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Authors
Hochman, Oded
Rausser, Gordon C.

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DIVISION OF AGRICULTURE AND NATURAL RESOURCES
UNIVERSITY OF CALIFORNIA AT BERKELEY

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ZONING AS A CONTROL OF POLLUTION IN A SPATIAL ENVIRONMENT

by

Oded Hochman and Gordon C. Rausser

California Agricultural Experiment Station
Giannini Foundation of Agricultural Economics
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Oded Hochman,[ǂ] Gordon C. Rausser[+]

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Abstract

Space matters not only because of the transportation costs it imposes on the economy but also because it can serve as an effective instrument to control pollution damages. Previous models of pollution either disregard space altogether or presume a predetermined separation between polluters and pollutees, usually into a CBD where the polluting industry is located and a residential ring where the city's laborers reside. All possible location combinations of housing and industry are considered in this study. The results demonstrate that the management of pollution must recognize the trade-off between two cost components: pollution costs and transportation costs. This trade-off along with the non-convexity inherent in spatial models results in multiple local optima. Negligible commuting costs combined with pollution emissions bearing ill effects at a rate declining with distance leads to an allocation with one industrial zone and one residential zone. As commuting costs increase, the optimal allocation passes through an endogenously determined series of increasing thresholds. Each time a threshold is crossed the number of zones of each type increases until the internal solution is reached after the final threshold has been crossed. In the internal solution, there is no commuting, and housing and industry assume adjacent locations. In such an economy, Pigouvian taxes are generally inefficient. Instead, the efficient tax is a per unit land tax equal to the additional damages contributed by that land unit's pollution.

Key Words: Location, Space, Pigouvian Corrective Taxes, Land, Non-convexities, Bid Rent Function, Threshold, Regulations.

JEL Classification: R13, H23.

Correspondence: Oded Hochman; Econ. Dept., Ben Gurion U., Beer Sheva, 84105 Israel
E-MAIL: oded@bgumail.bgu.ac.il Phone: 972-7-6472311 FAX: 972-7-6472310

ǂ Department of Economics and MCER, Ben Gurion University, Beer Sheva, Israel.

+ Dean, College of Natural Resources, Robert Gordon Sproul Distinguished Professor, Professor, Department of Agricultural and Resource Economics, and Member, Giannini Foundation of Agricultural Economics, University of California at Berkeley.

I. Introduction

Space matters not just because of the transportation costs incurred by land users, but also because it can and should be utilized as a means of controlling pollution. Pollution is defined as the external negative effect a particular land use has on other types of land use. Generally, separating polluter and pollutee reduces pollution damages but leads to increased commuting costs. Accordingly, an obvious trade-off arises between pollution damages and commuting costs; when commuting costs are sufficiently high it is uneconomic to control pollution damages by physical separation of the polluter and pollutee. Similarly, as the damages from pollution rise relative to commuting costs, increasing separation is justified.

Congestion is defined as the negative external effect a particular land use by one agent has on other agents or participants in the same land use. Space cannot have the same role in controlling congestion as pollution since both damaging and damaged parties coincide. Pollution, along with congestion, are the two major types of local externalities recognized in the literature. While both pollution and congestion effects have been extensively examined (e.g., Solow and Vickery, Mills and deFeranti, Arnott (1979b), Starrett, Tietenberg, Henderson, Baumol, Baumol and Oates, Coase, Hochman Pines and Zilberman, and others), the problem of pollution externalities has been inadequately and in some instances incorrectly analyzed. The purpose of this paper is to characterize the optimal resource allocation and joint location of a polluting industry and housing in which employees and laborers of industry reside.

In the existing literature on spatial pollution (Henderson 1977, 1985, 1996, Hochman and Ofek 1979, Baumol and Oates, Tietenberg 1974a and b,1978), separation between residential pollutees and polluting producers is taken as given and a single zone for each type of use is presumed. In the case of pollution, the issue of endogenous separation between types of land uses has never been investigated. Only recently has the endogenous location of a single activity in more than one zone been analyzed (e.g., Fujita and Ogawa), without, however, the consideration of pollution.

Certainly, many possible combinations of industry and residential locations exist that could be optimal. For example, the number of consecutive sets of residential and industrial zones can be greater than one, buffer zones can exist between industrial and residential zones thus separating residential pollutees and polluting producers, or residential and industrial land uses can be mixed. Aside from the recommended use of location differentiated pollution taxes (see Titenberg 1978, Seskin et al. 1983, Kolstat 1987), zoning has not been analyzed as an endogenous instrument for controlling pollution.

Even without any externalities, the inherent non-convexities in spatial models may lead to zoning and multiple optima. For example, in the basic Muthian city model there are two optima: one is the well known solution with residents/workers commuting from the residential zone to the industrial zone and back, and the second is the optimal internal solution in which there is no zonal distinction between residents and industry and there is no commuting see Mills (Ch.5). The second solution is the global optimum when commuting costs exceed a certain threshold level. Mills is one of very few who modeled the well known empirical fact that there are areas in which industry and housing are located in the same place. More recently, Dipasquale and Wheaton (Ch. 5) investigate this case empirically as well.

In the model advanced in this paper, the presence of pollution can lead to an infinite number of local optima. To eliminate any assumptions which are not essential for isolating the role of zoning for pollution control we follow Solow and Vickery by specifying a city without a predetermined center. We also assume a constant returns to scale production function and thus production processes will not be the source of any endogenous separation of housing and industry. Therefore, zoning will not emerge if ill effects of pollution do not exist. A ring-shaped city is also specified to avoid dealing with edge-of-city-effects. Accordingly, if pollution does not exist, a uniform layout of the city emerges from the specified model in which the work place and the household residences are adjacent in the same location.

The results of our model show that while there is only one global optimum for every specified level of commuting costs (except perhaps for a set of zero measures), there can be an infinite number of local optimum. When commuting costs are very low, maximum separation between polluters and pollutees is the optimal policy. Depending on the parameters of the system, empty buffer zones may exist between the occupied zones of industry and household residence. As commuting costs rise above a certain threshold level, the global optimum changes to an allocation of more than a single industrial zone and a single residential zone. As such costs rise, this process continues until a final threshold is reached above which the global optimum is a uniform allocation of mixed residential and industrial land uses without commuting.

Along with the endogenous land use patterns and zoning corner solutions, our model demonstrates that spatially differentiated Pigouvian taxes per unit emission levied on industrial polluters will not generally support the optimum either in the short or the long run. Only if the dispersion function is linear in emissions or if locations are predetermined and fixed and the dispersion function is convex in emissions, independent of whether the solution includes zoning or not, will the typical Pigouvian taxes offered in the literature (Baumol and Oates, Spulber) be optimal. Initially, Henderson (1977) showed the insufficiency of Pigouvian taxes, proposing an additional lump sum tax along with the Pigouvian tax. However, Hochman and Ofek proved that the correct tax levied on each unit of industrial land must equal the spatial aggregate of added damages contributed by that unit of land. In a non-spatial model, Polinsky demonstrated the failure of the Pigouvian tax and also derived a tax equal to the added damages caused by a firm. In this paper, we show that a spatially differentiated added-damages-tax is also sufficient to achieve the optimal number of zones.

In the following section the formal spatial model is specified. From the specification of the model, we derive the optimal and decentralized solutions in Section 3. To gain insight and intuitive understanding we characterize a number of special cases by the use of bid rent analysis in Section 4. Section 5 characterizes the general zoning local optima solutions based on the interpretation of the special cases and Section 6 describes the global optimum. Implications for effective pollution control are examined together with a few concluding remarks in Section 7.

2. Model Specification

Assume a ring-shaped featureless strip of land of unit width. Let L be the circumference of the inner circle located at equal distance from the two boundary circles of the ring (see Fig. 1). As a result, L is also the total area of the ring. We use this inner circle as the location axis in the ring. An arbitrary point O on this axis is chosen as the origin. The distance from the origin is designated by x and clockwise as the positive direction; $x=0$ and $x = L$ are the two coordinates of the origin and $0 \leq x \leq L$. As previously noted, a ring-shaped city avoids the edge-of-city-effects and thus allows us to concentrate on the interior structure of the city. We also impose the standard assumption by which commuting costs accrue only when traveling along the circumference of the ring and no horizontal traveling costs are incurred (Solow and Vickery).

For industrial output, a linear homogeneous production function is specified. The relevant inputs are: $a(x)$ = the fraction of land occupied by the industry at x ; $n(x)$ = number of workers per unit of industrial land at x; and $e(x)$ = amount of emissions of the industry per unit of industrial land at x. The production function is assumed to be an increasing function of these inputs, but at a decreasing rate. In addition $F(a,an,ae) \overset{\text{def}}{\equiv} af(n,e)$ is a constant returns to scale

(CES) function in inputs *a, an* and *ae*. Note that $f(\cdot)$ as such fulfills,

$f(\lambda n, \lambda e) < \lambda f(n,e)\ for\ \lambda \geq 1$.

A linear homogeneous production function along with a fixed and predetermined city size *L* allows pollution effects to be easily identified under varying conditions. The layout of the city under the constant returns to scale without pollution is simple and straightforward; hence, any deviation from this simple pattern when pollution is introduced is easily detectable and entirely due to its effects.

Free and costless population mobility in the economy as a whole is presumed, including mobility within the presumed city. Thus, utility level U_o must be fixed everywhere in the economy as well as in all residential locations within the city. It should be noted that mobility means moving from one residential location to another and it does not refer to commuting. Thus,

$$(1) \quad U[h(x),\ z(x),\ c(x)] = U_o$$

where

$U(\cdot)$ = the utility function

h(x) = the amount of housing consumed by a household at *x*

z(x) = the amount of composite good consumed by a household at *x* (the price of *z* is assumed to be a unit)

c(x) = the concentration of pollution at *x* which results from emissions of the industry throughout the city.

The utility function $U(\cdot)$ is assumed to be quasi concave in *h, z,* and *(-c)*.

Concentrations of pollution at location *x* are linked to industrial production activities through the generation of pollution emissions at location *y,* viz., *e (y)*. This linkage is specified by dispersion functions, $D[e(y), |y-x|]$, which convert contamination emissions at *y* per unit of land, *e (y),* to its contributor to pollution concentrations at *x* (Tietenberg (1974a, 1974b)).[1] Two different functions are specified to allow for the possibility of different dispersion effects depending upon the direction (North verses South) from the emission site. The positive direction of *x* is defined as north (N) and the opposite direction as south (S). Pollution concentrations at location *x* are determined by the emissions in all locations *y*, viz.

$$(2) \quad c(x) = \int_{x-L/2}^{x} a(y) D^N[e(y), x-y]dy + \int_{x}^{x+L/2} a(y) D^S[e(y), y-x]dy$$

The functions $D^i(e,y), i = S$ or *N*, have the following properties for positive pollution emissions *e* and distance *y*.

$$(3) \quad \frac{\partial D^i(e,y)}{\partial e} = D_1^i > 0^2, \quad \frac{\partial D^i(e,y)}{\partial y} = D_2^i < 0, \quad D^N(e,0) = D^S(e,0)$$

Note also that $D(e,y) = 0$, for all $y \geq L/2, \text{ or } y < 0$ prevents the same emissions affecting the same location more than once. The requirement $D^N(e,0) = D^S(e,0)$ prevents discontinuity at the emission source.[3] Instead of housing we assume *h(x)* to be the amount of

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[1] It should be noted that *D* is a function of the density of emissions and to capture the total contributions of a given location *x* to concentrations at *y* requires multiplication by *a(x)*. This makes concentrations additive across space in the direction of width and thus is the same as the additivity of integration specified in the direction of *x* (length). We thank an anonymous reviewer for pointing out the need of this assumption to achieve consistency.

[2] A subscripted function indicates the differential of the function with respect to the variable whose order is indicated by the subscript. Thus $\frac{\partial^2 D^i}{\partial e^2} = D_{11}^i$ and $\frac{\partial^2 D^i}{\partial y^2} = D_{22}^i$.

[3] As shown in Arrow, et. al. and references contained therein, economies or diseconomies of scale can exist in the assimilative powers of the environment when the density of concentrations at a given location gets closer to, or further away from a breakdown point of biological systems. This means that concentration at a given location is not just the addition of contributions from different sources, as implied from the use of integration in equation (2), but is a complex process of concentration and emissions levels at different locations. Indeed regulatory agencies have been employing complex nonlinear simulation models to represent the emission/dispersion process (see, for example, Allegrini and De Santis, 1996 and the NTIS, US Department of Commerce, 1997). Here we use a simplifying additivity assumption in the form of an integral in (2). We could not solve the model analytically without making this simplifying assumption. An assumption of the same nature would be that dispersion is a linear function of *e(x)*, the density of emissions at *x*, i.e., $D_{11} = 0$. However, in this case we can solve the model even if we assume more realistically that $D_{11} \neq 0$ (note that both Tietenberg (1974) and Henderson (1977) made the assumption of non linearity of the dispersion function). As we shall see in what follows assuming non linearity of *D*, if only with respect to emissions, has important policy implications. In view of our simplifying assumptions we shall discuss the robustness of these implications as well. Since according to Arrow et al., there are several biological systems, we assume D_{11} can change its sign as *e(x) increases* and be positive in parts of the emissions domain and in the rest of it negative. It is however, rarely zero.

land occupied by the household as if it consumes land only. This is a standard simplifying assumption in the urban economics literature which in most models, including ours, does not affect the outcome but saves complex calculations. Commuting costs are determined, in part, by the number of commuters, *T(x),* traveling from home northward crossing *x* minus the number of workers crossing *x* traveling from home to work southward. This addition to $T(x)$ by moving marginally north at location *x* is the number of residents at *x, b(x)/h(x)* minus the number of workers, *a(x)n(x)*, where *b(x)* is the proportion of land at *x* occupied by housing. Specifically,

(4a) $\dot{T}(x) = \frac{b(x)}{h(x)} - a(x)n(x)$,[4]

with

(4b) $T(0) = T(L)$ and

(4c) $T(0)=0$

The equality in equation (4b) implies that the total number of households in the city also equals the total number of city workers, with each household contributing a single worker to the labor force. The equality in equation (4c) forces the origin to be a point not crossed by commuters, i.e., commuters either both reside and work north or south of the origin, and thus do not cross the origin when commuting. This does not cause any loss of generality since, as shown later, every solution has at least two points commuters do not cross and equation (4b) implies that one of these points will be the origin. Accordingly, *T(x)* also equals the number of households minus the number of workers south of *x* and north of the origin and it can be either positive, negative or zero. Equations (4a) and (4c) also imply that:

(4d) $T(x) = \int_0^x \left[\frac{b(y)}{h(y)} - a(y)n(y) \right] dy$,

Operationally, the sign of D_{22} depends on the geography and weather conditions and can be either negative, positive or zero.

[4] A dot above the function indicates differentiation with respect to distance.

Since the optimum implies the minimum of total travel costs, crossing at the same point in opposite directions is ruled out. Hence, the gross and net crossings must be equal. Accordingly, *T(x)* is also the total number of commuters crossing x daily; all cross northbound if *T(x)* is positive and southbound if it is negative.

The relevant land-utilization constraints are:

(5a) $a(x)+b(x)-1\leq 0$ (5b) $a(x)\geq 0 \quad b(x)\geq 0.$

When these constraints are not binding, it means that at least some land at x is vacant. Moreover, the Solow and Vickrey specification of a city surrounded by an interstate highway is presumed so that differences in shipping costs of the city's export good due to differences in relative locations in the city are negligible. This means that the f.o.b. price of the export good in the city is independent of x, the mill's location in the city. If this specification is not imposed, separate industrial and residential zones may arise even without pollution.[5] The parameter P represents this given constant f.o.b. price of the city's export good. Finally, V will represent the cost of a single worker commuting a unit distance.

Given the above definitions, the net city surplus is given by

$$S=\int_0^L\left[P\cdot a\cdot f(n,e)+\frac{b}{h}(Y-z)-|T|\cdot V\right]dx \tag{6}$$

where Y, the non-earned income of a household, is independent of location.[6] Maximization of S in (6), subject to (1), (2), (4a), and (5) with (4b) and (4c) as terminal conditions, provides the necessary and sufficient conditions for local efficiency which is part of the necessary conditions for Pareto optimum for the economy as a whole (see Hochman, 1981). S may be interpreted as the net gains of a profit maximizing developer who owns the city and who provides workers with

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[5]Indeed, traditionally the Muthian model of a monocentric city assumes a central location in the city to which all products have to be shipped, thus making the product f.o.b. price differ from one location to another. These added transport costs may lead to separate designations for an industrial zone (CBD) and a residential zone. In such a model it is extremely difficult to distinguish between the pollution zoning effect and the transportation zoning effect. In order to avoid obscuring these separate effects, for the model presented in this paper zoning will not occur when there is no pollution.

[6] Note that for simplicity of notation, the variable x is omitted whenever there is no risk of confusion.

sufficient housing and composite consumption goods to sustain their predetermined utility level. In return she receives the residents' unearned income and their labor services. If the developer is a public company for which every household in the economy owns an equal share, the sum of all the S's of cities in the economy becomes the source of the unearned income in the economy. With free entry of cities, provided there is no shortage of land, S is driven to zero and each household's utility level is maximized.

To facilitate solving this problem, we define the function

$$(7) \qquad sign\ (x) = \begin{cases} +1 \ iff\ x>0 \\ 0 \ iff\ x=0 \\ -1 \ iff\ x<0 \end{cases}$$

Sign(x) is differentiable and its derivative equals zero everywhere except at $x = 0$ where the derivative is not defined. The function *sign* can be used to enable differentiation of $|T(x)|$ everywhere except at *zero*, i.e.,

$$(8) \quad |T(x)| = [sign\ (T(x))] \cdot T(x)$$

3. The Local Optimum Solution and its Supporting Price System

The necessary conditions for the resulting maximization problem are derived in Hochman and Rausser (Appendix A).

Definition 3.1: *A local maximum solution is an allocation which satisfies the necessary and sufficient conditions of the above maximization problem, and attains the highest value of the goal function in a sufficiently small neighborhood of the optimum contained in the initial domain of the problem.*

In our case the maximization problem is specified in the previous section. Since we are dealing with a non-convex problem there might be more than one such local optimum, even an infinite number of local optima. Our definition states that if we change slightly the variables of a local optimum, the value of the goal function will decrease.

Definition 3.2. *A global maximum solution is the local maximum solution with the highest value of the goal function S.*

It may occur that there are several global optimum solutions, all having the same value of *S*, in which case we are indifferent to the different global optima.

According to the Second Welfare Theorem each efficient allocation (Pareto Optimum) has a market equilibrium solution which yields the same allocation. We term this solution the *supporting market equilibrium* of the (Pareto) optimal allocation, and the price system in it as the *supporting price system*. In the presence of externalities, as is the case here, to achieve the optimum via market equilibrium a local government intervention in the form of corrective taxes or subsidies and/or regulations is required as well. In this and the following sections, together with the optimum, we investigate this supporting price system, which we also refer to as the market allocation, equilibrium, etc. Note that different allocations have different supporting price systems. In what follows, unless specified otherwise, we investigate *only allocations* which are locally optimal and their supporting price systems.

Let $\psi(x)$ be the costate of the commuter equation (4a). From the necessary and sufficient conditions for a local optimum,[7] employment is determined by the typical marginal productivity of labor equality to $\psi(x)$,

(9) $a(x)\{Pf_1[n(x),e(x)]-\psi(x)\}=0$,

The fact that *a(x)* multiplies the expression in (9) means that the equality of the expression in the brackets must hold only where industry is located (not necessarily exclusively).

[7] Khun-Tucker rules apply also for corner solutions of problems with non convexities. The corner solutions are of special

Choosing optimally the number of commuters yields

(10) $\dot{\psi}(x) = sign[T(x)]$.

The function $sign[T(x)]$ is constant and continuous as long as $T(x)$ does not change its sign. Therefore, along a segment where the sign of $T(x)$ remains constant, (10) indicates that $\psi(x)$ is a linear function of x and increases by V per unit distance in the direction of commuting.

<u>Definition 3.3</u>: *Let w(x) be the local net earnings (LNE) at location x in the supporting market solution. In a location where an industry is located, w(x) is the wage rate, and in a location where there is no industry, w(x) is the wage rate where the household works minus commuting cost to the work place.*

It follows that $w(x)=\psi(x)$ and that $w(x)$ is well defined. If the industry operates at x, equation (9) implies that $w(x)$ is the wage rate at x. However, if the industry does not operate at x, then *T(x) does not vanish* and equation (10) implies that the wages actually received by the residents at x exceed $\psi(x)$ by the amount of commuting costs to their workplace.

The next condition states that the marginal rate of substitution between housing and the composite good must never exceed $\rho(x)$, the shadow price of the land constraint (5a),

(11) $U_h / U_z = \rho(x) - \gamma_b(x); \ \gamma_b(x) \geq 0, \ \gamma_b(x) b(x) = 0$

and will be equal to this shadow value in residential areas. The slack γ_b is the shadow price of $b(x)$ in (5b). If we let $r(x)$ be the land rent in the supporting price system, it is quite clear from (11) that at least in residential areas $r(x)=\rho(x)$.

interest here because of their correspondence with the zoning solutions.

For the next condition, the expression in the brackets must be zero in residential areas. This expression is the household's budget at x and accordingly this condition implies that the budget constraint must equal zero in locations where $b(x)$ is positive, i.e., where residents live.

$$(12)\quad b(x)\left[Y+\psi(x)-z(x)-\frac{U_h}{U_z}h(x)\right]=0$$

For the remaining argument of the utility function (c), the relevant condition is

$$(13)\qquad \eta(x)=-\frac{\mathrm{b(x)}}{\mathrm{h(x)}}\frac{U_c(x)}{U_z(x)}$$

where $\eta(x)$ is the shadow price associated with pollution concentrations, (2) (and thus can be interpreted as the marginal damage of pollution concentration at location x), $b(x)/h(x)$ is the population residing at x, and $U_C(x)/U_Z(x)$ is the marginal rate of substitution between the composite good and pollution concentrations. In the case of pollution emissions,

$$(14a)\qquad a(x)[Pf_2-M(x)]=0$$

where

$$(14b)\quad M(x)=\int_x^{x+L/2}\eta(y)D_1^N[e(x),y-x]dy+\int_{x-L/2}^{x}\eta(y)D_1^S[e(x),x-y]dy.$$

is the marginal damages of pollution emitted at x. An increase of pollution emitted at x augments concentrations at y by $D_1^i[e(x),|y-x|]$, i=N or S. When these concentrations are multiplied by $\eta(y)$, the marginal damage caused by a unit concentration, and summed over all possible y, we obtain $M(x)$. In the supporting price system, $M(x)$ is equal to the well-known differential Pigouvian tax at x, and when it is levied on every unit of emissions at x, we obtain condition (14a).

For industrial land utilization, *a(x),* the residual condition emerges

(15a) $Pf[n(x),e(x)] - \psi(x)n(x) - Q(x) = \rho(x) - \gamma_a(x) \qquad \gamma_a \geq 0, \gamma_a a(x) = 0$

where

(15b) $Q(x) = \int_x^{x+L/2} \eta(y) D^N[e(x), y-x]dy + \int_{x-L/2}^{x} \eta(y) D^S[e(x), x-y]dy$

is the additional damages caused by the total emissions from a unit land at *x*. The first term of the left hand side (LHS) of (15a) stands for total industrial revenue per unit of land at *x* and the second is the total wage bill per unit land at *x*. The first term on the right hand side (RHS) of (15a) is the shadow price of the land utilization constraint (5a). The last term is the shadow price of industrial land utilization and its existence requires that the left hand side of (15a) will not exceed $\rho(x)$, which in turn must fulfill:

(16) $\rho(x) \geq 0; \; \rho(x)[1 - a(x) - b(x)] = 0$

If in the supporting equilibrium we levy *Q(x)* as a tax per unit of industrial land instead of the Pigouvian tax, $\rho(x)$ will be equal to the land rent *r(x).* The problem is that in order to satisfy (14a), *M(x)* must be levied as a per unit emission tax and only when the dispersion functions are linear is the tax burden the same in the two cases. The following proposition, however, resolves the relevant distinction.

Proposition 3.4: *To achieve efficiency in a market economy by taxing pollution emissions, a tax per unit of industrial land must be levied at every location x where the industry exists. This tax must be equal to the added damages caused by the pollution emissions from this unit of land, viz. Q(x).*

Proof: If an industrial producer pays *Q(x)* for emitting *e(x)* per unit land, wages of *w(x),* and land rent of *r(x)*, a long run equilibrium with zero profits will satisfy both (14) and (15) since

$$\frac{\partial Q(x)}{\partial(e(x))} = M(e(x)), \text{ QED.}$$

Corollary 3.4.1:[8] *Each local and each global optimal solution has a supporting equilibrium solution with its own price system and corrective pollution taxes.*

In what follows we shall use the supporting price system and the supporting equilibrium relations together with optimum relations to characterize the optimal local optimum. The key elements of the supporting equilibrium can be structured by both industrial and residential bid rent functions at the optimum.[9] Specifically,

Definition 3.5: Let R_I, *the bid rent function of the industry, be defined for all x by*

(17) $R_I(x) = Pf[n(x), e(x)] - \psi(x)n(x) - Q(x)$,

where n(x), e(x), $\psi(x)$ *and Q(x) are evaluated at the optimum.*

This bid rent equation follows from (15) and is the maximum amount the industry can pay for land at *x* without suffering losses (provided all variables are optimal and *Q(x)* in (15b) is imposed as a tax per unit of industrial land at every *x*).

Definition 3.6: Let $R_h(x)$ *be the household's bid rent function for land at x. It is defined to satisfy, for every x, given c(x),* $Y + \psi$ *(x), and* U_0,

(18) $$R_h(x) = \frac{1}{h(x)}(Y + \psi(x) - z(x)) = \frac{U_h(x)}{U_z(x)},$$

where the last equality in (18) holds in the optimum only where b(x)>0 and follows from (11).

[8] Another interesting equilibrium solution is the *laissez faire* allocation in which the government does not interfere with the market allocation and therefore no corrective taxes or regulations of any kind are imposed. This allocation is definitely not optimal. In this case, industry will always locate next to housing to avoid commuting costs, not realizing that by so locating it causes wage increases to compensate for added pollution damages. Thus there is no commuting and the marginal productivity of emissions is zero. Note that this allocation is the only one in this paper not related to a local optimum allocation and it is only mentioned in this footnote.

[9] Allonso (1964) first used these functions in depicting an equilibrium.

As with $R_I(x)$, in the supporting competitive solution $R_h(x)$ is the maximum amount households are willing to pay per unit of housing (land).

Using equations (15) and (17) we obtain

(19) $R_I(x) \le r(x), R_I = r(x) \Leftrightarrow a(x) > 0$.

From equations (11), (12) and (18), note also that

(20) $R_h(x) \le r(x); R_h(x) = r(x) \Leftrightarrow b(x) > 0$.

Equations (19) and (20) actually imply that an activity (of production or consumption) will take place at a given location if, and only if, its bid rent function equals the land rent. Finally, from (19) and (20) the land rent *r(x)* can be determined by:

(21) $r(x) = \max\left[0, R^h(x), R^I(x)\right]$.

The above definitions and relations imply the following bid rent rule (BRR),

<u>*Lemma 3.7. (Bid Rent Rule):*[10]</u> *Consider two non negative bid rents in a local optimum solution, one for housing and one for industry, that intersect at a location x. At the point of intersection, the land use with the larger derivative (with respect to x) of its bid rent function is located only north of the intersection point and the other land use is located only south. If the two derivatives are equal at the point of intersection, the two bid rent functions coincide at a neighborhood of this point and housing and industry coexist in this neighborhood. If the two bid rents do not intersect, but instead both are non positive between two different locations, then the area between these two points is an empty buffer zone.*

We can now demonstrate the following corollary.

<u>*Corollary 3.4.2:*</u> *Pigouvian taxes are distortive when* $D_{11} \neq 0$.

[10] See proof in Hochman and Rausser, Appendix B.

We first investigate the case of D_{11} <0, then necessarily $Q > eM$.. This, in turn, implies that levying an emission tax *M(x)* on the industry is insufficient to maintain the optimum. For a solution without zoning, in which industry and housing are located next to each other, when the Pigouvian tax *eM* replaces the optimal tax *Q,* the industry's bid rent function rises above its optimal value which in turn causes *r*, the land rent, to be too high as well. Since *r*, a supporting price, is higher than its optimal value, the allocation supported will not be optimal. However if the optimal proportion of the two land uses are imposed everywhere, the Pigouvian taxes support the optimum. Nevertheless this allocation can hardly be considered an equilibrium, not even in the short run, since at the same location housing and industry have different rents.

Consider the same situation of under-taxation in a zoning solution, i.e., the Pigouvian tax *eM* replaces *Q,* in an area where only industry is located and housing is located in a separate area. Once again, industry's bid rent function and hence land rents, are higher than their optimal values, but now only in the industrial area. This leads to an industrial zone larger than its optimal size. The residential zone is smaller and more heavily polluted than its optimal counterpart. From the restriction of the industrial zone to its optimal boundaries and in addition levying the Pigouvian taxes, the resulting decentralized solution will yield the optimum allocation. In this situation, over the short run when locations are fixed, Pigouvian taxes are optimal.[11]

[11] Henderson (1978) has shown that in a spatial setting over the short run, the period in which the location of activities is fixed, the Pigouvian taxes are efficient when the dispersion function is weakly convex in emissions $(D_{11} \leq 0)$. In a non spatial model Spulber (1985) and Baumol and Oates (1975) have shown that Pigouvian taxes provide the proper incentive for firms to produce the optimal output in the short run by using the optimal mix of inputs. Spulber has also argued that when the damage function is convex in emissions, Pigouvian taxes provide the proper incentives for entry and exit of firms in the long run. However, Pigouvian taxes fail to achieve efficiency in our spatial framework because the generated externality does not cause the actual damages. The emissions are the direct external effects of the production process, but what causes the damages are concentrations. Concentrations are created by emissions from different sources via non-linear (dispersion) functions. It is clear from equations (15) that if $\eta(y)$ could be levied as a tax per unit of concentration contributed by the firm, efficiency would be attained. This means that Pigouvian taxes are efficient when levied on concentrations rather than on emissions. However, producers create emissions, and only when the relation between emissions and concentrations is linear can optimal taxes on emissions be Pigouvian. Accordingly, in order for Pigouvian taxes to be effective, we need the accumulation process of concentrations from different sources to be additive in emissions,

When $D_{11}>0$, eM>Q, the industry's bid rent function is higher under the optimal supporting value than under the Pigouvian tax. Hence, so is the land rent *r*. Once again this implies that the price system resulting from the Pigouvian tax rate does not sustain the optimal allocation either in the case of both land uses coexisting in the same location or in the case of zoning. In both cases when $D_{11}>0$ the industry under Pigouvian taxes will occupy less land, produce less output and pay lower wages. Under these circumstances a short run zoning solution is inefficient with Pigouvian taxes.[12]

In what follows we discuss the robustness of the per unit land corrective tax *Q(x)* introduced in Proposition 3.4. What we tax is determined in this model by equation (15) to be a unit of land. This results from the assumptions of constant returns to scale (CRS) in production and the additivity of the dispersion function (DF) (see footnote 3). It seems that in general we should levy the tax not necessarily on a unit of land, and if our assumptions differ (e.g. external scale economies and lack of additivity of the DF) and lead to the formation of firms in the city, then the unit to tax will be a firm. In such a case the corrective tax will still be equal to the pollution damages added by the firm, i.e., total pollution damages with the firm's emissions minus pollution damages without it. This conjecture seems more plausible in view of the of Polinsky's results (see footnote 12). The mathematical formulation of *Q(x)* may be very different in other models from our *Q(x)* and will depend on the particular DF and the particular production function assumed.

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the external effect itself. This will occur only when $D_{11}=0$, a result rarely satisfied here (see footnote 3).

[12]Polinsky (1980) provides a non spatial example, where Pigouvian taxes fail to achieve efficiency. In his model for strict liability and negligence, Polinsky utilizes a partial equilibrium model almost identical in its mathematical exposition to that of Spulber's model, with one small difference. In Polinsky's model, 'care' (the equivalent of negative emissions in our and Spulber's models) reduces external damages caused by the individual firm, i.e. the amount of care provided by a firm is an argument with a negative effect in a separate damage function of the individual firm which transforms emissions of each firm into monetary terms. These individual money damages are then accumulated to the total social damages. In Spulber's model, the emissions of the individual firms are added first and the accumulated amount of emissions is then converted to monetary terms via a single social damage function. Both models are correctly specified and the differences in their specifications follow from differences in the issues examined. These differences lead to what appear to be contradictory results of the two models. Thus, while in Spulber's model Pigouvian taxes provide long run efficiency, in Polinsky's model they cause inefficiency in the long run. In Polinsky's model, separate damage functions introduce the non-linearity which in our model is introduced via the dispersion function.

4. Preliminaries and Special Cases[13]

The optimal land allocation may be an "empty" city, i.e., no households and no industry. This outcome will occur if the price of the export good produced within city limits fails to maintain a sufficiently low level of pollution emissions and pay sufficiently high wages so that the labor employed in the city can sustain its predetermined economy-wide utility level. In the following analysis, only non-empty allocations ($N>0$) which satisfy the necessary and sufficient conditions are considered.

There are two principal solution types of relevance. One is an interior solution in which land use is mixed; i.e., $a(x)>0$, $b(x)>0$ and $a(x)+b(x)=1$. Such an allocation satisfying the necessary conditions is always a local optimum.[14] In this case the two bid rent functions $R_I(\cdot)$ and $R_h(\cdot)$ coincide everywhere (see Fig. 2(c)). All other possible allocations are corner solutions and involve zoning.[15]

Definition 4.1: A zoning solution is an optimal allocation in which each land use is located in a separate zone--a continuous area (i.e., a segment of the ring) in which only one land use is located. Thus there are industrial zones or residential zones. An empty area with no land use is also included; such an area is termed a buffer zone.

The possible zone types of relevance are: (i) an industrial zone in which $a(x)=1$, $b(x)=0$ for some segment of x, (ii) a residential zone in which $a(x)=0$, $b(x)=1$, and (iii) a completely empty buffer zone[16] in which both $a(x)=0$ and $b(x)=0$. In each zoning allocation, designated

[13] Proofs and technical elaboration of some cases appear in Appendix B.

[14] Note that often in problems involving inequalities only one type of an extremum can result as in our case in which only local maxima can occur. To see this, note that a solution with a positive S cannot be a local minimum since $a(x)$ and $b(x)$ can be reduced continuously while maintaining their ratio intact and thus reducing S until it disappears. Since we can increase density and commuting distances indefinitely we can always increase a deficit ($-S$) indefinitely.

[15] There might be corner solutions which do not involve zoning, e.g. an optimal allocation in which $e(x)=0$. We consider only corner solutions that involve zoning.

[16] A buffer zone will exist between an industrial and a residential zone if there is a segment of land between the two zones in which the two bid rents are not positive. This may occur if, on one hand, at these locations, concentration levels are too high and wages are too low to support the predetermined economy-wide household utility level and, on the other hand, for

consecutive industrial and residential zones will emerge, but solutions can also include buffer zones.[17] To investigate these allocations, it will prove useful to define a no-crossing point and an associated lemma.

Definition 4.2: A no-crossing point (NC point) is a point in the industrial or residential zones not crossed by commuters. No one commutes from one side of this point to the other.

Lemma 4.3: The value of the function $T(\cdot)$ at an NC point is zero.

Proof: By definition, $T(x)$ equals the net number of commuters crossing x to the north. Since minimizing commuting costs is necessary for the optimization, no commuting path of one household will cross the path of another commuting in the opposite direction; the path of a household commuting south cannot intersect with that of a household commuting north. If this outcome is violated, the two households with crossing paths can be exchanged and commuting costs will be reduced accordingly. Therefore the net and gross number of commuters crossing a point is equal. Definition 4.1 makes this number zero at an NC point.

Lemma. 4.4 In each residential and each industrial zone there is one and only one NC point (which may sometimes be extended to an NC segment).

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the specified emission taxes and wages the industry suffers losses.

[17] The types of land use patterns considered in this paper, in which all zoned land has a specific use (residential zones and industrial zones), are not exhausted by empty buffer zones and completely occupy mixed land use areas. Zones with partially occupied and partially empty locations are also a possibility. Housing in a partially empty location cannot arise, since positive marginal utility of housing implies that households in a particular location will use all or none of the available land at that location. A similar outcome exists in the case of industry: the linear homogeneous production functions together with the diminishing marginal productivity, which implies that there will not be empty space where industry is located because by keeping constant overall emissions, $(a(x)e(x))$, as well as overall labor, $(a(x)n(x))$, and expanding industry across the entire space in that location x (i.e. $a(x)=1$) output could increase without changing inputs. Furthermore, when $D_{22}>0$, the reduction in emissions density also leads to a reduction in the contribution to concentrations which in turn strengthens the tendency to fill an empty space or leave it entirely empty. However, when $D_{22}<0$, reducing the density of emissions while keeping their total at the given location constant increases concentrations. In this case the last effect works to reduce the population's well-being and therefore should be avoided. Consequently we cannot rule out the possibility that increases in the concentrations will outweigh the effect of the constant returns of scale in production and

Proof: Since paths of commuters cannot cross, in the residential zone there must be a point where all those living north of it commute northward and all those living south commute southward and no resident crosses this point. In the industrial zone there must be a point at which commuters employed north come from the north and similarly from the south. Each of these NC points can extend to an empty segment, although this is not likely to happen at a global optimum. It should be noted that two or more NC points with occupied space between them cannot exist in the same non buffer zone, since such occupants would have to cross one of the NC points when commuting.

In respect to NC points, it is also helpful to define an autonomous area (AA).

Definition 4.5: An autonomous area (AA) is the area between two consecutive NC points

Thus an autonomous area includes part of a residential zone and part of an industrial zone and all households who reside in an AA also work there and vice versa. If the allocation includes buffer zones, each AA includes an empty buffer zone between its designated residential and industrial segments. *Without loss of generality, in what follows the origin will be placed at an NC point where residents are located.*

Armed with the above definitions we can now interpret diagramatically the bid rent rule (BRR) in three typical situations: Let x_0 and x_1 be two consecutive NC points, the former in the residential zone and the latter in the industrial zone. In Fig. 2(a) a case of zoning without buffer zones is depicted and $\bar{x}$ is the boundary between the residential and the industrial zones in which the two bid rent functions intersect. An industrial zone exists at the locations where $R_I > R_h$; where $R_h > R_I$, a residential zone is designated and at the point of intersection, i.e., where

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result in an optimal solution with industrial zones of only partially occupied areas near the boundary. In subsequent analyses we shall disregard this case.

$R_I = R_h$, the rents must be non negative. In Fig. 2(b) the bid rents result in a buffer. In this case both bid rents disappear at the boundaries of the buffer zone and remain non positive everywhere over these locations. In Fig. 2(c) the two bid rents are constants and coincide everywhere. Therefore industry and residency coexist everywhere, each at its own constant density.

Assumption 4.6.(Symmetric Dispersion Assumption): $D^S(e,y) = D^N(e,y)$ *for all e>0 and y>0*

From this stage on we shall restrict our analysis to a more specific case in which the symmetric dispersion assumption holds. The assumption is that pollution spreads to the north and to the south in the same way. This may happen in practice with respect to air pollution if the wind blows in each direction the same length of time at all seasons and at similar times of the day. The model can be solved for other assumptions (e.g., $D^S(\cdot) = 0 \, and \, D^N(\cdot) > 0$), but the solution under each assumption is different and since the scope of the paper is limited we chose only one such case.

From the above definitions, the solutions of three special cases can be characterized, each with one parameter having an extreme value (zero or infinity). Each case is simple and intuitive. These special cases capture the essence of the solution in general and specify the range of possible outcomes.

First Case (Base):

In this case all parameters are presumed to be finite and have positive absolute values except *V*, the commuting cost per unit distance, which is assumed to be zero. As in the general case, pollution causes positive damages that increase with concentrations at any level of consumption, i.e., $-\infty < U_C(h,z,c) < 0$. A superscript zero designates variables for this base case, e.g., $r^0(x), R_I^0(x)$ and $R_h^0(x)$ specify respectively the value of the rent function, the industrial bid rent function and the residential bid rent function.

Since the dispersion function decays with distance (i.e., $D_2 < 0$), the greater the distance between polluter and pollutee, the lower the concentrations experienced by the pollutee which contributes to a higher utility level for a given level of the composite good and housing. Since commuting costs are zero, i.e., *V=0,* this separation does not involve any loss of resources.

Under these conditions, the maximum possible separation between housing and production is obtained, implying separate industrial and residential zones. Hence, $a^0(x)b^0(x) = 0$ for all x.

Moreover, only one industrial zone and one residential zone should exist and there may or may not be a buffer zone between these zones. The distance between polluter and pollutee reduces concentrations and thus will generate benefits without increasing costs. Any solution with many industrial zones could be restructured as a single industrial zone without decreasing the distance between any residential and any industrial location with some distances actually increasing. The same could be carried out for many residential zones. This implies that concentration levels at residential locations will not increase and will decrease for locations where distances to the industrial zone increase. Since all other factors remain unchanged, the utility level must increase in the locations where concentrations decrease and hold steady elsewhere. This implies that the initial allocation cannot be Pareto optimal and that an efficient allocation with zero commuting costs will contain exactly one residential and one industrial zone, possibly with two additional empty buffer zones.

Essentially, if there is empty space in the midst of one of the occupied zones, land uses can be moved from the boundaries of the zone to fill it. Such reallocations increase the distances between the two land uses and thus reduce effective concentrations without incurring any cost. As a result, an initial allocation with empty space in the midst of an occupied zone cannot be optimal. Therefore for all x in an occupied area (a residential or an industrial zone), $a^0(x) + b^0(x) = 1$. This condition together with $a^0(x)b^0(x) = 0$ implies that if one of these two variables is positive, its value must be one.

Since $V = 0$, Eq. (10) implies $\dot{\psi}^0(x) = 0$, thus $\psi^0(x)$ is spatially constant. This is no surprise since when commuting costs are zero, workers are unaffected by employment location.

Two NC points emerge, one, the origin, in the residential zone and the other in the industrial zone. *T(x)* are zero at the NC points. For these specifications together with the symmetric dispersion assumption, $D^N(\cdot) = D^S(\cdot)$, Fig. 3 depicts the layout of the city ring with the boundaries of the different zones being designated by x_i^0, $0 \le x_i^0 \le L$, $i = 0,1,2,3$.

Specifically, x_0 is the southern boundary of the residential zone; x_1 the northern boundary of the residential zone and the southern boundary of the northern buffer zone; and x_2 and x_3 are the boundaries of the industrial zone and the northern and southern buffer zones, respectively. Note that x_0 is also the boundary between the southern buffer zone and the residential zone (if there are no buffer zones $x_1 = x_2$ and $x_3 = x_0$). The symmetric dispersion assumption $(D^N(\cdot) = D^S(\cdot))$ implies that the allocation has OO' as the symmetry axis.

The bid rent functions are equivalent to the residual income per unit land in each location (Eqs. (17) and (18)). The density of land use is an increasing function of the rent. When *V=0,* the rent together with the density of land use reach their peak at the center of each zone. The centers of each zone are also the NC points and the boundaries between the AAs. In the industrial zone, the center is the pollution-generating location furthest from all residential locations. At this location, the optimal tax Q will be at its lowest level, and R_I at its highest (see Eq. (17)). Similarly, the center of the residential zone is least affected by pollution since it is located farthest from any emission source. Accordingly, rents will expand at this location.

When V is positive the centers are also the least accessible to commuters because they are the most remote locations. Remoteness is a positive trait from the standpoint of pollution and a negative from the standpoint of commuting. When V is zero, commuting is of no consequence and only pollution counts; hence the highest rents are at the centers. Similarly, the lowest rents and densities are at the boundaries. In the residential zone the boundary is the closest to the polluting industry and hence suffers the most relative to other residential locations. Within the industrial zone, the boundary is the closest to all residential locations, and producers located here must pay the highest pollution taxes with corresponding reduced rents. When moving from the boundary to the center of the zone these outcomes change steadily and monotonously. In a buffer zone and its boundaries, rents vanish as does all economic activity.

Second Case (Test). This case presumes $U_C = 0$ and $0 < V < \infty$, namely there are no ill effects of pollution and there are positive commuting costs. This is a pollution free solution that serves as a natural test of comparison. We designate the solution of this case by superscript 1.

Since pollution causes no damages to households, households and industry will gravitate to the same location in order to mitigate commuting costs. For commuting costs to approach zero all workers at x also must live there. Since conditions are the same everywhere symmetry implies that: $a^1(x) = \bar{a} > 0$, $b^1(x) = \bar{b} > 0$ and $\bar{a} + \bar{b} = 1$. As a result,

$\eta^1(x) = \dot{T}^1(x) = M^1(x) = Q^1(x) = 0$. Clearly $T^1(x) = 0$ $\bar{r} = r^1(x) = R_l^1(x) = R_h^1(x)$, and thus $\bar{r}$, the land rent, is spatially constant. Note that $f_e(n^1(x), e^1(x)) = 0$, $pf_n(n^1(x), e^1(x)) = \overline{\psi}$, and $\overline{\psi}$, the wage rate is also spatially constant.

This pollution free outcome serves as the point of departure when assessing the implications of alternative solutions with pollution.

Third Case (Interior Solution): In this case we again assume pollution causes ill effects, i.e., for all positive arguments $-\infty < U_C(h, z, c) < 0$. In addition, we assume a finite V, but one that is sufficiently large to deter commuting. The superscript 2 distinguishes this solution.

Obviously, sufficiently large costs deter commuting, hence as in the second case laborers reside next to their working place, i.e., $\dot{T}^2 = T^2 = \dot{\psi}^2 = 0$. Actually, in this case, as in the test case, all variables are spatially constant. Since there is no travel, all points are NC points. The differences between the test case and this case is that commuting is no longer economically feasible. Also, contrary to the prior case, the ill effects of pollution are spatially constant, i.e., $\eta^2(x) = \eta_2 > 0$. Accordingly, zoning is ruled out as a means of controlling pollution and thus concentrations can be affected only through the production process.

This case satisfies the necessary conditions for any given level of commuting costs *V*. Under the specified assumptions, the solution for this case is a local optimum for all *V* and a global optimum when *V* is sufficiently large. Contrary to zoning solutions, this solution is an interior solution, since the location variables $a^2(x)$ and $b^2(x)$ are constants and positive everywhere, i.e. $a^2(x) = a_2 > 0$, $b^2(x) = b_2 > 0$, and $a_2 + b_2 = 1$ for all x. Hence, this solution is naturally referred to as the Internal Solution.

5. Local Optimum.[18]

In general, local optima can be either internal solutions (e.g., third case) or corner solutions of which we are interested in the solutions with zoning. There might be more than one internal solution and for a given number of zones there might be more than one local optimum. In this section we concentrate on characterizing the general zoning allocation by investigating the changes due to an increase of commuting costs in a local optimal solution with two symmetric AAs and its supporting price system simultaneously.

For the general zoning case, an AA consists of a residential zone portion, the equivalent portion of an industrial zone and, if one exists, a buffer zone. Obviously, there is no commuting in or out of an AA, and for symmetric pollution dispersion, each AA is the mirror image of the adjacent AA.[19] In Fig. 3, an allocation of two AAs is depicted. While the number of zones is two, as in the base case where *V* is zero, the remaining variables change continuously when *V* *increases*. These relationships will be investigated by allocating two AAs in which the origin is located at the NC point of the residential zone. Similar relationships exist when the number of zones is larger.

[18] Proofs of the Lemmas and Propositions of this section not proved in the text appear in Appendix C.

<u>*Lemma 5.1:*</u> *In an AA,* $w(x)$, *the local net earnings, is a linear function of x and its slope is equal to V, i.e.,*

$$(22)\quad w(x,V)=\begin{Bmatrix} \psi(V,x_2^0)+(x-x_2^0)\cdot V, & \text{for } 0\le x\le L/2 \\ \psi(V,x_3^0)+(x_3^0-x)\cdot V, & \text{for } L/2\le x\le L \end{Bmatrix}$$

Hence, $\psi(V,x_2^0)$ *(also equal to* $\psi(V,x_3^0)$*) is only a function of V, and in what follows we refer to it as the intercept of w(x).*

Note that the first line on the right hand side of (22) is the value of $\psi(x,V)$ in the northern AA and the second line is the value in the southern AA. In what follows, we deal only with the northern AA while keeping in mind that the southern AA is symmetric with OO' as the axis of symmetry (see Fig. 3).

Lemma 5.1 implies that if *V* is positive, the LNE (the wage rate) in the industrial zone increases at the rate of *V* per unit distance when moving from the boundary towards the NC point.[20] The opposite occurs when moving into the residential zone and away from its boundary. If there is no buffer zone, at the boundary of the industrial and residential zones the LNE has a single value. The difference between the LNEs on the respective boundaries of a buffer zone is equal to *V* times the length of the buffer zone. Lemma 5.1 is a standard result in models where separation into distinct industrial and residential zones occurs.

<u>*Corollary 5.1.1:*</u> *An increase of V augments the multiplier of x in* $\psi(x)$, *and moves the intercept* $\psi(V,x_2^0)$ *by the shift factor, defined as the derivative of* ψ *at the boundary of the industrial zone*

[19] The symmetry follows from assuming divisibility of the ring to the size of the optimal AA. If the solution of the optimal AA is not unique we may have asymmetric solutions as well, however here we consider only the symmetric case.

[20] NC points are the boundaries of an AA and are located in the middle of a zone. In an AA the residential portion has only one boundary and so does the industrial portion. If it exists, a buffer zone have two boundaries in an AA.

with respect to V, viz., $\frac{\partial \psi(x_2^0)}{\partial V}$. *The shift factor can be positive, negative, or zero, depending on the model's specifications.*

For ease of exposition, henceforth we restrict the analysis only to cases without buffer zones. If notable differences arise from the existence of buffer zones to zoning without buffers, such instances will be examined in footnotes.

Fig. 4 demonstrates the possible effects of an increase of *V* on $\psi(x)$ in the AA of the northern hemisphere of the ring that stretches from the origin in the positive direction of *x* to the next NC point at L/2, viz., the domain in which *T(x)* is positive, as in Lemma 5.1. There are two fundamental types of outcomes of $\psi(x,V)$ resulting from an increase in *V*. In Fig. 4, the line A'A' depicts $\psi(x,V)$ after it shifted by an increase of *V* from C'C', which depicts $\psi(x,V=0)$. For the first, the boundary point x_2^0 is at x_A, to the right of $\tilde{x}$, the location where $\psi(x,V=0)$ intersects $\psi(x,V>0)$. The shift factor, depicted in Fig. 4 by the segment at x_2^0 between A'A' and C'C', i.e., $x_2^0 = x_A$, is positive. The result is an increase in the wage rate in the AA throughout the industrial zone, while the LNE in the residential zone is increased near the boundary and decreased near the NC point 0. For the second, $x_2^0 = x_B$ is to the left of $\tilde{x}$, the shift factor is negative, and the LNE is decreased throughout the residential zone while the wage rate in the industrial zone is reduced near the boundary and increased near the NC point $L/2$.

The total derivative of the functions $R_k, k = h\, or\, I$ with respect to *V* is given by

$\frac{dR_k(x)}{dV} = \frac{\partial R_k(x)}{\partial V} + \sum_i \frac{\partial R_k(x)}{\partial t_i}\frac{\partial t_i}{\partial V}$, where t_i are the controls and shadow prices of the system.

Lemma 5.2. In the optimum allocation, both bid rents[21] *are functions of x and of V, and*

i. In the residential zone $\frac{dR_h(x)}{dV} = \frac{\partial R_h(x)}{\partial V} = \frac{(\partial\psi(x)/\partial V)}{h(x)}$

ii. In the industrial zone $\frac{dR_I(x)}{dV} = \frac{\partial R_I(x)}{\partial V} = -n(x)\frac{\partial\psi(x)}{\partial V}$.

Lemma 5.2 demonstrates that commuting costs affect the bid rent functions only through $\psi(x)$; the terms for the other controls disappear. The change in the industrial bid rent at a given location is inversely proportional to the change of the wage rate and the factor of proportionality equals *n,* the local labor density. The change in the residential bid rent at a given location is proportional to the change of the LNE and the factor of proportionality equals $(1/h)$, the local residential density.

Two patterns in which the optimal bid rent functions change when *V* changes, emerge s while the number of zones remains constant. From the bid rent rule (Lemma 3.7) the optimal AA changes with the bid rents. The results below follow directly when Lemma 5.2 is applied to Fig. 4 to obtain Fig. 5. Parts (a), (b) and (c) of Fig. 5 address pattern A, which results when the shift factor is positive. Part (d) of Fig. 5 depicts the synthesis of pattern B which emerges when the shift factor is negative.[22] Note that $\tilde{x}$ is the point in which the bid rent functions do not change with *V*. As seen from Lemma 5.2, this is the same point in which $\psi(x)$ does not change (see Fig. 4). An intersection point such as $\tilde{x}$ and the bid rent rule (Lemma 3.7) provide the basis for the results.

[21] Note that although the bid rent functions have their economic interpretation in the realm of market equilibrium, in definitions 3.5 and 3.6 they are well defined by variables of the optimum. In what follows we shall often analyze an optimum allocation through its supporting price system.

[22] In Fig. 5 all of these rent curves are depicted as straight lines only for the sake of convenience.

In Fig. 5(a), $R_h^0(x)$, the residential bid rent function in the base case, i.e. *V=0,* is depicted by the downward sloping line $C'CC'$, i.e., $\dot{R}_h^0(x) < 0$. The industrial bid rent function in the base case, $R_I^0(x)$, is depicted by the upward sloping line $C''CC''$ in 5(b), which intersects $C'CC'$ ($R_h^0(x)$) at $x_2^0 = x_A$ with a positive value *C.*

The patterns are as follows:

Pattern A (a positive shift factor) emerges when V increases from zero to V_A *>0 and the intersection point* $\tilde{x}$ *is left of the boundary point* $x_2^0 = x_A$ *(see Fig. 4).* The bid rent curves before and after the shift, which are of the same type (industrial or residential), also intersect at $\tilde{x}$. Thus, in this pattern the increase of *V* causes the residential bid rent function to increase near the boundary ($x_1 = x_2$) and decrease near the NC point (the origin). In Fig. 5(a), the line A'AA' represents $R_h^A(x, V_A)$ and C'CC' represents $R_h^0(x)$. The industrial bid rent function, however, is lower everywhere in the industrial zone for *V=0* relative to V_A, less so near the boundary (x_2) and more so near the NC point (L/2). The line A"AA" in Fig. 5(b) represents $R_I^A(x, V_A)$, and C"CC" is $R_I^0(x)$. The rent functions before and after the shift depicted in Fig. 5c are the upper envelope curves of the before and after bid-rent functions. The boundary moves from $x_2^0 = x_A$, the point of intersection of the residential and industrial bid rent curves before the shift to x_2^A, the point of intersection of the same two bid rents after the shift. The net result of pattern A is that relative to the case where commuting costs vanish, an increase of *V* causes the boundary to move towards the industrial NC point, the residential zone expands and the industrial zone shrinks by the same amount. In the industrial zone, the rent declines everywhere, less near the boundary (x_2) and more near the NC point (L/2). The rent function in the residential zone increases near the boundary (x_1) and decreases near the NC point (the origin).

Pattern B (a negative shift factor) in which V increases from zero to V_B *resulting in an intersection point* $\tilde{x}$ *to the right of the boundary point* $x_2^0 = x_B$. The increase of *V* causes the residential bid rent function to decline throughout the residential zone (relative to *V=0*), more near the origin than close to the boundary. In Fig. 5(d), the line $B'B$ represents $R_n^B(x, V_B)$, where $R_h^B(x, V_B) \geq R_I^B(x, V_B)$, and $C'C$ is the line where $R_h^0(x) > R_I^0(x)$.

When *V* increases, the industrial bid rent function increases near the boundary of the industrial zone and decreases near the NC point. In Fig. 5(d), BB'' represents $R_I^B(x, V_B)$, where $R_I^B(x, V_B) > R_h^B(x, V_B)$, and CC'' is $R_I^0(x)$, where $R_I^0(x) > R_h^0(x)$. As with $R_h(x, V_B)$ and $R_h^0(x)$, the difference between $R_I^B(x, V_B)$ and $R_I^0(x)$ reflects the shift in $R_I(x)$ due to the increase of *V* from zero to V_B. The boundary point between the two zones moves from $x_2^0 = x_B$ towards the origin to x_2^B so that the industrial zone expands and the residential zone shrinks. The rent reduces everywhere except near the boundary of the industrial zone.

In both patterns, the density of population in the residential zone and the density of employment in the industrial zone follow the same trends as the rents across the zones. Overall, the total AA's population falls when *V* increases. To be sure which of these patterns the optimum follows depends on the particular technology of the industry, the environment, and the tastes of the population.[23]

Note, in general, near the boundary, bid rents are increasing with *x* at a non decreasing rate as we move in the direction of the NC points. As we move from the boundary and closer to the

[23] The same results along with a shrinking of the buffer zone occur when a buffer zone is embedded in the AA. In this instance, however, an outcome in which both occupied zones shrink and the buffer zone increases (when *V* increases) cannot be ruled out. This will occur when $\psi(x, V + \Delta V)$ intersects $\psi(x, V)$ at the buffer zone. The intuition is that with an expanding buffer zone, both distances and emissions increase and thus cancel each other out while employment density decreases and wages increase. Indeed, within the residential zone the LNE and the rent may be lower but pollution concentrations can fall due to the increased distance.

NC points, the bid rent functions can become convex, decline, or even disappear altogether in the neighborhood of the NC points.

For zero commuting costs, i.e., *V=0* (the base case), the rent function is at a local maximum at the NC points, gradually declining with distance when moving towards the boundary where it reaches its lowest level (see the discussion of the base case in section 4)

Lemma 5.3: When V increases, the slope of the rent function in the direction away from the NC point and towards the boundary declines at every location except perhaps near the boundary, where it may increase with V.

Lemma 5.3 states that the slope of the rent function (see Fig. 5) becomes flatter everywhere except perhaps for pattern A at section $[\tilde{x}_A, x_2^0]$ and for pattern B at section $[\tilde{x}_B, x_2^0]$.

The above analysis pertains only to the case of two AAs. Symmetry dictates through Lemma 5.4 (see also footnote 19) that the analysis can apply with no significant changes to any even number of AAs.

Lemma 5.4: For each V>0, there may exist a local optimum solution with 2m zones, where m can be part of or all the integers fulfilling $1 \le m \le \infty$. If 2m, m=1,2,. . ., is the fixed number of AAs in an allocation, so is the number of NC points which are located at the boundaries of the AAs (and in the middle of the occupied zones). All AAs are of the same size with an area of L/2m and each AA is the mirror image of the adjacent AA. The qualitative results discussed previously in this section of the effects of an increase of V (from zero to infinity) on the internal structure of an AA hold for the general case as well.

Aside from symmetric solutions discussed in the lemma above, there may be solutions with AAs of varying sizes. Disregarding problems of indivisibility, each of the AAs in a mixed solution will also appear in a symmetric solution of a different number of zones. Of any two such symmetric solutions one is superior to the other and therefore superior to the solution where the two are mixed. We will ignore the case where we are indifferent to the two solutions and their mixture. Note that there always must be a symmetric solution in the optimum.

Also note that the parameter V does not always have a finite upper bound, above which the zoning local optimum does not exist (note that $m = \infty$ is equivalent to the internal solution). It might occur that as V grows larger, buffer zones disappear and the density of land use in the occupied areas become lower and more concentrated around the boundaries of the occupied zones. A further increase of V may cause the centers of the occupied zones to become empty, thus changing the no-crossing points to no-crossing segments, and when V approaches infinity, the actual occupied areas in the zones shrink towards the boundaries approaching zero, but never completely disappear while V is finite. It is clear that an allocation with a no-crossing segment in the middle of the two occupied areas cannot be a global optimum since a solution with a larger number of fully occupied smaller zones is clearly more efficient.

The following Lemma does not deal with the characterization of a solution but rather with the applicability of a local optimum solution.

<u>*Lemma 5.5:*</u> *A developer who wants to implement the allocation of a given local optimum solution has only to choose an origin and impose on each unit of land the optimal corrective tax of the supporting market allocation of this desired local optimum. Under this condition, market competition will lead to the desired local optimum allocation. Note that the optimal tax, $Q(x)$, is well-defined everywhere (see eq. (15B)) where $e(x)$ is the actual emissions emitted by the industry*

at location x and $\eta^*(y)$, the marginal damages of pollution concentrations at y, are evaluated at the optimum.

Proof: Throughout this section we showed that under the above conditions the industry outbids housing in the industrial zone and residents outbid the industry in the residential zone. Furthermore, we demonstrated that in a buffer zone both the industry and the residents do not bid. Since the bid rents reflect the highest amount each sector is willing to pay under these conditions, the desired local optimum is the only outcome which can result from the competition between the industry and housing. QED

It should be noted that a local government should behave like the developer and simply imposing the correct land taxes is enough to achieve a desired local optimum allocation.

6. Global Optimum.

Initially, when $V=0$, the global optimum consists of two AAs (the base case). In this case, $\dot{R}_I(x_2) > 0 > \dot{R}_h(x_1)$. It is possible that any increase in *V*, even an infinitesimal one, will cause the internal solution to become the global optimum. In this section, only cases in which zoning is the global optimum for at least some positive *V* are investigated. For simplicity, we assume each positive integer *m* has no more than one local optimum solution with *2m* occupied zones (or AAs)[24].

Definition 6.1: Let $S^m(n,V)$ designate the maximized surplus of a local zoning optimum solution with commuting cost $V \geq 0$ and 2m AAs, m being a positive integer.

The commuting cost and the second variable in S^x, *V*, is a parameter of the initial problem. The first variable in S^x is *m*, the number of occupied zones of the same type in the local optimum solution has to be determined endogenously in the global solution. In what follows we try to determine the number of zones in the global optimum by finding the m which maximizes S^x for a given V.

Lemma 6.2: The following are properties of the function $S^m(n,V)$:

(i) $\partial S^m(n,V)/\partial V = -\int_0^L |T(x)|\,dx \le 0$

(b) $\dfrac{\partial S^m(n_i,V_0)}{\partial V} > \dfrac{\partial S^m(n_j,V_0)}{\partial V}$ *if* $n_j > n_i$

and for all V_0 *for which both functions are positive and well-defined.*

(iii) $S^x(m_i,0) > S^x(m_j,0)$ *for all pairs fulfilling* $m_j > m_i$,

Lemma 6.2 reveals (see Fig. 6) that: (i) The slope of $S^m(n,V)$ in the (S,V) plane is non positive and strictly negative as long as m is finite (since $|T(x)|$ in a zoning solution is positive almost everywhere); *(ii)* In the (S,V) plane the slope of $S^m(n,V)$ at a given V is steeper for smaller m *(because* $|T(x)|$ attains higher values in larger zones); and *(iii)* The intercept on the S axis of $S^m(n,V)$ in the (S, V) plane is decreasing with m (since pollution damages increase and therefore the value of $S^x(m,0)$ decreases with the number of zones, see the base case for proof) .

The corollary below now follows directly from Lemma 6.2:

Corollary 6.2.1: In the (S,V) plane, two $S^m(n,V)$ *curves with different m's may intersect in the upper right quarter of the plane at most once (see Fig. 6).*

[24] In general, there may exist more than one local optimum for a given number of zones. The assumption made here of a single local optimum for each m simplifies the exposition. Yet it is easy to extend the analysis to the more general case and we leave this to the devices of the reader.

The above corollary follows directly from (ii) in Lemma 6.2 which implies that the smaller *m* is, the steeper the slope of S^x is for a given *V*. The lemma below actually follows from the definition of S^x and the nature of the global optimum for a given *V*.

<u>Lemma 6.3</u>: *For a given V, the global optimum allocation is the local optimum allocation in which the number of AAs, m*(V), is obtained from* $S^m(n^*, V) = \max_n S^m(n, V)$ *(note that m*(V) may be infinite for all V).*

We can now make the following definition,

<u>Definition 6.4</u>: *Let* $S^*(V) = S^x(m^*(V), V)$ *be the global optimum value of the surplus as a function of V.*

Lemma 6.3 implies that *S*(V)* is the upper envelope curve of all the $S^m(n,V)$ in the (S,V) plane. Lemma 6.2, Corollary 6.2.1 and Lemma 6.3 provide the basis for Proposition 6.5.

<u>Proposition 6.5</u>: *Let 2m*(V) be the number of AAs in the global optimum solution of the problem with commuting costs V. The function m*(V) is defined in Lemma 6.3; its range is in the set of natural numbers and it is a non decreasing step function of V,* $0 \le V \le \infty$.

The proof of proposition 6.5 is straightforward. Lemma 6.2 and Corollary 6.2.1 imply that two $S^m(n,V)$ curves intersect only once in the *(S, V)* plane, and the one with lower *m* intersects the one with the higher *m* from above. Thus if $n_i < n_j$ and both are in the global solution, n_i will be associated with lower *V*s than n_j. The remaining results follow immediately.

<u>Corollary 6.5.1</u>: *If* $V_i \overset{>}{\underset{<}{=}} V_j$, *then* $m^*(V_i) \overset{\geq}{\underset{\leq}{=}} m^*(V_j)$.

The proof of the corollary follows immediately from Proposition 6.5 and Fig.6

<u>*Corollary 6.5.2*</u>*: Let m_i be an integer such that $2m_i$ is the number of AAs in the global optimum for all $i \in (1,...,I)$. Then the set of all Vs, for which $m_i = m^*(V)$, is a segment of the non negative V axis for all $i \in (1,...,I)$. The intersection of each pair of segments is at most a single point and the union of all I segments of Vs exhaust the half line $V \geq 0$. (1,...,I) consists of increasing consecutive indices, s.t. $m^*(V) = m_i$. If $m^*(V_1) = m_i$, $m^*(V_2) = m_j$ and $i > j$, then $m_i > m_j$ and $V_1 > V_2$.*

The proof follows immediately from the proposition above and corollary 6.5.1.

Note that *I* in the corollary above--the number of different segments of *V*s and each *V* in a given segment has the same number of zones in the global optimum—can be any positive integer or infinity (see Fig. 6). In other words, the number of solutions which differ by their number of zones can be any positive integer including infinity.

To complete the characterization of the global solution, the concept of a threshold commuting cost can be introduced by means of Definition 6.6.

<u>*Definition 6.6*</u>*: Define $\underline{V}(m_i)$, $i = 1,2,...,I$, as the commuting cost threshold of an allocation with $2m_i$ AAs.. $\underline{V}(m_i)$ is the lowest commuting cost in which $2m_i$ zones are the number of AAs in the global optimum, i.e., $\underline{V}(m_i) = \min\{V / s.t.\ m^*(V) = m_i\}$.*

In Fig. 7, $\underline{V}(m_i)$, $i = 1,2,...,I$ are the jump points of the step function $m^*(V)$, and in Fig. 6 they are the value of *V* at the intersection points of the S^x curves in the global optimum. Note that the domain of $\underline{V}(m_i)$, i.e., $(1, m_1, ..., m_{I-1}, \infty)$ is a set of increasing, not necessarily consecutive positive integers whose number, *I,* may or may not be infinite.

<u>*Proposition 6.7*</u>*: The Threshold Theorem.*

(i) When V increases and reaches $\underline{V}(m_i)$, the number of zones in the global optimal allocation changes from m_{i-1} to m_i and remains at this level until V reaches $\underline{V}(m_{i+1})$

(ii) $m_I = \infty$ *always, even when I is finite and* $\underline{V}(\infty) \leq \infty$.

(iii) $m_1 = 1$ *and* $\underline{V}(1) = 0$.

In the proposition above (i) follows from the definition of $\underline{V}(\cdot)$ as the lower bound of all Vs in the group of solutions having the same number of zones; (ii) follows from the fact that the internal solution which is equivalent to a solution with infinite number of zones is always the solution when *V* becomes sufficiently large to deter commuting and the value of $s^x(V,\infty)$ is therefore independent of *V*;(iii) follows from the fact that the base case is always the solution when *V=0* .

The following proposition is once more about the implementation of optimal corrective taxes, this time in the global optimum.

Proposition 6.8: To achieve global efficiency, including optimal zoning, a developer (or a local government) has only to levy at every location x the global optimal corrective tax per unit of land. The global corrective tax is the corrective tax Q(x) of the particular local optimum solution which is in the global optimum for the given V.

The proof follows directly from Lemma 5.5.

7. Policy Implications and Concluding Remarks.

The results derived in this paper have a number of policy implications with respect to the optimal control of pollution. First, levying efficient taxes is sufficient to achieve a global optimum (Proposition 6.9). Then the optimal number of zones, the optimal level of emissions as well as the optimal level of the rest of the variables are determined by a competitive equilibrium.

Secondly, the correctly determined spatially differential tax per unit of land equals the additional damages caused by the total emissions from this unit of land (industrial or residential, see Lemma 5.5). The economic literature, spatially or non-spatially based, has recommended what appears to be contradictory pollution tax regulations. Some contributions argue that Pigouvian taxes are the correct policy (Baumol and Oates, Spulber) while others have argued that such taxes provide the wrong incentives (Henderson, Hochman and Ofek, Polinsky). Our results demonstrate that the correct tax policy is not a spatially differentiated Pigouvian tax, but instead a land tax, determined by a per unit of land additional contributions to total pollution concentrations *(Q(x), not M(x))*. These results may not all be robust, but it is certain that, in general, Pigouvian taxes are not sufficient. Taxing a unit of land does not seem to be a robust result; in other models the firm may be the unit of tax. Our result showing that the tax itself should be equal to the additional pollution damages contributed by the taxed unit is probably robust, however. Since in practice corrective taxes are difficult, if not impossible, to determine, a major policy instrument for controlling pollution might be the implementation of zoning regulations.

The trade-off between commuting and pollution costs along with the non convexity inherent in spatial models leads to multiple zoning optima. Zoning is therefore a critical means for controlling pollution. As commuting costs increase, the optimal land utilization passes through an endogenously determined series of increasing thresholds. Each time a threshold is crossed the number of zones of each type increases until an internal solution is reached once the final threshold has been crossed. Operationally, this solution can be pursued by first setting the long-run zoning allocations, whose boundaries, with or without geographic buffer designations, move with commuting costs through their effects on wages, local net earnings, and land bid rents, given a particular pollution emission and dispersion process. The second setting is the per unit land pollution tax. Some experimentation is possible with this instrument (Hochman and Ofek). Emission standards per unit land can be established which enhance land rents to include optimal pollution land taxes. Moreover, these and other instruments could be introduced and experimentally adjusted in the short-run to provide pollution abatement incentives (Rausser and Lapan) so long as the long-run global optimum zoning allocation has been appropriately determined by the local authority.

In simplifying the specifications advanced in this paper, the industry is assumed to exhibit constant returns to scale and there is no spillover of pollution between cities. In order to attain agglomeration in the long run, either scale economies in production or local public goods must be specified. Pollution itself has no agglomeration effects; on the contrary, it tends to enhance dispersion of activities. The optimal allocation of a single city is only a portion of the short-run economy-wide maximization problem with a fixed number of city sites. From the standpoint of the entire economy beyond the city in question, an internally consistent economy-wide goal is the maximization of the common utility level of the population. This goal is subject to both the overall population constraint and to the constraint requiring the sum of all the cities' surpluses to be non-negative and equal to total unearned income which becomes a choice variable. In this more generic setting and an infinite supply of city sites, optimal city size is one household.

In pursuing her self-interest, the developer-owner of the city can either provide residents with housing, consumption goods, commuting and can engage in production activities, or simply rent land to the highest bidder, and like a local authority, impose optimal taxes. We just showed that her gains are the same in both cases.

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Zoning As A Control Of Pollution Externalities In A Spatial Environment: Appendixes

by

Oded Hochman and Gordon C. Rausser

Introduction

This working paper contains appendixes of mathematical derivations and proofs of Lemmas and Propositions in the paper "Zoning As A Control Of Pollution Externalities In A Spatial Environment" by Hochman and Rausser.

In Appendix A the necessary and sufficient conditions are derived. In Appendix B proof of the bid rent rule (Lemma 3.1) is provided and some elaboration and proofs of the cases discussed in Section 4 of the paper. Proofs of lemmas of Sections 5 and 6 not proved in the text, are included in Appendix C.

Appendix A

Let **L** be the Lagrangian of the model, where the variables, constraints and shadow prices are as defined in sections 2 and 3 of the paper.

$$\text{(A.1)}\quad \mathbf{L} = \int_0^L [Paf(n,e) + \frac{b}{h}(I-z) - |T|V]dx + \int_0^L \lambda(x)\,[U(h,z,c) - u_0]\,dx$$
$$+\int_0^L dx\,\eta(x)\,\{c(x) - \int_{x-L/2}^{x} a(y)D^N[e(y), x-y]dy - \int_x^{x+L/2} a(y)D^S[e(y), x-y]dy\}$$
$$+\int_0^L \zeta(x)\{T(x)\ - \int_0^x [\frac{b(y)}{h(y)} - a(y)n(y)]dy\}\,dx - \int_0^L [\rho(a+b-1) - \gamma a - \mu b]\,dx$$

It should be noted that $\xi(x)$, the shadow price of the commuters constraint, is different from $\psi(x)$, the co-state of T(x) as defined in the text. We elaborate below on the relation between the two. The necessary conditions are as follows. (The variable of differentiation is noted on the left-hand side of each equation. Note that a function with a number as a subscript indicates derivations of the function with respect to the variable of the order of the subscript.)

$$n(x) \qquad (A2) \qquad a(x)[pf_1 + \int_x^L \xi(y)dy] = 0$$

$$e(x) \qquad (A.3) \qquad a(x)[\,Pf_2 - \int_x^{x+L/2} \eta(t)\,D_1^N(e(x)\,,\,t-x)\,dt$$
$$- \int_{x-L/2}^{x} \eta(t)\,D_1^S(e(x)\,,\,x-t)\,dt] = 0$$

$$a(x) \qquad (A.4) \qquad Pf - [\,\int_x^{x+L/2} \eta(t)\,D^N(e(x)\,,\,t-x\,)dt$$
$$+ \int_{x-L/2}^{x} \eta(t)\,D^S(e(x)\,,\,x-t)\,dt] + n(x)\int_x^L \zeta(t)\,dt$$
$$- \rho(x) + \gamma(x) = 0$$

$$h(x) \qquad (A.5) \qquad -\frac{\mathrm{b(x)}}{\mathrm{h(x)}^2}[I - z(x)] + \lambda(x)U_h + \frac{\mathrm{b(x)}}{\mathrm{h(x)}^2}\int_x^L \zeta(t)\,dt = 0$$

Appendix B.

Proof of Lemma 3.1:

Consider the boundary points of the zones in an optimal allocation, e.g. x_1 and x_2, $x_1 < x_2$, as deppicted in Fig.3 of the paper. The bid rent rule implies that at such boundary points the slopes with respect to distane (designated by a dot over the function) of thee bid rent functions must fulfill $\dot{R}_I(x_2) \geq \dot{R}_h(x_1)$, otherwise the allocation is not optimal. Suppose, only for the sake of proving a contradiction, that $\dot{R}_I(x_2) < \dot{R}_h(x_1)$, then industry outbids housing in the residential zone and housing outbids industry in the industrial zone. A single household at x_1 can then be transferred into the industrial zone in exchange of industry occupying the same amount of land at x_2. The fact that industry outbids housing in the residential zone and vice-versa in the industrial zone implies that this transfer increases total rents. It also increases total pollution damages since it shortens distances between polluters and pollutees. Total pollution taxes therefore increase as well. According to the Henry George rule ($\int_0^L [r(x) + Q(x)]dx = S$), total rents and optimal taxes together constitute the goal function. Since both are increased, the goal function of the initial allocation can be increased and is therefore not optimal in the first place; a contradiction. Hence $\dot{R}_I(x_2) \nless \dot{R}_h(x_1)$, QED

Calculating the Spatial Derivatives of the Bid Rent Functions

We differentiate the bid rent functions with respect to distance x,). First we differentiate Eq. (17) at locations where $a(x) > 0$ and substitute Eqs. (9) and (10) into the result to obtain,

(B1)
$$\begin{aligned}\dot{R}_I &= -n\dot{\psi} - \dot{Q} = \\ &- sign(T(x))Vn - \{\eta(x + L/2)[D^N(e(x), L/2) - D^S(e(x), L/2)] \\ &+ \int_{x-L/2}^{x} \eta(y) D_2^S(e(x), x - y)dy - \int_x^{x+L/2} \eta(y) D_2^N(e(x), y - x)dy\}\end{aligned}$$

$b(x) \quad (A.6) \quad \frac{1}{h}(I-z)-\frac{1}{h(x)}\int_x^L \zeta(t)\, dt-\rho(x)+\mu(x)=0$

$z(x) \quad (A.7) \quad -\frac{\mathrm{b}}{\mathrm{h}}+\lambda U_z=0.$

By using equation (8) from the text while differentiating **L** with respect to T(x)we obtain:

$T(x) \quad (A.8) \quad -[signT(x)]\cdot V+\zeta(x)=0.$

$c(x) \quad (A.9) \quad \lambda(x)U_c+\eta(x)=0$

Define the co-state of *T(x)* to be $\psi(x)$,

$$(\mathrm{A.10}) \quad \psi(x)\overset{def}{=}-\int_x^L \zeta(x)dt$$

then

$$(\mathrm{A.10'}) \quad \dot{\psi}(x)=\zeta(x)$$

By substituting $\zeta(x)$ in the above equations with *y(x)* and $\dot{\psi}(x)$ from (A.10), and then eliminating *l(x)* from the equations by substituting from (A.7), we obtain the necessary conditions as specified in the text.

where we obtained the second equality by first substituting $\dot{\psi}$ from Eq. (10) into the first equality. Then we differentiate Eq. (15b) and the result for $\dot{Q}$ we also substitute into the first equality to obtain the second. Use has also been made of the continuity assumption $D^N(e,0) = D^S(e,0)$. It should be noted that our assumptions also imply $\eta(x+L/2) = \eta(x-L/2)$ because $(x+L/2)$ *and* $(x-L/2)$ are the same point.

By differentiating (1) with respect to *x* and substituting the last equality of (18) into the result, we get the expression $R_h\dot{h} + \dot{z} + (U_c/U_z)\dot{c} = R_h\dot{h} + \dot{z} + h\eta\dot{c} = 0$, where the second equality is obtained by substitution of (13) with *b(x)=1* into the first equality. We then differentiate the first equality in the chain (18) and substitute the last equality of the chain above into the result to obtain the first equality of the chain below

(B2) $$\dot{R}_h = \frac{\dot{\psi}}{h} - \eta\dot{c} = \frac{1}{h(x)}[sign(T(x))V + \frac{U_c(x)}{U_z(x)}\dot{c}(x)]$$

Once again we obtained the second equality after substitution of (13) and (10) into the previous term. Differentiating (2) with respect to *x* yields

(B2a) $$\dot{c}(x) = \int_{x-L/2}^{x} a(y)D_2^N[e(y),x-y]dy - \int_{x}^{x+L/2} a(y)D_2^S[e(y),y-x]dy \\ - a(x+L/2)[D^N(e(x-L/2),L/2) - D^S(e(x+L/2),L/2)]$$

where once more we have made use of the fact that $x+L/2 = x-L/2$ and that $D^N(e,0) = D^S(e,0)$. Substituting (B2a) into (B2) yields the desired expression for $\dot{R}_h$.

Buffer Zones and Boundary Conditions In a Two AA's Case

In Fig.3 of the paper the following chain of inequalities holds:

(B3) $$0 < x_1 \le x_2 < x_3 \le x_0 < L$$

where x_i, $i = 0,1,2,3$ are the boundaries of the different zones.

A necessary condition for optimal boundaries is $R_I(x_2) = R_h(x_1)$ and $R_I(x_3) = R_h(x_0)$. If $x_1 = x_2$ and $x_3 = x_0$, buffer zones do not exist and there is only a residential zone and an industrial zone. However if in the optimum only strong inequalities hold between the boundaries specified in (B3), the solution also includes buffer zones.

A segment of the ring is a buffer zone if for all x of the segment, $R_I(x) \le 0,\ R_h(x) \le 0$ (by saying that $R_h(x) < 0$ we mean that if $\underline{z}(x)$ fulfills $U(\infty, \underline{z}(x), c(x)) = u_0$, then Y+ $\psi(x) - \underline{z}(x)$ *(=h(x)* $R_h(x)$*)<0.* At the boundary of a buffer zone and an industrial zone $R_I(x) = 0$, and at the boundary of a residential zone and a buffer zone $R_h(x) = 0$. Additional necessary conditions for the general zoning case are,

$$\text{(B4)}\quad \begin{array}{l} R_I(x) < R_h(x) > 0,\ \ for\ x_0 \le x \le L;\ \ and\ 0 \le x \le x_1 \\ 0 < R_I(x) > R_h(x),\ \ for\ x_2 \le x \le x_3 \end{array}$$

and the following conditions are specific to buffer zones.

$$\text{(B5)}\quad \begin{array}{l} R_h(x) \le 0,\ \ R_I(x) \le 0\ \ for\ x_1 \le x \le x_2\ \ and\ \ x_3 \le x \le x_0 \\ R_h(x_0) = R_h(x_1) = R_I(x_2) = R_I(x_3) = 0\ \ and \\ R_h(x_2) \le 0,\ \ R_h(x_3) \le 0\ \ R_I(x_0) \le 0\ \ R_I(x_1) \le 0 \end{array}$$

Since we use the assumption $D^N(e,y) = D^S(e,y)$ (Assumption 4.1) and disregard problems of indivisibility and multiple optima (see footnote 14 in the paper), there is complete symmetry between north and south. That is *OO'*, the line through the origin and the second fixed point, divides the circle into two halves and serves as an axis of symmetry between two mirror images. Thus $x_0 + x_1 = L = x_2 + x_3$, see Fig. 3 which depicts a case where this assumption holds.

Application Of The Bid Rent Rule To The Base Case:

Consider first the southern boundary of the residential zone, x_0^0. It is either an intersection point of the two bid rent curves and there is no buffer zone south of the residential zone, or $R_h(x_0^0) = 0,\ \ R_I(x_0^0) < 0$ and an empty buffer zone exists between the residential and industrial zones (see Fig. 3). Since in the base case *V=0,* the first term in the RHS of (B2) disappears. The second term there depends on $\dot{c}(x_0^0)$ given in (B2a). The assumption $D^N(e,y) = D^S(e,y)$ implies that the last term of $\dot{c}(x_0^0)$ is zero. Upon substitution of x=x_0^0 into the appropriate places in the integrals in the RHS of (B2a), we obtain

$$\dot{c}(x_0^0) = \int_{x_0^0 - L/2}^{x_0^0} a(y) D_2^N[e(y), x_0^0 - y]dy - \int_{x_0^0}^{x_0^0 + L/2} a(y) D_2^S[e(y), y - x_0^0]dy$$

Note that for y>L, *a(y)=a(y-L)*. Because there is no industrial activity in the residential zone $a(y) = 0, \ for\, x_0^0 \le y \le L + x_2^0$. Substituting this term into the second integral in the above expression yields

$$\int_{x_0^0}^{x_0^0 + L/2} a(y) D_2^S[e(y), y - x_0^0]dy = \int_{x_2^0}^{x_0^0 + L/2} a(y) D_2^S[e(y), y - x_0^0]dy = \int_{x_2^0 + L/2}^{x_3^0} a(y) D_2[e(y), x_0^0 - y]dy$$

Where the last equality is a result of the symmetry between the south and north directions due to assumption 4.1. It should be noted that if $x_2 \ge x_0 + L/2$ the last two terms in the above chain do not exist $\dot{c}(x_0^0) = \int_{x_0^0 - L/2}^{x_1^0 + L/2} a(y) D_2[e(y), x_0^0 - y]dy$ and the first term disappears. Substituting the above chain into $\dot{c}(x_0^0)$ yields $\dot{c}(x_0^0) = \int_{x_0^0 - L/2}^{x_1^0 + L/2} a(y) D_2[e(y), x_0^0 - y]dy$, and by substituting this expression into ($\dot{c}(x_0^0)$) we obtain,

(B6a) $$\dot{R}_h(x_0^0) = -\eta(x_0^0) \int_{x_0^0 - L/2}^{x_1^0 + L/2} a(y) D_2[e(y), x_0^0 - y]dy > 0$$

The sign in (B6a) results from $\eta(x)$ being positive (see (13)) and D_2 negative. Note that if $x_0^0 = x_3^0$, i.e. there is no buffer zone, (B6a) is also the value of $\dot{R}_h(x_3^0)$. However if $x_0^0 > x_3^0$, namely a buffer zone exists, $R_h(x_3^0) = 0$ *and* $\dot{R}_h(x_3^0) = 0$ since $b(x_3^0)=0$.

Consider now $\dot{R}_I(x_3^0)$. By substituting *V=0* and utilizing the symmetry assumption, the first line in the RHS of Eq. (B1) disappears. Next consider $\eta(y)$, the shadow price of concentrations. Equation (14) with *b(x)=0* outside the residential zone, imply

$\eta(y) = 0,\ for\, x_1^0 < y < x_0^0$. Upon substituting it into the second line in the RHS of (B1) we obtain,

(B6b) $$\dot{R}_I(x_3^0) = \int_{x_0^0}^{x_1^0+L} \eta(y) D_2^N[e(x_3^0), y - x_3^0]dy < 0$$

Note that $\dot{R}_I(x_0^0)$ is negative and equal to $\dot{R}_I(x_3^0)$ only when $e(x_0^0)$ is positive which happens when there are no buffer zones and $x_0^0 = x_3^0$. Otherwise $e(x_0^0) = 0,\ x_0^0 \neq x_3^0$, buffer zones exist and $\dot{R}_I(x_0^0) = 0$.

Since x_0^0 and x_3^0 are boundary points they are also points of intersection of bid rent curves and as such satisfy the bid rent rule. Equations (B6a) and (B6b) imply that north of x_0^0 housing outbids the industry and south of x_3^0 industry outbids housing. In complete symmetry to the case of x_0^0 and x_3^0, we obtain the expressions below for the slopes of the bid rent functions in x_1^0 and x_2^0,

(B6c) $$\dot{R}_h(x_1^0) = \eta(x_1^0)\int_{x_0^0 - L/2}^{x_1^0 + L/2} a(y) D_2[e(y), y - x_1^0]dy < 0$$

and if $x_2^0 = x_1^0$ and there is no buffer zone, (B6c) is also the value of $\dot{R}_h(x_2^0)$. However if $x_2^0 > x_1^0$, $\dot{R}_h(x_2^0) = 0$. Similar arguments also lead to

(B6d) $$\dot{R}_I(x_2^0) = -\int_{x_0^0}^{x_1^0+L} \eta(y) D_2^S[e(x_2^0), x_2^0 + L - y]dy \geq 0$$

where as before, for $y>L$, $\eta(y) = \eta(y - L)$ and $\dot{R}_I(x_1^0) = \begin{cases} \dot{R}_I(x_2^0) & if\ x_2^0 = x_1^0 \\ 0 & otherwise \end{cases}$. Eqs. (B6c) and (B6d) imply that since x_1^0 and x_2^0 are boundary points, hence left of x_1^0 housing outbids the industry and right of x_2^0 the industry outbids housing.

In a similar way it can be established that $\dot{R}_h(x)$ is positive for $x_0^0 \leq x < L$ and negative for $0 < x \leq x_1^0$ and $\dot{R}_I(x)$ is positive for $x_2^0 \leq x < L/2$ and negative for $L/2 < x \leq x_3^0$. It follows that $R_h(x)$ is positive in the residential zone and $R_I(x)$ is positive in the industrial zone. Consequently Eqs. (18) and (19) imply that all land is fully occupied in these two zones.

Appendix C.

Proof Of Lemma 5.1.

The commuting cost parameter *V*, appears in the necessary conditions explicitly, only in the expression of $\dot{\psi}$ (Eq. (10)). We already established that choosing the NC point of the residential zone as the origin, makes *T(x)* positive north of the origin up to the second NC point at *L/2*, from which point on *sign(T(x))* is negative, up to x=L. Substituting +1 and -1 for *sign(T(x))* in the appropriate places in (10) yields, $\dot{\psi}(x) = \begin{Bmatrix} V \text{ for } 0 < x < L/2 \\ -V \text{ for } L/2 < x < L \end{Bmatrix}$ which upon integration yields (22). From (9) we learn that $\psi(x)$ in the industrial zone is equal to the wages paid at x and from (12), the budget constraint, we learn that in the residential zone, $\psi(x)$ the LNE, is the household earned income after commuting costs have been deducted . From (22) it is clear that the highest wages are paid at $x=L/2$ (O' at Fig. 3). In the residential zone, $\psi(x)$ the LNE, is independent of work location and depends only on the place of residence.

Proof of corollary 5.1.1

By differentiating (22) with respect to *V*, we obtain

$$\text{(C1)} \quad \frac{\partial\psi(x)}{\partial V} = \partial\psi(x_2,V)/\partial V + (x - x_2)\cdot, \quad \text{for } 0 \le x \le L/2$$

where x_2 is the boundary of the industrial zone and the term $\partial\psi(x_2)/\partial V$ represents the change in the wage rate there. It is a shift parameter which has the same effect everywhere in the city and is therefore independent of location.

Proof of Lemma 5.2

The generalized Henry George rule (see Arnott 1979b) implies that the net city surplus *S*, satisfies $S = \int_0^L r(x)dx + \int_0^L Q(x)dx$, (see also Hochman and Ofek 1979) where

$r(x)$ the rent function, equals $R^h(x)$ in the residential zone and $R^I(x)$ at the industrial zone. Therefore the envelope theorem implies $\frac{\partial}{\partial t_i}\int_0^L r(x)dx = -\frac{\partial}{\partial t_i}\int_0^L Q(x)dx$ where t_i is any control variables or shadow prices except for *a(x)* and *b(x)* whose derivatives are everywhere zero except at the boundary points in which they are discontinuous. Out of these variables only $\eta(y)$ and $e(x)$ appear in *Q(x)*. Thus since in the residential zone, where *Q(x)* is zero, $r(x) = R_h(x)$ we have $\partial R_h(x)/\partial t(x) = -\partial Q(x)/\partial t(x)$=0, and in the industrial zone where $r(x) = R_I(x)$, the non zero differentials are $\partial R_I(x)/\partial \eta(y) = -\partial Q(x)/\partial \eta(y)$ and $\partial R_I(x)/\partial e(x) = -\partial Q(x)/\partial e(x)$. However we observe in (13) that $\eta(y)$ is independent of *V,* and from the rest of the production equations so is e(x) (Actually $\psi(x)$ is the only variable which depends on *V* and it does not appear in *Q).* From (15b) we learn that *Q* is also directly independent of *V*, hence $\partial Q/\partial V$ =0. Consequently by differentiating (17) with respect to V we get ,

(C2) $dR_I(x)/dV = \partial R_I(x)/\partial V = -n(x)\,\partial\psi(x)/\partial V$

Similarly, by differentiating (18) we obtain

(C3) $dR_h(x)/dV = \partial R_h(x)/\partial V = {(\partial\psi(x)/\partial V)}\big/{h(x)}$

We just proved that (i) and (ii) in the Lemma hold. QED

<u>Proof of Lemma 5.3</u>

By differentiating (C2) with respect to *x* we obtain,

(C4)
$$\begin{aligned}\partial \dot{R}_I(x)/\partial V &= -n(x)\,\partial\dot{\psi}(x)/\partial V - \dot{n}(x)\,\partial\psi(x)/\partial V \\ &= -n(x)sign(T(x)) + \frac{\dot{n}(x)}{n(x)}\partial R_I(x)/\partial V\end{aligned}$$

and by differentiating (c3) we obtain,

(C5)
$$\begin{aligned}\partial \dot{R}_h(x)/\partial V &= \frac{1}{h(x)}(\partial\dot{\psi}(x)/\partial V - \frac{\dot{h}(x)}{h(x)}(\partial\psi(x)/\partial V)) \\ &= \frac{sign(T(x)) - \dot{h}(x)\,\partial R_h(x)/\partial V}{h(x)}\end{aligned}$$

We are still looking at the northern hemisphere in the two AA's case. Consider the RHS of the second equality in (C4), the first term is always negative and the second term has the sign of $\partial R_I(x)/\partial V$. This sign is essentially negative except in Pattern B near the boundary where R_I has increased with V and with it the whole term. Thus the whole of the RHS of (C4) may or may not have increased as well in this location, depending on the relative size of the two terms in it.

A similar argument holds for (C5) but with an opposite sign. QED.

Proof of Lemma 6.1:

We obtain (i) in the lemma by differentiating (6) with respect to V and utilizing the envelop theorem. When n approaches infinity, AA's become infinitesimal and therefore commuting costs approach zero. The solution then approaches the internal solution of pollution without zones (case II). An informal proof of (ii)[1] is as follows: An increase of n implies shorter commuting distances and shorter distances for pollution dispersion before concentrations reach residential land use. This implies that in two allocations with the same V, overall commuting costs will be lower and overall concentration levels will be higher in the allocation with more zones. Subsequently, the negative slope of $S^x(n,V)$ with respect to V is steeper the smaller n is; the reason being that an increase of V when commuting distances are longer is costlier and therefore causes a larger reduction in the surplus. To prove (iii) we should note that the smaller n is, the higher is $S^x(n,0)$, because commuting costs are zero for all n while concentrations are lower when n is smaller.

[1] A formal proof of these statements can be devised along the following lines. Consider (i) in the Lemma. Increasing the number of zones while keeping V constant, shorten commuting distances thus the highest values of $|T(x)|$ are replaced with lower absolute values. Accordingly the total value of the integral is reduce.