

Lawrence Berkeley National Laboratory

LBL Publications

Title

hi_class: Horndeski in the Cosmic Linear Anisotropy Solving System

Permalink

<https://escholarship.org/uc/item/0qb774wk>

Journal

Journal of Cosmology and Astroparticle Physics, 2017(08)

ISSN

1475-7516

Authors

Zumalacárregui, Miguel
Bellini, Emilio
Sawicki, Ignacy
[et al.](#)

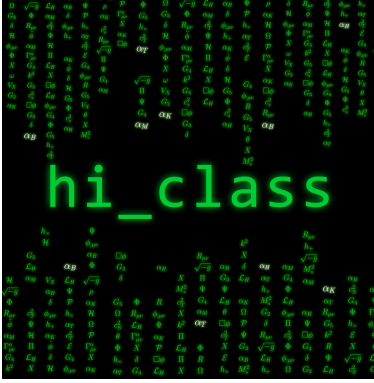
Publication Date

2017-08-01

DOI

10.1088/1475-7516/2017/08/019

Peer reviewed



hi_class: Horndeski in the Cosmic Linear Anisotropy Solving System

Miguel Zumalacárregui,^{a,b,c} Emilio Bellini,^{d,e} Ignacy Sawicki,^f
Julien Lesgourgues^g and Pedro G. Ferreira^e

^aNordita, KHT Royal Institute of Technology and Stockholm University
Roslagstullsbacken 23, SE-106 91 Stockholm, Sweden

^bBerkeley Center for Cosmological Physics, LBL and University of California at Berkeley
Berkeley, California CA94720, USA

^cInstitut für Theoretische Physik, Ruprecht-Karls-Universität Heidelberg
Philosophenweg 16, 69120 Heidelberg, Germany

^dInstitut de Ciències del Cosmos, Universitat de Barcelona, IEEC-UB
Martí i Franquè 1, E-08028 Barcelona, Spain

^eAstrophysics, University of Oxford,
Denys Wilkinson Building, Keble Road, Oxford OX1 3RH, United Kingdom

^fCentral European Institute for Cosmology and Fundamental Physics,
Fyzikální ústav Akademie věd ČR,
Na Slovance 2, 182 21 Praha 8, Czech Republic

^gInstitut für Theoretische Teilchenphysik und Kosmologie, RWTH Aachen University
D-52056 Aachen, Germany

E-mail: miguelzuma@berkeley.edu, emilio.bellini@physics.ox.ac.uk, ignacy.sawicki@fzu.cz,
lesgourg@physik.rwth-aachen.de, pedro.ferreira@physics.ox.ac.uk

Abstract. We present the public version of `hi_class` (www.hiclass-code.net), an extension of the Boltzmann code CLASS to a broad ensemble of modifications to general relativity. In particular, `hi_class` can calculate predictions for models based on Horndeski's theory, which is the most general scalar-tensor theory described by second-order equations of motion and encompasses any perfect-fluid dark energy, quint-essence, Brans-Dicke, $f(R)$ and covariant Galileon models. `hi_class` has been thoroughly tested and can be readily used to understand the impact of alternative theories of gravity on linear structure formation as well as for cosmological parameter extraction.

Keywords: Horndeski, public Boltzmann code, EFT of DE, modified gravity, dark energy, cosmology, scalar-tensor

Contents

1	Introduction	1
2	Horndeski’s theory	2
2.1	Background Evolution	3
2.2	Description of Perturbation Dynamics	4
2.3	Stability Conditions	7
2.4	Initial Conditions	8
3	Description of the Code	8
3.1	Implementation of Horndeski Dynamics	9
3.2	Available Models	11
3.3	Examples	12
4	Summary and Outlook	14
A	Equations and notation	17
A.1	Background	18
A.2	Linear perturbations in Synchronous Gauge	18
A.3	α Functions	20

1 Introduction

Numerical codes solving the Boltzmann equation in the presence of gravity have played a hugely important role in the development of modern cosmology. They are the theoretical and computational backbone for model building and precision constraints of cosmological parameters. The primary focus has been on standard cosmology – general relativity with a cosmological constant – where the modern era of fast, publicly available codes, began with CMBFAST [1] and was subsequently improved and extended with the Code for Anisotropies in the Microwave Background (CAMB) [2] and the Cosmic Linear Anisotropy Solving System (CLASS) [3]. These codes allow us to describe with percent-level precision the evolution of superhorizon initial conditions, through to the large-scale formation of structure and the propagation of photons. The output of these codes – a set of spatial and angular power spectra as a function of time – can be used to either constrain cosmological parameters directly [4] or to generate the initial conditions for simulations of more detailed and smaller-scale physics [5].

In this article, we present `hi_class`: *Horndeski in CLASS*, a publicly available extension of the CLASS Boltzmann-solver code which allows the user to consistently model the presence of an additional degree of freedom in the gravitational/dark-energy sector throughout the history of the universe, properly accounting for its effect on gravitational fields and matter and consistently connecting the early universe to the observables in these models. This code can then be used in the standard Monte Carlo Markov Chain module MontePython [6] to obtain constraints on parameters of the modifications of cosmology as well as to ascertain the degradation of the constraints on standard precision parameters resulting from our ignorance of the underlying theory for the dark sector.

The `hi_class` code has already been used to estimate non-linear effects such as the shift of the baryon acoustic scale [7], get constraints on scalar-tensor theories, using recent data [8], investigate relativistic effects on ultra-large scales and cross-correlations between the cosmic microwave background (CMB) and large-scale structure (LSS) [9].

In this first release of the code, we implement the necessary changes to allow for the modelling of the dynamics of all models of dark energy which belong to the class of Horndeski theories. These theories are the most general models of gravity extended by a single scalar degree of freedom featuring no more than second-order equations of motion and universally coupled to matter. This class is extremely wide and covers the substantial majority of models on the market, including $f(R)$ and $f(G)$ gravities, quintessence, perfect-fluid dark energy, Galileons, as well as many more exotic modifications. `hi_class` is under continuous development: features both extending the default choice of models and simplifying for the end-user the introduction of new ones are already in advanced stages of development and will be made public in due course.

The aim of this article is to present an overview of the modifications of gravity supported in `hi_class` and the way this support is implemented. Updated and more detailed information can be found on the website www.hiclass-code.net and the GitHub repository https://github.com/miguelzuma/hi_class_public. We start with a short review of Horndeski theories and the formulation we use to describe perturbations in Section 2. We then describe the modifications that have been introduced in CLASS and explain how to use all the features of the code in Section 3. The use of the public version of `hi_class` is free to the scientific community but conditional on the inclusion of references to at least this article and the CLASS paper [3].

2 Horndeski's theory

`hi_class` is a modification of the CLASS code which fully calculates the evolution of linear cosmological perturbations in dark energy/modified gravity (DE/MG) models belonging to the Horndeski class of theories. In these theories the gravitational sector contains one tensor and one scalar degree of freedom; their action was first written down in ref. [10], and then rediscovered as a generalisation of galileons [11] in ref. [12, 13].

Horndeski theories are specified through the action¹

$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left[\sum_{i=2}^5 \frac{1}{8\pi G_N} \mathcal{L}_i[g_{\mu\nu}, \phi] + \mathcal{L}_m[g_{\mu\nu}, \psi_M] \right], \quad (2.1)$$

$$\mathcal{L}_2 = G_2(\phi, X), \quad (2.2)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\square\phi, \quad (2.3)$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}(\phi, X) \left[(\square\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu} \right], \quad (2.4)$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}G_{5X}(\phi, X) \left[(\square\phi)^3 + 2\phi_{;\mu}{}^\nu\phi_{;\nu}{}^\alpha\phi_{;\alpha}{}^\mu - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\square\phi \right], \quad (2.5)$$

where the four Lagrangians \mathcal{L}_i encode the dynamics of the Jordan-frame metric $g_{\mu\nu}$ and the scalar field ϕ . They contain four arbitrary functions $G_i(\phi, X)$ of the scalar field and its canonical kinetic term, $2X \equiv -\partial_\mu\phi\partial^\mu\phi$; we use subscripts ϕ, X to denote partial derivatives, e.g. $G_{iX} = \frac{\partial G_i}{\partial X}$. This scalar field is a new degree of freedom when compared to general relativity and our implementation chooses appropriate initial conditions and then fully tracks its dynamical evolution. We restrict our code to scenarios where the weak equivalence principle holds, i.e. the matter, described by the Lagrangian \mathcal{L}_m , is universally coupled to the metric $g_{\mu\nu}$ and does not have direct couplings with the scalar.²

This general class of actions includes the majority of universally coupled models of dark energy with one scalar degree of freedom (see the review [15]): quintessence [16, 17], Brans-Dicke models [18], *k-essence*

¹Note that we have changed the normalisation of the functions G_i with respect to the usual convention by factoring out Newton's constant as measured on Earth today, G_N , and therefore reducing the mass dimension of the G_i by two. Since the Planck mass can vary in Horndeski models, see the discussion in section 2.1 for how we resolve the ambiguities. In our convention GR is $G_4 = 1/2$ and the rest zero.

²Modified gravity models may violate the equivalence principle via non-linear interactions [14]. However, these effects beyond the current scope of the `hi_class` code, which is based on linear perturbation theory.

[19, 20], kinetic gravity braiding [21–23], covariant galileons [11, 24], disformal and Dirac-Born-Infeld gravity [25–27], Chameleons [28, 29], symmetrons [30, 31], Gauss-Bonnet couplings [32] and models screening the cosmological constant [33, 34]. Archetypal modified-gravity models such as $f(R)$ [35] and $f(G)$ [36] gravity are within our purview.

We do not, at this time, include scalar-tensor extensions beyond Horndeski. These models always contain higher derivatives in the equations of motion, even though they cancel once all the constraints are solved, leaving second-order equations for the propagating degrees of freedom [37, 38] (see also [39–42]). Horndeski models can be extended even further to those which contain higher derivatives in space but not in time [43–46]. Modifications of gravity with non-scalar degrees of freedom, e.g. Einstein-Aether models [47] or ghost-free massive gravity [48–50] are not included either.

2.1 Background Evolution

The expansion rate of the universe H is determined by the energy density ρ_i of all the components of the universe in the standard manner,

$$H^2 = \sum_i \rho_i, \quad (2.6)$$

where we are using the CLASS normalisation for the components of the energy-momentum tensor of $3\rho_{\text{CLASS}} \equiv 8\pi G_{\text{N}}\rho_{\text{standard}}$, and G_{N} is fixed to be Newton’s constant as measured on Earth today. This sets the internal units for all dimensionful quantities in the code to Mpc^{-1} . In addition notice that `hi_class` and CLASS, and therefore this note, represents by H the physical-time Hubble parameter, but the time variable is nonetheless conformal time τ , with its associated $'$ notation for the derivative. Thus $a' = a^2 H$.

The evolution of the energy density for the Horndeski scalar field is determined by the energy density and pressure (rescaled by $8\pi G_{\text{N}}/3$), which are obtained from the action ((2.1)) through [51]

$$\rho_{\text{DE}} \equiv -\frac{1}{3}G_2 + \frac{2}{3}X(G_{2X} - G_{3\phi}) - \frac{2H^3\phi'X}{3a}(7G_{5X} + 4XG_{5XX}) \quad (2.7)$$

$$\begin{aligned} &+ H^2 \left[1 - (1 - \alpha_{\text{B}})M_*^2 - 4X(G_{4X} - G_{5\phi}) - 4X^2(2G_{4XX} - G_{5\phi X}) \right] \\ p_{\text{DE}} \equiv &\frac{1}{3}G_2 - \frac{2}{3}X(G_{3\phi} - 2G_{4\phi\phi}) + \frac{4H\phi'}{3a}(G_{4\phi} - 2XG_{4\phi X} + XG_{5\phi\phi}) - \frac{(\phi'' - aH\phi')}{3\phi'a}HM_*^2\alpha_{\text{B}} \quad (2.8) \\ &- \frac{4}{3}H^2X^2G_{5\phi X} - \left(H^2 + \frac{2H'}{3a} \right) (1 - M_*^2) + \frac{2H^3\phi'XG_{5X}}{3a}, \end{aligned}$$

where a is the scale factor of the Universe which appears as a result of the choice of definition of H . Note the presence of terms $\propto H^2, H^3$ stemming from the non-minimal coupling to the curvature in the action. The above expressions have been simplified by introducing M_*^2 , defined below, and α_{B} , defined in appendix A.3.

Cosmological backgrounds in Horndeski theories admit a time-evolving effective Planck mass, defined as the normalisation of the kinetic terms of the graviton. Horndeski models thus implicitly contain a free dimensionless function $M_*^2(\tau)$ — which we call the *cosmological strength of gravity* — which represents the square of the ratio of the cosmological Planck mass to the Earth-bound one and which is determined from the action through

$$M_*^2 \equiv 2 \left(G_4 - 2XG_{4X} + XG_{5\phi} - \frac{H\phi'}{a}XG_{5X} \right). \quad (2.9)$$

There are two subtleties which impact today’s value of M_*^2 :

- The measurement of G_{N} using Cavendish-like experiments in the context of a Horndeski gravity in fact measures the sum of the gravitational and the fifth forces and thus not quite the local value of the Planck mass. However, as the Solar-System constraints on the β and γ PPN parameters are of the

order 1 part in 10^5 [52], the difference between G_N and the underlying M_{Pl}^{-2} is negligibly small for the purposes of cosmological constraints and we neglect it.

- The non-linear screening of gravitational interactions (e.g. the chameleon [28] or Vainshtein [53] mechanisms) changes the strength of gravity between cosmological scales and the Solar System (it essentially is a change to the background solution, i.e. the local value of ϕ and ϕ' compared to these values in cosmology today). Thus Cavendish-like experiments determine M_{Pl} locally but this is not necessarily its value at cosmological scales today. Thus *if* there is a screening mechanism active, the cosmological strength of gravity today, $M_{*,0}^2$, is a free parameter to be constrained by observations. If no screening is active for the Solar System, $M_{*,0}^2 = 1$ up to the correction in the first bullet point and the initial conditions for M_*^2 should be set in such a way so as to achieve this. Note though that in the no-screening case, PPN constraints can be directly applied to cosmological scales and imply that the properties of gravity are very similar to GR, presumably preventing any significant modifications to cosmological observables.

We emphasise that the cosmological evolution does not actually depend on Solar-System measurements, but rather G_N sets the units necessary to convert the measurement of the dimensionful CMB temperature to an effect on space-time curvature that the radiation produces and thus the meaning that we ascribe to the Mpc distance unit and all dimensionful quantities in cosmological observations.

As a result of the evolving Planck mass, there also exists an ambiguity in the definition of the energy-momentum tensor. The particular choice of variables made here in eqs. (2.7-2.8) allows us to write down the conservation equation for the DE energy density and the equation of state in the standard manner

$$\begin{aligned}\rho'_{\text{DE}} &= -3aH(\rho_{\text{DE}} + p_{\text{DE}}) , \\ w_{\text{DE}} &\equiv \frac{p_{\text{DE}}}{\rho_{\text{DE}}} .\end{aligned}\tag{2.10}$$

This choice also implies that the density of e.g. pressureless matter scales as in the standard case, $\rho_{\text{m}} \propto a^{-3}$, despite the evolving masses. This matches the implicit assumption in all published observational constraints on w_{DE} . Note though that the equation of state w_{DE} would not necessarily be the ratio of pressure and energy density as measured by a comoving observer in the past if they had the means to probe the instantaneous energy density and pressure (see the discussion in Appendix A.1).

The flexibility of choosing arbitrary functions $G_i(\phi, X)$ and initial conditions for the scalar field means that essentially any choice of the function $w_{\text{DE}}(\tau)$ can be made. It is only the choice of the history of $\rho_{\text{DE}}(\tau)$ (equivalently $w_{\text{DE}}(\tau)$ and the density today $\rho_{\text{DE},0}$) that impacts the evolution of the background and thus specifying these is enough to describe all possible cosmological backgrounds. The only theoretical constraint is that such a background does not suffer from instabilities. We discuss this issue in Section 2.3.

Specifying the evolution of the background does not, however, determine the evolution of perturbations. Implementing the evolution of perturbations properly was our aim in writing `hi_class`, since it is discriminating between the various perturbation evolutions possible on the same observed background that allows us to differentiate between the models of modified gravity.

2.2 Description of Perturbation Dynamics

The primary focus of Einstein-Boltzman solvers is linear cosmological perturbations on a homogeneous and isotropic background. The evolution equations that completely describe the dynamics of the perturbed fields can be obtained from the linearized field equations arising from the action in Equation (2.1). An alternative, and enlightening, approach is to expand the action in Equation (2.1) to second order in linear perturbations of $g_{\alpha\beta}$, ϕ and the remaining matter fields. The action for perturbations consists of a sum of terms, quadratic in the perturbation fields, each of which is multiplied by a time dependent function which solely depends on the background cosmology. One can choose a basis for these time dependent coefficients and a particularly

useful one was given in [54], since they relate most directly to the physical observables and to the propagating degrees of freedom in the Horndeski models. Once the background $H(\tau)$ is specified through Eq. (2.6), the complete set of possible modifications of Einstein’s equations describing the evolution of linear perturbations in Horndeski theories is determined by four functions of time: the dimensionless cosmological strength of gravity $M_*^2(\tau)$ and three $\alpha_i(\tau)$. In this document we will refer to these four functions collectively as “ α functions”. The definitions of the α functions in terms of the action are given in Appendix A.3.

Working with the quadratic action arising from Equation 2.1 also allows us to connect with the approach of Effective Field Theory of Dark Energy (EFT) [55–57]. There the idea is to encode all the possible modifications of Einstein equations consistent with the symmetries of the cosmological background and gauge freedom in terms of coefficients which are functions of time only, and which multiply fixed operators which carry the information on scale dependence. In the EFT approach, a choice of basis for the operators must also be made (see ref. [54] for a translation of the EFT of DE operators into the α function basis).

`hi_class` solves the equations for the tensor (A.13) and scalar modes (A.14–A.18) of the modified gravitational sector, together with the Boltzmann hierarchy for the standard matter components, to give the full predictions for linear large-scale structure. The four functions describing the modification of gravity can be divided into two pairs: the first related to both the scalar and the tensor modes, the second only to the scalar. We refer the reader to ref. [54] for details of the physical impact of these parameters.

1. Non-minimal coupling of gravity. These functions modify both the scalar and the tensor propagation:

- (a) $M_*^2(\tau)$, *the cosmological strength of gravity*. M_*^2 is the dimensionless product of the normalisation of the kinetic term for gravitons and $8\pi G_N$ as measured on Earth. It thus encodes the difference between the solar system and cosmology in the gravitational force/space-time curvature produced by a fixed amount of energy. Large-scale structure is sensitive only to the time variation of the Planck mass,

$$\alpha_M \equiv \frac{d \ln M_*^2}{d \ln a}, \quad (2.11)$$

or the *Planck-mass run rate*. However, as a result of screening, it is possible that Newton’s constant as measured by local experiments, G_N , and that on cosmological scales are different. Thus only if screening in the Solar System is active, the value today of $M_{*,0}^2$ is free and largely unconstrained. If the Solar-System is unscreened, $M_{*,0}^2 = 1$. Also note that Planck observations of the recombination history imply that the value of the Planck mass at recombination cannot differ from that measured in the Solar System by more than 1% [4]. We give users the choice to employ as the principal parameterisation: either $M_*^2(\tau)$, or $\alpha_M(\tau)$ together with the initial value of M_*^2 .

- (b) $\alpha_T(\tau)$, *tensor speed excess*. This parameter denotes the difference in the propagation speed of gravitational waves compared to the speed of light, i.e. $\alpha_T = c_T^2 - 1$. It applies to modes propagating on cosmological scales and is currently the most weakly constrained parameter from physics other than cosmology (see ref. [58] and references therein).

In Horndeski theories one expects screening of high-density environments like the Earth or the location of a system that could produce gravitational waves (GWs) and thus no modification in the production or detection of GWs is likely [59, 60] (although see ref. [61]). The GW dispersion relation is only modified by the change in the sound speed and thus one would not expect any frequency-dependent modifications of the propagation speed. An observation of an indisputable electromagnetic counterpart to a gravitational wave event at cosmological distances would put a constraint on this parameter tight enough to render the remaining uncertainty irrelevant for structure formation [62, 63].

Note that either α_M or α_T must not vanish in order for gravitational slip to be generated [64] (e.g. modifying the spatial-traceless metric equation A.17). In such a case, the equation of motion for the propagation of gravitational waves (A.13) is also modified.

2. Kinetic terms. The scalar mode is also affected by the following two functions:

- (a) $\alpha_B(\tau)$, *braiding*. This operator gives rise to a new mixing of the scalar field and metric kinetic terms for the propagating degree of freedom [65]. This leads to a modification of the coupling of matter to the curvature, independent and additional to any change in the Planck mass. This is typically interpreted as an additional fifth force between massive particles and can be approximated as a modification of the effective Newton’s constant for perturbations. It is present in archetypal modified gravity models such as Brans-Dicke and $f(R)$ gravity (see [54] for details). A purely conformal coupling of the scalar to gravity leads to the universal property $\alpha_M + \alpha_B = 0$.
- (b) $\alpha_K(\tau)$, *kineticity*. Coefficient of the kinetic term for the scalar d.o.f. before demixing (see ref. [54]). Increasing this function leads to a relative increase of the kinetic terms compared to the gradient terms and thus a lower sound speed for the scalar field. This creates a sound horizon smaller than the cosmological horizon: super-sound-horizon the scalar does not have pressure support and clusters similarly to dust. Inside, it is arrested and eventually can enter a quasi-static configuration [66]. When looking only at the quasi-static scales, inside the sound horizon, this function cannot be constrained [67].³ This is the only term present in the simplest DE models, e.g. quintessence and in perfect-fluid dark energy.

Note that the quantity $D \equiv \alpha_K + \frac{3}{2}\alpha_B^2$ has to be *strictly* positive at all times to avoid strong coupling problems or ghosts. This comes from the fact that D represents the demixed kinetic term of the additional scalar degree of freedom. Moreover $D = 0$ is a pressure singularity and cannot be crossed in background dynamics, even though this might seem possible at the level of parameterised perturbations (see e.g. ref. [69] for example phase-space trajectories in cosmology).

The Horndeski class of theories together with the freedom to choose initial conditions is large enough to guarantee that there exists some action that would result in any choice of equation of energy density of dark energy $\rho_{DE}(\tau)$ and an arbitrary choice of the α functions, apart from the exception mentioned above. Such an arbitrary choice may be unstable and/or fine tuned and might thus not be a good solution in practice. Conversely, the only information relevant for linear Boltzmann codes that a particular model based on a full covariant action provides is exactly contained in this evolution history of the energy density of the dark energy and the α functions for that model. It is enough to calculate these as functions of time to then be able to obtain the predictions of that model at all scales with no loss of information. We can thus take two approaches to modelling:

1. Start by choosing a full covariant classical action within the Horndeski class according to some overarching principle. Then calculate the self-consistent predictions for this particular model for the background and the perturbations, given some initial conditions for the background scalar. This allows the user to constrain the values of particular parameters of the model.
2. Use a specification of some evolution history for the cosmological background and the α functions as the model, without further reference to the action. This allows the user to test whether there is any evidence for the departure of the behaviour of growth from the concordance predictions and what sort of physical properties of the gravity theory could be driving that growth.

Our implementation in `hi_class` allows us to cover both the approaches with the same code. Approach (1) requires an additional pre-computation of the background and the α -functions as part of an initialisation module which then feeds these quantities to the same code as (2) would use (see Figure 1). For the initial roll-out, we are enabling the parameterised EFT approach (2) in this version of `hi_class`, but will be bringing an automatised implementation beginning with a full covariant action in the full release.

³Constraints on α_K can be obtained from observations on ultra-large scales [68].

2.3 Stability Conditions

When specifying a model, it is possible to choose a set of α functions which lead to exponentially unstable perturbations. This is all the more likely when choosing a set of α s which are completely unrelated to a legitimate scalar field model, but can also happen for what seems like a valid background but which might actually have a kinetic term with the wrong sign (e.g. forcing a perfect fluid to evolve with $w_{\text{DE}} < -1$). Every background supplied to `hi_class` is tested to make sure that no such short-timescale instabilities are present and the background can be trusted over cosmological timescales. Only if such stability tests are passed, are the perturbation equations solved for that set of parameters.

There are essentially three types of instabilities that would disqualify a choice of background.

- *Ghost instabilities* occur when the sign of the kinetic term of a degree of freedom is wrong, giving negative energy modes and violating unitary evolution. If the mass of these ghosts is less than the cutoff of the theory, this would lead to rapid pair production into negative-energy modes, destroying the background.
- *Gradient instabilities* occur when the sound speed squared is negative. This leads to an exponentially growing instabilities with a rate corresponding to the shortest mode allowed by the effective theory. Importantly, such instabilities may not be visible until very short modes and/or higher-order (interaction) terms are included in the evolution and thus a seemingly unsuspect evolution of linear perturbations is not sufficient that the predictions are at all correct, necessitating the stability tests.
- *Tachyon instabilities* occur when the mass squared for the perturbations is negative. This leads to a power-law instability at *large* scales, which comes under control when the mode enters the sound horizon. Since this has a significant on the calculated observables, we choose to let the data exclude such cases, rather than creating a hard stability test.

The Horndeski class of models affects the evolution of both scalars and tensors. `hi_class` thus performs four tests to verify the stability of the background:

$$Q_S = \frac{2M_*^2 D}{(2 - \alpha_B)^2} > 0, \quad D \equiv \alpha_K + \frac{3}{2}\alpha_B^2 > 0 \quad (2.12)$$

$$c_s^2 = \frac{1}{D} \left[(2 - \alpha_B) \left(-\frac{H'}{aH^2} + \frac{1}{2}\alpha_B(1 + \alpha_T) + \alpha_M - \alpha_T \right) - \frac{3(\rho_m + p_m)}{H^2 M_*^2} + \frac{\alpha'_B}{aH} \right] > 0 \quad (2.13)$$

$$Q_T = \frac{M_*^2}{8} > 0 \quad (2.14)$$

$$c_T^2 = 1 + \alpha_T > 0, \quad (2.15)$$

Q_S and Q_T respectively represent the kinetic terms of scalar and tensor sectors after demixing; c_s^2 and c_T^2 are the sound speeds of these two degrees of freedom.

We should note that it is still possible that a background that passes the above tests, is nonetheless not healthy as a result of some non-linear instability, for example when the null energy condition is violated [70]. On the other hand, one may also argue that the appearance non-linear structure from the exponential instabilities at small scales might arrest the destabilisation and save a choice of a background that does not satisfy the stability checks. Strictly speaking, such a background would no longer be FRW and therefore the description in terms of the α functions would no longer be complete, but it is possible that the largest scales remain close enough to FRW to allow a description that we have implemented.⁴ In any case, should such a scenario be favoured by the user, or the user would like to check which part of the parameter space is

⁴But this approximate background might have different effective values of the α_i to the original ones we started with, presumably ones implying stability.

constrained on theoretical grounds as opposed to just data on large scales, we give them the freedom to relax or disable these checks entirely. See [71–73] for studies of the impact of stability conditions on parameterized models.

2.4 Initial Conditions

The formulation of perturbation equations that we have implemented depends on using as the variable for the fluctuation of the scalar field a velocity potential,

$$V_X \equiv a \frac{\delta\phi}{\phi'}. \quad (2.16)$$

Its dynamics is described by the second-order differential equation (A.18), which requires two initial conditions, to be solved.

The chosen initial conditions are specified superhorizon and typically evolve rapidly into the natural attractor of the theory, at least in the case of negligible modified gravity effects at early times (for the discussion of perfect fluid DE see [74]). Indeed, we have found that, for parameterisations enabled in this version of `hi_class`, the choice of initial conditions is not material to the study of the late universe. We have thus included a couple of simple choices to allow the user to verify the possible impact of initial conditions. Moreover, the code includes a flexible structure to allow the user to easily incorporate other choices.

The simplest possible choice is to set both the field perturbation and its derivative to zero

$$V_X(\tau_0) = 0, \quad V_X'(\tau_0) = 0. \quad (2.17)$$

In simple inflationary models the initial perturbations are given by *single-clock* initial conditions. In the case of the scalar field this scenario is implemented by assuming $\delta\phi(\tau, \vec{x}) = \phi(\tau + \delta\tau(\tau, \vec{x}))$, i.e. it is given by the background value of the scalar field evaluated on perturbed constant time hypersurfaces. In terms of the field perturbation and relating $\delta\tau$ to the perturbation in the photon density yields

$$V_X(\tau_0) = -\frac{1}{4H} \frac{\delta\rho_\gamma}{\rho_\gamma}, \quad V_X'(\tau_0) = 0. \quad (2.18)$$

The detailed initial conditions for Horndeski theories will be presented with the full version of the code.

3 Description of the Code

`hi_class` is a forked version of the Cosmic Linear Anisotropy Solving System (CLASS), a modern Boltzmann-Einstein code designed to compute cosmological predictions in the linear regime. CLASS has been developed from the outset to make it intuitive and simple to modify. It consists of several modules that perform tasks sequentially (reading parameters, computing the background, thermodynamics, perturbations, etc.; see [75]), with each module relying on the results generated by the previous ones. A major design goal of the code has been flexibility, so CLASS can easily incorporate non-standard cosmological models. Several extensions of the Λ CDM concordance scenario, such as massive neutrinos, warm dark matter, curvature [76–78], but also quintessence and perfect-fluid dark energy, have already been implemented in a manner compatible with the CLASS principles. These, as well as the standard components (photons, baryons, dark matter) are treated sequentially so that a block of instructions (to account for each component) is executed only if such a component is included in the model and ignored otherwise. This allows CLASS to remain both fast, flexible and self-consistent in the presence of many different modifications.

`hi_class` incorporates general Horndeski scalar-tensor modifications of gravity as yet another option in this scheme, allowing the user to investigate with ease the theory of gravity governing the Universe. Moreover, `hi_class` allows the user to investigate the new degeneracies that appear when modifications of gravity are consistently included together with other extensions of the concordance model. `hi_class` can be

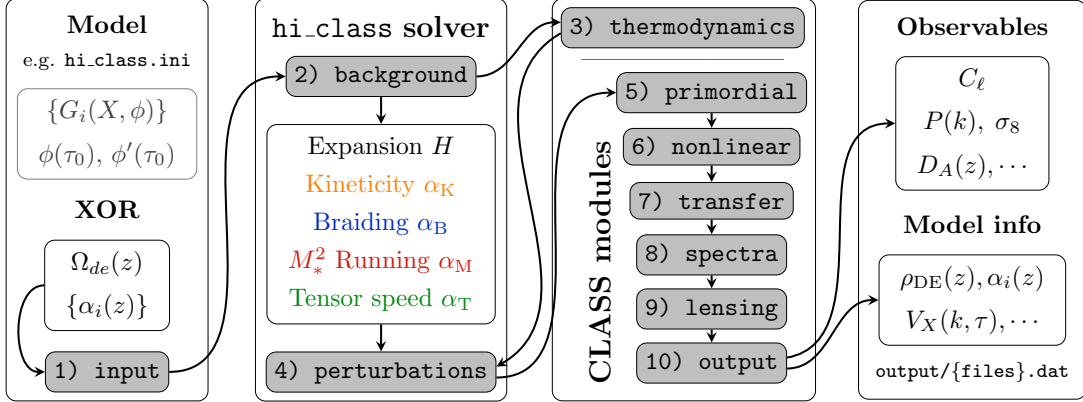


Figure 1: `hi_class` structure. The arrows represent the flow of information between modules, while the numbers represent the order of execution of the modules. The user has to specify a parameterisation for the expansion and the α functions (in the full version, a model based on a full covariant action is also possible) that is interpreted by the input module. Then the background is computed giving the Hubble rate and the α functions. The unmodified thermodynamics module computes the thermal evolution using this background expansion history. The code then uses these as inputs to determine the evolution of the perturbations. The rest of the code is unmodified with respect to the public version of CLASS. `hi_class` can be used to output both cosmological observables as well as details about the model and its evolution.

used with other cosmic components such as massive neutrinos, but not yet spatial curvature. Other features of CLASS such as the option to compute relativistic corrections to observables (e.g. the CLASSgal code [79]) are also compatible [9].

By using the description of the dynamics in terms of the α functions, as laid out in section 2, we have been able to write a common routine for all Horndeski models, leaving the user with the freedom to determine the particular choice of model, which boils down to a description of the evolution history of the cosmological background and the α functions, but not to concern themselves with the numerical algorithm. From the perspective of `hi_class`, it is immaterial if this evolution history originates from a model based on a full covariant action or if it is generated by assuming a parameterisation. In both cases, the code pre-computes the appropriate quantities from the given model automatically and evolves the predictions for large-scale structure (see Figure 1).

We describe the general algorithm used by `hi_class` in section 3.1, discuss the choice of models we are releasing to the public at this juncture in section 3.2 and demonstrate the impact on observables of some chosen modifications of gravity in the Horndeski class in section 3.3.

3.1 Implementation of Horndeski Dynamics

The code first runs the background module, solving for the background evolution. The scale factor as a function of conformal time is obtained by integrating the Hubble rate as given by the Friedmann equation (2.6), starting deep in the radiation era. The Hubble parameter depends on the matter components (DM, baryons, radiation, massive neutrinos) as well as the model for the energy density of the Horndeski scalar $\rho_{\text{DE}}(\tau)$ and its corresponding pressure $p_{\text{DE}}(\tau)$ specified by the user.

The user can specify either a parameterization for the density and pressure, possibly with multiple parameters (e.g. $\Omega_{\text{DE},0}$, w_0, w_a) or by specifying a model based on an action (not currently available in the public version). In the latter case, the energy density and pressure typically depend indirectly on the parameters of the action through the evolution of an underlying scalar field value and also its initial

conditions at the background level. `hi_class` employs CLASS’s automatic routine which iteratively solves for the evolution, making sure that the dark energy reaches the appropriate value for its density fraction today as required by the closure relation

$$\sum_i \Omega_{i,0} = 1. \quad (3.1)$$

The user is required to specify which of the background parameters is to be varied by the code to give the correct density fraction. Note that the degrees of freedom of the background parameterisation are always one fewer than the number of parameters, since the constraint (3.1) must always be satisfied: e.g. in flat Λ CDM, once the density fraction of CDM, baryons and radiation are specified, Ω_Λ is a derived parameter and there is no remaining freedom.

At this point, all background quantities — those that depend only on time (including indirectly through other background functions), but do not depend on scale — are computed and stored in an interpolation table for further use in subsequent modules. This includes the energy density and pressure of all matter species, including the Horndeski scalar field, as well as the α functions (as prescribed by the model chosen by the user) and all relevant combinations of quantities that enter the coefficients of the perturbation equations (see section A.2). Whenever these quantities depend on time derivatives (e.g. of the α functions), they are computed numerically at this stage. For instance eq. (2.9) defines the running of the effective Planck mass.

For parameterisations in which it is the cosmological strength of gravity $M_*^2(\tau)$ that is supplied by the user, α_M is obtained through a numerical derivative. On the other hand, whenever α_M is supplied by the user, M_*^2 is obtained by the background module using numerical integration, requiring an input of an initial value of M_*^2 ; this procedure gives the appropriate value of the cosmological strength of gravity today $M_{*,0}^2$ as a derived quantity. We note, however, that with the choice of definition for ρ_{DE} and p_{DE} , eqs (2.7-2.8), M_*^2 has no effect on background evolution since it can always be reabsorbed into an appropriate rescaling of both the Hubble constant H_0 and the physical energy densities of all the components of the Universe. It is only when perturbations are considered that it plays a role.

The code then moves on to evaluate the stability conditions given in section (2.3). This allows us to check if the particular background requested by the user is stable and consistent for the particular model of gravity chosen. If any of the conditions fails to be satisfied, the code interrupts execution and returns an error. If running on an MCMC, the sampler assigns a negligible likelihood to the parameters, indicating that the model is incompatible with data, and moves on to try different values. The user has the freedom to override this consistency check by setting alternative thresholds for each of the stability conditions (2.12-2.15) or skip this test altogether. Note that $M_{*,0}^2$ enters the stability conditions and is as of this point a physically meaningful quantity.

The perturbations module runs over the chosen range of scales k , solving the modified equations for the metric (A.1) and scalar-field in the synchronous gauge (c.f. appendix A.2). Numerical integrations are performed for the metric potential η through the first-order equation (A.15) and the scalar field eq. (A.18), while other quantities are determined algebraically from the remaining gravitational eqs (A.14,A.16,A.17). The effect of universally coupled Horndeski models is to modify just the gravitational sector. The effect on the dynamics of the matter components is solely through the modification of the gravitational potentials and thus the matter perturbations are evolved using the perturbed continuity/Euler equations or the Boltzmann hierarchy as appropriate, with no changes to the methods implemented in CLASS.

The initial conditions for all components are set in an early epoch that is both in the radiation era and guarantees that the scale in question is super-horizon, i.e. $k\tau \ll 1$ (see ref. [3] for details). `hi_class` by default allows for two possible initial conditions, as described in section 2.4. As mentioned above, we have checked that the choice of initial conditions does not affect the predictions whenever the α functions are negligible at early times.

For full consistency, we also modify the evolution of the tensor modes according to eq. (A.13). This does not significantly affect observables unless there is a significant primordial amplitude of tensor modes [80].

`hi_class` retains the full dynamics across cosmic history and does not invoke any further simplifications to solve the linear-perturbation equations. However, for some models, the computation of the evolution of the scalar field value can slow down considerably when the α functions are small and a hierarchy exists between them (in particular when $\alpha_K \ll \alpha_B^2$). This can happen for some models, typically at early times, and can make computations unfeasible as part of an MCMC chain. Such an issue is typically caused by a very large effective mass term for the mode in the equation of motion for the scalar-field perturbation, leading to very rapid oscillations in the solution, which the integrator attempts to track. Writing the equation of motion for the scalar (A.18) schematically,

$$V_X'' + \nu a H V_X' + (a^2 \mu^2 + c_s^2 k^2) V_X = S, \quad (3.2)$$

the physical origin of such a problem can be two-fold: either the speed of sound (2.13) is very high (extremely superluminal), $c_s^2 \gg 1$, so that modes which are outside of the cosmological horizon at early times nonetheless are inside the sound horizon and oscillating. This is probably an undesired feature for the model. Or: the mass of the scalar field is very high initially, $\mu \gg H$. In such a case, scales with $a\mu > c_s k$ are outside of the Compton length of the scalar and the scalar perturbation V_X is negligible. Such a scenario is typical, for example, in $f(R)$ models which always have $\alpha_K = 0$.

To accelerate the computation in these cases, we have introduced a user-defined parameter which regularises the kinetic term of the scalar field by adding a small constant value to the kineticity, $\alpha_K^{\text{tot}}(\tau) = \alpha_K(\tau) + \alpha_K^{\text{reg}}$. This artificially reduces the sound speed and mass at early times, leading the scalar-field to behave similarly to a dust component. Depending on the model, this might alter the configuration of the scalar during recombination sufficiently to have an effect on the predictions. However, we have tested that changing the value of α_K^{reg} does not significantly affect the results. We leave it to the user to test what maximal value for α_K^{reg} can be used for their chosen model and parameter range of interest without biasing the predictions. The next version of `hi_class` will incorporate suitable approximation schemes to avoid the necessity of kineticity regularization.

The output of the perturbations module consists of source functions dependent on time τ and scale k that store information (i.e. solutions of the perturbed variables and combinations thereof). This data is stored in memory and then used in subsequent modules to compute the observables. The CLASS code has been designed to avoid duplicated equations or implicit assumptions in latter parts of the code. This implies that the Friedmann equation appears only at one point in the background module, and similarly for the (modified) Einstein equations in the perturbations module. These are the only places where the dynamics of modified gravity enter, and there are essentially no changes in any subsequent module. Note however that certain options in CLASS might still be inconsistent with the modified-gravity dynamics in general. For example Halofit corrections to the matter power spectrum at small scales are calibrated from Λ CDM numerical simulations and are not necessarily meaningful in models departing from concordance.

3.2 Available Models

In this first public release of `hi_class`, we have only enabled a limited number of models. For the initial roll-out, we have focused on the EFT-like parameterisations, leaving models specified through full covariant actions for the next release. Moreover, the current implementation is limited to flat spatial sections for the cosmological background, $\Omega_k = 0$.

The user chooses the model using the initialization file `hi_class.ini`. This or a similar file is read in by the input module, which sets up the relevant variables and performs the appropriate tests.⁵ The user needs to specify the DE density and pressure and the four α functions. In general, they can be functions of any of the background quantities (e.g. H, a, τ , etc.).

For the background energy density for the Horndeski scalar we have enabled two choices in this first version of `hi_class` (see table 1a):

⁵See the `hi_class.ini` file included with the code for a detailed description of the available user options. The `hi_class` wiki contains extended and up-to-date information (https://github.com/miguelzuma/hi_class_public/wiki).

1. `lcdm`: the DE has constant energy density determined by the parameter $\Omega_{\text{DE},0}$ derived from the closure relation (3.1).
2. `wowa`: evolving equation of state with parameters $(\Omega_{\text{DE},0}, w_0, w_a)$ [81, 82] with the constraint that the density fraction today satisfies the closure relation (3.1).

These are sufficient to fully evolve the cosmological background. The user must then specify a parameterisation for the α functions needed to calculate the evolution of perturbations on this background. At this juncture we are enabling four parameterisations for the α functions (see table 1b):

1. `propto_omega`: the α functions are proportional to the density fraction of dark energy, Ω_{DE}
2. `propto_scale`: the α functions are proportional to the scale factor
3. `planck_linear`: *k-essence* conformally coupled to gravity studied by Planck [83]
4. `planck_exponential`: alternative *k-essence* conformally coupled to gravity studied by Planck [83]

All these parameterisations have $\alpha_i \rightarrow 0$ at early times, consistent with the expectation that gravity would not be modified during eras when the dark energy does not provide a significant contribution to the energy density of the Universe.

We have also implemented two models which were studied by the Planck team [83], `planck_linear` and `planck_exponential`, to provide a possibility of easy comparison with published constraints. Both these models are specified through a single function of time $\Omega(\tau)$ affecting three of the α functions:

$$\begin{cases} \alpha_{\text{K}} = \frac{3(\rho_{\text{DE}} + p_{\text{DE}})}{H^2} + \frac{3\Omega(\rho_{\text{m}} + p_{\text{m}})}{H^2(1+\Omega)} - \frac{\Omega'' - 2aH\Omega'}{a^2H^2(1+\Omega)} \\ \alpha_{\text{M}} = \frac{\Omega'}{aH(1+\Omega)} \\ \alpha_{\text{B}} = -\alpha_{\text{M}} \\ \alpha_{\text{T}} = 0. \end{cases} \quad (3.3)$$

The difference between the “linear” and the “exponential” models is just the parameterisation for $\Omega(\tau)$ (see table 1b).

3.3 Examples

In Figs. 2, 3 and 4 we show some illustrative plots of the CMB temperature power spectrum (left panels) and the matter power spectrum calculated at redshift $z = 0$ (right panels). In all these plots we have fixed the standard cosmological parameters to Planck-best-fit values [84] and we vary only the DE/MG parameters. They have been created using the `lcdm` parametrization for the background expansion history and `propto_omega` for the time evolution of the α functions. Constraints using this parametrization and recent data have been obtained in ref. [8]. Here the authors also show that variations of the kineticity have a negligible effect on this background. We thus fix $\hat{\alpha}_{\text{K}} = 1$ and explore the remaining α functions.

In particular, in Fig. 2 we fix $\hat{\alpha}_{\text{K}} = 1$, $\hat{\alpha}_{\text{M}} = \hat{\alpha}_{\text{T}} = 0$ and we vary $\hat{\alpha}_{\text{B}}$. With these restrictions, negative values of $\hat{\alpha}_{\text{B}}$ are not allowed by the stability conditions, Eqs. (2.12-2.15). It is possible to note that the relative difference between DE/MG models and the fiducial Λ CDM is large at small scales, i.e. low- ℓ 's and small k . In particular, the amplitude of the matter power spectrum is suppressed on these scales. However, $\hat{\alpha}_{\text{B}}$ has also a non-negligible effect at large k , i.e. $k \gtrsim 10^{-2}$, where we can see the opposite effect, i.e. the amplitude of the matter power spectrum is enhanced as $\hat{\alpha}_{\text{B}}$ increases. This results from the scalar field communicating an extra force and increasing the attractive interaction between perturbations at small scales [54]. Note that increasing $\hat{\alpha}_{\text{B}}$ pushes the low- ℓ CMB to lower values, but then becomes rapidly larger than in the standard case. This is because the braiding tends reduce the Integrated Sachs-Wolfe (ISW) and even make it negative

Parameterisations of Background		
expansion_model_smg	Expansion	Notes
lcdm	$\rho_{\text{DE}} = \Omega_{\text{DE},0} H_0^2$	CC-like
w0wa	$\rho_{\text{DE}} = \Omega_{\text{DE},0} H_0^2 a^{-3(1+w_0+w_a)} e^{3w_a(a-1)}$	Similar to omega_fld

(a) Parameterisations for dark energy density enabled in the first version of `hi_class`.

Parameterisations of Gravity					
gravity_model_smg	α_{K}	α_{B}	α_{M}	α_{T}	Additional Params.
propto_omega			$\alpha_i = \hat{\alpha}_i \Omega_{\text{DE}}(\tau)$		$M_{*,\text{ini}}^2$
propto_scale			$\alpha_i = \hat{\alpha}_i a(\tau)$		$M_{*,\text{ini}}^2$
planck_linear			Eq. (3.3) with $\Omega = \Omega_0 a(\tau)$		$M_{*,\text{ini}}^2$
planck_exponential			Eq. (3.3) with $\Omega = \exp\left[\frac{\alpha_{\text{M}0}}{\beta} a^\beta(\tau)\right] - 1$		$M_{*,\text{ini}}^2$

(b) Parameterisations of the α functions enabled in the first version of `hi_class`.

Table 1: Summary of models enabled in the first version of `hi_class`. For further details read the `hi_class.ini` file or visit https://github.com/miguelzuma/hi_class_public/wiki.

[9]. The initial decrease in the CMB spectrum is due to this initial reduction, while the subsequent increase for $\hat{\alpha}_{\text{B}} \gtrsim 1.5$ is due to the positive definite (ISW)² contribution.

In Fig. 3 we fix $\hat{\alpha}_{\text{K}} = 1$, $\hat{\alpha}_{\text{B}} = \hat{\alpha}_{\text{T}} = 0$ and we vary $\hat{\alpha}_{\text{M}}$, setting $M_{*,\text{ini}}^2 = 1$ initially. In this case negative values of $\hat{\alpha}_{\text{M}}$ lead to gradient instabilities. Then, assuming positive values of the Planck mass run-rate it is possible to note that the CMB temperature spectra amplitude are enhanced at low ℓ 's due to the ISW effect, while for large values of ℓ there are not substantial differences. An analogous result can be found looking at the matter power spectrum plots, where at large scales the relative differences between DE/MG models and Λ CDM appear maximised.

In Fig. 4 we fix $\hat{\alpha}_{\text{K}} = 1$, $\hat{\alpha}_{\text{B}} = \hat{\alpha}_{\text{M}} = 0$ and we vary $\hat{\alpha}_{\text{T}}$. Positive values of $\hat{\alpha}_{\text{T}}$ are unstable with this choice of the other parameters, and thus we have explored only the negative region. The effect of the tensor speed excess on the CMB temperature spectrum and the matter power spectrum is smaller w.r.t. the effects seen in Figs. 2 and 3 at all scales even for extreme values of $\hat{\alpha}_{\text{T}}$. This indicates that $\hat{\alpha}_{\text{T}}$ is expected to contribute less than the other parameters on CMB and LSS observables. In particular, $\hat{\alpha}_{\text{T}}$ by itself has no effect whatsoever on small scales.

We should also note that we have generated the plots for $w_{\text{DE}} = -1$. In this limit, many terms in the perturbation equations cancel and therefore the modifications to observables are suppressed. Adding in a background evolution that departs from Λ CDM can produce a much richer phenomenology. Similarly, considering simultaneously non-zero values of several α functions can enhance the observed effects and produce new ones, because of products of α functions appearing in the equations of motion. We emphasize that the spectra are the result of solving a complex system of differential equations (plus performing line-of-sight integrals in the case of the CMB) and therefore it is not to be expected for the deviations to be proportional to the coefficients $\hat{\alpha}_i$. As a result, the posteriors for the $\hat{\alpha}_i$ parameters are not expected to be Gaussian and are thus sensitive to the choice of the fiducial model.

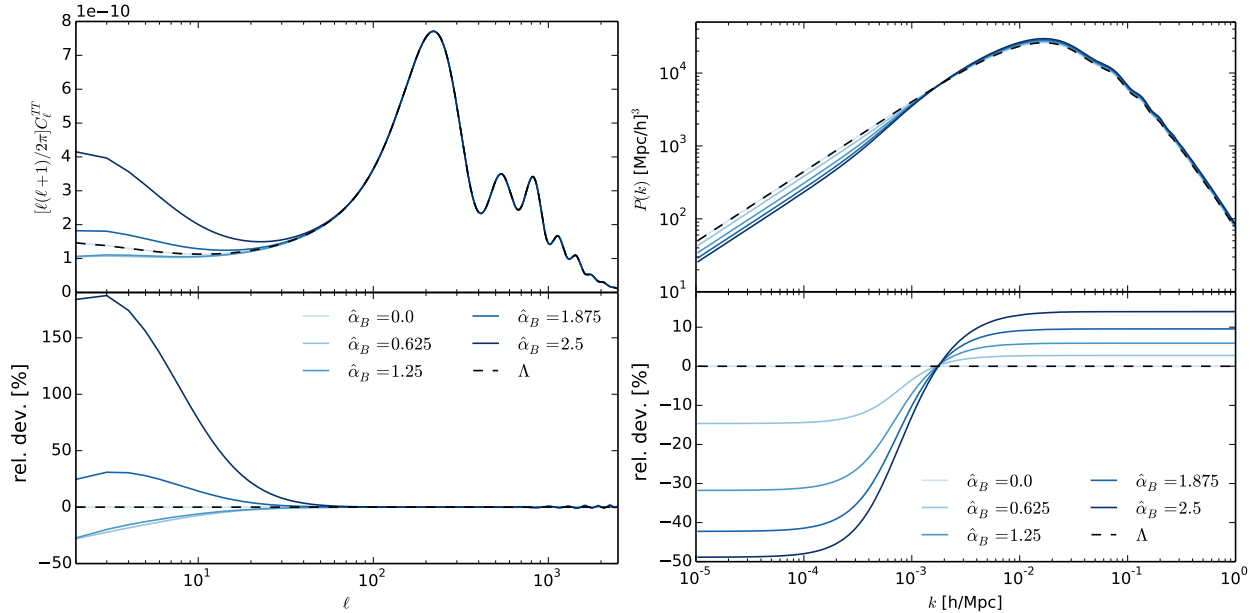


Figure 2: Effect of the braiding on CMB temperature power spectra (left panel) and the matter power spectrum at $z = 0$ (right panel). In the bottom panels we show the relative difference between the DE/MG models considered and a fiducial Λ CDM. In each model, we fix all the standard cosmological parameters to some fiducial value. We use the `lcdm` parametrization for the background expansion history (i.e. $w_{\text{DE}} = -1$) and `propto_omega` for the evolution of the alphas. We fix $\hat{\alpha}_K = 1$, $\hat{\alpha}_M = \hat{\alpha}_T = 0$ and we vary $\hat{\alpha}_B$ in the stable region.

4 Summary and Outlook

This paper accompanies the launch of the first public version of `hi_class`, an implementation of Horndeski’s theory in the Cosmic Linear Anisotropy Solving System. This initial version allows the user to freely specify parameterisations of the expansion history, as well as the functions that characterise the evolution of the perturbations at the linear level. The parameterised approach can be used to test gravity in a largely model-independent manner by directly modifying the fundamental properties of the gravitational degrees of freedom and calculating the subsequent impact on cosmological observables. Although technically more involved, this way of working neatly generalises the widely used parameterisations of the equation of state w_{DE} : just as detecting $w_{\text{DE}} \neq -1$ on the Hubble diagram would be a clear signature of physics beyond the cosmological constant, any significant data preference for $\alpha_i \neq 0$ would constitute model-independent evidence for dynamics beyond Einstein gravity.

The formulation of modified gravity as implemented in `hi_class` ensures that whatever the choice of parameters is made, there exists a model in the Horndeski class of scalar-tensor theories which would have exactly such a phenomenology at linear level in perturbations. The phenomenology of *any* particular scalar-tensor model of gravity or dark energy (such as quintessence, $f(R)$ or galileons) can be obtained by choosing particular α functions and background. In this way, `hi_class`, should be thought of as a unified code to test all such modifications of gravity for which separate codes exist currently.

The current version of `hi_class` represents just the first step towards a reliable, fast and flexible tool to address modifications of gravity on cosmological scales in a self-consistent manner. Further extensions and improvements to `hi_class` will be released publicly in the near future and which will include *inter alia* approximation schemes to optimise the computational performance, automatic predictions for full covariant

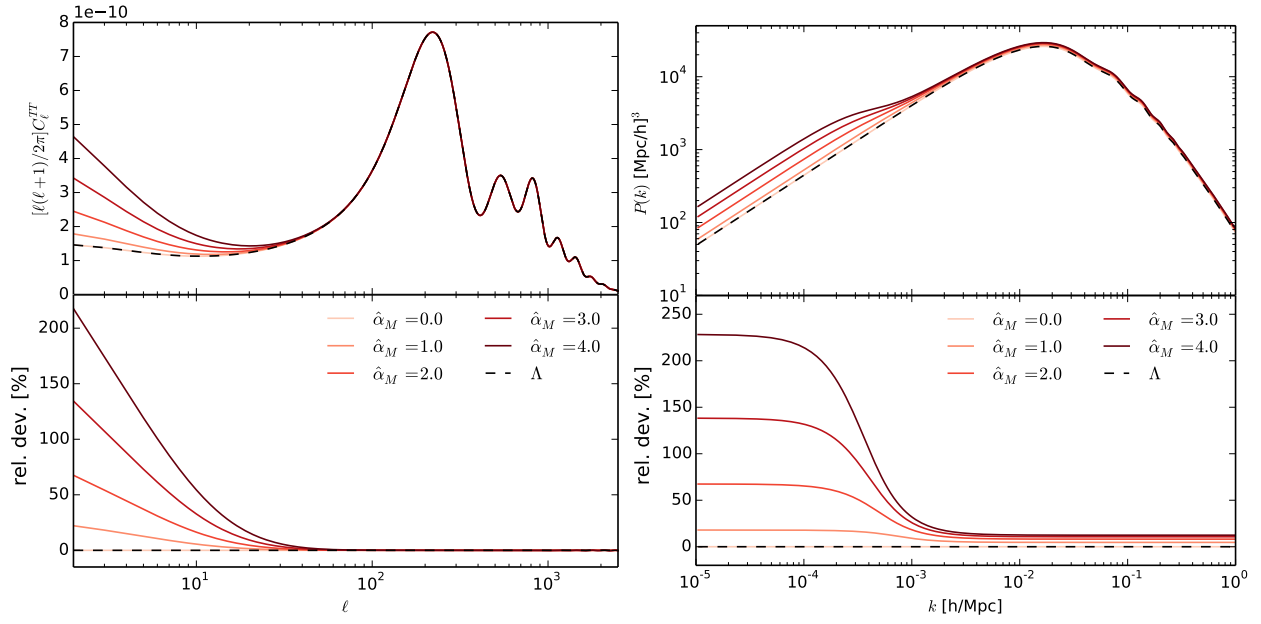


Figure 3: Same as Figure 2 but for the running of the Planck mass. We fix $\hat{\alpha}_K = 1$, $\hat{\alpha}_B = \hat{\alpha}_T = 0$ and we vary $\hat{\alpha}_M$ in the stable region, with $M_{*,0}^2 = 1$ initially.

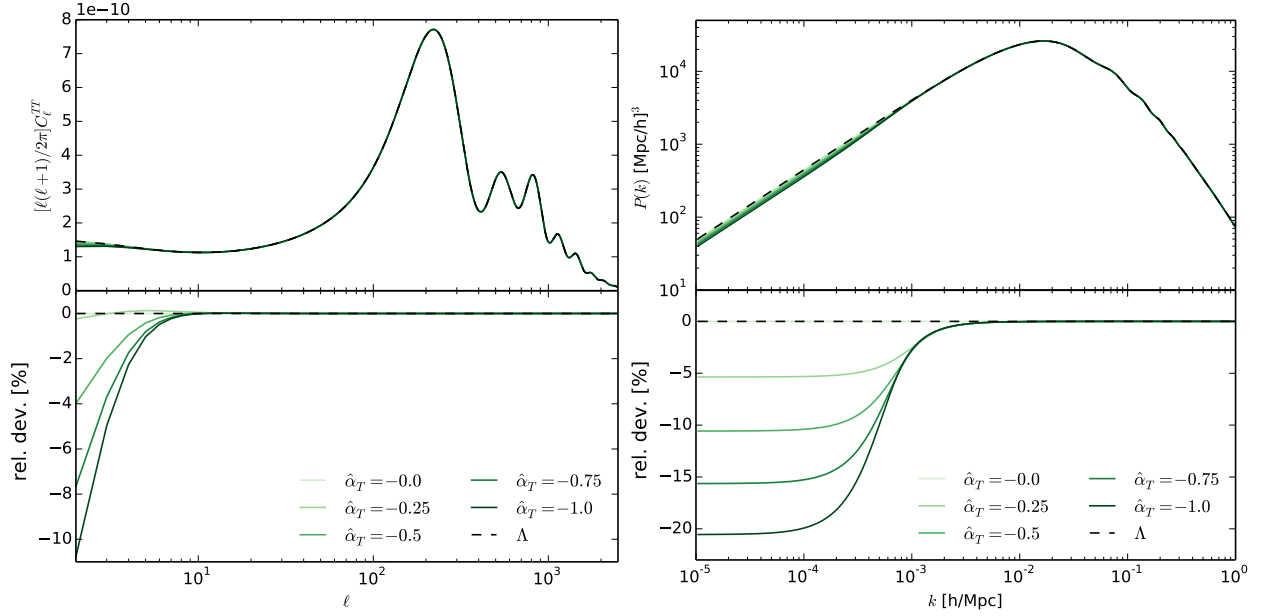


Figure 4: Same as Figure 2 but for the tensor speed excess. We fix $\hat{\alpha}_K = 1$, $\hat{\alpha}_B = \hat{\alpha}_M = 0$ and we vary $\hat{\alpha}_T$ in the stable region.

action-based models and also expand further the range of theories available to test.

The parameterised approach as implemented in the public `hi_class` currently relies neither on a specific choice for the action (2.1) nor on the initial conditions for the scalar field at the cosmological background, but rather is a very general method to test for evidence of non-GR effects in scalar tensor theories. This sort of study should be complemented by testing models arising from full covariant actions and placing limits on their parameters. One of the main features of the full version of `hi_class` currently in development is to allow the user to directly specify the Horndeski functions G_i in (2.2-2.5), integrate the background scalar equation given some initial conditions and automatically generate predictions in terms of the parameters that define the model. Starting with a covariant action-based model offers important advantages compared to the EFT approach:

1. *Background expansion and Self-Consistency.* In a particular model, the background and perturbations are related and controlled by the same parameters. In many Horndeski theories the background departs considerably from the Λ CDM or w CDM, leading to significantly stronger constraints from the outset (e.g. in galileons [85]). Moreover, certain time dependences and values of the α functions might require very fine tuned parameters, even when very general Lagrangians are considered (e.g. as in quintessence [86], also see [87]).
2. *Cosmology beyond linear order.* Extending the EFT approach to general models beyond linear order introduces a severely expanded set of functions at each order and rapidly spoils the economy possible in the linear description. When considering the Horndeski class of theories, this issue does not arise, since the structure of the theory is fixed and further relations are available in terms of the G_i functions [88]. In principle, the results of the linear predictions of `hi_class` can also be connected to the limit of very non-linear structure formation, e.g. through N-body simulations for specific models (e.g. galileons [89]) including understanding the effects of the particular screening built into the model.
3. *Relation to other regimes.* When working with a specified model, it becomes possible to link constraints from cosmology to tests of gravity in any other regime. This includes not only Solar-System tests, but also generation of gravitational waves, compact objects and other astrophysical tests [90] as well as, potentially, laboratory and collider experiments to connect tests of gravity on a broad range of scales [91]. Moreover, starting with a specific action allows one to understand the theoretical limits on the model, e.g. to compute the strong-coupling scale and to understand the range of validity of this description. Finally, it might even be possible to find connections with scenarios in fundamental physics such as extra dimensions and quantum gravity.

For these reasons, one should continue to test models defined through a covariant action which would connect predictions in various regimes. Nonetheless, the parameterised EFT allows for an exploration of the data for effects arising in a possibly wider set of models which remain consistent with the underlying symmetries assumed in cosmology but do not fit into the Horndeski scheme. The ultimate design goal of `hi_class` is to achieve full generality as a tool to test gravity with cosmological data. To this end future releases of `hi_class` will transcend the scope of Horndeski's theory and include the full EFT of DE and its generalizations [92, 93].

`hi_class` solves the full dynamical equations consistently across all cosmic epochs and scales, relying only on the validity of the linear description, but not making use of the so-called quasi-static approximation. A future design goal is to implement the QS approximation in `hi_class` in a controlled manner, not as a means to simplify the results, but rather as a tool to speed up the computations on scales and epochs in which these dynamical effects are below the desired precision. Future releases will also extend the choice of initial conditions for the scalar field, allowing for an exploration of models that lie beyond the usual scope of late time acceleration, including those featuring modifications of gravity in early epochs and connecting directly with inflationary physics.

In parallel, we are enhancing the code to compute observables beyond the linear regime, including higher-order effects [7] and improved perturbation-theory schemes suitably adapted to theories of gravity

beyond Einstein’s general relativity. These improvements will aim at making `hi_class` into modern tool able to match the requirements of the upcoming era of precision cosmology, helping the future generation of large LSS surveys to unveil the mysteries of the dark universe and shed light on the nature of gravity.

Acknowledgements: We are especially grateful to Thomas Tram for patiently answering many questions pertaining to the CLASS code, Alexandre Barreira for providing us with the output to test the code, and David Alonso, Nicola Bellomo, Antonio J. Cuesta, Carlos Garcia-Garcia, Francesco Montanari, Janina Renk, Angelo Ricciardone and Alessio Spurio Mancini, for using preliminary versions of `hi_class` and providing us with valuable suggestions leading to many improvements. We are also thankful to Jose María Ezquiaga, Eric Linder, Francesco Montanari and Janina Renk for comments on the first version as well as to Luca Amendola, Guillermo Ballesteros, Dario Bettoni, Alicia Bueno-Belloso, Santiago Casas, Juan García-Bellido, Cristiano Germani, Jérôme Gleyzes, Kurt Hinterbichler, Gregory Horndeski, Bin Hu, Luisa Jaime, Raul Jimenez, Valeria Pettorino, Marco Raveri, Laura Taddei, Lorenzo Tamellini, Licia Verde and Filippo Vernizzi for useful conversations during the completion of this work. The `hi_class` logo was designed by Marcos Vázquez Pingarrón (www.mywonderland.es).

This project was initiated and substantially carried out while E.B. and M.Z. benefited from being members of the Institut für Theoretische Physik of the University of Heidelberg. The work of E.B. is partially supported by the European Research Council under the European Community’s Seventh Framework Programme FP7-IDEAS-Phys.LSS 240117 and the ERC H2020 693024 GravityLS project as well as the Spanish MINECO through projects AYA2014-58747-P and MDM-2014- 0369 of ICCUB (Unidad de Excelencia María de Maeztu). I.S. is supported by the European Regional Development Fund and the Czech Ministry of Education, Youth and Sports (MŠMT) (Project CoGraDS — CZ.02.1.01/0.0/0.0/15_003/0000437). While in Heidelberg M.Z. acknowledges support from DFG through the TRR33 project “The Dark Universe”. PGF is supported by ERC H2020 693024 GravityLS project, the Beecroft Trust and STFC.

A Equations and notation

In this section, we list all the equations used in `hi_class`, both at the background and at the perturbation level. It is important to note that the notation of CLASS is slightly different than the notation of ref. [54]. The matter densities are redefined in such a way that the Friedmann constraint is written as $H^2 = \sum_i \rho_i$.

In addition, it should be noted that all the variables are expressed in conformal time τ (a prime denotes derivative w.r.t. conformal time), while the Hubble parameter is the physical one, i.e. $a' = a^2 H$.

The line element in synchronous gauge up to first order in perturbation theory reads [94]

$$ds^2 = a^2 \left[-d\tau^2 + (\delta_{ij} + \tilde{h}_{ij}) dx^i dx^j \right], \quad (\text{A.1})$$

where in Fourier space

$$\tilde{h}_{ij}(\tau, \vec{k}) = \hat{k}_i \hat{k}_j h + 6 \left(\hat{k}_i \hat{k}_j - \frac{1}{3} \delta_{ij} \right) \eta + h_{ij}. \quad (\text{A.2})$$

Here h and η are scalar perturbations and h_{ij} is the tensor perturbation. Sometimes in the code it is used the perturbation $\xi(\tau, \vec{k})$, which is related to h and η through⁶

$$\xi = \frac{h' + 6\eta'}{2k^2}. \quad (\text{A.3})$$

⁶Note that in ref. [94] our ξ is named α . We modified the notation to avoid confusion with the α functions of modified gravity.

A.1 Background

`hi_class` makes the choice of units such that the Friedmann equations read

$$H^2 = \rho_m + \rho_{\text{DE}} \quad (\text{A.4})$$

$$H' = -\frac{3}{2}a(\rho_m + p_m + \rho_{\text{DE}} + p_{\text{DE}}), \quad (\text{A.5})$$

where ρ_m and p_m are the energy density and pressure of the total matter content of the universe (except dark energy), while ρ_{DE} and p_{DE} are the energy density and pressure of the scalar field

$$\rho_{\text{DE}} \equiv -\frac{1}{3}G_2 + \frac{2}{3}X(G_{2X} - G_{3\phi}) - \frac{2H^3\phi'X}{3a}(7G_{5X} + 4XG_{5XX}) \quad (\text{A.6})$$

$$\begin{aligned} &+ H^2 [1 - (1 - \alpha_B) M_*^2 - 4X(G_{4X} - G_{5\phi}) - 4X^2(2G_{4XX} - G_{5\phi X})] \\ p_{\text{DE}} \equiv &\frac{1}{3}G_2 - \frac{2}{3}X(G_{3\phi} - 2G_{4\phi\phi}) + \frac{4H\phi'}{3a}(G_{4\phi} - 2XG_{4\phi X} + XG_{5\phi\phi}) - \frac{(\phi'' - aH\phi')}{3\phi'a}HM_*^2\alpha_B \quad (\text{A.7}) \\ &- \frac{4}{3}H^2X^2G_{5\phi X} - \left(H^2 + \frac{2H'}{3a}\right)(1 - M_*^2) + \frac{2H^3\phi'XG_{5X}}{3a}, \end{aligned}$$

(note that both eqs (A.6) and (A.7) contain α_B , defined in eq. (A.33)).

The manner in which we treat M_*^2 here is different than in ref. [54]. This allows us to ensure the usual conservation equations, both for the scalar field and the matter,

$$\begin{aligned} \rho'_{\text{DE}} + 3aH(\rho_{\text{DE}} + p_{\text{DE}}) &= 0, \quad (\text{A.8}) \\ \rho'_m + 3aH(\rho_m + p_m) &= 0. \end{aligned}$$

which replicates the meaning of the usual parameterisation of w_{DE} as a description of the expansion history and thus is much simpler to communicate. On the other hand, we note that an observer comoving with the cosmological background in the past would not be aware of the evolving strength of gravity. Thus if a local measurement of the energy density and pressure were possible and performed, the observer would rather see the quantities with \sim ,

$$\tilde{\rho}_m = \frac{3\rho_m}{M_*^2}, \quad (\text{A.9})$$

$$\tilde{p}_m = \frac{3p_m}{M_*^2}, \quad (\text{A.10})$$

$$\tilde{\mathcal{E}} = \frac{3\rho_{\text{DE}}}{M_*^2} + 3H^2 \frac{(M_*^2 - 1)}{M_*^2}, \quad (\text{A.11})$$

$$\tilde{\mathcal{P}} = \frac{3p_{\text{DE}}}{M_*^2} - \left(3H^2 + \frac{2H'}{a}\right) \frac{(M_*^2 - 1)}{M_*^2}, \quad (\text{A.12})$$

as defined in [54] and thus would have measured a different equation of state.

A.2 Linear perturbations in Synchronous Gauge

At linear order in perturbation theory, CLASS has the possibility to solve the evolution of the perturbations in both Newtonian and synchronous gauges. The present version of `hi_class` implements only the synchronous gauge option.

First, the Horndeski class of models modifies the propagation of gravitational waves h_{ij} (tensors). This has been taken into account in `hi_class` through the dynamical equation

$$h''_{ij} + (2 + \alpha_M) aHh'_{ij} + (1 + \alpha_T) k^2 h_{ij} = \frac{\sigma_{mij}}{M_*^2}, \quad (\text{A.13})$$

where σ_{mij} represents the source for the tensor modes arising from the matter.

The scalar sector is described through the perturbations of the metric tensor (η, h, ξ) (see eq. (A.1)), the scalar field perturbation V_X (defined in eq. (2.16)), and the perturbations of the matter density, velocity, pressure and anisotropic stress, ($\delta\rho_m, \theta_m, \delta p_m, \sigma_m$) respectively. The Einstein equations are

- Einstein (0,0)

$$h' = \frac{4k^2\eta}{aH(2 - \alpha_B)} + \frac{6a\delta\rho_m}{HM_*^2(2 - \alpha_B)} - 2aH \left(\frac{\alpha_K + 3\alpha_B}{2 - \alpha_B} \right) V'_X - 2 \left[3aH' + \left(\frac{\alpha_K + 3\alpha_B}{2 - \alpha_B} \right) a^2H^2 + \frac{9a^2}{M_*^2} \left(\frac{\rho_m + p_m}{2 - \alpha_B} \right) + \frac{\alpha_B k^2}{2 - \alpha_B} \right] V_X \quad (\text{A.14})$$

- Einstein (0,i)

$$\eta' = \frac{3a^2\theta_m}{2k^2M_*^2} + \frac{aH}{2}\alpha_B V'_X + \left[aH' + \frac{a^2H^2}{2}\alpha_B + \frac{3a^2}{2M_*^2}(\rho_m + p_m) \right] V_X \quad (\text{A.15})$$

- Einstein (i,j) trace

$$Dh'' = 2\lambda_1 k^2 \eta + 2aH\lambda_3 h' - \frac{9a^2\alpha_K\delta p_m}{M_*^2} + 3a^2H^2\lambda_4 V'_X + 2a^3H^3 \left[3\lambda_6 + \frac{\lambda_5 k^2}{a^2H^2} \right] V_X \quad (\text{A.16})$$

- Einstein (i,j) traceless

$$\xi' = (1 + \alpha_T)\eta - aH(2 + \alpha_M)\xi + aH(\alpha_M - \alpha_T)V_X - \frac{9a^2\sigma_m}{2M_*^2k^2}. \quad (\text{A.17})$$

In addition to the Einstein equations, the full system includes an equation for the evolution of the scalar field perturbations, i.e.

$$D(2 - \alpha_B)V_X'' + 8aH\lambda_7 V'_X + 2a^2H^2 \left[\frac{c_{sN}^2 k^2}{a^2H^2} - 4\lambda_8 \right] V_X = \frac{2c_{sN}^2}{aH} k^2 \eta + \frac{3a}{2HM_*^2} [2\lambda_2\delta\rho_m - 3\alpha_B(2 - \alpha_B)\delta p_m]. \quad (\text{A.18})$$

The definition of all these functions is

$$D = \alpha_K + \frac{3}{2}\alpha_B^2 \quad (\text{A.19})$$

$$\lambda_1 = \alpha_K(1 + \alpha_T) - 3\alpha_B(\alpha_M - \alpha_T) \quad (\text{A.20})$$

$$\lambda_2 = -\frac{3(\rho_m + p_m)}{H^2M_*^2} - (2 - \alpha_B)\frac{H'}{aH^2} + \frac{\alpha'_B}{aH} \quad (\text{A.21})$$

$$\lambda_3 = -\frac{1}{2}(2 + \alpha_M)D - \frac{3}{4}\alpha_B\lambda_2 \quad (\text{A.22})$$

$$\lambda_4 = \alpha_K\lambda_2 - \frac{2\alpha_K\alpha'_B - \alpha_B\alpha'_K}{aH} \quad (\text{A.23})$$

$$\lambda_5 = \frac{3}{2}\alpha_B^2(1 + \alpha_T) + (D + 3\alpha_B)(\alpha_M - \alpha_T) + \frac{3}{2}\alpha_B\lambda_2 \quad (\text{A.24})$$

$$\lambda_6 = \left(1 - \frac{3\alpha_B H'}{\alpha_K a H^2}\right) \frac{\alpha_K \lambda_2}{2} - \frac{D H'}{a H^2} \left[2 + \alpha_M + \frac{H''}{a H H'}\right] - \frac{2\alpha_K \alpha'_B - \alpha_B \alpha'_K}{2aH} - \frac{3\alpha_K p'_m}{2aH^3 M_*^2} \quad (\text{A.25})$$

$$\lambda_7 = \frac{D}{8}(2 - \alpha_B) \left[4 + \alpha_M + \frac{2H'}{aH^2} + \frac{D'}{aHD}\right] + \frac{D}{8}\lambda_2 \quad (\text{A.26})$$

$$\lambda_8 = -\frac{\lambda_2}{8} \left(D - 3\lambda_2 + \frac{3\alpha'_B}{aH}\right) + \frac{1}{8}(2 - \alpha_B) \left[(3\lambda_2 - D) \frac{H'}{aH^2} - \frac{9\alpha_B p'_m}{2aH^3 M_*^2}\right] - \frac{D}{8}(2 - \alpha_B) \left[4 + \alpha_M + \frac{2H'}{aH^2} + \frac{D'}{aHD}\right] \quad (\text{A.27})$$

$$c_{\text{sN}}^2 = \lambda_2 + \frac{1}{2}(2 - \alpha_B) [\alpha_B(1 + \alpha_T) + 2(\alpha_M - \alpha_T)]. \quad (\text{A.28})$$

Here c_{sN}^2 is the numerator of the sound speed of the additional degree of freedom, which is

$$c_s^2 = \frac{c_{\text{sN}}^2}{D}. \quad (\text{A.29})$$

A.3 α Functions

The time dependence of the α functions introduced in sec. 2.2 is uniquely specified for any particular Horndeski Lagrangian when taken together with the background initial conditions. The relation between α_i and the Horndeski functions $G_i(\phi, X)$ is given by

$$M_*^2 \equiv 2 \left(G_4 - 2XG_{4X} - \frac{H\phi'XG_{5X}}{a} + XG_{5\phi} \right) \quad (\text{A.30})$$

$$\alpha_M \equiv \frac{d \ln M_*^2}{d \ln a} \quad (\text{A.31})$$

$$H^2 M_*^2 \alpha_K \equiv 2X(G_{2X} + 2XG_{2XX} - 2G_{3\phi} - 2XG_{3\phi X}) \quad (\text{A.32})$$

$$\begin{aligned} & + \frac{12H\phi'X}{a} (G_{3X} + XG_{3XX} - 3G_{4\phi X} - 2XG_{4\phi XX}) \\ & + 12H^2 X [G_{4X} - G_{5\phi} + X(8G_{4XX} - 5G_{5\phi X}) + 2X^2(2G_{4XX} - G_{5\phi XX})] \\ & + \frac{4H^3\phi'X}{a} (3G_{5X} + 7XG_{5XX} + 2X^2G_{5XX}) \end{aligned}$$

$$HM_*^2 \alpha_B \equiv \frac{2\phi'}{a} (XG_{3X} - G_{4\phi} - 2XG_{4\phi X}) + 8HX(G_{4X} + 2XG_{4XX} - G_{5\phi} - XG_{5\phi X}) \quad (\text{A.33})$$

$$+ \frac{2H^2\phi'X}{a} (3G_{5X} + 2XG_{5XX})$$

$$M_*^2 \alpha_T \equiv 4X(G_{4X} - G_{5\phi}) - \frac{2}{a^2}(\phi'' - 2aH\phi')XG_{5X}. \quad (\text{A.34})$$

and ϕ, X and H are evaluated on their background solution to give the particular time-dependence of the α functions for that solution.

References

- [1] U. Seljak and M. Zaldarriaga, ‘‘A Line of sight integration approach to cosmic microwave background anisotropies,’’ *Astrophys. J.* **469** (1996) 437–444, [arXiv:astro-ph/9603033](#) [[astro-ph](#)].

- [2] A. Lewis, A. Challinor, and A. Lasenby, “Efficient computation of CMB anisotropies in closed FRW models,” *Astrophys. J.* **538** (2000) 473–476, [arXiv:astro-ph/9911177 \[astro-ph\]](#).
- [3] D. Blas, J. Lesgourgues, and T. Tram, “The Cosmic Linear Anisotropy Solving System (CLASS) II: Approximation schemes,” *JCAP* **1107** (2011) 034, [arXiv:1104.2933 \[astro-ph.CO\]](#).
- [4] **Planck**: Collaboration, P. A. R. Ade *et al.*, “Planck intermediate results - XXIV. Constraints on variations in fundamental constants,” *Astron. Astrophys.* **580** (2015) A22, [arXiv:1406.7482 \[astro-ph.CO\]](#).
- [5] W. Valkenburg and B. Hu, “Initial conditions for cosmological N-body simulations of the scalar sector of theories of Newtonian, Relativistic and Modified Gravity,” *JCAP* **1509** no. 09, (2015) 054, [arXiv:1505.05865 \[astro-ph.CO\]](#).
- [6] B. Audren, J. Lesgourgues, K. Benabed, and S. Prunet, “Conservative Constraints on Early Cosmology: an illustration of the Monte Python cosmological parameter inference code,” *JCAP* **1302** (2013) 001, [arXiv:1210.7183 \[astro-ph.CO\]](#).
- [7] E. Bellini and M. Zumalacarregui, “Nonlinear evolution of the baryon acoustic oscillation scale in alternative theories of gravity,” *Phys. Rev.* **D92** no. 6, (2015) 063522, [arXiv:1505.03839 \[astro-ph.CO\]](#).
- [8] E. Bellini, A. J. Cuesta, R. Jimenez, and L. Verde, “Constraints on deviations from Λ CDM within Horndeski gravity,” *JCAP* **1602** no. 02, (2016) 053, [arXiv:1509.07816 \[astro-ph.CO\]](#).
- [9] J. Renk, M. Zumalacarregui, and F. Montanari, “Gravity at the horizon: on relativistic effects, CMB-LSS correlations and ultra-large scales in Horndeski’s theory,” *JCAP* **1607** no. 07, (2016) 040, [arXiv:1604.03487 \[astro-ph.CO\]](#).
- [10] G. W. Horndeski, “Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space,” *International Journal of Theoretical Physics* **10** (Sept., 1974) 363–384. <http://dx.doi.org/10.1007/BF01807638>.
- [11] A. Nicolis, R. Rattazzi, and E. Trincherini, “The Galileon as a local modification of gravity,” *Phys.Rev.* **D79** (2009) 064036, [arXiv:0811.2197 \[hep-th\]](#).
- [12] C. Deffayet, X. Gao, D. Steer, and G. Zahariade, “From k-essence to generalised Galileons,” *Phys.Rev.* **D84** (2011) 064039, [arXiv:1103.3260 \[hep-th\]](#).
- [13] T. Kobayashi, M. Yamaguchi, and J. Yokoyama, “Generalized G-inflation: Inflation with the most general second-order field equations,” *Prog. Theor. Phys.* **126** (2011) 511–529, [arXiv:1105.5723 \[hep-th\]](#).
- [14] L. Hui, A. Nicolis, and C. Stubbs, “Equivalence Principle Implications of Modified Gravity Models,” *Phys.Rev.* **D80** (2009) 104002, [arXiv:0905.2966 \[astro-ph.CO\]](#).
- [15] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, “Modified Gravity and Cosmology,” [arXiv:1106.2476 \[astro-ph.CO\]](#).
- [16] C. Wetterich, “Cosmology and the Fate of Dilatation Symmetry,” *Nucl.Phys.* **B302** (1988) 668.
- [17] B. Ratra and P. J. E. Peebles, “Cosmological Consequences of a Rolling Homogeneous Scalar Field,” *Phys. Rev.* **D37** (1988) 3406.
- [18] C. Brans and R. Dicke, “Mach’s principle and a relativistic theory of gravitation,” *Phys.Rev.* **124** (1961) 925–935.
- [19] C. Armendariz-Picon, T. Damour, and V. F. Mukhanov, “k-Inflation,” *Phys. Lett.* **B458** (1999) 209–218, [arXiv:hep-th/9904075](#).
- [20] C. Armendariz-Picon, V. F. Mukhanov, and P. J. Steinhardt, “Essentials of k-essence,” *Phys. Rev.* **D63** (2001) 103510, [arXiv:astro-ph/0006373](#).
- [21] C. Deffayet, O. Pujolas, I. Sawicki, and A. Vikman, “Imperfect Dark Energy from Kinetic Gravity Braiding,” *JCAP* **1010** (2010) 026, [arXiv:1008.0048 \[hep-th\]](#).
- [22] T. Kobayashi, M. Yamaguchi, and J. Yokoyama, “G-inflation: Inflation driven by the Galileon field,” *Phys.Rev.Lett.* **105** (2010) 231302, [arXiv:1008.0603 \[hep-th\]](#).

- [23] O. Pujolas, I. Sawicki, and A. Vikman, “The Imperfect Fluid behind Kinetic Gravity Braiding,” *JHEP* **1111** (2011) 156, [arXiv:1103.5360 \[hep-th\]](#).
- [24] C. Deffayet, G. Esposito-Farese, and A. Vikman, “Covariant Galileon,” *Phys.Rev.* **D79** (2009) 084003, [arXiv:0901.1314 \[hep-th\]](#).
- [25] C. de Rham and A. J. Tolley, “DBI and the Galileon reunited,” *JCAP* **1005** (2010) 015, [arXiv:1003.5917 \[hep-th\]](#).
- [26] M. Zumalacárregui, T. S. Koivisto, and D. F. Mota, “DBI Galileons in the Einstein Frame: Local Gravity and Cosmology,” *Phys.Rev.* **D87** (2013) 083010, [arXiv:1210.8016 \[astro-ph.CO\]](#).
- [27] D. Bettoni and S. Liberati, “Disformal invariance of second order tensor-scalar theories: framing the Horndeski action,” *Phys.Rev.* **D88** (2013) 084020, [arXiv:1306.6724 \[gr-qc\]](#).
- [28] J. Khoury and A. Weltman, “Chameleon fields: Awaiting surprises for tests of gravity in space,” *Phys.Rev.Lett.* **93** (2004) 171104.
- [29] J. Khoury and A. Weltman, “Chameleon cosmology,” *Phys.Rev.* **D69** (2004) 044026, [arXiv:astro-ph/0309411 \[astro-ph\]](#).
- [30] K. Hinterbichler and J. Khoury, “Symmetron Fields: Screening Long-Range Forces Through Local Symmetry Restoration,” *Phys.Rev.Lett.* **104** (2010) 231301, [arXiv:1001.4525 \[hep-th\]](#).
- [31] K. Hinterbichler, J. Khoury, A. Levy, and A. Matas, “Symmetron Cosmology,” *Phys.Rev.* **D84** (2011) 103521, [arXiv:1107.2112 \[astro-ph.CO\]](#).
- [32] J. M. Ezquiaga, J. García-Bellido, and M. Zumalacárregui, “Towards the most general scalar-tensor theories of gravity: a unified approach in the language of differential forms,” *Phys. Rev.* **D94** no. 2, (2016) 024005, [arXiv:1603.01269 \[hep-th\]](#).
- [33] C. Charmousis, E. J. Copeland, A. Padilla, and P. M. Saffin, “General second order scalar-tensor theory, self tuning, and the Fab Four,” *Phys.Rev.Lett.* **108** (2012) 051101, [arXiv:1106.2000 \[hep-th\]](#).
- [34] P. Martín-Moruno, N. J. Nunes, and F. S. N. Lobo, “Horndeski theories self-tuning to a de Sitter vacuum,” *Phys. Rev.* **D91** no. 8, (2015) 084029, [arXiv:1502.03236 \[gr-qc\]](#).
- [35] S. M. Carroll, V. Duvvuri, M. Trodden, and M. S. Turner, “Is cosmic speed - up due to new gravitational physics?,” *Phys.Rev.* **D70** (2004) 043528, [arXiv:astro-ph/0306438 \[astro-ph\]](#).
- [36] S. M. Carroll, A. De Felice, V. Duvvuri, D. A. Easson, M. Trodden, *et al.*, “The Cosmology of generalized modified gravity models,” *Phys.Rev.* **D71** (2005) 063513, [arXiv:astro-ph/0410031 \[astro-ph\]](#).
- [37] M. Zumalacárregui and J. Garcia-Bellido, “Transforming gravity: from derivative couplings to matter to second-order scalar-tensor theories beyond the Horndeski Lagrangian,” *Phys.Rev.* **D89** (2014) 064046, [arXiv:1308.4685 \[gr-qc\]](#).
- [38] J. Gleyzes, D. Langlois, F. Piazza, and F. Vernizzi, “Healthy theories beyond Horndeski,” *Phys. Rev. Lett.* **114** no. 21, (2015) 211101, [arXiv:1404.6495 \[hep-th\]](#).
- [39] D. Langlois and K. Noui, “Degenerate higher derivative theories beyond Horndeski: evading the Ostrogradski instability,” *JCAP* **1602** no. 02, (2016) 034, [arXiv:1510.06930 \[gr-qc\]](#).
- [40] M. Crisostomi, M. Hull, K. Koyama, and G. Tasinato, “Horndeski: beyond, or not beyond?,” *JCAP* **1603** no. 03, (2016) 038, [arXiv:1601.04658 \[hep-th\]](#).
- [41] C. Deffayet, G. Esposito-Farese, and D. A. Steer, “Counting the degrees of freedom of generalized Galileons,” *Phys. Rev.* **D92** (2015) 084013, [arXiv:1506.01974 \[gr-qc\]](#).
- [42] G. D’Amico, Z. Huang, M. Mancarella, and F. Vernizzi, “Weakening Gravity on Redshift-Survey Scales with Kinetic Matter Mixing,” [arXiv:1609.01272 \[astro-ph.CO\]](#).
- [43] P. Creminelli, M. A. Luty, A. Nicolis, and L. Senatore, “Starting the Universe: Stable Violation of the Null Energy Condition and Non-standard Cosmologies,” *JHEP* **12** (2006) 080, [arXiv:hep-th/0606090 \[hep-th\]](#).

- [44] P. Horava, “Spectral Dimension of the Universe in Quantum Gravity at a Lifshitz Point,” *Phys. Rev. Lett.* **102** (2009) 161301, [arXiv:0902.3657 \[hep-th\]](#).
- [45] D. Blas, O. Pujolas, and S. Sibiryakov, “On the Extra Mode and Inconsistency of Horava Gravity,” *JHEP* **10** (2009) 029, [arXiv:0906.3046 \[hep-th\]](#).
- [46] X. Gao, “Unifying framework for scalar-tensor theories of gravity,” *Phys. Rev.* **D90** (2014) 081501, [arXiv:1406.0822 \[gr-qc\]](#).
- [47] T. Jacobson and D. Mattingly, “Gravity with a dynamical preferred frame,” *Phys.Rev.* **D64** (2001) 024028, [arXiv:gr-qc/0007031 \[gr-qc\]](#).
- [48] C. de Rham and G. Gabadadze, “Generalization of the Fierz-Pauli Action,” *Phys.Rev.* **D82** (2010) 044020, [arXiv:1007.0443 \[hep-th\]](#).
- [49] C. de Rham, G. Gabadadze, and A. J. Tolley, “Resummation of Massive Gravity,” *Phys.Rev.Lett.* **106** (2011) 231101, [arXiv:1011.1232 \[hep-th\]](#).
- [50] S. F. Hassan and R. A. Rosen, “Bimetric Gravity from Ghost-free Massive Gravity,” *JHEP* **02** (2012) 126, [arXiv:1109.3515 \[hep-th\]](#).
- [51] A. De Felice, T. Kobayashi, and S. Tsujikawa, “Effective gravitational couplings for cosmological perturbations in the most general scalar-tensor theories with second-order field equations,” *Phys.Lett.* **B706** (2011) 123–133, [arXiv:1108.4242 \[gr-qc\]](#).
- [52] C. M. Will, “The Confrontation between General Relativity and Experiment,” *Living Rev. Rel.* **17** (2014) 4, [arXiv:1403.7377 \[gr-qc\]](#).
- [53] A. Vainshtein, “To the problem of nonvanishing gravitation mass,” *Phys.Lett.* **B39** (1972) 393–394.
- [54] E. Bellini and I. Sawicki, “Maximal freedom at minimum cost: linear large-scale structure in general modifications of gravity,” *JCAP* **1407** (2014) 050, [arXiv:1404.3713 \[astro-ph.CO\]](#).
- [55] G. Gubitosi, F. Piazza, and F. Vernizzi, “The Effective Field Theory of Dark Energy,” *JCAP* **1302** (2013) 032, [arXiv:1210.0201 \[hep-th\]](#). [JCAP1302,032(2013)].
- [56] J. K. Bloomfield, E. E. Flanagan, M. Park, and S. Watson, “Dark energy or modified gravity? An effective field theory approach,” *JCAP* **1308** (2013) 010, [arXiv:1211.7054 \[astro-ph.CO\]](#).
- [57] J. Gleyzes, D. Langlois, F. Piazza, and F. Vernizzi, “Essential Building Blocks of Dark Energy,” *JCAP* **1308** (2013) 025, [arXiv:1304.4840 \[hep-th\]](#).
- [58] D. Blas, M. M. Ivanov, I. Sawicki, and S. Sibiryakov, “On constraining the speed of gravitational waves following GW150914,” [arXiv:1602.04188 \[gr-qc\]](#).
- [59] C. de Rham, A. J. Tolley, and D. H. Wesley, “Vainshtein Mechanism in Binary Pulsars,” [arXiv:1208.0580 \[gr-qc\]](#).
- [60] C. de Rham, A. Matas, and A. J. Tolley, “Galileon Radiation from Binary Systems,” *Phys. Rev.* **D87** no. 6, (2013) 064024, [arXiv:1212.5212 \[hep-th\]](#).
- [61] J. Beltran Jimenez, F. Piazza, and H. Velten, “Evading the Vainshtein Mechanism with Anomalous Gravitational Wave Speed: Constraints on Modified Gravity from Binary Pulsars,” *Phys. Rev. Lett.* **116** no. 6, (2016) 061101, [arXiv:1507.05047 \[gr-qc\]](#).
- [62] L. Lombriser and A. Taylor, “Breaking a Dark Degeneracy with Gravitational Waves,” *JCAP* **1603** no. 03, (2016) 031, [arXiv:1509.08458 \[astro-ph.CO\]](#).
- [63] D. Bettoni, J. M. Ezquiaga, K. Hinterbichler, and M. Zumalacárregui, “Speed of Gravitational Waves and the Fate of Scalar-Tensor Gravity,” *Phys. Rev.* **D95** no. 8, (2017) 084029, [arXiv:1608.01982 \[gr-qc\]](#).
- [64] I. D. Saltas, I. Sawicki, L. Amendola, and M. Kunz, “Anisotropic Stress as a Signature of Nonstandard Propagation of Gravitational Waves,” *Phys. Rev. Lett.* **113** no. 19, (2014) 191101, [arXiv:1406.7139 \[astro-ph.CO\]](#).

- [65] D. Bettoni and M. Zumalacárregui, “Kinetic mixing in scalar-tensor theories of gravity,” *Phys. Rev.* **D91** (2015) 104009, [arXiv:1502.02666 \[gr-qc\]](#).
- [66] I. Sawicki and E. Bellini, “Limits of quasistatic approximation in modified-gravity cosmologies,” *Phys. Rev.* **D92** no. 8, (2015) 084061, [arXiv:1503.06831 \[astro-ph.CO\]](#).
- [67] J. Gleyzes, D. Langlois, M. Mancarella, and F. Vernizzi, “Effective Theory of Dark Energy at Redshift Survey Scales,” *JCAP* **1602** no. 02, (2016) 056, [arXiv:1509.02191 \[astro-ph.CO\]](#).
- [68] D. Alonso, E. Bellini, P. G. Ferreira, and M. Zumalacárregui, “Observational future of cosmological scalar-tensor theories,” *Phys. Rev.* **D95** no. 6, (2017) 063502, [arXiv:1610.09290 \[astro-ph.CO\]](#).
- [69] D. A. Easson, I. Sawicki, and A. Vikman, “G-Bounce,” *JCAP* **1111** (2011) 021, [arXiv:1109.1047 \[hep-th\]](#).
- [70] I. Sawicki, I. D. Saltas, L. Amendola, and M. Kunz, “Consistent perturbations in an imperfect fluid,” [arXiv:1208.4855 \[astro-ph.CO\]](#).
- [71] V. Salvatelli, F. Piazza, and C. Marinoni, “Constraints on modified gravity from Planck 2015: when the health of your theory makes the difference,” *JCAP* **1609** no. 09, (2016) 027, [arXiv:1602.08283 \[astro-ph.CO\]](#).
- [72] L. Perenon, C. Marinoni, and F. Piazza, “Diagnostic of Horndeski Theories,” *JCAP* **1701** no. 01, (2017) 035, [arXiv:1609.09197 \[astro-ph.CO\]](#).
- [73] S. Peirone, M. Martinelli, M. Raveri, and A. Silvestri, “The importance of being stable: the role of stability conditions in single field Quintessence,” [arXiv:1702.06526 \[astro-ph.CO\]](#).
- [74] G. Ballesteros and J. Lesgourgues, “Dark energy with non-adiabatic sound speed: initial conditions and detectability,” *JCAP* **1010** (2010) 014, [arXiv:1004.5509 \[astro-ph.CO\]](#).
- [75] J. Lesgourgues, “The Cosmic Linear Anisotropy Solving System (CLASS) I: Overview,” [arXiv:1104.2932 \[astro-ph.IM\]](#).
- [76] J. Lesgourgues and T. Tram, “The Cosmic Linear Anisotropy Solving System (CLASS) IV: efficient implementation of non-cold relics,” *JCAP* **1109** (2011) 032, [arXiv:1104.2935 \[astro-ph.CO\]](#).
- [77] J. Lesgourgues and T. Tram, “Fast and accurate CMB computations in non-flat FLRW universes,” *JCAP* **1409** no. 09, (2014) 032, [arXiv:1312.2697 \[astro-ph.CO\]](#).
- [78] T. Tram and J. Lesgourgues, “Optimal polarisation equations in FLRW universes,” *JCAP* **1310** (2013) 002, [arXiv:1305.3261 \[astro-ph.CO\]](#).
- [79] E. Di Dio, F. Montanari, J. Lesgourgues, and R. Durrer, “The CLASSgal code for Relativistic Cosmological Large Scale Structure,” *JCAP* **1311** (2013) 044, [arXiv:1307.1459 \[astro-ph.CO\]](#).
- [80] L. Amendola, G. Ballesteros, and V. Pettorino, “Effects of modified gravity on B-mode polarization,” *Phys. Rev.* **D90** (2014) 043009, [arXiv:1405.7004 \[astro-ph.CO\]](#).
- [81] M. Chevallier and D. Polarski, “Accelerating universes with scaling dark matter,” *Int. J. Mod. Phys.* **D10** (2001) 213–224, [arXiv:gr-qc/0009008 \[gr-qc\]](#).
- [82] E. V. Linder, “Exploring the expansion history of the universe,” *Phys. Rev. Lett.* **90** (2003) 091301, [arXiv:astro-ph/0208512 \[astro-ph\]](#).
- [83] **Planck** Collaboration, P. A. R. Ade *et al.*, “Planck 2015 results. XIV. Dark energy and modified gravity,” [arXiv:1502.01590 \[astro-ph.CO\]](#).
- [84] **Planck** Collaboration, P. A. R. Ade *et al.*, “Planck 2015 results. XIII. Cosmological parameters,” [arXiv:1502.01589 \[astro-ph.CO\]](#).
- [85] A. Barreira, B. Li, C. Baugh, and S. Pascoli, “The observational status of Galileon gravity after Planck,” *JCAP* **1408** (2014) 059, [arXiv:1406.0485 \[astro-ph.CO\]](#).
- [86] D. J. E. Marsh, P. Bull, P. G. Ferreira, and A. Pontzen, “Quintessence in a quandary: Prior dependence in dark energy models,” *Phys. Rev.* **D90** no. 10, (2014) 105023, [arXiv:1406.2301 \[astro-ph.CO\]](#).

- [87] E. V. Linder, G. Sengör, and S. Watson, “Is the Effective Field Theory of Dark Energy Effective?,” [arXiv:1512.06180 \[astro-ph.CO\]](#).
- [88] E. Bellini, R. Jimenez, and L. Verde, “Signatures of Horndeski gravity on the Dark Matter Bispectrum,” *JCAP* **1505** no. 05, (2015) 057, [arXiv:1504.04341 \[astro-ph.CO\]](#).
- [89] B. Li, A. Barreira, C. M. Baugh, W. A. Hellwing, K. Koyama, S. Pascoli, and G.-B. Zhao, “Simulating the quartic Galileon gravity model on adaptively refined meshes,” *JCAP* **1311** (2013) 012, [arXiv:1308.3491 \[astro-ph.CO\]](#).
- [90] E. Berti *et al.*, “Testing General Relativity with Present and Future Astrophysical Observations,” *Class. Quant. Grav.* **32** (2015) 243001, [arXiv:1501.07274 \[gr-qc\]](#).
- [91] T. Baker, D. Psaltis, and C. Skordis, “Linking Tests of Gravity On All Scales: from the Strong-Field Regime to Cosmology,” *Astrophys. J.* **802** (2015) 63, [arXiv:1412.3455 \[astro-ph.CO\]](#).
- [92] M. Lagos, T. Baker, P. G. Ferreira, and J. Noller, “A general theory of linear cosmological perturbations: scalar-tensor and vector-tensor theories,” [arXiv:1604.01396 \[gr-qc\]](#).
- [93] D. Langlois, M. Mancarella, K. Noui, and F. Vernizzi, “Effective Description of Higher-Order Scalar-Tensor Theories,” [arXiv:1703.03797 \[hep-th\]](#).
- [94] C.-P. Ma and E. Bertschinger, “Cosmological perturbation theory in the synchronous and conformal Newtonian gauges,” *Astrophys. J.* **455** (1995) 7–25, [arXiv:astro-ph/9506072](#).