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J. P. Davidson and M. G. Davidson

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I. INTRODUCTION

Low lying energy levels of positive and negative parity in deformed even-even nuclei have been explained with some success by the work of Davydov[1,2] and Davidson[3,4] respectively, and co-authors. Reduced transition probability ratios predicted by the adiabatic or pure rotational models[1,3] are in reasonable agreement with experiment at least for transitions within and between what are usually called the ground state and "γ-vibrational" bands for the positive parity systems and their analogues in the negative parity systems (although in the asymmetric rotator models used here, both of these bands merge into a common rotational level sequence). Discrepancies between theory and experiment are usually accounted for by the perturbation mixing of the various bands both for energy differences and for ratios of reduced transition probabilities. However, recent experimental investigations of the high spin levels of deformed even-even nuclei indicate that such a perturbation approach will not account for the observed level structure[5] and that it is necessary to take the beta (or deformation) vibrations into account more exactly as is done in Refs. 2 and 4. Other observations of level structure and gamma ray branching ratios both in the rare earth deformed region[6] and in the actinide deformed region[7] suggest that the influence of the beta band mixing is an order of magnitude greater than that of the gamma band mixing. In particular in Sm^{152} the experimental branching ratio from the beta band to the ground state, $B(E2{:}212\rightarrow 211)/B(E2{:}212\rightarrow 011)$[8] is greater by a factor of two than predicted by the simple collective model.[6] This in itself would suggest that a perturbation approach to handling these vibrations is probably not adequate. Since the effects of γ-band admixtures are much smaller, at least in the ground state band, perturbation methods will be more nearly adequate to describe any deviations from theory for them. It is the purpose of this paper to examine the affects on gamma ray branching ratios of the beta-vibrations in deformed even-even nuclei by a more exact method than perturbation theory. We deal here principally with E2 transitions both within the positive parity and the negative parity bands; however, the analysis is sufficiently general that the numerical calculations reported can be easily extended to other electric transitions.

In Sec. II we outline the vibrational treatment of the problem and describe the resulting state functions both as a review and to fix the notation, while in Sec. III we examine the reduced transitions probabilities and appropriate electric quadrupole operators for both parities. Section IV is a comparison of theory and experiment.

II. THE VIBRATION PROBLEM

We begin by expanding the nuclear surface in the laboratory coordinate system

$$R(\theta,\phi) = R_0 \, [1 + \sum_{\mu} \alpha^*_{\lambda\mu} Y_{\lambda\mu}(\theta,\phi)] \qquad (1L)$$

λ being 2 for π^+ and 3 for π^- states. Applying small oscillation theory to such a surface yields a Hamiltonian of the form[9]

$$H_\lambda = \frac{1}{2} B_\lambda \sum_{\mu} |\dot{\alpha}_{\lambda\mu}|^2 + \frac{1}{2} C_\lambda \sum_{\mu} |\alpha_{\lambda\mu}|^2 , \quad H = \sum_{\lambda} H_\lambda \qquad (2)$$

We now transform to the body-fixed reference system where the surface (1L) is given by

$$R(\theta',\phi') = R_0[1 + \sum_\nu a^*_{\lambda\nu} Y_{\lambda\nu}(\theta',\phi')] \tag{1B}$$

and the expansion coefficients are related by

$$\alpha_{\lambda\mu} = \sum_\nu D^{\lambda *}_{\mu\nu}(\theta_i) a_{\lambda\nu}$$

the $D^\lambda_{\mu\nu}(\theta_i)$ being components of the (2I+1)-dimensional representation of the rotation group[10] and are functions of the Euler angles θ_i. It is helpful if the body expansion coefficient $a_{\lambda\mu}$ are parametrized as[4]

$$a_{\lambda\mu} = \beta_\lambda \sigma_{\lambda\mu} \tag{3}$$

the β_λ being the λ^{th} order deformation parameter and the asymmetry parameters, $\sigma_{\lambda\mu}$, can be subjected to the further requirement

$$\sum_\nu \sigma^2_{\lambda\mu} = 1. \tag{4}$$

For the quadrupole and octupole cases the $\sigma_{\lambda\mu}$ have the familiar form[9,3]

$$\sigma_{20} = \cos\gamma, \sqrt{2}\,\sigma_{2\pm2} = \sin\gamma, \quad \lambda = 2$$

$$\sigma_{30} = \cos\eta, \sqrt{2}\,\sigma_{3\pm2} = \sin\eta, \quad \lambda = 3$$

the others in each case being zero. (For $\lambda = 2$ this is a consequence of the degrees of freedom available while for $\lambda = 3$ it is a sufficient condition to diagonalize the inertial tensor.[4] While this latter reduction of the degrees of freedom may appear arbitrary it is supported by a recent calculation of

Soloviev et al.[11] which shows that the states associated with the $\lambda = 3$, $\mu = \pm 3$ degrees of freedom in deformed nuclei are almost pure two quasi-particle states and thus not to be associated with a collective model.

The Hamiltonian (2) so transformed consists, as is well known, of three terms: one representing rigid rotations, one vibrations and the third a rotation-vibration cross term. For the cases $\lambda = 2,3$ the latter term vanishes identically. In keeping with the desire to treat the deformation vibrations exactly leaving the effects of asymmetry vibrations to be treated as a perturbation the β_λ are taken to be variables while the $\sigma_{\lambda\mu}$ remain as fixed fitting parameters. Thus the generalized curvilinear coordinate space with respect to which the system is quanitzed contains the four variables θ_i, β_λ. The transformed Hamiltonian is[4]

$$H = -\frac{\hbar^2}{2B_\lambda}\left\{\frac{1}{\beta_\lambda^3}\frac{\partial}{\partial\beta_\lambda}\left(\beta_\lambda^3\frac{\partial}{\partial\beta_\lambda}\right) - \frac{1}{4\beta_\lambda^2}\sum_k I_k^2/\mathcal{J}_k^\lambda\right\} + \frac{1}{2}C_\lambda(\beta_\lambda - \beta_{\lambda o})^2$$

where the potential term of the Hamiltonian (2) has been generalized to permit oscillations about a non-spherical deformation specified by $\beta_{\lambda o}$, $\underset{\sim}{I}$ is the angular momentum operator in the body and $\mathcal{J}_k^\lambda = 4B_\lambda\beta_\lambda^2\mathcal{J}_k^\lambda$ are the principal moments of inertia whose form is known.[9,3]

The Schrödinger equation separates into a rotational part

$$\left[\frac{1}{2}\sum_k \frac{I_k^2}{\mathcal{J}_k^\lambda} - \epsilon_{IN}^\lambda\right]\phi_{IN}(\theta_i) = 0$$

where the rotational eigenvalues have been given in tabular form for various values of the spin as a function of γ for the quadrupole case[12] and as a function of η for the octupole case.[4]

The vibrational Schrödinger equation is

$$\left[-\frac{\hbar^2}{2B_\lambda}\frac{1}{\beta_\lambda^3}\frac{\partial}{\partial\beta_\lambda}\left(\beta_\lambda^3\frac{\partial}{\partial\beta_\lambda}\right)+\frac{\hbar^2}{4B_\lambda\,\beta_\lambda^2}\,\epsilon_{IN}^{\lambda}+\frac{1}{2}C_\lambda\,(\beta_\lambda-\beta_{\lambda 0})^2\right]\Phi_n^{IN}(\beta_\lambda)$$

$$=E_{INn}^{\lambda}\,\Phi_n^{IN}(\beta_\lambda)\quad . \tag{5}$$

By expanding the second and third terms of (5) about the new equilibrium position β_λ (IN), keeping only the harmonic term for the new potential and defining a new independent variable y by

$$y = Z_1\,\frac{\beta_\lambda-\beta_\lambda(IN)}{\beta_\lambda(IN)}$$

and dependent variable $D_\nu(\sqrt{2}\,y)$ by

$$D_\nu(\sqrt{2}\,y)=\beta_\lambda^{3/2}\,\Phi_n^{IN}(\beta_\lambda)$$

then (5) can be placed in the form

$$\frac{d^2\,D_\nu(\sqrt{2}\,y)}{d\,y^2}+(2\nu+1-y^2)\,D_\nu(\sqrt{2}\,y)=0$$

which is just Weber's equation.[13] Here ν is a real, but not necessarily integral, quantum number determined by the boundary condition

$$\Phi_n^{IN}(\beta_\lambda=0)\neq\infty\quad ,$$

and Z_1 is a known function of ν.[4]

III. REDUCED E2 TRANSITION PROBABILITIES

For the reduced transition probability we use[14]

$$B_{\ell}\ (INn \rightarrow I'N'n') = \frac{1}{2I+1} \sum_{MM'} |\langle I'N'n'M'| Q^{Lab}_{\ell_\mu} |INnM\rangle|^2 \qquad (6)$$

where B_ℓ is the ℓ^{th} order transition from the state INn to the state I'N'n' and $Q^{Lab}_{\ell\mu}$ is the appropriate transition operator as seen in the space-fixed or laboratory coordinate system. The state vectors used are products of the rotation and vibration functions discussed above. The laboratory transition operator is related to the body-fixed operator by

$$Q^{Lab}_{\ell\mu} = \sum_{\nu} Q^{body}_{\ell\nu}\ D^{\ell *}_{\mu\nu}(\theta_i) \ .$$

The electric quadrupole operator in the body-fixed coordinate system is

$$Q_{2\mu} = [\frac{4\pi}{5}]^{1/2} \int r^2\ Y_{2\mu}(\theta,\phi)\ \rho_e(r)\ d\tau \qquad (7)$$

where ρ_e is the static charge density and the integration is over the nuclear volume, or $\rho_e = 3Ze/4\pi R_o^3$, and integrating from zero to the nuclear surface in r and using (1B) Eq. (7) becomes

$$Q_{2\mu} = \frac{3ZeR_0^2}{4\pi} \sqrt{\frac{4\pi}{5}}\ \{a_{\lambda\nu}\ \delta_{2\lambda} + 2 \sum_{\mu'} a_{\lambda\mu'}\ a_{\lambda\mu'-\mu}$$

$$\times\ C(2\lambda\lambda:\mu,\mu'-\mu,\mu')\ C(2\lambda\lambda:000)\} \qquad (8)$$

where the $C(I_1I_2I:\mu_1\mu_2\mu)$ coefficients are Clebsch-Gordan Coefficients.[10] For the positive parity case (8) becomes

$$Q^{(2)}_{2\mu} = \frac{3ZeR_0^2}{4\pi}\left[\frac{4\pi}{5}\right]^{\frac{1}{2}} a_{2\mu} \tag{9a}$$

while for the negative parity case

$$Q^{(3)}_{2\mu} = -\sqrt{\frac{3}{5}}\,\frac{ZeR_0^2}{\pi}\sum_{\mu'} a_{3\mu'}\, a_{3\mu-\mu'}\, C(233;\mu,\mu'-\mu,\mu'). \tag{9b}$$

The $a_{\lambda\mu}$ are taken as real and written in the form (3). Substituting the quadrupole operator (9a) or (9b) into equation (6)

$$B(E2:INn\rightarrow I'N'n') = \frac{1}{2I+1}\sum_{MM'}\left|\sum_{\nu}\langle I'N'M'|D^2_{\mu\nu}g_{\nu}(\sigma_{\lambda\rho})\, INM\rangle\right|^2$$

$$\times\ \left|\langle^{I'N'}_{n'}|f(\beta_\lambda)|\phi^{IN}_{n}\rangle\right|^2 \tag{10}$$

$$\equiv B_a\,(E2:\ IN\rightarrow I'N')\ S_{\nu\nu'}\ .$$

Here B_a (E2: IN→ I'N') is the adiabatic or pure rotational reduced transition probability and $S_{\nu\nu'}$ is the vibration contribution. The functions $g_\nu(\sigma_{\lambda\rho})$ and $f(\beta_\lambda)$ are those functions of the asymmetry and deformation parameters respectively which result from expressing the quadrupole operator in terms of the collective parameters: in particular $f(\beta_2) = \beta$ and $f(\beta_3) = \beta_3^2 = \zeta^2$.

The rotational contribution is well known as a sum over Clebsch-Gordan coefficients in each case,[1,3] and has been machine calculated for numerous sets (IN,I'N') as a function of the appropriate asymmetry parameters.

The vibrational contribution can be written in the form

$$S^{1/2}_{\nu\nu'} = N_\nu N_{\nu'}\int_0^\infty D_\nu\left(\sqrt{2}\left[\frac{Z_1}{\beta_\nu}\beta_\lambda - Z_1\right]\right)\beta_\lambda^M\, D_{\nu'}\left(\sqrt{2}\left[\frac{Z_1}{\beta_{\nu'}}\beta_\lambda - Z_1'\right]\right) d\beta_\lambda$$

where

$$M = \begin{cases} 1, & \lambda = 2, \pi^+ \\ 2, & \lambda = 3, \pi^- \end{cases}$$

N is a normalization constant and can be written in terms of these same parameters and a normalization integral I_ν as

$$N_\nu^2 = Z_1/\mu\beta_0 \, Z \, I_\nu$$

μ being the stiffness parameter of the nucleus, being (apart from a factor $\sqrt{2}$) the ratio of the deformation of a pure vibrator to that of a rotor-vibrator and Z is the positive real root of

$$Z^4 - \frac{1}{\mu} Z^3 - \frac{1}{2} (\epsilon_{IN}^{\lambda} + \frac{3}{2}) = 0,$$

and $\beta_\nu = \beta_\nu(IN)$ the new equilibrium deformation. By defining the ratio $R_z \equiv Z_1'Z/Z_1Z'$ we can rewrite the vibrational contribution as

$$S = (\frac{Z_1 Z_1'}{ZZ' I_\nu I_{\nu'}})(\frac{Z}{Z_1})^{2M+2} (\mu\beta_0)^{2M} I_{\nu\nu'}^2,$$

with

$$I_{\nu\nu'} = \int_0^\infty D_\nu(\sqrt{2}[y-Z_1]) \, y^M \, D_{\nu'}(\sqrt{2}[R_z y - Z_1']) \, dy$$

In actual practice we calculate only the ratios of the reduced matrix elements so that we need evaluate only

$$\frac{S_{\nu\nu'}}{S_{\nu\nu''}} = (\frac{Z_1' \, Z''}{Z_1' \, Z_1''}) \frac{I_{\nu''}}{I_{\nu'}} \frac{I_{\nu\nu'}^2}{I_{\nu\nu''}^2} \quad .$$

Since the Weber functions are in general not available in tabular form these integrals have been calculated numerically by computer.

IV. DISCUSSION

In Figs. 1, 2, 3 and 4 are displayed the ratio of complete reduced matrix elements for E2 transitions both within a vibrational band ($\Delta n = 0$) and between two adjacent bands ($\Delta n = 1$) both for positive and negative parity states. They are plotted as a function of γ and μ for transitions between positive parity states and as a function of η and μ for transitions between negative parity states. For μ zero the curves represent the ratios for a rigid nucleus.[1,3] Figure 1 is the ratio of the reduced matrix elements $B(E2{:}221 \rightarrow 011)/B(E2{:}221 \rightarrow 211)$ from the "gamma" to the ground state band. It is a strong function of γ but shows only a slight μ dependence. It is plotted only to $\gamma = 5^{\circ}$ since the 221 energy becomes infinite as γ vanishes. Figure 2 is the transition ratio $B(E2{:}212 \rightarrow 011)/B(E2{:}212 \rightarrow 411)$ from the beta-vibrational band to the ground state band. This ratio shows only a slight γ dependence but a strong μ dependence. This is a general feature of transition ratios between the bata and ground bands.

Figures 3 and 4 are similar ratios of E2 transitions between negative parity bands. Figure 3 is the ratio $B(E2{:}321 \rightarrow 311)/B(E2{:}321 \rightarrow 111)$ from the negative parity analog of the "gamma"-band (sometimes called the "g"-band) to the ground state negative parity band. This ratio is a strong function of the octupole asymmetry parameter η but shows only a slight μ dependence. The opposite situation is shown in Fig. 4 which gives the interband transition ratio $B(E2{:}312 \rightarrow 111)/B(E2{:}312 \rightarrow 112)$, that is for transitions from the zeta-vibrational band (the octupole analogue of the beta-vibrational band) to the ground state negative parity band. A strong μ and a slight η dependence is evident.

Figure 5 represents the ratio of Coulomb excitation from the ground state to the first 2+ states in the beta and ground state bands, that is the ratio B(E2:011→ 212)/B(E2:011→ 211). As with other interband transitions the μ dependence is much more marked that is the γ dependence. This figure also shows several recently measured Coulomb excitation ratios and quoted errors[20] for nuclei near the lower edge of the rare earth deformed region. The values of μ have been assigned in each case from the ratio of the energy of the beta band to the energy of the first excited state (i.e. from E(012)/E(211)).

In table I we have compared this theory with experiment[6,15,16,17,20] and the adiabatic ratios for several E2 transitions in both positive and negative parity bands in W^{182} and Th^{228} and interband transitions in Sm^{152}. In W^{182} there are two high lying 2+ slates below the first 3+ state either of which could be identified with the second 2+ state of the ground state-vibrational band. Choosing the lower 2+ state as the (212) level and the upper as the (221) state, which satisfies the model criterion

$$E(21) + E(22) \geq E(31), \tag{11}$$

yields a better fit to the level energies than the opposite choice; however, the fit to the E2 branching ratios, especially the ratios B(E2:221→ 211)/B(E2:221→ 411) and B(E2:212→ 211)/B(E2:212→ 411), are very poor. Thus we have chosen the lower 2+ state as the (221) state, which violates (11), and the other as the (212) state and obtained only a slightly poorer fit to the level energies while bringing the branching ratio predictions into line with the experimental data. The level designated here as the (221) level has been interpreted as a 1-level,[18] however, this assignment does not fit into the negative parity, collective model systematics.[3] Also Harmatz et al.[15] have made the assignment as we have and for similar reasons.

In Os^{186}, it has been noted,[19] one can fit the level energies at gamma of about 16.5° but the fit to the E2 branching ratios is quite poor. On the other hand, one can obtain reasonable branching ratio values including the vibrational effects with γ between 10° and 13° but then the fit to the energy levels is very poor. Unfortunately μ has been obtained only from the ground and gamma band energies which is the poorest method of determining this parameter. It is more uniquely fit from a knowledge of the (012) or (212) levels which are not identified experimentally. Until this is done it is only possible to state that for this end of the rare earth deformed region the model is not consistent with experiment. However, for the other end of this same region it is as is seen from the comparison between theory and experiment for Sm^{152} in Table I, and the Coulomb excitation data[20] shown in Fig. 5. It is clear then that an adequate test of these collective models must include the vibrational contributions both to the energy level systematics and the electromagnetic transition probabilities.

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APPENDIX

Here we outline the numerical methods used to evaluate the integrals involving the Weber's functions. The first integral considered is the normalization integral

$$I_\nu = \int_{-Z_1}^{\infty} D_\nu^2 (\sqrt{2}y)\, dy = \left[\int_{-Z_1}^{0} + \int_{0}^{\infty} \right] D_\nu^2 (\sqrt{2}y)\, dy \;.$$

The second integral on the right is available in closed form,[22] however, the first must be done numerically--the trapezoidal method is sufficient here.

The overlap integral

$$I_{\nu\nu'} = \int_0^\infty D_\nu (\sqrt{2}[y-Z_1])y^M D_{\nu'} (\sqrt{2}[R_Z y - Z_1']) \, dy$$

is written as

$$I_{\nu\nu'} = 2^{\frac{\nu+\nu'}{2}} \pi \int_{-Z_1}^{\infty} \exp - \frac{1}{2} (x^2 + [R_Z(x+Z_1) - Z_1']^2)(x + Z_1)^M$$

$$\times X_\nu (x) X_{\nu'} (R_Z[x+Z_1] - Z_1') \, dx \tag{A1}$$

where

$$X_\nu(x) = \frac{1}{\Gamma(-\nu/2)\,\Gamma(\frac{1}{2}-\nu/2)} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma(\frac{1}{2}k-\nu/2)(2x)^k}{k!} \;. \tag{A2}$$

The integral (A1) may be written as

$$\int_{-Z_1}^{\infty} e^{-x^2} g(x)\, dx = I$$

with

$$g(x) = \exp \frac{1}{2} (x^2 - [R_Z(x+Z_1)-Z_1']^2)(x+Z_1)^M$$

$$\times X_\nu(x)\, X_{\nu'}(R_Z[x+Z_1]-Z_1') \quad .$$

This integral is then evaluated by the Gauss-Hermite method of quadratures as

$$I = \sum_{j=1}^{N} H_j\, g(A_j)$$

where the weights H_j and the points A_j are available in tabular form for various values of N.[23]

The series form (A2) converges very slowly for large x; therefore for $x \gtrsim 4.5$ the asymptotic expression[13]

$$X_\nu(x) \sim \frac{x^\nu}{\sqrt{\pi}} [1 - \frac{\nu(\nu-1)}{x^2} + \frac{(\nu-1)(\nu-2)(\nu-3)}{2x^4} + \ldots]$$

has been used. For $x \lesssim -4.5$ it has been found that ν is integral so that the functions X_ν are, apart from a numerical factor, just the Hemite polynomials so that the series (A2) terminates.

FOOTNOTES AND REFERENCES

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Table I

	Exp.		Thy.	
Transitions	B(E2)	Ref.	B_a(E2)	B(E2)
Sm^{152}, π^+, $\gamma = 11.3^\circ$, $\mu = 0.396$				
$\frac{011\to 212}{011\to 211}$	0.023 ± 0.006	20	1.0	0.036
$\frac{212\to 411}{212\to 211}$	6.7 ± 1.8	6	1.81	4.22
$\frac{212\to 011}{212\to 411}$	0.048 ± 0.015	6	0.386	0.088
$\frac{221\to 011}{221\to 211}$	0.44 ± 0.02	21	0.502	0.453
W^{182}, π^+, $\gamma = 10.93^\circ$, $\mu = 0.28$[a]				
$\frac{221\to 011}{221\to 211}$	.69 .70	15 16	.522	.507
$\frac{221\to 211}{221\to 411}$	1. /b	15	11.52	10.96
$\frac{212\to 011}{212\to 211}$	.57	15	.708	.600
$\frac{212\to 211}{212\to 411}$	.833	15	.547	.405
$\frac{421\to 211}{421\to 411}$	.2	15	.158	.150
$\frac{421\to 411}{421\to 611}$	1. /b	15	5.21	4.98
$\frac{621\to 411}{621\to 611}$	.17	15	.068	.066
W^{182}, π^-, $\eta = 83.5^\circ$, $\mu = 1.0$				
$\frac{411\to 211}{411\to 311}$	.883 .556	15 16	.631	.563
$\frac{421\to 211}{421\to 311}$	1. 1.28	15 16	6.86	6.11

(continued)

Table I continued.

	Exp.		Thy.	
Transitions	B(E2)	Ref.	$B_a(E2)$	B(E2)
Th^{228}, π^+, $\gamma = 9.1^\circ$, $\mu = 0.30$				
$\frac{221\to 411}{221\to 211}$	.09 ± .02	17	.073	.080
$\frac{221\to 211}{221\to 011}$	2.32 ± .28	17	1.72	1.81
$\frac{421\to 611}{421\to 411}$	≤ .25	17	.151	.165
$\frac{421\to 411}{421\to 211}$	6.25 ± 0.8	17	4.66	5.12
$\frac{311\to 411}{311\to 211}$	.66 ± .08	17	.600	.658
Th^{228}, π^-, $\eta = 12.3^\circ$, $\mu = 0.258$				
$\frac{211\to 311}{211\to 111}$	<.3	17	.212	.215
$\frac{321\to 111}{321\to 311}$	0.36 ± .04	17	.502	.495
$\frac{411\to 511}{411\to 311}$	0.75 ± .02	17	.371	.379

[a]Experimental error are not given for the transition ratios of W^{182}.

[b]Unobserved.

TABLE CAPTION

Table I. Comparison between experiment and theory for E2 branching ratios of positive and negative parity bands of W^{182} and Th^{228} and for positive parity bands of Sm^{152}. The first column gives the initial and final states for the transitions where $I_i N_i n_i - I_f N_f n_f$, I is the spin of a level, N the ordinal of the level and n the ordinal of the vibration band. The second and third columns give experimental values and references for Sm^{152}, W^{182} and Th^{228}, the fourth column the adiabatic ratio while the fifth column gives the ratio including the vibrational contribution.

FIGURE CAPTIONS

Fig. 1. Ratio of reduced E2 transition probabilities for the two π^+ transition 221 to 011 and 221 to 211 plotted against the asymmetry parameter γ for various values of μ.

Fig. 2. Ratio of reduced E2 transition probabilities for the two π^+ interband transitions 212 to 011 and 212 to 411 plotted against the asymmetry parameter γ for various values of μ.

Fig. 3. Ratio of reduced E2 transition probabilities for the two π^- transitions 321 to 311 and 321 to 111 plotted against the asymmetry parameter η for various values of μ.

Fig. 4. Ratio of reduced E2 transition probabilities for the interband transition 312 to 111 and 312 to 112 plotted against the stiffness parameter μ for different values of η.

Fig. 5. Ratio of Coulomb excitation transition probabilities for the excitation of the lowest 2+ states in the ground and beta vibrational bands from the 0+ ground state plotted as a function of μ for different values of γ. The experimental values and errors for Nd^{150}, Sm^{152}, Gd^{154} and Gd^{156} are from ref. 20. The value of μ has been assigned from the energy ratio E(012)/E(211).

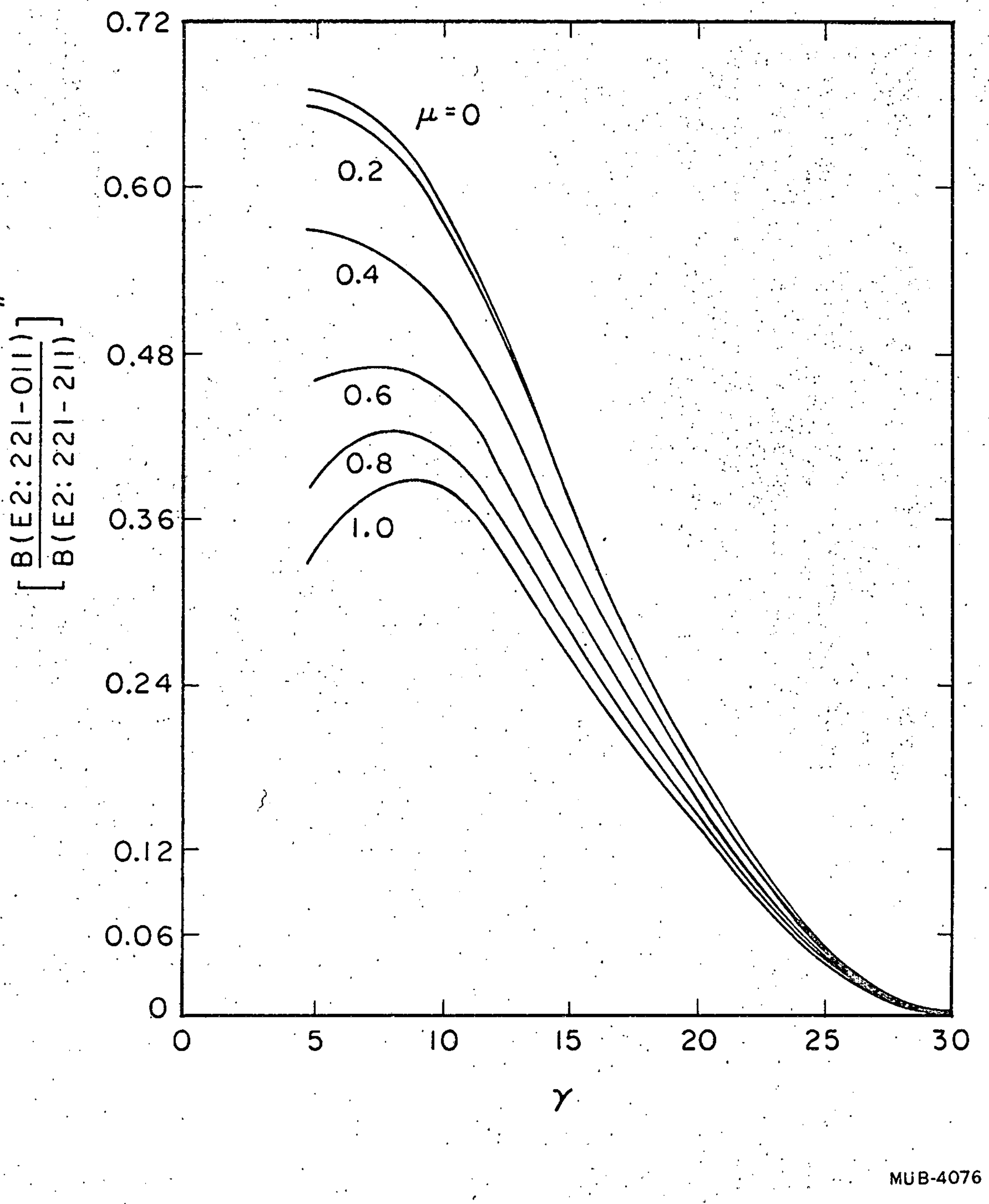
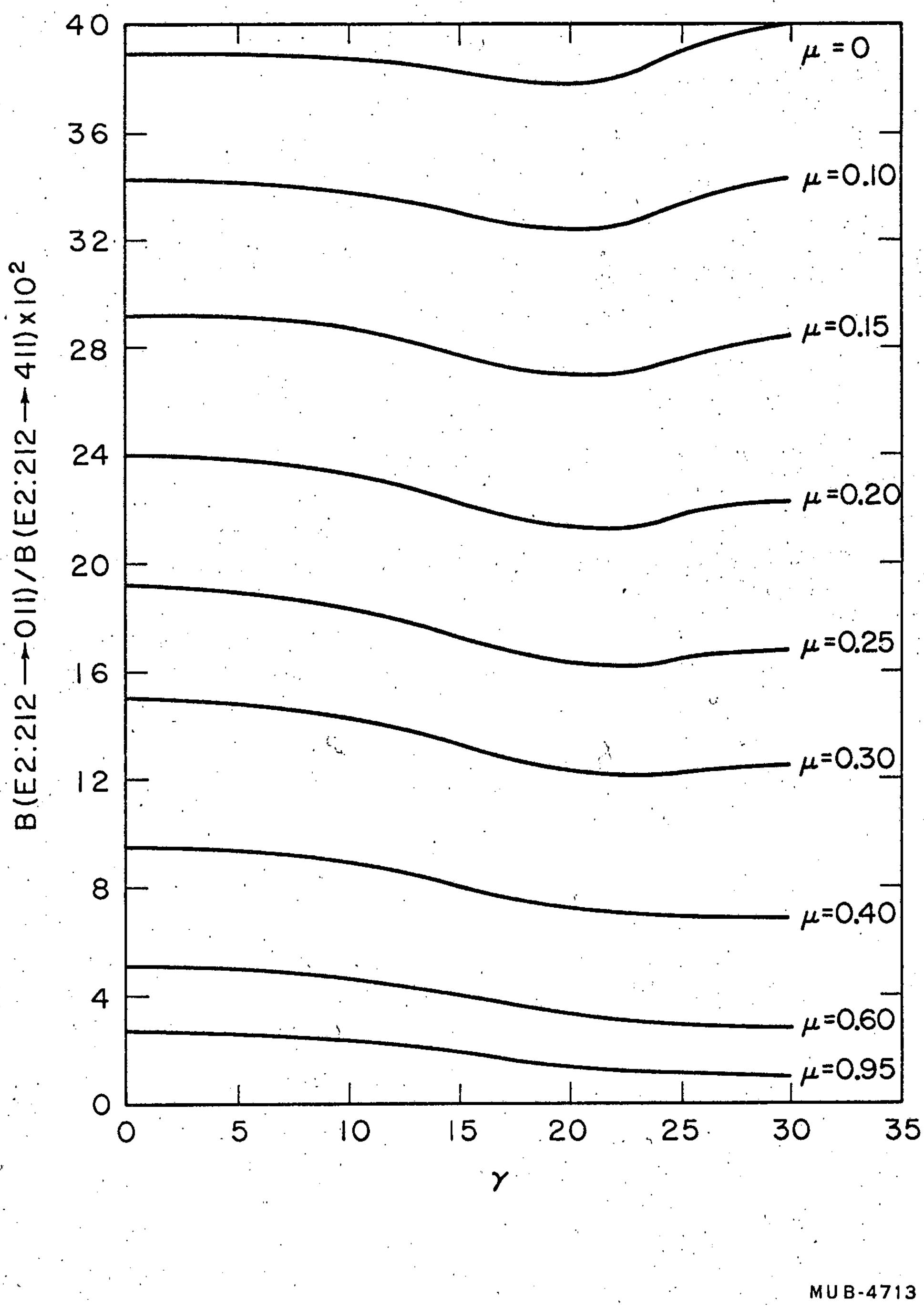

0.72
0.60
0.48
0.36
0.24
0.12
0.06
0
0
5
10
15
20
25
30
γ
$\left[\frac{B(E2: 221-011)}{B(E2: 221-211)}\right]^{\pi^+}$
μ = 0
0.2
0.4
0.6
0.8
1.0
MUB-4076

Fig. 1.



40
36
32
28
24
20
16
12
8
4
0
B(E2:212 ⟶ 011)/B(E2:212 ⟶ 411) x 10²
μ = 0
μ=0.10
μ=0.15
μ=0.20
μ=0.25
μ=0.30
μ=0.40
μ=0.60
μ=0.95
0
5
10
15
20
25
30
35
γ
MUB-4713

Fig. 2.

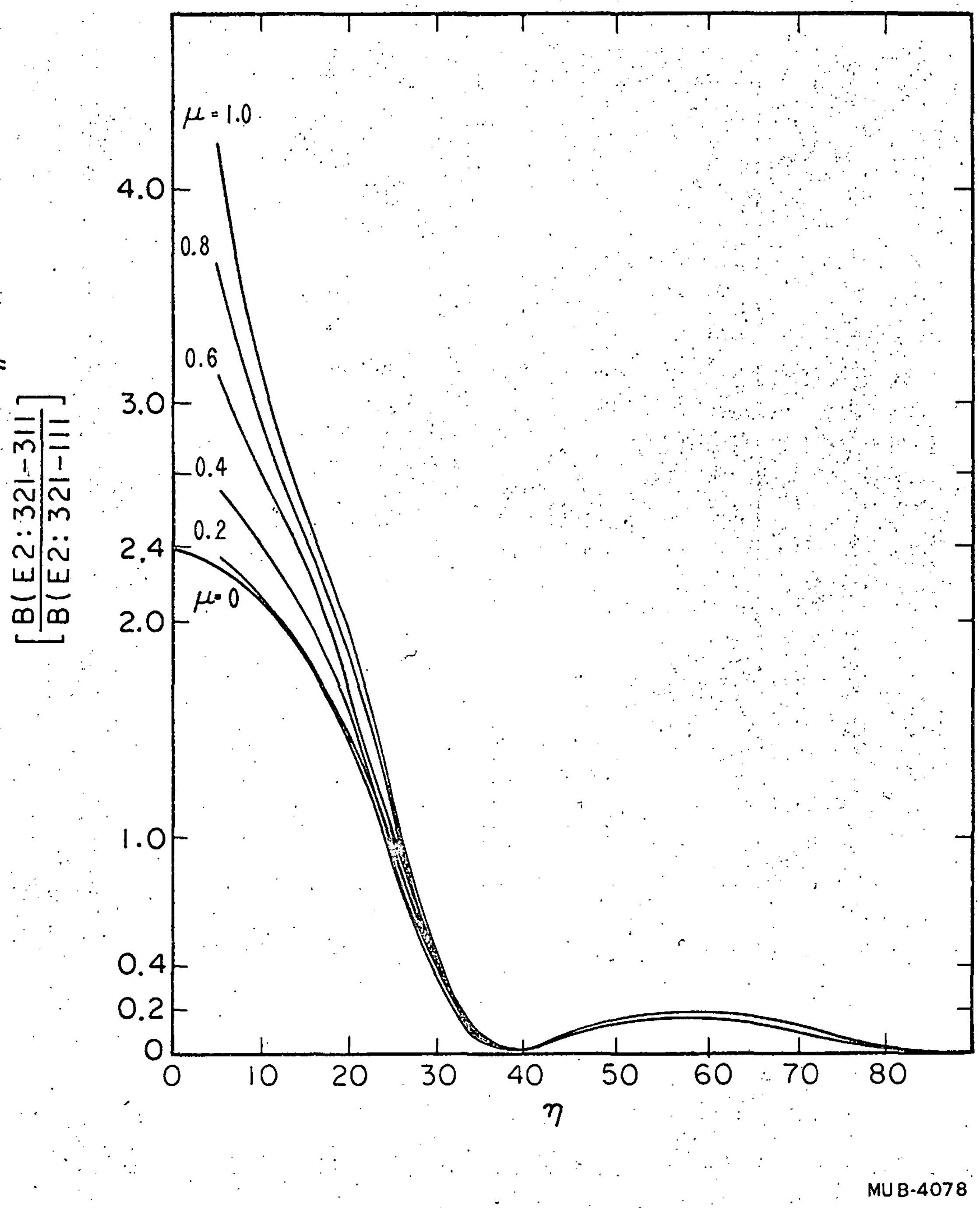

μ = 1.0
0.8
0.6
0.4
0.2
μ = 0
4.0
3.0
2.4
2.0
1.0
0.4
0.2
0
0
10
20
30
40
50
60
70
80
η
[B(E2: 321→311) / B(E2: 321→111)] π⁻
MUB-4078

Fig. 3.

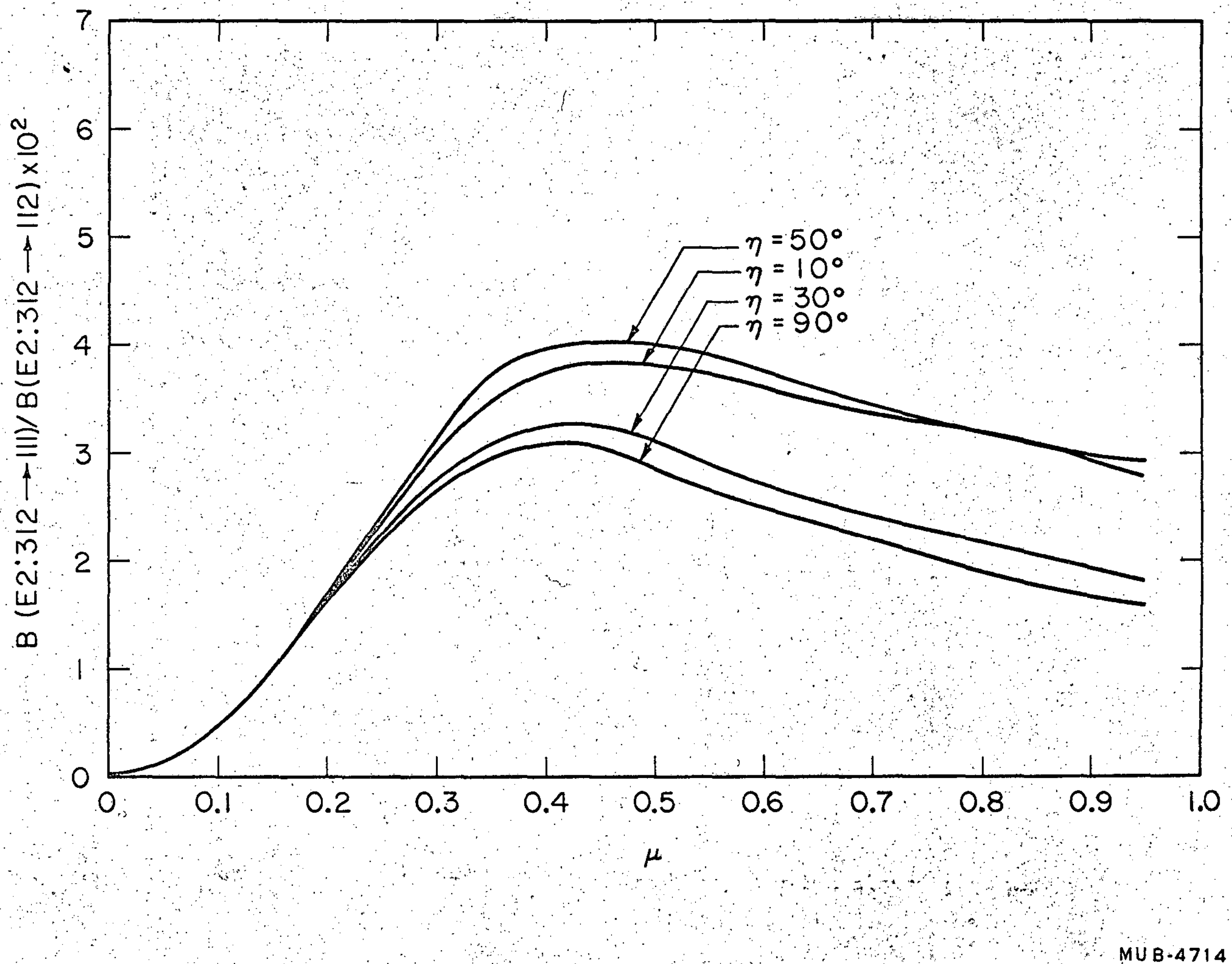

B (E2:312 ⟶ 111)/ B(E2:312 ⟶ 112) x 10²
0
1
2
3
4
5
6
7
0.
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
μ
η = 50°
η = 10°
η = 30°
η = 90°
MUB-4714

Fig. 4.

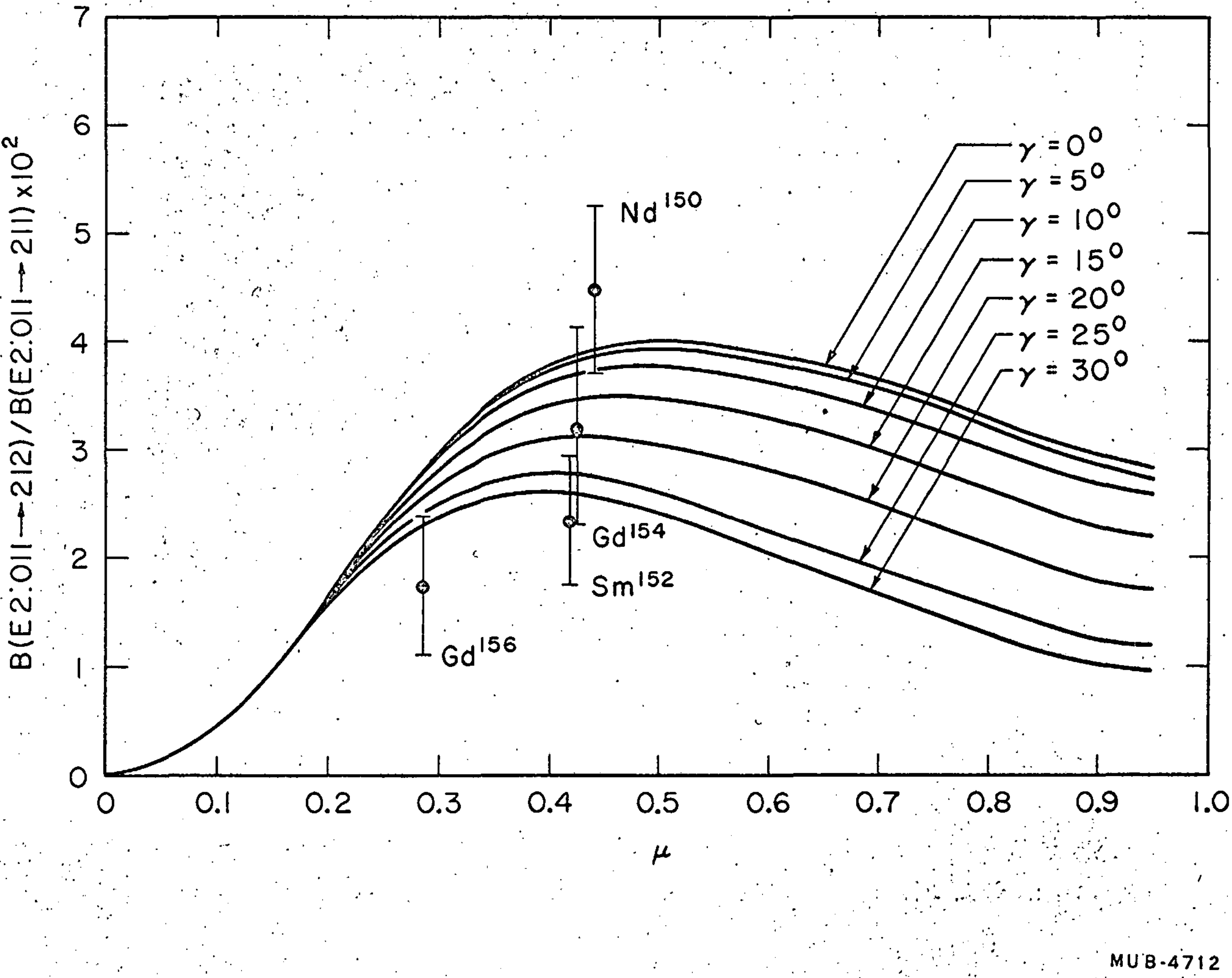

7
6
5
4
3
2
1
0
B(E2:011 → 212)/B(E2:011 → 211) x10²
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
μ
Nd¹⁵⁰
Gd¹⁵⁴
Sm¹⁵²
Gd¹⁵⁶
γ = 0°
γ = 5°
γ = 10°
γ = 15°
γ = 20°
γ = 25°
γ = 30°
MUB-4712

Fig. 5.