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I. INTRODUCTION

It has now become apparent that the strength of the "strong nuclear interactions" of the strange particles must be considered comparable to that of the pion-nucleon and nucleon-nucleon interactions, despite the fact that strange-particle production occurs with relatively small branching ratio even at the highest energies studied. This conclusion is particularly evident from the data on Λ-hypernuclear binding energies[1] and on the low-energy reaction processes[2] for strange particles, as will be pointed out again in Sections III and V. It is also indicated by the occurrence of resonances with substantial half-widths in a number of strange-particle systems, for example in the K^--p system[3] at approx 1850 Mev, the $\bar{K}$-π system[4] at approx 875 Mev, and the π-Λ system[5,6,7] at approx 1485 Mev. With such strong interactions, the restrictions of the unitarity condition --that is, of probability conservation--on the cross sections for competing processes and on their energy dependence are of the greatest importance, as we know from experience in the nonrelativistic domain of low-energy nuclear reaction phenomena. Their effect on the energy dependence of reaction amplitudes and cross sections is especially marked in the neighborhood of

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new two-particle thresholds where, for example, they give rise to the cusp behavior now observed[8] particularly in the angular distribution of the $\pi^- + p \rightarrow \Lambda + K^0$ reaction at the $\Sigma + K$ threshold.

In this situation it is particularly appropriate to describe these reaction processes in terms of the elements of the reaction matrix K, since the scattering matrix constructed from them necessarily satisfies the unitarity conditions. The theory appropriate to the definition and application of the reaction matrix is reviewed briefly in Section II, with special reference to coupled two-particle systems and to the occurrence of and description of resonant states in this formalism. In Section III (and Appendix A), the analysis of the low-energy K^--proton data is reviewed in terms of this formalism, and the "$\bar{K}$-nucleon virtual bound state" interpretation of the π-Λ resonance is compared with the experimental data.

Although the reaction-matrix formalism is quite general, it has proved convenient in the dispersion-theory treatment[9,10,11] of elementary particle processes to adopt a specific method of solution, which involves separating the scattering matrix for a state of definite angular momentum and parity into two factors ND^{-1}, each with characteristic analytic properties as functions of the barycentric energy. The function D may be determined explicitly from N, and the singularities of N are directly related to the dynamical mechanisms which lead to the observed processes. The function N either may be regarded as a function to be determined experimentally or, more satisfactorily, it may be used to include in the form of the scattering amplitudes those specific features which would arise from particular mechanisms that could influence these reaction processes. The use of this formalism, and its relationship with the reaction-matrix formalism, is discussed briefly in Section IV, with special reference to the description of resonant states. These remarks are illustrated in Appendix B by discussion of a simple example.

In Sec. V, the present evidence bearing on the validity of the global symmetry hypothesis of Gell-Mann[12] and Schwinger[13] is reviewed, and the interpretation of the π-Λ resonance as an analogue of the (3,3) isobar state in the π-N system is discussed and compared with the data.

II. THE REACTION MATRIX AND RESONANCES FOR MULTICHANNEL SYSTEMS

We consider explicitly a system with n two-particle channels labeled $i = 1, \cdots, n$, channel i describing a spinless and a spinor particle of rest masses m_i, M_i respectively, with c.m. momentum k_i. For the total energy E we have, then,

$$E = (m_i^2 + k_i^2)^{1/2} + (M_i^2 + k_i^2)^{1/2} . \tag{2.1}$$

The elements of the reaction matrix K are now defined in terms of the asymptotic form of the wavefunction for an incident wave of unit amplitude in one channel, together with standing waves in all channels. Explicitly, with orbital angular momentum ℓ_i for channel i, the elements of K are defined by the form of the wavefunction $\psi_j^{(i)}$ for the $\underline{j}$th channel:

$$\psi_j^{(i)}(r) \sim \delta_{ij} \frac{\sin (k_j r - \ell_j \frac{\pi}{2})}{k_j r} + \Lambda_j K_{ji} \Lambda_i \frac{\cos (k_j r - \ell_j \frac{\pi}{2})}{r} , \tag{2.2}$$

where Λ_i is a normalization coefficient given by $(\pi \rho_{ii}/k_i)^{1/2}$ and ρ_{ii} is the $\underline{i}$th diagonal element of the phase-space density (which is diagonal in the present representation), namely:

$$\rho_{ii} = \frac{M_i}{E} \cdot \frac{k_i}{\pi} . \tag{2.3}$$

The wavefunction (2.2) corresponds to a configuration in which there is an incident wave (the part of $\exp(i\underline{k}_i \cdot \underline{r})$ with orbital angular momentum ℓ_i) in channel i, together with a standing wave of cosine form in all channels. The matrix element K_{ij} may be written in the form

$$K_{ij} = k_i^{\ell_i} R_{ij} k_j^{\ell_j} , \tag{2.4}$$

where the two energy-dependent factors remove the leading term of the energy dependence of K_{ij} in the neighborhood of the threshold energies E_i, E_j for channels i and j. When both channels i, j are open, that is when $E > E_i$, E_j, we have, from the hermiticity of the Hamiltonian,

$$K_{ij} = K_{ji}^{*} . \tag{2.5}$$

If, in addition, the Hamiltonian is invariant with respect to time reversal, the elements of the K matrix are real, and K is a real symmetric matrix in this region.[16]

The K-matrix elements are analytic functions of the total energy, with a branch cut at each threshold energy E_i where ℓ_i is odd. The elements R_{ij} are real and do not have these branch cuts at threshold energies. When the energy is sufficiently far below threshold in channel i, even the R_{ij} will generally become complex when the dynamical singularities are reached:[17] this last point is brought out rather clearly in the dispersion formalism discussed in Section IV. It is in dealing with this region of unphysical energy values that the dispersion formalism has great advantage over the present discussion and there is no doubt that this formalism, or some modification of it in the same spirit, will become the more appropriate procedure for the discussion of the more complicated situations which will arise in the future.

For multiparticle channels, the defining boundary conditions are more conveniently expressed in momentum-space variables (see Ref. 14). The energy E, the angular momentum, and the parity are then no longer sufficient to characterize the configuration completely, and further continuous variables are necessary to characterize the sharing of the total energy and the total angular momentum among the particles. In this case the label i becomes continuous and the reaction matrix becomes an integral operator.

In principle, for complete theoretical expressions with the correct analyticity properties, it is necessary to give the K-matrix elements for all possible channels, open or closed. In practice, however, we aim only to obtain expressions valid over some limited energy range; in this case, attention may be confined to the open channels and to those channels whose thresholds lie close to this energy range, as discussed by Dalitz and Tuan.[2] In the cases discussed here, the three-particle channels either are weak or have thresholds outside the range of interest, and we shall not have to consider multiparticle channels explicitly. For this reason, and because of the mathematical complexity of situations involving multiparticle channels, we shall not go into further detail about this here.[18]

The formal relation of the scattering matrix T to the reaction matrix K is given by the equation

$$T = K[1 - i\pi \rho K]^{-1} ; \tag{2.6}$$

where ρ denotes the matrix of phase-space densities. In terms of T, the cross section for the reaction $i \to j$ in a state of total angular momentum J and definite parity is then given by

$$\sigma(i \to j) = \frac{4\pi^2}{k_i} \cdot (J + \frac{1}{2}) \frac{M_i}{E_i} \left| \langle i \mid T \mid j \rangle \right|^2 \rho_{jj} . \tag{2.7}$$

For any two-particle channel i, the elastic scattering in this channel may be described by a complex phase shift δ_i, such that

$$\langle i \mid T \mid i \rangle = e^{i\delta_i} \sin \delta_i / \pi \rho_{ii} \quad , \tag{2.8}$$

or by a complex scattering length $A_i = a_i + ib_i$, such that

$$k_i \cot \delta_i = A_i^{-1} = (a_i + ib_i)^{-1} \quad , \tag{2.9}$$

and

$$\langle i \mid T \mid i \rangle = A_i/(1 - ik_i A_i) \; . \tag{2.10}$$

We will now distinguish channel i from the other channels f and subdivide the reaction matrix K as follows,

$$K = \begin{pmatrix} \alpha_i & \beta_i \\ \beta_i^{\dagger} & \gamma_i \end{pmatrix} \quad , \tag{2.11}$$

where α_i denotes $\langle i \mid K \mid i \rangle$, β_i denotes the row matrix whose elements and $\langle i \mid K \mid f \rangle$ for $f \neq i$, $\beta_i^{\dagger}$ its Hermitian conjugate (equal to the transpose of β_i , if time-reversal invariance holds), and γ_i denotes the submatrix obtained by excluding from K the row and column labeled i. In other words, β_i includes the elements of K describing transitions from i to all other channels, and γ_i consists of all those elements describing transitions between the channels f . Then the following expression[21] relates $a_i + ib_i$ to these matrices,

$$a_i + ib_i = \frac{M_i}{E} \{\alpha_i + i\pi \beta_i (1 - i\pi \rho_f \gamma_j)^{-1} \rho_f \beta_i^{\dagger}\} \; , \tag{2.12}$$

In a similar way, the transition amplitude may be expressed in the form

$$\langle i \mid T \mid f \rangle = \frac{1}{1 - ik_i A_i} \cdot \langle i \mid \mathcal{M}_i \mid f \rangle \; , \tag{2.13}$$

where the first factor results from the damping effect of the competing channels on the incident channel, and the second factor is the appropriate element of the transition matrix $\mathcal{M}_i$ from initial state i to all final states, which includes the effects of the scattering processes in the final states, namely:

$$\mathcal{M}_i = \beta_i (1 - i\pi\, \rho_f\, \gamma_i)^{-1} . \tag{2.14}$$

The imaginary part of A_i is directly related to $\mathcal{M}_i$, since

$$\begin{aligned} A_i &= \frac{M_i}{E} \{\alpha_i + i\pi\, \beta_i (1 - i\pi\, \rho_f \gamma_i)^{-1} \rho_f (1 + i\pi\, \gamma_i\, \rho_f)(1 + i\pi\, \gamma_i\, \rho_f)^{-1} \beta_i^{\dagger}\} \\ &= \frac{M_i}{E} \{\alpha_i + i\pi\, \mathcal{M}_i (\rho_f + i\pi\, \rho_f\, \gamma_i\, \rho_f) \mathcal{M}_i^{\dagger}\} , \end{aligned} \tag{2.15}$$

so that we have

$$b_i = \pi \frac{M_i}{E} \{\mathcal{M}_i\, \rho_f\, \mathcal{M}_i^{\dagger}\} = \pi \frac{M_i}{E} \Sigma_{f \neq i} \mid \langle\, i \mid \mathcal{M}_i \mid f\, \rangle \mid^2 \rho_f . \tag{2.16}$$

Finally, an important expression for the partial cross section for the reaction $i \to f$ may now be obtained, using (2.13) and (2.7),

$$\begin{aligned} \sigma(i \to f) &= (J + \tfrac{1}{2}) \frac{4\pi}{k_i} \cdot \frac{M_i}{E_i} \cdot \frac{1}{(1 + k_i b_i)^2 + (k_i a_i)^2} \cdot |\langle i\, | \mathcal{M}_i \,|\, f\rangle|^2 \rho_f , \\ &= \left\{ \frac{\pi}{k_i^2} (J + \tfrac{1}{2})(1 - \eta_i^2) \right\} \left\{ \frac{\frac{M_i}{E} |\langle i\, |\mathcal{M}_i \,|\, f\rangle|^2 \rho_f}{b_i} \right\} , \end{aligned} \tag{2.17}$$

where the last bracket may be abbreviated as $\{b_{if}/b_i\}$, and $b_i = \Sigma_{f \neq i}\, b_{if}$. This expression (2.17) has a simple physical interpretation. The first factor is the total absorption cross section for incident particles in channel i, for η_i denotes the usual absorption parameter

$$\eta_i = \exp(-2 \text{ Im } \delta_i) = |(1 + ik_i A_i)/(1 - ik_i A_i)|^2 . \tag{2.18}$$

The second factor then gives the fraction of the absorption transitions out of channel i which lead to the final channel f.

We shall now restrict our discussion to the case of coupled $\bar{K}$-nucleon and π-hyperon channels of definite J, parity, and isotopic spin I. For channel i, we take the $\bar{K}$-N channel, so that α denotes the diagonal element of K for the $\bar{K}$-N channel, β is the row matrix $(\beta_\Sigma, \beta_\Lambda)$ for the transitions $\bar{K} + N \to \pi + Y$, and γ is the submatrix of K referring to the π-Y channels. The scattering length a + ib now refers to the $\bar{K}$-N channels. According to (2.16), its imaginary part, b, is proportional to the square of $\mathcal{M}$, the transition amplitude defined by (2.14) carrying the $\bar{K}$-N channel to the π-Y channels. Its real part, a, is given by

$$a = \frac{M_N}{E} \{\alpha - \pi^2(\mathcal{M}\, \rho_Y\, \gamma\, \rho_Y\, \mathcal{M}^\dagger)\} . \tag{2.19}$$

Unless γ is particularly large, the values of a and $M_N\alpha/E$ are rather close when the imaginary part b is small.

In the low-energy region for the $\bar{K}$-N system, the simplest possible assumption for the s-wave interaction is that (a + ib) is constant. This is generally referred to as the "zero-range approximation" and corresponds to the assumption of a constant K matrix and the neglect of the variation of ρ_Y with energy. This is not unreasonable if the $(K\Lambda)$ and $(K\Sigma)$ parities are odd, since the π-Y channels are then s-wave; however, for odd (KY) parities, the centrifugal barrier effect causes the elements of β_Y to have the energy dependence $B_Y q_Y$ (at least for sufficiently low momentum q_Y)

where q_Y is the c.m. momentum in the relevant π-Y channel, and the elements $\gamma_{YY'}$ to have the form $C_{YY'}\, q_Y\, q_{Y'}$, where B and C denote smoothly varying real functions of E. In the latter case, it would be surprising if the imaginary part of A did not have quite appreciable energy dependence.

An effective range theory has been developed for the representation of the K matrix by Ross and Shaw.[22] For this purpose, the appropriate quantity to consider is the reciprocal matrix K^{-1}. Assuming first that $\ell_i = 0$ for all channels, the effective range expansion improves on the assumption of a constant K matrix by making a linear approximation to the energy dependence of K^{-1},

$$K^{-1} = K_0^{-1} + B(E - E_0) = A + BE . \tag{2.20}$$

The discussion given by Ross and Shaw makes it apparent that the symmetric matrix R, given by

$$R = M_r^{-1/2}\, B\, M_r^{-1/2} , \tag{2.21}$$

where the matrix M_r of reduced masses for each channel has been introduced for dimensional reasons, may be interpreted as an effective range matrix in exactly the same sense as is well known for the one-channel case, and also makes it plausible that the off-diagonal elements of R are generally somewhat smaller than the diagonal elements in the representation in which the phase-space density ρ is diagonal. With (2.6), the T matrix is then given by

$$T = (A + BE - i\pi\rho)^{-1} . \tag{2.22}$$

More generally, the quantity appropriate for expansion in powers of E is the matrix $(k^\ell)K^{-1}(k^\ell)$, where (k^ℓ) denotes the diagonal matrix with elements $(k_i)^{\ell_i}$, thus

$$K^{-1} = (k^\ell)^{-1}(A + BE)(k^\ell)^{-1} . \tag{2.23}$$

We remark next that the discussion given above (following Eq. 2.11) of the structure of transition amplitudes $\langle i \mid T \mid j \rangle$ is equally valid if the label i is extended to refer to a group of channels; in this case the label f refers to all the remaining channels. For the channels i, the submatrix T_{ii} of the scattering matrix T may be obtained by exactly the same methods (see Ref. 21), with the form

$$T_{ii} = \mathcal{A}(1 - i\pi\, \rho_i \mathcal{A})^{-1} , \tag{2.24}$$

where $\mathcal{A}$ is the matrix analogous to (2.12),

$$\mathcal{A} = \alpha_i + i\pi\, \beta_i(1 - i\pi\, \rho_f\, \gamma_i)^{-1} \rho_f\, \beta_i^{\dagger} . \tag{2.25}$$

It is of interest to note that the expression (2.24) has again the form (2.6), and that $\mathcal{A}$ plays the role of an "equivalent reaction matrix" for the channels i considered alone. In general, this "equivalent reaction matrix" $\mathcal{A}$ has complex elements, although it remains a symmetric matrix. Physically, this feature corresponds to the use of a different boundary condition for the channels f from that used for channels i in the definition of the reaction matrix K, namely that there are now only outgoing waves in all the channels f. For energies such that some of the channels f are open, this modification has no particular virtue.[23] However, if group i is chosen such that at energy E all the channels i are open and all the channels f are closed, then this modified boundary condition is especially appropriate to the physical situation. In each channel f, the c.m. momentum is then imaginary, with the value $k_f = i\,|k_f|$, and the condition of outgoing waves becomes the condition that the wavefunction falls off exponentially with increasing distance in the closed channels f. Further, since $i\rho_f = -\bar{\rho}_f$ in this energy range, the matrix $\mathcal{A}$ is real and symmetric and does have the form of a reaction matrix for the channels i. We shall refer[24] to $\mathcal{A}$ as the "reduced reaction matrix" for channels i. Finally, we remark that, if the channels f were isolated from the channels i (that is, the off-diagonal elements β_i of the K matrix were replaced by zeros) and with interactions corresponding to a reaction matrix γ_i, the condition for a bound state in the system of channels f is given by the eigenvalue equation

$$\det(1 + \pi\, \bar{\rho}_f\, \gamma_i) = 0 . \tag{2.26}$$

This may be seen in a number of ways. For example, in the case that matrix β_i is taken zero, the scattering matrix T_{ff} reduces to

$$T_{ff} = \gamma_i (1 - i\pi\, \rho_f\, \gamma_i)^{-1} , \tag{2.27}$$

and Eq. (2.26) is the condition for T_{ff} to have a pole on the real axis below all the thresholds E_f for the channels f. Alternatively, Eq. (2.26) represents the condition that it is possible, for an energy value $E < E_f$, to form a linear combination $\psi = \Sigma_f\, c_f\, \psi^{(f)}$ of the states $\psi^{(f)}$ given by Eq. (2.2), such that the asymptotic form of ψ is exponentially damped in all channels f. We note that Eq. (2.26) is also the condition for the vanishing of the denominator of the second term of expression (2.25) in the energy region $E < E_f$.

In our present example of coupled $\bar{K}$-N and π-Y channels, this transformation to the reduced reaction matrix is appropriate for the discussion of the π-Y channels below the $\bar{K}$-N threshold $E_t = M_N + M_K$. The scattering matrix for the π-$\bar{Y}$ system then takes the form

$$T_{YY} = \Gamma(1 - i\pi \rho_Y \Gamma)^{-1} , \tag{2.28}$$

where, in terms of the matrices α, β, γ defined above for the $\bar{K}$-N and π-Y channels, the reduced reaction matrix Γ has the form

$$\Gamma = \gamma + i\pi \beta^\dagger (1 - i\pi \rho_K \alpha)^{-1} \rho_K \beta . \tag{2.29}$$

The eigenphases of the scattering matrix are most conveniently defined in terms of a modified matrix T', directly related to T by the equation

$$T' = \pi\rho^{1/2} \; T\rho^{1/2} = K'(1 - iK')^{-1} , \tag{2.30}$$

where the matrix $K' = \pi\rho^{1/2} K\rho^{1/2}$ is again a symmetric matrix and the submatrix of K' referring to open channels is both real and symmetric. The eigenvalues of T' may be written in the form $e^{i\delta_s} \sin \delta_s$: for the submatrix of T' referring to the i open channels, these i eigenphases are all real. In the present review, we define a <u>resonance energy E_r</u> as an energy E at which one of these eigenphases passes through a value $(n + \frac{1}{2})\pi$, for some integer n. From the relation between K and T, it then follows that, at these energies E_r, the reduced reaction matrix K_R for the open channels becomes infinite. The corresponding resonant state is the eigenstate of the scattering matrix corresponding to this particular eigenphase.

For a multichannel system, these resonances can arise in two distinct ways:

(a) The complete reaction matrix may have a pole in E at the real energy value E_r. This is the situation usually discussed in nuclear reaction theory. In this case, each of the matrices γ, β, and α in Eq. (2.25), for example, will have a pole at the same energy value E_r.

A simple example of this situation is given at once by the effective-range approximation of Ross and Shaw.[22] The matrix

$$K = (A + BE)^{-1}$$

has poles only on the real axis, since $(A + BE)$ is a symmetric real matrix; these poles occur at energy values for which $\det(A + BE) = 0$, and are then common to all elements of K. Of course, only those energy eigenvalues which lie within the region of validity of the effective range expansion may be expected to represent physical resonances.

A well-known example of such a resonance is the (3,3) π-N resonance, for which

$$K = \frac{k^2}{E} \left(\frac{3}{4f^2} - r(E - M_N)\right)^{-1} , \tag{2.31}$$

where $f^2 \approx 0.08$ denotes the pion-nucleon coupling constant and the effective range parameter r has been determined empirically.

(b) The reduced reaction matrix K_R may have a pole occurring in the terms which arise from the closed channels, that is, at energies for which $\det(1 + \pi \bar{\rho}_f \gamma_i)$ is zero. At these energies, elements α_i, β_i, and γ_i of the complete reaction matrix K do not become infinite. The physical interpretation of these resonances is that they would correspond to bound states in the closed channels f if it were not for their coupling to the open channels i, as a result of which they appear as resonances in the open channels i. For this reason we have referred to these resonances as "virtual bound-state resonances".[25] The possibility of such resonant states obviously arises only in multichannel situations. Their occurrence is due primarily to interactions existing between the particles in the closed channels and they will generally be located not too far below the threshold energy for a new channel.

These possibilities can be illustrated conveniently by reference to the coupled $\bar{K}$-N and π-Y systems. For $E < E_t = M_N + m_K$, the reduced reaction matrix Γ of Eq. (2.28) becomes

$$\Gamma = \gamma - \pi \tilde{\beta}(1 + \pi \bar{\rho}_K \alpha)^{-1} \bar{\rho}_K \beta \quad , \tag{2.32}$$

where $\bar{\rho}_K$ denotes the modulus of ρ_K and we have assumed that time reversal invariance holds in replacing $\beta^\dagger$ by $\tilde{\beta}$. A resonance of the first type will occur in this energy region if α, β, γ have a common pole on the real axis in E. This would occur for the $I = 1$ $p_{3/2}$ π-Y state (cf. Section V), if the π-Y interaction is analogous to the π-N isobar interaction, as envisaged in the global symmetry hypothesis. This situation could occur either above or below the $\bar{K}$-N threshold. Although there would, in either case, be a component of the $\bar{K}$-N state in the resonant state, this would not be a dominant component here. A resonance of the second type is possible only below the $\bar{K}$-N threshold and will then appear as a pion-hyperon resonance. It can occur only if α is such that $(1 + \pi \bar{\rho}_K \alpha)$ vanishes between the π-Λ and the $\bar{K}$-N thresholds. This requires that α be negative and sufficiently large. If the imaginary part of the $\bar{K}$-N scattering length A is small, this condition is essentially equivalent to the requirement that the real part of A be large and negative. In order that the resonant phase shift pass through 90 deg rapidly enough to give rise to a marked resonance bump in π-Y scattering, it is necessary that the coefficient $(\pi \tilde{\beta} \bar{\rho}_K \beta)$ of the resonance term in (2.32) be sufficiently small: this is equivalent to the requirement of a sufficiently small value for the imaginary part of A at the resonance energy.

The eigenphases of the π-Y system are most conveniently obtained by the diagonalization of

$$\Gamma' = \pi \rho_Y^{1/2} \Gamma \rho_Y^{1/2} , \tag{2.33}$$

whose eigenvalues give the values of $\tan \delta_s$. Near a resonance of the second type, where the term γ may be neglected in first approximation, this diagonalization is particularly simple. In the neighborhood of resonance, the resonant eigenstate has the form

$$\psi_r = \left\{ \beta_\Sigma^r (\rho_\Sigma^r)^{1/2} \mid \Sigma \pi \rangle + \beta_\Lambda^r (\rho_\Lambda^r)^{1/2} \mid \Lambda \pi \rangle \right\} , \tag{2.34}$$

and the resonant phase shift is given by

$$\tan \delta_r = \frac{\pi(\beta_\Sigma^2 \rho_\Sigma + \beta_\Lambda^2 \rho_\Lambda)\bar{\rho}_K}{1 + \pi \bar{\rho}_K \gamma \alpha} + C . \tag{2.35}$$

The correction term C may be obtained near resonance by taking the expectation value of Γ' in the resonant state (2.34), thus

$$C_r = \pi(\gamma_{\Sigma\Sigma} \beta_\Sigma^2 \rho_\Sigma + 2\gamma_{\Sigma\Lambda} \beta_\Sigma \beta_\Lambda(\rho_\Sigma \rho_\Lambda)^{1/2} + \gamma_{\Lambda\Lambda} \beta_\Lambda^2 \rho_\Lambda)/(\beta_\Sigma^2 \rho_\Sigma + \beta_\Lambda^2 \rho_\Lambda) . \tag{2.36}$$

In this region, the nonresonant phase shift is given by

$$\tan \delta_{nr} = \pi(\rho_\Sigma \rho_\Lambda)^{1/2}(\beta_\Lambda^2 \gamma_{\Sigma\Sigma} + \beta_\Sigma^2 \gamma_{\Lambda\Lambda} + 2 \beta_\Lambda \beta_\Sigma \gamma_{\Lambda\Sigma})/(\beta_\Sigma^2 \rho_\Sigma + \beta_\Lambda^2 \rho_\Lambda) . \quad (2.37)$$

These expressions are valid, of course, only when the resonance energy lies above the π-Σ threshold. If the resonance energy lay below this threshold, it would be necessary to take $\rho_\Sigma = + i \bar{\rho}_\Sigma$ and to include the π-Σ channel among the closed channels f.

If β_Σ and β_Λ are relatively small, and α relatively large (and negative), the phase shift δ_r passes rapidly through 90 deg at the resonance energy defined by the relation $(1 + \pi \bar{\rho}_K \alpha) = 0$. The shape of the cross section in the resonant state then depends on the value of C_r, as is well known in the parallel case of resonances observed in the scattering of low-energy neutrons by nuclei. "Potential scattering" in the pion-hyperon system, which the term C_r represents, would have quite a marked effect on the symmetry of the resonance curve. If C_r were large, the cross section would generally fall to zero for an energy near the resonance energy before rising to the resonance maximum. For the observed π-Λ resonance, the degree of symmetry in the resonance curve shows that there can be at most quite moderate potential scattering, indicating that the term C_r (and correspondingly the elements of γ) corresponds to quite a small phase angle. The curves shown in Fig. 1 illustrate the effect of potential scattering on the π-Λ scattering for a particular situation.

At resonance, the structure of the resonant state is given by the wavefunction (2.34), from which we conclude the following expression for the branching ratio Σ/Λ for the resonant state,

$$(\Sigma/\Lambda)_r = (\beta_\Sigma{}^r/\beta_\Lambda{}^r)^2 (\rho_\Sigma{}^r/\rho_\Lambda{}^r) \quad . \tag{2.38}$$

When the resonant state is produced, the nonresonant π-Y state orthogonal to (2.34) will generally be produced also. At resonance the matrix elements for these two contributions will be approximately 90 deg out of phase so that, provided the resonance is reasonably sharp, there will be little interference between the resonant and nonresonant Λ (or Σ) production, and production through the resonant state will generally be dominant. In this case, the (Σ/Λ) ratio observed in production processes may be generally expected to be given by the ratio (2.38), although it is quite possible for deviations to occur in special circumstances.

III. THE $\overline{K}$-NUCLEON INTERACTION AND THE INTERPRETATION OF THE Y^* RESONANCE AS A $\overline{K}$-N BOUND STATE

The data available on the scattering and reaction cross sections for K^--p collisions at low energies (lab momentum $\leqslant$ 200 Mev/c) are still rather limited.[29] For the K^- mesons that come to rest in liquid hydrogen, the branching ratios for the reactions

$$K^- + p \rightarrow \Sigma^- + \pi^+ \tag{3.1a}$$

$$\rightarrow \Sigma^0 + \pi^0 \tag{3.1b}$$

$$\rightarrow \Sigma^+ + \pi^- \tag{3.1c}$$

$$\rightarrow \Lambda + \pi^0 \tag{3.1d}$$

are known. From the arguments concerning the role of the Stark distortion of the K^--p atom in these K^--p capture processes, as discussed by Day, Snow, and Sucher,[30] we assume that these ratios are characteristic of the $\overline{K}$-N s-wave interaction at zero energy. From these, we obtain three parameters of interest,

$$\sigma_1/\sigma_0 = (\Sigma^+ + \Sigma^- - 2\Sigma^0 + \Lambda)/3\,\Sigma^0 , \tag{3.2}$$

the relative intensity of the I = 1 and I = 0 final pion-hyperon states in zero-energy K^--p capture,

$$\epsilon = (\Lambda/\Sigma + \Lambda)_1 = \Lambda/(\Sigma^+ + \Sigma^- - 2\,\Sigma^0 + \Lambda), \tag{3.3}$$

the proportion of I = 1 absorptions which lead to Λ hyperons, and ϕ_t, the relative phase between the I = 0 and I = 1 amplitudes for the $\Sigma + \pi$ reactions. As discussed in Appendix A, the parameters ϵ and ϕ_t are rather poorly determined, mainly because of the dominance of the I = 0 reaction channel over the $I = 1$ channel.

In the (lab) momentum range 100 to 200 Mev/c, total cross sections are available for K^--p elastic scattering, for the charge-exchange reaction

$$K^- + p \rightarrow \overline{K^0} + n , \qquad (3.4)$$

and for the absorptive reactions (3.1a) and (3.1c) leading to charged hyperons. The total absorption cross section is not yet available; separation between reactions (3.1b) and (3.1d) is difficult and has not yet been achieved in this energy range. For elastic scattering, the statistics are sufficient to show that the angular distribution is quite isotropic, except at forward angles where the Coulomb scattering becomes important. In the angular distribution at 175 Mev/c, the Coulomb-nuclear interference is quite weak, showing that the real part of the elastic scattering amplitude is rather small at this energy. The value obtained at 175 Mev/c is $\mathrm{Re}(f) = 0.3 \pm 0.3$ fermi, corresponding to a weakly constructive interference, but a more careful analysis of the data is necessary and is at present under way.[31] Within statistics, the other angular distributions are all consistent with isotropy. Since the absorption cross sections show the rapid decrease with increasing energy characteristic of s-wave absorption, and since the elastic cross section varies slowly over this energy range, the evidence is strong that the K^--p interaction is predominantly s-wave[32] below 200 Mev/c.

These data on total cross sections in the region 100 to 200 Mev/c and on the zero-energy reactions is just sufficient for a rough determination of the s-wave scattering lengths A_0 and A_1, provided that these are assumed to have negligible energy dependence between zero and 175 Mev/c momentum (lab). In this analysis (discussed in Appendix A), the $(K^-, \overline{K^0})$

mass difference must be taken into account, as it has a quite strong effect on the expressions for the reaction rates in the low-energy region.[33,34,2] The four sets of scattering amplitudes (A_0, A_1) obtained are listed in Table I. It will be seen that their values are not yet accurately determined. On the other hand, for each set, the outstanding qualitative features are now rather definite. For example, for the (a-) set, A_1 has a large negative real part and a small imaginary part; A_0 has a large imaginary part, whereas its real part is rather poorly determined and may be either large or small.

From the discussion in Section II, it is apparent that an interpretation of the Y^* resonance in terms of an $I = 1$ $\bar{K}$-N bound state requires that the (a-) set, the only set for which the I = 1 amplitude A_1 has a large negative real part, be the physically correct set. We note that the (a-) amplitude A_1 gives a low rate for the absorption process $\bar{K} + N \rightarrow Y + \pi$, which is in good correlation with the relatively narrow width ($\Gamma/2 \approx 20$ Mev) reported for the Y^* resonance.[5,6] (We note also that, with the (a-) amplitudes, it is conceivable that there might exist also an $I = 0$ $\bar{K}$-N bound state which would appear as a $\pi^{\pm} - \Sigma^{\mp}$ resonance. Because of the large value of b_0, this resonance would necessarily be rather broad and correspondingly difficult to detect. Although this possibility exists, there is no compelling reason at present to expect this to be the case; a_0 may well be quite small, and may correspond to a repulsive interaction.)

Since the resonance is narrow, it is sufficient for the determination of the parameters of this resonant state to consider the I = 1 elastic scattering amplitude (2.8),

$$\langle \bar{K}N \mid T \mid \bar{K}N \rangle = \frac{E}{M_N} \cdot \frac{e^{i\delta} \sin \delta}{k} = \frac{E}{M_N} \frac{A_1}{1 - ik A_1} , \qquad (3.5)$$

TABLE I. $\bar{K}$-N Scattering Lengths[a]

Set	A_0 (fermis)	A_1 (fermis)
(a+)	$0.05 \pm 0.2 + i(1.10 {+0.2 \atop -0.3})$	$1.45 \pm 0.2 + i(0.35 {+0.09 \atop -0.07})$
(a-)	$-0.75 {+0.35 \atop -0.45} + i(2.0 \pm 0.35)$	$-0.85 \pm 0.15 + i(0.21 \pm 0.04)$
(b+)	$1.25 \pm 0.4 + i(2.0 \pm 0.3)$	$0.75 \pm 0.2 + i(0.24 \pm 0.05)$
(b-)	$-1.85 \pm 0.15 + i(1.10 {+0.9 \atop -0.3})$	$-0.10 \pm 0.2 + i(0.65 \pm 0.15)$

(a) Note that the sign convention is chosen such that $k \cot \delta = 1/A$, so that a positive real part for A corresponds to constructive interference with Coulomb scattering, a negative real part to destructive interference, and the imaginary part of A is necessarily positive.

in the unphysical region of negative $\bar{K}$-N kinetic energy. In this region we have $k = +i\kappa$, where $\kappa = [2\mu_K (E_t - E)]^{1/2}$, $E_t = M_N + m_K$, and μ_K is the $\bar{K}$-N reduced mass. We make a linear approximation of the Breit-Wigner resonance form $(E - E_r + i\Gamma/2)^{-1}$ to the denominator factor of (3.5) and are then led to conclude that the resonance energy E_r corresponds to $\kappa_r = [2\mu_K(E_t - E_r)]^{1/2} = |a_1|^{-1}$, that is,

$$E_r = M_N + m_K - (2\mu_K a_1^2)^{-1}, \tag{3.6}$$

as expected from the expression (2.35) for the π-Y scattering phase and from the smallness of b, and that the width is given by[35]

$$\Gamma/2 = b_1\kappa_r^2/(\mu_K |a_1|) = b_1/(\mu_K |a_1|^3). \tag{3.7}$$

In these expressions, the values of a_1 and b_1 which appear should be taken at the momentum $k = +i\kappa_r$ corresponding to the resonance energy E_r. If the energy dependence of a_1 and b_1 is neglected, the I = 1 (a-) amplitude leads to the value $\kappa_r = 230(\pm 40)$ Mev/c. This corresponds to a resonance energy at 82(±30) Mev below the K^--p threshold, i.e. at mass value $M^* = 1350(\pm 30)$ Mev, which is not in disagreement with the observed location of the Y^* resonance. According to (3.7), the corresponding half-width of this resonance is $\Gamma/2 = 21 \pm 4$ Mev. In making this estimate, we have adjusted $a_1(\kappa_r)$ to the observed value, that is to the value giving the observed resonance location M^* according to Eq. (3.6). The narrowness of this resonance is due partly to the smallness of b, which reflects the slowness of the I = 1 $\bar{K} + N \rightarrow Y + \pi$ transition rate, and partly to the largeness of a_1, as a result of the corresponding diffuseness of the $\bar{K}$-N bound-state system.

A more adequate discussion of the resonance shape may be based on the expression (2.35) for the π-Y scattering phase in the resonant state. However, the matrix γ is not known. If we first consider the approximation of taking $\gamma = 0$, the parameters β_Σ and β_Λ can be related to the zero-energy data, if we assume them to have the simple energy dependence appropriate to the angular momentum of each channel, and the expression

$$\sigma(\pi + \Lambda \rightarrow \pi + \Lambda) = \frac{4\pi}{q_\Lambda^2} \frac{(\kappa b_\Lambda)^2}{|1 + \kappa a|^2 + (\kappa b)^2} \quad (3.8)$$

is then obtained for the s-wave π-Λ elastic scattering cross section, where

$$b = b_\Sigma + b_\Lambda = \beta_\Sigma^2 \rho_\Sigma + \beta_\Lambda^2 \rho_\Lambda \quad (3.9)$$

is to be taken as energy-dependent.

The resonance shape given by (3.8) is shown in Fig. 1 for several cases of interest. For s-wave π-Λ resonance, the shape is somewhat asymmetric, with a long tail on the low-energy side; the full width at half maximum is 42 Mev, in agreement with that given by expression (3.7). Although, as remarked in Section II, a large value for γ does appear excluded by the degree of symmetry observed for the resonant state, a moderate value of γ would not distort the resonance curve unreasonably. In fact, the rather symmetric resonance curve (b) has been drawn by taking the (arbitrarily chosen) value $C/q_\Lambda = -0.33$ fermi in the expression (2.35); it will be noticed that this assumption of a moderate finite value for γ has not appreciably affected either the half-width or the location of the resonance. If it is supposed that the (Σ, Λ) parity is odd, so that the corresponding π-Σ system is $p_{1/2}$, the corresponding curve for $\sigma(\pi + \Sigma \rightarrow \pi + \Sigma)$ is given by (d); owing to the centrifugal barrier this resonance curve is displaced upward in energy a little (~ 7 Mev). If the π-Λ resonant state is $p_{1/2}$ (corresponding to even (KΛ) parity), the resonance shape is rather symmetric for $\gamma = 0$, the lower side being suppressed by the decrease in b_Λ as the energy falls, but the half-width is rather smaller (≈ 14 Mev) as a result. These variations illustrate the uncertainties inherent in any attempt to predict the width of the Y^* resonance on the basis of this model at present. Rather, a definite measurement of the resonance half-width would be of value for the interpretation of the detailed character of the $\bar{K}$-N bound state and its outgoing channels.

This interpretation of the π-Λ resonance naturally requires that $j = \frac{1}{2}$ hold for the Y^* spin. The experimental evidence appears consistent with this assignment,[4,6] and the evidence on the polarization properties of the Λ decay when the Y^* resonance produced in a polarized state further suggests that the π-Λ system resulting from Y^* decay is in an $s_{1/2}$ state. The latter situation requires that the (KΛ) parity be odd, and this requirement is consistent with what other indications exist concerning the (KΛ) parity.[3,36]

Finally, this interpretation allows a simple explanation[37] of the excitation function observed[6] for Y^* production in $K^- + p$ collisions. On this view, there is quite a close analogy between the Y^* production reaction,

$$\bar{K} + N \rightarrow \pi + Y^* , \tag{3.10}$$

and the well-known nucleon-nucleon reaction

$$N + N \rightarrow \pi + d . \tag{3.11}$$

For the latter reaction, it is known that the pion production is predominantly p-wave for final c.m. momentum above about 50 Mev/c, and it is now believed the this is a direct consequence of the pseudoscalar nature of the pion.[16] Regarding the Y^* as a $\bar{K}$-N bound state analogous to the deuteron, the pion in reaction (3.6) can be emitted only from the nucleon (the interaction $\bar{K} \to \bar{K} + \pi$ being forbidden by angular momentum and parity conservation), and the analogy[38] between these reactions leads to the expectation that p-wave pion production should be dominant in the Y^* production reaction also, sufficiently far above the threshold energy.

The Σ/Λ ratio observed for Y^* decay is quite small; in fact, there is at present almost no clear evidence for a resonance in the Σ-π system at the Y^* energy and an upper limit at approx 10% has been placed[7] on the ratio $(\Sigma^0\pi^- + \Sigma^-\pi^0)/(\Lambda\pi^-)$. In terms of the present interpretation, it is difficult to make any prediction of this ratio, except by an extrapolation from the Σ/Λ ratio in $I = 1$ absorption at the K^--p threshold, which depends on some additional assumptions. As discussed in Appendix A, this threshold ratio is known rather poorly, but it is shown that a lower limit of 0.25 can be placed on it from the observed Σ^-/Σ^+ ratio and the $(\Lambda + \pi^0)$ rate at threshold. In terms of the reaction matrix elements, this threshold ratio is given by[2]

$$\left(\frac{\Sigma}{\Lambda}\right)_1 = \left(\frac{\beta_\Sigma^{\;t}}{\beta_\Lambda^{\;t}}\right)^2 \frac{\rho_\Sigma^{\;t}}{\rho_\Lambda^{\;t}} \frac{1 + \left(\pi\rho_\Lambda^{\;t}\,(\beta_\Lambda^{\;t}\,\gamma_{\Lambda\Sigma}^{\;t} - \beta_\Sigma^{\;t}\,\gamma_{\Lambda\Lambda}^{\;t})/\beta_\Sigma^{\;t}\right)^2}{1 + \left(\pi\rho_\Sigma^{\;t}\,(\beta_\Sigma^{\;t}\,\gamma_{\Sigma\Lambda}^{\;t} - \beta_\Lambda^{\;t}\,\gamma_{\Sigma\Sigma}^{\;t})/\beta_\Lambda^{\;t}\right)^2}, \tag{3.12}$$

where very little is known of these elements of γ. As remarked above, there is no indication from the resonance shape that these elements are at all large. If we first neglect them, the relationship between the Σ/Λ ratio at threshold and at resonance may be discussed as follows:

(a) $s_{1/2}$ resonance for both π-Λ and π-Σ channels. Here the natural approximation is to neglect the energy dependence of β_Σ and β_Λ, so that the Σ/Λ ratio falls from the threshold value only because of the fall in the phase-space ratio ρ_Σ/ρ_Λ. Since $(\rho_\Sigma/\rho_\Lambda)$ is $\approx 0.8(\rho_\Sigma/\rho_\Lambda)_t$, it is clear that a ratio as small as that observed can be accounted for only if the parameters β_Σ, β_Λ have quite appreciable energy dependence over this energy range, or if the elements of γ are sufficiently large at threshold to modify this comparison. Neither of these possibilities gives a simple interpretation of the data.

(b) $p_{1/2}$ resonance for the π-Σ channel. If the $(K\Sigma)$ parity were even, the final π-Σ system would be $p_{1/2}$, and the natural assumption on the energy dependence of β_Σ is that of proportionality to q_Σ. Since the Y^* energy is about 55 Mev about the π-Σ threshold, $q_\Sigma^{\ r} \approx 120$ Mev/c, and $(\beta_\Sigma^{\ r}/\beta_\Sigma^{\ t}) \approx 0.45$. In this case it is conceivable that a $(\Sigma/\Lambda)_r$ ratio as low as $0.25 \times 0.8 \times 0.45 = 0.09$ is compatible with the threshold data, and this is comparable to the upper limit quoted by Dahl et al.[5] If it is supposed that the $(K\Lambda)$ parity also is even, so that the π-Λ resonant state is $p_{1/2}$ and $(\beta_\Lambda^{\ r}/\beta_\Lambda^{\ t})^2 \approx 0.66$, a somewhat less favorable ratio $(\Sigma/\Lambda)_r \approx 0.14$ results from these simple assumptions on the energy dependence of β_Σ and β_Λ.

At this point, we must emphasize that there is no clear-cut experimental evidence which otherwise requires that the (a-) amplitudes are the physically correct ones. Not even the sign of the real parts of the amplitudes is definitely established. In principle, this last could be achieved from the observation of the Coulomb-nuclear interference in K^--p elastic scattering at low energies. At lab momentum of 172 Mev/c, the scattering amplitudes corresponding to the (a+) and (a-) solutions of Table I are

$$f(a\pm) = \pm 0.35 + 0.74 i . \tag{3.13}$$

The real parts of these scattering amplitudes arise almost entirely from a_1 and are moderately well determined (within about 20%). Owing to the dominance of the absorptive part of f, however, a clear-cut decision between the two sign possibilities is difficult and will probably not be achieved until the statistics on K^--proton scattering are greatly improved. For K^--nucleus scattering, the optical-model potential is known to be attractive. This conclusion was convincingly argued by Alles et al.[40] several years ago from observations on the inelastic scattering of low-energy K^- mesons by nuclei, and has also been reached in the study of small-angle scattering of K^- mesons by emulsion nuclei.[41] However, if we think in terms of potential interactions, we must realize that the existence of a $\overline{K}$-N bound state means that the potential corresponding to the $I = 1$ (a-) amplitude must actually be strongly attractive. In this situation there is some doubt[42] whether the sign of the K^--nucleus potential at low energies really provides any clear indication of the sign of the real part of the $\overline{K}$-N scattering amplitudes.

The (b-) solution differs most markedly from the other amplitude sets in the behavior it predicts for the absorption cross sections. For this solution, the $I = 0$ absorption cross section falls more rapidly with increasing momentum, while the $I = 1$ absorption cross section falls less rapidly, than for any of the other solutions. Thus, whereas the other solutions give values for $\sigma(\Lambda)$ between 7 and 9 mb., and for $\sigma(\Sigma^0)$ between 12 and 14 mb., at (lab) momentum 172 Mev/c, the (b-) solution leads to $\sigma(\Lambda) \approx 16$ mb and $\sigma(\Sigma^0) \approx 9$ mb. This corresponds also to a much stronger energy dependence of the $\Lambda/(\Sigma^0 + \Lambda)$ ratio for the (b-) solution than for the others; the ratio predicted at 172 Mev/c (for value 0.21 at zero energy) is 0.63 for the (b-) solution, compared with the predictions 0.37 for (a+), 0.31 for (a-), and 0.40 for (b+). It is expected that data will soon be available on this ratio in this momentum region.[31] Another experimental parameter of particular interest is the Σ^-/Σ^+ ratio, whose mean value averaged over the (lab) momentum interval 100 to 200 Mev/c is 0.95 ± 0.3. The energy dependence of this ratio depends on the energy dependence both of the absorption cross sections σ_0 and σ_1 and of the phase angle ϕ between the corresponding matrix elements $M_0(\Sigma)$ and $M_1(\Sigma)$. If the KYN parity is odd, or if the final state scattering is weak, it is natural to assume that the energy dependence of ϕ arises entirely from the initial state scattering.[2] With the value 2.18 for the Σ^-/Σ^+ ratio at zero energy, the mean Σ^-/Σ^+ ratio predicted for the 100 to 200 Mev/c (lab) momentum interval is 0.83 for the (a-) solution, 1.45 for the (b+) solution, in agreement with the data, whereas the values predicted with the (a+) and (b-) solutions are 2.15 and 3.24, respectively. It must be borne in mind, however, that, especially if the KYN parity is even, the neglect of energy dependence for the final state scattering may be an uncertain assumption.

There are considerable data available on K^--deuterium scattering and reactions in the low-energy region.[29,43] The analysis of this in terms of the $\bar{K}$-N interaction amplitudes is complicated, however, by the strong initial- and final-state interactions which occur in the initial and final three-body systems. The discussions which have been given for the capture reactions from rest[44] and for the elastic and inelastic scattering at approx 200 Mev/c [45] are not yet sufficiently complete to give any clear-cut indications for preferring a particular set of $\bar{K}$-N amplitudes, although there is every reason to expect that such data will become valuable in this respect as the experiments and the theoretical calculations each become more refined.

There is also promise that the study of K_2^0-p scattering and reaction processes in the low energy region will give some direct indications concerning the $\bar{K}$-N amplitudes in the near future.[46] On the one hand, the $\bar{K}$-N interactions in K_2^0-p collisions are entirely in the $I = 1$ state, so that the observation of K_2^0-p reactions will allow a very direct determination of the (Σ/Λ) ratio in the $I = 1$ channel, and the measurement of a total absorption cross section for the $I = 1$ channel will help greatly in distinguishing between the (a) and (b) sets of amplitudes. On the other hand, as pointed out by Biswas,[47] the s-wave cross section for the reaction

$$K_2^0 + p \rightarrow K_1^0 + p \tag{3.14}$$

is given by the expression

$$\sigma(K_2^0 + p \rightarrow K_1^0 + p) = \pi \left| \frac{\alpha_1}{2(1 - ik\alpha_1)} + \frac{\alpha_0}{2(1 - ik\alpha_0)} - \frac{A_1}{1 - ikA_1} \right|^2 , \tag{3.15}$$

where α_0, α_1 are the (real) scattering lengths for the $I = 0, 1$ K-N channels. The scattering length α_1 is well known, α_0 is less well known but is smaller, with the same sign, and there is some hope of discriminating quite strongly between the (a+) and (a-) $\bar{K}$-N amplitudes as a result of the interference between the real parts of the two terms of (3.15). Unfortunately, such an experiment appears feasible only down to (lab) momenta of about 300 Mev/c, a momentum region where there will be some question concerning the importance of p-wave contributions to the expression (3.15).

At the present stage, any further indications of the sign of the $\bar{K}$-N scattering amplitudes, and whether they are of the (a) or the (b) type, will be of the greatest importance in establishing the relationship of the Y^* resonance observed with the possible existence of an $I = 1$ $\bar{K}$-N virtual bound state.

IV. THE DISPERSION-RELATION FORMALISM AND THE $\bar{K}$-N INTERACTION

In order to go beyond a strictly phenomenological approach and to discuss what energy dependences may be expected for the parameters we have introduced, the use of the dispersion-relation formalism is the most complete and convenient procedure for including specific physical mechanisms, such as the exchange of pions between the $\bar{K}$ meson and nucleon. As Bjorken[10] and Nauenberg[11] have pointed out, the method used by Chew and Mandelstam[9,48] for one-channel problems may readily be extended for multichannel situations. For a state of definite angular momentum and parity, the scattering matrix T(E) for a system of n two-particle channels may be written in the form

$$T(E) = N(E)\, D^{-1}(E) , \tag{4.1}$$

where the elements of the n-by-n matrix D(E) are analytic functions of the total energy E, each in a cut E plane where the branch cut is chosen to run from an appropriate threshold along the positive real axis. The elements of the n-by-n matrix N(E) are analytic functions, for each of which the branch cuts and singularities lie to the left of the corresponding thresholds, their location and character reflecting the nature of the dynamical influences affecting the corresponding reaction processes. Since the elements of N(E) are real on the branch cut of the corresponding element of D(E), the unitarity condition

$$\mathrm{Im}[T^{-1}(E)] = -\pi\, \rho(E) \tag{4.2}$$

leads to the result that

$$\mathrm{Im}\, D(E) = -\pi\, \rho(E)\, N(E) \tag{4.3}$$

along the upper side of the right-hand branch cut. Assuming that Re D(E) approaches a constant as $E \to \infty$, it is then convenient to normalize $D(E)$ in such a way that $\text{Re}\, D(E)$ approaches the unit matrix at infinity, which is possible because $T(E)$ is expressed in the form of a ratio by Eq. (4.1). Then, following Bjorken and Nauenberg, $D(E)$ may be determined, leading to the form[49]

$$T(E) = N(E) \left\{ 1 - \int_{E_t}^{\infty} \frac{\rho(E')\, N(E')}{E' - E}\, dE' \right\}^{-1} . \qquad (4.4)$$

The matrix $N(E)$ may now be regarded as a quantity to be determined in terms of its singularities, either in a semiphenomenological way or in terms of some dynamical principles. For arbitrary $N(E)$, the expression $T(E)$ is not generally a symmetric matrix, as is required by time-reversal invariance, but Bjorken and Nauenberg[50] have demonstrated that if $N(E)$ is determined from the condition

$$\text{Im}\, N(E) = [\, \text{Im}\, T(E)]\, D(E) \qquad (4.5)$$

on its dynamical singularities, then $T(E)$ will be symmetric as long as the matrix $[\text{Im}\, T(E)]$ on these dynamical singularities is itself symmetric.

This formalism is, of course, very closely related to the K-matrix formalism. In fact, the explicit relationship is given by the equation

$$K(E) = N(E) \{ D(E) + i\pi\, \rho(E)\, N(E) \}^{-1} , \qquad (4.6)$$

where $D(E)$ is given by the denominator of expression (4.4). The elements of the denominator of (4.6) are real functions of E along the real E axis, both for physical energies and for energies below the thresholds, until E reaches the first dynamical branch cuts appropriate to the matrix element considered. Thus, in this region to the right of all branch cuts, the elements of the K matrix are real, as they should be; K(E) has the correct symmetry when the condition specified following Eq. (4.5) is satisfied. Further, as expected,[51] these elements are analytic functions of E, and in particular they are analytic functions of E along the real axis, in both physical and unphysical regions, to the right of these branch cuts. To the left, however, the function N(E) generally becomes complex below the onset of the first branch cut and the K-matrix elements become complex in this region, as remarked in Ref. 2. The form (4.6) has the advantage that it makes explicit the cause and nature of this behavior.[52]

In terms of the form (4.6), resonances of the first type discussed in Section II correspond to zeros of the determinant of the denominator, i.e., they occur for real energies such that

$$\det\,(D(E) + i\,\pi\,\rho(E)\,N(E)) = 0\,. \tag{4.7}$$

At these resonance energy values, all elements of the complete K matrix for the n systems become infinite.

As discussed in Section II, the more convenient procedure is to confine explicit attention to the subset i of channels which are energetically available at the energy of interest and to make use of the "reduced K matrix" $K_R(E)$. This matrix $K_R(E)$ is related to the scattering matrix T(E) by the relation

$$T(E) = K_R(E)\,\{\,1 - i\,\pi\,\rho(E)\,\theta_i(E)\,K_R(E)\,\}^{-1}\,, \tag{4.8}$$

where $\theta_i(E)$ denotes a projection operator which is unity for the energetically available channels at energy E, and zero otherwise. Only the submatrix $K_R{}^i(E)$ of $K_R(E)$ which refers to the open channels is of direct physical interest, since the scattering amplitudes for the energetically permitted reactions obviously depend on the elements of $K_R^i(E)$ alone. Remembering the condition (4.3) and that its right-hand side is simply to be taken zero for an energy below the appropriate threshold, comparison of (4.8) with (4.1) leads to the following expression for $K_R(E)$:

$$K_R(E) = N(E) \{ \text{Re } D(E) \}^{-1} . \tag{4.9}$$

From its definition, and the discussion in Section II, it is clear that $K_R(E)$, although continuous along the real axis, is not an analytic function of E but has, in fact, a cusp-like behavior with a change of analytic form at each threshold.

From the expression (4.9), the location of all resonances of the system are given by the real roots of the equation

$$\det \{ \text{Re } D(E) \} = 0 . \tag{4.10}$$

Those roots of (4.10) which lie below all thresholds represent stable bound states of the system. Those which lie between the ith and (i+1)th threshold represent resonances in the set of i channels; these resonances include the "virtual bound-state" resonances arising from interactions in the closed channels as well as the resonances of the first type. To determine the structure of such a resonance state, the scattering matrix $T_i(E)$ for the open channels is then considered; the eigenvalues of $\pi \rho_i^{1/2} T_i(E) \rho_i^{1/2}$ are the set $\{ e^{i\delta_s} \sin \delta_s \}$, where the $\{\delta_s\}$ are (with $s = 1, 2, \cdots i$) the (real) eigenphases for the open channels. At the resonance energy, one of these eigenphases (s = r, say) passes through 90 deg; the eigenstate corresponding to this eigenphase δ_r,

$$| r, E \rangle = \sum_{\alpha=1}^{i} c_r^{\alpha}(E) \, | \alpha, E \rangle , \tag{4.11}$$

then represents the resonant state, the relative intensity of the channel α being $| c_r^{\alpha}(E) |^2$ at energy E in this state.

For coupled $\bar{K}$-N, π-Σ, and π-Λ channels, the branch cut which lies closest to the physical region is that arising from the exchange of two pions between $\bar{K}$ and N. For exchange of a system of mass m, the corresponding branch cut begins at energy E(m) given by

$$E(m) = (M_N^2 - \frac{1}{4} m^2)^{1/2} + (m_K^2 - \frac{1}{4} m^2)^{1/2} . \qquad (4.12)$$

For the exchange of a pion pair, the cut begins only about 30 Mev below the $\bar{K}$-N threshold. This situation certainly raises questions concerning the validity of extrapolation from the threshold to the Y^* resonance energy, as is discussed again below and in Appendix B. If the emission of a pair of s-wave pions by the $\bar{K}$ meson is not an especially strong coupling, it is possible that this branch cut may not have an important effect on the K-matrix elements in this energy region. However, more serious branch cuts may well arise from exchange of the $I = 0$ ω^0 particle[53] (if it is strongly coupled with K mesons), or of a resonating pion pair.

Ferrari et al.[54,55,56] have taken the first step in a more general discussion of these $\bar{K}$-N reaction processes following dispersion-theory methods, by including a simple pole in the $\bar{K}$-N diagonal element of [Im T] as a rough representation of the terms arising from the exchange of a particle or resonant system (nominally a pion-pion resonance) between $\bar{K}$ and N. This pole has residues R_0 and R_1 for the $I = 0$ and $I = 1$ systems, respectively. These residues are related by $R_0 = R_1$, or $R_0 = -3 R_1$, according as the isotopic spin of the system exchanged is $i = 0$ or 1. The magnitudes of these residues are otherwise not known, unless some specific dynamical theory is adopted,[57] and are generally to be regarded as parameters to be determined empirically.[60] At present, it appears a very difficult proposition to determine further such parameters from the low-energy K^--p data, since these have proved barely sufficient for the determination of constant scattering amplitudes A_0 and A_1. Using a rather speculative estimate of R_0 and R_1 (with a ratio $R_0/R_1 \approx -3$, corresponding to the exchange of a $j = 1$, $I = 1$ π-π resonance), Ferrari et al.[56] have calculated the energy dependence of the (a+) amplitudes A_0 and A_1, due to $\overline{K}$-N interactions corresponding to the exchange of mass $m = 3.6\ m_\pi$, and have concluded that an extremely strong energy dependence can result, even though the range parameter of this interaction is only $(3.6\ m_\pi)^{-1} \approx 0.4$ fermi. Since their calculated scattering lengths vary by as much as a factor of 2 between zero and 150 Mev/c momentum (c.m.), it is apparent that such a strong energy dependence would completely invalidate any attempt to relate the Y^* resonance to the low-energy K^--p data without a rather complete theory of the mechanisms giving rise to this energy dependence.

A simplified treatment of the situation discussed by Ferrari et al. is given in Appendix B. It appears that the strong energy dependence they obtained for the (a+) amplitudes is largely a consequence of the great strength assumed for the interaction of the pion pair with the K meson. In this case, there is a question whether, for consistency, further branch-cut terms corresponding to the exchange of two, three, and more resonating pion pairs should not also be included at the same time; although these more complicated singularities are more distant from the physical energy region, their strength may be very great, and they may play a significant part in determining the energy dependence of even the low-energy scattering.[61] Although the dispersion-theory formalism represents a tremendous step forward in the technique and understanding of strong-interaction problems, the way in which it is used at the present preliminary stage does often represent a new kind of perturbation approach, involving the assumption that a strong near-by singularity can be introduced to represent some particular physical mechanism without need for the inclusion of any related, further distant singularities.

On the other hand, for mechanisms of moderate strength, the dispersion method provides a convenient method for the semiphenomenological inclusion of their effects in the theoretical expressions to be compared with the experimental data. Since, for the strange particles, nothing is known concerning the strengths of the many possible vertices that play a role even in the simplest situations, it is clear that a phenomenological approach of this kind places a very severe demand on the data. At the present stage, the guidance of some framework of dynamical principles, such as those of global symmetry (cf. Section V) or of the vector theory of strong interactions discussed by Sakurai,[58] would be exceedingly advantageous.

Ferrari et al.[55] have also discussed the relationship between the K-N and the $\overline{K}$-N interactions which arise from pion-exchange processes. In the interpretation of the Y^* resonance as $\overline{K}$-N bound state, these processes appear of the greatest importance, since they can give rise to potential interactions which have relatively long range, and which can therefore be especially effective in binding the $\overline{K}$-N system. For this discussion, we need the relationship between the vertices for the interactions

$$\text{(a)} \quad K \rightarrow K + n\pi , \tag{4.13a}$$

$$\text{(b)} \quad \overline{K} \rightarrow \overline{K} + n\pi , \tag{4.13b}$$

in corresponding configurations. For the derivation of this relationship, the operation of G conjugation is appropriate. For the pions, this operation simply multiplies their wavefunction by $(-1)^n$; for the K mesons, this operation changes each K meson to its antiparticle and multiplies the matrix element by $(-1)^i$ where i is the isotopic spin transferred by the $(n\pi)$ system. Since the $(n\pi)$-N interaction is common to the K-N and the $\overline{K}$-N interactions, we have at once

$$M(K^- -N) = (-1)^{n+i} M(K^+ -N) . \tag{4.14}$$

By taking this relation (4.14) in turn for p and n, we deduce the following relationships between the amplitudes $\overline{M}_I$, M_I for definite I-spin states of the $\overline{K}$-N and K-N systems,

$$\overline{M}_1(\overline{K}-N) = (-1)^{n+i} (M_0 (K-N) + M_1(K-N))/2 , \tag{4.15a}$$

and

$$\overline{M}_0(\overline{K}-N) = (-1)^{n+1} (3 M_0(K-N) - M_1(K-N))/2 . \tag{4.15b}$$

Finally, we recall that for $i = 0$, we have $\overline{M}_1 = \overline{M}_0$ and $M_1 = M_0$; for $i = 1$, $\overline{M}_0 = -3 \overline{M}_1$ and $M_0 = -3 M_1$. Inserting these relations into Eqs. (4.15) for the case $i = 0$ and $i = 1$ in turn, we derive the general result,[63] independent of the I-spin state of the interacting particles and of the i-spin transferred between them,

$$\overline{M}_I(\overline{K}-N) = (-1)^n M_I(K-N) . \tag{4.16}$$

This relation[64] has the same form as the well-known relation connecting the pionic contributions to the N-N and the $\bar{N}$-N interactions.

If we denote by (X_e, X_0) and (Y_e, Y_0) the contributions to the I = 1 K-N interaction due to the exchange of systems with even and odd G-conjugation parity (e.g., for even and odd n), with total isotopic spin i = 0 and 1, respectively, then we have for the other K-N and $\bar{K}$-N states the following interactions:

$$V_1(K\text{-}N) = X_0 + X_e + Y_0 + Y_e, \quad V_1(\bar{K}\text{-}N) = -X_0 + X_e - Y_0 + Y_e ,$$

and

$$V_0(K\text{-}N) = X_0 + X_e - 3Y_0 - 3Y_e , \quad V_0(\bar{K}\text{-}N) = -X_0 + X_e + 3Y_0 - 3Y_e .$$

The interactions $V_1(K\text{-}N)$ and $V_1(\bar{K}\text{-}N)$ are known to be strongly repulsive and strongly attractive (with the interpretation of the Y^* resonance as an I = 1 $\bar{K}$-N bound state), respectively, whereas $V_0(K\text{-}N)$ is weakly repulsive and $V_0(\bar{K}\text{-}N)$ may be repulsive, of uncertain strength, or very strongly attractive. These facts could be fitted qualitatively by these expressions if the dominant contributions were from the exchange of an I = 0 particle (ω^0 ?) with odd G-conjugation parity, and of an I = 1 system with odd G. In the attempt to understand what interactions could give rise to a bound $\bar{K}$-N state, it is natural, as remarked above, to consider first the processes of pion exchange between the $\bar{K}$ meson and the nucleon, since, for given coupling strength, the $\bar{K}$-N interactions of longest range are those which will be the most effective in binding. However, in terms of the pion configurations at present conjectured to be of particular importance, there does not yet appear an obvious and simple interpretation of the character of the observed (K-N) and $\bar{K}$-N potentials.[65]

V. GLOBAL SYMMETRY AND PION-HYPERON RESONANCES

The charge-independent Yukawa interaction of the pion with Λ and Σ hyperons may be written

$$g_{\Sigma\Lambda\pi}(\Lambda^\dagger\, O\underset{\sim}{\Sigma} - \underset{\sim}{\Sigma}^\dagger\, O\Lambda)\cdot\underset{\sim}{\pi} + g_{\Sigma\Sigma\pi}(\underset{\sim}{\Sigma}^\dagger \times O\underset{\sim}{\Sigma})\cdot\underset{\sim}{\pi}\ , \tag{5.1}$$

where Λ, $\underset{\sim}{\Sigma}$, and $\underset{\sim}{\pi}$ denote the isotopic-spin components of the Λ, Σ, and pion wave functions, and O denotes the relevant space-spin operators. It was pointed out independently by Gell-Mann[12] and by Schwinger[13] that, for

$$g_{\Lambda\Sigma\pi} = g_{\Sigma\Sigma\pi} = g\ , \tag{5.2}$$

the interaction (5.1) can be written in a form whose structure parallels that of the pion-nucleon interaction, namely

$$g_{NN\pi}(N^\dagger\, \underset{\sim}{\tau}\, O\, N)\cdot\underset{\sim}{\pi}\ . \tag{5.3}$$

This was achieved by replacing the Λ singlet and Σ triplet by two doublets, which we may denote by N_2 and N_3,

$$\begin{pmatrix} N_2^{\ +} \\ \\ N_2^{\ 0} \end{pmatrix} = \begin{pmatrix} \Sigma^+ \\ \\ (\Lambda - \Sigma^0)/\sqrt{2} \end{pmatrix},\ \begin{pmatrix} N_3^{\ 0} \\ \\ N_3^{\ -} \end{pmatrix} = \begin{pmatrix} (\Lambda + \Sigma^0)/\sqrt{2} \\ \\ \Sigma^- \end{pmatrix}, \tag{5.4}$$

in terms of which the pion-hyperon interaction (5.1) takes the form

$$g(N_2^{\ \dagger}\, \underset{\sim}{\tau}\, O\, N_2 + N_3^{\ \dagger}\, \underset{\sim}{\tau}\, O\, N_3)\cdot\underset{\sim}{\pi}\ . \tag{5.5}$$

For this to be possible, it must, of course, be assumed that Λ and Σ hyperons have the same parity and that the operators O associated with

$g_{\Lambda\Sigma\pi}$ and $g_{\Sigma\Sigma\pi}$ are of identical form. If the Λ and Σ hyperons had the same mass, and (5.5) represented their only strong interaction, then, regardless of the coupling strength g, this doublet symmetry would be exact. N_2 and N_3 would then represent two independent doublets of the same mass, which could not be transformed one into the other by any pion processes.

This doublet representation of the Λ and Σ hyperons actually corresponds to a representation of their isotopic spin $\tilde{I}_Y$ as the sum of two half-integer isotopic spins $\tilde{i}$ and $\tilde{k}$, such that $\tilde{I}_Y = \tilde{i} + \tilde{k}$. The form of (5.5) then corresponds to the situation where the pion field is coupled with only one of these (say $\tilde{i}$); the N_2 and N_3 doublets are then the $+\frac{1}{2}$ and $-\frac{1}{2}$ substates, respectively, of the 3-component of the other isotopic spin $\tilde{k}$. As Pais[66] has pointed out, the experimental evidence on K-meson processes shows that the N_2 and N_3 doublets are actually linked quite strongly through the K couplings. In fact, the large Λ-Σ mass difference already represents a large deviation from the doublet approximation, which has often been attributed to the nonsymmetry of the K couplings and which itself must lead to substantial mixing between the N_2 and N_3 doublets.

The "global symmetry hypothesis" of Gell-Mann and Schwinger supposes further that the coupling parameters $g_{\Sigma\Sigma\pi}$ and $g_{\Sigma\Lambda\pi}$ (as well as $g_{\Xi\Xi\pi}$) are all equal to $g_{NN\pi}$, the space-spin operator O being assumed the same for all of these interactions. With this hypothesis, the N_2 and N_3 states will behave exactly like nucleons as far as their interactions with pions are concerned, at least in the limit that those interactions for which the doublet symmetry does not hold do not strongly disturb this symmetry for the pion processes.

The global-symmetry hypothesis then leads directly to relationships between the hyperon-nucleon and the nucleon-nucleon potentials, at least for that part of these potentials which arises from the exchange of pions. If the N-N potential is written in the form

$$V = \frac{1}{4} V_0(\underline{r}_{12}, \underline{L}_{12}, \underline{\sigma_1}, \underline{\sigma_2})(1 - \tau_1 \cdot \tau_2) + \frac{1}{4} V_1(\underline{r}_{12}, \underline{L}_{12}, \underline{\sigma_1}, \underline{\sigma_2})(3 + \tau_1 \cdot \tau_2) , \tag{5.6}$$

where V_0 and V_1 denote the $I = 0$ and $I = 1$ potentials, then the hyperon-nucleon potentials may be deduced in terms of V_0 and V_1 on the basis of this hypothesis. Thus, in the Σ^--n configuration, the isotopic spins are aligned to total $\frac{3}{2}$, so that the N_3-N configuration which is effective is that with parallel isotopic spins, that is with $I = 1$; generally, for the $I = \frac{3}{2}$ Y-N states, we have then

$$V_{3/2}(\Sigma,\Sigma) = V_1 \quad . \tag{5.7}$$

For the $I = \frac{1}{2}$ Y-N states, both Σ-N and Λ-N systems contribute, so that the $I = \frac{1}{2}$ interaction takes a matrix form, as follows:

$$\begin{pmatrix} V_{1/2}(\Lambda,\Lambda) & V_{1/2}(\Lambda,\Sigma) \\ V_{1/2}(\Sigma,\Lambda) & V_{1/2}(\Sigma,\Sigma) \end{pmatrix} = \begin{pmatrix} \frac{1}{4}(V_0 + 3V_1) & \frac{\sqrt{3}}{4}(V_0 - V_1) \\ \frac{\sqrt{3}}{4}(V_0 - V_1) & \frac{1}{4}(3V_0 + V_1) \end{pmatrix} . \tag{5.8}$$

The Schrödinger equation for the $I = \frac{1}{2}$ system then consists of the coupled equations,

$$-\frac{1}{2M_\Sigma}\nabla^2\psi_\Sigma + V_{\Sigma\Sigma}\psi_\Sigma + V_{\Sigma\Lambda}\psi_\Lambda = (E - \Delta)\psi_\Sigma , \tag{5.9a}$$

$$-\frac{1}{2M_\Lambda}\nabla^2\psi_\Lambda + V_{\Lambda\Sigma}\psi_\Sigma + V_{\Lambda\Lambda}\psi_\Lambda = E\,\psi_\Lambda , \tag{5.9b}$$

where E denotes the kinetic energy in the Λ-N system, and the inclusion of the mass difference $\Delta = M_\Sigma - M_\Lambda$ is, of course, quite essential.

The main difficulty of principle in the use of these relations to predict the hyperon-nucleon interactions in terms of the global-symmetry hypothesis lies in the fact that the Pauli exclusion principle limits the states available for the N-N system, but not for the Y-N system. For example, consider the Σ^--n ^3S interaction. Equation (5.7) states that this is given by the $I = 1$ ^3S N-N interaction, but this interaction cannot be measured directly, since the exclusion principle forbids the ^3S state for the $I = 1$ N-N system. The $I = 1$ triplet N-N potential must be deduced from measurements on the ^3P, ^3F, and ^3H states; if it is possible to identify the angular momentum dependence of the potential, we can then extrapolate to zero angular momentum and deduce the form of the $I = 1$ ^3S potential. Such an extrapolation will be possible in practice only if it is justified to confine attention to potentials of sufficiently simple angular momentum dependence, such as tensor forces, spin-orbit $((\sigma_1 + \sigma_2) \cdot L_{12})$ forces, and perhaps forces depending on $\sigma_1 \cdot L_{12}\, \sigma_2 \cdot L_{12}$ or $L_{12}^{\;2}$. Fortunately, the pion theory of nucleon forces gives us some reason to believe that, outside a strongly repulsive central region whose details are not of particular importance, the N-N forces have a dependence on $\underline{r}_{12}$, $\underline{L}_{12}$ and spin in which more complicated terms than these will not play a major role. Further, the theory gives considerable guidance concerning the spin, isotopic spin, and radial dependence of various terms of the nuclear potential in the outer region. On this basis it appears reasonable to believe that, given sufficient experimental data on the N-N system, one could make a fairly reliable extrapolation to determine the potential appropriate to states forbidden for the N-N system. Such an extrapolation procedure would appear particularly plausible for the $I = 1$ singlet and the $I = 0$ triplet potentials, where the potential is obtained empirically in the S, D, and G states and extrapolation is required to the P, and F states lying between them since the S scattering explores the inner regions of the potential while the higher partial waves are particularly sensitive to the outer regions and to the angular-momentum-dependent parts of the interaction. The extrapolation to the S interaction for the $I = 1$ triplet or the $I = 0$ singlet potentials is much less certain, for the experimental data then refer only to the P, F, H, states. The inner region of the central potential, which is particularly important for the S scattering, cannot really be so well established from the study of the higher partial waves, and this may be a source of appreciable uncertainty in the applications now to be discussed.

De Swart and Dullemond[67] have recently carried out detailed calculations on the S-wave hyperon-nucleon interactions, based on an N-N potential which gives a good fit to the data available at present. This potential consisted of the $I = 1$ potential deduced by Bryan[68] from the p-p data and the $I = 0$ n-p potential of Gartenhaus.[69]

The S-wave scattering amplitudes for the Λ-N system at low energies have been calculated by using Eqs. (5.9), including the coupling to the (energetically unavailable) Σ-N channel. For the ^1S state, the zero-energy scattering length obtained was -2.1 fermis, with an effective range of 2.24

fermis. The Yukawa potential corresponding to these parameters has a range parameter of 0.78 fermi, close to $(2m_\pi)^{-1}$, as expected. This equivalent central potential has volume integral 370 Mev f^3, in good agreement with the ^1S Λ-N potential strength deduced by Dalitz and Downs[70] from the data on light Λ hypernuclei. For the ^3S state, the scattering length obtained was 0.12 fermi (with effective range 85 fermis, which corresponds to a range parameter of approx 0.65 fermi), corresponding to a weakly repulsive equivalent potential of volume integral approx -55 Mev f^3. The latter is compatible with the data on hypernuclei if a three-body Λ-N-N potential[71] is included which is attractive and of reasonable strength.[72]

Calculations have also been made by de Swart and Dullemond on the rates for the competing reactions

$$\Sigma^- + p \to \Sigma^0 + n \tag{5.10a}$$

and

$$\to \Lambda + n , \tag{5.10b}$$

for Σ^--proton collisions at very low energies. For Σ^- hyperons which came to rest in liquid hydrogen, these reactions have been studied by Ross,[75] who found the ratio

$$(\Sigma^- + p \to \Lambda + n)/(\Sigma^- + p \to \Sigma^0 + n) = 2.0 \pm 0.5 . \tag{5.11}$$

If these reactions are assumed to occur through the S-wave Σ-N interaction, as would follow from the discussion by Day, Sucher, and Snow[30] of mesic absorption from high-lying levels of hydrogen-like mesic atoms in consequence of the Stark-mixing mechanism, then these calculations can be compared with Ross's data. The amplitudes for Σ^--p elastic scattering and for the Σ^0

reaction (5.10a) are given by

$$M(\Sigma^- + p \rightarrow \Sigma^- + p) = \frac{1}{3}(a_3(\Sigma,\Sigma) + 2\, a_1(\Sigma,\Sigma)), \quad (5.12a)$$

$$M(\Sigma^- + p \rightarrow \Sigma^0 + n) = \frac{\sqrt{2}}{3}(a_3(\Sigma,\Sigma) - a_1(\Sigma,\Sigma)), \quad (5.12b)$$

where $a_3(\Sigma,\Sigma)$ and $a_1(\Sigma,\Sigma)$ denote the zero-energy elastic-scattering amplitudes for the Σ-N system in the $I = \frac{3}{2}$ and $I = \frac{1}{2}$ states, respectively. At zero energy, the amplitude $a_3(\Sigma,\Sigma)$ is real, but $a_1(\Sigma,\Sigma)$ is complex, because of the absorption due to the competing reaction (5.10b), confined to the $I = \frac{1}{2}$ channel. The amplitude for the Λ reaction (5.10b) is given by

$$M(\Sigma^- + p \rightarrow \Lambda + n) = \frac{\sqrt{2}}{3}(a_1(\Sigma,\Lambda)), \quad (5.12c)$$

where the amplitudes $a_1(\Sigma,\Sigma)$, $a_1(\Sigma,\Lambda)$ are calculated together from the $I = \frac{1}{2}$ equations (5.9). For the ^1S state, the $I = \frac{3}{2}$ interaction is almost resonant at zero energy, since it is equal to the ^1S N-N interaction, and the amplitude $a_3(\Sigma,\Sigma)$ is very large, whereas $a_1(\Sigma,\Sigma)$ and $a_1(\Sigma,\Lambda)$ have only moderate values. As a result, the Σ^0 reaction is strongly dominant in the ^1S state, the calculated Λ/Σ^0 ratio[76] being $\approx 1/40$. For the ^3S state this near-resonant situation does not hold, and the Λ/Σ^0 ratio obtained is closer to the phase-space ratio of 4.6, [76] although somewhat smaller than this for the reasons discussed previously;[77] the calculated Λ/Σ^0 ratio for the ^3S state is $3.6\ (\pm\ 0.4)$ where the error given reflects the present uncertainty in the (Σ^-, Σ^0) mass difference. Assuming that the processes of atomic capture and Stark mixing discussed by Day et al.[30] do not depend on the relative spin orientation of Σ^- and proton, the Λ/Σ^0 ratio predicted for Σ^- capture in hydrogen is predicted to be

$$\left(\frac{\Lambda}{\Sigma^0}\right)_{\text{s-wave capture}} = \frac{(\Lambda/\Sigma^0 + \Lambda)_{S=0} + 3(\Lambda/\Sigma^0 + \Lambda)_{S=1}}{(\Sigma^0/\Sigma^0 + \Lambda)_{S=0} + 3(\Sigma^0/\Sigma^0 + \Lambda)_{S=1}} \tag{5.13}$$

$$= 1.55\ (\pm\ 0.1)\ ,$$

which is in essential agreement with the observed ratio (5.11).

These two comparisons with the experimental data really provide quite different tests of global symmetry. For the Λ-N potential, the dominant terms arise from two-pion exchange and are proportional to $g^2_{\Sigma\Lambda\pi}$; from the ^1S comparison here, we conclude that the values of $g^2_{\Sigma\Lambda\pi}$ and $g^2_{NN\pi}$ must be very comparable. However, the longest-range potentials which give rise to the $\Sigma^- + p$ reactions (5.10a,b) are those arising from exchange of one pion, and are therefore proportional to $g_{\Sigma\Sigma\pi}$ and $g_{\Sigma\Lambda\pi}$, respectively. The ratio of the $(\Sigma^0 + n)$ and $(\Lambda + n)$ transition rates therefore provides a rough measure of the ratio $(g_{\Sigma\Sigma\pi}/g_{\Sigma\Lambda\pi})^2$. The two comparisons discussed above therefore indicate,[78] at least qualitatively, $g^2_{\Sigma\Lambda\pi} \approx g^2_{\Sigma\Sigma\pi} \approx g^2_{NN\pi}$, in accord with the global symmetry hypothesis.

The main argument against global symmetry was that given by Salam,[79] concerning the nature of the final-state interactions in the $K^- + p \rightarrow \pi + \Sigma$ reactions in the low-energy region. The early data indicated that the phase difference ϕ_t between the $I = 0$ and $I = 1$ matrix elements at zero energy was large, $\phi_t \approx 60$ deg. Since the π-Σ scattering states involved are $s_{1/2}$ or $p_{1/2}$, according as the $(K\Sigma)$ parity is odd or even, it was difficult to understand in terms of global symmetry how these could be so large, even when the kinematic effects of the (Λ,Σ) mass difference were included. More recent data[29] have shown that, although the phase difference $|\phi_t|$ could be as large as 60 deg, uncertainties in the data are such that they are also compatible with any angle ϕ_t down to $\phi_t \approx 0$ deg, and this argument against global symmetry loses much of its force, at least until a more certain determination of ϕ_t is achieved.

For pion-hyperon scattering, as pointed out by Gell-Mann,[12] global symmetry requires $j = \frac{3}{2}$, $I = \frac{3}{2}$ resonances in the π-N_2 and π-N_3 systems, corresponding to the π-N (3,3) resonance. After the Σ-Λ mass difference Δ is taken into account, these resonances are expected to appear as separated I = 1 and I = 2 resonant states in the pion-hyperon scattering. Their final location has been estimated by Amati et al.[80] in terms of a static model of the pion-hyperon interaction. They have also considered the effect of a disturbance of $g_{\Sigma\Lambda\pi}$ and $g_{\Sigma\Sigma\pi}$ from the global symmetry value $g_{NN\pi}$ by nonsymmetric forces, measured by the parameters

$$\delta = (g^2_{\Sigma\Lambda\pi} - g^2_{\Sigma\Sigma\pi})/(g^2_{\Sigma\Lambda\pi} + g^2_{\Sigma\Sigma\pi}), \quad g_Y^{\,2} = \frac{1}{2}(g^2_{\Sigma\Lambda\pi} + g^2_{\Sigma\Sigma\pi}) \;. \tag{5.14}$$

For small values of δ, Amati et al. find that the resonance energies are given by

$$E_r^{\,1} = M_\Lambda + \Omega - \frac{1}{2}\Delta - \frac{5}{6}\Delta\delta \,, \tag{5.15a}$$

$$E_r^{\,2} = M_\Lambda + \Omega + \frac{3}{2}\Delta + \frac{1}{2}\Delta\delta \,, \tag{5.15b}$$

where Ω is given by

$$\Omega = g_Y^{-2}\, \frac{P}{12\pi} \int_{m_\pi}^{\infty} \frac{d\omega'\, q'^3\, u^2(q')}{\omega'^2(\omega' - \omega)} \,, \tag{5.16}$$

and depends on $g_Y^{\,2}$ and the cutoff energy. As expected, the location of the resonance is rather sensitive to the value of $g_Y^{\,2}$. The I = 2 resonance is predicted to lie higher than the $I = 1$ resonance, which is reasonable, since, to a first approximation, the resonance location is expected to correspond to a definite momentum for the incident system, which must be entirely π-Σ for the $I = 2$ case.

Amati et al. have suggested recently that the observed π-Λ resonance may represent this $I = 1$, $j = \frac{3}{2}$ resonance. They calculate the half-width of this resonance as

$$\Gamma_1/2 = \frac{4}{9} g_Y{}^2 (2q_\Lambda{}^3 + q_\Sigma{}^3) , \tag{5.17}$$

with a correction factor of $(1 + 0.66\ \delta)$ for $g_{\Sigma\Lambda\pi} \neq g_{\Sigma\Sigma\pi}$. Taking the same value of $g_Y{}^2$ as for the (3,3) resonance, $\delta = 0$, and the value $q_{\pi N} = 230$ Mev/c, the (3,3) resonance half-width[16] $\Gamma_{\pi N}/2 = 50$ Mev leads to a half-width $\Gamma_1/2 = 28$ Mev for the π-Λ resonance, quite compatible with the present experimental evidence. The branching ratio at resonance is given by

$$(\Sigma\pi/\Lambda\pi)_1 = \frac{1}{2} \left(\frac{q_\Sigma}{q_\Lambda} \right)^3 \frac{1}{(1 + \delta)^2} , \tag{5.18}$$

which takes the value 0.11 for $\delta = 0$. This prediction is also compatible with the data.

These predictions are in remarkable agreement with the data on the Y^* resonance. The conclusion above that the hyperon-nucleon interactions are in good general agreement with the global symmetry hypothesis gives further weight to the identification of the Y^* resonance with this $j = \frac{3}{2}$ resonance. Obviously, a clear-cut spin determination would distinguish most clearly between this possibility and the " $\bar{K}$-N bound-state" interpretation discussed in the earlier sections. At present, although the Adair analyses which have been made are all consistent with isotropic decay of the Y^* and a $j = \frac{1}{2}$ spin assignment,[81] these are still somewhat unsatisfactory in that these Adair plots show considerable backward-forward asymmetry,[84] a feature which could not be present if the Y^* decayed in isolation, as the use of the Adair analysis assumes.

With this identification of the Y^* resonance, the prediction of an $I = 2$ π-Σ resonance becomes rather specific. The $I = 2$ resonance is expected to occur at a mass value close to 1545 Mev and to have a half-width of about $50(288/230)^3 \approx 100$ Mev. It is of obvious importance to investigate whether this resonant state is produced in reactions such as

$$K^- + p \rightarrow (\Sigma^- + \pi^+) + \pi^0 \tag{5.19a}$$

and

$$\pi^+ + p \rightarrow (\Sigma^+ + \pi^+) + K^0 \tag{5.19b}$$

at higher energies than have been investigated to date.

In conclusion, it must be emphasized that the calculations by Amati et al. ignore the effect of the coupling between the pion-hyperon system and the $j = \frac{3}{2}$ $\bar{K}$-N channel. Since the KYN coupling is strong, it is quite possible that, even with global symmetry, these interactions could modify appreciably the location of these resonances, quite beyond their influence on the effective values of $g_{\Sigma\Lambda\pi}$ and $g_{\Sigma\Sigma\pi}$. It may well be that the agreement between the observed Y^* resonance and this predicted $j = \frac{3}{2}$ resonance is fortuitous, and that an analogue of the (3,3) resonance may lie in some higher energy region. (Although our intuitive expectation, based on lowest-order perturbation theory, would be that this additional coupling to a higher-energy configuration would depress, rather than raise, the resonance energy.) In this event, it could appear as an $I = 1$ $\bar{K}$-N resonance, but its influence on the $\bar{K}$-N channel need be marked only if the matrix elements coupling the resonant state with the $\bar{K}$-N system were sufficiently large. As we have seen above in the discussion of the $\bar{K}$-N system, the coupling between open channels for strongly interacting systems need not always be large. Since such a pion-hyperon resonance would influence strongly the phase of the $I = 1$ reaction amplitude $M(\bar{K} + N \rightarrow \pi + \Sigma)$, the $(\Sigma^+ + \pi^-)/(\Sigma^- + \pi^+)$ ratio, which depends sensitively on the relative phases of the $I = 0$ and $I = 1$ reaction amplitudes, may provide a sensitive indicator for such a pion-hyperon resonance. It is quite probable that further surprises are in store for us concerning resonances in these strange-particle systems.

APPENDIX A

THE ZERO-ENERGY $\bar{K}$-N SCATTERING LENGTHS

A brief discussion is given here of the derivation of the scattering lengths A_0 and A_1 given in Table I and of the uncertainties in this derivation. The data used were those summarized in the Kiev Conference Report of Alvarez.[29]

First consider the in-flight data. All evidence concerning the data in the (lab) momentum range 100 to 200 Mev/c is consistent with the assumption that the interaction is effective dominantly in the s wave.[34] Instead of attempting an elaborate least-squares fitting to the data in various momentum ranges, we concentrated the available data at a mean energy of 172 Mev/c in the following way. Since the elastic scattering cross section is slowly varying, a weighted average of the available cross sections was used, giving $\sigma_{e\ell} = 79 \pm 10$ mb. The charge-exchange cross section was taken as 15 ± 4 mb. A value for $\sigma_{abs}(\Sigma^{\pm})$ was obtained by taking a weighted average of $k_L \times \sigma_{abs}(\Sigma^{\pm})$ over this momentum range; from this mean value, the estimate $\sigma_{abs}(\Sigma^{\pm}) = 45 \pm 7$ mb was obtained for $k_L = 172$ Mev/c. At this energy, $\pi\lambda\!\!\!^{-2} = 98.5$ mb, and this partial absorption cross section is therefore to be considered rather large; in fact, its upper limit comes relatively close to the geometrical limit allowed by the other cross sections, taken together with the zero-energy parameters. For this reason it was decided to make some rough allowance for the amount of p-wave absorption included in this cross section, as follows. At 400 Mev/c, the angular distributions show clear evidence of strong p-wave interactions and the total absorption cross section for all hyperon production is observed to be 33.5 mb, to be compared with an s-wave geometrical limit of $\pi\lambda\!\!\!^{-2} = 20$ mb at this energy. Rather arbitrarily, it was assumed that about half of the absorption cross section at 400 Mev/c was from the p wave, and that the p-wave cross section for $\Sigma^{\pm}$ production at this energy was about 9mb. This estimate was scaled in proportion to the momentum, to give a corresponding estimate for 172 Mev/c, which was then subtracted from the above figure for $\sigma_{abs}(\Sigma^{\pm})$ at this momentum. This procedure led to the estimate of 40.5 ± 7 mb adopted for $\sigma_{abs}(\Sigma^{\pm})$.

As Kruse and Nauenberg have discussed,[85] the knowledge of the s-wave cross sections for all hyperon-production reactions at a given energy E, together with the elastic and charge-exchange cross sections, would allow a determination of the scattering amplitudes $A_0(E)$ and $A_1(E)$ appropriate to that energy. However, such complete data are not yet available, and, in order to obtain an estimate of the scattering amplitudes, it is necessary to make use of the "at rest" data and to make some specific assumption concerning the energy dependence of A_0 and A_1. For example, one could assume the energy dependence of effective range theory, $A_I(1 + \frac{1}{2} R_I A_I k^2)^{-1}$, with some physically appropriate choice for the effective ranges R_0 and R_1. For simplicity, we have made the choice of zero effective range, that is, of energy-independent values for A_0 and A_1.

At c.m. momentum k, it is convenient to write the expressions for elastic and charge-exchange cross sections as

$$\sigma_{e\ell} + \sigma_{ce} = \pi \{(a_0^2 + b_0^2)/2D_0 + (a_1^2 + b_1^2)/2D_1\} , \tag{A1}$$

$$\sigma_{ce} = \pi \{(a_0 - a_1)^2 + (b_0 - b_1)^2\}/(D_0 D_1) , \tag{A2}$$

where D denotes $\{(1 + kb)^2 + (ka)^2\}$. In these expressions, the modifications due to the $(K^-, \overline{K}^0)$ mass difference and to the K^--p Coulomb interaction have been neglected, as they do not represent major corrections at 172 Mev/c and can be allowed for subsequently. For the absorption cross section, we have

$$\sigma_{abs}(\Sigma^{\pm}) = \frac{2\pi}{k} \{2b_0/3D_0 + ((1 - \epsilon)/2)(b_1/D_1)\} , \tag{A3}$$

where ϵ denotes the fraction of $I = 1$ absorption which leads to Λ hyperons. The value of ϵ was also assumed energy-independent and was taken from the zero energy data (see below). The equations (A1) and (A3) could then be solved algebraically for D_0 and D_1 in terms of b_0 and b_1, and therefore for a_0 and a_1. By a systematic procedure of trial and error, for assigned values of b_0, all values of b_1 (together with the corresponding values of a_0 and a_1) which satisfy the equation (A3) were then determined by an electronic computer.

At zero energy, the quantities determined directly from the "at rest" events are

$$R = (\Sigma^+ + \Sigma^-)/(\Sigma^0 + \Lambda) = 1.79 \pm 0.18,$$

$$S = \Lambda/(\Sigma^0 + \Lambda) = 0.214 \pm 0.04,$$

$$T = \Sigma^-/\Sigma^+ = 2.18 \pm 0.06.$$

These numbers allow an estimate to be obtained for ϵ, or for $(\Sigma/\Lambda)_1$,

$$(\Sigma/\Lambda)_1 = \left(\frac{1}{\epsilon} - 1\right) = (R - 2(1 - S))/S . \tag{A4}$$

The value obtained, $\epsilon = 0.5 \begin{smallmatrix} +0.35 \\ -0.15 \end{smallmatrix}$, is rather poorly determined at present. In terms of M_0 and M_1, the zero-energy amplitudes for absorption leading to $(\pi + \Sigma)$ states with $I = 0$ and 1, the expression for T is

$$T = (M_0^2 + \frac{3}{2} M_1^2 + \sqrt{6}\ M_0 M_1 \cos \phi_t)/(M_0^2 + \frac{3}{2} M_1^2 - \sqrt{6}\ M_0 M_1 \cos \phi_t), \tag{A5}$$

where ϕ_t is the relative phase between M_0 and M_1. Comparison of this expression with the observed value for T allows a lower limit of 0.025 to be placed on the ratio M_1^2/M_0^2; combining this with the value of $N_1^2/M_0^2 = 0.091$ (where N_1 denotes the $I = 1$ amplitude leading to the $(\pi + \Lambda)$ channel) obtained from S leads to the lower limit of 0.28 ± 0.05 for the ratio $(\Sigma/\Lambda)_1$, which corresponds to an upper limit of about 0.8 for ϵ. The ratio σ_0/σ_1 of the $I = 0$ and $I = 1$ zero energy absorption rates (given by $M_0^2/(M_1^2 + N_1^2)$) may be determined from

$$\sigma_0/\sigma_1 = \epsilon(3\Sigma^0/\Lambda) = \epsilon(3(1 - S)/S) . \tag{A6}$$

Since the second factor is relatively well determined (11.0 ± 3) uncertainties in σ_0/σ_1 and ϵ are quite strongly correlated. The relationship between σ_0/σ_1 and the amplitudes A_0, A_1 is given by[2]

$$\frac{\sigma_0}{\sigma_0} = \left(\frac{b_0}{b_1}\right) \left\{ \frac{(1 + \kappa a_1)^2 + (\kappa b_1)^2}{(1 + \kappa a_0)^2 + (\kappa b_0)^2} \right\}, \tag{A7}$$

where κ denotes $(2\mu_K \Delta)^{1/2}$ and Δ is here the $(K^-, \overline{K}^0)$ mass difference. Since the solutions obtained have the feature that either a_0 or a_1 is large, this second factor has a considerable effect on the determination of A_0 and A_1; in particular, the magnitude of this factor depends quite strongly on the absolute sign chosen for the pair (a_0, a_1).

The procedure for determining A_0 and A_1 from these data was then as follows. For specified values of $\sigma_{e\ell}$, $\sigma_{abs}(\Sigma^{\pm})$, σ_{ce}, and ϵ, the parameters a_0, a_1, and b_1 were determined as function of b_0. Generally, two solutions were obtained in each of which the relative sign of a_0 and a_1 was definite but the absolute sign of (a_0, a_1) was not determined. For all four solutions (i.e., with both choices for the absolute sign of (a_0, a_1) for both cases), the right-hand side of (A7) was calculated as a function of b_0, and the value of b_0 (and with it, the values of a_0, a_1, and b_1) was then determined by comparison of σ_0/σ_1 with the physical value determined from (A6).

The mean values given in Table I for the (a±) and (b-) amplitudes were obtained in this way from the best values for the input data. The uncertainties to be associated with these amplitudes were then estimated by considering the sum

$$\chi^2 = \Sigma_{i=1}^{4} (X_i (a_0, b_0, a_1, b_1) - \bar{X}_i)^2/\sigma_i^2 , \tag{A8}$$

where X_1, X_2, X_3, X_4 denote the expressions (A1), (A2), (A3), (A7), respectively, and σ_1, σ_2, σ_3, σ_4 denote the standard deviations associated with the experimentally observed values $\bar{X}_1$, $\bar{X}_2$, $\bar{X}_3$, and $\bar{X}_4$. The relative probability for a given set (a_0, b_0, a_1, b_1) on the basis of this data is then proportional to $\exp(-\chi^2/2)$. For a mean value set, $\chi^2 = 0$; the surface $\chi^2 = 1$ in the (a_0, b_0, a_1, b_1) space then defines the uncertainty on these parameter values to the confidence level of one standard deviation. The error quoted for each parameter in Table I was obtained from the intersections of this surface with the corresponding co-ordinate axis when the other three parameters were held at their mean values. This procedure ignores the possibility of large off-diagonal elements in the error matrix and may underestimate the uncertainty of the parameter in certain cases.

For the (a+) set, the amplitudes appear to be relatively well determined. For the (a-) set, a_0 is quite poorly determined in comparison with a_1. This insensitivity of the data to the value of a_0 is due to the large value of b_0, for the contribution from b_0 generally dominates the contribution from a_0 in the expressions above; the value of b_0 itself is also no more accurately determined. It is of interest to note that the Σ^-/Σ^+ ratio in the region 100 to 200 Mev/c, which has not been used in the above analysis, is also insensitive to the value of a_0. As a_0 varies from -1.15 to -0.35, the value calculated for the average Σ^-/Σ^+ ratio over this interval varies from 0.87 to 0.80, the experimental value being 0.95 ± 0.3. For the (b-) set, the amplitudes are quite well determined except for b_0, for which any value between 0.8 and 1.8 is acceptable; the probability curve for b_0 is very asymmetric and falls very gradually on the upper side of the best value for b_0.

No solution of the (b+) type exists for the best values of the input data. However, a solution of this type existed if σ_{abs}, ϵ or λ were reduced by one standard deviation. These solutions were used as starting values in a systematic search for the set (a_0, b_0, a_1, b_1) giving the least value of χ^2. This set of (b+) amplitudes is given in Table I and corresponds to $\chi^2 = 0.12$, which is a quite acceptable value. This minimum is quite well-defined and gives a satisfactory (b+) set of scattering amplitudes.

In concluding this Appendix, we wish to express our appreciation for the assistance of Mr. J. Dick, Applied Mathematics Division, Argonne National Laboratory, and of Mr. J. Schwartz, Physics Department, Lawrence Radiation Laboratory, with the programming of the computer calculations which were necessary here.

APPENDIX B

A SIMPLE DISPERSION-THEORETIC DISCUSSION OF PION-EXCHANGE IN $\overline{K}$-N PROCESSES

In order to illustrate some of the points made in Section IV, we consider here, following Ferrari et al.,[54,55,56] a simplified nonrelativistic treatment of the effect of the exchange of a vector boson B between $\overline{K}$ mesons and nucleon on the energy dependence of the scattering amplitudes. The diagram of interest is shown in Figure 2. Its amplitude is given by

$$F = f_K f_N \Omega(\underset{\sim}{k} + \underset{\sim}{k}')/(-(k - k')^2 + m_B^2) , \tag{B1}$$

where $\Omega = \tau_N \cdot \tau_K/4$ or 1, according as the boson B has $I = 1$ or $I = 0$, and f_K, f_N denote the coupling strength of B with the $\overline{K}$ meson and nucleon, respectively. This boson may represent a pair of $J = 1-$, $I = 1$ resonating pions, or perhaps the $I = 0$ ω^0 particle, or some other resonant $j = 1-$, $I = 0$ pion configuration. In the nonrelativistic limit, (B1) reduces to

$$F_{n.r.} = f_K f_N \Omega \, 2m_K/(m_B^2 + (\underset{\sim}{k} - \underset{\sim}{k}')^2) . \tag{B2}$$

Averaging over angles to obtain the s-wave amplitude leads to a logarithmic branch cut in $F_{n.r.}{}^s(k^2)$ as a function of k^2, running from $k^2 = -m_B^2/4$ to the left. Following Ferrari et al., we replace this branch cut by a simple pole at the point $k^2 = -m_B^2/2$. For the scattering matrix T, this corresponds to the assumption that on the left-hand cut, the imaginary part of T on the upper side of the cut may be approximated by

$$\text{Im}\, T_{KK} = \pi R \, \delta(k^2 + k_0^2) , \tag{B3}$$

where $k_0^2 = -m_B^2/2$ and $R = m_K \Omega f_K f_N/2\pi$, recalling the relationship $T = F/2\pi$ for the T matrix as defined in the text.

Consider first the two-channel case. The analytic function N(E) which satisfies the condition (4.5) on the left-hand cut, and which is finite at infinity, may be written

$$N = \begin{pmatrix} \alpha + \frac{R}{k^2 + k_0^2} D_{KK}(0) & \beta + \frac{R}{k^2 + k_0^2} D_{KY}(0) \\ \beta^\dagger & \gamma \end{pmatrix} . \qquad (B4)$$

The elements α, β, and γ are constants which represent the contributions to N from more distant singularities. When $R = 0$, the matrix N must reduce to the K matrix used in the text and must then be symmetric. The factors $D_{KK}(0)$ and $D_{KY}(0)$ denote the values of the corresponding elements of D at the point $k^2 = -k_0^2$.

The elements of the denominator matrix D are analytic functions in the k^2 plane, whose imaginary parts along the right-hand cuts are given by

$$\text{Im } D = -\pi \rho N . \qquad (B5)$$

Approximating the $\bar{K}$-N phase-space density by $\pi\rho_K = Ck$, and the π-Y phase space density by corresponding expressions, we may write down by inspection the analytic functions which satisfy Eq. (B5) and which agree with the normalization condition that Re D → 1 at infinity. The result is

$$D = \begin{pmatrix} 1 - iCk\alpha - iCRD_{KK}(0)\dfrac{k - ik_0}{k^2 + k_0^2} & -iCk\beta - iCRD_{KY}(0)\dfrac{k - ik_0}{k^2 + k_0^2} \\ - i\pi\rho_Y\beta^\dagger & 1 - i\pi\rho_Y\gamma \end{pmatrix} \tag{B6}$$

We now have two equations of consistency for the determination of $D_{KK}(0)$ and $D_{KY}(0)$. Thus, for $k = + ik_0$ in (B6), we have

$$D_{KK}(0) = 1 + Ck_0\alpha - \frac{CR}{2k_0} D_{KK}(0) , \tag{B7}$$

from which we obtain

$$D_{KK}(0) = (1 + Ck_0\alpha)/(1 + CR/2k_0) . \tag{B8}$$

Similarly,

$$D_{KY}(0) = Ck_0\beta/(1 + CR/2k_0) , \tag{B9}$$

so that the matrices N and D, modified for the effect of exchange of the boson B, are now obtained.

As pointed out by Bjorken and Nauenberg,[50] it is not immediately apparent that the scattering matrix T thus obtained is symmetric, but this may be verified quite readily by direct calculation of

$$\tilde{D}(T - \tilde{T})D = \tilde{D}N - \tilde{N}D , \tag{B10}$$

by showing that the right-hand side vanishes identically. It is now of interest to calculate the K matrix, by means of the relation (4.6). This leads to the result

$$K = Z \begin{pmatrix} \alpha + \phi(1 + Ck_0\alpha) & \beta \\ \beta^\dagger & \gamma Z^{-1} + \phi(Ck_0)^2 \beta^\dagger \beta \end{pmatrix} , \tag{B11}$$

where $Z = (1 - \phi Ck_0(1 + Ck_0\alpha))^{-1}$ and $\phi = [R/(1 + CR/2k_0)]/(k^2 + k_0^2)$. The result (B11) has been written in such a form that it is valid at once for the three-channel case also, when β is replaced by a 1-by-2 matrix. The $\bar{K}$-N scattering amplitude may now be obtained from (B11), with the result

$$A = Z \frac{M}{E} \Big\{ \alpha + \phi(1 + Ck_0\alpha) + i\pi\beta(1 - \phi Ck_0(1 + Ck_0\alpha) - \\ - i\pi\rho_Y(\gamma Z^{-1} + (Ck_0)^2\phi\beta^\dagger\beta))^{-1} \rho_Y \beta^\dagger \Big\} . \tag{B12}$$

In the unphysical region for the $\bar{K}$-N channel, the reduced K matrix may be obtained by using expression (4.9),

$$K_R = \gamma - \beta^\dagger\beta C(\kappa - \frac{Ck_0R}{1 + CR/2k_0} \frac{1}{\kappa + k_0})(1 + C\kappa\alpha - \frac{CR(1 + Ck_0\alpha)}{1 + CR/2k_0} \frac{1}{\kappa + k_0})^{-1} . \tag{B13}$$

The location of a $\bar{K}$-N bound state is then to be determined from Eq. (4.10), which, as can be seen from (B13), reduces here to the simple equation

$$1 + C\kappa\alpha - \frac{CR(1 + Ck_0\alpha)}{1 + CR/2k_0} \frac{1}{\kappa + k_0} = 0 , \tag{B14}$$

where the replacement $k = +i\kappa$ has been made in this region. It is of interest to note that, despite the occurrence of a pole in T at the point $k^2 = -\kappa^2 = -k_0^2$, the condition (B14) for a bound state shows no singular behavior even at the point $\kappa = k_0$. This is in accord with expectation, as discussed in Section IV. The Eq. (B14) is of course identical with the equation $(1 + C\kappa K_{KK}) = 0$.

The energy dependence of (B11) and (B12) arises from the energy dependence of the term ϕ. If the coupling parameters f_K and f_N are small, so that R is also small, this energy dependence is generally quite weak.

The case of most interest is that in which the coupling parameters f_K and f_N are large and most of the real part of the large scattering amplitude (a_1 for the (a±) solutions, a_0 for the (b±) solutions) can be attributed to the attractive potential generated by the exchange of the boson B between $\bar{K}$ meson and nucleon. For this case the parameter R is large and positive and the coefficient $R/(1 + CR/2k_0)$ which appears in ϕ is not strongly sensitive to its precise value. We shall illustrate the situation for the (a±) solutions by choosing k_0 to correspond to the mass 305 Mev (roughly the ω^0 mass), the lowest mass which may be relevant, since this may be expected to lead to a correspondingly large effective range and to the strongest energy dependence for A.

First we consider the (a-) situation. Here the potential term is sufficiently strong for binding, so a coupling strength $f_K f_N/4\pi \approx 1.4$ was chosen (sufficient to give about the observed $\bar{K}$-N binding for the static potential). Since the value of b_1 is small relative to a_1, we neglect the small contributions from the last term of (B12), neglecting also the element γ for the reasons discussed in Section III, and determine the value of α from the observed value $a = -0.85$ f, with the result $\alpha = -1.06$ f. To illustrate the energy dependence of $A = a + ib$, we then calculate these parameters at lab momentum 175 Mev/c, with the result $A_1 = (-0.93 + 0.15\ i)$f. The energy dependence found for this case is relatively slight. Finally, we substitute in Eq. (B14) to determine the location of the $\bar{K}$-N bound state for this model. The value obtained for κ is 1.27 f^{-1}, to be compared with the value $\kappa = 1.18\ f^{-1}$ computed from the zero-range approximation. This corrected value of κ would place the $\bar{K}$-N bound-state resonance at about 97 Mev below the $\bar{K}$-N threshold, compared with the estimate of -80 ± 30 Mev, with the zero-range assumption. When we recall that, in this case, the location of this $\bar{K}$-N bound state almost coincides with the location of the pole inserted into N(E) to represent the exchange of this boson, it is quite remarkable that this extrapolation into the unphysical region deviates so little from the extrapolation carried out with the zero-range approximation.

To illustrate the (a+) situation, we have chosen (rather arbitrarily) a weaker coupling, $f_K f_N/4\pi = 0.7$, to correspond to the absence of a bound state. The value of α corresponding to $a_1 = +1.45$ f is then found to be -0.27f. In this case the energy dependence of a and b is found to be much stronger; at 175 Mev/c, the value $A_1 = (0.45 + 0.15\ i)$f results, a very substantial fall from the zero-energy value assumed. With such a rapid variation of the parameters, it would be quite essential to modify the procedure of the analysis given in Appendix A, to relate the "at-rest" data and the 175 Mev/c data correctly. It is not easy to understand physically why the effective range for this case turns out to be so large, and a more detailed study of the effect of "long-range pion exchange" on the $\bar{K}$-N interaction certainly appears desirable at this stage.

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14. More formally, the complete wave-function of the system is written

$$\psi = \phi + \frac{P}{E - H_0} H_{int} \psi , \qquad \text{(i)}$$

where ϕ is an eigenfunction of the Hamiltonian H_0 without interactions between channels and has here been taken to be the projection of a plane wave, P denotes a principal value integral at the singularity $E = H_0$, and H_{int} is the interaction Hamiltonian causing the reaction processes. It is this second term of (i) which corresponds to the cosine wave of (2.2) for a two-particle channel. The K-matrix elements are then defined, apart from normalization factors, by the relation

$$K_{ij} = \langle \phi^{(i)}, K \phi^{(j)} \rangle = \langle \phi^{(i)}, H_{int} \psi^{(j)} \rangle$$

where $\psi^{(j)}$ is the wavefunction obtained for inhomogeneous term $\phi^{(j)}$ in Eq. (1). For a more complete reference to these formal points, we refer to the well-known paper of Lippmann and Schwinger (Ref. 15).

15. B. Lippmann and J. S. Schwinger, Phys. Rev. 79, 469 (1950).
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17. For example, with a potential interaction in a one-channel case, this point is reached for an energy such that $\psi^2 V(r) \sim e^{2\kappa r} e^{-\alpha r} \to \infty$ as $r \to \infty$, where κ denotes $|k|$. For the case where the potential interaction $V(r)$ is asymptotically proportional to $e^{-\alpha r}$, this occurs for total energy $E = (M^2 - \alpha^2/4)^{1/2} + (\mu^2 - \alpha^2/4)^{1/2}$.
18. The main mathematical problem lies in carrying out the matrix inversion specified in Eq. (2.6) for the evaluation of the scattering matrix from the reaction matrix. For a system with multiparticle channels, this involves the inversion of an integral operator, that is, the solution of an integral equation for T,

$$T(1 - i\pi \rho K) = K. \qquad \text{(i)}$$

This is the integral equation first discussed by W. Heitler (Ref. 19)

in connection with "radiation damping," the term used at that time to describe the effect of the unitarity condition on scattering and reaction amplitudes. An exactly analogous situation arises in the dispersion-theory formalism for multiparticle reaction channels, of course, and this has been discussed recently by R. Blankenbecler (Ref. 20).

19. W. Heitler, The Quantum Theory of Radiation, 2nd ed. (Oxford University Press, 1944), Sec. 25. See also W. Pauli, Meson Theory of Nuclear Forces (Interscience Publishers Inc., New York, 1946).

20. R. Blankenbecler, On the Construction of Unitary Scattering Amplitudes, (to be published, 1961).

21. Derivations of these formulae, and most of the subsequent formulae of this Section, are found in Ref. 2, for two-particle channels. However, these formulae are in fact quite general and hold valid when the channels f include any number of multiparticle channels. They are most readily derived by first expanding (2.6) as a series of powers of K, then separating out sub-summations of terms accordingly as they link i with i, i with f, f with i, or f with f, and finally summing up all the subseries obtained in this way. For example, the result (2.11) may be derived very directly, as follows:

$$T = K + (i\pi)\, K\rho K + \cdots \qquad + (i\pi)^n\, K\rho K\rho K \cdots \rho K + \cdots \qquad \text{(i)}$$

$$= K + i\pi\, K\,(P_i + Q_i)\rho K + \cdots$$

$$+ (i\pi)^n\, K(P_i + Q_i)\,\rho K\,(P_i + Q_i)\,\rho K \cdots \rho K + \cdots, \qquad \text{(ii)}$$

where P_i is the projection operator for channel i and Q_i its complement $(1 - P_i)$. To obtain the diagonal element of T, that is, $P_i T P_i$ in this notation, the sum (ii) may be regrouped (recalling $P_i^2 = P_i$ and $Q_i^2 = Q_i$) to give the form

$$P_i T P_i = A + i\pi A \rho_i A + \cdots + (i\pi)^n A\rho_i A\rho_i A \cdots \rho_i A + \cdots$$

$$= A(1 - i\pi \rho_i A)^{-1}, \qquad \text{(iii)}$$

where A is given by

$$A = P_i \{K + i\pi K Q_i \rho K + (i\pi)^2 KQ_i \rho K Q_i \rho K + \cdots$$
$$+ (i\pi)^n KQ_i \rho K Q_i \rho K \cdots \rho K + \cdots\} P_i$$

$$= P_i \{K + i\pi KQ_i \frac{1}{1 - i\pi Q_i \rho K Q_i} Q_i \rho K\} P_i, \qquad \text{(iv)}$$

which is precisely the curly bracket of expression (2.12). Other examples of the use of this technique for situations of interest here are to be found in Ref. 2, pp. 330-333.

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23. There are, however, many particular situations where explicit reference to the open channels f is very complicated (for example, if the channels f are multiparticle) and not of interest to the matter at hand. For such cases, the use of the "equivalent-reaction matrix" a would be convenient for the discussion of the energy dependence of the cross sections relating channels i, for example for the discussion

of cusp behavior at thresholds for some of the channels i. Such a situation arises in the discussion of cusps for Λ-K production at the Σ-K thresholds, where it is not of interest to specify in detail the features of the competing $\pi + N \rightarrow N + \pi + \pi$ processes.

24. This transformation to a "reduced reaction matrix" with the elimination of explicit reference to the closed channels is well known in nuclear reaction theory. For a particularly clear discussion in a standard text, we refer to R. G. Sachs, Nuclear Theory (Addison-Wesley, Cambridge, Mass., 1953), p. 295.

25. Resonances of this type have frequently been discussed in particular contexts. For example, Karplus and Rodberg (Ref. 26) have referred to this possibility for the $I = \frac{1}{2}$ Σ-N system in their discussion of K^--deuterium reactions. Dalitz and Tuan (Refs. 2, 27) first discussed this possibility for the $\bar{K}$-N and π-Y coupled systems. A general discussion for coupled two-particle systems has been given by Fonda and Newton (Ref. 28).

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27. R. H. Dalitz and S. F. Tuan, Phys. Rev. Letters 2, 425 (1959).

28. L. Fonda and R. G. Newton, Ann. Phys. (N. Y.), 10, 490 (1960).

29. Luis W. Alvarez, The Interactions of Strange Particles, Lawrence Radiation Laboratory Report UCRL-9354, Aug. 1960 (unpublished).

30. T. Day, G. Snow, and J. Sucher, Phys. Rev. Letters 3, 61 (1959). See also G. Snow, Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers Inc., New York, 1960), p. 407.

31. R. Ross and W. Humphrey, Lawrence Radiation Laboratory, private communication.

32. At higher energies (400 Mev/c), it is known (Ref. 29) that there is a strong p-wave interaction, both from the non-isotropic angular distributions (especially for elastic scattering) and from the magnitude of the total absorptive cross section (which exceeds the s-wave limit).

33. J. D. Jackson and H. W. Wyld, Phys. Rev. Letters 2, 355 (1959).

34. R. H. Dalitz and S. F. Tuan, Ann. Phys. (N. Y.) 8, 100 (1959).

35. A derivation of this result for a simple zero-range model of the $\bar{K}$-N bound state is given in Ref. 37. When b_1 is replaced by $\frac{1}{2}(b_0 + b_1)$ and $|a_1|$ by the Bohr radius B of the K^--p atom, the expression (3.7) corresponds to the well-known result for the lifetime width of the 1s K^--p atom (see, for example, T. L. Trueman, Energy Level Shifts in Atomic States of Strongly-Interacting Particles, submitted to Nuclear Phys. (1960) and Refs. quoted there), apart from a factor of 2 which has its origin in the difference in form between the $\bar{K}$-N wavefunction ($\sim \exp(-r/|a_1|)/r$) for zero-range interaction and the 1s Coulomb wavefunction ($\sim \exp(-r/B)$).

36. M. M. Block, F. Anderson, A. Pevsner, E. M. Harth, J. Leitner, and H. Cohn, Phys. Rev. Letters 3, 291 (1959).

37. R. H. Dalitz, Phys. Rev. Letters 6, 239 (1961).

38. This analogy also suggests that the state in which $\pi + Y^*$ production is dominant is that in which the π-N (3,3) resonance can be effective. Then, the $\pi + Y^*$ production may occur predominantly in the I = 1 K^--p channel, and mainly from the $p_{3/2}$ incident wave. The first of these conclusions is consistent with the observation that the (Y^{*+}/Y^{*-}) ratio is unity for $K^- + p$ interactions up to approx 1000 Mev/c lab momentum, and that Y^* production is strong in

$K_2^0 + p$ collisions (Ref. 39).

39. R. K. Adair, Physics Department, Yale University, private communication, 1961.

40. W. Alles, N. N. Biswas, M. Ceccarelli, and J. Crussard, Nuovo cimento 6, 571 (1957).

41. P. B. Jones, Phys. Rev. Letters 4, 35 (1960); M. Melkanoff, D. J. Prowse, and D. H. Stork, Phys. Rev. Letters 4, 183 (1960); R. D. Hill, J. H. Hetherington and D. G. Ravenhall, K^- Meson Scattering in Nuclear Emulsion, Phys. Rev. (to be published, 1961).

42. R. Karplus, L. Kerth, and T. Kycia, Phys. Rev. Letters 2, 510 (1959).

43. O. Dahl, N. Horwitz, D. Miller, J. Murray and M. Watson, Proceedings of the 1960 Annual International Conference on High Energy Physics at Rochester (Interscience Publishers Inc., New York, 1960), p. 415.

44. R. Karplus and L. Rodberg, Phys. Rev. 115, 1058 (1959); T. Kotani and M. Ross, Nuovo cimento 14, 1282 (1959).

45. T. Day, G. Snow, and J. Sucher, Nuovo cimento 14, 637 (1959); Phys. Rev. (to be published, 1961).

46. W. J. Willis, Brookhaven National Laboratory, private communication, 1961.

47. N. N. Biswas, Phys. Rev. 118, 866 (1960).

48. G. F. Chew and S. Mandelstam, Phys. Rev. 119, 467 (1960).

49. This expression (4.4) is not limited to the case of reactions with two-particle channels, but may be used also for multiparticle channels. In this case, however, the matrix inversion involved requires the solution of a linear integral equation. This point has been discussed recently by R. Blankenbecler (Ref. 20) in a more general representation of the scattering matrix, not limited to a state of definite angular momentum and parity.

50. J. Bjorken and M. Nauenberg, Phys. Rev. 121, 1250 (1961).

51. For a two-channel situation, the analytic properties of the relativistic K matrix on both the physical and unphysical sheets of the Riemann surface, as well as their relation with the singularities of the scattering matrix, have recently been discussed by R. Oehme, Resonance Poles and the Reaction Matrix, Nuovo cimento (to be published, 1961).

52. This behavior will occur also for potential interactions, provided that these do not fall off more rapidly than exponential functions with increasing radial distance.

53. A. Abashian, N. E. Booth, and K. M. Crowe, Phys. Rev. Letters 5, 258 (1960).

54. F. Ferrari, G. Frye, and M. Pusterla, Phys. Rev. Letters 4, 215 (1960).

55. F. Ferrari, G. Frye, and M. Pusterla, Long-Range Interaction in $\bar{K}$-Nucleon and K-Nucleon Elastic Amplitudes, Lawrence Radiation Laboratory Report UCRL-9421, Oct. 1960.

56. F. Ferrari, G. Frye, and M. Pusterla, Energy Dependence of the Low-Energy K^--Proton and K^+-Proton Cross Sections, Lawrence Radiation Laboratory Report UCRL-9434, Oct. 1960.

57. Such as the Vector Theory of Strong Interactions, discussed recently by Sakurai (Ref. 58). In this theory the strong interactions are mediated by a number of vector bosons related to specific conservation laws. For example, as first proposed by Yang and Mills,[59] there is an $I = 1$ vector boson B_I coupled with the total isotopic spin operator, so that there are definite equalities connecting the strength of its coupling with the pion field, with the K-meson field, and with the nucleon, irrespective of the nature of other strong interactions

that may exist (as long as these are compatible with I-spin conservation). As a result, if the strength of this coupling (and the mass of the boson B_I) is determined from the analysis of pion-nucleon phenomena (for example, from the electromagnetic structure of the nucleons), then this theory would require a definite strength for the coupling between $\bar{K}$ and N due to the exchange of this boson B_I.

58. J. J. Sakurai, Ann. Phys. (N. Y.) 11, 1 (1960).

59. C. N. Yang and R. Mills, Phys. Rev. 96, 191 (1954).

60. It is important, however, that residues for the exchange of the same system between K and N are directly related with these residues, in a way depending on the system exchanged (see below). This provides an important consistency constraint in their determination, which relates quite distinct physical processes.

61. Blankenbecler et al. (Ref. 62) have made similar observations in their discussion of the use of dispersion relations for potential scattering.

62. R. Blankenbecler, M. L. Goldberger, N. Khuri, and S. Treiman, Ann. Phys. (N. Y.) 10, 62 (1960).

63. Although (4.14) agrees with the statements made in Sec. VI of Ref. 55, this result (4.16) does not agree with the relationship used in Sec. VI of Ref. 56. The result (4.16) does agree with the explicit calculations of Sakurai (Ref. 58).

64. We note that this relation is not limited to pion-exchange processes, but holds generally in the form $\bar{M}_I = G\, M_I$, where G denotes the G-conjugation parity of the system exchanged.

65. There is one notable exception to this remark. As pointed out by Sakurai (private communication; see also Ref. 58), the exchange of the $i = 1$ B_I

vector meson (identified with the π-π resonance) and the $i = 0$ B_Y vector meson (identified tentatively with ω^0) gives rise to an attractive Y_e and a repulsive X_0. If X_0 were larger than Y_e (by a factor $\gtrsim 3$), the potentials given above would all fit the data qualitatively except for $V_0(\bar{K}\text{-}N)$, which would be more strongly attractive than $V_1(\bar{K}\text{-}N)$ and would lead to a deeply bound state $(\bar{K}\text{-}N)_0$. It is of interest to note that such a state $(\bar{K}\text{-}N)_0$ with mass below the π-Σ threshold would be difficult to detect. It would be stable with respect to charge-independent strong interactions, so that its dominant decay modes would be $\Sigma^0 + \gamma$ and $\Lambda + \gamma$. On the other hand, its mass is too great for it to be formed in strange-particle reactions studied most intensively to date, namely the K^--p "at rest" reactions and the π^--p reactions up to 1.3 Bev/c; furthermore, in sufficiently high-energy reactions it will be difficult to distinguish the production of this state from the production of $(\Lambda + \pi^0)$ continuum states. At present, the existence of such an $I = 0$ $\bar{K}$-N state cannot be excluded!

66. A. Pais, Phys. Rev. 110, 574 (1958).

67. J. de Swart (Enrico Fermi Institute for Nuclear Studies, University of Chicago) and C. Dullemond (University of Rochester), private communication, 1960.

68. R. A. Bryan, Nuovo cimento 16, 895 (1960).

69. S. Gartenhaus, Phys. Rev. 100, 900 (1956).

70. B. Downs and R. H. Dalitz, Phys. Rev. 114, 593 (1959).

71. The existence of a three-body Λ-N-N potential is a necessary consequence of the form of the potential (5.8), since the operation of $V_{\Lambda\Sigma}$ between the Λ hyperon and one nucleon followed by the operation of $V_{\Sigma\Lambda}$

between the resulting Σ hyperon and the other nucleon automatically generates such an interaction. In a more complete treatment of the structure of Λ-hypernuclei, potentials of the form (5.8) would be used, with a hypernuclear wavefunction extended to include a component describing explicitly the Σ configuration; this procedure would automatically include most (although not all) of these three-body potential effects.

72. The binding-energy difference between $_{\Lambda}He^{7}$ and $_{\Lambda}Li^{7}$ allows a rough estimate of the difference between singlet and triplet Λ-N potential strengths largely independent of the details of the three-body potential. The most naive interpretation[73] of this comparison corresponds to the existence of an attractive three-body potential together with a weak ^{3}S potential, which is certainly not in disagreement with the results above. See also Ref. 74.

73. R. H. Dalitz, Hyperon-Nucleon Interactions, unpublished report presented to the 1959 Annual International Conference on High Energy Physics at Kiev.

74. A. R. Bodmer and S. Sampanthar, Binding Energies of Λ Particles and the Λ-N Interaction, (to be published, 1961).

75. R. R. Ross, Bull. Am. Phys. Soc., Ser. II, 3, p. 335 (1958).

76. It is of interest to note that, if the mass difference could be neglected in the calculation of these matrix elements (5.12), global symmetry would imply the equality of (5.12b) and (5.12c). Then, since the phase-space ratio $(\Lambda\text{-N})/(\Sigma\text{-N})$ is approx 4.6, a ratio Λ/Σ^{0} of approx 4.6 would be expected for both ^{1}S and ^{3}S states. The strong effect of Δ on the relation between these matrix elements is particularly marked in the ^{1}S state here, because there is a near-resonant interaction which

is thrown far off resonance in the $I = \frac{1}{2}$ channel (but not in the $I = \frac{3}{2}$ channel) by the effect of Δ.

77. R. H. Dalitz, in 1958 Annual International Conference on High Energy Physics at CERN (CERN, Geneva, 1958), p. 187.

78. It is not clear at present how sensitive these comparisons are to the relative signs of these coupling constants. The global-symmetry hypothesis requires that $g_{\Sigma\Lambda\pi}$ and $g_{\Sigma\Sigma\pi}$ have the same sign, of course, but leaves open the question of their sign relative to $g_{NN\pi}$.

79. Abdus Salam, Strange-Particle Interactions, unpublished report presented to the 1959 Annual International Conference on High Energy Physics at Kiev.

80. D. Amati, A. Stanghellini, and B. Vitale, Nuovo cimento 13, 1143 (1959); Phys. Rev. Letters 5, 524 (1960).

81. Block et al. (Ref. 82) have given an independent argument supporting this conclusion, based on the study of angular correlations in Y^* decay following the $K^- + He^4 \rightarrow He^3 + Y^{*-}$ reaction. However, since the Y^* travels a mean distance of only ≈ 1.2 fermis before decay, there is a possibility that the Λ-He^3 and π-He^3 final-state interactions may complicate the interpretation of these data. For a unique prediction of the angular correlation for a given Y^* spin, the K^--He^4 capture must also be assumed to occur through an s-orbital interaction, an assumption which is by no means certain (Ref. 83).

82. Martin M. Block, Duke University, private communication, 1961.

83. T. Day, Nuovo cimento 18, 381 (1960).

84. It seems probable that most of this asymmetry arises from interference between the primary pion and the decay pion as a result of Bose

statistics, rather than from a dynamical interference between them (such as might arise from a pion-pion force or from simultaneous interaction of the two pions with the Λ hyperon). It should be remembered that the primary pion travels a distance of approx 4.5 fermi in one mean Y^* lifetime. The Bose-statistics interference is expected to diminish with increasing production energy, so that clearer results may be obtained in experiments now planned for higher production energies.

85. U. Kruse and M. Nauenberg, S-Wave $\bar{K}$-N Scattering Amplitudes, Lawrence Radiation Laboratory Report UCRL-8888, Sept. 1959 (unpublished).

FIGURE LEGENDS

Fig. 1 Pion-hyperon scattering cross-sections calculated for the (a-) set of $\bar{K}$-N scattering amplitudes, with a_1 adjusted to locate the resonance energy at 1382 Mev and b_1 = 0.20 fermi. The curves shown are as follows: (a) the total π-Λ elastic scattering cross section, with π-Λ and π-Σ systems both $s_{1/2}$, and with zero potential scattering ($\gamma = 0$); (b) the same, with the potential scattering chosen to give a potential scattering phase of $\delta = -15$ deg. at the resonance energy; (c) the total π-Λ elastic scattering cross section with π-Λ and π-Σ systems both $p_{1/2}$, and with zero potential scattering; and (d) the energy dependence of the total π-Σ elastic scattering cross section (arbitrary normalization), with the assumptions of case (c).

Fig. 2 Graph showing schematically the exchange of vector boson B between $\bar{K}$ and nucleon, as considered in the model calculation of Appendix B.

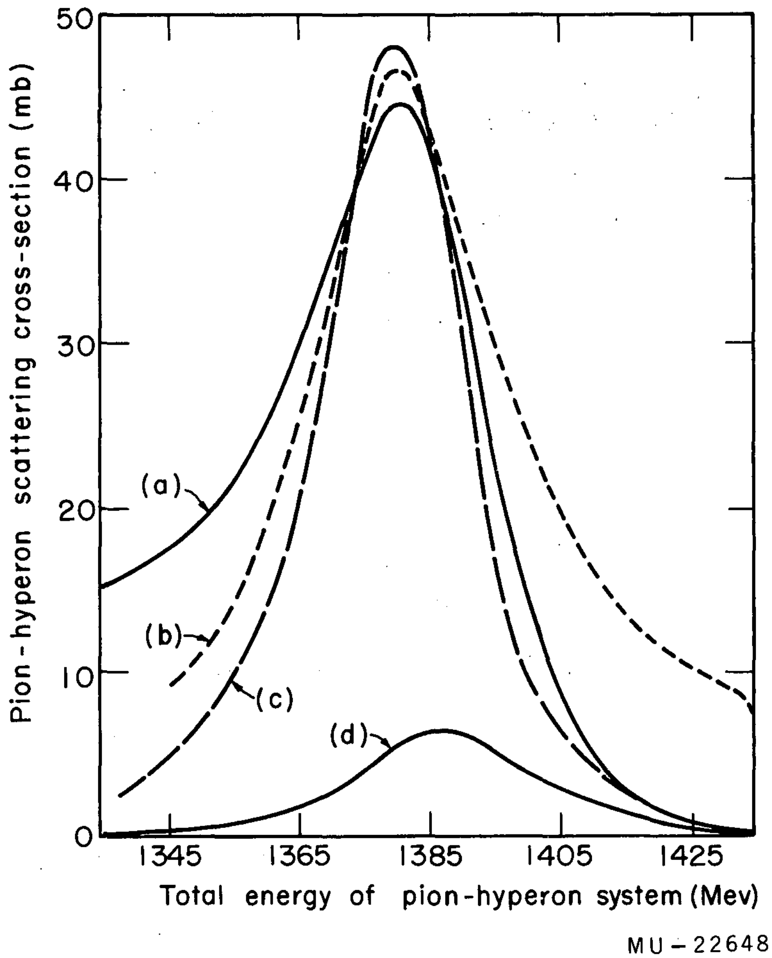
Pion-hyperon scattering cross-section (mb)
50
40
30
20
10
0
(a)
(b)
(c)
(d)
1345
1365
1385
1405
1425
Total energy of pion-hyperon system (Mev)
MU-22648

Fig. 1

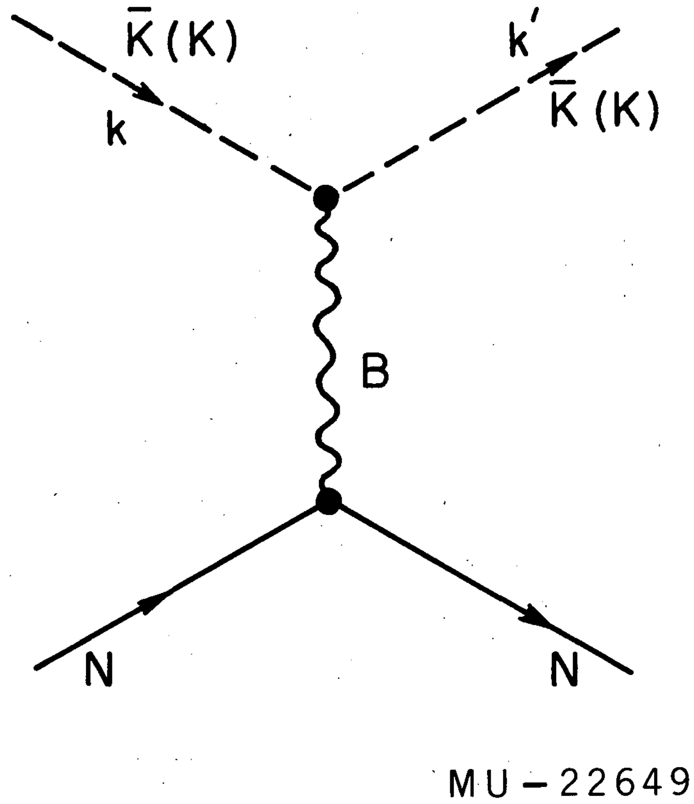

Fig. 2