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# Review of Laguerre-Gaussian Mode Laser Heater for Microbunching Instability Suppression in Free-Electron Lasers

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**Abstract:** We formulate the mode conversion of Hermite Gaussian (HG) to Laguerre Gaussian (LG) beams as an alternative to generation using spiral phase plates. Additionally, the methods used by the researchers for Microbunching Instability (MBI) suppression to increase Free-Electron Laser (FEL) performance are discussed.

## INTRODUCTION

Free-Electron Lasers (FELs) are essential tools for research within condensed matter physics, chemistry, and structural biology, providing coherent and extremely bright radiation at short wavelengths. To achieve the high current required for FEL operation, magnetic compression is applied to the electron beams within the accelerator. This process often leads to Microbunching Instability (MBI), where density and energy modulations amplify along the beamline, degrading the beam quality and introducing excessive energy spread. These effects reduce the FEL's performance and coherence [1].

To mitigate MBI, laser heaters (LHs) introduce controlled energy spread into the electron beam, suppressing the instability downstream via Landau damping. Researchers at the Linac Coherent Light Source (LCLS) have traditionally utilized a simple Gaussian-shaped laser beam matched to the transverse electron beam shape [1]. However, this configuration frequently results in a two-horn energy distribution, which is less effective for MBI suppression.

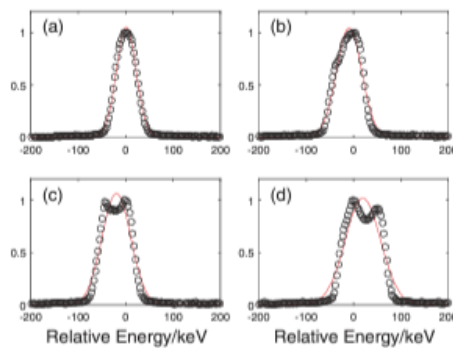


Fig 1. Gaussian fitting based on energy spread at laser heater for LG01 and Gaussian Modes, energy spreads of (a) 20.5, (b) 26.7, (c) 30.1, (d) 37.1 KeV [1]

Exploring alternative laser modes, such as the Laguerre-Gaussian (LG) modes, offers a pathway to better MBI suppression. Specifically, the LG01 mode has shown promise for creating Gaussian-shaped energy distributions, eliminating the inefficiencies associated with the double-horn energy profile. In their experimental setup, researchers converted a Gaussian mode into the LG01 mode using a spiral phase plate (SPP). The SPP imparts a total phase change of  $2\pi$ , creating a null at the beam center and forming the characteristic donut-shaped intensity profile. While effective, the SPP is not the only method for generating LG modes, and alternative approaches such as Hermite-Gaussian (HG) to LG conversion may provide additional flexibility and efficiency.

## METHODS

The study demonstrated that the LG01 laser heater effectively suppressed MBI by avoiding the double-horn energy distribution commonly observed with Gaussian modes. The conversion of the laser heater into the LG01 mode was achieved using an SPP, which transformed the transverse phase profile of the input beam into a helical phase with cylindrical symmetry.

While the SPP is a well-established method for generating LG modes, other methods, such as Hermite-Gaussian to Laguerre-Gaussian mode conversion, can achieve similar results. The HG modes, characterized by their Cartesian symmetry, can be mathematically decomposed into LG modes, which exhibit cylindrical symmetry. This conversion can be facilitated using optical systems such as cylindrical lenses or spatial light modulators (SLMs), providing additional flexibility in beam shaping.

Hermite-Gaussian (HG) modes are solutions to the paraxial wave equation in Cartesian coordinates and are defined as:

$$\text{HG}_{m,n}(x, y, z) = H_m\left(\frac{\sqrt{2}x}{w(z)}\right) H_n\left(\frac{\sqrt{2}y}{w(z)}\right) e^{-\frac{x^2+y^2}{w^2(z)}} e^{i\Phi(x,y,z)}$$

Where  $H_m$  and  $H_n$  are Hermite polynomials,  $w(z)$  is the beam waist, and  $\Phi(x,y,z)$  includes phase terms like the Gouy phase [2]. Then we describe Laguerre-Gaussian modes slightly differently, this time in cylindrical coordinates to give:

$$\text{LG}_{p,l}(r, \theta, z) = \sqrt{\frac{2p!}{\pi(|l| + p)!}} \left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|} L_p^{|l|}\left(\frac{2r^2}{w^2(z)}\right) e^{-\frac{r^2}{w^2(z)}} e^{i(l\theta)} e^{i\Phi(r,\theta,z)}$$

Where  $L_p$  represents generalized polynomials, and  $l$  is the azimuthal index associated with orbital angular momentum [3].

To illustrate the conversion, consider the first-order Hermite-Gaussian mode:

$$\text{HG}_{1,0}(x, y) = 2xe^{-\frac{x^2+y^2}{w^2}}$$

This is the first Hermite-Gaussian mode that can be manipulated to reach the desired beam mode. By introducing the phase shift:

$$\theta(x, y) = \arctan\left(\frac{y}{x}\right)$$

The resulting electric field becomes:

$$E'(x, y) = \text{HG}_{1,0}(x, y)e^{i\theta(x,y)}$$

When represented in cylindrical coordinates this becomes:

$$E'(r, \theta) = re^{-\frac{r^2}{w^2}} e^{i\theta}$$

And corresponds to the first Laguerre-Gaussian mode  $\text{LG}_{0,1}$  which is characterized by its helical phase and donut shaped intensity profile. Researchers have found that the interconversion

between HG and LG can be done with a cross-phase lens under certain conditions (with  $\theta = 0$  and  $\mu = \pm 2/w^2$ ) to change the optical field.

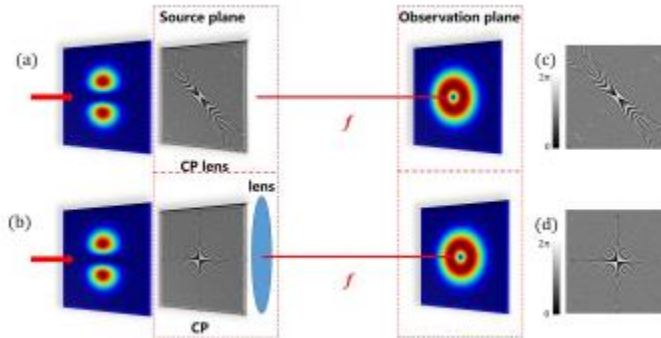


Fig 2. Schematic diagram of a Cross Phase (CP) lens system [2]

With these results and mathematical intuition, there are many methods for forming the desired intensity beam and mode, including the SPP used by researchers at LCLS.

## RESULTS AND INTERPRETATION

This study shows that the Laguerre-Gaussian (LG<sub>0,1</sub>) mode effectively suppresses Microbunching Instability (MBI) in Free-Electron Lasers (FELs). Its donut-shaped intensity profile creates a Gaussian-like energy distribution, avoiding the double-horn profile of traditional Gaussian modes. This improves FEL coherence and reduces MBI through enhanced Landau damping.

The research validates Hermite-Gaussian (HG) to LG mode conversion as a flexible alternative to spiral phase plates. Using cylindrical lenses, spatial light modulators, or cross-phase lens systems, this method reliably produces LG<sub>0,1</sub> beams with helical phase structure and simplified experimental setups. These findings confirm the practical and theoretical feasibility of alternative beam shaping techniques for MBI suppression.

## CONCLUSIONS

This work has significant implications for the future of FELs and other high-energy laser applications. By demonstrating that LG<sub>0,1</sub> modes improve MBI suppression and beam coherence, the study paves the way for more efficient FEL designs. The flexibility of Hermite-Gaussian to LG mode conversion offers opportunities for further optimization, potentially enabling higher-order modes tailored to specific operational needs.

Beyond FELs, this research contributes to advancing technologies in optical trapping, particle accelerators, and laser-material interactions. The alternative mode-shaping methods introduced here, such as CP lenses, promise adaptable solutions for various high-intensity laser applications. These findings lay the groundwork for future innovation in tailored beam shaping and energy distribution control, addressing challenges across multiple fields.

for tailored beam designs and innovations in laser heating technologies, cementing its significance in advancing the field of high-intensity laser applications.

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