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Competition with Social Externalities

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Competition with Social Externalities (preliminary version)

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Abstract

1   Introduction

In a large number of markets and transactions, the agents, when choosing between suppliers, have preferences over the identity of the other agents using the same suppliers. The examples are abound:

When consuming goods that involve social interaction, consumers naturally have preferences for the identity of other customers. This can be various types of clubs and dating sites. Another example regards choice of schools, where the other customers (students) is a pool both for social interaction and as basis for a social network that may be valuable in professional life. Even in restaurants and bars, low-key social interaction (as well as social identification) may imply that the customers have preferences over the types of consumers that visit the establishment.

Similar effects take place when choosing between platform providers. For instance, the attractiveness of connecting to Windows relative to Macintosh is influenced by the choices of other consumers. First there is a direct effect; it may be more convenient to use the same system as colleagues and business partners with whom the person in question has direct contact with. In addition, increasing returns to scale in providing applications imply that the number and types of applications available will depend on the number and preferences of the consumers connected to that platform. Thus, not only the number, but also the preferences of the other consumers matter for the attractiveness of a given platform.

Another industry where the "type" of consumers is important is in the transactions industry, like the banking and credit card industry. When choosing credit card, it is important not only to buy one with a large customer base, but also (for a given size) to chose one where the customers have the same trading habit as you have. This will increase the likelihood that the sellers you approach will accept that credit card (is Diners a good example here?). In banking it matters for a firm if its present and future trading partners, suppliers, customers etc. are likely to use the same bank, as this may reduce direct transaction costs as well as

problems associated with asymmetric information (the bank will receive more information about your trading partner and thereby learn about your credit worthiness. The bank may also be able to internalize external effects of default between suppliers and manufacturers. In addition, the customers may care about previous customers, as this may give information about the firms' areas of expertise.

Finally, the identity of consumers may be of importance in classical network industries, like telecommunication. Several telecommunication firms (particularly mobile phones) set different on-and off net prices. As a result, consumers have a preference for staying in the same network as the people they are communicating with.

Social preferences can naturally be divided into a vertical and a horizontal part. The vertical part, which is often the focus in for instance housing markets and models for provision of local public goods, relates to variables for which agents have the same preferences. The preferences may for instance relate to the income, social capital, human capital, beauty etc. of the potential peers. The horizontal part reflects idiosyncratic taste parameters and history. In a club setting, personal interests and taste are example of horizontal differences, occupation when choosing platform, industry when choosing bank, and family and friends when choosing mobile phone.

Existing literature: social interaction

Existing literature, network effects

2 Modelling social externalities

We analyze competition between two suppliers (or firms) of a good, supplier A and supplier B. The suppliers are horisontally differentiated along two dimensions. First, they are differentiated along a technological dimension. This is exogenous in the model. Second, they are differentiated regarding the set of consumers they attract. This will be determined endogenously.

We want to capture horizontal social preferences as described in the introduction. We do this two steps. First we assume that each consumer has a social location on the Salop circle, with circumference equal to two.[1] . Denote by $z_i \epsilon \Omega$ agent i's social location, where $\Omega = [-1, 1]$. Finally, let d denote a distance measure on Ω, defined as

$$d(z_i, z_j) = \min[|z_i - z_j|, 2 - |z_i - z_j|]$$

Thus $d(z_i, z_j)$ is the shortest distance between the two agents along the circle.

Let us give some examples. If the application at hand relates to membership in clubs, social location reflects preferences and interests. If it relates to the choice of platform (like Macintosh and Windows), the social location will be influenced by occupation and education. If the application at hand relates to banking, social location may reflect industry and business niche, while if it relates to mobile telephony it may be related to the position of your personal friends.

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[1] The motivation behind letting agents be distributed on the circle is to avoid the asymmetry associated with consumers on the end of a line that only communicate in one direction. Each consumer has a set of other agents with which he or she interact, hereafter called friends.

The second step regards the utility obtained by "social interaction" with the peers choosing the same supplier. The function $g : [-1, 1] \rightarrow [0, 1]$ shows agent i's preference for being in the same network as an agent at social distance d. We assume that g is strictly decreasing in d, reflecting that agents gain more from "being together" with people that are socially close than socially distant. We do not allow g to be negative. Finally, we assume that the value of social interaction is additiv, in the following sense: suppose a fraction $H(z)$ of the agents of social location z belongs to network A. Then the social utility of joining firm A for a person of location z_i can be written as $\int_\Omega g(d(z, z_i))H(z)dz$. We refer to this as the network utility associated with joining firm A.

Hence there is no crowding-out effects of membership. This seems to be a reasonable assumption for platforms, banks, and telephony, but maybe less so for social clubs, where the average member "type" may matter. Note also that this additivity property gives rise to increasing return to scale on the demand side (reference), but in contrast with the existing models with increasing returns to scale not only the number, but also the social location of the other customers matter for own utility.

The second element in our analysis relates to technological differentiation. We assume that two rivaling suppliers A and B offer horisontally differentiated products. We model technological preferences by the Hotelling line, where the suppliers are located at the end points of a line of unit length, while the consumers are located between them. Technological differences may reflect pure technological features, user-friendliness, and design. Macintosh and PCs have chosen different solutions, as have Playstation and X-box. Different mobile phone operators also offer services with different features that appeal to different segments of the market, and the Hotelling model is a workhorse when modelling competition within telecommunication. Finally, schools may offer different currucila and students may differ in their preference for these.[2]

A driving assumption in our analysis is that social and technical preferences may be related. People who are socially close are then more likely to share the same technological preferences. For instance, when choosing between Macintosh and PC, the technological solutions the platforms may be better suited for some professional tasks than others, and thus suit members of some professions better than others. People like one would prefer to socialize with may have similar interests as oneself regarding curriculum (schools) activities (clubs) and calling plans (mobile phones). More specifically, we assume that we can write the technological preference of an agent with social location z_i is given by

$$y_i = a|z_i| + (1 - a)\varepsilon_i \tag{1}$$

where ε_i is drawn from a uniform distribution on $[0, 1]$, i.i.d. for all agents, and the parameter a satisfies $0 \leq a \leq 1$. If $a = 0$ then y and z are independent, there is no correlation between social location and and if $a = 1$ the two variables are perfectly correlated. Finally, in technology space, all agents are located between the two platforms at 0 and 1. It follows

---

[2]Note that the social positions of the customers may shape the technological properties of the supplier. For instance, we argued that professionals of a given type prefer to stay on the same platform as their peers since this may increase the number of applications that will developed. In our set-up this is embodied in the g-function. Technological differences refer to pre-existing technological differences, i.e., differences between the products that are prsent before the social location of the customers are determined.

that the expected social location (conditioned on z) can be written as

$$Ey|z_i = a|z_i| + (1-a)/2$$

Thus $Ey|0 = (1-a)/2$ and $Ey|1 = (1+a)/2$, while $Ey|1/2 = 1/2$. Note the symmetry around $1/2$. The cumulative distribution function of y conditional on z, $F(y|z)$ can thus be written as

$$
\begin{aligned}
F(y|z) &= 0 && \text{if } y < a|z| \\
&= \frac{y - a|z|}{1 - a} && \text{if } a|z| \le y \le a|z| + 1 - a \\
&= 1 && \text{if } y > a|z| + 1 - a
\end{aligned}
\tag{2}
$$

Or, more compactly,

$$F(y|z) \equiv \max\left[\min\left[\frac{y - a|z|}{1-a}, 1\right], 0\right]$$

The pair (y_i, z_i) completely characterizes any given agent i.

The utility of an agent (y_i, z_i) by joining network A at price p_A is given by

$$u^A(y_i, z_i) = \alpha - ty_i + \int_\Omega g(d(z, z_i))H(z)dz - p_A \tag{3}$$

We have already defined the third term. The parameter t reflects the intensity of technological preferences, while α denote the intrinsic value of being connected to a platform. In what follows we assume that α is sufficiently big so that the entire market is covered. Analogously, we have that

$$u^B(y_i, z_i) = \alpha - t(1 - y_i) + \int_\Omega g(d(z, z_i))(1 - H(z))dz - p_B \tag{4}$$

Finally, define $\bar{g}$ as

$$\bar{g} = \int_\Omega g(d(z, z_i))dz$$

Note that $\bar{g}$ denotes the maximum network utility obtainable, obtained if all agents in the economy join the same supplier.

The timing of the model goes as follows:

1. The two firms A and B simultaneously and independently choose prices p_A and p_B, respectively. The firms are not able to price discriminate by setting different prices for agents with different locations at the circle.

2. The agents independently decide which firm to go to, given the prices and given their expectations about the choice of the other agents in the economy. In equilibrium, expectations are rational.

3   Equilibrium

In this section we derive the equilibrium of the model. We first solve the second stage of the game, which we refer to as the assignment game. Then we solve for the optimal prices given the equilibrium of the assignment game.

3.1   The assignment game

In this section we focus on the agents choice of network for given prices p_A and p_B.

For any given distribution $H_0(z_i)$ defined on $[-1,1]$,[3] let $y^m(z_i)$ denote the technological preference of an agent that is indifferent between the two platforms. From (3) and (4) it follows that

$$u^A(y^m(z_i), z_i) = u^B(y^m(z_i), z_i)$$

$$\Longleftrightarrow$$

$$ty^m(z_i) - \int g(d(z, z_i))H_0(z)dz = \frac{p_B - p_A - \overline{g} + t}{2} \tag{5}$$

or

$$y^m(z_i) = [\int g(d(z, z_i))H_0(z)dz + \frac{p_B - p_A - \overline{g} + t}{2}]/t \tag{6}$$

Let $H_1(z)$ denote the fraction of agents at social localization z that prefers the A-network given H_0, and write $H_1(z) = \Gamma H_0(z)$. In order to characterize Γ we use the fact that there is a close relationship between y^m and H_1. From (2) it follows that

$$
\begin{aligned}
\Gamma H(z_i) &= 0 & &\text{if } y^m(z_i) < a|z_i| \\
&= \frac{y^m(z_i) - a|z_i|}{1-a} & &\text{if } a|z_i| \le y \le a|z_i| + 1 - a \\
&= 1 & &\text{if } y^m(z_i) > a|z_i| + 1 - a
\end{aligned}
\tag{7}
$$

Or, more compactly,

$$H_1(z_i) = \Gamma H_0(z_i) = \max\left[\min\left[\frac{y^m(z_i) - a|z_i|}{1-a}, 1\right], 0\right] \tag{8}$$

Since (3) and (4) are continuous in y, it follows that $y^m(z_i)$ and thus $H_1(z_i)$ are continuous.

For given prices p_A and p_B, an *equilibrium distribution function* $H^e(z)$ is a fixed-point satisfying

$$H^e(z) = \Gamma H^e(z)$$

or by combining (6) and (8)

---

[3] We do ont require that $H(1) = H(-1)$. Below we show that for the equilibrium distribution, this is always the case.

$$\Gamma H(z_i) = \max \left[\min \left[\frac{\int g(d(z,z_i))H(z)dz + \frac{p_B - p_A - \overline{g} + t}{2} - ta|z_i|}{t(1-a)}, 1 \right], 0 \right] \quad (9)$$

Proposition 1 *Suppose $\overline{g} < t(1-a)$. Then Γ is a contraction mapping with modulus $\frac{\overline{g}}{t(1-a)}$. Hence, for any given prices p_A and p_B, the fixed point $H(z) = \Gamma H(z)$ exists and is unique.*

Proof. We apply Blackwell's sufficient condition[4]. It follows from Blackwell's sufficient condition that Γ is a contraction if it satisfies i) a monotonicity condition, and ii) discounting. Denote by S the set of all bounded continuous functions on $[-1,1]$. Then Γ is a mapping from S into S. It is bounded above by 1 and below by 0, and continuous as $H(z)$ is continuous. The monotonicity condition requires that if $H_i, H_j \epsilon S$ and $H_i(z) \leq H_j(z)$ all z, then $\Gamma H_i(z) \leq \Gamma H_j(z)$ all z. Since the RHS of (9) is increasing in $H(z)$ for all z, the monotonicity condition is satisfied. Consider next the discounting condition. The discounting condition requires that there exists some α in $(0,1)$ such that for all H_i in S, all $v \geq 0$, and all z_i we have $\Gamma(H_i + v)(z_i) \leq \Gamma(H_i)(z_i) + \alpha v$. It follows from (9) that

$$\Gamma(H_i + v)(z_i) = \max \left[\min \left[\frac{\int g(d(z,z_i))(H_i(z) + v)dz + \frac{p_B - p_A - \overline{g} + t}{2} - a|z_i|}{t(1-a)}, 1 \right], 0 \right]$$

$$= \max \left[\min \left[\frac{\int g(d(z,z_i))H_i(z)dz + \frac{p_B - p_A - \overline{g} + t}{2} - a|z_i|}{t(1-a)} + v\frac{\overline{g}}{t(1-a)}, 1 \right], 0 \right]$$

Hence, if neither the requirement that $H \leq 1$ (the minimum operator) or the requirement that $H \geq 0$ (the max operator) binds, it follows that $\Gamma(H_i + v)(z_i) = \Gamma(H_i)(z_i) + v\frac{\overline{g}}{t(1-a)}$. If either the minimum operator or the maximum operator strictly binds, then $\Gamma(H_i + a)(z_i) < \Gamma(H_i)(z_i) + v\frac{\overline{g}}{t(1-a)}$. It follows that Γ is a contraction mapping with modulus $\frac{\overline{g}}{t(1-a)}$. ∎

Thus, whenever $\overline{g} < t(1-a)$, the coordination game between the agents has a unique solution. In order to understand the result, note that the assumption on parameter values imply that the technology preferences are strong compared with the network effect. Assume for the moment that $H(z) < 1$ for all z and suppose as an example that all types increase their threshold value $y^m(z)$ with Δ units. This increases the density function H with $\Delta/(1-a)$ units. The increased utility of joining network H due to network externalities is thus $\Delta\overline{g}/(1-a)$. The increase in transportation cost for the marginal agent however is Δt, which is greater than $\Delta\overline{g}/(1-a)$ by assumption.

As a result, self-fulfilling prophesies is not an issue in this model: an increase in the number of agents going to one network increases the attractiveness of the network, but not sufficiently much to compensate for the increased transportation costs for the new agents.

Given proposition 1, we can easily show that $H^e(z)$ has the following properties:

---

[4]See e.g. Sydsæter, Strøm and Berck (2005) or Stokey and Lucas (1989).

Lemma 1 *The equilibrium function $H^e(z)$ has the following properties*

i) $H^e(z)$ is symmetric around $z = 0$, $H^e(z) = H^e(-z)$. If $p_A = p_B$ then $H(z_i) = 1 - H(1 - z_i)$ for all $z_i \epsilon [0,1]$

ii) For all values of z where $0 < H^e(z) < 1$, $H^e(z)$ is strictly decreasing in z for $z > 0$ and strictly increasing in z for $z < 0$ (except in the special case where $H^e(z) = 0.5$ everywhere, see below).

iii) H can be written as a function of $p_B - p_A$ and is increasing in $p_B - p_A$ for all z

Proof in appendix.

The equilibrium function $H^e(z)$ is rather difficult to analyze. However, in the case with $p_A = p_B$ we can derive some nice properties for the function, and since we are mostly interested in the symmetric equilibrium this is also the most interesting case. With $p_A = p_B$ and $\bar{g} = 0$ it follows that an agent chooses the A network if and only if $y \leq 0.5$. Denote the equilibrium distribution in this special case by $H^t(z)$. It follows that

$$
\begin{aligned}
H^t(z) &= 1 & \text{if} \quad |z| \leq \frac{1}{2a} - \frac{1-a}{a} \\
H^t(z) &= \frac{1/2 - a|z|}{1-a} & \text{if} \quad \frac{1}{2a} - \frac{1-a}{a} \leq |z| < \frac{1}{2a} \\
H^t(z) &= 0 & \text{if} \quad |z| > \frac{1}{2a}
\end{aligned}
$$

For $\bar{g} > 0$, the equilibrium H is as follows: for $|z| < 1/2$ is H a concave function above H^t, for $|z| > 1/2$ it is a convex function below H^t.

Our next concern is how the H function depends on the underlying parameters. Our first concern regards the spread of g. Define a $\bar{g}$-preserving increase in the spread of g as a transformation where mass is moved from the center to the periphery in a symmetric fashion, analogous to mean-preserving increase in spread. We can then show the following result

Lemma 2 *The equilibrium function $H^e(z)$ has the following properties:*

a) A $\bar{g}$-preserving increase in the spread of g reduces $H^e(z)$ for $|z| < 1/2$ and the reduction is strict if $0 < H^e(z) < 1$. The opposite holds for $|z| > 1/2$.

b) An increase in $\bar{g}$ or a decrease in t increases $H^e(z)$ for $|z| < 1/2$, and the decrease is strict if $H^e(z) < 1$. The opposite holds for $|z| > 1/2$.

c) An increase in a (a reduction in $1 - a$) increases $H^e(z)$ for $|z| < 1/2$, and the increase is strict if $H^e(z) < 1$. The opposite holds for $|z| > 1/2$.

Proof in appendix.

3.2 Equilibrium prices

In this section we derive the equilibrium prices p_A and p_B. To this end, define

$$
\begin{aligned}
N_A(p_B - p_A) &= \int H(z; p_B - p_A)dz \\
N_B(p_A - p_B) &= \int [1 - H(z; p_B - p_A)]\, dz = 2 - N_A(p_B - p_A)
\end{aligned}
$$

Thus N_A and N_B denote the total number of agents in the two networks. Suppose the per agent connection cost is $c_i, i = A, B$. The profit of firm i can be written

$$\pi_i = (p_i - c_i)N_i(p_{-i} - p_i)$$

with first order conditions

$$N_i(p_{-i} - p_i) - (p_i - c_i)N_i'(p_{-i} - p_i) = 0 \tag{10}$$

and second order conditions

$$-2N_i'(p_{-i} - p_i) + (p_i - c_i)N_i''(p_{-i} - p_i) < 0 \tag{11}$$

With identical costs, the first order conditions for maximum are given by

$$p_A = p_B = c + \frac{1}{N'(0)} \tag{12}$$

Due to symmetry, $N(\cdot)$ is odd, and thus has an inflection point at zero. Hence $N''(0) = 0$, and the second order conditions are satisfied locally. Finally, it follows directly from (10) that (12) is unique.[5] Note the similarity with the standard Hotelling model. In that model, $N'(0) = 1/t$, which gives $p_A = p_B = c + t$ as usual.

It is hard to show generally that the second order conditions for the firms maximization problem is satisfied globally. If not there may exist mixed-strategy equilibria. In the symmetric case (with $c_A = c_B$) or close to the symmetric case, the second order conditions are always satisfied locally. Furthermore, we are able to demonstrate uniqueness in the general case in the two market configurations analyzed below, referred to as global and local competition.

4   Characterizing equilibrium

In what follows we want to characterize the equilibrium in some detail. To simplify the exposition we assume that $c_A = c_B = c$. The general case is briefly discussed in footnotes. We distinguish between three types of equilibria; with global competition, with local competition and a intermediate case referred to as having hybrid competition. Global competition refers to a situation where $H(z) < 1$ for all z, so that there are marginal consumers for all social locations. We show that in this case, global network externalities play an important role. Local competition requires among other things that $H(z) = 1$ around $z = 0$ (and $H(z) = 0$ around 1). In this case, global network externalities play no role at all, and network effects play no role for the degree of competition.

---

[5] It follows from (10) that $\frac{p_A - c}{p_B - c} = \frac{N_A(p_B - p_A)}{2 - N_A(p_B - p_A)}$. If p_A exceeds p_B, the left hand side exceeds one whereas the right hand side is strictly below one.

4.1 Global competition

With global condition we refer to equilibria where $H^e(z)$ is strictly between zero and one at all social locations z. Thus, competition is global in the sense that firms are competing for agents (there are indifferent agents) for all locations z. In the appendix we show that a sufficient condition for global competition is that

$$\overline{g} < t(1 - 2a)$$

The left-hand side is an upper bound on the social gain of being in the A-network, while the right-hand side shows the largest technological preference for the B-network over the A-network when located at $z = 0$ (obtained when $\varepsilon = 1$, see equation 1). The condition states that if the latter dominates the former, $H(z) < 1$. A neccesary condition for existence of global competition is that

$$1 - a > 1/2$$

The latter follows from the fact that at $z = 0$, the highest possible value of y is $1 - a$. As the social value for this person of joining the A-network is bigger than the social value of joining the B-network, a necessary condition is that this person has a technological preference for the B-network, i.e. that $1 - a > 1/2$.

Furthermore, global competition is more likely if g is sufficiently close to the uniform distribution on $[0, 1]$, in the sense that a bigger set of other parameter values will lead to global competition (social location does not matter for social interaction). It is trivial to show that if g is uniform on $[0, 1]$, there is global competition whenever $1 - a > 1/2$.

Lemma 3 *Suppose $0 < H^e(z) < 1$ for all z. Then*

$$N'(\cdot) = -\frac{1}{t(1 - a) - \overline{g}} \tag{13}$$

Proof. We want to show by construction that $dH^e(z)/dp_A$ is the same for all z. Suppose this is the case, and differentiate (5). This gives

$$t dy^m(z) - \overline{g}dH = -dp_A/2$$

From (9) it follows that $dy^m = (1 - a)dH$, which inserted gives

$$[t(1 - a) - \overline{g}]dH = dp_A/2$$

By construction, $H^e + dH$ is an equilibrium distribution, and as the equilibrium distribution is unique it is also the only one. As the social circle has a circumference of 2, $dN_A = 2dH$, and this gives (13). $\blacksquare$

Inserted into (12) it follows that the equilibrium price is given by (with topscript G indicating global competition)

$$p_A^G = p_B^G = c + t(1 - a) - \overline{g} \tag{14}$$

9

This is analogous to the equilibrium price level with global network externalities, see Laffont, Rey and Tirole (1998). Most importantly, the existence of network externalities increases competition and decreases prices. The point is as demand becomes more price sensitive: a reduction in price brings in new agents. This makes the network even more attractive, and even more agents are attracted to the network, and it is the existence of transportation costs that keep demand from exploding.[6] Furthermore, note also that the shape of g does not influence for network pricing, only $\overline{g}$

The factor $1 - a$ reflects that the transportation costs becomes less important when technology preference y is more affiliated with social location z. To be more specific, note that for any given z, the "distance" between the most extreme agents in technology space is exactly $1 - a$, not 1 as in the standard model. Absent global network effects, an increase in p_A will therefore imply that a larger fraction of the customers at any z switches supplier. The reduced maximum distance between the two alternatives has exactly the same effect as using the transportation cost parameter t proportionally. As a consistency check, note that when $1 - a = 1$ (no relationship between social location and technological preferences), $p = c + t + \overline{g}$ as in LRT.

4.2   Local competition

With local competition we refer to equilibria for which $H(z) = 1$ on sufficiently large intervals around $z = 0$ and $H(z) = 0$ around $z = 1$. To be more precise, define $z_0 > 0$ to be the smallest value such that $g(2z_0) = 0$. In equilibria with local competition, we require that $H(z) = 1$ on an interval around 0 containing $[-z_0, z_0]$. The interval at which $H(z) = 1$ is thus sufficiently large so that the social value of bringing in more customers in one end of the interval for persons at the other side of the interval is zero. Due to symmetry, it then follows that $H(z) = 0$ on an equally large interval around 1.

Note that z_0 is exogenously determined by the shape of g. For local competition to exist, it must be so that $z_0 < 1/2$. As the social value of joining supplier A is greater than that of joining supplier B, a sufficient condition for local competition to exist (in the symmetric case) is that all agents with social location at z_0 has a technical preference for the A network, or from (1) that $az_0 + (1 - a) \leq 1/2$. This is always satisfied if $1 - a$ is sufficiently small. [Note that local competition can only exist when $1 - a < 1/2$.]

With local competition, the equilibrium distribution H has some remarkable properties. Let $H^0(z)$ denote the equilibrium distribution of customers when $\Delta p = 0$, and let $H^{\Delta p}(z)$ denote the equilibrium distribution of H for a small $\Delta p = p^B - p^A$. Furthermore, denote by z^1 the highest value of z such that $H(z^1) = 1$. Then the following holds:

Lemma 4 *With local competition for both $\Delta p = 0$ and $p_B - p_A = \Delta p$, the following holds*
   a) For all $z_i > 0$, $H^{\Delta p}(z_i) = H^0(z_i - \delta)$, where $\delta = \frac{\Delta p}{ta}$
   b)

$$N'(0) = \frac{1}{ta} \tag{15}$$

---

[6]Consider briefly the general case where c_A may differ from c_B. Under global competition $N''(\cdot) = 0$, hence (10) yields a unique equilibrium. Thus equilibrium prices are $p_i = c_i + t(1 - a) - \overline{g}$.

Proof. a) Consider any value $z_i > 0$. We want to show that $H^{\Delta p}(z_i) = H^0(z_i - \delta)$ is an equilibrium. Suppose it is true. For $z_i < \delta$ it then follows directly from (9) that $H^{\Delta p}(z_i) = 1 = H^0(z_i - \delta)$. We therefore concentrate on the case where $z_i > \delta$. Then

$$
H^{\Delta p}(z_i) = \max\left[\min\left[\frac{\int_\Omega g(d(z,z_i))H^{\Delta p}(z)dz + \frac{\Delta p - \bar{g} + t}{2} - ta z_i}{t(1-a)}, 1\right], 0\right] \tag{16}
$$

$$
= \max\left[\min\left[\frac{\int_\Omega g(d(z, z_i - \delta))H^0(z)dz + \frac{\bar{g}+t}{2} - ta(z_i - \delta)}{t(1-a)}, 1\right], 0\right] \tag{17}
$$

Consider then the integral. Inserting $H^{\Delta p}(z_i) = H^0(z - \delta)$ and taking into account that $g(d) = 0$ whenever $d > 2z_0$, gives

$$
\int_\Omega g(d(z, z_i))H^{\Delta p}(z)dz = \int_{z_i - 2z_0}^{z_i + 2z_0} g(d(z, z_i))H^{\Delta p}(z)dz
$$
$$
= \int_{z_i - 2z_0}^{z_i + 2z_0} g(d(z, z_i))H(z - \delta)dz = \int_{z_i - \delta - 2z_0}^{z_i - \delta + 2z_0} g(d(z, z_i - \delta))H(z)dz
$$
$$
= \int_\Omega g(d(z, z_i - \delta))H(z)dz
$$

It thus follows that $H^0(z - \delta)$ satisfies the fixed point $H^p = \Gamma^p(H^p)$ and is thus an equilibrium. Furthermore, since we know that the equilibrium is unique it is also the only equilibrium.

b) Due to symmetry we have

$$
N(\cdot) = 2\left[\int_0^{z_1 - \frac{\Delta p}{2t}} 1dz + \int_{z_1 - \frac{\Delta p}{2t}}^{1 - z_1 + \frac{\Delta p}{2t}} H(z - \frac{\Delta p}{2t})dz + \int_{1 - z_1 + \frac{\Delta p}{2t}}^{1} 0dz\right]
$$

Differentiating with respect to Δp yields the result. ∎

By inserting (15) into (12) it follows that (with topscript L indicating local competition)

$$
p_A^L = p_B^L = c + at \tag{18}
$$

Proposition 2 *Suppose the network externalities are local. Then the network externalities have no effect on equilibrium prices.*

As mentioned above, the additional value for the marginal agent of increasing the network's market share is positive and proportional to $\bar{g} > 0$. Still, this will not influence the pricing decision of the firm.

To gain intuition for the proposition, first note that global network externalities tend to increase price competition, because they increase the price elasticity of demand. Reducing the price will then increase the size of the network, and this will make the network even more attractive. This mechanism does not hold with local externalities. A reduction in price will increase the network size, and this has a substantial effect on the utility of the agents that previously were marginal. However, these agents are now inframarginal. The utility

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of joining the network for the marginal agents is unchanged (taking into account that the identities of the marginal agents also change).

Finally, note that the the technology preference influences prices in a different way with local than with global competition (at in 18 and $(1-a)t$ in 14). Again this reflects how different competition works in the two cases. With global competition, there are marginal customers for all z. For a given z, the technology preferences are spread over an interval of length $1-a$, and the relevant "transportation cost" is thus $(1-a)t$. With local competition it is different, as a price increase in this case shifts the entire distribution to the right. Consider a person at $z = z^1 > 0$ (the highest value of z such that $H(z) = 1$. An increase in the price shifts this point to the right, without changing the social value of joining the network. It follows that z^1 must fall sufficiently much so that the person with noise parameter drawn at $\varepsilon = 1$ still is indifferent between joining the network and not. Given $\varepsilon = 1$, technological preferences are spread over an interval with length a (as z moves from 0 to 1), and the relevant transportation cost is ta.

4.3   Hybrid competition

Hybrid competition occurs if there is neither global nor local competition in equilibrium, i.e., when $H(z) = 1$ for $|z|$ close to one while $H(z_0) < 1$. Hybrid equilibria may exist for a wide range of parameter values. A sufficient condition for the existence of hybrid competition is that $a < 1/2$ (which rules out global competition) and $z_0 > 1/2$ (which rules out local competition).

4.4   Type of competition and competition intensity

Recall that $z^1 < 1/2$ is the highest value of z^1 such that $H(z^1) = 1$. Consider a shift in parameters. We say that the equilibrium is getting closer to the global competition whenever a shift in parameters reduces z_1. Analogously, we say that the equilibrium is getting closer to local competition whenever a shift in parameters increases z^1 and decreases z^0. We want to analyze how such a shift influence competition. To get clean result we look at shifts that does not influence pricing with pure local and pure global competition, that is, $\overline{g}-$ preserving increase in spread as defined above. From lemma 2 the following lemma is immediate

Lemma 5 *A $\overline{g}$-preserving increase in spread implies that the equilibrium is getting closer to global competition and further away from local competition*

We can then show the following proposition

Proposition 3 *Suppose the equilibrium is getting closer to global competitions (and further away from local competition) as a result of a $\overline{g}$-preserving increase in spread. Then the equilibrium price is always larger with local competition than with hybrid competition, and always lower with global competition than with hybrid competition.*

Proof. In appendix. ∎

5 Efficiency

In this section we derive the optimal distribution of agents over networks, and refer to this as composition efficiency. An important issue here is the total social value created in the two networks, defined as

$$
\begin{aligned}
V_A &= \iint g(d(z, z_i)) H(z) H(z_i) dz dz_i \\
V_B &= \iint g(d(z, z_i))(1 - H(z))(1 - H(z_i)) dz dz_i
\end{aligned}
$$

where V_A and V_B are the total social value created in A and B, respectively.

Let us first derive the distribution of agents on networks that maximize the number of connections, see appendix for proofs. Given that the two networks are equally large (that is, $\int_\Omega H(z) dz = 1$), we show that the social value is maximized if $H(z) = 1$ for all $z \in (-\frac{1}{2}, \frac{1}{2}]$ and $H(z) = 0$ otherwise. The social value is minimized if $H(z) = \frac{1}{2}$ for all z.

The number of connections is *minimized* if $H(z) = 0.5$ for all z, in which case each agent can communicate with exactly half of her friends. The number of connections is *maximized* if $H(z)$ equals 1 up to a certain z value and then jumps to zero. However, in that case some of the marginal agents bear a high cost associated with a strong technological preference for the other network than they are attached to. Hence there is a trade-off between the social benefit of increasing the number of connections and costs associated with technology preferences.

Similarly, for a given distribution $H(z)$ let T_A and T_B denote the aggregate "travel cost". It follows that

$$
\begin{aligned}
T_A &= \int_\Omega \int_0^{y^m(H(z_i))} ty \, dF(y; z_i) dz_i \\
T_B &= \int_\Omega \int_{y^m(H(z_i))}^1 t(1 - y) \, dF(y; z_i) dz_i
\end{aligned}
$$

where $F(y; z_i)$ is defined in equation (2) and where $y^m(H(z)) = a|z_i| + (1-a)H(z)$ (from 8).

We say that $H^*(\cdot)$ is *composition efficient* if it coincides with the social planner's allocation of agents on networks. A composition efficient distribution $H^*(z)$ maximizes the aggregate agent utility and aggregate profits. Clearly, the optimal allocation solves the sum of connections net of total transport costs,

$$
\max_{H(z_i)} W = \max_{H(z_i)} V_A + V_B - T_A - T_B \text{ all } z_i \epsilon [0, 1]
$$

The first order conditions for maximum writes

$$
\frac{dW}{dH(z_i)} = 2 \int g(d(z, z_i)) H(z) dz - t y^m(z_i) - \left[2 \int g(d(z, z_i))(1 - H(z)) dz - t(1 - y^m(z_i)) \right] = 0
\tag{19}
$$

which can be expressed

$$\frac{t}{2}y^m(z_i) - \int g(d(z,z_i))H(z)dz = \frac{p_B - p_A - \bar{g} + \frac{t}{2}}{2}$$

Thus $H^*(z)$ is a fixed-point to the mapping Γ^g given by

$$\Gamma^g H^*(z_i) = \max\left[\min\left[\frac{\int g(d(z,z_i))H^*(z)dz + \frac{p_B - p_A - \bar{g} + \frac{t}{2}}{2} - \frac{t}{2}a|z_i|}{\frac{t}{2}(1-a)}, 1\right], 0\right] \qquad (20)$$

If we compare (9) and (20) we see that the only difference between Γ and Γ^g is that t in Γ is replaced with $t/2$ in Γ^g. Hence following proposition is immediate

Proposition 4 *The equilibrium distribution is not composition efficient. The social efficient composition profile $H^*(\cdot)$ is steeper than the equilibrium profile $H(\cdot)$. Thus, for $|z| < 1/2$ it follows that $H^*(z) \geq H(z)$ with strict inequality whenever $H(z) < 1$. The opposite is true for $|z| > 1/2$.*

The result follows from lemma 2 b) and the fact that the planner's solution is equivalent with the market solution with t replaced by $t/2$.

The efficiency result is quite intuitive. The consumers, when choosing between suppliers, trade off travel cost and social gains. However, there is an externality associated with the latter but with the latter: the social value of joining a network gives rise to an equally large social gain for the agents that have already chosen the same supplier. As a result, the planner puts twice as much weight on social value relative to transportation cost as the market, or equivalently half as much weight on travel costs.

For $z_i < 1/2$, $H^e(z) > 1/2$. Thus, the agent located at z obtains more social value by joining the A-network than the B-network. For the same reason, the positive externality of joining the A-network is larger than the positive externality associated with joining the B−network, and it follows that $H^*(z_i) = H^e(z_i)$.

To be even more precise, by inserting from (5) in (19) we find the marginal social value of increasing the A network's market share at z_i evaluated at $H^e(z_i)$ is given by

$$\frac{dW}{dH(z_i)} = \int g(z_i - z)H^e(z)dz - \int g(z_i - z)(1 - H^e(z))dz$$

which captures the net externality associated with the choice of network. Again observe that the net externality is positive if the marginal agent at z_i has a majority of friends in the A-network. Hence, when a agent located at $z_i < 0.5$ (hence has her majority of friends in the A-network) joins the B-network (due to her technological preference for B), she exerts a negative *net* externality since the majority of her friends suffer. Thus, compared to first best composition efficiency, too many agents with social location below 0.5 choose the B network, and too many above 0.5 choose the A network. Hence the welfare maximizing distribution $H^*(z)$ is steeper than the equilibrium contribution.

6 Endogenous agent heterogeneity

It is well known that differences between marginal and average agents may give rise to distortions. This was first explored in Spence (1975). He studied a monopolist's choice of product quality level, and showed that this will depend on the marginal consumer's preferences for quality. A social planner's choice of quality, by contrast, depends on the average preference for quality among the consumers. If marginal and average valuation differ, the quality level chosen by the monopolist is not socially optimal.

Local network externalities give rise to a certain structure on the difference between the marginal and the average agents in a network. The marginal agents obtains less utility from social interacting than the average agent. If we extend the model by including more choice variables for consumers this difference between the marginal and the average consumer will lead to new distortions, which are absent with global network externalities.

6.1 Communication intensity

In this subsection we assume that consumers, when connected to a network, choose how intensively to use the network. A very natural example here is communications, where usage depends on the number of people a person communicates with. With a club interpretation usage may be how many times a member uses the club, and with platform competition it may be the number of applications purchased. The firms compete by offering two-part tariffs, with a fixed fee (connection or membership fee K) and a usage price q. Both are set by the two firms simultaneously and independently.

Our driving assumption is that utility of usage depends positively on the social value of the network to the consumer. This is obviously the case in a communication network, where $g()$ can be interpreted as the number of "friends" in the network. In what follows we therefore use communication networks as our reference. We assume that the utility obtained by communicating with one friend is $\omega(x)$, where x is usage. If the consumer has N friends in the network, we assume that total utility is $\omega(x)N$. As all friends are equally valuable, all consumers choose the same communication intensity x with all friends (the exact specification of preferences at this point is not so important. For simplicity, we assume that only communication paid by the agent gives rise to utility.[7] Finally, an agent can only communicate with the agents in the same network. Compatibility is discussed in the next section.

The net surplus $v(q_A)$ for a consumer attached to network A for each "friend" in that network is given by

$$v(q_A) = \underset{x}{Max}[\omega(x) - q_A x]$$

We write the optimal usage as a function of q_A, $x(q_A)$. Note that $x(q_A) \equiv -v'(q_A)$.

Then we turn to the assignment game. Given prices p_A and q_A, the net surplus for a agent (z_i, y_i) of joining the A network is

$$u^A(y_i, z_i) = \alpha - ty_i + v(q_A)\int g(d(z, z_i))H(z)dz - p_A \qquad (21)$$

---

[7]Note that the social externality identified in the previous section is still presnet: If a person joins a network, her "friends" in that network obtains utility from having one more person to contact.

and similarly, the net benefit of joining the B network is

$$u^B(y_i, z_i) = \alpha - t(1 - y_i) + v(q_B) \int g(d(z, z_i))[1 - H(z)]dz - p_B \tag{22}$$

As above, if all z_i types prefer the A network then $H(z_i) = 1$, and if all prefer the B network $H(z_i) = 0$. Otherwise, z_i types are divided in two groups. Those with technology preference $y_i < y^m(z_i)$ who prefer the A network, and $y_i > y^m(z_i)$ who prefer the B network. From equations (21) and (22) it follows that the indifferent consumer has technology preferences given by agents with technology preference $y(z_i)$ are indifferent, that is $u^A(y(z_i), z_i) = u^B(y(z_i), z_i)$ or

$$ty(z_i) - \frac{v(q_A) + v(q_B)}{2} \int g(d(z, z_i))H(z)dz = \frac{p_B - p_A - v(q_B)\bar{g} + t}{2} \tag{23}$$

By following exactly the same procedure as when deriving (8), it follows that we can write the equilibrium of the assignment game as a fixed point $H = \Gamma^x H$, where the mapping Γ^x is defined as

$$\Gamma^x H(z_i) = \max\left[\min\left[\frac{\frac{v(q_A)+v(q_B)}{2} \int g(d(z, z_i))H(z)dz + \frac{p_B - p_A - v(q_B)\bar{g} + t}{2} - ta|z_i|}{t(1 - a)}, 1\right], 0\right] \tag{24}$$

Note that for given q_A and q_B, $v(q_A)$ and $v(q_B)$ are constants, hence we can show existence and uniqueness of the fixed point in exactly the same way as above. We refer to the equilibrium distribution as $H^x(z)$ (which of course depends on prices)

The profit of firm A is given by

$$\pi_A = (p_A - c) \int H(z)dz + x(q_A)(q_A - c) \int\int g(d(z, z_i))H(z)H(z_i)dzdz_i \tag{25}$$

The first integral is the size of the network (the number of customers). The double integral shows the aggregate number of communication links in the network. Note that the firm not only care about the size of its network, but also its composition (the social location of its customers), as this influences the amount of communication that takes place within the network.

We want to characterize equilibrium in the symmetric case. Since the optimizing with respect to p_A corresponds to the simpler case above, we focus on the choice of usage price q_A. Maximizing π_A with respect to q_A yields the following first order condition (see appendix for details):

$$[1 - \gamma]x(q_A) + (q_A - c)x'(q_A) = 0 \tag{26}$$

where

$$\gamma := \frac{\frac{1}{2}\bar{g}}{\int\int g(d(z, z_i))H(z)H(z_i)dzdz_i}$$

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Note that in the symmetric equilibrium, the agent located at $z = 1/2$ has half of its friend in both networks. For marginal customers at $z < 1/2$ they have more than half of their friends in the network, and vice versa for $z > 1/2$. Due to symmetry it follows that the nominator shows the average number of friends for the marginal agents. The denominator shows the number of communication links, which is equal to the average number of friends in the entire network (since the measure of agents in the network is 1 in the symmetric equilibrium). The variable γ thus measures the average number of friends for the marginal agents relative to the average number of friends of all agents attached to the network. Note that $\gamma \in (1/2, 1]$.

The second term in (26) captures the demand effects of the usage price, which can be divided in two parts. The first part is the standard direct negative demand effect $x'(q_A)$. The second part is the indirect negative demand effect from the change in network composition. A higher usage price implies hurts the marginal agents with many friends in the network (z low) more than those with a few friends in their network (z high).The H function thus decreases for values of z above $1/2$ (with many friends) and decreases for $z > 1/2$ (with few friends in the network). As a result the total amount of communication decreases.

It follows that the profit-maximizing traffic price exceeds marginal cost. Local externalities creates agent heterogeneity, and traffic price can be used as a rent extraction device. In this case the marginal agent has a lower level of exchange than the inframarginal. Hence, the network owner is better off increasing the usage price slightly and compensate the marginal agent by reducing the fixed fee. The firm thus trades off efficiency for the "low-type" (marginal) agents and rent extraction for the "high-type" (inframarginal) agents.

Proposition 5 *The firms set the communication price q_k, $k = A, B$ above marginal cost. Thus, the communication price exceeds the price level that induces a static first best level of traffic represented by marginal cost pricing .*

It can be show that this result does not depend on the particular specification of two part tariffs. With an optimal general contract, increasing the usage price for marginal agents relaxes the incentive compatibility constraint of the inframarginal workers, and hence enables to firm to extract more rents from the latter. Finally, the effect is weakened by the negative effect increased usage price has on the composition of the network. As long as the network has a positive margin on usage, this is costly for the network.

The network owner price internal traffic as if he had some degree of market power, where the degree of market power is captured by the relative deviation between the marginal and average intensity of exchange. With global network externalities, symmetry between agents prevails (hence $\gamma = 1$), which means that the network adopts marginal cost pricing. Note that $\gamma > 1/2$. Denoting the inverse demand elasticities of $x(q_A)$ by ε, the equilibrium usage price (assuming constant elasticity) is

$$q_A = \frac{1}{1 - \varepsilon \left[1 - \gamma\right]} c \qquad (27)$$

Note that γ decreases in network size, and approach $1/2$ when the network goes to infinity. Thus, large network will have a stronger incentive than small networks to overprice traffic. BRA

Consider the socially efficient usage price. In the proposition we referred to static first best usage price as marginal cost pricing. Define the *constrained efficient* usage price as

the usage price that maximizes net welfare given that agents are distributed according to individual optimization (i.e., the price that emerges if a planner could set the usage price but make no other decisions). Then the following holds:

Proposition 6 *The constrained efficient usage price is below marginal cost under pure local network externalities.*

The proposition is almost like a corollary to proposition 4. Note that there are no externalities related to communication intensity (since only the payer gets utility from communication). However, the externality identified in a previous section related to the agents' choice of network carries directly over to this setting. It is trivial to show, analogous with the results above, that the socially optimal H (for given v) solves (24) with $t/2$ substituted in for t. As we have seen, a low usage price increases the steepness of the H function. It follows that by subsidizing usage, the planner can make the distribution function steeper and thus closer to the socially optimal distribution.

It thus follows that the market solution for usage pricing distorts the distribution of H in the wrong direction, and leads to a distribution of agents on the networks that are even further away from the optimal distribution.

6.2 Compatibility

We will now discuss the agents' incentives to undertake investments in order to make the networks compatible. We focus on the situation with one-way compatibility. Thus, network A may give its members (inferior) access to network B by undertaking an investment C. Let $\theta_A \leq 1$ denote the degree at which the agents in network A can utilize network B, and write the cost of compatibility as $C(\theta_A)$.

The timing of the game is as follows: Firms first set prices p_i, $i = A, B$ and the degree of compatibility θ_i, $i = A, B$, independently and simultaneously. Then agents choose which platform to assign to. The utility of a agent in network A is then given by

$$u^A(y_i, z_i) = \alpha - ty_i + \int g(d(z, z_i))H(z)dz + \theta_A \int g(d(z, z_i))(1 - H(z))dz - p_A \qquad (28)$$

By doing the exact same reasoning as in the previous subsection it follows that the equilibrium distribution H^c is a fixed-point to the mapping

$$\Gamma^C H(z_i) = \max\left[\min\left[\left(1 - \frac{\theta_A + \theta_B}{2}\right)\int g(d(z, z_i))H(z)dz + \frac{p_B - p_A - (1 - \theta_A)\overline{g} + t}{2}, 1\right], 0\right] \qquad (29)$$

Network A's net profit equals

$$\pi_A = p_A \int H(z)dz - C(\theta_A) \qquad (30)$$

verdien til konnektivitet As with two-part tariff case, it is convenient to distinguish between the composition effect and the scale effect when solving the network's optimization

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problem in fixed fee p_A and degree of compatibility θ_A. Increasing p_A for a given θ_A shifts the $H(z)$ profile to the right with no effects on its slope. The optimal degree of compatibility can then be characterized by maximizing (30) w.r.t. θ_A and simultaneously adjusting p_A such that the average agent is indifferent (who has half of her friends in the network). Analogously with the former section, we have the required adjustment $dp_A = (\bar{g}/2)d\theta_A$.

Simple manipulations yield the following first order condition for compatibility

$$\frac{\bar{g}}{2} - C'(\theta_A) = 0 \tag{31}$$

Increased compatibility is valuable to the agents and allow the network to raise access price without making the marginal agents (on average) worse off - this is captured by the first term of (31). In optimum the network's marginal gain from compatibility exactly balances the marginal cost of compatibility.

The socially efficient degree of compatibility (contingent on equal market shares), by contrast, maximizes welfare given by

$$W = \int \int_{-y(z_i)}^{y(z_i)} u^A(y_i, z_i)f(y_i - z_i)dy_i dz_i + \int \int_{y(z_i)}^{-y(z_i)} u^B(y_i, z_i)f(y_i - z_i)dy_i dz_i \tag{32}$$
$$-C(\theta_A) - C(\theta_B)$$

Maximizing (32) w.r.t. θ_A, given that $\theta_A = \theta_B$ yields the first order condition,

$$2\int \int g(d(z, z_i))H(z_i)(1 - H(z))dz dz_i + \frac{\partial W}{\partial(H(z))}\frac{dH(z)}{d\theta_A} - C'(\theta) = 0 \tag{33}$$

The first term in (33) is the average agent value of compatibility. The second term captures the composition effect of a higher degree of compatibility.

If the network externality is global, a comparison of (31) and (33) shows that the market solution is optimal: Average agent value coincides with marginal agents value due to anonymity, and the composition effect is zero. With local externalities both terms matter.

With local network externalities, the marginal agents value compatibility higher than the average agent, since the marginal agent communicates more with the agents in the other network than does the average agent. Since the firms compete for the marginal agent, it is his/her preferences that governs the choice of compatibility, and therefore too much resources are spent on making the systems compatible compared with the socially optimal level. Hence the first term in (33) is strictly lower than $\bar{g}/2$.

Consider then the composition effect. Increasing θ_A has a partial negative effect on composition efficiency since it attracts agents that communicate intensively with the other network (that is types $z_i > 0.5$) and punish agents with most of their friends in the A-network (types $z_i < 0.5$). Hence the $H(z)$ profile becomes flatter. Thus, the second term of (33) is negative. This yields the following result.

Proposition 7 *Suppose the network externalities are local. Then the firms have too strong incentives to make the networks (one-way) compatible.*

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6.3   Interconnection fees

In this section we generalize the model by introducing internetwork exchange. We maintain the linear price structure. We assume that the networks can discriminate between internal and external traffic, and we denote by q_A^T the unit price charged by agents in network A for communication with the agents in network B. We assume that the termination price is exogenously given (for instance set by the regulator) and equal to c. Our main finding is that as marginal agents communicate more intensively with agents outside the network, it is optimal for the firms to set the price for external traffic *below* marginal cost.

The game structure corresponds to the previous section on two-part tariffs. In stage one of the game the networks independently and simultaneously determine their inter- and infranetwork prices q_i and q_i^T and the fixed parts of the tariffs p_i, $i = A, B$, whereas agents independently choose which network to join in stage 2. The utility of agent y if he joins network A is

$$u^A(y_i, z_i) = \alpha - t y_i + v(q_A) \int g(d(z, z_i)) H(z) dz + v(q_A^T) \int g(d(z, z_i))(1 - H(z)) dz - p_A$$

with corresponding expressions for the utility obtained if he becomes a member of the B network.

Network A's profit can be written

$$\pi_A = p_A \int H(z) dz + x(q_A)(q_A - c) \int \int g(d(z, z_i)) H(z) H(z_i) dz dz_i \qquad (34)$$
$$+ x(q_A^T)(q_A^T - c) \int \int g(d(z, z_i))(1 - H(z)) H(z_i) dz dz_i$$

As in previous sections we characterize the optimal inter- and intranet usage prices by maximizing (34) with respect to q_A and q_A^T conditioned on a given network scale. The optimal value of q_A is still given by (27). Maximizing (34) with respect to q_A^T gives the analogous expression

$$[1 - \gamma^T] x(q_A^T) + (q_A^T - c) \left[x'(q_A^T) + noe \right] = 0, \qquad (35)$$

where γ^T is given by

$$\gamma^T := \frac{0.5 \overline{g}}{\int \int g(d(z, z_i))(1 - H(z)) dz dz_i} > 1$$

Thus, γ^T is the number of intranet transactions for the marginal agent relative to that of the average agent, and with local externalities this fraction is greater than unity. It thus follows immediately that the firm subsidizes traffic out of the network. Solving (35) gives

$$q_A^T = \frac{1}{1 - \varepsilon \left[1 - \gamma^T \right]} c$$

We have thus shown the following:

Proposition 8 *With local network externalities, the firms set the price for traffic out of the net, q_k, $k = A, B$ below marginal cost. Thus, the traffic price is below the price level that induces a first best level of traffic out of the net.*

Observe that network composition is not a concern in this section due to full compatibility. Hence the socially efficient usage prices are represented by marginal cost pricing.

7 Concluding remarks

This paper contributes to the small but growing literature on local externalities. We demonstrate the local externalities do not stiffen competition. Furthermore, due to the coordination problem that arises with non-anonymity, the equilibrium suffers from composition efficiency. Finally we derive the effects of local externalities on price setting when networks charge two parts tariffs.

8 Appendix

Proof of Lemma 1

i) Suppose the equilibrium is not symmetric around 0. Then there exists a strictly positive number z' such that $H(z') \neq H(-z')$. But since the model is symmetric, there must exists another equilibrium distribution H' defined as $H'(z') = H(-z')$ and $H'(-z') = H(z')$. Since the equilibrium is unique we have thus derived a contradiction. The claim that if $p_A = p_B$ then $H(z_i) = 1 - H(1 - z_i)$ for all $z_i \epsilon [0, 1]$ can be proved by exactly the same argument

ii) Suppose $H(z)$ is strictly increasing in z at an interval in $[0, 1]$. It follows that H has a local maximum for some $z^* \neq 0$. Define z' as the highest value of z less than z^* such that $H(z') = H(z^*)$. If z^* is also a global maximum, let $z' = -z^*$. Now define a new distribution function $\widetilde{H}(z)$ such that $\widetilde{H}(z) = H(z^*)$ on $[z', z^*]$ and $\widetilde{H}(z) = H(z)$ otherwise. Since, by construction, $\widetilde{H}(z) \geq H(z)$ for all z and strictly greater on the interval (z', z^*) it follows that $\Gamma^T \widetilde{H}(z) \geq \Gamma^T \widetilde{H}(z)$, with strict inequality on $[z', z^*]$, $T = 1, 2....$ Since $H(\cdot)$ is a contraction there exists a fix point $H_2(z) = \lim_{T \to \infty} \Gamma^T \widetilde{H}(z) \geq \widetilde{H}(z)$, which is a contradiction due to uniqueness.

Finally, suppose H is decreasing but not strictly, and constant at some interval $[z_1, z_2]$, and strictly decreasing otherwise. This cannot be an equilibrium either. The agent localized at z_1 obtains stronger network effects than one localized at z_2, hence $y^m(z_1) > y^m(z_2)$. From equation (7) it then follows that $\Gamma H(z_1) > \Gamma H(z_2)$, and H cannot be a fixed point

iii) From (9) it follows that Γ, and thus the fixed-point H, depends on the difference $p_B - p_A$. Let $H^*(z)$ denote the initial equilibrium, and consider an increase in $p_B - p_A$. It follows that $\Gamma H^* \geq H^*$, with strict inequality for all z where $1 > H^*(z) > 0$. The result thus follows from monotonicity, see proof Proposition 1.

Proof of Lemma 2.

a) The following lemma is used in the proof:

Lemma 6 *Assume $H_0(z)$ is symmetric and that $H_0(z) \leq \Gamma H_0(z)$ for all $z\epsilon[0,1/2)$ with strict inequality for some z. Then $H_0(z) \leq \Gamma H_0(z) \leq \Gamma^{T-1}H_0(z) \leq \Gamma^T H_0(z) \leq H(z)$ and $\Gamma^\infty H_0(z) = H(z)$. For $z\epsilon[1/2,1)$ and $z\epsilon[-1,-1/2)$ the signs are reversed.*

Proof of lemma: Due to symmetry we have that $\Gamma H_0(\frac{1}{2}) = H_0(\frac{1}{2}) = H(\frac{1}{2}) = \frac{1}{2}$ and similar for $z = -\frac{1}{2}$. Also, due to symmetry $\Gamma H_0(z) - H_0(z) = \Gamma H_0(1-z) - H_0(1-z)$ for all z. Observe that all marginal agents with social location $z\epsilon(-\frac{1}{2}, \frac{1}{2})$ are at least as good off with $\Gamma H_0(z)$ as with $H_0(z)$, and some are strictly better off, thus $\Gamma^2 H_0(z) \geq \Gamma H_0(z)$ with strong equality for some. This holds for each step Γ^T. QED

Let us then return to the proof of lemma 2. We first provide intuition for the proof. As will be shown, H is concave on $[-\frac{1}{2}, \frac{1}{2}]$ and convex on $[\frac{1}{2}, 1]$ and $[-1, -\frac{1}{2}]$. A $\bar{g}$-preserving increase decreases the number of friends an agent has on concave segments and increases the number of friends on convex segments However, since agents with social location on $[-\frac{1}{2}, \frac{1}{2}]$ have a majority of their friends on $[-\frac{1}{2}, \frac{1}{2}]$, the concave segment weights more. Thus H reduces on $[-\frac{1}{2}, \frac{1}{2}]$. On $[\frac{1}{2}, 1]$ and $[-1, -\frac{1}{2}]$ H is convex, and with the same argument H increases. This reasoning is shown formally below.

We first prove that *if $H(z)$ is concave on $[-\frac{1}{2}, \frac{1}{2}]$*, then it is strictly concave on every segment on $(-\frac{1}{2}, \frac{1}{2})$ at which $H(z) < 1$. Afterwards we prove concavity.

Consider segments where $H(z) < 1$. Assume on the contrary that $H(z)$ is linear on a subinterval $[z_i, z_j] \subset (0, \frac{1}{2})$, and on $[-z_j, -z_i]$ by symmetry. It is now convenient to change notation. Denote by $\gamma(v) := g(d(z, z+v))$, the number of friends located at distance v from z. Due to symmetry, $\gamma(v)$ does not depend on the agent's own location as such, only on the distance. Let $\widehat{g}(z)$ denote the equilibrium number of friends of an agent located at z,

$$\widehat{g}(z) = \int_{-1}^{1} \gamma(v)H(z+v)dv$$

Consider any two marginal agents $z' < z''$ such that $z'\epsilon[z_i, z_j]$ and $z''\epsilon[z_i, z_j]$. It follows from (5) and (8) that

$$\widehat{g}' - \widehat{g}'' = t\left[(1-a)\frac{H(z') - H(z'')}{z'' - z'} - a\right](z'' - z')$$

$\frac{H(z')-H(z'')}{z''-z'}$ is the slope (in absolute value) of $H(z)$ over the linear segment $[z_i, z_j]$. Thus, comparing agents at z' and z'', the difference in their number of friends is proportional to the distance $(z'' - z')$. Hence, considering an in between agent $z_k = \lambda z_i + (1-\lambda)z_j$, we have under the assumed linearity of the $H(z)$ function that

$$\widehat{g}(\lambda z_i + (1-\lambda)z_j) = \lambda\widehat{g}(z_i) + (1-\lambda)\widehat{g}(z_j)$$

We will now show that $\widehat{g}(\lambda z_i+(1-\lambda)z_j) > \lambda\widehat{g}(z_i)+(1-\lambda)\widehat{g}(z_j)$ contradicting the assumed linearity. We have that

$$\lambda\widehat{g}(z_i) + (1-\lambda)\widehat{g}(z_j)$$
$$= \lambda\int_{-1}^{1}\gamma(v)H(z_i+v)dv + (1-\lambda)\int_{-1}^{1}\gamma(v)H(z_j+v)dv$$
$$= \int_{-1}^{1}\gamma(v)\left[\lambda H(z_i+v) + (1-\lambda)H(z_j+v)\right]dv$$

Denote by $\Delta = \frac{z_i + z_j}{2}$. Let $\overline{v}$ and $\underline{v}$ be defined as follows (see figure below)

$$\overline{v} = \frac{1}{2} - \Delta, \quad \underline{v} = -\frac{1}{2} - \Delta$$

Let $\Omega_1 := [\overline{v}, -\overline{v}]$ and $\Omega_0 := [-1, -\overline{v}] \cup [\overline{v}, 1]$.
Then we can write

$$\lambda \int_{-1}^{1} \gamma(v) \left[\lambda H(z_i + v) + (1 - \lambda) H(z_j + v) \right] dv$$

$$= \lambda \int_{\Omega_1} \gamma(v) \left[\lambda H(z_i + v) + (1 - \lambda) H(z_j + v) \right] dv$$

$$+ \lambda \int_{\Omega_0} \gamma(v) \left[\lambda H(z_i + v) + (1 - \lambda) H(z_j + v) \right] dv$$

Since $H(z)$ is concave on $[-\frac{1}{2}, \frac{1}{2}]$ by assumption, we have that

$$\int_{\Omega_1} [\lambda H(z_i + v) + (1 - \lambda) H(z_j + v)] dv \leq \int_{\Omega_1} [H(\lambda z_i + (1 - \lambda) z_j)] dv$$

Observe that $\overline{v}$ is defined such that $\lambda H(z_i + \overline{v}) + (1 - \lambda) H(z_j + \overline{v}) = H(1/2)$, which follows directly from the symmetry of H around $1/2$. The equivalent result holds for $\underline{v}$.

On $[-\frac{1}{2}, -1]$ and $[\frac{1}{2}, 1]$ $H(z)$ is assumed convex, thus

$$\int_{\Omega_0} [\lambda H(z_i + v) + (1 - \lambda) H(z_j + v)] dv \geq \int_{\Omega_0} [H(\lambda z_i + (1 - \lambda) z_j)] dv$$

Finally, due to symmetry the following must hold

$$\int_{\Omega_0 \cup \Omega_1} [\lambda H(z_i + v) + (1 - \lambda) H(z_j + v)] dv = \int_{\Omega_0 \cup \Omega_1} [H(\lambda z_i + (1 - \lambda) z_j)] dv = \frac{1}{2} \qquad (36)$$

Let $\Omega_1^{pos} := [0, \overline{v}]$, $\Omega_1^{neg} := [\underline{v}, 0]$, $\Omega_0^{pos} := [\overline{v}, 1]$, $\Omega_0^{neg} := [-1, \underline{v}]$. Since $\gamma(v)$ is single peaked with maximum at $v = 0$, we have that $\gamma(v') > \gamma(v'')$ for any pair (v', v'') such that $v' \epsilon \Omega_1^{pos}$ and $v'' \epsilon \Omega_0^{pos}$, and $\gamma(v') > \gamma(v'')$ for any $v' \epsilon \Omega_1^{neg}$ and $v'' \epsilon \Omega_0^{neg}$. Thus, integrating over the circle, the concave segment on Ω_1 has a higher weight than the negative segment Ω_0, that is

$$\int_{\Omega} \gamma(v) \left[\lambda H(z_i + v) + (1 - \lambda) H(z_j + v) \right] dv$$

$$< \int_{\Omega} \gamma(v) [H(\lambda z_i + (1 - \lambda) z_j)] dv$$

This contradicts linearity.

It remains to show that $H(z)$ is concave on $[-\frac{1}{2}, \frac{1}{2}]$. Assume not. We can now prove concavity on $[-\frac{1}{2}, \frac{1}{2}]$, by symmetry this also proves convexity on remaining parts of the circle. Assume $H(z)$ has strictly convex segments on $[-\frac{1}{2}, \frac{1}{2}]$. Denote by $H^t(z)$ the envelope of $H(z)$ on $[-\frac{1}{2}, \frac{1}{2}]$. Accordingly $H^t(z)$ has linear segments everywhere concavity on $H(z)$ is

not satisfied. As above refer to such intervals as $[z_i, z_j]$. For all $z \notin [-\frac{1}{2}, \frac{1}{2}]$, segments that are strictly concave are replaced by linear segments. Hence $H^t(z)$ is symmetric. Clearly, all z on the intervals $[-\frac{1}{2}, \frac{1}{2}]$, who are not on $[z_i, z_j]$, are better off with $H^t(z)$ than with $H(z)$, thus $\Gamma H^t(z) \geq H^t(z)$ on these intervals. Consider then intervals $[z_i, z_j]$. As shown above, $\Gamma H^t(z) > H^t(z)$ for all $z\epsilon[z_i, z_j]$. It follows now directly from the reasoning above that $\Gamma H^t(z) \geq H^t(z)$ for all $z\epsilon[-\frac{1}{2}, \frac{1}{2}]$ and $\Gamma H^t(z) \leq H^t(z)$ otherwise. Then it follows from Lemma x that there exists an equilibrium $H(z) \geq H^t(z)$ contradicting the suggested equilibrium.

b) and c) Due to Lemma (6) above it is sufficient to consider the first round effect of parameter changes. Let $H_0(z)$ denote the initial equilibrium, and consider an increase in $\bar{g}$, a decrease in t or an increase in a. Then it follows that $\Gamma H_0(z) \geq (>)H_0(z)$ for all $z \leq (>)1/2$.

Existence of global competition.

Recall that $H(z)$ reaches a maximum at 0. In a symmetric equilibrium with $p_A = p_B$ it follows from (9) that we have global competition if

$$\int g(d(z,0))H(z)dz + \frac{-\bar{g}+t}{2} < t(1-a)$$

ref . Since $\int g(d(z,0))H(z)dz \leq \bar{g}$, a sufficient condition for global competition is

$$\bar{g} + \frac{-\bar{g}+t}{2} \; < \; t(1-a)$$
$$\bar{g} \; < \; t(1-2a)$$

Proof of Proposition 3
Proof. The following Lemma is used:

Lemma 7 *Assume $H_1(z) = \Gamma H_0(z) \geq H_0(z)$. Then $\Gamma H_1(z) \geq H_1(z)$*

■

Proof. Assume on the contrary that $\Gamma H_1(z) < H_1(z)$ for some z. This is only possible if the increase in A's market share associated with the mapping from H_0 to H_1 has a negative impact on the welfare of some members of the A-network. Since an increase in market share has either a neutral or a positive impact on every network's members' welfare this is not possible. QED

Under global competition a reduction in p_A generates a vertical parallel shift in the H-function of size $1/(t(1-a) - \bar{g})$, thus after the price change $\Gamma(H_0 + 1/(t(1-a) - \bar{g})) = H_0 + 1/(t(1-a) - \bar{g})$. Assume the equilibrium is characterized by local or hybrid competition, thus $H = 1$ around 0 and $H = 0$ around 1. Denote by H_0 the equilibrium before the decrease in p_A. If H were allowed to take values strictly above 1, then from (9) the following would hold: $\Gamma(H_0 + 1/(t(1-a) - \bar{g})) = H_0 + 1/(t(1-a) - \bar{g})$. Consider the function $H_d :=$ $H_0 + \min[1/(t(1-a)-\bar{g}), 1-H_0]$. Then it follows directly from (9) that $\Gamma[H_0 + \min[1/(t(1-a)-\bar{g}), 1-H_0]] \leq [H_0 + \min[1/(t(1-a)-\bar{g}), 1-H_0])$, with strict inequality for some z. Due to

Lemma (7) we thus now that there is an equilibrium below $\Gamma[H_0+\min[1/(t(1-a)-\overline{g}), 1-H_0]]$ which due to uniqueness is the only equilibrium. Since N' is monotonically increasing in the derived change in H, it follows that N' is always larger with global competition than with hybrid or local.

Under local competition a reduction in p_A generates a horizontal parallel shift in the H-function of size $1/ta$. Assume the equilibrium is characterized by hybrid competition, and refer to $H_0(z)$ as the equilibrium before the price decrease. Consider the function $H_d(z) := H_0(z-1/ta)$. By construction, the parallel shift captures the effect from the increased number of friends "on the same side of the market". However, under hybrid competition some agents get access to new friends on the "other side of the market", thus $\Gamma H_0(z - 1/ta) \geq H_0(z - 1/ta)$ with strict inequality for some z. Thus N' is always larger under hybrid than under local competition. $\blacksquare$

Maximizing versus minimizing social value

In the appendix we show that the social value in the the following: With two symmetric networks this is equivalent to maximizing V_A with respect to the distribution $H(z)$ subject to $\int H(z)dz = 1$, that is

$$\max_{H(z_i)} \iint g(z_i - z)H(z)H(z_i)dzdz_i \ s.t. \int H(z_i)dz_i = 1 \text{ all } z_i\epsilon[0, 1]$$

with the associated Langrangian

$$\int \left[\int g(z_i - z)H(z)dz - \lambda\right] H(z_i)dz_i$$

Point-wize maximization yields the first order condition

$$\int g(z_i - z)H(z)dz - \lambda \ > \ 0 \rightarrow H(z_i) = 1$$

$$\int g(z_i - z)H(z)dz - \lambda \ < \ 0 \rightarrow H(z_i) = 0$$

$$\int g(z_i - z)H(z)dz - \lambda \ = \ 0 \rightarrow H(z_i) \text{ undetermined}$$

Obviously there are two solutions satisfying the first order conditions, either $H(z) = 0.5$ all z, or $H(z) = 1$ for all $z\epsilon[z', -(1 - z')]$ where z' is arbitrary, and $H(z) = 0$ otherwise.[8] The two solutions are referred to as the maximum and minimum solutions respectively.

Two part tariff

It is convenient to characterize the optimal usage price (hence optimal subscriber composition) holding network size fixed (this requires an adjustment in connection fee), and then afterwards characterize the optimal network size (which is to determine the optimal connection fee). Due to symmetry, an increase in q_A matched with a decrease in p_A such

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[8]Observe from the first order conditions that the number of friends in the A network, $\int g(z_i - z)H(z)dz$, must be equal for all z_i at which $H(z_i)$ is strictly between 0 and 1. Then it follows trivially that H can be interior only if $H = 0.5$ everywhere.

that $-p_A + v(q_A)\overline{g}/2$ is constant will not change $H(1/2)$ and will not change the size of the network (but will reduce $H(z)$ for $z < 1/2$ and increase $H(z)$ for $z > 1/2$ in such a way that symmetry is preserved). Increasing q_A and adjusting p_A such that the scale effect is neutralized requires

$$\frac{dp_A}{dq_A} = \frac{v'(q_A)\overline{g}}{2} = \frac{-x(q_A)\overline{g}}{2} \tag{37}$$

Maximizing (25) with respect to q_A subject to (37) yields the first order condition

$$-\int H(z)dz\frac{x(q_A)\overline{g}}{2} + p_A \int \frac{dH(z)}{dq_A}dz + [x(q_A) + x'(q_A)(q_A - c)] \int\int g(d(z, z_i))H(z)H(z_i)dzdz_i$$

$$+x(q_A)(q_A - c)\int\int g(d(z, z_i))\frac{dH(z)}{dq_A}H(z_i)dzdz_i = 0$$

Since $\frac{dH(z)}{dq_A}$ and $H(z)$ are odd it follows that $\int\int g(d(z, z_i))\frac{dH(z)}{dq_A}H(z_i)dzdz_i = 0$ and that $\int \frac{dH(z)}{dq_A}dz = 0$. Hence

$$[1 - \gamma]x(q_A) + (q_A - c)x'(q_A) = 0$$

where $\gamma := \frac{1}{2}\overline{g}/\int\int g(d(z, z_i))H(z)dzdz_i$.

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