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Immanuel Kant's Logically Perfected Natural Science

DISSERTATION

submitted in partial satisfaction of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in Philosophy

by

Toni Queck

Dissertation Committee:

Professor Jeremy Heis, Chair

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2023

DEDICATION

TO

the great people that joined me on this fascinating journey; in Irvine, Munich, and beyond

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ABSTRACT OF THE DISSERTATION

Immanuel Kant's Logically Perfected Natural Science

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My dissertation investigates the relation between Immanuel Kant's *Metaphysical Foundations of Natural Science* (MAN) and the *Critique of Pure Reason*. While it is clear that Kant somehow draws on the first *Critique* in MAN, the nature of the connection is mysterious. I solve the puzzle by arguing that Kant's central philosophical objective in MAN is Leibniz-Wolffian: to express apodictically certain knowledge (in regards to empirical physics) in a systematic philosophy; rather than to extend the distinct kind of "a priori" knowledge that the first *Critique* proposes. Even though Kant's core philosophical objective is Leibniz-Wolffian, his methods are not. The dissertation explores three crucial dimensions of this theme. First, I distinguish Kant's conception of a priori knowledge in MAN from that in the first *Critique*. Second, I clarify the role of the "construction" of the concept matter in MAN through a comparison with his views on construction in geometry. Third, I examine Kant's view on true judgment in MAN, i.e., the analytic/synthetic distinction.

My first step towards establishing the main thesis consists in showing that Kant's conception of a priori knowledge in MAN differs from that in the first *Critique* (Chapter 1). To see this, I begin by articulating Kant's hierarchy of logical perfection, which is his proposal for how different cognitive faculties unify previously indistinct representations. This hierarchy serves as Kant's overall epistemic order on which the first *Critique* (i.e., the Analytic) precedes empirical concept formation (e.g., the empirical concept matter), which, in turn, precedes Kant's "pure" natural science in

MAN. The latter approach reflects Kant's distinct conception of a "particular" logic, a "logic" of empirical physics. I argue that while a priori knowledge in the first Critique functions to unify indistinct perceptions, "a priori" knowledge as developed in MAN isolates the necessary elements of the empirical concept matter, in particular the trajectories of objects in pure intuition, and (Kantian) absolute space as a first ground.

The second chapter clarifies Kant's suggestion that he imitates the "mathematical method" and proceeds according to a "construction" of the concept matter in pure intuition in MAN. I develop Kant's position through comparison with his conception of Euclidean geometry, in particular his view on mathematical definitions and proofs. My thesis is that Kant's geometrical construction serves as his model for isolating constructable component concepts through which the concept matter can be securely cognized. Kant modifies Leibniz-Wolffian philosophy by proposing that a proper "reflection" on empirical science draws not only on the pure representations from the understanding but also on those from the sensibility. Kantian construction in MAN can hence be thought of as a distinct form of "analysis" of an empirical concept, one that leads to "spatially" encoded (and thereby apodictically certain) determinations of the concept matter.

The third chapter investigates Kant's view of true judgment in MAN. The analytic/synthetic distinction Kant proposed in the first Critique manifests his departure from Leibniz-Wolff's approach that all truths can be derived through an application of the principle of contradiction by showing that the predicate is "contained" in the subject concept. I argue that the three criteria Kant gives for the distinction in the first Critique (containment, identity, and differing epistemic uses) express a unified view when understood in regards to the judgments Kant defends in MAN. A key move is to interpret his fundamental Explications of the concept matter as paradigmatic instances of his critical conception of analytic judgment. On this view, the containment criterion reflects containment in the constructed concept, not the "mere" concept.

Overall, my approach to MAN differs from the literature in the following respects. Kant's essential philosophical moves can, I suggest, be interpreted in a way that MAN comes out as a consistent and interesting project, at least by Kant's own standards, not as a partial or even a complete

failure. In essence, Kant's programmatic suggestions to derive a priori knowledge and to construct the concept matter in pure intuition have a consistent interpretation. The reading entails that the relation between the first Critique and MAN is rather loose. In the literature, one typically finds the view that Kant's core philosophical objectives and conceptions in those two works align closely. My interpretation of MAN centers on a fine-grained comparison with Leibniz-Wolffian philosophy, in particular the joint goal of logically perfected cognitions. Despite the clear evidence for such a connection—Kant's appeal to "metaphysical foundations" and the mathematical method—this angle is underappreciated in the literature.

1 Kant's material conception of apriority

But nature is also taken otherwise in its material meaning, not as a constitution [Beschaffenheit], but as the purest concept [Inbegriff] of all things, insofar as they can be objects of our senses, and thus also of experience. Nature, in this meaning, is therefore understood as the whole of all appearances, that is, the sensible world, excluding all nonsensible objects.

- From the Preface of Kant's *Metaphysical Foundations of Natural Science*, 4:467, translation modified¹

At the same time permit me to explain that the efforts at criticism I have heretofore made are in no way meant (as they might appear to be) to attack the Leibniz-Wolffian philosophy (for I find the latter neglected in recent times). My aim is rather to pursue the same track according to a rigorous procedure.

- Kant in a letter to A.G. Kästner in 1790, 11:186

1.1 Introduction

At the center of the first *Critique* is a very distinct conception of apriority. A priori cognitions are known “absolutely independently of all experience” (B3), says Kant. While he uses the term “a priori” a lot, he does not explicitly spell out what independence of experience really amounts to. To get clear on it, work through the actual arguments in the first half of the *Critique*, especially the *Deduction*, Kant might say. In his *Metaphysical Foundations of Natural Science* (MAN), which was published in-between the first and second edition of the *Critique*, Kant seemingly has the same target, that is, derive a priori knowledge. This time it concerns natural science in the form of the concept of matter.

This chapter investigates the relation between Kant's conception of apriority in the *Critique* and

¹The German sentence is: “Sonst wird aber auch Natur in materieller Bedeutung genommen, nicht als eine Beschaffenheit, sondern als der Inbegriff aller Dinge, so fern sie Gegenstände unserer Sinne, mithin auch der Erfahrung sein können, worunter also das ganze aller Erscheinungen, d. i. die Sinnenwelt mit Ausschließung aller nicht sinnlichen Objekte verstanden wird”.

MAN. The essential question is: does Kant have the same conception of apriority in MAN as he does in the *Critique*? The answer we give to this question determines the kinds of philosophical objectives we take Kant to pursue with respect to natural science, in essence, his view on the relation between metaphysics and science. If we answer the question with “yes”,² we could, for instance, interpret Kant’s claims in MAN such as the laws of motion as “instantiations” of his principles of the understanding from the *Critique*.³ The laws of motion would inherit the “constitutive” status of those general principles with respect to “objective experience”. To put the point differently, we might say that Kant’s cognitions in MAN amount to further “conditions for the possibility of experience”. We need to presuppose those more specific cognitions from MAN for the possibility of proper spatiotemporal objects, a more specific type of object than treated in the *Critique*.⁴ In short, to approach MAN from this overall perspective means to assimilate Kant’s arguments in MAN to the a priori justifications that he gives in the *Analytic* for the categories and principles.

If we answer the question with “no”, the next questions are: what is Kant’s conception of apriority in MAN and how does it differ from that in the *Critique*. This will be my approach in the following.⁵ Contrary to the literature, I argue that Kant has a distinct conception of apriority in MAN and that his core philosophical objective is different than in the first *Critique*. I will take my clues from what Kant calls a “logic of the particular use of the understanding” (A52/B76). To put it directly, on my view, MAN can be read as a “logic” of empirical physics. Kant presupposes empirical facts regarding matter—what Newton called the universal qualities of bodies that conform to all experiments—at the outset. In the main body of MAN, Kant *represents* this empirical knowledge in a “system”. More specifically, MAN can be thought as a system of propositions, a system that expresses the apodictically certain parts of empirical physics. Such a view entails that Kant’s explications and justifications in MAN are epistemically secondary to empirical investigation.

For the concept motion as the fundamental determination of the concept matter, the thought

²See, for instance, Friedman (1992a,b, 2013), Plaass ([1965], 1994), Förster (2012) and Watkins (2019) for versions of this approach.

³See, for instance, Friedman (1992b, 2013).

⁴See, for instance, Watkins (2019, Ch.4).

⁵Buchdahl (1969) defends a somewhat similar approach.

will be this. On Kant’s view on pure intuition and the categories, it is necessary that relative motion (in empirical intuition) involves a pure part, that is, the mere trajectory of the object in space. To know a priori (henceforth: material a priori)⁶ in this context means to know the necessary “intuitive” features of the empirical concept matter. Kant’s material conception of apriority builds on but also modifies Leibniz’s (and Wolff’s) conception of apriority or truths of reason.⁷ It is key to recognize that Kant has his own way of accounting for the “reason” of a necessary proposition through “analysis”. Accordingly, his conception of a complete series of conditions for a given conditioned—drawn from the *Dialectic* and applied to his conception of absolute space—completes my proposal.

I first survey the literature regarding Kant’s conception of apriority in MAN and show that it is a dominant theme to explicate apriority in MAN in terms of Kant’s conception from the first *Critique* (Sect. 1.2). After that, I will introduce Kant’s sevenfold hierarchy of logical perfection from the *Jäsche Logic*. This hierarchy will help me to distinguish different cognitive projects that Kant pursues in his theoretical philosophy; in particular, it will serve as Kant’s overall epistemic order (Sect. 1.3). After clarifying Kant’s conception of apriority in the first *Critique* in terms of this hierarchy (Sect. 1.4), I will suggest that apriority in MAN (the material a priori) consists of a different type of approach. The key thought will be that material a priori cognitions are epistemically secondary to empirical theorizing. To see this, I highlight the relevant aspects of Kant’s view of a “particular” logic and compare the resulting view with Leibniz’s conception of truths of reason (Sect. 1.5). My proposal will be completed through a discussion of Kant’s view of cognition through universal principles (Sect. 1.6). I will draw on Kant’s positive suggestions on the faculty of reason from the *Dialectic* to give a reading of Kant’s notion of absolute space as a proper universal principle for the *Phoronomy*.

⁶This term is inspired from Kant’s comment at 4:467, as quoted above.

⁷There is an illuminating literature that connects aspects of Kant’s critical conception of apriority to ideas from German rationalism. See, for instance, Burge (2000), Smit (2009), Hebbeler (2015), Stang (2019), and Messina (2022). Those scholars tend to assume that the conception of apriority in MAN and the *Critique* is the same.

1.2 Literature on apriority in MAN

1.2.1 A very influential view

It is a natural first thought that Kant's conception of apriority in MAN is identical with his conception of independence of experience from the first *Critique*. After all, Kant models the four chapters of MAN on the table of categories.⁸ His guiding thought is that the "transcendental principles" from the *Critique* "are applied" to objects of our senses (MAN, 4:470).

To see the shape of this proposal more clearly, it is useful to start with Kant's definition of apriority from the beginning of the *Critique*:

In the sequel therefore we will understand by a priori cognitions not those that occur independently of this or that experience, but rather those that occur absolutely independently of all experience. (B2-3)

Kant defines a priori cognitions as independent of all experience. The first half of the *Critique* is where Kant develops what those cognitions are and how we know them. Roughly, he explicates independence of experience in terms of "constitution". That is, a priori cognitions are independent of experience because they constitute the "form" of a philosophically powerful notion of experience. As I will explain in sect. 1.3-4 in more detail, the best way to think about this suggestion is in terms of an epistemic primacy of a priori cognitions relative to all empirical concepts. That is, on Kant's hierarchical reconstruction of experience, we state and justify the categories and principles prior to the formation of empirical concepts. Kant, of course, would grant that, pre-theoretically, we acquire and "learn" about concepts like cause through our "experiences". On Kant's philosophical proposal, however, the proper epistemic order requires that we state and justify concepts like cause prior to forming ordinary empirical concepts and propositions like "The sun warms the stone".

In the literature, scholars heavily rely on this "constitution" approach when explicating Kant's conception of apriority in MAN. Here is a first approximation of this view:

⁸See MAN, 4:473-4.

Constitutivity-Thesis Kant's conception of apriority in MAN corresponds to that from the *Critique*. The cognitions regarding the concept matter are known independently of experience of objects of outer sense. They thereby amount to additional conceptual conditions on objective experience, that is, on spatial experience. (Friedman (1992a,b, 2013), Plaass [1965] (1994), Förster (2012), Watkins (2019))

On this view, Kant's argumentative strategy in MAN would be very similar to that in the *Analytic*. He would establish that the concept matter is properly known prior to a consideration of experience of objects of outer sense. That is, he would give justifications for the partial concepts of the concept matter of the sort he gives for the categories and the principles of the understanding.⁹ In essence, it would mean that the concept matter (as articulated in MAN) can be justified prior to any empirical concept. I want to turn to the literature now to show that this approach is very influential.

1.2.2 Friedman (1992)

Michael Friedman's (1992a,b) work in the nineties, especially his *Kant and the Exact Sciences*, subscribes to the Constitutivity-Thesis. This is especially clear in Friedman's treatment of Kant's laws of motion, which Kant derives in the *Mechanics* chapter of MAN.

Friedman characterizes Kant's laws of motion as "realizations" and "specifications" of the corresponding principles of the understanding from the *Critique*, i.e., the *Analogies*. The resulting thought is that the laws of motion inherit the status of Kant's principles as constitutive conceptual conditions on experience. Here are three passages:

1) The dynamical principles of pure understanding, and their specifications given in the *Metaphysical Foundations* (that is, the laws of motion), are nonetheless "constitutive with respect to experience" [...] The dynamical principles of pure understanding,

⁹Förster (2012, pp.66-71) in fact proposes that Kant's arguments in MAN are required to complete the project of the *Analytic*, in particular his thought is that MAN secures the objective reality of the categories. In more detail, Förster suggests that Kant's *Phoronomy* and *Dynamics* together provide a "schematism of outer sense", which complements the schematism in time provided in the *Critique*, thereby securing "the objective reality of transcendental philosophy" (p.68). Förster's reading is a clear instance of assimilating Kant's objective in MAN to that in the *Critique*. Plaass ([1965] 1994) has a somewhat similar approach. One of his key moves is a "metaphysical deduction" of the (content of the) concept motion/matter in analogy with Kant's approach towards the categories and space and time (see pp.298-300, 272-77, 303-308).

and their specifications given in the *Metaphysical Foundations*, are rules which govern a procedure for constructing objective experience from given perceptions if such experience is possible at all. (Friedman (1992b, pp.163-4))

2) [T]he principles of special metaphysics that correspond to, and thus exemplify and realize, the analogies of experience. These are given in Chapter 3 or *Mechanics of the Metaphysical Foundations* [...] Each is derived by an a priori “proof” which proceeds by starting with the corresponding principle from the *Critique* and substituting in, as it were, the empirical concept of matter. (Friedman (1992b, p.168))

3) the laws of motion are not empirical facts about true motions but a priori conditions of the possibility of such motions—just as the analogies of experience, which the laws of motion are intended to instantiate or realize, are not facts of objective experience but a priori conditions of the possibility of such experience. (Friedman (1992b, p.171))

Friedman takes Kant’s laws of motion to have a “constitutive” character with respect to objective experience (1). They are part of the a priori conditions for the possibility of “objective experience” (1, 3). In particular, they serve as presuppositions for a conception of true motions (3). However, in light of the involvement of the empirical concept matter as the concept of the object of outer sense (2), the notion of experience that the a priori cognitions in *MAN* establish is more specific than in the first *Critique*.

The underlying thought is that the laws of motion are instantiations of the *Analogies* (1, 2, 3). Friedman in fact suggests that the laws can be derived “a priori” from the corresponding *Analogy* (2). For example, consider the first *Analogy*: “In all change of appearances substance persists, and its quantum is neither increased or diminished in nature” (B224). The idea is that we substitute “matter” for “substance” and “change of corporeal nature” for “change of appearances”. This gives us Kant’s first law of motion: “In all changes of corporeal nature the total quantity of matter remains the same, neither increased nor diminished” (*MAN*, 4:541). Just as the first *Analogy* functions as a necessary condition on experience in general, the first law of motion is one essential presupposition for us to experience objects of outer sense and thereby constitutive with respect to this more restricted conception of experience.

1.2.3 Friedman (2013)

Friedman (2013) articulates apriority in MAN in terms of measurability: a cognition is known a priori if it makes the measurement of an empirical concept possible.¹⁰ Here is a long passage where Friedman explicates this approach in more detail:

So it is the principle of the equality of action and reaction (Kant's Third Law of Mechanics), applied to interactions in accordance with universal gravitation, which, for the first time, allows us to extend the static measure of weight to a universal measure of quantity of matter valid for all bodies in general on the basis of the new (Newtonian) mechanical quantity of mass. We presuppose the universal applicability of the mechanical laws of motion, the equality of inertial and gravitational mass, and the universally penetrating character of gravitational force in this procedure. And it is for precisely this reason that Kant builds all three into the characteristically dynamical concept of matter that he articulates in the *Metaphysical Foundations*. It is for this reason, too, that the mechanical laws of motion and the above properties of gravitational force (the fundamental force of attraction) count as a priori for Kant. This emphatically does not mean, however, that he attempts to demonstrate by pure reason, independently of experience, what Newton has discovered by observation and experiment. The point is rather, on my reading, that Kant attempts to isolate just those features of the concept of matter in virtue of which Newton has successfully mathematized its quantity, and he analyzes or explicates this concept (in terms of motion, the filling of space, inertia, and so on) precisely to reflect these features. (pp.568-69)

Friedman argues that the laws of motion (especially the third law), the equality of inertial and gravitational mass, and gravitational force are necessary presuppositions for Newton's universal procedure for measuring the quantity of matter. It is for this reason that the laws of motion (as treated in Kant's *Mechanics*) and attractive force (as treated in Kant's *Dynamics*) are a priori for Kant. As the second half of the passage makes clear, Friedman operates not with Kant's classical conception of apriority in terms of independence of *all* experience here. Rather, Friedman's Kant articulates and justifies the conditions that explain how central physical concepts receive their rigorous quantitative structure.¹¹ In other words, Kant's a priori cognitions in MAN trace the implicit

¹⁰See, for instance, Friedman (2013, p.32): "Kant aims, more generally, to explain how the quantitative structure of each of the concepts required by the mathematical theory of motion becomes possible – and, in this way, to explain the application of mathematics in all of (Newtonian) mathematical physics. One of the most important goals of the *Mechanics* chapter, for example, is to explain how the concept of mass or quantity of matter becomes possible as a mathematical magnitude – to explain how, in Kant's terms, the quantity of matter of any body may be (mechanically) measured or 'estimated'".

¹¹See Friedman (2013, pp.32-3).

presuppositions of Newton's physical theorizing.

One natural way to make sense of what Friedman says here is to draw on a conception of apriority that is defended by some Neo-Kantians.¹² The idea is to keep separate two aspects of Kant's original conception, that is, its eternal validity and its constitutive character for the objects of experience. Reichenbach and Cassirer have maintained the second element but have rejected the first. A priori knowledge on this view has a constitutive character with respect to objective experience but is not meant to be valid eternally. On this modified conception, a priori cognitions consist of presuppositions of a particular scientific theory. A priori knowledge is derived "bottom-up" from scientific theories, not "top-down" from the principles of the understanding. Friedman's (2013) Kant accordingly isolates "just those features of the concept of matter in virtue of which Newton has successfully mathematized its quantity, and he analyzes or explicates this concept (in terms of motion, the filling of space, inertia, and so on) precisely to reflect these features" (as quoted above). I say that Friedman's explication of a priori knowledge in terms of necessary presuppositions for scientific theorizing suggests a commitment to the Constitutivity-Thesis. That is, on Friedman's reconstruction of Kant, the cognitions defended in MAN are epistemically prior to empirical physics.

1.2.4 Watkins (2019)

Eric Watkins's (2019, Ch.4) suggestion is to think of the justifications in MAN in terms of transcendental arguments. A transcendental argument, as Watkins conceives it, has the following form:

(1) it is an argument that reveals the necessary conditions for (2) a general type of experience. (pp.77-8, n.21)

Building on this conception, Watkins explicates the argumentative structure of MAN in terms of a two-step transcendental argument. For the first step (pp.78-81), Watkins's Kant argues that movability in space is a necessary condition for us to experience an object of outer sense. Watkins then proceeds to the *Mechanics* chapter since it is the "clearest case" for his approach (p.82).

¹²See Heis (2014b).

The suggestion is that Kant adds the communication of motion as a new determination to the concept of matter in the *Mechanics*.¹³ Watkins (p.81 ff.) argues that Kant's laws of motion are necessary conditions for us to experience the communication of motion. For instance, Kant's second law "Every change in matter has an external cause" (MAN, 4:543) establishes that a causal interaction must hold between two parts of matter in order for us to experience the communication of motion. On Kant's conception of an object of outer sense, there is just no way for matter to have internal grounds for changing its state of motion. So if the state of motion of a body changes, that presupposes some kind of causal interaction between two distinct parts of matter. In light of their status as necessary conditions of spatial experience, the laws of motion are a priori and apodictic, says Watkins (p.87).

This line of reasoning leads Watkins to suggest the following view concerning the relation between the Critique and MAN:

The transcendental part (in the first Critique) explains how experience in general (or temporal experience) is possible by establishing certain necessary conditions for experience considered under each categorical heading. The special part argues analogously, but attempts to explain the possibility of experience that is more specific and richer than that of the first Critique (i.e., experience in general). For it presupposes a concept of matter as an object of outer sense (i.e., outer or spatial experience) that clearly has more content than "experience in general". Since the notion of experience it attempts to explain is richer in content, the principles it presupposes will naturally have more content as well. (p.87)

Watkins assimilates Kant's justifications for movability and the laws of motion in MAN to his reasoning in the *Analytic*, especially to the proofs for the *Analogies* in terms of transcendental arguments. The only difference is that we establish a more restricted type of experience than in the Critique. Kant's a priori cognitions in MAN for this reason function as necessary presuppositions for experience of objects of outer sense. I therefore say that Watkins defends a version of the

¹³The thought is that the communication of motion instantiates the relevant categorical heading, i.e. relation.

Constitutivity-Thesis here.

1.3 Kant's hierarchy of logical perfection

The focus of this section is Kant's hierarchy of "logical perfection" from his *Jäsche Logic* (Logik). This hierarchy provides a useful framework in which we can understand his distinct conceptions of apriority. I will quote the full hierarchy and then show in the remainder of this chapter that it has significant exegetical value regarding Kant's views on apriority. To anticipate a bit, it will pave the way for approaching apriority from a more Leibniz-Wolffian point of view than is common in the secondary literature. One crucial dimension of this suggestion is that belief and concept *acquisition* have very little formative force for Kant on this issue.

The following sevenfold hierarchy of logical perfection appears in sect. VIII of Kant's *Logik*:¹⁴

In regard to the objective content of our cognition in general, we may think the following degrees, in accordance with which cognition can, in this respect, be graded:

The *first* degree of cognition is: to represent something;

The *second*: to represent something with consciousness, or to perceive (*percipere*);

The *third*: to be acquainted [*kennen*] with something (*noscere*), or to represent something in comparison with other things, both as to sameness and as to difference.

The *fourth*: to be acquainted with something with consciousness, i.e., to cognize [*erkennen*] it (*cognoscere*). Animals are acquainted with objects too, but they do not cognize them.

The *fifth*: to understand something (*intelligere*), i.e., to cognize something through the understanding by means of concepts, or to conceive. One can conceive much, although one cannot comprehend [*begreifen*] it, e.g., a *perpetuum mobile*, whose impossibility is shown in mechanics.

The *sixth*: to cognize something through reason, or to have insight [*einsehen*] into it (*perspicere*). With few things do we get this far, and our cognitions become fewer and fewer in number the more that we seek to perfect them as to content.

The *seventh*, finally: to comprehend [*begreifen*] something (*comprehendere*), i.e., to cognize something through reason or a priori to the degree that is sufficient for our purpose. For all our comprehension is only relative, i.e., sufficient for a certain purpose; we do not comprehend anything without qualification. Nothing can be comprehended more than what the mathematician demonstrates, e.g., that all lines in the circle are proportional. And yet he does not comprehend how it happens that such a simple figure has these properties. The field of understanding or of the understanding is thus

¹⁴Karl Schafer (2022) also highlights this passage in his work on Kant.

in general much greater than the field of comprehension or of reason. (Logik, 9:64-5, my emphasis)

The overall hierarchy captures different kinds of representations, ones that consist of an increasing level of distinctness or “unity”.¹⁵ This results from the structuring activities of the faculty of the understanding (at stage four and five), and ultimately, the faculty of reason (at stage six and seven).¹⁶ At the first stage, we consider unconscious or blurred representations; it involves merely to “represent something”.¹⁷ At the second stage, we consider representations that are conscious, what Kant calls “perceptions”. At the third stage, we have representations that amount to “acquaintance” with distinct objects. For Kant, that consists of the ability to compare perceptions of objects with one another as regards to sameness and difference. On the fourth stage, the nature of representations amounts to a *conscious* acquaintance with proper objects, or to “cognize” [erkennen]. Kant famously argues in the *Deduction* that it is the understanding by means of the categories that makes (comparatively) indistinct representations “come together in one consciousness” (B143). So, at this level, we consider representations that have the “form” that the *Analytic* of the Critique attributes to “proper” representations of objects, that is, cognition. This involves basically the schematized categories and the principles of the understanding.¹⁸ At the fifth stage, we consider representations that have a proper conceptual form, like the empirical concept gold as a representation of a list of general marks. Representations on the sixth and seventh stage consist of even higher degrees of unity or distinctness through the contribution of the faculty of reason.

I will employ this hierarchy heavily in the following. In particular, I will compare representations by their respective degree of objective content. This will lead me to make claims that representations are “epistemically primary” relative to other kinds of representations. This talk is meant to refer to this hierarchy here; representations that have a lesser degree of objective content

¹⁵See A838-9/B866-7 and the second *Critique*, 5:40-1 for further emphasis on the role of logical perfection for theoretical philosophy.

¹⁶Cf.: “All our cognition starts from the senses, goes from there to the understanding, and ends with reason, beyond which there is nothing higher to be found in us to work on the matter of intuition and bring it under the highest unity of thinking” (A298-9/B355).

¹⁷Jauernig (2021, pp.52-58) convincingly argues that Kant’s account of perception has some crucial similarities with that of Leibniz. So, there is at least some plausibility to attributing to Kant a version of Leibniz’s thought that we confusedly represent the whole world in the first stage.

¹⁸I assume here that we talk about humans with their distinct “forms” of sensibility.

will be called “epistemically primary”. The suggestion that, on Kant’s view, some representations are epistemically primary relative to other kinds, in essence, reflects the involvement of different cognitive faculties.

Since the terminology of “epistemic primacy” is borrowed from Carnap’s *Logical Structure of the World* [Aufbau], I want to compare and contrast Kant’s hierarchy with that of Carnap. The purpose of this is to make a first step towards understanding Kant’s philosophical objectives. One thing that is essential to recognize beforehand is the following. My upcoming argument intends to show that Kant’s epistemic story concerning the transition from stage three to four, which I will associate with apriority in the *Critique*, and from five to six, which I will associate with apriority in MAN, differ in an interesting way. The idea that Kant’s hierarchy reflects a *uniform* epistemic order is thus only a first and rough approximation and will be refined in the following.

Now, I turn to Carnap for a short excursion to enable a comparison. In the *Aufbau*, Carnap proposes and analyzes a method of rational reconstruction for different scientific disciplines. One core concern is to study how knowledge from different scientific areas can be expressed in a unified, hierarchical system, using the powerful logical resources of his time. He shows how a small basis (of relations and elements) can suffice to constitute different scientific objects, for instance, those associated with physics and psychology. The system he mainly studies is the “autopsychological” system, which aims to constitute physical and psychological objects (i.e., the mental states of other people) in terms of subjective experiences of the individual. What makes this system particularly interesting for Carnap is that it embodies a notion of “epistemic primacy”. It reconstructs different kinds of objects in accordance with how the subject can cognize them. Here is Carnap’s definition of epistemic primacy: an object is “called epistemically primary [erkenntnismäßig primär] relative to another one, which we call epistemically secondary, if the second one is recognized through the mediation of the first and thus presupposes, for its recognition, the recognition of the first” (§54). He gives the example of high air pressure and a high barometer reading (§49). The high barometer reading is epistemically primary relative to a high air pressure because it is cognized first

and identifies a state of high air pressure.¹⁹

One key difference, as mentioned, is that Kant's hierarchy of logical perfection does not reflect such a uniform epistemic order in light of the different cognitive faculties that are involved. In addition, while Carnap's motivation in the *Aufbau* is at least in part to transform or "reduce" sentences about physical or psychological objects to more basic ones,²⁰ i.e., subjective experiences;²¹ Kant's hierarchy has no such aim. Kant's goal is rather to show how various cognitive faculties contribute in making representations of the world more distinct or perfect. One thing both proposals have in common, though, is that they provide "rational reconstructions" of (scientific) experience and so intend to abstract from the order of actual acquisition.

The remainder of this chapter employs Kant's hierarchy as a guiding thread and adds distinctions and refinements to it along the way. My upcoming discussion focuses mainly on stages 3-6 of the hierarchy. I now turn to Kant's conception of apriority in the Critique, which I take to be connected to stage three and four. After that, I move on to Kant's conception of apriority in MAN, which I take to be connected to stage five, six and seven. The seventh stage, in particular Kant's suggestion that "[n]othing can be comprehended more than what the mathematician demonstrates" will be the topic of the second chapter.

1.4 Kant's conception of apriority in the first Critique

In order to shed some light on the conception of apriority in MAN and its relation to that in the Critique, I want to start with Kant's original conception of apriority in the Critique.

Here is a first statement from the third paragraph of the Introduction to the Critique:

It is therefore at least a question requiring closer investigation, and one not to be dismissed at first glance, whether there is any such cognition independent of all experi-

¹⁹See Pincock (2005, pp.524-530) for a more detailed discussion of the role of epistemic primacy in the *Aufbau*.

²⁰See, for instance, §2, 35, 47, 49.

²¹One strand in the recent literature on the *Aufbau* is to emphasize virtues other than reduction and objectivity. On Pincock's (2005) view, for instance, the aim of Carnap's various constitution systems—he discusses the autopsychological and the physical in depth—is to "clarify and unify a body of knowledge" (p.523). The point of the autopsychological system is "the rational reconstruction of our actual cognitive processes, smoothed out and appropriately clarified" (p.526).

ence and even of all impressions of the senses. One calls such cognitions a priori, and distinguishes them from empirical ones, which have their sources a posteriori, namely in experience. (B2)

Kant defines a priori cognitions in terms of independence of “all experience” here. In the fifth paragraph, he adds the following:

In the sequel therefore we will understand by a priori cognitions not those that occur independently of this or that experience, but rather those that occur absolutely independently of all experience. Opposed to them are empirical cognitions, or those that are possible only a posteriori, i.e., through experience. Among a priori cognitions, however, those are called pure with which nothing empirical is intermixed. Thus, e.g., the proposition "Every alteration has its cause" is an a priori proposition, only not pure, since alteration is a concept that can be drawn only from experience. (B2-3)

He now draws a further distinction between the pure and the impure a priori. In the former, “nothing empirical is intermixed”. The proposition that every alteration has its cause is impure a priori. Initially, this further distinction is confusing because independence of experience seems to rule out that the concept alteration can be drawn “only from experience”.²² To make sense of this, I want to distinguish the following three ways of knowing according to Kant,²³ focusing on conceptual representations.²⁴

pure-apriority A representation is known purely a priori if it is known through the pure understanding and functions to provide unity/order to (all kinds of) empirical intuitions. Example:

²²See Cramer (1985) for an illuminating take on Kant’s puzzling claim at (B3) that the concept alteration can only be drawn from experience.

²³My treatment of apriority in the first Critique has similarities with Burge (2000, e.g. p.364). Burge also emphasizes that Kant explicates apriority in terms of the functions of cognitive capacities in enabling cognitive access to empirical objects.

²⁴Pure sensible representations like the pure intuition of space will be treated in Chapter 2. To properly account for them here would require a more fine-grained discussion of Kant’s conception of knowledge and cognition (see, e.g., A293-5/B249-51), as well as the status of pure intuition. I do not treat those issues here because my goal is eventually to compare the first Critique with MAN, which does not involve purely sensible representations in isolation from the understanding.

The category cause instantiated in the object of possible experience.²⁵

impure-apriority/empirical-1 A representation is known impurely a priori if it is known through the pure understanding and functions to provide unity/order to *spatiotemporal intuitions*. Examples: the category cause instantiated in *spatiotemporal* objects, the *schematized* categories, the principles of the understanding.²⁶

empirical-2 A representation is known empirically-2 if it is known through comparison of actual “experiences” (i.e., ones that conform to the established “form” of experience) as provided by human sensibility. Example: instances of the category cause like “The sun warms the stone”.²⁷

I suggest that the distinction between a priori—both pure and impure—and empirical-2 cognitions reflects different stages in Kant’s intended epistemic order. In particular, a priori cognitions are epistemically primary relative to empirical-2 cognitions.

Let me first clarify the status of a priori cognitions. A priori cognitions function to provide a determinate order to sequences of indistinct representations like perceptions that, on Kant’s view, do not have any of that yet. Recall that at the third stage of the logical hierarchy, Kant considers representations that amount to mere acquaintance with distinct objects. That involves the ability to compare conscious representations with one another in regards to sameness and difference. Kant’s programmatic suggestion is that a priori cognitions put order on those blurred perceptual representations by means of a joint representation.²⁸

²⁵This definition of apriority takes Kant’s introduction to the Transcendental Logic (A50/B74 ff.) as formative. See, in particular, A68-9/B93-4 for the functional aspect. Kant engages in this kind of justification project, for instance, in the first half of the B Deduction: “A manifold that is contained in an intuition that I call mine is represented as belonging to the necessary unity of self-consciousness through the synthesis of the understanding, and this takes place by means of the category” (B144).

²⁶Kant engages in this kind of justification project, for instance, in the second half of the B Deduction: “Now the possibility of cognizing a priori through categories whatever objects may come before *our* senses, not as far as the form of their intuition but rather as far as the laws of their combination are concerned, thus the possibility of as it were prescribing the law to nature and even making the latter possible, is to be explained. For if the categories did not serve in this way, it would not become clear why everything that may ever come before our senses must stand under the laws that arise a priori from the understanding alone” (B159, my emphasis).

²⁷See Logik, §3, n.1; A220/B267.

²⁸See A68/B93.

For instance, I think that Kant’s strategy concerning the second Analogy—“all alterations occur in accordance with the law of the connection of cause and effect” (B232)—conforms well with the suggested account of apriority. The key premise in Kant’s “proof” is that at the level of perceptions, we do not have any knowledge of a determinate time order. He observes that “mere sense and intuition” provides no knowledge of succession (B233). For Kant, we acquire perceptions in the following form: a perception of a warm stone, a perception of a cold stone, a perception of a yellow ball in a certain location.²⁹ At this stage, we can only say that there are six possible orderings of those three perceptions. Kant’s suggestion is that in order to pick out an “objective” sequence, we need to presuppose the (schematized) categories.³⁰ To understand this move at least roughly, the following needs to be added. We have to assume a world where appearances reflect the successive states of substances that persist.³¹ The only way to represent an “objective” order of states is to presuppose a rule that expresses a necessary connection between them. This is the relation of cause and effect and is drawn from the pure understanding according to Kant.³² This argument, I suggest, is paradigmatic for Kant’s thinking about a priori cognitions. It shows how they function to provide order to sequences of perceptions.³³

It is worthwhile to make explicit the sense of independence of experience that underlies my proposal. In the literature, there has been a lot of discussion on this issue.³⁴ It really comes down to interpreting what Kant tries to get at when he says that our cognitive faculties need to be “awakened” (B1) first, that all cognition “commences with experience” (ibid.). A paradigmatic position

²⁹If we wanted to give a precise characterization of those Kantian perceptions, we should rather use relative terms like “warmer” or “colder” and refer to objects in terms of their basic qualities like shape, impenetrability and so on.

³⁰Anderson (2002) and Watkins (2005, Ch.3) influenced my thinking on the second Analogy. For instance, both scholars emphasize that Kant’s justification presupposes subjective/indistinct representations and moves towards (more) objective/distinct representations by invoking the categories.

³¹This was established in the context of the first Analogy.

³²See B233-4.

³³The second Analogy is known impurely a priori because its justification depends also on the pure intuition of time. See especially B233-4, and A177/B219 for Kant’s “modi of time” like succession. Since Kant recognizes the possibility of other forms of intuition, the dependence on a specific form of intuition introduces a contingent element. The (unschematized) category cause instantiated in an object of possible experience, as treated in the first half of the *B Deduction*, does not depend on those specific forms of sensibility and is therefore known purely a priori. See Plaass ([1965] 1994, Ch. 4&5, esp. pp.283-7, 296-8) and Parsons (1984) for alternative readings of the “impurity” of the concept alteration/motion.

³⁴See, e.g., Kitcher (1980), Van Cleve (1999), and Smit (2009).

in the literature is Kitcher's (1980, p.5) suggestion that "knowledge is independent of experience if any experience which would enable us to acquire the concepts involved would enable us to have the knowledge".³⁵ This captures the idea that a representation is independent of experience if it is independent of the sequence of experiences of a single knower, as long as the sequence is rich enough to acquire the concepts.

One shortcoming of this proposal, to my mind, is that it lacks textual support from the *Analytic*. On Kant's suggested notion of experience, a priori cognitions are known, in essence, through their function to unify (relatively indistinct) representations like empirical intuitions and perceptions, as we see in the *Deduction* or his arguments for the principles of the understanding.³⁶ The idea that we can properly know concepts like cause in all generality through a consideration and comparison of various particular causal sequences in experience, on the contrary, does not appear to be relevant to what Kant does in the *Analytic*.³⁷ In light of Kant's hierarchy of logical perfection, we can give a stronger, more interesting reading of independence of experience.

Kant, on my view, wants to suggest that any sequence of actual "experiences" needs to presuppose concepts like cause. While he, of course, would grant that *pre-theoretically*, we can "learn" about the concept cause through a comparison of our "experiences". However, based on his suggested epistemic order, the categories and principles are known prior to empirical objects like "The sun warms the stone". To form those kinds of empirical claims requires that representations can be attributed to a single consciousness, which requires the categories to impose that structure. Any proper empirical concept, for Kant, therefore consists of representations that are necessarily structured according to the categories and principles. This underlies our ability to form empirical claims in terms of changes of states of persisting things like stones and suns. So, when Kant in the first and second paragraph of the B Introduction says that cognition starts with experience, he refers, I think, to the *indistinct* representations from the first three stages of the hierarchy. He says that

³⁵See also Van Cleve (1999, pp.15-7).

³⁶Kant starts his argument in the *Deduction* with the presupposition of "a manifold that is given in a sensible intuition" (B143). The latter lacks "combination" or distinctness (B129-31).

³⁷On Nunez's (2014) view of Kant's categories, they are not explicated through a consideration of examples but can actually be defined in terms of their function in unifying empirical intuitions.

“objects that stimulate our senses” (B1) awaken our cognitive faculty, that we “work up the raw material of sensible impressions” (ibid.), that “[a]s far as time is concerned [...] no cognition in us precedes experience” (ibid.).

To sum up for now, my suggestion is that representations are known a priori if their content is drawn from the pure understanding and if they function to impose a determinate order on perceptions. I interpret their independence of experience in the sense that those representations are epistemically primary relative to experience. That is, a priori representations are stated and justified prior to all empirical concepts on Kant’s reconstruction of experience. This completes my discussion of Kant’s conception of apriority in the *Critique*.

1.5 Kant’s material conception of apriority in MAN

In this section, I propose a conception of apriority for MAN that is different from that attributed to the first *Critique*. In a nutshell, I will show that material a priori representations are epistemically secondary to empirical theorizing; they do not function as (further) presuppositions of experience.³⁸

My proposal involves three claims:

1. The concept matter (as presupposed at the outset of MAN) is known empirically-2.
2. MAN is a system of propositions (a “particular” logic) that *represents* the necessary elements of the presupposed empirical physics.
3. Kant’s material conception of apriority has similarities with that of Leibniz-Wolff.

The final piece in the puzzle—Kant’s view of a complete series of conditions—will be discussed in the next section.

³⁸In this chapter, I will continue to use the general term “representation”. In Chapter 2, the material a priori representations will be specified as constructable component concepts of the concept matter. Kant uses those constructable component concepts to form judgments, or holdings-to-be-true about the concept matter. The latter is the topic of Chapter 3.

1.5.1 The concept matter is known empirically-2

First, I show that the concept of matter that Kant treats in MAN is known empirically-2. Recall that a representation is known empirically-2 if it is known through comparison of actual “experiences” (as provided by human sensibility).

Kant is very clear that MAN relies on an “empirical concept”. It “takes the empirical concept of matter [...] as its basis” (4:470), it concerns “itself with a particular nature [...] for which an empirical concept is given” (ibid.).³⁹ Although the fact that the concept matter is an empirical one is recognized widely in the literature, the real nature of this claim is rather opaque in light of Kant’s concern with a priori knowledge in MAN.

One natural way to further explicate this claim, in my view, is through the levels of distinctness/epistemic primacy. Recall that the categories state the conceptual conditions on empirical objects in general. They are known prior to any form of empirical cognition for Kant. So the suggestion would be that the empirical concept matter derives its content from actual human experiences, which conform to the overall conceptual structure that the Analytic develops. The concept matter would have a higher degree of objective content for Kant and be epistemically downstream from the categories.

Here is how Kant glosses those two distinct projects in the *Preface* of MAN:

Thus proper natural science presupposes metaphysics of nature. Now this latter must always contain solely principles that are not empirical [...] but it can still either: first, treat the laws that make possible the concept of a nature in general, even without relation to any determinate object of experience [bestimmtes Erfahrungsobjekt], and thus undetermined with respect to the nature of this or that thing in the sensible world, in which case it is the transcendental part of the metaphysics of nature; or second, concern itself with a particular [besonderen] nature of this or that kind of thing, for which an empirical concept is given, but still in such a manner that, outside of what lies in this concept, no other empirical principle is used for its cognition (for example,

³⁹Similar claims can also be found in the Architectonic A845-8/B873-6.

it takes the empirical concept of matter or of a thinking being as its basis, and it seeks that sphere of cognition of which reason is capable a priori concerning these objects), [...] it is then not a general, but a special [besondere] metaphysical natural science (physics or psychology), in which the above transcendental principles are applied to the two species of objects of our senses. (MAN, 4:469-70)

The “transcendental” part of metaphysics is concerned with representations that do not have the degree of distinctness that a “determinate object of experience” has, says Kant. They are “undetermined with respect to the nature of this or that thing in the sensible world”. The “special” metaphysics, on the contrary, deals with determinate “objects of our senses”, that is, things for which an “empirical concept is given”. It amounts to an instance of cognition through the understanding by means of concepts.⁴⁰

Kant also characterizes the empirical basis of the explanatory project as “given facts” [gegebene Facta], which will eventually be explained through reason (MAN, 4:468). It is worth noting that this type of terminology is also central to Christian Wolff’s discussion of the preliminary stage of “philosophy”, that is, what he calls “historical knowledge”. To bring out crucial continuities between Kant and Wolff in regards to the status of empirical physics, I want to quickly introduce Wolff’s view.

Here is how Wolff characterizes the difference between historical and philosophical knowledge in his *Preliminary Discourse on Philosophy in General* (PD):

Philosophical knowledge differs from historical knowledge. The latter consists in the bare knowledge of the fact (#3). The former progresses further and exhibits the reason of the fact so that it be understood why something of this sort could occur. (PD, sect.7)⁴¹

For Wolff, we first have “knowledge of the fact”, which is acquired by means of the senses.⁴² This is historical knowledge for him. The next step in the progression of knowledge is philosophical

⁴⁰That is the fifth stage in the order of objective content from Logik, 9:64-5.

⁴¹See also PD, sect.17.

⁴²See PD, sect.1.

knowledge, i.e. “to acquire certain knowledge of the things which we know confusedly by the senses and by reflection on ourselves” (PD, sect.56). Philosophical knowledge consists of explanations of those facts. Wolff’s suggested progression of knowledge also applies with respect to physics:

Experimental physics must precede dogmatic physics. (PD, sect.109)

Here is why I think Wolff’s progression of knowledge is relevant. Kant draws a very similar distinction in the second paragraph of MAN:

the doctrine of nature can be better divided into historical doctrine of nature [historische Naturlehre], which contains nothing but systematically ordered facts about natural things [Facta der Naturdinge] [...] and natural science. Natural science would now be either properly or improperly so-called natural science, where the first treats its object wholly according to a priori principles, the second according to laws of experience. (MAN, 4:468)

So I think Kant wants to maintain a similar *progression* of knowledge as Wolff does from historical knowledge to philosophical knowledge. While there are some notable differences between Kant and Wolff/Leibniz regarding the status of the faculty of sensibility (which I will address in 1.5.3), they can be ignored for now.

Here are four passages that support the primacy of empirical knowledge over the knowledge developed in MAN:

1. one need merely consider the various propositions that come forth at the outset of proper (empirical) physics, such as those of the persistence of the same quantity of matter, of inertia, of the equality of effect and counter-effect, etc., and one will quickly be convinced that they constitute a *physica pura* (or *rationalis*), which well deserves to be *separately established*, as a science of its own, in its whole domain, whether narrow or wide. (B20, note, my emphasis)
2. But indeed there is also much in it that is not completely pure and independent of *sources in experience*, such as the concept of motion, of impenetrability (on which the empirical concept of matter is based), of inertia. (Prol, 4:295, my emphasis)

3. I further remark that, since the movability of an object in space cannot be cognized a priori, and without *instruction through experience*, I could not, for precisely this reason, enumerate it under the pure concepts of the understanding in the Critique of Pure Reason; and that this concept, as *empirical*, could only find a place in a natural science, as applied metaphysics, which concerns itself with a concept *given through experience*, although in accordance with a priori principles. (MAN, 4:482, my emphasis)⁴³
4. However, according to our discussion of this property, impenetrability rests on a *physical* basis [physischen Grunde]. For expanding force first makes matter itself possible, as an extended thing filling its space [...] the original attractive force [...] does not present itself so immediately to the senses as impenetrability, so as to furnish us with concepts of determinate objects in space. (MAN, 4:502, my emphasis; MAN, 4:513)

Those passages clearly show that empirical facts precede Kant's "rational" physics. In the first passage, Kant mentions propositions drawn from "empirical physics" that he will eventually explicate "rationally" in MAN as his laws of motion. In the second passage, Kant says that the concepts motion, impenetrability, and inertia depend on "sources in experience". The third and the fourth passage provide support for the claim that the concepts motion (of an object) and impenetrability have an empirical basis, they are concepts whose content is "given through experience". Those comments on the empirical origin of those concepts appropriately show up in Remark sections in MAN. The main body of the text, i.e., the Explications and Propositions, treats those concepts "rationally".

To conclude the discussion of the empirical basis of Kant's concept of matter, it is useful to put this view into historical context. I think the connection to Newton is especially clear. In Rule 3 of his "Rules for the study of natural philosophy" in the *Principia* (P) (pp.794-5), Newton states "universal qualities" of bodies. On his view, those qualities are known through experiments, universal qualities of bodies are those "that square with experiments universally" (P, p.795). He states the following universal qualities of bodies: extension, impenetrability, mobility, and perseverance in motion or rest by means of "force of inertia" (P, p.795).⁴⁴ The list of qualities and the associated universal agreement with experience fits nicely with Kant's approach. Recall that in the four passages quoted above, Kant states that the concepts motion, impenetrability and inertia are drawn

⁴³See the note to B155 for the same point regarding the concept of motion of an object.

⁴⁴Hardness is also mentioned in this section but seems to have a slightly different status for Newton than the other qualities.

from experience. Building on Newton's arguments, Kant can claim that those three qualities are empirically certain, which is a necessary (but not sufficient) condition for what Kant will call pure natural science.⁴⁵ The extension of bodies has a different status for Kant in light of his conception of pure intuition.

1.5.2 MAN as a "logic" of empirical physics

In this section, I articulate what I take to be Kant's framework for philosophical knowledge (to use Wolff's term), that is, his conception of what it means to "explain" empirical facts.⁴⁶ At the center is Kant's view on logic, what he calls a "particular" use of the understanding.

A natural first step is to get clear on Kant's distinction between "general" and "particular" logic. The following two passages are from introductory sections on logic in the Critique and the Jäsche Logic. He draws a distinction between general logic and particular logic and spells out what a particular logic, also characterized as an "organon", is:

Now logic in turn can be undertaken with two different aims, either as the logic of the general or of the particular [besonderen] use of the understanding. The former contains the absolutely necessary rules of thinking, without which no use of the understanding takes place, and it therefore concerns these rules without regard to the difference of the objects to which it may be directed. The logic of the particular [besonderen] use of the understanding contains the rules for correctly thinking about a certain kind of objects. The former can be called elementary logic, the latter, however, the *organon* of this or that science. In the schools the latter is often stuck before the sciences as their propaedeutic, though in the course of human reason they are certainly the latest to be reached, *once the science is already long complete, and requires only the final touch for its improvement and perfection*. For one must already know the objects rather well if one will offer the rules for how a science of them is to be brought about. (A52/B76-7, my emphasis)

By an organon we understand, namely, a directive as to how a certain cognition is to be brought about. This requires, however, that I already be acquainted with the object of the cognition that is to be produced according to certain rules. An organon of the sciences is thus not mere logic, because an organon presupposes exact acquaintance

⁴⁵See MAN, 4:468.

⁴⁶Kant's critical conception of apriority is approached through the perspective of German rationalism also by Burge (2000), Smit (2009), Hebbeler (2015), Stang (2019), and Messina (2022), for instance. My treatment of this issue differs from previous authors in that I distinguish two critical conceptions of apriority, as well as emphasize (in sect. 1.6) the influence of Kant's distinct conception of reason from the *Dialectic* for the material a priori.

with the sciences, their objects and sources. [...] It is only a universal art of reason [allgemeine Vernunftkunst] (canonica Epicuri) for making cognitions in general conform to the form of the understanding in general, and hence is only to this extent to be called an organon, which serves of course merely for passing judgment and for correcting our cognition, but not for expanding it. (Logik, 9:13)

General logic contains the “absolutely necessary rules of thinking”. Kant clarifies this by saying that general logic ignores the “difference of the objects”.⁴⁷

A particular logic expresses the rules for “correctly” thinking about a particular kind of object. To develop a particular logic requires that the respective object is already fully known. It presupposes an “exact acquaintance with the sciences, their objects, and sources”, that the “science is already long complete”, says Kant. The main function of such a logic is to “improve” and “perfect” existing scientific cognitions, not to expand them. It involves an investigation of how scientific cognitions “conform to the form of the understanding in general”.

With this conception of a particular logic in mind, I now turn to the *Preface* of MAN to show that the highlighted features are indeed essential aspects of Kant’s approach. Kant in the first paragraph clarifies the particular object as “objects of the outer senses” (4:467). More specifically, he wants to systematize “all things, insofar as they can be objects of our senses” (ibid.), the “whole of all appearances, that is, the sensible world” (ibid.). Those types of basic objects (or rather, concepts on Kant’s thinking about extension) are provided by empirical science, in our specific case, physics. Although Kant initially considers also empirical psychology, he eventually argues that it cannot be “systematized” in the way that physics can.⁴⁸

Kant’s overall aim is then to express the objects of empirical physics in a systematic way. For Kant, a system is a “whole of cognition ordered according to principles, is called a science” (4:467). A further desideratum for the kind of system that Kant intends is that the connection of cognitions is an “interconnection of grounds and consequences” (4:468), what he calls a “rational science”. Finally, Kant intends a rational science that has grounds or principles that carry with them a “consciousness of their necessity” (ibid.), are “cognized a priori” (ibid.). Such a rational science

⁴⁷ See also A54/B78, A55/B80 on this point.

⁴⁸ See MAN, 4:471.

is called “pure” or “proper” by Kant and contains only “apodictically certain” cognitions (ibid.).⁴⁹

So much for Kant’s explicit characterization of his aims. How does he think he can achieve a system that satisfies those very high standards? At the most fundamental level, his suggestion is to compare the concepts of empirical physics with the form of the pure understanding. He thinks that such an approach allows us to recognize a layer of empirical objects/concepts that reflects the unifying acts of the pure understanding.⁵⁰ Those kinds of representations satisfy the desideratum that they carry a consciousness of necessity with them.

The following three passages from the Preface indicate such a comparative approach:

1) Hence all natural philosophers who have wished to proceed mathematically in their occupation have always, and must have always, made use of metaphysical principles (albeit unconsciously) [...] Undoubtedly they have understood by the latter the folly of contriving possibilities at will and playing with concepts, which can perhaps not be presented in intuition at all, and have no other certification of their objective reality than that they merely do not contradict themselves. All true metaphysics is drawn from the *essence of the faculty of thinking itself*, and is in no way fictitiously invented on account of not being borrowed from experience. Rather, it contains the *pure actions of thought*, and thus a priori concepts and principles, which first bring the manifold of empirical representations into the law-governed connection through which it can become empirical cognition, that is, experience. (MAN, 4:472, my emphasis)

2) However, [in this second case] it is then not a general, but a special [besondere] metaphysical natural science (physics or psychology), in which the above transcendental principles are applied to the two species of objects of our senses. (MAN, 4:470)

3) in metaphysics the object is only considered in accordance with the general laws of thought [...] the object has to be compared always with all the necessary laws of thought. (MAN, 4:473)

In his “particular” logic/metaphysics of natural science,⁵¹ Kant wants to compare the empirical concept matter with the necessary concepts and principles of the understanding from the *Analytic*,

⁴⁹The apodictic certainty of Kant’s a priori cognitions in MAN, on my view, needs to be understood in terms of construction in pure intuition. This issue will be discussed in depth in Chapter 2 and 3.

⁵⁰See Logik, §5 for Kant’s view on the “origin of concepts as to mere form”. This kind of investigation is characterized for Kant by the question: “Which acts of the understanding constitute a concept?” (9:93). It relies on “reflection and on abstraction from the difference among things that are signified by a certain representation” (ibid.).

⁵¹Kant, just as we would expect in light of his characterization of a “particular” use of the understanding, tends to use the term “besondere”, translated as “particular” or “special”, in the Preface: “a particular nature [besonderen Natur] of this or that kind of thing, for which an empirical concept is given” (MAN, 4:470); “special metaphysical natural science [besondere metaphysische Naturwissenschaft]” (MAN, 4:470); “special doctrine of nature [besonderen Naturlehre]” (MAN, 4:470); “to place each in a special system [besonderes System] so that it constitutes a science of its own kind” (MAN, 4:472-3). See also MAN, 4:477, 478.

as the three passages suggest. In light of my previous argument (in sect.1.4), it should be clear that Kant's "transcendental principles" from the *Analytic* are not susceptible to the concern he raises in the first half of the first passage, that the metaphysical principles may not be applicable to intuition.

Let me add two thoughts to further clarify Kant's suggestion to compare the empirical concept matter with all the necessary laws of thinking. First, we should recall that Kant's systematic representation of the concept matter in *MAN* perfects existing empirical knowledge, it does not extend empirical knowledge.⁵² The following comments from the *Preface* indicate this concern with distinctness and perfection:

All determinations of the general concept of a matter in general must be able to be brought under the four classes of [pure concepts of the understanding] [...] There is no more to be done, or to be discovered, or to be added here, except, if need be, to improve it where it may lack in distinctness [Deutlichkeit] or exactitude [Gründlichkeit]. (MAN, 4:475-6, translation modified)

In this treatise, although I have not followed the mathematical method with thoroughgoing rigor (which would have required more time than I had to spend thereon), I have nonetheless imitated that method—not in order to obtain a better reception for the treatise, through an ostentatious display of exactitude, but rather because I believe that such a system would certainly be capable of this rigor, and also that such perfection [Vollkommenheit] could certainly be reached in time by a more adept hand. (MAN, 4:478)

Those comments indicate that his objective is to make existing cognitions (more) "distinct" or "perfect". To consider the table of categories can be understood as a heuristic in this procedure. It helps to isolate those parts of the concept matter that are constituted by the acts of the pure understanding.

⁵²Cf.: "one finds that metaphysics has busied so many heads until now, and will continue to do so, *not in order thereby to extend natural knowledge (which takes place much more easily and surely through observation, experiment, and the application of mathematics to outer appearances)*" (MAN, 4:477, my emphasis).

Secondly, we can “test” candidate representations by applying the law of contradiction in the distinct way Kant defends it in the Critique. Consider the following formulation:

But one can also make a positive use of it, i.e., not merely to ban falsehood and error (insofar as it rests on contradiction), but also to cognize truth. For, if the judgment is analytic, whether it be negative or affirmative, its truth must always be able to be cognized sufficiently in accordance with the principle of contradiction. *For the contrary of that which as a concept already lies and is thought in the cognition of the object is always correctly denied, while the concept itself must necessarily be affirmed of it, since its opposite would contradict the object.* (A151/B190-1, my emphasis)

On Kant’s special conception of the principle of contradiction, it enables us to draw out the necessary components of existing empirical cognitions. As I will explain in more detail in Chapter 3, the principle of contradiction is essential for a recognition of Kant’s explications in MAN. Since those explications function as the first principles/definitions of Kant’s system, the principle of contradiction plays a fundamental role in Kant’s view that material a priori cognitions are necessary. His explanation for the necessity of synthetic judgments in MAN is more complex and derivative from his explanation of the necessity of the explications.⁵³

To further illustrate those two features, I want to discuss Kant’s explication of motion from the *Phoronomy*. We start from the assumption that the empirical concept matter consists in part of the concept of relative motion. Now, in light of Kant’s view on pure intuition and the categories, it is necessary that bodily motion as presented in empirical intuition is constituted by a “pure part”, that is, the trajectory of the object in space.⁵⁴ The trajectory in pure intuition is conceptualized in terms of the category of quantity, the addition of homogeneous units in space (henceforth: subjective motion). Subjective motion is hence rectilinear and uniform.⁵⁵ This kind of subjective motion is

⁵³Just as for apriority, it appears that Kant’s conception of necessity in MAN is different from the one in the first Critique and resembles Leibniz’s thinking on this issue. I cover Leibniz’s view in the next section.

⁵⁴To complete this proposal, we also need Kant’s notion of absolute space. I will turn to this final element in Sect. 1.6.

⁵⁵Other kinds of motion like rotation also depend at least in part on subjective motion because they can be analyzed in part through rectilinear motions.

necessary for the following reason. To state the opposite, that is, that there is no trajectory in space, is to reject the motion of the object in empirical intuition, which is impossible. So the necessity that is involved here is relative to the assumed empirical concept.

To summarize, Kant's "particular" logic of natural science consists of the necessary components of the empirical concept matter. It results from a comparison of the concept matter with the necessary laws of the understanding from the Analytic. Kant's proposed "philosophical" knowledge is therefore epistemically secondary to empirical theorizing. The argumentative structure that is associated with the Constitutivity Thesis—that the concept matter is epistemically primary relative to empirical knowledge—is hence false.

1.5.3 Leibniz's truths of reason

Based on what I have said so far, it is natural to think that Kant's material a priori has continuities with Leibniz-Wolff. I want to first highlight the core features of Leibniz's account and eventually state similarities and dissimilarities with Kant.

Just as we saw above for Wolff, Leibniz's conception of truth involves two distinct layers. First, a factual layer and second, a rational layer. Let's start with the factual layer. Imagine you watch someone performing a Euclidean proof on a piece of paper. On Leibniz's view, this would establish sensible knowledge that a certain figure, say a triangle, has a certain property. At the rational level, Leibniz requires that the connection between subject and predicate is also established through a proper proof. For this, we would rely only on universal propositions like definitions, axioms and theorems. Diagrams cannot be invoked according to Leibniz since they are not universally valid.⁵⁶

Here is a passage from the *Preface of Leibniz's New Essays on Human Understanding* (NE),⁵⁷

⁵⁶Cf. Leibniz (NE, pp.360-1): "But I do not agree with what seems to be your view, that this kind of general certainty is provided in mathematics by 'particular demonstrations' concerning the diagram that has been drawn. You must understand that geometers do not derive their proofs from diagrams, although the expository approach makes it seem so. The cogency of the demonstration is independent of the diagram, whose only role is to make it easier to understand what is meant and to fix one's attention. It is universal propositions, i.e. definitions and axioms and theorems which have already been demonstrated, that make up the reasoning, and they would sustain it even if there were no diagram".

⁵⁷There is reason to think that reading this book around 1769 was Kant's first direct contact with Leibniz's thought (see p.xiii in the introduction by the editors of the *New Essays*).

where he summarizes this position:

necessary truths, such as we find in pure mathematics and particularly in arithmetic and geometry, must have principles whose proof does not depend on instances nor, consequently, on the testimony of the senses, even though without the senses it would never occur to us to think of them. This distinction must be thoroughly observed, and Euclid understood that so well that he demonstrates by reason things that experience and sense-images make very evident. Logic also abounds in such truths, and so do metaphysics and ethics, together with their respective products, natural theology and natural jurisprudence; and so the proof of them can only come from inner principles, which are described as innate. It would indeed be wrong to think that we can easily read these eternal laws of reason in the soul [...] without effort or inquiry; but it is enough that they can be discovered within us by dint of attention: the senses provide the occasion, and successful experiments also serve to corroborate reason. (NE, p.50)

Leibniz emphasizes a twofold approach in regards to necessary truths. Through empirical investigation, we first learn about connections between physical objects and their properties. In the case of Euclidean geometry, we get the idea that certain geometrical figures have certain properties by studying diagrams. “[E]xperience and sense-images” make those Euclidean connections “very evident”, says Leibniz.

The rational layer makes patent the underlying “eternal laws of reason”. This kind of justification is supposed to be independent of the sequence of discoveries, which obviously differs among knowers. We are rather concerned with “the connection and natural order of truths, which is always the same” (NE, p.412). In the *Monadology*, Leibniz articulates the basic features of those necessary truths:

There are also two kinds of truth: those of reasoning, and those of fact. Truths of reasoning are necessary, and their opposite is impossible; those of fact are contingent, and their opposite is possible. When a truth is necessary, the reason for it can be

found by analysis, by resolving it into simpler ideas and truths until we arrive at the basic ones. [...] Thus mathematicians use analysis to reduce speculative theorems and practical canons to definitions, axioms, and postulates. (§§33-4)

Necessary truths are founded on the principle of contradiction (only), their opposite is impossible.⁵⁸ We recognize the truth of those propositions through an analysis of the subject concept, through a demonstration that the predicate is contained in the subject concept.

To work towards a comparison with Kant, it is useful to see how Leibniz applies this approach to natural philosophy in his *Specimen Dynamicum* (SD).⁵⁹ In particular, SD is relevant for my purposes because in it, Leibniz is concerned in his own way with a “science” of physics, i.e. with proper knowledge that originates in secure principles in regards to body/matter.⁶⁰

Empirical physics for Leibniz, in essence, deals with relative motions of bodies.⁶¹ The thought is to conceive motion in terms of changes of relative positions of bodies. On Leibniz’s conception of empirical knowledge, it is not determinate (in general) which bodies specifically we should attribute motion and which rest if their relation to one another changes.⁶² As a result, it is not certain whether or not the predicate motion is contained in the respective subject (concept); there can be only truths of fact or contingent knowledge in this domain.⁶³

Leibniz’s notion of force as developed in the SD is meant to address this shortcoming by attributing forces to bodies:

I recognize that all corporeal action arises from motion, and that motion itself

⁵⁸See also *Monadology*, §31.

⁵⁹The first part (i.e., par.1-34) of the *Specimen* was published in the journal *Acta Eruditorum* (Apr. 1695). It is therefore plausible to say that Kant could have known this work of Leibniz.

⁶⁰See SD, par.1, 5, 17-20.

⁶¹See SD, par.9 for Leibniz’s claim that all other material phenomena can be understood in terms of relative motions.

⁶²See DM, §18.

⁶³Garber’s (1995) illuminating account of Leibniz’s physics influenced my thinking. He (pp.305-8) makes the same point on the relationship between Leibniz’s notion of motion and force. Most of the details of Leibniz’s view on physics are not really relevant for my purposes because my goal is eventually to clarify Kant’s view, which is embedded in a more Newtonian conceptual framework. What is relevant for me here is Leibniz’s mainly implicit conception of secure knowledge or science with respect to physics. As for Wolff, I take it that Leibniz’s science of dynamics is epistemically secondary to empirical physics, it reflects a later stage in the progression of knowledge. In this respect, Garber emphasizes different aspects than I do, especially the ways in which Leibniz’s laws of motion—despite Leibniz giving “demonstrations” for them—should be taken as contingent (see pp.319–21).

comes only from other motion, either already in the body or impressed from outside. But when we analyse it, motion, like time, does not really exist [...] Thus there is nothing real in motion but the momentary state which a force endowed with an effort for change must produce. Therefore, whatever there is in corporeal nature besides the object of geometry, or extension, reduces to this. (SD, par.3)

So we analyze motions in terms of forces that physical bodies contain.⁶⁴ It is key to recognize that at this level of analysis, we can attribute actions/motions to specific bodies: “the force or immediate cause behind those changes is something which is more real, and there is enough of a basis for attributing it to one body rather than to another; and it is, moreover, only by this that we can know to which one the motion better belongs” (DM, §18).⁶⁵ The consequence appears to be that we can derive necessary knowledge that grounds the empirical knowledge (such as the laws of motion) by drawing on causes: “my dynamics, which gives an account of the laws of nature and of motion which is true and in accordance with the facts” (NI, par.7).⁶⁶ In short, Leibniz gives a causal grounding of empirical physics in terms of a notion of derivative force that inheres in bodies.⁶⁷

To transition to Kant’s view, it is essential to recognize the following key difference. Kant maintains that sensibility is an independent faculty. So Leibniz’s story that confused sensible representations are made distinct through reflection needs modification. Kant clearly articulates this difference with respect to Leibniz:

The conditions of sensible intuition, which bring with them their own distinctions, [Leibniz] did not regard as original; for sensibility was only a confused kind of representation for him, and not a special source of representations. (A270/B326)⁶⁸

⁶⁴See SD, par.2: forces are “implanted” in bodies, they constitute “the inmost nature of bodies”.

⁶⁵Adams (1994, Ch.9 & 13, esp. pp.225-32, 253-261) provides a useful explanation of Leibniz’s talk of different levels of reality of corporeal phenomena in connection with his notion of force. My focus in the quoted passage is the conceptual or epistemic dimension of Leibniz’s view, not his metaphysical doctrines.

⁶⁶See also SD, par.8: “we must now move on to deal with the theory of derivative powers and resistances, showing how bodies act on and resist each other to differing extents in virtue of their different levels of effort. The laws of action which deal with these things must not only be understood rationally, but must also be confirmed by experience of the phenomena”.

⁶⁷The relation between MAN and Leibniz’s *Specimen* will be discussed from another angle in Chapter 2.

⁶⁸Here is how this passage continues: “for him appearance was the representation of the thing in itself, although

Kant recognizes sensibility as a “source of representations”. The overall move towards “logical perfection” thus has a distinct meaning for Kant. It involves isolating those representations of the empirical concept matter that have a “general” (or “intuitive”) form from those that have a “particular” form.⁶⁹ This is the (only) sense in which the former are more “distinct”.

In general, Kant relies here on the following distinction: “With every concept we are to distinguish matter and form. The matter of concepts is the object, their form universality” (Logik, §2). The spatial/intuitive determinations of the concept matter provide its “form” (MAN, 4:481). The “matter” of the concept matter is “that in the outer intuition which is an object of sensation, and thus the properly empirical element of sensible and outer intuition, because it can in no way be given a priori” (ibid.). So, Kant makes the empirical concept matter distinct through a separation of its form from its matter, as he clarifies in the *Preface* of MAN:

Therefore, a complete analysis of the concept of a matter in general will have to be taken as the basis, and this is a task for pure philosophy – which, for this purpose, makes use of *no particular experiences*, but only that which it finds in the isolated (although intrinsically empirical) concept itself, *in relation to the pure intuitions in space and time* (in accordance with laws that already essentially attach to the concept of nature in general) and is therefore a genuine metaphysics of corporeal nature. (MAN, 4:472, my emphasis, translation modified)

The suggested analysis of the concept matter abstracts from all “particular experiences”, what he later calls “objects of sensation”.⁷⁰ Kant characterizes the form of the concept as “that which it finds in the isolated (although intrinsically empirical) concept itself, in relation to the pure intuitions in

distinguished from cognition through the understanding in its logical form, since with its customary lack of analysis the former draws a certain mixture of subsidiary representations into the concept of the thing, from which the understanding knows how to abstract. In a word, Leibniz intellectualized the appearances” (A270-1/B326-7).

⁶⁹See A43/B60-1 where Kant says “The difference between an indistinct and a distinct representation is merely logical, and does not concern the content”. Kant would hence disagree with Leibniz’s thought (as quoted above from SD, par.3) that the analysis of motions can reveal that they do not really exist, that, ultimately, there are only forces. That is, Leibniz seems to hold the view that the difference between indistinct and distinct representations concerns the content of the representation.

⁷⁰Kant’s conception of analysis and the precise nature of those intuitive determinations of the concept matter will be discussed in Chapter 2.

space and time”. Subjective motion as treated in sect. 1.5.2—uniform, rectilinear motion of a point in pure intuition—is the most fundamental instance of the form of the concept matter.

This leads to the following first take on the material a priori, which Kant glosses in the *Preface* as: “to cognize [...] from [...] mere possibility” (MAN, 4:470).

[MATERIAL] A representation is known materially a priori if it consists of the necessary intuitive features of the empirical concept matter. Example: subjective motion.

This proposal will now be further refined.

1.6 Kant’s truths of reason

I have suggested so far that, for Kant, motion in empirical intuition has a necessary component, that is, the trajectory in space. However, this is not yet a proper *Anfangsgrund* [foundational ground] for Kant because the space in which the object is perceived is material and can itself move. This leads to the following thought: “a movable space, if its motion is to be capable of being perceived, presupposes in turn an enlarged material space, in which it is movable; this latter presupposes in precisely the same way yet another; and so on to infinity” (MAN, 4:481). Kant’s solution is to postulate a distinct conception of absolute space: “that in which all motion must finally be thought (and which is therefore itself absolutely immovable) is called pure, or also absolute space” (MAN, 4:480). I want to explicate this move because it speaks to what Kant takes to be the fundamental *Anfangsgrund* in regards to motion.

1.6.1 The blueprint for *Anfangsgründe* in the Dialectic

Reason is the faculty of principles according to Kant. In light of the very frequent and central use of the term “principle” [Prinzip] in MAN, especially in the *Preface*, a discussion of Kant’s faculty of reason is natural. In particular, we should keep in mind Kant’s definition of a system: “a whole of cognition ordered according to principles” (MAN, 4:467). Recall also that the stage of the hierarchy of logical perfection that we get after cognition through concepts is to “cognize something through reason, or to have insight into it” (Logik, 9:65).

Kant articulates his account of reason in the *Dialectic*. My upcoming discussion has a limited scope in this regard. It highlights only aspects that reflect Kant's positive and final account of principles with respect to pure intuition. I will not discuss Kant's development of the view nor his illuminating critique of other philosophers. That is because my goal is only to clarify the nature of the principles in MAN.

At the center of Kant's positive view of reason is a "principle of reason in general".⁷¹ Here is what I take to be Kant's final version in regards to cosmological ideas like the beginning of the world in space:

the principle of pure reason we are thinking of retains its genuine validity only in a corrected significance: not indeed as an axiom for thinking the totality in the object as real, but as a problem for the understanding, thus for the subject in initiating and continuing, in accordance with the completeness of the idea, the regress in the series of conditions for a given conditioned. For in sensibility, i.e., in space and time, every condition to which we can attain in the exposition of given appearances is in turn conditioned, because these appearances are not objects in themselves in which the absolutely unconditioned might possibly occur, but only empirical representations, which must always find in intuition their condition, which determines them as regards space or time. [...] thus it is a principle of reason which, as a rule, postulates what should be effected by us in the regress, but does not anticipate what is given in itself in the object prior to any regress. (Section: "The regulative principle of pure reason in regard to the cosmological ideas", A508-9/B536-7)

Kant characterizes the principle of reason as a rule for the "regress in the series of conditions for a given conditioned" and tends to use the term "condition" where Leibniz would say "reason". To put the thought in more Leibnizian terms, Kant describes his own approach to the task of accounting for the reason of a necessary proposition through "analysis". For conditioning relations in regards to space and time, Kant observes that every further condition (in the ascending series) we find is

⁷¹The initial version is stated at A307-8/B364.

itself an “empirical representation”, that is, a representation that has a spatiotemporal form. It is thereby itself conditioned in space and time.

Kant’s intended principles have the form of a “problem” for the subject, they postulate “what should be effected *by us* in the regress” (my emphasis). What guides this regress is the “completeness of the idea”, says Kant. To get clearer on this, let’s look at Kant’s corresponding notion of an idea:

Thus the idea of reason will only prescribe a rule to the regressive synthesis in the series, a rule in accordance with which it proceeds from the conditioned, by means of all the conditions subordinated one to another, to the unconditioned, even though the latter will never be reached. (A510/B538)

So, what an idea does is to specify how the subject is supposed to continue the regress. Kant’s thought is that the subject “believes” that a condition in intuition can be found for every conditioned.⁷² This belief has a subjectively sufficient ground because of the form of the pure intuition of space as an infinite given magnitude.⁷³ That is, in light of Kant’s notion of pure intuition, it is certain that it is not a contradiction to think that there is a condition in intuition for every conditioned. The subject hence performs a regress by assuming that there is a spatial condition for every spatial conditioned. This leads us naturally to Kant’s view on absolute space.

1.6.2 Kant’s absolute space

Here is how Kant describes absolute space in the *Phoronomy*:

Absolute space is thus in itself nothing, and no object at all, but rather signifies only any other relative space, which I can always think beyond the given space, and which I can only defer to infinity beyond any given space, so as to include it and suppose it to

⁷²See A470-1/B498-9 where Kant says that we have “belief” in the completeness of the idea. Believing for Kant amounts to “holding-to-be-true based on a ground that is objectively insufficient but subjectively sufficient, relates to objects in regard to which we not only cannot know anything but also cannot opine anything, indeed, cannot even pretend there is probability, but can only be certain that it is not contradictory to think of such objects as one does think of them” (Logik, 9:67).

⁷³See A25/B39.

be moved. [...] To make this into an actual thing is to transform the logical universality of any space with which I can compare any empirical space, as included therein, into a physical universality of actual extent, and to misunderstand reason in its idea. (MAN, 4:481-2)

Absolute space is an idea for Kant. It signifies “only any other relative space, which I can always think beyond the given space” and captures “the logical universality of any space with which I can compare any empirical space”. So absolute space somehow involves the representation of a universality. To think of absolute space is to think of a space that encompasses *any* given space, without any restriction or condition.⁷⁴ It is hence the *idea* of absolute space that is “complete” or unconditioned. As a consequence, all members in the regressive series of conditions for any kind of relative motion can/are “given” by means of absolute space. To have such a universal condition establishes for Kant that subjective motion can be reached by reason, that is, is possible a priori:

For since in the [ascending series of conditions] the cognition (the conclusio) is given only as conditioned, we cannot reach it by means of reason except at least on the presupposition that all members of the series are given on the side of the conditions (totality in the series of premises), because only under this presupposition is the judgment before us possible a priori. (A331/B388)

This leads me to the following refined conception of material apriority:

MATERIAL* A judgment is known materially a priori if it can be analyzed in terms of a “complete” series of intuitive conditions. Example: the explication of matter in terms of subjective motion thought in absolute space.

1.6.3 Comparison with Buchdahl’s view

Since my proposal has on the surface similarities with Buchdahl’s approach, it is useful to compare the two views. On Buchdahl’s (1969, chapter VIII) reading, there is a considerable “looseness of

⁷⁴Kant’s understanding of “absolute” is the following: “It is in this extended meaning that I will make use of the word absolute, opposing it to what is merely comparative, or valid in some particular respect; for the latter is restricted to conditions, while the former is valid without any restriction” (A326/B382).

fit” between the *Analytic* and Kant’s view of natural science.⁷⁵ This looseness reflects the difference between the understanding and reason. In essence, the two faculties support distinct types of necessity: the faculty of the understanding in terms of conceptual presuppositions of experience or constitution. The faculty of reason to the extent that propositions are part of “theoretical systems, involving the addition of theoretical concepts in no way reducible to the pure observational basis of the empirical domain of science” (p.476). Kant’s pure natural science, on Buchdahl’s reading, depends on those two conceptions.⁷⁶

I agree with Buchdahl that making explicit the contribution of the different cognitive faculties is essential in order to understand apriority in Kant. Moreover, there are indeed distinct conceptions of apriority. However, what Buchdahl underestimates, in my view, is that the *progression* of knowledge has a formative role for Kant on this issue. My argument also suggests that we need a further conception of apriority, in addition to the transcendental one and what Kant grants to empirical sciences like chemistry that invoke ideas of reason.⁷⁷

1.7 Conclusion

I defended the view that there is a distinct conception of apriority in MAN. It was key to show how the *Analytic*, empirical (i.e., Newtonian) physics, and pure natural science reflect different stages in Kant’s suggested epistemic order. In particular, Kant’s pure natural science expresses the necessary elements of empirical science. It does so by isolating the intuitive features of the concept matter from the elements that belong to sensation. Those judgments are “a priori” in the sense that they can be reached through reason by a “complete” series of intuitive conditions on the side of the thinker.

⁷⁵See Buchdahl (1969, pp.475-6, 501-2, 512-4, 672-674).

⁷⁶See Buchdahl (1969, pp.475-6, 672-4).

⁷⁷See McNulty (2015) for such a view on the necessity of chemical laws in Kant.

2 Kant's account of construction

It remains clear that something can be a phenomenon for Kant by virtue of its appearing to the intellect and not to sensation—though of course it must appear in such a way as to be placed by the intellect within the spatiotemporal “form” of sensation.

- Robert M. Adams (1994, p.227)

2.1 Introduction

Kant's methodology in the *Metaphysical Foundations of Natural Science* (MAN) is puzzling, to say the least. One essential reason for this is that he attributes a central role to the “construction” of the concept of matter in pure intuition.⁷⁸ This is confusing at first and second glance because the concept matter is not a “mathematical” concept for Kant, and so, in light of his arguments in the first *Critique*,⁷⁹ not capable of being “constructed”. (Henceforth, I use the term “construction” without any modifier for Kant's conception of construction in MAN.)

Not surprisingly, there is a rather far-ranging disagreement in the literature on the sense of construction at play, especially on the overall function it plays in Kant's arguments. One influential approach is to argue that MAN in fact does not contain a single construction in pure intuition.⁸⁰ Views of this sort typically rely in some form on the assumption that construction coincides with mathematical construction. On this view, construction is an evidential notion. The other influential trend in the literature is to defend the constructability of the concept matter in a limited fashion. Typically, scholars defend it with respect to Propositions where Kant reasons diagrammatically and/or appeals to “laws of space”.⁸¹ Obvious examples are the Proposition in the *Phoronomy*, the inverse-square law in the *Dynamics*, and the Third law of motion in the *Mechanics*. At the same time, those readings emphasize that certain other features of matter, say the actual forces themselves, or the “real” content of the laws, cannot be constructed.

⁷⁸See especially 4:470-72.

⁷⁹See A729/B757.

⁸⁰Peter Plaass ([1965] 1994) and Marius Stan defend versions of this view.

⁸¹Eckart Förster (2000), Michael Friedman (2013) and James Messina (2022) defend versions of this position.

While both approaches attribute to Kant philosophically interesting views, they lack exegetical plausibility because they have to explain away the clear methodological comments that Kant makes in the *Preface*. Kant emphasizes there that MAN is formulated according to the “mathematical method” and involves construction in intuition across-the-board.⁸²

The following chapter aims to resolve the puzzle that

- construction in pure intuition is Kant’s suggested method in MAN,
- but only few of Kant’s actual arguments in MAN, if at all, are diagrammatic.

My solution to this puzzle will go like this. I argue that Kant’s conception of geometrical construction serves as his *model* for isolating constructable component concepts that function as “inner” (i.e., explanatory) determinations of the concept matter. What construction has in common with mathematical construction is that it captures marks through which the object of the concept can be *securely* cognized. However, construction does not function to establish that the concept matter is “objectively real” prior to empirical cognition, as mathematical construction does, because the concept matter is known through experience. Rather, Kantian construction should be seen as a modification of Leibniz’s explanatory approach. Accordingly, Kant’s “transcendental reflection” on the concept matter isolates those features of the empirical concept matter that appear to the “intellect and not to sensation [...] placed by the intellect within the spatiotemporal “form” of sensation”, to use Adams’s formulation (as quoted above).

To see this, I will clarify how the *Amphiboly* provides Kant’s philosophical motivation for construction in MAN. We will see that Kant shares with Leibniz the idea that reflection starts with the material that empirical investigation provides. Kant, however, departs from Leibniz’s approach because for him, reflection draws also on the original representations from sensibility, not only those from the pure understanding. Kant’s transcendental reflection on the concept matter therefore results in spatially encoded determinations of matter, as opposed to non-relational determinations like Leibnizian force. This manifests one sense in which Kant’s theoretical philosophy amounts to

⁸²See especially 4:470-72, 4:478.

a qualification of Leibniz's view.⁸³

A key advantage of my reading is that it attributes to Kant a consistent view. It is consistent because it entails that each and every Explication and Proposition in MAN necessarily relies on construction. Kant, in essence, follows Christian Wolff's programmatic suggestion that the proper philosophical method imitates the mathematical method with respect to apodictically certain knowledge.⁸⁴ Kant's construction of the concept matter in MAN is his distinct contribution to this research program. In fact, in light of the status of pure intuition and the suggestion that we can only cognize appearances, Kant has a clear path for how to establish the apodictically certain parts of natural science. So his appeal to the mathematical method and construction in pure intuition as a *general* methodological feature of MAN can in fact be supported. As a consequence, I say that Kantian construction provides an original way of doing "mathematical" natural philosophy; he is not assimilating pure natural science to mathematics, as commentators tend to read him.

I first discuss the literature on construction in MAN, focusing on what Kant's philosophical objective is supposed to be (Sect. 2.2). After that, I work towards a more Leibnizian way of understanding the motivation for construction through a discussion of Leibniz's use of reason in mathematics (Sect. 2.3). I then discuss Kant's conception of geometrical construction in depth because it serves as his model for establishing the secure components of the concept matter (Sect. 2.4). With this model in hand, I turn to his motivation for construction in the *Amphiboly* and explicate his distinct account of analysis, i.e. transcendental reflection (Sect. 2.5). I finally put this account to use and clarify the nature of Kant's constructions in the *Phoronomy* (Sect. 2.6).

2.2 Literature

If there is one assumption about construction that existing readings have in common, then it is the idea that construction has the same characteristics and/or the same function that Kant attributes to construction regarding mathematics.

⁸³See, for instance, Langton (1998), Ameriks (2003), and Jauernig (2008) for context and related senses in which Kant's theoretical philosophy can be interpreted as a modification of Leibniz's theoretical philosophy. My argument mainly concerns the conceptual or epistemic dimension of the relation between Leibniz and Kant.

⁸⁴Wolff highlights this point, for instance, in his *Preliminary Discourse on Philosophy in General*.

As we will see below in more detail, mathematical construction according to Kant exhibits the object of the mathematical concept, say a triangle, a priori in pure intuition. For Kant, the overall function of mathematical construction is to establish with apodictic certainty that the object of the mathematical concept is possible independently of experience.⁸⁵ Accordingly, an influential trend in the literature is to suggest that construction in MAN is Kant's attempt to establish the objective reality of the concept matter (or, similarly, of its partial concepts motion and impenetrability) independently of experience.⁸⁶

In this fashion, Michael Friedman (2013), for example, argues on the one hand that Kant intends to show that the partial concept motion can be mathematically constructed in pure intuition.⁸⁷ The partial concept treated in the *Dynamics*, i.e. the filling of space or impenetrability, on the other hand, cannot be mathematically constructed.⁸⁸ Friedman says that the “real possibility of a fundamental force – whether repulsive or attractive – can never be established a priori by mathematical construction. And he also holds, for the same reason, that his preferred dynamical concept of matter [...] cannot be constructed in pure intuition” (p.223). Friedman also recognizes that Kant demonstrates certain “purely geometrical” laws regarding the diffusion of attraction and repulsion, which depend on the ratios of geometrical shapes (p.224). Such exhibitions of the mathematical content of force laws in pure intuition, however, are not sufficient for Kant's overall purpose. The suggested mathematical constructions cannot “facilitate an a priori proof of the real possibility or objective reality of the dynamical concept of matter”, says Friedman (p.225).

James Messina (2022) is more optimistic than Friedman on the value of a construction of the mathematical content of laws/propositions in MAN. He seems to say such constructions are actually sufficient for Kant's overall purpose:

special metaphysics provides principles and elements of construction by analyzing the concept of matter into concepts of matter and then showing how those partial concepts admit of a sort of mathematical construction in principle. [...] In the case

⁸⁵See A729/B757.

⁸⁶See, for instance, Plaass (1994, pp.257-9, 279, 285), Förster (2000, pp.62-5), and Friedman (2013, e.g. pp.27-31).

⁸⁷See, e.g., Friedman (2013, p.31, 67).

⁸⁸See Friedman (2013, p.31 & pp.222-5).

of the movable in space, Kant shows in the *Phoronomy* chapter how this concept can be in principle constructed by showing how composite motion admits of construction in pure intuition. In the case of the movable insofar as it fills space, I think Kant is trying to show how it is in principle possible to construct the concept, insofar as it is in principle possible to construct a model and formal laws that explain how the mathematical content of the dynamical laws is both possible and necessary. (Messina (2022, pp.111-2))

The suggestion is that the constructability of the composition of motion and of the mathematical content of force laws establishes the constructability of the partial concepts treated in the *Phoronomy* and *Dynamics*. The underlying assumption is that the constructability of the laws of the partial concepts suffices for establishing the constructability of the partial concepts themselves.⁸⁹ On Kant's view on the function of mathematical construction, such a position would entail, I think, that the corresponding object of the concepts of matter can be given in pure intuition. That the latter is Kant's view can be doubted because the concept matter is empirical, its objective reality can only be established through experience.

It is in part for this reason that other scholars that take Kant to aim for a proper mathematical construction of the concept matter reject the cogency of his appeal to construction. Peter Plaass ([1965] 1994, p.258), for instance, associates Kant's mathematical construction of the concept matter with an a priori demonstration of its objective reality. He reaches the conclusion that a mathematical construction of the concept matter, and thereby an a priori proof of its objective reality, is not possible in Kant's system.⁹⁰ Here is his summary: "The MF [i.e., Kant's *Metaphysical Foundations*] does not contain any mathematical knowledge at all, no single construction in intuition" (p.314).⁹¹ On such an interpretation, Kant is rather confused about his method.

Here is what I think is missing in the literature. First, while philosophically stimulating, I think the discussed readings make Kant look very confused, especially in the *Preface* of MAN.

⁸⁹See Messina (2022, pp.105-6) for a defense of this assumption.

⁹⁰See Plaass (1994, pp.285-9, 314).

⁹¹Marius Stan (unpub. ms.) draws the same inference but articulates the position differently.

Kant attributes an essential role to the construction of the concept matter in pure intuition and the mathematical method.⁹² The discussed approaches do not come anywhere near explaining Kant's suggestion that there is construction across-the-board in MAN. Secondly, I want to suggest that Kant's *motivation* for construction can be understood well by seeing what it is meant to replace, that is, Leibniz's use of reason through "mere" concepts. On such a conception of Kant's overall objective, the function of construction is different from a defense of the objective reality of the concept matter.

Those two concerns set the agenda for the rest of this chapter. That is, I will motivate Kant's conception of construction through comparison with Leibniz. That will result in an account on which construction in pure intuition is essential to the arguments in MAN, not merely to the propositions where Kant reasons diagrammatically. It thereby supports Kant's comments on the centrality of construction.

2.3 Leibniz's mathematical reasoning

A proper account of construction in MAN, on my view, should include what construction is meant to modify or improve on, according to Kant. I want to suggest that through his conception of construction, Kant, in essence, proposes a modification to Leibniz's conception of truths of reason, what the latter also referred to as "acts of reflection".⁹³ To see this, I will first clarify Leibniz's use of reason with respect to mathematics and Kant's response to that. The mathematical example is crucial because it serves both thinkers as the benchmark for a conception of necessary knowledge.

Leibniz modeled his conception of proper knowledge on mathematics, especially arithmetic and geometry.⁹⁴ To illustrate this conception, I want to discuss his proof for the proposition that equiangular triangles have proportional sides from his *On Analysis Situs*.⁹⁵ Leibniz's proof builds centrally on an improved definition of sameness of form ("similarity"), which he takes to be better

⁹²See especially 4:470-72.

⁹³See, for instance, §§29-30 in Leibniz's *Monadology*.

⁹⁴See, e.g., *Monadology*, §§33-5; NE, p.50, pp.85-6.

⁹⁵Leibniz's writings on the foundations of geometry, what he called "analysis situs", were not publicly available during Kant's time. I nevertheless use them because Leibniz drew on those arguments in his (later) published essays, which were accessible to Kant. For discussion of this point and an overview of analysis situs, see De Risi (2013).

“adapted to mathematical investigation” than previous definitions (p.255). The definition is: “things are similar which cannot be distinguished when observed in isolation from each other” (ibid.). With this new definition in play, the proof of the proposition is straight-forward.

We consider two triangles whose angles are, respectively, equal.⁹⁶ To know the base and the two adjacent angles of a triangle is sufficient for our purpose: two triangles can be compared by comparing their bases and corresponding adjacent angles. We first note that the two bases, that is, the straight lines, are similar to one another. They cannot be distinguished when observed in isolation from one another. Second, the angles that are adjacent to the base are, respectively, the same. Therefore, there is no feature that can be observed in one triangle but not in the other when observing the triangles in isolation from each other. Such a proof relies only on universal propositions: definitions and axioms;⁹⁷ no appeal to particular diagrams or lengths is required.⁹⁸ It therefore establishes that equiangular triangles have proportional sides on Leibniz’s view.

Kant’s response to reasoning of this kind is well documented. He thinks that it is incapable of explaining what he calls “synthetic” a priori propositions; in essence, the axioms and theorems of Euclidean geometry. Those kinds of truths, so he thinks, cannot be known merely through clarification of concepts and unpacking definitions, as Leibniz does. Consider the following crucial passage where Kant compares Leibniz’s use of reason in mathematics with his own approach regarding the proof of Euclid I.32 (i.e., that the interior angles of a triangle add up to two right angles):

Give a philosopher the concept of a triangle, and let him try to find out in his way how the sum of its angles might be related to a right angle. He has nothing but the concept of a figure enclosed by three straight lines, and in it the concept of equally many angles. Now he may reflect on this concept as long as he wants, yet he will never

⁹⁶See *On Analysis Situs*, p.256.

⁹⁷One axiom Leibniz appeals to in his proof is: “things which cannot be distinguished through their determinations (or through data adequate to define them) cannot be distinguished at all” (p.256). Proper definitions for Leibniz are real definitions, they guarantee the possibility of the defined; see DM, §24.

⁹⁸In the *New Essays*, Leibniz denies that a proper mathematical demonstration depends on a particular diagram. He notes that the “cogency of the demonstration is independent of the diagram, whose only role is to make it easier to understand what is meant and to fix one’s attention. It is universal propositions, i.e. definitions and axioms and theorems which have already been demonstrated, that make up the reasoning” (NE, p.360). In his *On Analysis Situs* (pp.254-5), he remarks that the intended comparison of figures does not depend on quantity/magnitude.

produce anything new. He can analyze and make distinct the concept of a straight line, or of an angle, or of the number three, but he will not come upon any other properties that do not already lie in these concepts. But now let the geometer take up this question. He begins at once to construct a triangle. [...] In such a way, through a chain of inferences that is always guided by intuition, he arrives at a fully illuminating and at the same time general solution of the question. (A7I6-7/B744-5)

Kant observes that to prove the proposition, a “philosopher” would reflect on and unpack the concept triangle: she would analyze the marks straight line, angles, and number three. As we saw above, this is, more or less, Leibniz’s suggested approach to the proposition that equiangular triangles have proportional sides.⁹⁹ Kant’s response is that the proposition cannot be derived from this type of analysis of the marks of the concept. His alternative suggestion involves the construction of the concept triangle in pure intuition and inferences that are “always guided by intuition”.

2.4 Kant’s geometrical construction

I now present Kant’s conception of geometrical construction. As I will argue below, this *method* of bringing concepts to intuitions (rather than the resulting character of geometrical proof itself) serves as an inspiration for Kant’s proposed pure natural philosophy.¹⁰⁰ It is worth noting that Kant also talks about construction regarding arithmetic and algebra.¹⁰¹ I take geometrical construction to be Kant’s ideal case which is why I won’t say much about arithmetic and algebra. Geometrical construction is ideal because it establishes a priori fully general results (contrary to arithmetical constructions that establish only singular propositions)¹⁰² through the construction of the geometrical objects themselves (contrary to algebra, whose “symbolic” construction abstracts from the constitution of the object it studies)¹⁰³.

⁹⁹The key ingredient was an improved definition of sameness of form.

¹⁰⁰Cf. A724-5/B752-3.

¹⁰¹See B15-7, A164-5/B204-5, A239-40/B298-9, A717/B745, A720/B748, 4:370-1.

¹⁰²See A164/B204-5.

¹⁰³See A717/B745.

2.4.1 A first approximation on geometrical construction

Let me start with a detailed passage from the *Discipline of Pure Reason* chapter, which contains the gist of Kant's conception of geometrical construction:

But to construct a concept means to exhibit a priori the intuition corresponding to it. For the construction of a concept, therefore, a nonempirical intuition is required, which consequently, as intuition, is an individual object, but that must nevertheless, as the construction of a concept (of a general representation), express in the representation universal validity for all possible intuitions that belong under the same concept. (A713/B741)

Kant thinks that geometrical concepts like triangle can be constructed in pure intuition. What that means is that we combine the general representation of a triangle (i.e., a list of marks) with a representation of its associated object, i.e., the representation of a particular triangle in intuition.

We should note that Kant thinks that the constructed instance somehow stands in for all possible triangles. At first sight, it is a bit strange that Kant talks about the intuition that results from the construction of the concept as an individual object even though it captures all possible objects that belong under the concept, i.e. each and every triangle. Kant's own attempt to clarify the constructed object is to say it is the schema of the concept.¹⁰⁴ What this suggests is that we should not think of the constructed object as a representation of a particular triangle, say the one on p.317 of my copy of Euclid's *Elements*. It is better to think of it as a *rule* for drawing each and every triangle in a three-dimensional Euclidean space.¹⁰⁵

2.4.2 The nature and role of pure intuition in geometry

This brings us naturally to Kant's conception of pure intuition. As I will explain in this section, pure intuition is the most fundamental element of Kant's account of geometrical construction, at

¹⁰⁴Here is Kant: "mathematical cognition considers the universal in the particular, indeed even in the individual, yet nonetheless a priori and by means of reason, so that just as this individual is determined under certain general conditions of construction, the object of the concept, to which this individual corresponds only as its schema, must likewise be thought as universally determined" (A714/B742).

¹⁰⁵Friedman (2012) defends a view of this sort. See also Shabel (2006) on this.

least from the perspective of the faculty of sensibility. In particular, Kant presupposes the pure intuition of space in his conception of geometrical definitions and theorems.

For my purposes, it is key to have at least some working conception of the status and function of pure intuition with respect to geometry. To give a full account of the role of pure intuition in Kant's philosophy is, obviously, beyond the scope of my argument here. In order to shed some light on the status and function of the pure intuition of space with respect to geometry,¹⁰⁶ I want to focus on the third and fourth argument in *Metaphysical exposition of space*, the following *Transcendental exposition of space* (B40-1), and Kant's remarks on Kästner's *Treatises* (20:410-423).¹⁰⁷

What the passages in the first *Critique* and the remarks on Kästner have in common is that they distinguish a "metaphysical" and a "geometrical" conception of space. Metaphysics considers space as "original and only one (unitary) space" (20:419),¹⁰⁸ as "infinite, subjectively given space" (20:420), space is considered "in the way it is given" (20:419),¹⁰⁹ that is, "before all determination of it in conformity with a certain concept of object" (20:419).¹¹⁰ In a nutshell, metaphysical space for Kant is an immediate, singular representation of an infinitely extended three-dimensional space.

Geometrical space is "derived and hence there are (many) spaces" (20:419); geometrical spaces determine metaphysical space in accordance with a specific concept (20:419).¹¹¹ Therefore, "all [geometrical] spaces are only possible and thinkable as parts of one single space" (20:419). While geometrical operations and constructions involve at most a potential infinite (20:421), original metaphysical space involves an "actu infinitum" (20:421), an infinite in actuality.¹¹²

On such a view, (metaphysical) space cannot be accounted for in terms of a *conceptual* representation, but is what Kant calls an "intuition". The key observation is that metaphysical space is logically prior to geometrical spaces, which is why geometrical spaces are not constituents of

¹⁰⁶Cf. Friedman (1992, Ch.1 and 2), Parsons (1992), Carson (1997), Shabel (2006).

¹⁰⁷I use the English translation of Kant's remarks on Kästner's *Treatises* from the *Kantian Review* (2014, pp.305-14).

¹⁰⁸See also A25/B39.

¹⁰⁹Cf.: "the exposition is metaphysical when it contains that which exhibits the concept as given a priori" (A23/B38).

¹¹⁰Cf.: "The bare universal form of intuition called space is therefore certainly the substratum of all intuitions determinable upon particular objects, and, admittedly, the condition for the possibility and variety of those intuitions lies in this space" (Prol, 4:322).

¹¹¹See also A25/B39.

¹¹²Cf.: "space is represented as an infinite given magnitude" (A25/B39).

metaphysical space. This indicates that geometrical spaces cannot jointly make up metaphysical space, which is what we would expect if metaphysical space was capable of a conceptual form, that is, if it could be determined through its component concepts (of objects).¹¹³

A lot can be said about Kant's arguments in the Metaphysical exposition for the claim that metaphysical space is an intuition. Since Kant's conception of geometrical construction is my overall target, I do not want to discuss those arguments here in more depth but rather clarify pure intuition through a discussion of its role in Kant's thinking about geometrical definitions and theorems. So I want to turn now to the function of metaphysical space with respect to geometry.

Here is the core thought: to conceive of metaphysical space as an a priori intuition enables Kant to explain the possibility of Euclidean geometry. In both the Metaphysical exposition (followed in the B edition by the Transcendental exposition where Kant discusses geometry) and the Kästner treatise, I say Kant establishes a grounding claim between metaphysical space and geometry.¹¹⁴ To be more specific, Kant's conception of metaphysical space as an intuition—an immediately given, infinitely extended three-dimensional space—is supposed to explain why, or rather, how it is possible to construct geometrical concepts, why straight lines can be extended indefinitely (as Euclid's second postulate states), and so on. Kant articulates such a grounding view just before the Metaphysical exposition of space where he says that the Transcendental Aesthetic will show that there are “two pure forms of sensible intuition *as principles [Prinzipien] of a priori cognition*, namely space and time” (A22/B36, my emphasis).

Here is how I think the overall reasoning goes. In the Metaphysical exposition, Kant suggests a conception of space that can serve as a first principle with respect to Euclidean geometry. The next step is to show that and how this principle actually explains geometry, that is, we move from the ground to the grounded. Kant calls this move “transcendental exposition of the concept of space”:

I understand by a transcendental exposition the explanation of a concept as a principle from which insight into the possibility of other synthetic a priori cognitions can

¹¹³See A25/B39.

¹¹⁴Carson (1997, p.497) defends a similar view regarding the relationship between metaphysical space and geometry. Friedman (1992b, Ch.1 and 2) defends a different view on this.

be gained. For this aim it is required 1) that such cognitions actually flow from the given concept, and 2) that these cognitions are only possible under the presupposition of a given way of explaining this concept. (B40)

So, the thought is that the articulated conception of metaphysical space serves as a “principle” from which insight into the possibility of geometry, in particular the possibility of synthetic a priori cognitions, can be gained. In order to establish this, it needs to be shown that geometrical cognitions “actually flow from the given concept”. The details of that explanation are the focus of the next three sections.

2.4.3 Geometrical definitions

The definition is another fundamental ingredient of geometry for Kant. In order to understand the full scope and originality of Kant’s conception of definition, in particular how it provides the next step in his explanation of the syntheticity of geometry, it is helpful to start again with Leibniz. Recall my earlier discussion of Leibniz’s proof from *On Analysis Situs* for the proposition that equiangular triangles have proportional sides. In his reasoning, Leibniz is very clear on the key innovation: an improved, more general definition of the equality of form, or similarity. This case illustrates that mathematical definitions, for Leibniz, result from a clarification and reflection on the marks of a concept.

Kant’s conception of (mathematical) definition in both the first *Critique* and the *Logik* is formulated as a response to this Leibnizian conception of definition by analysis.¹¹⁵ What Kant calls “philosophical definitions” are “expositions of given concepts” (A730/B758) while his own mathematical definitions involve “constructions of concepts that are originally made” (ibid.). Accordingly, in “philosophy”, on Kant’s view, we start with the confused concept and work towards the definition; “in mathematics we do not have any concept at all prior to the definitions” (A731/B759). The former approach has the shortcoming that its “completeness [...] is never apodictically certain” (A730/B758); mathematical definitions, on the contrary, “come about synthetically, and therefore

¹¹⁵See A727/B755 ff., *Logik*, §§99-109.

make the concept itself” (A730/B758). Mathematical definitions are hence apodictically certain for Kant.

Here is how Kant formulates his resulting conception of definition:

to define properly means just to exhibit originally the exhaustive concept of a thing within its boundaries.

[note:]

Exhaustiveness signifies the clarity and sufficiency of marks; boundaries, the precision, that is, that there are no more of these than are required for the exhaustive concept; original, however, that this boundary-determination is not derived from anywhere else and thus in need of a proof, which would make the supposed definition incapable of standing at the head of all judgments about an object. (A727/B755)

Only mathematical concepts conform to this notion according to Kant.¹¹⁶ To see why that is, I want to work through the four types of concepts that Kant distinguishes. First, empirically given concepts like gold rely on experience for their boundary-determination. New observations can always reveal new properties, and hence, the boundaries of the concept are never stable.

Second, there are concepts that are given *a priori* like cause and substance, which cannot be defined either. Kant’s reasoning here resembles his view on Leibniz’s conception of definition (as discussed above): we can never be fully certain that the “definition” is exhaustive with respect to its object. On this view, we might say in regards to Leibniz’s definition of similarity in *Analysis Situs* that it is valid to a certain degree because it allows us to prove a number of propositions. However, for Kant, it can never be apodictically certain that the underlying analysis of the concept is complete (729/B757). The same goes for concepts like cause and substance.

Third, there are concepts that are empirical and “made” according to Kant, e.g. chronometer. In this case, the potential “definition” (i.e. the combination of marks) is not directly derived from experience or the understanding. Still, the component concepts (e.g., that concern the material of a watch) require experience to establish their possibility. Kant characterizes the issue in the following

¹¹⁶See A729/B757.

way: “the object and its possibility are not given through this arbitrary concept; from the concept I do not even know whether it has an object” (A729/B757). Accordingly, there cannot be a definition of the concept because the corresponding object cannot be exhibited a priori in pure intuition, as Kant’s “originality” condition demands.

Geometrical concepts satisfy this latter condition because their definition establishes the possibility of the object without proof. Kant says: “the object that [the mathematical concept] thinks it also exhibits a priori in intuition, and this can surely contain neither more nor less than the concept, since through the explanation of the concept the object is originally given” (A729-30/B757-8). Consider the class of all triangles. One natural way to make Kant’s view precise is to say that the definition amounts to a rule for determining “metaphysical space”, a rule for generating each and every triangle, which establishes with apodictic certainty the possibility of each and every triangle a priori.¹¹⁷ This can be seen as Kant’s explanation for why mathematical definitions “suffice for cognition of the object according to its inner determinations, [...] they present the possibility of the object from inner marks” (Logik, §106), i.e. that definitions are real.¹¹⁸ In short, Kantian mathematical definitions are rules according to which all instances of the concept can be constructed in metaphysical space, which means they establish the possibility of the object that corresponds to the concept with apodictic certainty. Such a conception of definition gives rise to a modified view on what it means to make concepts distinct in comparison with the Leibnizians.

2.4.4 Kant’s conception of synthetic distinctness

Before I elaborate the role of construction in Kant’s view on mathematical axioms and theorems, I think it is useful to zoom out and see what Kant’s overall method is. His distinct conception of mathematical definition overcomes a crucial hurdle that a rationalist like Leibniz or Wolff inevitably faces. The latter cannot establish with certainty that an analysis of a concept is complete.

¹¹⁷Friedman (2012, e.g. p.237) defends a similar view on Kant’s conception of geometrical concepts and proofs. On his view, the schema that corresponds to a geometrical concept plays a central role. His interpretation of the role and conception of metaphysical space is different than what I suggest (see pp.242-7). See also Friedman (1992b, Ch.1, esp.94-5) on this.

¹¹⁸See Heis (2014a) for Kant’s theory of real definitions in geometry.

Kant's geometrical construction in pure intuition ultimately demonstrates mathematical results by clarifying not so much individual concepts but the structure of "objects of possible experience" in general. It is this feature that essentially characterizes Kant's "improved" mathematical method through the construction of concepts, which puts mathematics on a more secure footing than what a rationalist can ever achieve, on Kant's view. It also explains why geometrical construction can establish fully *general* results, as opposed to arithmetical construction, where the constructed concepts do not coincide with the "objects of possible experience".¹¹⁹

In this context, I want to consider the following comment on the overall role that construction plays from the Introduction to the *Logik*:

Logicians of the Wolffian school place the act of making cognitions distinct entirely in mere analysis of them. But not all distinctness rests on analysis of a given concept. It arises thereby only in regard to those marks that we already thought in the concept, but not in respect to those marks that are first added to the concept as *parts of the whole possible concept*. The kind of distinctness that arises not through analysis but through synthesis of marks is synthetic distinctness. [...] For when I make a distinct concept, I begin with the parts and proceed from these toward the whole. Here there are no marks as yet at hand; I acquire them only through synthesis. From this synthetic procedure emerges synthetic distinctness, then, which actually extends my concept as to content through *what is added as a mark beyond the concept in (pure or empirical) intuition*. The mathematician and the natural philosopher make use of this synthetic procedure in making distinctness in concepts. [...] To synthesis pertains the making distinct of objects, to analysis the making distinct of concepts. (Logik, 9:63-4, my emphasis)

It is key to recognize that Kant's construction brings out marks of a concept that are "first added to the concept as parts of the whole possible concept". He characterizes this as a shift from a Wolffian conception of making given concepts distinct to "mak[ing] a distinct concept", or "synthetic distinctness". On his proposal, a consideration of the proper marks of a (mathematical or physical)

¹¹⁹See A163-5/B204-6.

concept presupposes a prior consideration of the intuition that corresponds to the concept.¹²⁰

What is implicit in this passage but deserves to be highlighted is that construction is a presupposition not only for synthetic but also for “analytic” judgments in mathematics. The underlying thought is that “there are no marks as yet at hand; I acquire them only through synthesis”. This comment would seem to apply also to judgments like a triangle has three straight lines, where the predicate is seemingly “contained” already in the “mere” concept. What Kant’s conception of real definition ultimately suggests is that even containment relations need to be understood in terms of the constructed object, not the “mere” concepts. To consider a mathematical concept in abstraction from its constructed object is a contradiction on Kant’s view; there are no “mere” concepts in mathematics for Kant. The real definition comes necessarily first for Kant; all other judgments about the object are proven from them, even “analytic” ones.

Such a view regarding “analytic” geometrical propositions is also expressed in the B Preface when he says:

To be sure, a few principles that the geometers presuppose are actually analytic and rest on the principle of contradiction; but they also only serve, as identical propositions, for the chain of method and not as principles, e.g., $a = a$, the whole is equal to itself, or $(a + b) > a$, i.e., the whole is greater than its part. And yet even these, although they are valid in accordance with mere concepts, are admitted in mathematics only because they can be exhibited in intuition. (B16-7)

Kant clearly says that even identities, which are valid in accordance with mere concepts, are admitted to mathematics only because they can be exhibited in intuition.

To summarize, construction in intuition—which underlies both analytic and synthetic judgments in geometry—reflects a more powerful procedure than a Wolffian analysis of given concepts, on Kant’s view, because it draws on apodictically certain parts of the intuition that corresponds to a concept. That gives rise to a modified conception of what it means to make concepts distinct on

¹²⁰To understand the full scope of this point, we need to take into account that intuition is “immediately related to the object and is singular” (A320/B377).

which the constructed concept precedes the cognition of its proper marks.¹²¹

2.4.5 Geometrical axioms and theorems

It is now natural to ask whether construction in pure intuition has the same function in Kant's reasoning regarding geometrical axioms and theorems as it has for definitions. Interestingly, those functions are different. Since scholars often draw on Kant's conception of mathematical proof to illuminate his approach in MAN, I think it is useful to also discuss the role of construction in this context.

Kant conceives mathematical axioms as “synthetic a priori principles, insofar as they are immediately certain” (A732/B760). So just like mathematical definitions, axioms are immediately certain. However, in contrast to definitions, axioms are synthetic,¹²² which means that their certainty is “not grounded on [the] identity of concepts” (Logik, §36). Rather, Kant characterizes their certainty as “intuitive certainty”, due to “evidence” [Evidenz] provided by pure intuition (A734/B762).

To see how this works in more detail, I want to present how Kant's disciple Johann Schultz (1792), in his attempt to popularize the first *Critique*, discusses the geometrical axiom that no two straight lines enclose a space. Kant also discusses this axiom but Schultz's treatment is more detailed.¹²³ Here is Schultz's explanation for how the truth of the axiom can be recognized:

I construct the general concept of the subject, that is, draw through the imagination two individual straight lines that intersect in space. By means of the pure intuition of them, not only will I immediately acquire the representation that they have only one point in common (as through the intuition of a piece of gold [I would acquire] the representation yellow), but this also comes with the consciousness of absolute necessity; since the representation that they had a second point in common would contradict the intuition of those two lines, and only the intuition of a single [line] would remain.

¹²¹This aspect is discussed in more detail in Chapter 3.

¹²²The view that Kant's mathematical definitions are analytic is contentious. What I say in Chapter 3 can be taken as indirect support for this, at least if we recognize Kant's idiosyncratic way of understanding analyticity in the critical period.

¹²³See, e.g., A47/B65, A163/B204.

(Schultz (1792, §31, p.64))¹²⁴

Schultz's explanation starts with the construction of the concept of intersecting straight lines through the faculty of imagination. His reasoning makes it clear that we consider an "object" that pure intuition encodes, in essence the set of all possible instances of intersecting straight lines in space.¹²⁵ Accordingly, to make patent a general proposition about this object, we can only draw on characteristics of the object that apply to all specified geometrical spaces; not those that would capture (say) only a single instance of intersecting straight lines (which would involve, for instance, particular lengths and angles).

The immediacy of the axiom can be explained through an appeal to an auxiliary construction—one that Schultz does not spell out—namely by rotating one line segment around the point of intersection until another point on that line comes to lie on the other line segment.¹²⁶ Such an auxiliary construction is appropriate because it relies only on the constructed concept intersecting straight lines, it does not invoke any additional propositions.¹²⁷ This reasoning establishes with immediate certainty that the representation that two intersecting lines had a second point in common would contradict the constructed concept because it would mean that the two lines coincide. It follows that no two straight lines enclose a space.

In short, Schultz suggests that geometrical axioms are intuitively certain because they can be known solely through the set of geometrical spaces associated with the constructed subject concept. Auxiliary constructions licensed by the constructed concept are one natural way to make precise the aspects of the geometrical object that are used to make the truth of the axiom patent.

Kant's conception of geometrical theorems can be thought of as an extension of the approach towards axioms. A theorem requires a proof, which also involves other constructions (licensed

¹²⁴Translations from Schultz are mine.

¹²⁵Kant puts this step in the argument for the immediacy of the axiom as "you see yourself forced to take refuge in intuition, as indeed geometry always does. You thus give yourself an object in intuition [...] You must therefore give your object a priori in intuition, and ground your synthetic proposition on this" (A47-8/B65).

¹²⁶Shabel (2006, p.106) and Heis (2020, esp. pp.178-9) emphasize the role of auxiliary constructions when accounting for the syntheticity of geometrical cognitions. Another option would be to draw on general phenomenological facts regarding pure intuition, the kinds of facts Kant relies on in the *Metaphysical Exposition of the concept space* (A23/B37 ff.). See, e.g., Parsons (1992, pp.67-80).

¹²⁷The real definition of the concept intersecting straight lines already contains the property that the lines can intersect at all angles. This establishes the possibility that one line segment can be rotated (fully) around the point of intersection.

by previously established propositions), not just those associated with the subject concept. Proper “demonstrations”, for Kant, are intuitive, which means they depend at each step on the object in pure intuition.¹²⁸ Consider the famous passage where Kant describes the proof of Euclid I.32 that the sum of the interior angles of a triangle adds up to two right angles:

But now let the geometer take up this question. He begins at once to construct a triangle. Since he knows that two right angles together are exactly equal to all of the adjacent angles that can be drawn at one point on a straight line, he extends one side of his triangle, and obtains two adjacent angles that together are equal to two right ones. Now he divides the external one of these angles by drawing a line parallel to the opposite side of the triangle, and sees that here there arises an external adjacent angle which is equal to an internal one, etc. In such a way, through a chain of inferences that is always guided by intuition, he arrives at a fully illuminating and at the same time general solution of the question. (A7I6-7/B744-5)

Kant describes a sequence of (auxiliary) constructions in space that are licensed by the constructed concept triangle and other propositions (e.g., the proposition that through a given point a straight line parallel to a given straight line can be drawn, i.e. Euclid I.31). His proof relies crucially on equalities of certain shapes—especially angles, which can be securely cognized in pure intuition.¹²⁹ On the basis of those kinds of inferences, Kant establishes that the sum of the interior angles conforms to two right angles with a certainty, a kind of certainty that derives from his notion of metaphysical space and the principle of contradiction.¹³⁰

Let me conclude my discussion of geometry with a comparison of the exact function that construction in pure intuition has for Kant. In the case of geometrical definitions, construction in pure intuition functions to establish the real possibility of the object of the concept with apodictic cer-

¹²⁸See A734/B762.

¹²⁹Cf.: “Geometrical construction requires that one quantity be the same as another or that two quantities in composition be the same as a third, not that they produce the third as causes, which would be mechanical construction. Complete similarity and equality, insofar as it can be cognized only in intuition, is congruence. All geometrical construction of complete identity rests on congruence” (MAN, 4:493).

¹³⁰See A10/B14 for the role of the principle of contradiction in mathematical reasoning.

tainty. In the context of synthetic judgments, licensed (auxiliary) constructions in pure intuition function to provide intuitively certain evidence for the truth of the judgment. Those are related but not identical functions. I now turn to Kant's *Metaphysical Foundations*, in particular the question of how the construction of the concept matter is connected to geometrical construction.

2.5 Kant's philosophical motivation for construction in MAN

2.5.1 A puzzle

Kant suggests in the Preface that MAN follows the “mathematical method”, that is, somehow depends on construction in pure intuition.¹³¹ One concrete proposal for how this is supposed to work is to suggest that construction in pure intuition has a similar *evidential* function as in Kant's reasoning regarding geometrical axioms and theorems. In particular, those who defend Kant's claim regarding the constructability of the concept matter in a limited fashion find support for their position where Kant actually appeals to diagrams or “laws of space”: in the proof of the Proposition in the *Phoronomy*,¹³² the inverse square law in the *Dynamics*,¹³³ and the Third law of motion in the *Mechanics*.¹³⁴ Accordingly, scholars then also highlight other features like the forces themselves, which cannot be geometrically constructed in pure intuition.¹³⁵

While such a reading has philosophical virtues, I think it does not go far enough and misses the function that construction plays in the domain of natural science. In order to see this, consider the following comment from the *Preface*:

I assert, however, that in any special doctrine of nature there can be only as much proper science as there is mathematics therein. For, according to the preceding, proper

¹³¹See MAN, 4:470-72, 4:478.

¹³²MAN, 4:490-3.

¹³³MAN, 4:519-22.

¹³⁴MAN, 4:544-47.

¹³⁵See MAN, 4:525 for Kant's comment that forces cannot be constructed. When Kant says about the “metaphysical-dynamical” mode of explanation that “we lack all means for constructing this concept of matter” (MAN, 4:525), I think he refers to Leibniz's view, not his own. It is Leibniz's view in the *Specimen Dynamicum* that “the material itself is transformed into fundamental forces” (ibid.), as I will discuss below. For Kant, matter is not transformed into fundamental forces. Rather, he isolates the aspects of motion and their underlying moving forces that can be presented in intuition.

science, and above all proper natural science, requires a pure part lying at the basis of the empirical part, and resting on a priori cognition of natural things. Now to cognize something a priori means to cognize it from its mere possibility. But the possibility of determinate natural things cannot be cognized from their mere concepts; for from these the possibility of the thought (that it does not contradict itself) can certainly be cognized, but not the possibility of the object, as a natural thing that can be given outside the thought (as existing). Hence, in order to cognize the possibility of determinate natural things, and thus to cognize them a priori, it is still required that the intuition corresponding to the concept be given a priori, that is, that the concept be constructed. (MAN, 4:470)

Kant suggests that there is a strict correspondence between proper natural science and mathematics. By “mathematics” in the first sentence of the quote, Kant means constructed concepts, as the end of the quote suggests. In combination with his further claim that the possibility of natural things cannot be cognized from “mere concepts”, it seems as though the suggestion to read only *some* parts to rely on construction is not consistent with the text.

This reading is confirmed by the continuation of the passage:

Now rational cognition through construction of concepts is mathematical. Hence [...] a pure doctrine of nature concerning determinate natural things (doctrine of body or doctrine of soul) is only possible by means of mathematics. And, since in any doctrine of nature there is only as much proper science as there is a priori knowledge therein, a doctrine of nature will contain only as much proper science as there is mathematics capable of application there. (MAN, 4:470)

Once again, Kant says that the scope of pure natural science coincides with the extent to which mathematics can be applied. My interpretation of this is that pure natural science is the outcome of an analysis of empirical physics, an analysis that isolates constructed component concepts through which the concept matter can be securely cognized. This view is encoded in Kant’s programmatic

comment from the *Amphiboly*: “Observation and *analysis* [Zergliederung] of the appearances penetrate into what is *inner* in nature, and one cannot know how far this will go in time” (A278/B334, my emphasis). What that means is that at least in MAN, analysis and construction are closely connected processes.¹³⁶ Since MAN presupposes empirical theorizing, the construction of the concepts is not first (as in mathematics) but rather characterizes the outcome of the analysis of the empirical concept matter. The essential questions that need to be addressed in the following are hence: What is Kant’s conception of analysis? What are those constructable component concepts? How do those concepts differ from mathematical concepts?

2.5.2 Kant’s motivation for construction in the *Amphiboly*

To better understand the point of construction, I want to develop further the sense in which construction in pure intuition introduces a modification to Leibniz’s conception of reflection or cognition through reason. A key source where Kant articulates this view is the *Amphiboly* chapter of the first *Critique*. That is, one purpose of Kant in the *Amphiboly* is to motivate how his preferred conception of reflection differs from Leibniz’s “reflections” regarding the concept matter in the *Specimen Dynamicum* (SD). Kant in fact talks about the concept matter here as a “concept of reflection” [Reflexionsbegriff].¹³⁷

It is useful to start with what Leibniz means by reflection. Here are two passages from the *New Essays*:

[Theo.] We are aware of many things, within ourselves and around us, which we do not understand; and we understand them when we have distinct ideas of them accompanied by the power to reflect and to derive necessary truths from those ideas. [...] So ‘understanding’ in my sense is what in Latin is called *intellectus*, and the exercise of this faculty is called ‘*intellection*’, which is a distinct perception combined

¹³⁶In regards to MAN, both analysis and construction appear to be procedural underpinnings of the analytic/synthetic distinction and thereby on a par with one another if what I say about the unity of the three characterizations of the distinction in Chapter 3 holds. Analysis is connected to the principle of contradiction and construction explains Kant’s notion of containment in MAN.

¹³⁷See A277/B333.

with a faculty of reflection. (NE, p.173)

[Theo.] for intellectual ideas, or ideas of reflection, are drawn from our mind. I would like to know how we could have the idea of being if we did not, as beings ourselves, find being within us. [Phil.] What do you say, sir, to this challenge which a friend of mine has offered? If anyone can find a proposition whose ideas are innate, let him name it to me (he says) [...] [Theo.] I would name to him the propositions of arithmetic and geometry, which are all of that nature. (NE, pp.85-6)¹³⁸

According to Leibniz, we “understand” perceptions as soon as we “reflect” on their underlying intellectual ideas and derive necessary knowledge from them. It is worth keeping in mind that for Leibniz, ideas of reflection are the immediate “objects” of thought and express the nature of things.¹³⁹ The mathematical case serves as a paradigmatic example. Diagrammatic reasoning, on Leibniz’s view, helps us to become aware of properties of certain figures.¹⁴⁰ To properly understand and demonstrate those connections, however, requires to have distinct mathematical ideas and show what they contain.

This paves the way for a discussion of Kant. Here is how he starts the *Amphiboly*:

Reflection (*reflexio*) does not have to do with objects themselves, in order to acquire concepts directly from them, but is rather the state of mind in which we first prepare ourselves to find out the subjective conditions under which we can arrive at concepts. It is the consciousness of the relation of given representations to our various sources of cognition, through which alone their relation among themselves can be correctly determined. (A260/B316)

Kant departs from Leibniz’s approach and suggests that reflection is about finding the subjective conditions that are involved in concepts. Ultimately, this investigation helps to determine the re-

¹³⁸See also NE, pp.51-2, 111.

¹³⁹Consider: “an idea is an immediate inner object, and that this object expresses the nature or qualities of things. If the idea were the form of the thought, it would come into and go out of existence with the actual thoughts which correspond to it, but since it is the object of thought it can exist before and after the thoughts. Sensible outer objects are only mediate, because they cannot act immediately on the soul” (NE, p.109). See Adams (1994, pp.221-2) for such a reading concerning Leibniz’s conception of material things.

¹⁴⁰This point was discussed in section 2.3.

lation of “given representations” to one another. The kinds of relations that Kant considers are: identity/difference, agreement/opposition, inner/outer, and determinable/determination.¹⁴¹ Kant likely drew them from Leibniz. For instance, Leibniz, on Kant’s view, held that there can be no opposition of different realities in a subject concept, only agreement.¹⁴²

Kant’s suggestion is that the exact characteristics of the four relations depend on the faculty in which “they subjectively belong to each other, whether in sensibility or in understanding” (A261/B317). Since Kant believes that given representations belong (also) to sensibility, those four relations of concepts have different characteristics than for Leibniz. That departure will ultimately result in different kinds of reflections. Here is a programmatic statement that captures his departure from Leibniz: “we cannot understand anything except that which has something corresponding to our words in intuition” (A277/B333).¹⁴³

Accordingly, Kant states his preferred conception of reflection in comparison with what he calls “logical reflection”:

logical reflection is a mere comparison, for in its case there is complete abstraction from the cognitive power to which the given representations belong [...] transcendental reflection, however, (which goes to the objects themselves) contains the ground of the possibility of the objective comparison of the representations to each other, and is therefore very different from the other, since the cognitive power to which the representations belong is not precisely the same. This transcendental reflection is a duty from which no one can escape if he would judge anything about things a priori. (A262-3/B318-9)

In general, Kant agrees with Leibniz on the overall outcome of reflection, namely that it allows us to form judgments about things a priori. Logical reflection involves a comparison of concepts, in essence, to decide whether they agree or differ. For example, one concept might be contained in the

¹⁴¹See A261/B317.

¹⁴²See A273/B329. Leibniz discusses those themes, for instance, in his *New Essays*.

¹⁴³See also A278-9/B334-5. The sevenfold hierarchy of logical perfection obviously abstracts from the influence of sensibility: “The fifth: to understand something (intelligere), i.e., to cognize something through the understanding by means of concepts, or to conceive” (Logik, 9:65).

other concept. Kant recommends a different kind of analysis (“transcendental reflection”), which enables an “objective comparison” of concepts because it takes into account the faculty to which those concepts “belong”.

Before I spell out the details of Kant’s conception of transcendental reflection in MAN, I think it is informative to quickly look at Leibniz’s concern for intellection regarding natural science in the *Specimen*.¹⁴⁴ Leibniz operates in this text with a conception of intellection on which relational determinations of matter at the phenomenal level, in essence relative motions,¹⁴⁵ are explained in terms of the underlying forces. On Leibniz’s view of forces—he distinguishes active and passive force (both in primitive and derivative versions), they are not conceived as relations between bodies but as non-relational (or intrinsic) determinations of bodies. A particular body has a certain amount of (derivative) active and passive force, for instance. According to Leibniz, an analysis of natural phenomena in terms of forces helps us to understand the observed empirical regularities rationally.¹⁴⁶

For Kant, the analysis of empirical regularities does not draw on non-relational (purely inner) determinations of matter.¹⁴⁷ His explanatory basis involves only relations:

I will always have to compare my concepts in transcendental reflection only under the conditions of sensibility, and thus space and time [...] I proceed in the same way with the other concepts of reflection. Matter is *substantia phaenomenon*. What pertains to it internally I seek in all parts of space that it occupies and in all effects that it carries out, and which can certainly always be only appearances of outer sense. I therefore

¹⁴⁴My focus here is on the conceptual or epistemic level. That is, I highlight Leibniz’s suggested hierarchy of concepts in connection with his project in the SD, in particular, which concepts are (more) primitive. A full account of why certain concepts are primitive in relation to others would also require discussion of Leibniz’s system more broadly. For more metaphysical discussions of Leibniz’s view on relations and his conception of different levels of reality, in particular his tiered conception of the phenomenal level that underlies his explanatory project in SD, see Adams (1994, Ch.9) and Jauernig (2010).

¹⁴⁵Leibniz conceives motion as a relationship: “motion, in so far as we experience it (*quoad phaenomena*) is nothing but a relationship” (SD, par.37). He also holds that all “material phenomena are explicable in terms of local motion” (SD, par.9). See DM, §18 for Leibniz’s point that an analysis of motions leads to forces that can be attributed to specific individual bodies. Garber (1995, esp. pp.305-9) gives a useful overview of Leibniz’s conception of motion and its laws.

¹⁴⁶See SD, par.6, 20; NI, par.3-4, 7.

¹⁴⁷The view that Kantian appearances are, in essence, relational determinations of things is at the heart of Langton’s (1998) argument. I suggest that there is a hierarchy among the various relational determinations of appearances, that is, I put a more fine-grained structure on Kant’s appearances.

have nothing absolutely but only comparatively internal, which itself in turn consists of outer relations. (A276-7/B332-3)

For Kant, the privileged set of cognitions in his analysis of matter are nothing but “outer relations”, ones that capture “all parts of space that [matter] occupies and in all effects that [matter] carries out”. So what is meant by an inner (i.e., explanatory) determination in this context appears to be a spatially encoded determination. For instance, it involves the extension of body as captured in pure intuition, motion as conceived in terms of lines in pure intuition, opposing directions in pure intuition to capture the *effects* of fundamental forces.¹⁴⁸ As for Leibniz, it is natural to think that Kant’s more metaphysical doctrines from the first *Critique* provide the foundation for his specific conception of reflection or philosophical analysis.¹⁴⁹

In particular, his conception of philosophical analysis takes as primitives the pure representations from the *Aesthetic* and the *Analytic*, as well as the nature of their interrelatedness.¹⁵⁰ We are supposed to analyze the empirical concept matter in terms of the previously secured cognitions, the table of categories and pure intuition. “The concept of matter had therefore to be carried through all four of the indicated functions of the concepts of the understanding (in four chapters), where in each a new determination of this concept was added” (MAN, 4:476), says Kant.¹⁵¹ In this context, it is also useful to consider the following detailed comment from the *Preface*:

a complete analysis [vollständige Zergliederung] of the concept of a matter in general will have to be taken as the basis, and this is a task for pure philosophy – which, for this purpose, makes use of no particular experiences, but only that which it finds in the isolated (although intrinsically empirical) concept itself, in relation to the pure

¹⁴⁸The following passage from the *Aesthetic* captures the pure intuition part, but abstracts from the original representations from the understanding: “everything in our cognition that belongs to intuition [...] contains nothing but mere relations, of places in one intuition (extension), alteration of places (motion), and laws in accordance with which this alteration is determined (moving forces)” (B66-7). For Kant’s idea to consider opposing directions/lines to capture the effects of opposing forces, see also A273/B329.

¹⁴⁹Maudlin’s (2007, esp. pp.5-7) useful discussion on the methodology of philosophical analysis helped me to formulate my ideas on Kant’s conception of analysis.

¹⁵⁰See especially MAN, 4:473-4.

¹⁵¹Cf.: “The understanding traces back all other predicates of matter belonging to its nature to [motion], and so natural science, therefore, is either a pure or applied doctrine of motion” (MAN, 4:476-7). See also Prol, 4:325-6.

intuitions in space and time (in accordance with laws that already essentially attach to the concept of nature in general). (MAN, 4:472, translation modified)

Kant here states that his conception of analysis consists of a comparison of the concept matter with the pure representations from the *Aesthetic* and *Analytic* (“laws that already essentially attach to the concept of nature in general”). It is ultimately those previously secured concepts (and principles) that confer a privileged, i.e. explanatory, status to the corresponding elements of the concept matter, that is, the constructable component concepts.

2.6 Kant’s constructions in MAN

2.6.1 Motion

In this section, I finally clarify Kant’s construction of motion and his resulting view on the identity/difference relation of (constructable) concepts. To appreciate Kant’s positive results in regards to the concept matter, it is useful to start with an example where Kant’s suggested analysis does not work, i.e. for chemistry. Here is a passage:

So long, therefore, as there is still for chemical actions of matters on one another no concept to be discovered that can be constructed, that is, no law of the approach or withdrawal of the parts of matter can be specified according to which, perhaps in proportion to their density or the like, their motions and all the consequences thereof can be made intuitive and presented a priori in space [...] its principles are merely empirical, and allow of no a priori presentation in intuition. Consequently, they do not in the least make the principles of chemical appearances conceivable with respect to their possibility, for they are not receptive to the application of mathematics. (MAN, 4:470-1)

Kant reflects on chemistry. His observation is that, at least so far, it contains no concept that can be constructed. What this means is that no intuitive determination can be extracted from chemistry, it involves nothing that can be “made intuitive and presented a priori in space”.

For physics, the story is different. A reflection leads to intuitive determinations, most fundamentally, motion conceived as the trajectory in pure intuition, that is, a concept that can be constructed:

(1) Matter is the movable in space. (Explication 1, MAN, 4:480)

(2) our present business, in which we must *necessarily treat space as a property of the things under consideration*, namely, corporeal beings, because these things are themselves only appearances of the outer senses, and only require to be *explicated as such* here. (MAN, 4:484, my emphasis)

(3) phoronomy has first to determine the construction of motions in general as quantities, and, since it has matter merely as something movable as its object, in which no attention at all is therefore paid to its quantity, [it has to determine] these motions a priori solely as quantities, with respect to both their speed and direction, and, indeed, with respect to their composition. For so much must be *constituted wholly a priori, and indeed intuitively*, on behalf of applied mathematics. (MAN, 4:487, my emphasis)

(4) In phoronomy, since I am acquainted with matter through no other property but its movability, and may thus consider it only as a point, motion can only be considered as the *describing of a space* – in such a way, however, that I attend not solely, as in geometry, to the space described, but also to the time in which, and thus to the speed with which, a point describes the space. Phoronomy is thus the pure theory of quantity (mathesis) of motions. The determinate concept of a quantity is the concept of the generation of the representation of an object through the composition of the homogeneous. (MAN, 4:489, my emphasis)

Kant “explicates” motion as an intuitive, i.e. spatially encoded determination of corporeal things (1, 2, 3). More specifically, the concept can be constructed in terms of a straight line of a certain length in space, which captures the representation of “quantity” through composition of homogeneous parts (4); the construction abstracts from the mass and shape of bodies (3). In essence, Kant explains bodies in terms of their speed and direction (3).

Such a conception of construction gives rise to a richer notion of the equivalence of concepts than one that relies on the identity of component concepts. To see this, consider Kant’s approach to the “Principle” [Grundsatz] in the *Phoronomy*:

Every motion, as object of a possible experience, can be viewed arbitrarily as motion of the body in a space at rest, or else as rest of the body, and, instead, as motion of the space in the opposite direction with the same speed. (MAN, 4:487)

I would argue that this Principle is immediately certain because it follows directly from the construction of the concept(s). The thought is that the concepts <motion of a body for time t in a space at rest> and <rest of the body and motion of the space in the opposite direction for time t > are congruent when constructed in pure intuition. The two concepts have different conceptual marks but agree on their corresponding pure intuitions. That is, the length of their respective constructed straight lines is the same. This renders the two *constructed* concepts equivalent. Kant says in this context:

For any concept is entirely the same as a concept whose differences from it have no possible example at all, being only different with respect to the connection we wish to give it in the understanding. (MAN, 4:488)

So constructable concepts that do not differ in regards to their corresponding pure intuitions are equivalent. The relativity principle is thereby “analytic”. Its certainty is grounded on the identity of the *constructed* concepts.

2.6.2 Comparison between construction and geometrical construction

To conclude, it is helpful to compare construction with mathematical construction. Kant’s reflection in MAN starts with the empirical concept matter and isolates the intellectual or explanatory parts of it: “Observation and analysis of the appearances penetrate into what is inner in nature, and one cannot know how far this will go in time” (A278/B334). To do so, he compares the concept matter with the original representations defended in the *Analytic* and the *Aesthetic*. The constructed partial concepts of matter in MAN are the outcome of this analysis, they amount to inner (but relational) determinations of the concept matter. A constructed component concept like (pure) motion in the *Phoronomy* can be thought of as a “mark by means of which the object (definitum) can always be securely cognized” (A241-2, note),¹⁵² on my reading. So I take the interaction between pure

¹⁵²Nunez (2014, p.635) emphasizes the note from A241-2 in his defense of possible Kantian definitions of the categories in the Critique. On his view, the secure marks of the schematized categories are discursive (p.635). In light of my previous points on the negligible role for objective reality in MAN and my distinction of different kinds of a priori knowledge in Chapter 1, the suggested proper definitions of the categories and Kant’s explications in MAN belong to

understanding and pure intuition that Kant's constructed concepts in MAN embody to be at the heart of his explanation for why pure natural science is apodictically certain.¹⁵³ In light of the involvement of pure intuition, it appears that the notion of certainty at issue is indeed intuitive certainty, although Kant uses the notion of intuitive certainty mainly to characterize the evidence in proper demonstrations/synthetic judgments in mathematics, not the marks of a concept.¹⁵⁴ Still, construction connects with mathematical construction in the sense that the inner marks of a concept are exhibited in space, as entailed by his view on real mathematical definitions. Construction, however, does not establish that the object of the concept matter is objectively real a priori.

It is worth emphasizing that my focus here has been mainly on the function of construction in comparison with mathematical construction. As I will show in Chapter 3, the constructable component concepts of matter serve as secure grounds for proper knowledge claims about natural science, which are relevant on Kant's view of cognition through reason. Construction is hence a key part of his view for how higher, more objective forms of cognition than intellection, that is, "insight" and "comprehension" are possible (Logik, 9:64-5).¹⁵⁵

2.7 Conclusion

Kant describes the method of his *Metaphysical Foundations of Natural Science* as an "imitation" of the "mathematical method". I argued that his conception of geometrical construction serves as a model for isolating constructed component concepts through which matter can be securely cognized. The function of construction is to explain the concept matter in terms of the previously secured original representations from the understanding and sensibility (which together make up the "intellect" in Kant's theoretical philosophy), not to confer objective reality to its partial con-

rather different cognitive projects and involve different argument patterns. For my argument, it is also less problematic that Kant removes this passage in the second edition of the Critique (cf. Nunez (2014, p.644)).

¹⁵³See Logik, 9:70-1 for the notion of apodictic certainty at play. For instance, Kant says: "We are rationally certain of that into which we would have had insight a priori even without any experience" (Logik, 9:71). The suggested explanation for apodictic certainty in MAN differs from Kant's conception of certainty in the Analytic and the Aesthetic since he there treats the faculties in isolation from one another.

¹⁵⁴See A734/B762; A161-2/B200-1.

¹⁵⁵A comprehensive treatment of this topic would require a more detailed presentation of the faculty of reason than I can give here.

cepts. Since Kantian intellection necessarily depends on pure intuition, it differs from Leibnizian intellection in terms of mere concepts. The latter intellectualizes empirical physics (i.e., matter in motion) in terms of the concept force, which is a property of things-in-themselves, and so not acceptable for critical Kant. Instead, construction isolates spatially encoded (and thereby relational) determinations of matter. Kant's approach in MAN is epistemically humbler than Leibniz's approach because it does not depend on knowledge of things-in-themselves.

3 Kant's analytic/synthetic distinction in MAN

In an analytic judgment, reference to [the common object] X is otiose.

–L.W. Beck (1955, p.76)

3.1 Introduction

Kant's *Metaphysical Foundations of Natural Science* (MAN) is a systematic philosophy, which shares with a Leibniz-Wolffian system of knowledge the objective to express apodictically certain knowledge. In the *Critique of Pure Reason*, Kant famously suggested that almost all the relevant kinds of knowledge—in mathematics, pure natural science, and metaphysics—cannot be derived by showing that the predicate is “contained” in the subject concept, as Leibniz and Wolff think. To make room for this kind of knowledge, Kant draws a distinction between two kinds of judgments, analytic and synthetic judgments. Kant gives three characterizations of the analytic/synthetic distinction in the *Introduction* to the first *Critique*—in terms of containment, the principle of contradiction, and differing epistemic uses (A6-7/B10-11).

It is a very common theme in the literature to argue that those three characterizations are incompatible with one another and/or to suggest that one of them should be seen as fundamental.¹⁵⁶ In light of Kant's *systematic* approach to knowledge in MAN, I think it provides an ideal context to investigate and clarify the distinction. In fact, MAN is arguably the only critical text we have where Kant expresses a body of proper theoretical knowledge [Wissen] in a systematic and complete way. (The first *Critique* is supposed to be a doctrine of method, not a systematic philosophy.) I argue that in regards to the judgments Kant defends in MAN, the three criteria for the analytic/synthetic distinction are independent and compatible with one another. In particular, Kant requires both containment and the principle of contradiction to specify his position.

To see this, I suggest that Kant's conception of real mathematical definition serves as his *model* for analytic judgments. When applying this model to pure natural science, the Explications of the concept matter come out as paradigmatic instances of analytic judgment. Contrary to Beck's

¹⁵⁶See, for instance, L.W. Beck (1955), Allison (2004, pp.89-93), Proops (2005), and Anderson (2015).

(1955) influential point (as quoted above),¹⁵⁷ the reference to the common object is *not* otiose in light of the explications's dependence on "construction" in pure intuition. Kant's conception of containment is hence not logical in the Leibniz-Wolffian sense.¹⁵⁸ His conception of containment is "logical" in the sense that it builds on his notion of Transcendental Logic, which requires both sensibility and understanding. My reading complements existing views by bringing out how the distinction serves as Kant's essential vehicle for expressing mathematical and physical knowledge in a complete and apodictically certain manner. The distinction therefore reflects Kant's positive account of how a proper system of knowledge should be conceived and less his attempt to criticize or even refute his rationalist predecessors.

I start by presenting Kant's analytic/synthetic distinction from the first *Critique* and clarify the way Kant applies this distinction to different kinds of judgments by drawing on his notion of holding-to-be-true [Fürwahrhalten] from the *Jäsche Logic* (Sect. 3.2). After that, I trace Kant's conception of analytic judgment in mathematics, which I take to be his ideal case of holding-to-be-true, i.e. *Wissen* [proper knowledge]. The developed conception will then help to explain his fundamental analytic judgments in MAN, i.e. his Explications (Sect. 3.3). I show that Kant's three criteria for the analytic/synthetic are independent and jointly characterize Kant's view (Sect. 3.4). Finally, I examine the character of synthetic judgments through a discussion of the Proposition from Kant's *Phoronomy* (Sect. 3.5).

3.2 The distinction in the first Critique

Kant characterized the distinction between analytic and synthetic judgments in the *Introduction* to the first *Critique* in the following way:

In all judgments in which the relation of a subject to the predicate is thought (if I consider only affirmative judgments, since the application to negative ones is easy) this relation is possible in two different ways. Either the predicate B belongs to the subject

¹⁵⁷Allison (2004, p.91), for example, draws on Beck's point for his account of Kantian analyticity.

¹⁵⁸Contrary to Anderson (2015).

A as something that is (covertly) contained in this concept A; or B lies entirely outside the concept A, though to be sure it stands in connection with it. In the first case I call the judgment analytic, in the second synthetic. Analytic judgments (affirmative ones) are thus those in which the connection of the predicate is thought through identity, but those in which this connection is thought without identity are to be called synthetic judgments. One could also call the former judgments of clarification, and the latter judgments of amplification, since through the predicate the former do not add anything to the concept of the subject, but only break it up by means of analysis into its component concepts, which were already thought in it (though confusedly); while the latter, on the contrary, add to the concept of the subject a predicate that was not thought in it at all, and could not have been extracted from it through any analysis. (A6-7/B10-11)

Kant explicates the analytic/synthetic distinction in respect to affirmative judgments of the form A is B here. If the relation between the subject concept and the predicate concept is thought through identity, the judgment is called “analytic”. If it is not, then it is called “synthetic”. In the former case, the predicate B is “contained” in the subject concept A. For synthetic judgments, the predicate is somehow connected to the subject concept but not contained in it. The third criterion Kant proposes for the distinction is in terms of differing epistemic uses. Analytic judgments “clarify” the subject concept through the specification of a component concept that was “thought in it (though confusedly)”. Synthetic judgments, in turn, “amplify” the subject concept.

The example Kant uses in his official characterizations of analytic judgment in the first *Critique* and the *Prolegomena* is “All bodies are extended”.¹⁵⁹ This is a very illuminating choice in my view. To see why that is, let’s start with Kant’s reasoning on this example from the first *Critique*:

For it would be absurd to ground an analytic judgment on experience, since I do not need to go beyond my concept at all in order to formulate the judgment, and therefore need no testimony from experience for that. That a body is extended is a proposition

¹⁵⁹See A7/B11, 4:266-7, as well as the characterization in the *Jäsche Logic (Logik)*, 9:111 and his discussion in *OD*, 8:235.

that is established a priori, and is not a judgment of experience. For before I go to experience, I already have all the conditions for my judgment in the concept, from which I merely draw out the predicate in accordance with the principle of contradiction, and can thereby at the same time become conscious of the necessity of the judgment, which experience could never teach me. (B11-2)

Kant articulates his conception of analytic judgment with a different twist here; he uses the terminology of “ground” and “condition”.¹⁶⁰ The judgment that body is extended is grounded solely in the concept body, not on “experience”. “[A]ll the conditions for my judgment [are] in the concept”, says Kant. The principle of contradiction is the means for drawing out the predicate from the subject concept; its application confers a consciousness of necessity to the judgment according to Kant.

Scholars often clarify Kant’s reasoning in terms of the Leibniz-Wolffian view on which a true judgment (in general) is one where the predicate concept is a component of the subject concept. One natural motivation for this view are identities, which are ideal cases of true judgment according to Leibniz.¹⁶¹ The connection of the predicate concept with the subject concept is certain in this case because it is identical with the subject concept. For all other kinds of true judgment, the connection between predicate and subject is established through an “analysis” of the subject concept, by showing that the predicate is included in the subject concept.

Based on this conception of true judgment, the proposition that body is extended is true because

¹⁶⁰It is natural to think that Kant’s discussion here makes patent (his version of) the principle of sufficient reason. Kant’s view on the principle of sufficient reason is quite subtle and cannot be discussed in depth here. Kant arguably wants to distinguish between a logical and a transcendental version of it. The former appears more relevant for my purposes here. Kant formulates this principle as “every proposition has its reason” and says that it follows directly from the principle of contradiction in his response to Eberhard (OD, 8:197).

¹⁶¹Here is Leibniz in the *New Essays* (NE): “truth is always grounded in the agreement or disagreement of ideas, but it is not generally the case that our knowledge of truth is a perception of this agreement and disagreement” (p.357); “primary truths of reason are the ones to which I give the general name ‘identities’, because they seem to do nothing but repeat the same thing without telling us anything [...] these can be established with certainty, without any proof, i.e. without bringing them down to an opposition (i.e. down to the principle of contradiction), when the ideas are well enough understood not to need any analysis at this point” (pp.361-3). See Anderson (2015, esp. Ch.1) for discussion of those kinds of passages in Leibniz and Wolff and the relation to Kant. My emphasis in the following differs from Anderson in the sense that I take Kant’s component concepts/proper marks (for cases where the subject concept can be presented in intuition) to be logically secondary to the subject concept. In other words, what is at stake is Kant’s distinct view on the whole-part structure of objective representations.

the mark extension is connected to the concept body: extension is a component concept of the concept body. Here is how Van Cleve (1999, p.18), for instance, clarifies Kant's position:

'All bodies are extended' expresses an analytic judgment, because by 'body' we mean among other things an extended, impenetrable thing. The proposition we express is therefore equivalent to 'all extended and impenetrable things are extended', in which the inclusion of the predicate in the subject concept is visible to the mind's eye.

An influential challenge to such a view of true judgment during Kant's time (and beyond) has been that it is subjective, it depends on what a particular reasoner includes in her subject concept.¹⁶² For instance, it depends on what a particular physicist includes in her concept body. To fully address this concern will take some time. For now, I want to start with the particular example. It seems to me that the way Van Cleve reconstructs Kant's reasoning is not entirely right. For one, the concepts extension and impenetrability are not on par with one another in regards to the concept body:

So if I separate from the representation of a body that which the understanding thinks about it, such as substance, force, divisibility, etc., as well as that which belongs to sensation, such as impenetrability, hardness, color, etc., something from this empirical intuition is still left for me, namely extension and form. These belong to the pure intuition, which occurs a priori, even without an actual object of the senses or sensation, as a mere form of sensibility in the mind. (A20-1/B35)

So while the mark impenetrability "belongs" to sensation, extension belongs to pure intuition.¹⁶³ This point seems related to the second part of Kant's explanation from B11-2. Namely to his comment that the proposition that bodies are extended comes with a consciousness of necessity.

Here is my explanation of those two observations. Since the extension of bodies occurs prior to a consideration of body in empirical intuition, the connection between the concept body and the

¹⁶²Maaß (1789, p.189) expressed a view of this sort. Anderson (2015, pp. 45-49) provides an illuminating overview of different versions of the objection and its later modifications.

¹⁶³See also B5-6 on this.

concept extension is necessary.¹⁶⁴ The connection of the concept body with the mark impenetrability, on the contrary, is grounded on the accumulation of empirical intuitions of bodies.¹⁶⁵ It is therefore not thought with necessity. The underlying point is that Kant wants to draw a distinction between certain and uncertain (true) judgments.¹⁶⁶

Here is a general statement from his *Logik* that helps to make this point:

Truth is an objective property of cognition; the judgment through which something is represented as true, the relation to an understanding and thus to a particular subject, is, subjectively, holding-to-be-true. Holding-to-be-true is in general of two kinds, certain or uncertain. Certain holding-to-be-true, or certainty, is combined with consciousness of necessity, while uncertain holding-to-be-true, or uncertainty, is combined with consciousness of the contingency or the possibility of the opposite. The latter is again either subjectively as well as objectively insufficient, or objectively insufficient but subjectively sufficient, The former is called opinion, the latter must be called belief. Accordingly, there are three kinds or modi of holding-to-be-true: opining, believing, and knowing. (*Logik*, 9:65-6)

Kant distinguishes between certain and uncertain holding-to-be-true. If the ground of a judgment is the pure understanding or pure intuition, we have an instance of knowledge [Wissen]. Mathematics and metaphysics belong to this category for Kant.¹⁶⁷ If the ground is drawn from experience,

¹⁶⁴In other words, the connection between subject and predicate is not grounded on actual experiences, which for Kant are inevitably linked to a particular point in time. Here is Kant on this point in *On a Discovery*: “That all bodies are extended is necessarily and eternally true, whether they exist now or not, and whether that existence is brief or lengthy, or goes on throughout all time, i.e., eternally. The proposition says only: these truths do not depend upon experience (which must occur at one time or another), and are therefore not limited by any temporal conditions” (OD, 8:235).

¹⁶⁵The connection between body and impenetrability, on Kant’s view, is empirically certain, which resembles Newton’s view. So my disagreement with the quoted passage from Van Cleve is that it does not capture the different conceptions of certainty that Kant’s account of analyticity depends on. I agree that the judgment that all bodies are impenetrable is analytic. See also Chapter 1 on the status of impenetrability in MAN.

¹⁶⁶In his German metaphysics (“Vernünfftige Gedanken - Deutsche Metaphysik”, DM), Wolff draws a distinction between two ways of recognizing truth, through the senses and through the understanding (DM, §372). He also notes that if we are working only with “experience” [Erfahrung]/sensible propositions, we at most have “opinion” [Meinung], which is cognition based on grounds that are not certain (DM, §§384-5). Science, according to Wolff, has “secure grounds” [ungezweifelte Gründe] and is derived through “correct” inferences (DM, §361). So, Wolff distinguishes between certain and uncertain holding-to-be-true, that is, between proper knowledge and opinion.

¹⁶⁷See *Logik*, 9:67-71.

or more precisely, if it consists of the “matter” of sensation, the holding-to-be-true amounts to opinion.¹⁶⁸ The judgment that all bodies are extended is an instance of knowledge because it is grounded on the pure representations. It therefore comes with a consciousness of necessity for Kant.

To draw a distinction between certain and uncertain holding-to-be-true can resolve some of the dissatisfaction in the literature on Kant’s conception of analytic judgment. Lewis White Beck (1955, p.78), for instance, notices a “fundamental failure on Kant’s part to distinguish the logical from the phenomenological aspects of thought. Where definitions or fairly complete analyses are available, he thinks of the distinction between analytic and synthetic judgment as logical; where they are not, but rather the objects of search, he has recourse to a phenomenological criterion, by virtue of which he seeks definitions through analysis of what, in the plainest sense, is ‘actually thought’ in a concept”. Beck suggests that Kant maintains incompatible conceptions of analytic judgment, one “logical” and one “phenomenological”. I suggest to read Kant’s arguments that Beck (and others in a similar vein) associates with the phenomenological criterion as uncertain holding-to-be-true, thereby reflecting a unified Kantian approach. For example, when Kant considers the proposition that gold is yellow,¹⁶⁹ he wants to say that this is an instance of uncertain holding-to-be-true because color representations belong to sensation, not to intuition.¹⁷⁰

One might object here that Kant explicitly uses the term “know” when presenting this example: “Gold is a yellow metal; for in order to know [wissen] this, I need no further experience outside my concept of gold, which includes that this body is yellow and a metal” (Prol, 4:267). It is true that, for Kant, our observation that all objects of the concept gold are yellow provides a sufficient ground for the proposition that gold is yellow, for instance. It amounts to knowledge in a sense, that is, *empirically certain* holding-to-be-true.¹⁷¹ This kind of certain holding-to-be-true is based on actual

¹⁶⁸Kant, for example, says “matters of opinion can only be objects of a cognition by experience [Erfahrungserkenntnis], a cognition which is possible in itself but impossible for us in accordance with the restrictions and conditions of our faculty of experience and the attendant degree of this faculty that we possess” (Logik, 9:67).

¹⁶⁹See Prol, 4:267.

¹⁷⁰Cf.: “the sensations of colors, sounds, and warmth, which, however, since they are merely sensations and not intuitions, do not in themselves allow any object to be cognized, least of all a priori” (B44).

¹⁷¹See Logik, 9:70-1; MAN, 4:468.

experiences, not Kant's forms of experience.¹⁷² It seems that Kant accepts empirical certainty as a sufficient ground for knowledge claims if rational certainty is not possible.¹⁷³

This suggestion receives confirmation from Kant's comment in MAN on the distinction between proper and improper knowledge:

[w]hat can be called proper science [Wissenschaft] is only that whose certainty is apodictic; cognition that can contain mere empirical certainty is only knowledge [Wissen] improperly so-called. (MAN, 4:468)

Kant distinguishes proper from improper knowledge in terms of the associated form of certainty, apodictic versus empirical. The truth of the proposition that gold is yellow rests on the empirical certainty of the identity of the predicate yellow with the concept gold and is thereby an instance of improper knowledge.

To sum up, Kant's conception of analytic judgment comes in different versions depending on the kind of certainty with which the predicate is grounded in the subject concept. If the certainty is apodictic—reflecting Kant's forms of experience, holding-to-be-true amounts to proper knowledge [Wissen], e.g. the proposition that body is extended. If the certainty of the connection of the predicate with the subject concept is empirical, holding-to-be-true amounts to improper knowledge, e.g. the proposition that gold is yellow. From now on, I focus on Kant's judgments and reasoning regarding mathematics and pure natural science because both are arguably instances of proper knowledge.

3.3 Analyticity in MAN

3.3.1 Mathematical judgments as the model

In light of Kant's frequent comments on the virtues of mathematics in the first *Critique*, it is a natural starting point to say that his conception of mathematics influenced his conception of analytic

¹⁷²Kant departs here from Wolff (e.g., DM, §385) who denies that there can be a science [Wissenschaft] that is based purely on experience. We might think of this feature as an "empiricist" dimension of Kant's approach.

¹⁷³Cf.: "[w]e cannot have rational certainty of everything, but where we can have it, we must put it before empirical certainty" (Logik, 9:71).

judgment, in particular on proper knowledge claims.

Here is his characterization of the distinct nature of mathematical knowledge from the B Preface:

A new light broke upon the first person who demonstrated the isosceles triangle (whether he was called "Thales" or had some other name). For he found that what he had to do was not to trace what he saw in this figure, or even trace its mere concept, and read off, as it were, from the properties of the figure; but rather that he had to produce the latter from what he himself thought into the object and presented (through construction) according to a priori concepts, and that in order to know something securely a priori he had to ascribe to the thing nothing except what followed necessarily from what he himself had put into it in accordance with its concept. (Bxi-ii)¹⁷⁴

Kant distinguishes proper mathematical knowledge from an empirical investigation of diagrams, as well as from the analysis of a "mere" geometrical concept (as, for instance, Leibniz would do)¹⁷⁵. The mathematician neither investigates the properties of empirical figures nor analyzes the marks of a geometrical concept. Instead, she derives "secure" knowledge through analyzing the constructed concept in pure intuition. Kant refers to necessary propositions that follow in accordance with the constructed concept.

Before I can unpack the underlying conception of true judgment here, we need to understand Kant's conception of mathematical definition.¹⁷⁶ On his view, the mathematical definition always comes first. There is no mathematical concept prior to the definition. Accordingly, there is no proper mathematical knowledge prior to the definition of the concept. Here is Kant's view on mathematical definitions from the *Doctrine of Method*:

Thus there remain no other concepts that are fit for being defined than those containing an arbitrary synthesis which can be constructed a priori, and thus only mathe-

¹⁷⁴There is some debate on whether Kant wanted to refer to equilateral or isosceles triangles here. For my purposes, that has little relevance. My concern in this passage is his description of what secure geometrical knowledge consists in.

¹⁷⁵I discussed Leibniz's view on this in Chapter 2.

¹⁷⁶I gave a more detailed presentation and references on Kant's view of geometrical concepts in Chapter 2.

matics has definitions. For the object that it thinks it also exhibits a priori in intuition, and this can surely contain neither more nor less than the concept, since through the explanation of the concept the object is originally given, i.e., without the explanation being derived from anywhere else. (A729-30/B757-58)

Mathematical concepts can be defined, they involve an “arbitrary synthesis which can be constructed a priori”. The thought is that the object that corresponds to a mathematical concept can be exhibited in pure intuition, prior to considering particular experiences of figures. This view on the relation between mathematical concepts and their objects explains why Kant takes mathematical definitions as paradigmatic instances of secure cognition of mathematical objects, namely, as real definitions. Real definitions, for Kant, are “ones that suffice for cognition of the object according to its inner determinations, since they present the possibility of the object from inner marks” (Logik, §106).

With this background on mathematical concepts, we can now turn to Kant’s comments on analytic judgments in mathematics:

To be sure, a few principles that the geometers presuppose are actually analytic and rest on the principle of contradiction [...] e.g., $a = a$, the whole is equal to itself, or $(a + b) > a$, i.e., the whole is greater than its part. And yet even these, although they are valid in accordance with mere concepts, are admitted in mathematics only because they can be exhibited in intuition. What usually makes us believe here that the predicate of such apodictic judgments already lies in our concept, and that the judgment is therefore analytic, is merely the ambiguity of the expression. We should, namely, add a certain predicate to a given concept in thought, and this necessity already attaches to the concepts. But the question is not what we should think in addition to the given concept, but what we actually think in it, though only obscurely, and there it is manifest that the predicate certainly adheres to those concepts necessarily, *though not as thought in the concept itself, but by means of an intuition that must be added to the concept.* (B16-7, my emphasis)

The overall sentiment we should get from this passage is that the difference between a mathematical concept conceived merely as a list of conceptual marks and one that is constructed in pure intuition is subtle but crucial for Kant's conception of analytic judgment.

Judgments like $A=A$ are valid in "accordance with mere concepts", says Kant. What he means is that the certainty of the connection rests on the identity of the predicate with the subject concept; the judgment is thereby valid in accordance with two concepts ("this necessity already attaches to the concepts"). This differs from synthetic judgments where the necessity attaches not merely to the concepts but to something else as well.¹⁷⁷ Second, Kant notes that the predicate adheres to the intuition that corresponds to the subject concept, not the "mere" concept.¹⁷⁸ To use the ground terminology, we can say that for analytic judgments in mathematics, the ground is the constructed subject concept.

To make this conception of analytic judgment more vivid, I want to discuss a geometrical example. Kant defines a triangle as "a figure enclosed by three straight lines" (A716/B744). As a consequence, he would say that the apodictic certainty of the judgment that a triangle has three straight lines is based on the identity of the predicate three straight lines with the concept triangle. It is crucial to recognize that the predicate three straight lines adheres with certainty to the intuition that corresponds to the concept triangle, which precedes a recognition of the proper marks of the concept triangle.¹⁷⁹ Therefore, the analytic judgment in question, in a more precise formulation, is:

Geometry A triangle has three straight lines in (metaphysical) space.

Let me summarize Kant's conception of analytic judgment in regards to mathematics. A judgment whose certainty rests on the identity of the predicate with the constructed subject concept (in accordance with its definition) is called analytic.

¹⁷⁷I will discuss synthetic judgments in 3.5.

¹⁷⁸Kant makes the same general point when he notes regarding mathematics that "it is occupied, to be sure, with objects and cognitions only so far as these can be exhibited in intuition. This circumstance, however, is easily overlooked, since the intuition in question [gedachte Anschauung] can itself be given a priori, and thus can hardly be distinguished from a mere pure concept" (A4/B8).

¹⁷⁹See Logik, 9:63 for Kant's point on the priority of a synthesis (i.e. construction) of the concept over the recognition of its proper marks in mathematics and natural philosophy.

3.3.2 Analytic judgments in MAN

In the following, I want to elaborate how Kant's conception of analytic judgment in MAN is modeled on the geometrical case. Kant's disciple Jacob Sigismund Beck in his Explanatory abstract (1794) for Kant's *Metaphysical Foundations of Natural Science* provides a very useful articulation of what I take to be Kant's view.¹⁸⁰ So my approach is to develop Kant's general view through a discussion of a passage from Beck's Explanatory abstract and then show how such a view fits with Kant's fundamental analyticities in the *Phoronomy* and *Dynamics*.

Here is Beck (1794) in the *Introduction* of his commentary on MAN:

Although the concept matter is empirical and distinct from the original concepts of the understanding, this object can only be thought in terms of those conditions of all cognition. We reach this systematic unity if we represent what is object of the outer senses in accordance with the table of categories, and accordingly analyze this concept.

(pp.393-4)¹⁸¹

Beck describes an analysis of the concept matter in accordance with the table of categories. It involves to "think" the object of outer sense in terms of the "conditions of all cognition". The underlying thought is that the table of categories provides the rules for a "systematic" analysis of the concept matter.

J.S. Beck continues on the relation of the intended analysis of the concept matter to mathematical definitions:

We will use the mathematical method in this project. What is distinct about this method is that the concepts are arbitrarily made through the definition. It might hence

¹⁸⁰Kant recommended the mathematician J. S. Beck for the task of writing an explanatory abstract of his critical writings to the publisher Hartknoch (11:289). Kant and Beck had frequent exchanges on this topic in the following, which indicate Beck's deep level of comprehension of complicated but crucial issues related to Kant's transcendental philosophy and his view on physics (see 11:376-7; 11:313-5; 11:360-65; 11:394-5). The first volume of the Abstract concerns the first and second Critique and came out in 1793. The third Critique and the *Metaphysical Foundations of Natural Science* are treated in the second volume from 1794. Beck's text on MAN has about 150 pages. The first two volumes of the Explanatory abstract were published with the recommendation of Kant (11:338), as indicated on the cover page of the books. All in all, I think Beck's Explanatory abstract is a useful source to clarify Kant's thinking in MAN.

¹⁸¹All translations of Beck are mine.

appear that this method cannot be applied to the object [of outer sense] that we treat because we deal here with given concepts whose marks are determined by the object. Such an exposition needs to be demonstrated; it does not involve an arbitrary synthesis. (p.394)

Beck describes Kant's view on mathematical definitions and asks whether such a view can be useful for the concept matter. He observes that the concept matter is not made through the definition, its explication presupposes an empirical investigation (its "exposition needs to be demonstrated").

Still, Beck thinks that there is a crucial sense in which the explications of the concept matter resemble mathematical definitions:

Our science [Wissenschaft] can nevertheless start with definitions [Definitionen]. Just like definitions in mathematics, though for a different reason, it must not prove those definitions. The reason for this is that the explications of the concept matter should capture those marks that are immediately [unmittelbar] thought in the action through which the manifold of the object is connected through the categories. Those definitions will hence agree with geometrical axioms in that they are indemonstrably certain [unerweislich gewisse] propositions, although they are analytic propositions, while the latter [i.e. geometrical axioms] are synthetic. (pp.394)

For Beck, the definitions of the concept matter that come at the beginning of "science" share one crucial feature with mathematical definitions. They consist of marks through which the object of the concept can be securely cognized. In the specific case, marks that reflect the categories's action in determining the object of the concept matter. In analogy with analyticities in mathematics, we can say that their truth rests on the apodictic certainty of the identity of the predicate with the constructed subject concept. For Kant's approach in MAN, a constructable concept, I suggest, coincides with the combined contribution of pure understanding and pure intuition to the empirically determined representation of the concept matter.

To make this proposal more concrete, I turn to Kant's first two fundamental definitions of the

concept matter in MAN.¹⁸² The starting point for Kant's analysis of the concept matter is his characterization of the object of the concept matter in terms of the empirical concepts motion, impenetrability, and inertia.¹⁸³ At this initial stage of Kant's investigation, the judgment that matter is movable amounts to empirically certain holding-to-be-true or improper knowledge, just as we saw for the proposition that gold is yellow. Recall that the truth of such a proposition consists in the empirical certainty of the identity of the predicate concept with the subject concept.

Kant, however, thinks that for the concept matter, there are also instances of apodictically certain holding-to-be-true, of proper knowledge/science. His fundamental Explications belong to this category:

Phoronomy Matter is the movable in space. (Explication 1, MAN, 4:480)¹⁸⁴

Dynamics Matter is the movable insofar as it fills a space. To fill a space is to resist every movable that strives through its motion to penetrate into a certain space. (Explication 1, MAN, 4:496)¹⁸⁵

Both judgments reflect the features that J.S. Beck attributes to Kant's physical definitions. Movability in space is an immediately certain mark of the concept matter because it coincides with the action of the schematized category quantity in space (i.e., the composition of the object in regards to homogeneous spatial units) in the representation of the concept matter.¹⁸⁶ The thought is that the representation of matter in terms of relative motion (in empirical intuition) has an apodictically certain part: motion along a straight line of a certain length in a certain amount of time. So Kant's phoronomical explication is a case of proper knowledge, its certainty rests on the identity of the mark movability in space with the constructable concept matter (in terms of the category quantity).

¹⁸²Kant (e.g., MAN, 4:536) sometimes uses the term "definition" for his fundamental explication in the *Phoronomy*, *Dynamics*, and *Mechanics*.

¹⁸³See, for instance, Prol, 4:295; MAN, 4:482, 4:513. I discussed this aspect in Chapter 1.

¹⁸⁴The German text reads: "Materie ist das Bewegliche im Raume".

¹⁸⁵In German: "Materie ist das Bewegliche, so fern es einen Raum erfüllt".

¹⁸⁶On the category quantity, Kant says: "The pure schema of magnitude (quantitatis), however, as a concept of the understanding, is number, which is a representation that summarizes the successive addition of one (homogeneous) unit to another" (A142/B182).

A similar thing can be said for Kant's fundamental dynamical explication that matter is the movable insofar as it fills a space. The explication captures the action of the category quality in space (i.e., an intensive magnitude, a degree in space) in regards to the concept impenetrability, which is a part of the cognition of the object of the concept matter.¹⁸⁷ Kant will conceive of this filling of a space in terms of an opposition of the previously specified motions in pure intuition.¹⁸⁸ The truth of the dynamical explication thereby rests on the certainty of the identity of the mark filling a space with the constructable concept matter, here, in terms of the category quality.¹⁸⁹

The following can be said in connection with Kant's characterization of analytic judgment in the first *Critique*. For both explications, the predicate is "contained" in the constructable part of the concept matter. That is, both predicates are intuitive (movable *in space*, fill a *space*), which explains why their connection with the subject concept comes with a consciousness of necessity, to use Kantian terminology. The thought is that it would be a contradiction with respect to the concept matter (Kant presupposes) if there was no trajectory of the object in space (given that it is a universal property of matter that it moves), or if it would not fill a determinate space (given that matter is impenetrable).

Those explications also satisfy the desideratum that they "clarify" the concept matter: they involve predicates that were already thought in the concept, albeit not clearly. More precisely, the explications involve similar concepts as the concept matter that Kant takes from empirical physics, i.e. motion, impenetrability, and inertia. Still, the predicates in Kant's explications in MAN coincide with those concepts only to the extent that they reflect the action of the categories

¹⁸⁷Here is Kant on the category quality and its schema: "Hence there is a relation and connection between, or rather a transition from reality to negation, that makes every reality representable as a quantum, and the schema of a reality, as the quantity of something insofar as it fills time, is just this continuous and uniform generation of that quantity in time" (A143/B182-3). See also A166-9/B207-210. For our specific case, the reality in question are impenetrable bodies, which involves for Kant, among other things, the representation of opposing motions (which reflect opposing forces) in space and thereby an intensive magnitude.

¹⁸⁸Cf.: "in the sensible intuition in which reality (e.g., motion) is given, there are conditions (opposed directions) [...] that make possible a conflict" (A282/B338).

¹⁸⁹The concept to fill a space (conceived in MAN in terms of an opposition of the previously specified motions in space) is not the same as the concept impenetrability (see A173-6/B215-8). The former captures merely the intensive quantity of impenetrability and can therefore be cognized a priori with certainty. The concept impenetrability, as other qualities of sensation like warmth and color is empirical. Kant, for instance, says in this context: "in all quality (the real of appearances) we can cognize a priori nothing more than their intensive quantity, namely that they have a degree, and everything else is left to experience" (A176/B218).

in space. For instance, the judgment that matter is the movable in space clarifies on Kant's view the representation of matter as movable in empirical intuition. Kant also puts this by saying that analyticities increase our cognition in a formal, not material sense.¹⁹⁰

I now want to highlight one crucial sense in which the developed Kantian account of analytic judgment in mathematics and natural science differs from the Leibnizian view. At the heart is a difference in how to account for certainty. Leibniz draws on identities at the fundamental level. Kant's notion of analytic judgment also depends on identities, but, to my mind, not at the fundamental level. For Kant, there is a single cognition of the object, one that consists in a "synthesis" of different representations,¹⁹¹ a cognition that precedes the derivation of its proper marks.¹⁹² This feature is an essential aspect of Kant's *critical* view on analytic judgments.

As we saw in Kant's treatment of mathematics and natural science, he centrally relies on construction in pure intuition. For mathematics, the definition comes first and involves a construction of the object in pure intuition, an a priori or "arbitrary" synthesis of different representations (e.g., different shapes) according to Kant.¹⁹³ Proper mathematical knowledge is secondary to the definition.¹⁹⁴ For proper knowledge in regards to natural science, a synthesis of the empirical object that corresponds to the concept matter logically precedes the explications in MAN, which extract the intuitive elements of that empirical cognition. Overall, what that suggests is that Kant accounts for the certainty of knowledge in terms of (pure) intuition, which has a different part-whole structure than a concept.¹⁹⁵ His view thereby differs from Leibniz who accounts for the certainty of true

¹⁹⁰Kant suggests in his *Logik* that analytic propositions increase cognition "merely formaliter" (9:111).

¹⁹¹Cf.: "By synthesis in the most general sense, however, I understand the action of putting different representations together with each other and comprehending their manifoldness in one cognition" (A77/B103).

¹⁹²Here is one of the first passages on "synthesis" in the body of the first *Critique*: "Prior to all analysis of our representations these must first be given, and no concepts can arise analytically as far as the content is concerned. The synthesis of a manifold, however, (whether it be given empirically or a priori) first brings forth a cognition, which to be sure may initially still be raw and confused, and thus in need of analysis; yet the synthesis alone is that which properly collects the elements for cognitions and unifies them into a certain content; it is therefore the first thing to which we have to attend if we wish to judge about the first origin of our cognition" (A77-8/B103, my emphasis).

¹⁹³Recall A729-30/B757-5, as quoted above.

¹⁹⁴On the suggested view, Kant's seemingly mysterious formulation concerning mathematical knowledge that the predicate is "added" to the subject concept as in: "What usually makes us believe here that the predicate of such apodictic judgments already lies in our concept, and that the judgment is therefore analytic, is merely the ambiguity of the expression. We should, namely, add [*hinzudenken*] a certain predicate to a given concept in thought, and this necessity already attaches to the concepts" (B17, my emphasis, as quoted above) makes some sense.

¹⁹⁵See A25/B39-40. I will look at this point on the priority of a unified cognition that involves a synthesis over proper

judgments in terms of an equality of the predicate with a constituent of the subject concept.

3.4 Compatibility of the three criteria

What has been implicit in my discussion so far and deserves to be made more explicit is the relation of the three criteria for analyticity to one another. In my view, the three criteria are independent and compatible. In the literature, such a move is typically resisted. One influential thought has been that Kant's containment criterion (and with that his conception of analyticity) is a logical notion as understood by Leibniz and Wolff, concerned essentially with the component concepts of the subject concept. On such a view, it is, for instance, hard to see how the containment criterion could be compatible with the third criterion on whether the judgment clarifies or amplifies our knowledge of its real object.

Allison (2004, pp.91-2), for instance, says:

since the truth or falsity of the [analytic] judgment is determined merely by analyzing the concept of the subject, the reference to the [common] object *x* is otiose. [Footnote that refers to L.W. Beck's (1955) paper.] That is why it is possible to form analytic judgments about non-existent, even impossible objects, and why all analytic judgments are a priori. [...] Kant's conception of analyticity [...] rests upon his conception of a concept as a set of marks (themselves concepts), which are thought together in an "analytic unity," and which can serve as a ground for the cognition of objects.

Allison attributes to Kant the view that the truth of an analytic judgment can be recognized through an analysis of the component concepts that jointly constitute the subject concept. As a consequence, no reference to the common empirical object of the two concepts is required. This characteristic indicates according to Allison that analytic judgments can be formed about other kinds of objects like "non-existent" ones.¹⁹⁶ In essence, Allison thereby suggests that sensibility does not ground the truth of an analytic judgment.

knowledge from another angle in the next section.

¹⁹⁶I return to this point towards the end of this section.

My previous argument, contrary to Allison's (and L.W. Beck's) point, suggests that the truth of the explications of the concept matter depends in part on the common, empirical object. More specifically, their truth depends on intuitive components of the concept matter. To see this point more clearly, it is useful to zoom out a bit and understand the way Kant thinks about the relation between the empirical concept matter and its corresponding judgments.

In general, Kant starts his treatment of concepts and judgments with the formation and/or justification of concepts. The formed concepts are then used to derive judgments. For instance, in the *Analytic* of the first *Critique*, Kant first introduces and justifies the applicability of the categories ("The Analytic of Concepts", A65/B90ff.). After that, he articulates certain principles that he derives from those concepts (The "Analytic of Principles", A130/B169ff.).¹⁹⁷ To transition from concepts to judgments, Kant introduces two principles in-between his justification of the categories and his derivation of the principles of the understanding, a "supreme principle of all analytic judgments" (A150/B189ff.) and a "supreme principle of all synthetic judgments" (A154/B193).

The principle of all analytic judgments is the principle of contradiction. Kant says:

if the judgment is analytic [...] its truth must always be able to be cognized sufficiently in accordance with the principle of contradiction. For the contrary of that which as a *concept already lies and is thought in the cognition of the object* is always correctly denied, while the concept itself must necessarily be affirmed of it, since its opposite would contradict the object. Hence we must also allow the principle of contradiction to count as the universal and completely sufficient principle of all analytic cognition. (A151/B190-1, my emphasis)

Kant suggests that the truth of an analytic judgment can always be cognized in accordance with the principle of contradiction. The application of the principle allows us to recognize concepts that are "thought in the cognition of the object". It is key to notice that the principle of contradiction allows us to draw out concepts from the cognition of the object. While the Leibnizian would say

¹⁹⁷Similarly, in the main body of the *Logik*, Kant (or rather Jäsche) begins with a discussion of different kinds of concepts (§§1-4), their formation and content (§§5-16), and then moves on to discuss judgments (§§17-40). After that, he treats inferences (§§41-93).

that the component concepts are logically prior to the cognition of the object, for Kant, one unified cognition of the object is fundamental.¹⁹⁸ This is one way to see that containment cannot be Kant's fundamental criterion, but is rather logically on a par with the principle of contradiction.

The overall procedure for analytic judgments of empirical concepts like matter hence works like this. First, we form the empirical concept, it "arises from the senses through comparison of objects of experience" (Logik, §3), says Kant.¹⁹⁹ With the concept in hand, we form judgments about what the cognition of the object contains, both with empirical and apodictic certainty.

It is highly informative to compare Kant's conception here with Leibniz's approach. Consider, for instance, Leibniz's comment in the *New Essays*:

Intellectual ideas, from which necessary truths arise, do not come from the senses [...] Now, it is true that explicit knowledge of truths is subsequent (in temporal or natural order) to the explicit knowledge of ideas; as the nature of truths depends upon the nature of ideas, before either are explicitly formed, and truths involving ideas which come from the senses are themselves at least partly dependent on the senses. But the ideas that come from the senses are confused; and so to, at least in part, are the truths which depend on them. (NE, p.81)

Leibniz and Kant share the overall view that knowledge of concepts precedes explicit knowledge of true judgments.²⁰⁰ Their views differ on the status of sensible and intellectual components. Leibniz distinguishes intellectual ideas (and their corresponding truths), which are distinct, from sensible ideas (and their corresponding truths), which are confused.

Kant, in my view, transforms Leibniz's distinction between intellectual and sensible ideas (and their corresponding truths) in a distinction between empirical concepts that have *constructable* component concepts and those that do not have them. The former concepts give rise to necessary

¹⁹⁸We saw this, for instance, for the case of mathematics in the previous section.

¹⁹⁹See also Logik, §6 on this. I argued in Chapter 1 that for the formation of empirical concepts, Kant relies on representations that conform to the categories and his forms of sensibility, i.e. "cognitions" as specified on the fourth stage of the hierarchy of logical perfection. In essence, it means that we consider representations of persisting substances that change their state in accordance with cause and effect relations, and so on. And not entities that, say, come into and go out of existence all the time.

²⁰⁰As argued above, what Kant and Leibniz mean by the subject concept differs.

truths because the corresponding analyticities reflect the concept's relation to intuition, and thereby its necessary components. The partial concepts of the concept matter treated in the four chapters of MAN belong to this category and give rise to necessary judgments.

On the other side, there are empirical concepts that have no constructable component concepts. An example would be the concept gold. Such a concept gives rise to (at best) empirically certain judgments. Kant would not say that those kinds of concepts and their corresponding truths are confused, as Leibniz speaks about sensible ideas. Rather, Kant would say that those concepts and their corresponding judgments, like gold is a yellow metal, do not come with a consciousness of necessity.

My discussion demonstrates that Kant's containment criterion requires a reference to the real object of the concept.²⁰¹ Since the constructable component concepts manifest only the action of the categories in combining the empirical object of the concept matter, Kant's analyticities in MAN have no subjective flavor, as scholars often claim. To say, as Maaß (1789, p.189) did, that whether a given judgment is analytic or synthetic is relative to what a particular reasoner includes in their subject concept is wrong for Kant's explications in MAN. The two discussed explications are both grounded on the understanding, that is, a specific category, which for Kant involves its function in unifying empirical intuition. Since those categories are general conditions of experience and valid for everyone, there is no question of whether or not all subjects would recognize those judgments as analytic. While Kant grants that empirical physics progresses and may give rise to additional proper knowledge over time, the judgments defended in MAN continue to be considered apodictically certain and necessary. With such a notion of containment at play, Kant is right to think that the three criteria for the analytic/synthetic distinction are closely connected. All three criteria revolve around his distinct ways of thinking about concepts of empirical objects, concepts that are formed in accordance with his overall argument in the body of the first *Critique*.

The necessary reference to the common object for analytic judgments can also be motivated through Kant's distinction between theoretical and practical knowledge. "Those propositions that

²⁰¹I will distinguish the reference to the real object in analytic judgments from Kant's treatment of (parts of) the real object as a condition/reason for synthetic judgments in the next section.

relate to the object and determine what belongs or does not belong to it are called theoretical”, says Kant (Logik, §32).²⁰² This conception of theoretical knowledge, it seems to me, indicates again that analyticities in regards to empirical concepts like matter cannot do without a reference to an empirical object. As a result, I say that the analyticities about non-existent and impossible objects that Allison mentions have a different status than the analyticities about empirical objects that Kant treats in MAN.

3.5 Synthetic judgments

To complete my discussion of the analytic/synthetic distinction, I want to clarify Kant’s appeal to the intuition of the concept as a *condition* for the truth of synthetic judgments.

Here are three passages:

1. the principle of synthetic judgments in general, which follows necessarily from their definition: that they are not possible save under the condition of an intuition underlying the concept of their subject, which, if they are judgments of experience, is empirical, and if they are synthetic judgments a priori, is pure intuition a priori. (OD, 8:241)
2. The supreme principle of all synthetic judgments is, therefore: Every object stands under the necessary conditions of the synthetic unity of the manifold of intuition in a possible experience. [...] The conditions of the possibility of experience in general are at the same time conditions of the possibility of the objects of experience. (A158/B197)
3. In synthetic propositions the relation among the concepts is not properly represented immediately (for this occurs only in analytic propositions), but only in the conditions of their concrete representation in the subject, whether intuition or appearance. [...] x always signifies the object of the concept a. But there can be no object except that of either pure or empirical intuition. As far as the latter is concerned, the concept can pertain either to a given object of the senses x or to conditions of sensibility under which an object, insofar as it merely corresponds to the concept x, must be given, and under which alone it can be cognized as subject to a. (Note 4674, from the Duisburg Nachlass, 17:644-5)

In his reply to Eberhard (OD), Kant defines synthetic judgments in terms of “the condition of an intuition underlying the concept of their subject”. In the passage from the first Critique, Kant

²⁰²On the distinction between theoretical and practical knowledge, see also Bix-x; 5:19-20, 44. As far as I understand, Allison (2004, e.g. p.93) takes seriously the idea that it matters that we consider theoretical as opposed to practical reason only in his account of synthetic judgments, not for analytic judgments.

makes more precise what the nature of that intuition is. It is distinct and unified.²⁰³ In Kantian terms, the intuition of the concept that can serve as a condition for synthetic judgments conforms to the conditions of the “synthetic unity” of experience, the conditions of proper “cognition”.

As we see in the first and third passage, the condition for a synthetic judgment can either be an empirical or a pure intuition. Geometry is the paradigmatic case for pure intuition serving as a condition of synthetic judgments. To repeat, the syntheticity of geometry can be traced (either) to the diagram—especially auxiliary constructions (in analogy with Euclid’s reasoning)—or general facts about the phenomenology of mathematical objects in pure intuition.²⁰⁴ In the *Duisburg Nachlass*, Kant accordingly characterizes synthetic judgments as involving a relation among concepts that cannot be represented “immediately” (as in analytic judgments) but in the conditions of their “concrete representation”.

The passage from the *Duisburg Nachlass* also bifurcates the grounding role of empirical intuition in two cases. We either rely on a “given object of the senses” or the “conditions of sensibility under which an object, insofar as it merely corresponds to a concept x, *must* be given, and under which it can be cognized as subject to a” (my emphasis). I suggest that the latter case concerns Kant’s synthetic judgments in MAN.²⁰⁵ In light of the empirical nature of the concept matter, it is an empirical intuition that serves as a condition for those judgments. However, what makes the Propositions in MAN special (i.e. synthetic *a priori*) is that they involve only those conditions of sensibility under which the object of the concept matter *must* be given.²⁰⁶

We see this approach at work in Kant’s proof of the Proposition in the *Phoronomy*. Here is the proposition:

The composition of two motions of one and the same point can only be thought in such a way that one of them is represented in absolute space, and, instead of the other, a motion of the relative space with the same speed occurring in the opposite direction

²⁰³In opposition to, say, an indistinct perception.

²⁰⁴See Chapter 2 for discussion.

²⁰⁵The former case concerns judgments of experience, which are by default synthetic *a posteriori* for Kant.

²⁰⁶This is a point where the modified conception of apriority defended in Chapter 1 helps to see how Kant’s view can be given a consistent and interesting interpretation.

is represented as the same as the latter. (MAN, 4:490)

In the first Remark to this proposition Kant, I suggest, makes clear that we cannot think of his proof as “geometrical”, as drawing on pure intuition as a condition.²⁰⁷ More specifically, the proof does not establish an identity in pure intuition—which he calls congruence.²⁰⁸ To understand this, it is worth recalling Kant’s strategy for geometric propositions. On his view, the demonstration for Euclid I.32 relies on claims about the identity of angles, ultimately, that the interior angles of a triangle are identical to two right angles.²⁰⁹

In regards to the Proposition in the *Phoronomy*, Kant notes in his Remark:

Now this congruence of two combined motions with a third (as with the motus compositus itself) can never take place if these two combined motions are represented in one and the same space, for example, in relative space. Therefore, all attempts to prove the above Proposition in its three cases were always only mechanical analyses – namely, where one allows moving causes to produce a third motion by combining one given motion with another—but not proofs that the two motions are the same as the third, and can be represented as such a priori in pure intuition. (MAN, 4:493)

I take this comment to suggest that his derivation of the Proposition relies on those “conditions of sensibility” under which the object of the concept composite motion must be thought. Those conditions can be represented “as such a priori in pure intuition”, remarks Kant. This procedure differs from conditioning on pure intuition itself (and establishing an identity in terms of a congruence) as appealed to in his conception of geometry because the combination of motions presupposes moving causes, which cannot be represented in pure intuition. Kant’s proposition thereby has a hypothetical character. If two uniform, rectilinear motions (and their underlying causes) exist, the combination of those motions follows a version of the parallelogram law.

²⁰⁷Contrary to Friedman (2013, p.67).

²⁰⁸Cf.: “Geometrical construction requires that one quantity be the same as another or that two quantities in composition be the same as a third, not that they produce the third as causes, which would be mechanical construction. Complete similarity and equality, insofar as it can be cognized only in intuition, is congruence. All geometrical construction of complete identity rests on congruence” (MAN, 4:493).

²⁰⁹See A716/B744 ff.

In more detail, we can think of the syntheticity in the following way. Kant's proposal involves a distinct way in which the Proposition consists of a "mediate" relation between two motions:

It is impossible to think two equal motions in the same body in opposite directions *immediately, that is, in relation to precisely the same space at rest*. But the representation of the impossibility of these two motions in one body is not the concept of its rest, but rather of the impossibility of constructing this composition of opposite motions, which is nonetheless assumed as possible in the Proposition. (MAN, 4:494, my emphasis)

this construction is possible in no other way than through the mediate [mittelbare] composition of two equal motions. (MAN, 4:494)

So the motions that are combined have different relative spaces, they cannot be represented in relation to the same space. Their connection is for this reason not immediate. The underlying point is that the combination of motions has a temporal element. We consider two uniform rectilinear motions successively, not simultaneously.

Overall, Kant's argument relies on the a priori conditions of sensibility: facts about objects in space (like orientation and equalities of shapes) and what he calls "modi of time", in essence, succession,²¹⁰ not specific points in time. Accordingly, a key ingredient appears to be the second Analogy, which guarantees that the combined motion can always be generated, that is, that there is a corresponding moving force. To appeal to such conditions preserves the apodictic certainty of the proposition, on Kant's view. The proposition is thereby synthetic a priori, its truth is grounded on intuition.

²¹⁰"The three modi of time are persistence, succession, and simultaneity. Hence three rules of all temporal relations of appearances, in accordance with which the existence of each can be determined with regard to the unity of all time, precede all experience and first make it possible" (A177/B219).

Abbreviations

For Kant's texts, I mainly rely on the translations from the Cambridge Edition and cite according to the Academy Edition, except for quotations from the first Critique and the numbered sections from Kant's Logik. The following abbreviations will be used:

Logik Jäsche Logic

MAN Metaphysical Foundations of Natural Science

Prol Prolegomena

OD On Discovery

For Leibniz's texts, I use the following abbreviations:

DM Discourse on Metaphysics (1686)

NE New Essays on Human Understanding (1765)

NI Nature Itself (1698)

SD Specimen Dynamicum (1695)

For Wolff's texts, I use the following abbreviations:

DM German Metaphysics ("Deutsche Metaphysik")

PD Preliminary Discourse on Philosophy in General

For Newton's texts, I use the following abbreviations:

P Principia

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