- 148
Total	16,926	89,078	83,513	129,119	217,221	39,610	6,671	8,290

B. Out-Migration

Age Group (in 1960)	Division of Destination							
	NE	MA	WNC	SA	ESC	WSC	MT	PAC
10-19	4,269	11,120	18,553	33,992	6,488	9,016	20,372	51,905
20-29	6,787	19,409	30,039	40,502	10,476	14,215	20,685	61,729
30-39	3,088	12,191	10,606	32,906	5,136	8,585	17,059	59,277
40-49	1,690	4,536	3,814	23,753	1,929	2,782	11,203	29,916
50-59	487	624	-94	24,338	747	1,165	6,442	17,023
60-69	266	-1,209	-3,304	30,029	- 483	922	3,953	13,601
70 and over	229	- 592	-20,383	15,693	-60	-3,230	-2,698	-5,743
Total	16,816	46,079	39,230	201,214	24,233	33,456	77,016	227,707

C. Net Migration

Age Group (in 1960	Division of Exchange							
	NE	MA	WNC	SA	ESC	WSC	MT	PAC
10-19	-361	13,604	-576	-2,043	48,618	3,613	-17,937	-48,118
20-29	1,746	19,881	8,652	12,569	76,346	3,495	-16,275	-54,787
30-39	444	9,163	8,497	-6,059	37,910	-1,782	-17,253	-59,630
40-49	543	5,548	505	-11,110	18,444	-1,143	-10,962	-30,601
50-59	-473	- 634	3,682	-20,069	8,140	-99	-6,896	-18,427
60-69	-839	-1,965	507	-30,369	1,536	-1,524	-4,008	-13,448
70 and over	-950	-2,597	23,017	-15,014	1,994	3,595	2,987	5,595
Total	110	42,999	44,282	-72,094	192,988	6,154	-70,346	-219,417

8. A Note on Stable Growth

We concluded Section 6 by observing that the condition for perfect aggregation that is expressed by (58) is met only if the interregional population system is stable. It may be appropriate, therefore, to conclude this paper by identifying some of the properties of such interregionally stable systems.

First, we note that if the place-of-residence-by-place-of-birth vector $\underline{u}^{(t)}$ in (14) is undergoing stable growth, then it must be growing at an intrinsic k-year growth rate of $\lambda - 1$, where λ is the dominant characteristic root of Q. It follows, then, that the place-of-birth-by-place-of-residence vector $\underline{v}^{(t)}$ in (15) must be undergoing stable growth at precisely the same rate, since in (16) and (17) Q and R were shown to be similar, and similar matrices have the same characteristic function:

$$Q - \lambda I = P^{-1}RP - \lambda I = P^{-1}(R - \lambda I)P;$$

whence

$$|Q - \lambda I| = |P^{-1}||R - \lambda I||P| = |P^{-1}||P||R - \lambda I| = |R - \lambda I|.$$

Next, observe that during the period of stable growth, a constant proportional relationship will prevail among and between the elements of ${}_A\underline{w}_A$, ${}_A\underline{w}_B$, ${}_B\underline{w}_A$, and ${}_B\underline{w}_B$. Finally, since the value of the dominant characteristic root is preserved in perfectly aggregated consolidations [Ara (1959)], if ${}_AG_{ij} = {}_BG_{ij}$, then $\lambda - 1$ also will be the intrinsic k-year growth rate of the population vector $\underline{w}$. Note, however, that if neither of the two conditions for perfect aggregation is satisfied, then the A-born population ultimately will grow at a different intrinsic growth rate than the B-born population.

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