Prominence in Multi-Attribute Choice: A Drift Diffusion Analysis

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Abstract

We use hierarchical drift diffusion models to investigate the effect of prominence in two-alternative multi-attribute preferential choice. We find that two types of prominence effects, option-based and attribute-based, both increase choice probabilities for options favored by prominence. However, model fits suggest that the two effects work through different mechanisms. Altering choice option prominence leads to a response bias for the prominent option, whereas altering attribute prominence leads to an evaluation bias for the option that is dominant on the prominent attribute. Our results illustrate how seemingly identical contextual factors can be distinguished with the use of drift-diffusion modelling.

Keywords: drift diffusion model; multi-attribute choice; prominence effect

Introduction

Prominence effects are well documented in multi-attribute choice research (Armel et al., 2008; Atalay et al., 2012; Li & Epley, 2009; Nedungadi, 1990). These effects pertain to changes in choice behavior as the salience of information in the choice environment is altered. Understanding the cognitive underpinnings of prominence effects is necessary to fully characterize the psychological mechanisms involved in multi-attribute choice.

In this paper we study prominence through the lens of a drift diffusion model (DDM). The DDM is a mathematical model capable of predicting choice probabilities, reaction time (RT) distributions, and their relationships in two-option forced choice (e.g., Ratcliff, 1978). It has been successfully applied in a variety of perceptual and lexical choice tasks and is compatible with key insights in neuroscience (a recent review: Forstmann et al., 2016).

The core assumption of the DDM is that noisy evidence is accumulated dynamically over the course of the decision process. Figure 1 is a schematic representation of such a process. Evidence accumulation begins at a starting point (z), and increments based on a normally distributed signal. The mean accumulation speed is determined by the drift rate, v. The diffusion process is terminated as soon as one of the decision boundaries ($+a$ or $-a$) is hit. The specific decision boundary to be hit determines the chosen option. The time to hit the boundary, plus a non-decisional time, t_{ND}, corresponds to the reaction time (RT) of that trial. Note that $z = 0$ indicates a neutral starting point.

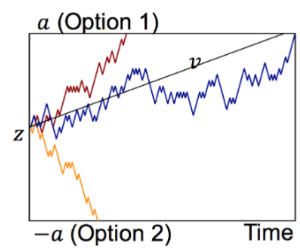

Figure 1 The drift diffusion model.

Each of the DDM parameters has an intuitive interpretation in terms of the psychological processes involved. Due to space constraints we summarize the parameter-process relationships as follows:

a: accuracy vs. speed motivation
z: a priori response bias
v: relative signal strength for evaluation
t_{ND}: non-decisional process

In this paper, we use these relationships to infer the cognitive components that are influenced by prominence manipulations in multi-attribute preferential choice. Our use of the DDM framework to model multi-attribute choice (rather than perceptual or lexical choice) necessitates some minor modifications: Instead of accumulating signal strengths, we assume that the DDM in preferential choice accumulates the relative utility of option 1 over option 2, i.e. $v = \Delta U = U_1 - U_2$.

Our use of the DDM reflects existing insights regarding the psychological determinants of multi-attribute choice. Firstly, the model assumes that preference is not static but constructed on the spot, as is argued by a number of existing theories (AAM: Bhatia, 2013; DFT: Busemeyer & Townsend, 1993; Query theory: Dinner et al., 2011, etc.) Secondly, some process components (e.g. the a priori bias) quickly influence the decision, prior to the stimuli being evaluated, whereas other components (e.g. the drift rate v) involve the gradual evaluation and aggregation of stimuli values over a period of time. This distinction mirrors the automatic vs. deliberative dichotomy proposed by dual process theories (e.g. Stanovich & West, 2000). Thirdly the choice probability predicted by the DDM is identical to the well-known logit choice rule (Luce, 1959) when $z = 0$, implying that the DDM can be seen as a dynamic extension of established approaches for modelling multi-attribute choice. Finally, past work has also demonstrated the descriptive power of the DDM in preferential choice research. For example, the DDM outperforms static models in predicting intertemporal choices (Dai & Busemeyer, 2014). Models that share several core assumptions with the DDM have also been shown to account for a set of important contextual effects (e.g. decoy effects and compromise effects) in multi-attribute choice (Bhatia, 2013; Roe et al., 2001).

The Prominence Task

The task we study involves a forced choice between two options (options 1 and 2) that vary on two attributes (X and Y). We denote the value of option i on attribute X as X_i, and the value of option i on attribute Y as Y_i. ΔX and ΔY denote the attribute value differences between option 1 and 2 on the two attributes. The choice information can be displayed in an attribute-by-option matrix as in Figure 2 (left panel).

We vary information prominence with a bright orange frame, and use this manipulation to test for two types of prominence effects. The first type of prominence manipulation involves highlighting both the attribute values of a single option. We refer to this as option-based prominence. The second type involves highlighting the attribute values for one attribute for both options. We refer to this as attribute-based prominence. Figure 2 (middle and right panels) illustrate these two types of manipulations.

As discussed earlier, the drift rate for preferential DDMs can be written as $v = \Delta U = U_1 - U_2$. Subjective utilities in multi-attribute choices are typically modeled as a weighted sum of the attribute values (e.g. Keeney & Raiffa, 1993). Therefore, without prominent options or attributes we obtain

$$\Delta U = (\omega_X X_1 + \omega_Y Y_1) - (\omega_X X_2 + \omega_Y Y_2) = \omega_X \Delta X + \omega_Y \Delta Y ,$$

where ω_X and ω_Y denote weights for attributes X and Y.

Before using the DDM to model prominence effects, we define three variables. The first (P_O) is an indicator for the option that is prominent. The second (P_A) is an indicator for the attribute that is prominent. Finally, as attribute-based prominence can disproportionality impact the option that is superior on the prominent attribute, we define a 3rd variable (P_{AD}) to indicate the option that is dominant on the prominent attribute.

$$P_O = \begin{cases} 1 & \text{if option 1 is prominent} \\ 0 & \text{if neither option is prominent} \\ -1 & \text{if option 2 is prominent} \end{cases}$$

$$P_A = \begin{cases} 1 & \text{if attribute X is prominent} \\ 0 & \text{if neither attribute is prominent} \\ -1 & \text{if attribute Y is prominent} \end{cases}$$

$$P_{AD} = \begin{cases} 1 & \text{if option 1 dominates on the prominent attribute} \\ 0 & \text{if neither attribute is prominent} \\ -1 & \text{if option 2 dominates on the prominent attribute} \end{cases}$$

In the left panel of Figure 2, $P_O = P_A = P_{AD} = 0$. In the middle panel, $P_O = 1$ and $P_A = P_{AD} = 0$. In the right panel, $P_O = 0$, $P_A = 1$, and $P_{AD} = \begin{cases} 1 \text{ if } X_1 > X_2 \\ -1 \text{ if } X_1 < X_2 \end{cases}$.

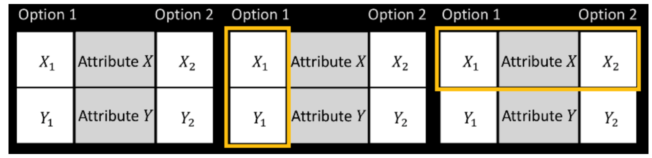

Figure 2 Choice task presentation. Left panel: no visual prominence manipulations. Middle panel: option-based prominence. Right panel: Attribute-based prominence.

Models

Next, we discuss how to model prominence effects in the DDM. There are two possibilities. First, prominence can be assumed to alter how prominent attributes are weighted. The effect of prominence within this approach depends on the attribute values offered in the choice problem. For this reason, we call this a stimuli-dependent approach. Second, the influence of prominence on the decision process can be assumed to influence choice independently of the attribute values. We refer to this as a stimuli-independent approach.

Stimuli-Dependent Approach

For option-based prominence, we assume that attribute values for the prominent options are given higher weights compared to attribute values for the non-prominent options. Suppose the weight change for attribute X is α and the weight change for attribute Y is β, then

$$\begin{aligned} v = {} & \omega_X\big((1+\alpha P_O)X_1 - (1-\alpha P_O)X_2\big) \\ & +\omega_Y\big((1+\beta P_O)Y_1 - (1-\beta P_O)Y_2\big) \\ = {} & \omega_X \Delta X + \omega_X \alpha P_O (X_1 + X_2) + \omega_Y \Delta Y + \omega_Y \beta P_O (Y_1 + Y_2). \end{aligned}$$

This way $\alpha, \beta > 0$ would indicate increased weights for the two attribute values of the prominent option and decreased weights for the two attribute values of the non-prominent option.

Next, we consider attribute-based prominence, which may increase the weights for the prominent attribute and decrease the weights for the non-prominent attribute. We denote the weight change for attribute X as δ and the weight change for attribute Y as γ. Therefore,

$$\begin{aligned} v = {} & \omega_X\big((1+\delta P_A)X_1 - (1+\delta P_A)X_2\big) \\ & +\omega_Y\big((1-\gamma P_A)Y_1 - (1-\gamma P_A)Y_2\big) \\ = {} & \omega_X \Delta X + \omega_X \delta P_A \Delta X + \omega_Y \Delta Y - \omega_Y \gamma P_A \Delta Y. \end{aligned}$$

In this case $\delta, \gamma > 0$ would suggest weight changes for attribute X and Y in the hypothesized directions.

Stimuli-Independent Approach.

In essence, the stimuli-dependent approach assumes a multiplicative effect on attribute values. However, in addition to this multiplicative effect, the DDM can also incorporate additive stimuli-independent effects. These effects pertain to biases in the starting point and the drift rate, biases that can be distinguished both theoretically and empirically (e.g. Ashby, 1983, White & Poldrack, 2014).

A shift in the starting point indicates an a priori bias that prepares the response before information evaluation. A shift in the drift rate influences how information is accumulated throughout the decision process. Hereafter we denote the response bias as B_R and the evaluation bias as B_E. Note that although we use the word *bias* here, we have no intention to suggest that the respective processes are irrational.

For option-based prominence, the model specification for a response bias can be written as $z = P_O B_R$. Hence $z = B_R$ when Option 1 is prominent (i.e. $P_O = 1$), $z = -B_R$ when option 2 is prominent (i.e. $P_O = -1$), and $z = 0$ when neither option is prominent (i.e. $P_O = 0$). Therefore, $B_R > 0$ indicates that the decision maker has a response bias towards choosing the prominent option, and has a neutral starting point when neither option is prominent.

Similarly, the evaluation bias B_E can be written as $v = P_O B_E + \Delta U$. Hence $v = B_E + \Delta U$ when option 1 is prominent, $v = -B_E + \Delta U$ when option 2 is prominent, and $v = \Delta U$ when neither option is prominent. Therefore, $B_E > 0$ indicates that the decision maker has an evaluation bias towards choosing the prominent option, and has no evaluation bias when neither option is prominent.

For attribute-based prominence, a stimuli-independent effect should favor the option dominant on the prominent attribute. Therefore, similarly to the approach in option-based prominence, a response bias can be modelled by specifying $z = P_{AD} B_R$, whereas the evaluation bias can be modelled by specifying $v = P_{AD} B_E + \Delta U$.

Summary

To summarize, the DDM can incorporate prominence effects using a stimuli-dependent (multiplicative) approach and/or a stimuli-independent (additive) approach. Table 1 provides a model summary with different combinations of possible mechanisms. The most flexible model allows for both stimuli-dependent and stimuli-independent influences of prominence (SDSI). Nested in that, we have models that either include only stimuli-dependent effects (SD) or only stimuli-independent effects (SI). Further nested in the SI model, we have a model with only evaluation biases (SIBE) and a model with only response biases (SIBR).

These models can be evaluated using goodness of fit to observed datasets. Additionally, as the DDM predicts qualitatively distinct choice-RT patterns for changes in drift rates (rows 1&2 of Table 1) vs. changes in starting points (row 3 of Table 1), choice-RT data can be used to rule out certain models. This alleviates the problems of using goodness-of-fit as a single piece of evidence for theory testing (Roberts & Pashler, 2000). Intuitively, a change in the drift rate is persistent throughout the evaluation process and hence can be observed in choices with both short and long RTs. However, a change in the starting point gets gradually washed out by the evaluation process. Therefore, the influence of a change in the starting point on choice probabilities is mainly observable in choices with shorter RTs. In the following sections, we test the models summarized in Table 1 using behavioral data collected from lab experiments, considering both quantitative goodness-of-fits measures and corresponding qualitative choice-RT patterns. Experiment 1 examined option-based prominence, and experiment 2 examined attribute-based prominence.

Table 1 Model specifications.

	SDSI	**SD**	**SI**	**SIBR**	**SIBE**
Stimuli-dependent weight change in v	X	X			
Stimuli-independent evaluation bias in v	X		X		X
Stimuli-independent response bias in z	X		X	X	

Experiment 1: Option-Based Prominence

Methods

47 participants were recruited from a paid participant pool at the University of Pennsylvania. Their task was to choose hotel options from choice pairs. The hotel options varied on two attributes (Attribute X: comfort; Attribute Y: Location). The two attributes were measured on a 6-10 scale. Participants chose their preferred options using keyboard presses. Responses and reaction times were recorded. Participants could take as long as they wanted before making choices, and were instructed to press keys as soon as they reached a decision.

In this experiment, the set of stimuli was pre-determined for all participants. As each choice problem can be uniquely characterized by the two attribute values of the two options, we randomly chose four one-decimal numbers from 6 to 10 for each problem. There were 72 unique problems in total. To control for the position effect, we counterbalanced the position of the two hotel options for each unique choice problem. To investigate the option-based prominence effect in this experiment, we implemented three conditions for each unique choice problem: (1) Option 1 was prominent ($P_O = 1$). (2) Neither of the options were prominent ($P_O = 0$). (3) Option 2 was prominent ($P_O = -1$). Thus, each unique choice problem repeated 6 times for a participant (2 positions * 3 prominence conditions). The 72*6=432 trials were distributed into blocks of 50 trials (the last block had 32).

Descriptive Results

We first compared the group-level choice probabilities in the three different prominence conditions. Note that as the display positions of the choice problems were counterbalanced, the choice probabilities for each option should have been 50% (the chance level). However, when option 1 was prominent, the mean choice probability for option 1 across participants was 52.2%, which deviated from the chance level significantly ($t(46) = 2.86, p = .006$). When option 2 was prominent, the mean choice probability for option 1 was 47.6%, again significantly deviating from the chance level ($t(46) = -4.13, p < .001$). When neither option was prominent, the mean choice probability for option 1 was 49.7%. The preliminary analysis suggests that making an option prominent increases choice probabilities for that option. To further understand the nature of this effect, we fit the DDMs to data.

Modeling results

The models were fit to choice and RT data using HDDM (Wiecki et al., 2013), a Python package for hierarchical Bayesian estimation of drift-diffusion models. This approach estimates group and individual level parameters simultaneously, with group-level parameters forming the prior distributions from which individual subject estimates are sampled. A recent study comparing HDDM with alternative estimation approaches showed that hierarchical fitting requires fewer data to recover parameters (Ratcliff & Childers, 2015; Wiecki et al., 2013). Moreover, the Bayesian approach permits direct inferences for parameter variability and parameter distribution. To fit the models, 4 chains of 50,000 samples were generated, where the first 25,000 were burn-ins, and a thinning of 2 was applied. The Gelman-Rubin convergence statistics for model parameters were all close to 1, suggesting that the sample size was sufficient for the chains to converge. Note that we also controlled for the position effect (left vs right) on choices in the starting point and the drift rate parameters, but the position results are not reported here for brevity.

Model comparisons: goodness-of-fit. The DDMs with different prominence effect specifications were compared using the deviance information criterion (DIC; Spiegelhalter et al., 2002), which measures the model fits while penalizing model complexity to avoid over-fitting. Smaller DICs indicates better model performance. Besides DICs, we also report the average deviance (the posterior mean of the model deviance) and the effective number of parameters p_D. p_D is calculated as the difference between the average deviance and the deviance of the model with the parameter posterior mean substituted in, and is a measure of model complexity.

The results are presented in Table 2. Across the model tested, SD and SIBE had relatively larger DICs and hence worse fits. Among these two models, SD included only stimuli-dependent effects of prominence, indicating that it is essential to include a stimuli-independent component to adequately fit data. SIBE included only a stimuli-independent evaluation bias. The bad fit of this model indicates that the response bias (B_R) was a more important stimuli-independent component than the evaluation bias (B_E). Unlike SD and SIBE, the other three models (SDSI, SI and SIBR), which all had a relatively good fit, all included a response bias. Moreover, the most complex model (SDSI) had a larger DIC and thus worse fit than the most constrained model (SIBR). The next most complex model (SI) had an almost identical DIC as SIBR ($\Delta < 0.9$). These results suggest that using only response bias (B_R) to account for option-based prominence effects in the DDM was sufficient, and that adding more mechanisms might led to over fitting.

Model comparisons: choice-RT patterns. As discussed before, a shift in the starting point has a larger influence in trials with shorter RTs compared to trials with longer RTs. In contrast, a shift in the drift rate influences choice probabilities in trials with both shorter and longer RTs (White & Poldrack, 2014). This qualitative distinction for choice-RT relationships could help disentangle whether option-based prominence influences the drift rate or whether it influences the starting point.

For each participant, we plot how choice probabilities vary across trials with shorter vs longer RTs under different prominence conditions (Figure 3 upper panels). As can be seen from the graphs, the observed choice probability differences were mainly present in trials with smaller RTs. In other words, the choice probabilities for option 1 were larger than 50% when option 1 was prominent (solid red lines), and smaller than 50% when option 2 was prominent (solid blue lines), only for quicker choices. The observed choice probabilities were almost 50% in trials with longer RTs across all 3 prominence conditions. This choice-RT relationship has the characteristics of a response bias effect. Not surprisingly, the relationship can be captured by the SI model (Figure 3 top-left panel dashed lines), and the constrained model with only a response bias (Figure 3 top-right dashed panel lines). However, the constrained model with only an evaluation bias fails to capture this relationship, and thus performs worse than the other two models (Figure 3 top-middle panel dashed lines). The choice-RT relationship serves as a behavior marker showing that the responses biases are the more important mechanism in option-based prominence, and explaining why including only the response bias in the DDM was sufficient to generate a good fit to the data.

Table 2 Model comparisons: DIC, average deviance and pD for the five candidate models in experiment 1 and experiment 2.

		SDSI	SD	SI	SIBR	SIBE
	DIC	59064	59144	59061	59062	59144
Exp 1	**Avg. Dev.**	58791	58886	58795	58800	58890
	p_D	273	258	266	263	254
	DIC	57133	57360	57140	57402	57136
Exp 2	**Avg. Dev.**	56864	57084	56882	57160	56890
	p_D	270	276	258	242	246

Experiment 2: Attribute-Based Prominence

Methods

In this experiment, we investigated attribute-based prominence. 42 participants were recruited. To make the experimental results comparable to experiment 1, the stimuli and the procedure of the experiment was kept the same, except for the information prominence manipulation, which was changed from option-based to attribute-based. Again, there were three conditions: (1) Attribute X (Comfort) was prominent ($P_A = 1$). (2) Neither of the attributes were prominent ($P_A = 0$). (3) Attribute Y (Location) was prominent ($P_A = -1$). In total, there were 432 trials (72 unique choice problems * 2 position counterbalance * 3 prominence conditions).

Descriptive Results

We first examined how group-level choice probabilities varied with the prominence conditions and the choice attributes. Both stimuli-dependent and stimuli-independent models predicted that the option dominating on the prominent attribute should be selected more frequently. Therefore, we clustered choice problems into 3 groups, on the basis of the option that was dominant on the prominent attribute (i.e. $P_{AD} = 1, -1$, or 0). When option 1 was dominant on the prominent attribute, it was chosen 56.9% of the time across participants. This is significantly larger than 50% ($t(41) = 6.08, p < .001$). When option 2 was dominant on the prominent attribute, it was chosen 43.1% of the time across participants, which is significantly smaller than 50% ($t(41) = -5.41, p < .001$). When neither option was dominant, the choice probability for option 1 was 50.1%. This preliminary result showed that attribute-based prominence made choice probabilities deviate from the chance level (50%).

Modeling results

The model fitting procedures were the same as experiment 1.

Model comparisons: goodness-of-fit. As Table 2 shows, SD and SIBR had relatively larger DICs and thus worse fits. As in experiment 1, the SD model with only stimuli-dependent effects failed to fit the data well. However, unlike experiment 1, the SIBR model with only a stimuli-independent response bias was not a good model either. The other three models performed closely in terms of DIC, and they all included the evaluation bias. These results suggest that using only evaluation bias (B_E) to account for attribute-based prominence effects might be sufficient, and other additional model specifications are not very useful once the evaluation bias has been included.

Model comparisons: choice-RT patterns. Observed choice probabilities for option 1 across the three conditions were different for the entire range of RTs. This is illustrated by the near parallel solid lines in Figure 3. This choice-RT relationship is compatible with a drift rate effect in the DDM. In line with this, the observed pattern is best captured by the SI model (Figure 3 bottom-left panel dashed lines), and the constrained model with only an evaluation bias (Figure 3 bottom-middle panel dashed lines). The constrained model with only a response bias fails to capture this relationship (Figure 3 bottom-middle panel dashed lines). The choice-RT relationship suggests that evaluation biases in the drift rate accounted for the effects of attribute-based prominence.

Comparison with experiment 1. We have shown that response biases are essential for explaining option-based prominence (experiment 1) and that evaluation biases are essential for explaining attribute-based prominence (experiment 2). Because the two experiments used essentially the same design and the same set of stimuli values, we could compare the parameter estimates across experiments. For brevity, we only compare individual-level posterior means of response biases and evaluation biases in the SI models (Figure 4). Response biases in experiment 1 were positive for most participants, and their magnitude was larger than responses biases in experiment 2. The opposite is the case for evaluation biases in experiments 1 and 2. This again shows that option-based prominence operates through response biases and attribute-based prominence operates through evaluation biases.

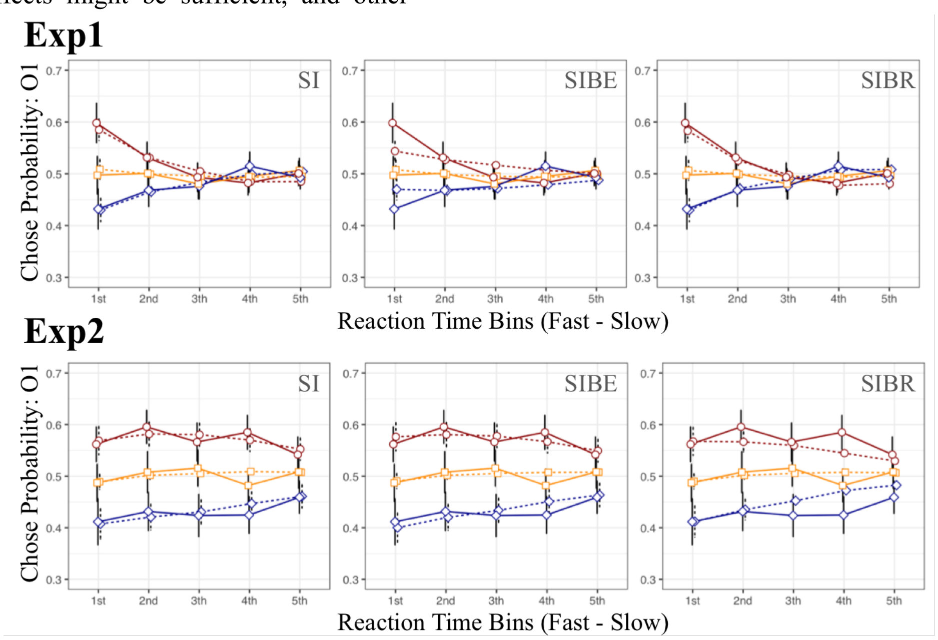

Figure 3 Choice-RT relationships for observed data (solid lines) and model simulated data (dashed lines). Here the x-axis indicates RTs, which were adjusted for choice attribute value differences and assorted into 5 bins. Trials with smaller (longer) adjusted RTs are on the left (right). The y-axis indicates choice probabilities for option 1. Red (blue) lines indicate the condition where option 1 (2) was favored by prominence. Yellow lines are for the neutral condition. The SI, SIBE, and SIBR model predictions are presented in the left, middle, and right panels respectively, and are based on 500 samples from their posterior predictive distributions. The error bars are 95% confidence intervals.

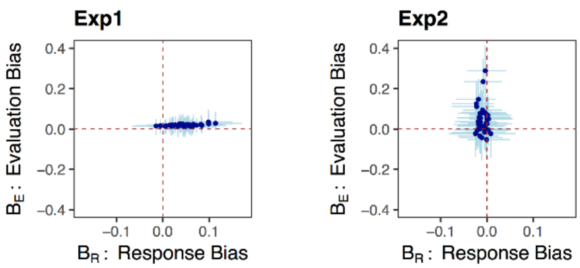

Figure 4 Individual level estimates for response biases and evaluation biases. Dots represent parameter posterior means and error bars indicate 95% credible intervals.

Discussion

In this paper we used hierarchical drift-diffusion models to investigate the effect of visual prominence on two-alternative multi-attribute preferential choices. Our results highlight the value of using mathematical models to simultaneously analyze choice probabilities and RTs in decision research. For both option-based and attribute-based visual prominence, the stimuli-independent models outperformed the stimuli-dependent ones. Both types of prominence effects increased choice probabilities for options favored by prominence. However, model fits indicated that the two effects worked through different mechanisms. Option-based visual prominence led to a response bias while attribute-based prominence led to an evaluation bias. These quantitative comparisons were accompanied by qualitatively different choice-RT patterns across the two types of prominence effects. Option-based prominence influenced choice primarily in trials with shorter RTs, whereas attribute-based prominence effects were present in trials with both shorter and longer RTs.

Although our study was on prominence effects in mufti-attribute choices, our methods could be easily extended to study other incidental factors and other types of preferential choices. The distinct choice-RT patterns found in our data may also be useful to classify the effects of different types of incidental factors in preferential choice.

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