# Cooperative control of VAV air-conditioning systems

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**Abstract:** A dynamic model for a heating ventilating air-conditioning (HVAC) system is developed. Assuming the power of air supply fan is fixed, the air pressure in different rooms and air transmission ducts can be controlled by adjusting variable air volume (VAV) box and its terminals and air diffusers. Since the inlet air pressures in different rooms are interactive, traditional VAV controller, which is to control the dampers individually, will unavoidably generate a large overshoot. In this paper, we consider a novel VAV control strategy which is to control the air diffusers and VAV dampers in a cooperative way. This consensus-based approach can lead to a smoother air pressure change comparing with the traditional approaches. The simulation validates the effectiveness of the new control strategy.

**Key Words:** VAV, HVAC, Cooperative control, Consensus

## 1 Introduction

Energy saving and environmental protection problem has become one of the critical concerns of human being. The world's growing energy demands require developing a sustainable living plan. The largest sector of energy consumption in a modern city is buildings. Usually, a great portion of the energy consumption of buildings is attributed to heating and ventilating air-conditioning (HVAC) systems. In fact, a lot of energy is wasted in HVAC systems. Therefore, the controller design of HVAC systems is of great significance.

There are two classes of models for HVAC systems: static models for real-time optimization and dynamic models for stabilization control. Most of the existing works are based on static models. [1] is mainly focused on developing a simplified model of cooling coils. The model parameters are determined on-line based on commission or catalog information by linear or nonlinear least squares methods. For overall HVAC systems, [2] and [3] formulate the minimization of the total power consumption, which is mainly caused by chillers, pumps and fans, as a global optimization problem. A modified genetic algorithm was used to set the optimal operating point of each component. When we consider the dynamic properties of HVAC systems, the control problem becomes more difficult. A new dynamic simulation model for air-handling unit (AHU) is developed by [4]. The parameters can be easily determined from the measurement of the total fan energy consumption. Since dynamic models can describe the dynamic changing at different time in a day, a dynamic control strategy would often lead to a better performance comparing with static control strategy. By using Kalman filtering, [5] presents a temperature prediction algorithm based on a simple time varying zone model. The optimal performance is reached by applying a genetic algorithm.

Other than minimizing energy consumption while keeping thermal comfort, there are some works focusing on thermal comfort while keeping low energy consumption. The temperature can be affected by adjusting the cool air flow rate into rooms, which is controlled by dampers. There are two temperature control strategies: pressure dependent control and pressure independent control. Pressure dependent control means to control the damper based on the room temperature. According to the temperature difference between the room temperature and the desired one, we can adjust the air flow by controlling the damper. The reader can refer to [6], [7], [8], [9] and [10] for detail.

Pressure independent control strategy contains two control loops. The inner loop is for temperature control which can provide set point for outer loop based on the temperature difference. The outer loop is for air flow control. Pressure independent control leads to better control performance. This is because a pressure dependent controller would take action when the air flow changes affect the room temperature. Therefore the response of the system suffers from a time delay comparing with systems with pressure independent controllers. Keeping the static pressure of the room inlet to be constant is the key step to air flow control. In this paper, we consider the static pressure control problem by cooperatively adjusting the VAV box damper and its terminals. Comparing with the traditional PID controller, the cooperative controller can reduce the pressure differences among each room inlet while tracking the desired pressure, which makes the whole tracking process smoother. A numerical example is given to demonstrate our cooperative control scheme.

## 2 Preliminaries in Graph Theory

A directed graph is denoted by $\mathcal{G} = \{\mathcal{V}, \mathcal{E}_\mathcal{G}, A_\mathcal{G}\}$, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of node with $i$ representing the $i$th agent; $\mathcal{E}_\mathcal{G}$ is the set of edges which are represented by a pair of node indices $(i, j)$. We consider that $(i, j) \in \mathcal{E}$ if and only if node $i$ can send its information to node $j$. In this case, node $i$ is called the parent node and node $j$ is called a child node. The set of neighbors of the $i$th agent is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} \mid (j, i) \in \mathcal{E}\}$. If $(i, i) \in \mathcal{E}$, we say that node $i$ has self-loop. In this paper, we assume that no self-loop exists. $A_\mathcal{G} = [a_{i,j}] \in \mathbb{R}^{N\times N}$ is the the adjacency matrix associated with $\mathcal{G}$. If $j \in \mathcal{N}_i$, $a_{i,j} > 0$, otherwise $a_{i,j} = 0$. If matrix $A$ is symmetric, then the corresponding graph is called undirected graph.

A graph is balanced if the in-degree $deg_{in}(i) \triangleq \sum_{j\in\mathcal{V}\setminus\{i\}} a_{i,j}$ and the out-degree $deg_{out}(i) \triangleq \sum_{j\in\mathcal{V}\setminus\{i\}} a_{j,i}$ are equal for all $i \in \mathcal{V}$. For example, an undirected graph is a kind of balanced graph.

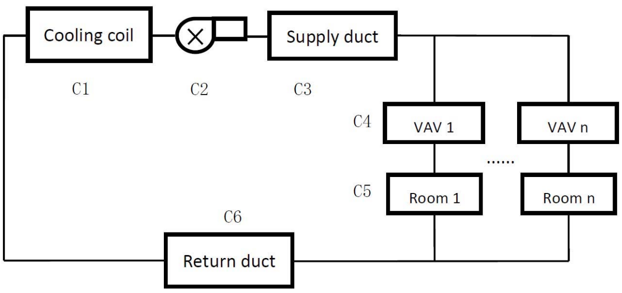


Fig. 1: Pressure-flow balance

There is a path from node $i$ to node $j$ if there exists a sequence $l_1, \ldots, l_p \in \mathcal{V}$ satisfying $(i, l_1), (l_1, l_2), \ldots, (l_p, j) \in \mathcal{E}_\mathcal{G}$ where $i, l_1, \ldots, l_p, j$ are distinct vertices. Given a graph $\mathcal{G}$, it contains a spanning tree if there exists at least one node $i$ such that for any other node $j$, there is a path from $i$ to $j$. If an undirected graph contains a spanning tree, it is connected.

The Laplacian matrix $L = [l_{i,j}]$ of the graph $\mathcal{G}$ is defined as that for any $i, j \in \mathcal{V}$ and $i \neq j$, $l_{i,j} = -a_{i,j}$ and $l_{i,i} = \sum_{j \in \mathcal{V}\backslash\{i\}} a_{i,j}$. If $a_{i,j} > \delta$ for all $a_{i,j} > 0$, the corresponding digraph is called $\delta$-graph. The following result is recalled [11].

**Lemma 2.1** *Consider system*

$$\dot{x}(t) = -L(t)x(t), \tag{1}$$

*with L a Laplacian matrix. If the corresponding graph is $\delta$-graph and contains a spanning tree, all components of any solution $\zeta(t)$ of* (1) *converge to a common value as $t \to \infty$.*

## 3 Mathematical Models of HVAC systems

In this section, we will provide mathematical model of the HVAC components. Figure 1 shows a pressure-flow balance model of a simplified HVAC system. C1 is the air filter and cooling coil; C2 is the air supply fan which can provide positive air pressure to the air flow; C3 is the air supply duct; C4 are VAV boxes and air diffusers which can be adjusted to control the air pressure through the ducts; C5 are rooms and the ducts of outlet; C6 is the return air duct. We assume that the pressure resistances of ducts, rooms and cooling coil are constant. Sometimes, a return air fan is added for better controlling the air exfiltration of the rooms. For simplicity, we do not consider the return air fan.

The governing equation of the pressure drop of a damper ($\Delta P$) is based on Detailed Damper Model/Valve Model presented in HVAC 2 Toolkit [12], which is given below.

$$\Delta P = R\dot{m}^2, \tag{2}$$

where $\dot{m}$ is the mass flow rate, $R$ is the the flow resistance coefficient, which varies with the damper position $\theta$ in the following way [13]

$$R = R' \left( \frac{W_f}{[(1-\lambda)\theta + \lambda]^2} + (1 - W_f)\lambda^{2(\theta - 1)} \right). \tag{3}$$

The damper is fully open when $\theta = 1$ and the damper is closed when $\theta = 0$. $R'$ is a constant corresponding to the flow resistance with the damper fully open. $W_f$ is a weighting factor for linear damper and exponential damper. $\lambda$ is a leakage parameter and is defined as the ratio of flow through closed damper to flow through an open damper at a fixed pressure difference.

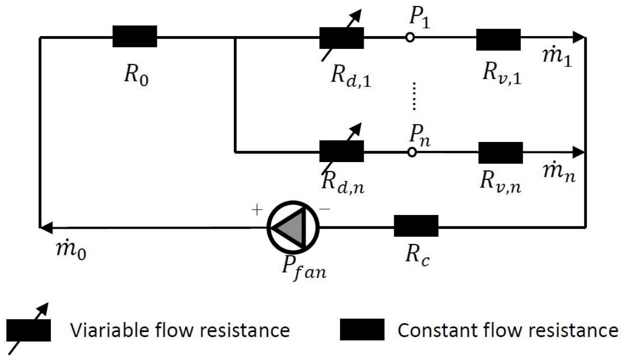


Fig. 2: Equivalent Circuit Diagram

The above diagram can be simplified as figure 2. $P_{fan}$ is the air pressure difference across the supply fan. $R_{v,i}$, $i = 1, 2, \ldots, n$ are the pressure resistances of room $i$ and their outlet ducts. Although we describe them as constant flow resistances, they may not be constant all the time due to the changes of states of windows, doors. $R_{d,i}$, $i = 1, \ldots, n$ are flow resistances of dampers which can be designed to control the static air pressure at inlet of each room ($P_i$, $i = 1, 2, \ldots, n$). $R_0$ is the flow resistance of supply air duct. $R_c$ is the flow resistance of return air duct as well as air filter and cooling coil. $\dot{m}_i$, $i = 0, 1, \ldots, n$ are mass flow rates of supply air ducts. The objective is to control the dampers such that the pressures at the inlets of all the rooms are equal, i.e.

$$P_1 = P_2 = \cdots = P_n = P_0, \tag{4}$$

where $P_0$ is a desired value to be tracked.

## 4 Consensus-based controller design

Most of the existing VAV dynamic controllers are PID controller [14], which locally controls a damper $i$ and therefore $R_{d,i}$ to compensate the change of $R_{v,i}$. However, this approach does not consider the coupling of the air pressure in each air duct. When damper $i$ is changing, the air pressure in each inlet duct of the rest rooms will be affected. This may cause a bad transient response of the whole system. In this section, we propose a consensus-based approach. By employing cross-coupling error technology, the proposed approach can guarantee that all the static pressures converge to the desired one more smoothly. This is because the dampers work in a cooperative manner.

We assume that $R_0$, $R_c$, $R_{d,i}$, $i = 1, \ldots, n$ and $R_{v,j}$, $j = 1, 2, \ldots, n$ are positive and uniformly bounded away from zero. According to pressure balance and flow balance principle, we have

$$(R_0+R_c)\dot{m}_0^2+(R_{d,1}+R_{v,1})\dot{m}_1^2=P_{fan}, \tag{5}$$

$$(R_{d,1}+R_{v,1})\dot{m}_1^2=(R_{d,2}+R_{v,2})\dot{m}_2^2= \cdots=(R_{d,n}+R_{v,n})\dot{m}_n^2, \tag{6}$$

$$\sum_{i=1}^{n}\dot{m}_i=\dot{m}_0. \tag{7}$$

$R_{d,i},\ i=1,\dots,n$ are given in the form of (3). We can change the value of $R_{d,i}$ by changing the corresponding damper position $\theta_i$. From (3) we can see that the flow resistances of the dampers satisfy $R'<R_{d,i}<R'\lambda^{-2}$, $i=0,1,\dots,n$. In fact, we can choose different $R'$ and $\lambda$ for different dampers. As this will not increase any difficulty but make the notations confusing, we assign a same value to each damper.

Since the air flow resistances of dampers are bounded, which will cause a input constraint problem, we introduce the following transformation which is motivated by [15]

$$R_{d,i}=\frac{R'}{\pi}(\lambda^{-2}-1)\arctan(c\cdot r_i)+\frac{R'}{2}(1+\lambda^{-2}), \quad i=1,\dots,n, \tag{8}$$

where $c>0$ is the coefficient. In fact, we can choose any positive value for $c$.

We can introduce a graph $\mathcal{G}(\mathcal{V},\mathcal{E},A)$ for distributed controller. $A=[a_{i,j}]$ is the corresponding adjacency matrix. The $i$th damper controller can access the pressure information of room $j$ if and only if $a_{j,i}>0$. From (8) we can see that the range of $r_i$ is $(-\infty,+\infty)$. Then the controller can be designed as

$$\dot{P}_{fan}=\sum_{i=1}^{n}(P_0-P_i), \qquad \dot{r}_i=\sum_{k\in\mathcal{N}_i}(P_i-P_k),\ i=1,\dots,n. \tag{9}$$

Then we have the following result.

**Theorem 4.1** *For the HVAC system as described in Figure 1, in the case that $R_{v,i},\ i=1,2,\dots,n$ are time-invariant, all the static pressures $P_i,\ i=1,2,\dots,n$ as shown in Figure 2 converge to $P_0$ asymptotically based on the controller given in* (9)*, where $R_{d,i},\ i=0,1,\dots,n$ can be adjusted according to* (3).

**Proof**. From the equations (5)-(7), we can find the following relationship

$$P_i=\frac{R_{v,i}P_{fan}}{(R_0+R_c)\left(\sum_{j=1}^{n}\sqrt{\frac{R_{d,i}+R_{v,i}}{R_{d,j}+R_{v,j}}}\right)+R_{d,i}+R_{v,i}}, \quad i=1,2,\dots,n. \tag{10}$$

According to (10), $\frac{\partial P_i}{\partial P_{fan}}$, $\frac{\partial P_i}{\partial R_{d,i}}$, $i=0,1,\dots,n$, $j=0,1,\dots,n$ can be calculated as follows:

$$\frac{\partial P_i}{\partial P_{fan}}=\frac{R_{v,i}}{(R_0+R_c)\left(\sum_{j=1}^{n}\sqrt{\frac{R_{d,i}+R_{v,i}}{R_{d,j}+R_{v,j}}}\right)+R_{d,i}+R_{v,i}}, \tag{11}$$

$$\frac{\partial P_i}{\partial R_{d,i}}= \frac{-V_iP_{fan}\left(1+\frac{1}{2}\sum_{j=1,j\neq i}^{n}\sqrt{\frac{1}{(R_{d,i}+R_{v,i})(R_{d,j}+R_{v,j})}}\right)}{\left[(R_0+R_c)\left(\sum_{j=1}^{n}\sqrt{\frac{R_{d,i}+R_{v,i}}{R_{d,j}+R_{v,j}}}\right)+R_{d,i}+R_{v,i}\right]^2}, \tag{12}$$

$$\frac{\partial P_i}{\partial R_{d,j}}= \frac{\frac{1}{2}R_{v,i}P_{fan}\sqrt{\frac{R_{d,i}+R_{v,i}}{(R_{d,j}+R_{v,j})^3}}}{\left[(R_0+R_c)\left(\sum_{j=1}^{n}\sqrt{\frac{R_{d,i}+R_{v,i}}{R_{d,j}+R_{v,j}}}\right)+R_{d,i}+R_{v,i}\right]^2}, \quad j\neq i. \tag{13}$$

It is easy to see that $\frac{\partial P_i}{\partial P_{fan}}>0$, $\frac{\partial P_i}{\partial R_{d,i}}<0$, $\frac{\partial P_i}{\partial R_{d,j}}>0$, $j\neq i$. Since $\dot{R}_{v,i}=0$, $i=1,2,\dots,n$, we have

$$\dot{P}_i=\frac{\partial P_i}{\partial P_{fan}}\dot{P}_{fan}+\sum_{j=1}^{n}\frac{\partial P_i}{\partial r_j}\dot{r}_j,\ \ i=1,2,\dots,n. \tag{14}$$

By substituting (9) into the above equation, the closed-loop system can be written into the following compact form

$$\dot{P}=-L(t)P, \tag{15}$$

where

$$P=\begin{bmatrix}P_0\\P_1\\\vdots\\P_n\end{bmatrix},\ L(t)=\left[\begin{array}{c|ccc}0&0&\cdots&0\\ l_{1,0}&l_{1,1}&\cdots&l_{1,n}\\ \vdots&\vdots&\ddots&\vdots\\ l_{n,0}&l_{n,1}&\cdots&l_{n,n}\end{array}\right],$$

and

$$\begin{aligned} l_{i,0}(t)&=-\frac{\partial P_i}{\partial P_{fan}},\\ l_{i,i}(t)&=\sum_{j=1}^{n}\left[\frac{\partial P_i}{\partial R_{d,j}}\frac{dR_{d,j}}{dr_j}-\frac{\partial P_i}{\partial R_{d,i}}\frac{dR_{d,i}}{dr_i}\right]+\frac{\partial P_i}{\partial P_{fan}},\\ l_{i,j}(t)&=\frac{\partial P_i}{\partial R_{d,i}}\frac{dR_{d,i}}{dr_i}-\frac{\partial P_i}{\partial R_{d,j}}\frac{dR_{d,j}}{dr_j},\\ &\quad j=1,2,\dots,n,\ j\neq i.\end{aligned}$$

Since $\frac{dR_{d,i}}{dr_i}>0$ for $i=0,1,\dots,n$, it is straightforward that $\sup_{t\geq 0}l_{i,0}(t)<0$, $\sup_{t\geq 0}l_{i,j}(t)<0$, $\inf_{t\geq 0}l_{i,i}(t)>0$, and $\sum_{j=0}^{n}l_{i,j}(t)=0$. Therefore, $L(t)$ is a valid Laplacian matrix corresponding to a leader-follower graph $\bar{\mathcal{G}}$. Since the damper communication graph $\mathcal{G}$ is connected, $\bar{\mathcal{G}}$ contains a spanning tree. According to Lemma 2.1, the state will reach consensus. Due to that $P_0$ is a constant, all the pressures will converge to $P_0$, i.e.

$$\lim_{t\to\infty}P(t)=P_0\begin{bmatrix}1&1&\cdots&1\end{bmatrix}^T. \tag{16}$$

□

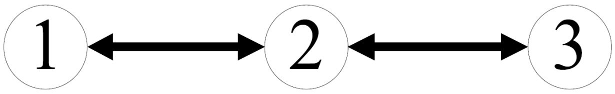

Fig. 3: Control topology

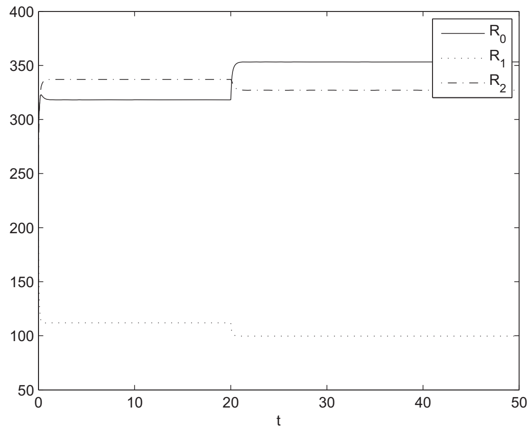


Fig. 4: Damper controller

**Remark 4.1** *The controller is simple as shown in* (9) *and is easy to be implemented based on the air pressure measurements. The damper positions are controlled according to* (3). *The pressure of each room inlet will simultaneously and asymptotically converge to the desired value. The coefficient c can be chosen according to the control purpose. A larger value we choose for c, a faster convergence rate we can get and a larger overshoot the system generates.*

## 5 Numerical Example

We consider the HVAC systems with 3 rooms. The flow resistance of supply duct, return duct and the cooling coil are $R_0 = 300$, $R_c = 300$. The flow resistances of room 1 and 2 are given as follows:

$$R_{v,1} = \begin{cases} 250, & 0 \leq t \leq 20, \\ 270, & 20 < t \leq 50, \end{cases} \quad R_{v,2} = 250, \; R_{v,3} = 230, \tag{17}$$

where $t$ is time. The setpoint for $P_i$, $i = 1, 2, 3$ is $P_0 = 400$ Pa.

For the damper, we choose $K' = 10$, $\lambda = 1/7$. The initial value of $R_i$, $i = 0, 1, 2$ and $P_{fan}$ are $R_0(0) = 234.77$, $R_1(0) = 250$, $R_2(0) = 242.37$ and $P_{fan}(0) = 3000$ Pa, respectively. The control topology $\mathcal{G}$ is shown in Fig. 3.

Then the flow resistances of the dampers can be controlled according to (8) and (9). We choose the coefficient $c$ involved in (8) as $0.1$. Figure 4 shows the trajectories of $R_i$, $i = 1, 2, 3$. The air pressure generated by air supply fan is shown in Figure 5. The trajectories of $P_1$, $P_2$ and $P_3$ are given in Figure 6.

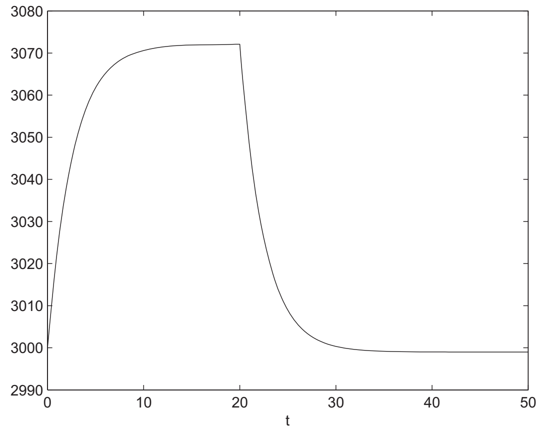


Fig. 5: Air pressure generated by air supply fan

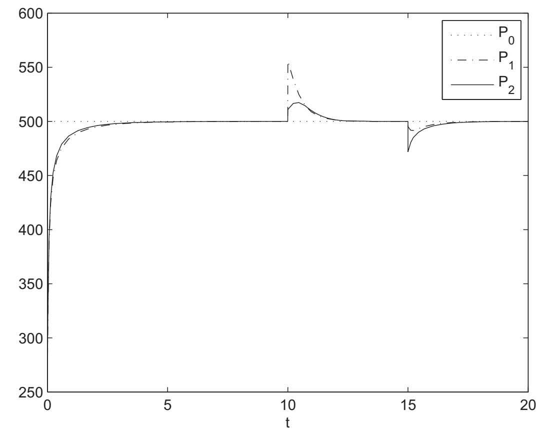


Fig. 6: Air pressure of room inlets

## 6 Conclusion

In this paper we have considered static air pressure control for HVAC systems. The VAV is controlled in a cooperative way which makes the system response smoother. The numerical example showed the effectiveness of the algorithm. In the future, we can consider how to minimize the energy cost of supply fan while keeping the same system transient performance.

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