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REPORT OF THE STORAGE RING DESIGN GROUP

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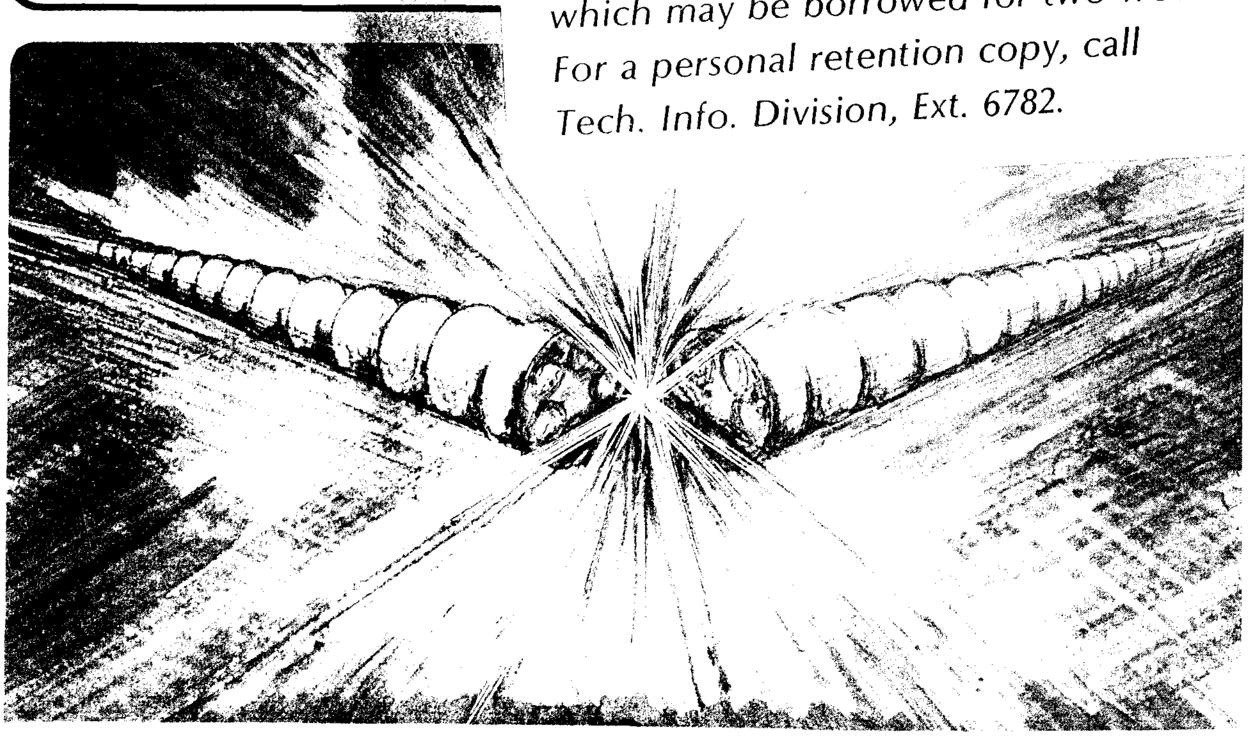

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REPORT OF THE STORAGE RING DESIGN GROUP

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INTRODUCTION

Members of the Storage Ring Design Group were R. Blumberg, K-J. Kim, S. Krinsky, J. Madey, C. Pellegrini, J. Peterson, M. Preger, A. Sessler and A. van Steenbergen.

The group set itself the tasks of (1) agreeing on the basic formulas with which one designs (conceptually) a Free Electron Laser (FEL) ring, (2) making three examples employing these formulas, and (3) studying the performance of these rings under the assumption that they have been built and that there are operating parameters that can be varied.

The study group made good progress on Task No. 1, and the work is described in Section I. We had time under Task No. 2 to produce only one example and this is described in Section II. The group was unable to attack Task No. 3 in the limited time which we had available. However, the subject is of sufficient interest that the two of us, and C. Pellegrini, intend to go on and do a more extensive job on Task No. 2 and some work on Task No. 3, with the thought that out of this survey will come some generally interesting conceptual designs for FEL rings.

Although this group was able to produce only one example, it was a most interesting example. We have shown that a FEL ring designed for 500 Angstroms lasing appears to be within the capabilities of storage ring builders. The consequences of our example is commented upon, a bit, in Section III.

I. BASIC FORMULAS AND CONSTRAINTS

The Working Group discussed at length the question of what set of relationships and constraints should be used to design a storage ring optimized for operation with a free-electron laser. The following collection of relations was taken largely from the papers of Krinsky and Madey at this workshop, but with significant modifications by the Working Group.

IA Storage Ring Formulas

The following formulas pertaining to the storage ring were adopted:

IA1 Emittance

The horizontal emittance is that of a storage ring made up of

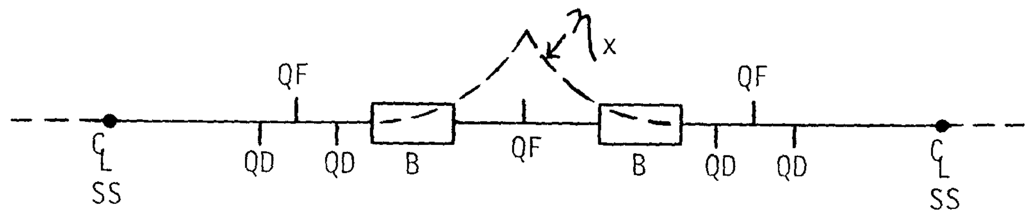

Figure 1. An Achromatic Cell of the Chasman-Green Lattice

M achromatic periods of the Chasman-Green type (Figure 1). FODO-type lattices having the same number of cells generally have considerably larger emittance (at any given beam energy). The minimum horizontal emittance possible in Chasman-Green lattice is:

$$E_{x,min} = 7.7 \times 10^{-13} \frac{\gamma^2}{M^3} \quad \text{(Rad-M)} \tag{1.1}$$

where γ is the electron beam energy in units of the rest mass.

IA2 Compaction Factor (α)

The corresponding compaction factor in a Chasman-Green lattice is:

$$\alpha = \frac{1}{6} \left(\frac{\pi}{M}\right)^2 \frac{\rho}{R} \tag{1.2}$$

where ρ is the bending radius in the bend magnets and R is the average radius (circumference/2π).

IA3 Longitudinal Phase Space

The relation between rms bunch length (σ_s) and rms fractional energy spread (σ_E/E_0) is determined by the dynamics of the RF bucket

$$\frac{\sigma_s}{R} = \left(\frac{\alpha}{\nu_s}\right) \frac{\sigma_E}{E_0} \tag{1.3}$$

where ν_s is the synchrotron tune (number of longitudinal phase oscillations per revolution).

$$\nu_s^2 = \frac{\alpha e h V_{RF} \cos\phi_s}{2\pi E_0} \tag{1.4}$$

where h is the RF harmonic number, V_{RF} the peak RF voltage, ϕ_s the synchronous phase, and E_0 the beam energy.

IA4 The Microwave Instability

At beam-current levels of interest to FEL operation, the energy spread, and, hence, the pulse length and peak current (I_p), are likely to be determined by the microwave instability. The equilibrium condition is:

$$I_p \left| \frac{Z_n}{n} \right| = \frac{2\pi E_0 \alpha}{e} \left(\frac{\sigma_E}{E_0} \right)^2 \tag{1.5}$$

where $|Z_n|$ is the absolute value of the longitudinal impedance seen by the beam at the nth harmonic of the revolution frequency. This impedance is determined by the geometry of the vacuum chamber, the RF cavities, and insertion devices. For a smooth chamber of typical radius b, and

for $\sigma_s > b$; $|Z_n/n| = Z_0$ = constant (typically a few ohms)

and for $\sigma_s < b$; $\left| \frac{Z_n}{n} \right| = Z_0 \left(\frac{\omega}{\omega_c} \right)^{-1.68}$ (for $\omega > \omega_c$) (1.6)

$$\omega_c \equiv 2\pi c/b \equiv \text{"cut-off" frequency} \tag{1.7}$$

(Note added in write-up: If at $\omega > \omega_c$, the peak radiation resistance $(Z_n/n)_{rad} \simeq 300$ b/R ohms is larger than the reactive impedance $Z_0(\omega/\omega_c)^{-1.68}$; then the radiation resistance should be used for $|Z_n/n|$.) If the longitudinal beam distribution is gaussian, the peak current per bunch I_p is related to the average current per bunch I_0 as:

$$I_p = \sqrt{2\pi} \frac{R}{\sigma_s} I_0 \tag{1.8}$$

IA5 Touschek Lifetime

The beam lifetime (τ_T) due to intra-beam scattering that leads to particles being knocked out of the RF bucket is given by the Touschek formula:

$$\tau_T = \frac{\gamma^3 \varepsilon_{RF}^2 \sigma_{x'} V}{\sqrt{\pi}\, r_0^2 c N_e F(x)} \qquad (1.9)$$

where ε_{RF} is the fractional RF bucket half height.

$$\varepsilon_{RF}^2 \cong \frac{2eV_{RF}}{\pi E_0 h\alpha}, \quad (\phi_s << 1) \qquad (1.10)$$

$\sigma_{x'}$ is the rms horizontal angular width.

$$V = 8\pi^{3/2}\sigma_s\sigma_x\sigma_y \quad \text{(bunch volume)} \qquad (1.11)$$

N_e = number of electrons per bunch.

$$x \equiv \left(\frac{\varepsilon_{RF}}{\gamma\sigma_{x'}}\right)^2$$

$$F(x) \simeq \int_1^\infty \frac{dt\, e^{-xt^2}}{t^3}(t^2 - \ln t - 1) \qquad (1.12)$$

at x =	0	10^{-3}	10^{-2}	0.1	1	2	5
$F(x)$ =	3.83	2.30	1.32	0.424	0.023	0.0034	0.000042

IA6 Gas Scattering Lifetime

The beam lifetime due to scattering from the residual gas at a vacuum of p torr is approximately:

$$\tau = \frac{4 \times 10^{-9}\gamma^2 b^2}{\langle\beta\rangle\beta_{max} p} \quad \text{(sec)} \qquad (1.13)$$

where b is the half-height (cm) of the chamber; $\langle\beta\rangle$ is the average value (meters) of the beta function in the plane corresponding to the limiting aperture, and β_{max} the maximum value (meters).

IB FEL Formulas

IB1 Resonance Condition

The photon wavelength λ with which an electron of energy $\gamma m_0 c^2$ will stay in proper phase in a wiggler of wavelength λ_w is:

$$\lambda = \frac{\lambda_w}{2\gamma^2}(1 + K^2) \qquad (1.14)$$

where K is the wiggler parameter

$$\left. \begin{aligned} K &= eB^* \lambda_w / 2\pi m_0 c^2 \\ &= 0.934\ \lambda_w\ (\text{cm})\ B^*\ (\text{T}) \end{aligned} \right\} \qquad (1.15)$$

B* is the rms magnetic field strength (in Teslas).

IB2 Wiggler Strength

The obtainable peak magnetic field B_p (Tesla) in a purely rare-earth-cobalt (REC) wiggler of the Halbach type is related to the full gap g and the wiggler wavelength as:

$$Bp \simeq 1.55\ e^{-\pi g/\lambda_w} \quad (\text{Teslas}) \qquad (1.16)$$

for $0.07 < g/\lambda_w < 0.7$
and for $B_r = 0.9$ Tesla (remanent)

For a hybrid wiggler using the same REC material,

$$Bp \simeq 3.33\ e^{-g/\lambda_w\ (5.47\ -\ 1.8\ g/\lambda_w)} \qquad (1.17)$$

IB3 FEL Gain Formula

At low signal strengths, the gain (G) in a transverse wiggler with N_w periods is:

$$G = 32\sqrt{2}\pi^2 (\lambda^3 \lambda_w)^{1/2} \left[\frac{K^2}{(1 + K^2)^{3/2}} \frac{I_P}{\Sigma I_A} \right] N_w^3 f(\delta) \qquad (1.18)$$

$$\text{where } f(\delta) = \frac{(\cos\delta - 1 + \frac{\delta}{2}\sin\delta)}{\delta^3}$$

$$\delta = \frac{4\pi N_w (\gamma_0 - \gamma_R)}{\gamma_R}$$

γ_0 = the initial electron energy

γ_R = the energy at which equation 1.14 holds for given λ and λ_w

$f(\delta)_{max} \simeq 0.07$ at $\delta = 2.5$

$I_A = 17 \times 10^3$ Amperes (Alfvén Current)

Σ = beam-optical-mode area

$$= \pi \left[\left(\sigma_{xe}^{2} + \sigma_{xL}^{2} \right) \left(\sigma_{ye}^{2} + \sigma_{yL}^{2} \right) \right]^{1/2}$$

where subscript e refers to the electron beam and subscript L refers to the optical mode.

IB4 Diffraction Limit

The minimum value of Σ in a wiggler of length L_W is the diffraction limit:

$$\Sigma \geq \frac{L_W \lambda}{2\sqrt{3}} \tag{1.19}$$

IB5 Energy-Spread Limitation

For a given energy spread (σ_E/E_0) in the electron beam, the proper phase relation between the electrons and the photons is effectively smeared out after N_{eff} periods of the wiggler.

$$\sigma_E/E_0 = \frac{1}{4} N_{eff} \tag{1.20}$$

In order to operate in the high-gain regime, the required value of N_{eff} is defined by

$$G(N_{eff}) \equiv 1.5. \tag{1.21}$$

IB6 Limitations on Transverse Velocities

Because transverse velocities have phase-smearing effects similar to that due to energy spread, the transverse emittances are limited:

$$E_y \leq \frac{1}{4\pi N_{eff}} \left[\frac{\lambda \lambda_W}{2} \frac{(1 + K^2)}{K^2} \right]^{1/2} \tag{1.22}$$

$$\sigma_{x'} \leq \left[\frac{\lambda}{\lambda_W N_{eff}} \right]^{1/2} \tag{1.23}$$

IB7 Wiggler Vertical Focussing

To allow for the vertical focussing due to the wiggler, the vertical betatron function β_y should be limited to

$$\beta_y = \rho_W \tag{1.24}$$

if $2\pi\rho_W << L_W$, where ρ_W is the bending radius corresponding to the peak wiggler field B_p.

II. AN EXAMPLE

In examining the equations of Section I, we see immediately that there are more parameters than equations. Thus, we can choose some of the variables "arbitrarily." Of course, we must choose "reasonable" values for these variables or we will obtain unreasonable values for some of the output parameters. Based upon its past experience, and fully aware that we were not producing an optimized ring, but only an "existence proof" example, the group made a number of arbitrary choices.

We consider a FEL lasing at 500 Angstroms:

$$\lambda = 500 \text{ Angstroms.} \tag{2.1}$$

We took the ring energy as 1 GEV:

$$E = 1 \text{ GeV } (\gamma = 2000). \tag{2.2}$$

We took the wiggler full gap as 1 inch:

$$g = 2.5 \text{ cm.} \tag{2.3}$$

We took the peak RF voltage per turn as 1 million volts:

$$V_{RF} = 1 \text{ MV/turn.} \tag{2.4}$$

We took the ring in race track form and with two 20 meter straight sections. The average arc radius was taken (about) 10 meters:

$$C = 2\pi R = 100 \text{ meters.} \tag{2.5}$$

We took the RF frequency to be 500 MHz, and hence:

$$h = 166. \tag{2.6}$$

The wiggler was taken to pretty much fill up one straight section:

$$L_W = 15 \text{ meters.} \tag{2.7}$$

We took, on the basis of experience with modern rings,

$$|Z_n|/n = 2 \text{ Ohms.} \tag{2.8}$$

This value is small, but should be realizable. Application of this formula for $|Z_n|/n$ is only valid if the bunch length is larger than the gap and we will have to check, later, that this is in fact so.

We took the lattice to have four periods:

$$M = 4 \tag{2.9}$$

The momentum compaction is, now, from equation 1.2:

$$\alpha = 2.57 \times 10^{-2} \tag{2.10}$$

Finally, we took the average current per bunch to be 500 mA which is large, but should be attainable:

$$I_0 = 0.5 \text{ A}. \tag{2.11}$$

From the properties of magnets, equation 1.17, we have

$$\lambda_w = 6.6 \text{ cm},$$

$$B_p = 5.1 \text{ kG}. \tag{2.12}$$

Hence, the radius of curvature of a 1 GeV electron in the wiggler is:

$$\rho_w = 6.4 \text{ meters}. \tag{2.13}$$

Also, trivially,

$$K^2 = 5.0,$$

$$K = 2.24. \tag{2.14}$$

Also, clearly, our wiggler has 326 periods:

$$N_w = 326. \tag{2.15}$$

From the microwave instability, equation 1.5, and the machine properties, equation 1.3, we have:

$$\frac{\sigma_s}{R} = 2.5 \times 10^{-3}. \tag{2.16}$$

Hence, $\sigma_s = 3.9$ cm which is comfortably larger than the zero-current bunch length of 1.0 cm; i.e., there is significant "bunch lengthening." Note, also, that $\sigma_s > g$ so we are self-consistent in using equation 2.8. Now from I_0 and σ_s, we can obtain the peak current:

$$I_p = 510 \text{ A}. \tag{2.17}$$

From the Rayleigh range, equation 1.19, given the length of the wiggler, equation 2.7, and the wavelength of the light, equation 2.1, we have:

$$\sigma_{xL} = \sigma_{yL} = 2.6 \times 10^{-4} \text{ meters.} \qquad (2.18)$$

The horizontal emittance is determined by the lattice, equation 1.1:

$$\mathcal{E}_x = 4.8 \times 10^{-8} \text{ meters.} \qquad (2.19)$$

Taking β_x = 10 meters (in a 20 meter straight section, this is about the best one can do unless there is focusing in the straight section), we have:

$$\sigma_{xe} = 6.9 \times 10^{-4} \text{ meters.} \qquad (2.20)$$

We take the beam vertically equal to the laser light cross section:

$$\sigma_{ye} = \sigma_{yL} = 2.6 \times 10^{-4} \text{ meters.} \qquad (2.21)$$

Now we are in a position to calculate the overlap cross section $\mathbf{\Sigma}$. By equation 1.18:

$$\mathbf{\Sigma} = 8.5 \times 10^{-7} \text{ meters.} \qquad (2.22)$$

At this point, we can check that the Coulomb gas scattering lifetime and the Touschek lifetime are sufficiently long. We find, using equation 1.9, that the Touschek lifetime is 1.0 hour and that the gas scattering lifetime, at a pressure of 10^{-9} Torr, is much longer.

Thus, we have all the ingredients in the gain formula, equation 1.18, and thus, we can determine:

$$N_{eff} = 140. \qquad (2.23)$$

Now we must apply a self-consistency check on the energy spread. From the microwave instability, we need an energy spread

$$\frac{\sigma_E}{E} = \leq 2.0 \times 10^{-3}. \qquad (2.24)$$

On the other hand, for the FEL, the energy spread must be less than 1/4 N_{eff} or

$$\frac{\sigma_E}{E} \leq 1.8 \times 10^{-3}. \qquad (2.25)$$

These two results are in substantial agreement and thus the design is consistent.

The full wiggler, equation 2.15, has a gain which is much larger than unity. The small signal gain is:

$$G = 19. \tag{2.26}$$

Finally, the energy lost to synchrotron radiation is:

$$V_0 = 22 \text{ keV/turn}, \tag{2.27}$$

with a damping time of 15 milliseconds. The radiated power:

$$W_0 = 11 \text{ kW}, \tag{2.28}$$

which means that the laser has a power

$$P_{laser} = 17 \text{ Watts}. \tag{2.29}$$

III. A PROGRAM

Based upon the small amount of work which the group did, it appears possible to design a FEL ring for λ = 500 Angstroms. The electron-beam energy has not yet been optimized, but 1 GeV electrons seem adequate.

Because the performance is such a strong function of wavelength, we feel that such a complex will "easily" work at (say) 750 Angstroms. After some operational experience is obtained, hopefully, one could get to 500 Angstroms.

Because the low harmonics of a FEL can be expected to be significantly excited, one can obtain coherent radiation down to (say) 100 Angstroms. Note, however, that the power in these harmonics will be greatly reduced from that in the fundamental.

All-in-all the group was excited by the prospects of a VUV-FEL ring.

ACKNOWLEDGEMENT

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