DYNAMIC BEHAVIOR OF FOUNDATIONS: AN EXPERIMENTAL STUDY IN A CENTRIFUGE

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1984-07-18

Proceedings of the

SYMPOSIUM ON RECENT ADVANCES IN GEOTECHNICAL CENTRIFUGE MODELING

A symposium on Recent Advances in Geotechnical Centrifuge Modeling was held on July 18-20, 1984 at the University of California at Davis. The symposium was sponsored by the National Science Foundation's Geotechnical Engineering Program and the Center for Geotechnical Modeling at the University of California at Davis.

The symposium offered an opportunity for a meeting of the International Committee on Centrifuges of the International Society for Soil Mechanics and Foundation Engineering. The U.S. participants also met to discuss the advancement of the centrifuge modeling technique in the U.S. A request is being transmitted to the American Society of Civil Engineers to establish a subcommittee on centrifuges within the Geotechnical Engineering Division.

PART III

SOIL DYNAMIC AND EARTHQUAKE APPLICATIONS

DYNAMIC BEHAVIOR OF FOUNDATIONS: AN EXPERIMENTAL STUDY IN A CENTRIFUGE

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SUMMARY

Correctly-scaled rigid model structures with foundations of different shapes and sizes resting on the surface or embedded in a cohesionless soil were subjected to forced vibrations in their coupled rocking-sliding mode in a centrifuge. A medium-dense fine sand was used in the experiments and the centrifugal acceleration was equal to 50 g. For the dynamic loading of the model structures, a miniature shaker was designed to produce steady-state shaking of the models in a low to high amplitude vibration range. The response of the model structures to the input dynamic motion was measured by accelerometers and pressure and displacement transducers. Results of the experiments indicate that almost in every test, the foundation lifts off the soil surface and the soil at the edge of the foundation-soil contact area yields. The yielded zone gradually moves inward as the number of vibration cycles or amplitude of vibration increases. Therefore, the foundation-soil system behaves non-linearly under the effects of both lift-off and plastic deformation of the contact soil.

1. INTRODUCTION

Recently, the response of dynamically loaded foundations has received a good deal of attention. This arises from the growing need to design dynamic load-resistant foundations and to include the effect of the ground behavior on the overall response of the structures.

Analysis and design of foundations under low-amplitude dynamic loads such as those generated by machine vibrations and vehicular traffic were the original reasons for consideration of this problem. However, research in earthquake engineering and dynamic behavior of structures under other sources of high-amplitude loads such as those generated by bomb blasts, or loading due to sea wave action has demonstrated a need to improve the dynamic models of buildings and other structures by including the soil-structure interaction effect.

Many investigations of both an analytical and experimental nature on the dynamic response of footings have been conducted. For analysis, different approximate methods have been used in studying foundation vibration problems, such as "In-Phase Mass" method (Crockett and Hammond 1949, Rao 1961), or

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"Dynamic Subgrade Reaction" (Hayashi 1921, Terzaghi 1943, 1955, Hetenyi 1946, Barkan 1962). However, elastic-half-space theory has received the greatest attention. Because of its idealized nature, certain mathematical simplifications which are not quite realistic had to be introduced. Over many years, the gap between the results of this theory and real-life behavior of foundations has been narrowed. These half-space studies are mainly concerned with dynamic response of rigid foundations which can be divided into two basic groups: (1) resting on the soil surface, and (2) embedded in the soil. In the majority of these investigations circular foundations are considered and the soil medium is simulated by a homogeneous isotropic elastic half-space. The early analytical solutions were based on the classic work by Lamb who formulated and solved the problem of a harmonically varying point force acting on the surface of an elastic half-space. Using his solution many investigators solved the foundation vibration problem by considering a prescribed stress distribution over contact areas of regular geometry such as circular, strip, or rectangular shapes (e.g. Reissner 1936, Sung 1953, Bycroft 1956, Thomson and Kobori 1963). These solutions were followed by mixed boundary-value treatment of the problem in which an oscillating displacement is prescribed in the loaded region and stresses are defined over the rest of the boundary surface (e.g. Veletsos and Wei 1971, Luco and Westmann 1971). To improve the mathematical model of the problem, other continuum models such as viscoelastic, layered, poroelastic subgrade, etc. have been considered (e.g., Hoskin and Lee 1959, Veletsos and Verbic 1973, Kausel 1974, Luco 1974, Halpern and Christiano 1982).

Foundations of most structures such as buildings, machines, nuclear power plants, etc., are usually placed partially below the soil surface. This has a great effect on the vibration characteristics of these footings and has been much investigated in recent years. Several analytical methods, including the finite element method, have been used (e.g., Baranov 1967, Novak and Beredugo 1971, 1972, Bielak 1975, Kuhlemeyer 1969, Kausel 1974).

All these theoretical methods for analyzing the dynamic response of foundation-soil systems are based on a number of simplifying assumptions regarding soil properties and system geometry. In particular, real non-linear hysteretic soil properties are generally not included, or are approximated only. As a result, the application of theoretical results is questionable in many cases, such as, in particular, high amplitude vibration of the foundation soil system during strong earthquake ground motion. Therefore there has existed a great need for experimental studies to calibrate theoretical techniques and to clarify the ambiguities produced by using simplified mathematical models.

Experimental studies on the dynamic behavior of surface and embedded foundations have been reported by many investigators. Barkan 1962, Novak 1960, 1970, Fry 1963, Drnevich and Hall 1966, Beredugo 1971, Stokoe 1972, Tiedemann 1972, Erden 1974 and many others studied the vertical, torsional, rocking and sliding modes of vibration of surface and embedded footings.

All the experimental work cited above has been performed on model or small prototype footings in the field or in the laboratory. Since the stress conditions on a soil element have a considerable effect on its behavior under both static and dynamic loadings it is expected that a soil mass behaves differently under full-scale and model conditions. Running full scale tests on foundations comparable in size and weight with foundations of real structures is very expensive and in some cases even impossible. Because of the limitations in the one-g testing of model or full-scale foundations the best approach appears to be to conduct properly-scaled shaking tests on model foundations in a centrifuge. This technique is described more in the next sections.

2. CENTRIFUGE MODEL TESTING

Most soil properties depend on continuum stresses which are generally gravity-induced. Thus, in order to have the same stress conditions on soil elements at two homologous points in model and prototype unit weight of model material should be scaled properly to produce same confining stresses. For a geometrically similar model N times smaller than its prototype we should have

$$\gamma_M = N\gamma_p$$

where

γ_M = unit weight of model material

γ_p = unit weight of soil in prototype

This can be achieved either by choosing a model material with a density N times the density of the prototype soil or by increasing the gravitational acceleration on the soil at the model scale. It is more convenient to use prototype material, but to increase the gravitational acceleration by the lineal scale factor N. Thus, if a 1/50th scale model, made of the same material as the prototype is subjected to a gravitational acceleration 50 times that of the prototype, the confining stresses, and thus the properties and behavior of the model, are the same as in the prototype. A centrifuge is a machine that can provide model gravity as desired (Bucky 1931, Pokrovsky and Fyodorov 1975, Scott et al 1977).

Since in this research rigid prototype footings of different shapes and sizes were simulated using model footings with high rigidity compared with soil stiffness, only the geometrical and inertial characteristics of the models were scaled to the required ratios. Table I lists the relations between prototype and model (centrifuge) parameters when a centrifuge is employed (Rowe 1975, Scott 1977).

In the experiments described here, N was chosen to be 50, so that model dimensions were 1/50 of the prototype linear dimensions, and the model (centrifugal) acceleration was 50 times normal terrestrial gravity.

3. EQUIPMENT

The centrifuge used is shown in Figure 1 and has been described elsewhere (Scott et al 1982). The capacity (payload) of the centrifuge is about 10,000 g-lbs. This means that at 50g's, the maximum load of model structure, soil and container that it can sustain is about 200 lbs. A miniature air-driven counterrotating mass shaker was designed for steady-state forced excitation of the model structures in their coupled rocking-sliding mode of vibration. Because of its small size, reasonable force amplitude output, and high value of frequency response, it forms ideal loading equipment for any dynamic test in centrifuge. The main part of the shaker is a three gear arrangement (Figure 2), two of them in parallel in the horizontal plane and the third one a vertical gear that transfers motion of the lower horizontal gear to the upper one in the opposite direction. Two flywheels made from phenolic, a light and strong composite material, are assembled on the parallel gears and have counterrotating motion. Compressed air was used as the source of energy to run the shaker. Air flows with very high velocity from two nozzles on the sides of the shaker pushing forward circular cups machined on the circumference of the lower flywheel. Different eccentric masses can be used to adjust the force of vibration independently of the shaker speed. The maximum frequency output of the shaker at 50g centrifugal acceleration was about 750 Hz which is equivalent to 15 Hz prototype frequency and its output force amplitude varied from very small values up to few pounds depending on the amount of eccentric mass and speed of the shaker. Other properties of the

TABLE I. Scaling Relations

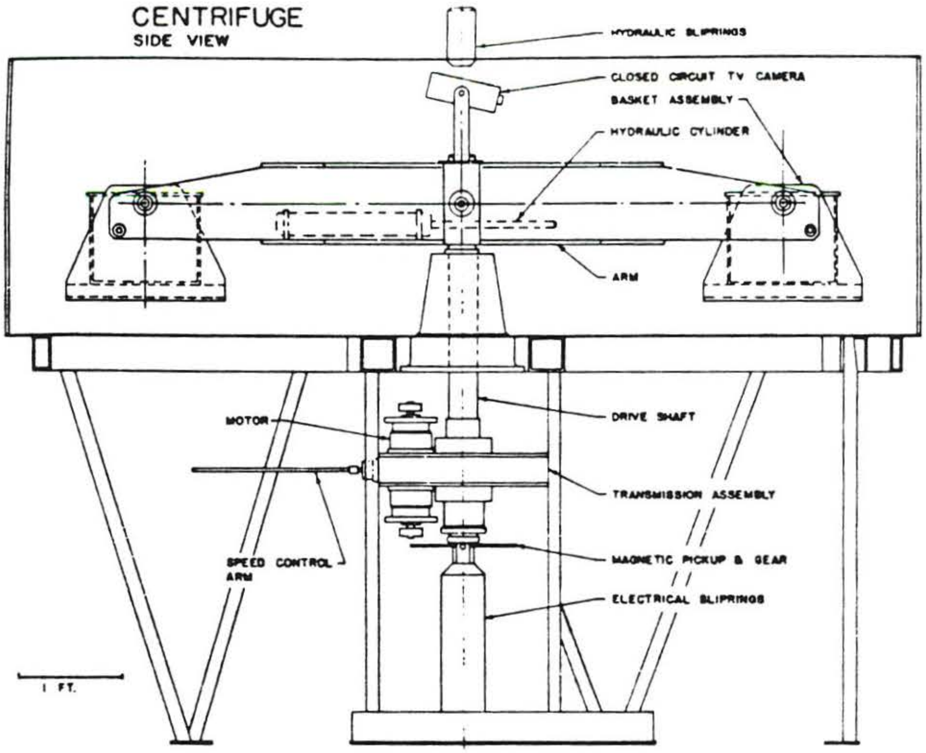
Parameter	Full Scale (Prototype)	Centrifugal Model at Ng's
Acceleration	1	N
Linear dimension	1	1/N
Area	1	$1/N^2$
Volume	1	$1/N^3$
Stress	1	1
Strain	1	1
Force	1	$1/N^2$
Mass	1	$1/N^3$
Mass density	1	1
Weight density	1	N
Time (dynamic)	1	1/N
Time (consolidation)	1	$1/N^2$
Frequency	1	N

TABLE II. Properties of the Air-Driven Shaker

Diameter (in)	Height (in)	Eccentricity (in)	Height of Center of Gravity (in)	Weight (lbf)	Mass Moment of Inertia (lb - in^2)
1.73	1.38	0.74	0.67	0.23	0.08

TABLE III. Properties of the Models Used in Different Parametric Studies

Soil-Foundation Parameters	Model Properties			
	Weight (lbf)	Mass Moment of Inertia (lb-in^2)	Height of Center of Gravity (in)	Footing Semi-Dimension (in)
Footing Shape & Size:				
Circular Model I	0.97	8.71	3.66	1.0, 1.25, 1.50
Model II	1.85	11.08	3.75	1.0, 1.25, 1.50
Square	1.85	11.08	3.75	1.0, 1.25, 1.50
Rectangular	1.85	11.08	3.75	See Fig. 12
Ecentric Mass				
of Shaker Model I	1.75	9.08	2.75	3.00
Model II	1.85	11.08	3.75	2.50
Depth Of Embedment	1.47	7.85	3.10	3.00



SIDE VIEW OF CALTECH CENTRIFUGE
FIGURE 1

COMPONENTS OF THE AIR-DRIVEN SHAKER
FIGURE 2

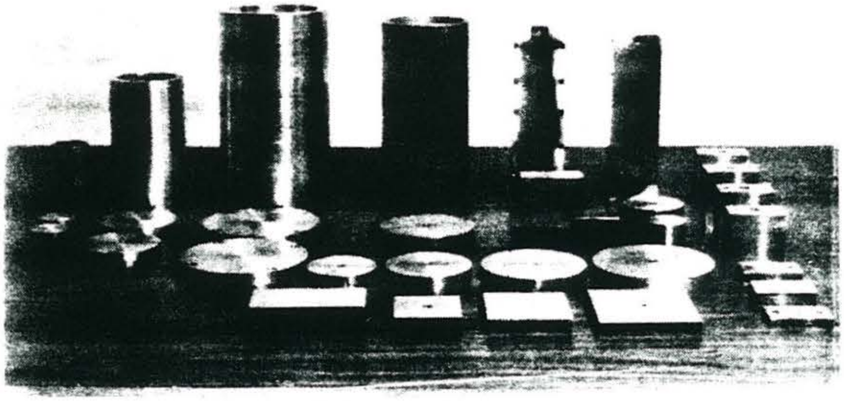
shaker are summarized in Table II. Frequency of the shaker was measured by a frequency counter, an LED, a photocell, and a number of reflecting silver strips bonded to the upper surface of the shaker.

The model structures used were all rigid hollow or solid aluminum cylindrical and rectangular tubes attached to rigid footings of different shapes and sizes. At 50g the models simulated prototype structures with footings of 8 ft to 15 ft diameter, masses of 121 kips to 266 kips, and mass moment of inertias (with respect to an axis passing through the base) ranging from 47,000 kips-ft to 92,000 kips-ft. Figure 3 shows the model structures and the model footings used in the experiments. The characteristics of the models are summarized in Table III.

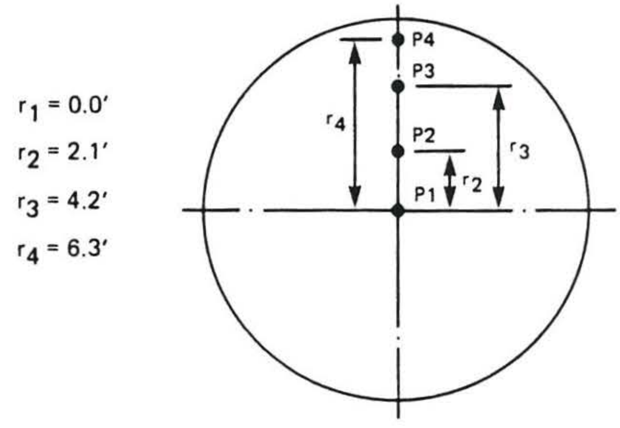
The model towers were instrumented with three Entran model EGA-125F-500D miniature accelerometers to record the tower motion at different elevations in order to measure the acceleration components of the coupled rocking-sliding mode of vibration. Horizontal displacement components of the models in the direction of applied force and the transverse direction were measured at the top of the tower using a double axis optical transducer, Model PIN-SC/10D obtained from United Detector Technology, Inc. In addition four Entran Model EPF-200-50 pressure transducers were mounted on the bottom surface of the footings to measure the contact pressure distribution along the diameter (Figure 4). The accelerometers were calibrated by placing them in the centrifuge at known rotation speeds and recording their outputs at corresponding centrifugal acceleration. Calibration of pressure transducers in order to measure the absolute pressure amplitudes introduces some difficulties because of soil-structure interaction effect between transducer and soil (Weiler and Kulhawy, 1982). Pressure transducers were calibrated by placing them on the floor of a bucket full of water or a uniformly dense sand while spinning the centrifuge at different centrifugal accelerations. This calibration technique provides an approximate measure of absolute pressures and a precise measure of relative pressure amplitudes. Thus, pressure distribution configuration can be reliably evaluated.

All the signals were suitably amplified and filtered to eliminate high frequency noise, passed through slip rings, and then recorded digitally by an analog-to-digital convertor (ADC) and a microcomputer system.

The soil used in the tests was Nevada 120 silica sand in a dense (104 ± 1 pcf) dry state. The soil container was a 12 inch diameter 10 inch deep aluminum cylindrical bucket.



COLLECTION OF TOWERS, FOOTINGS AND MASSES USED IN TESTS
FIGURE 3



PATTERN OF PRESSURE TRANSDUCER HOUSINGS ON FOOTINGS
FIGURE 4

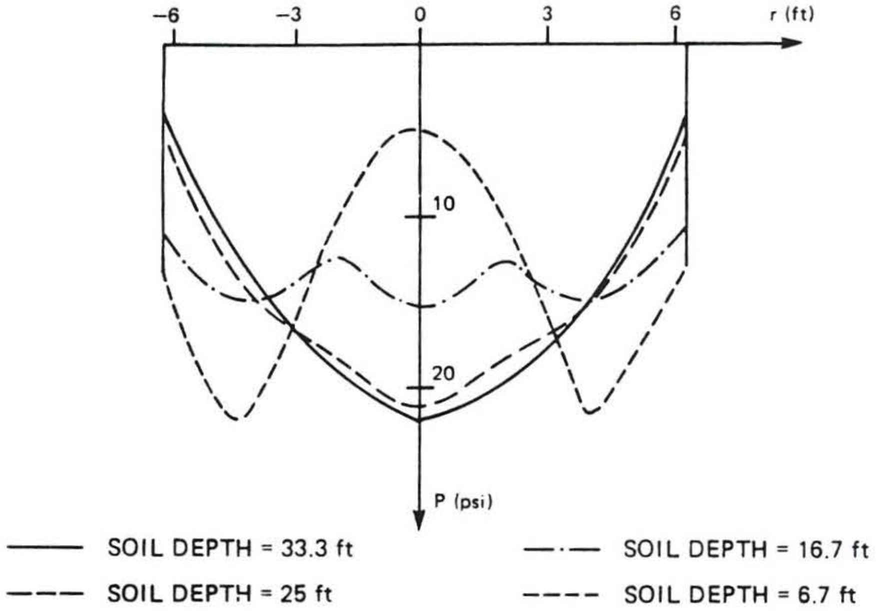
4. EXPERIMENTAL PROCEDURE

Dry Nevada Fine Sand (NFS) was placed in the centrifuge bucket to a predetermined depth and density. The soil surface was leveled and smoothed particularly where the footing was to be located. Next, the model structure with appropriate transducers and the air-driven shaker was securely placed on the sand. In the embedded foundation tests, after locating the tower on the soil surface, more sand was placed around the tower and compacted carefully to the required density and depth of embedment. After the centrifuge was brought up to required speed the shaking machine was run through a range of frequencies while the tower motions were recorded at different frequencies of the oscillation. The signals were recorded by the data acquisition system (ADC) and then accessed by the micro-computer and stored on disks. Data reduction included plotting the raw data, calculation of Fourier transforms, filtering of noise, sine fitting to calculate amplitude and phase, plotting amplitude versus frequency curves for each test, and fitting the curves by the response of a single degree of freedom damped oscillator to determine resonant frequency and equivalent damping of the system.

5. RESULTS

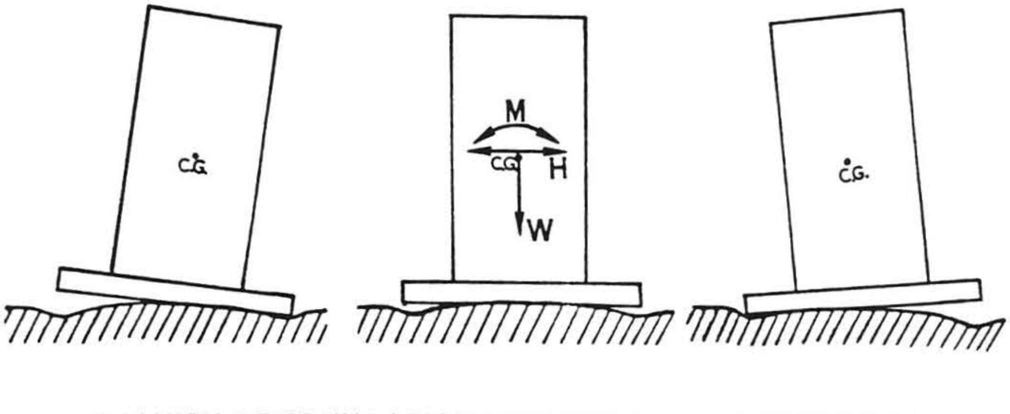
The effect of different soil-footing parameters on the dynamic characteristics of the model structures and contact pressure distribution measurements are presented in the following sections. Results are presented by response amplitude curves and plots showing variation of resonant frequency with the change of different parameters of the soil-foundation system. The y-axis of the response amplitude curves is expressed in terms of displacement and pressure amplitude per frequency squared which is equal to amplitude per unit force ratio for an imaginary shaker of unit mass-times-eccentricity product. The unit for the y-axis labeled as "Amplitude/Force" is in "in-sec^2" for displacement data and "psi-sec^2" for pressure signals. In the following sections all parameters are given in prototype scale.

The static pressure distributions along the diameter of a rigid circular footing, 12.5 feet diameter, resting on the surface of sand were measured for different soil depths (Figure 5). The static pressure distribution for the footing on maximum soil depth of 33.3 feet has an approximately parabolic shape which is comparable with the theoretical triangular distribution shape



STATIC PRESSURE DISTRIBUTIONS ALONG FOOTING DIAMETER AT DIFFERENT SOIL DEPTHS

FIGURE 5

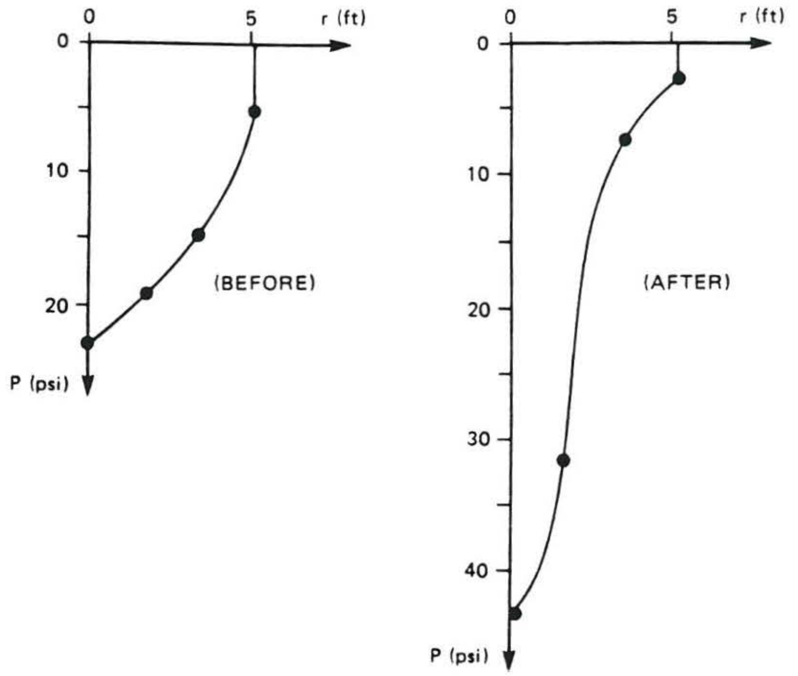


MECHANISM OF FOUNDATION LIFT-OFF AND ROUNDING OF THE CONTACT SOIL SURFACE

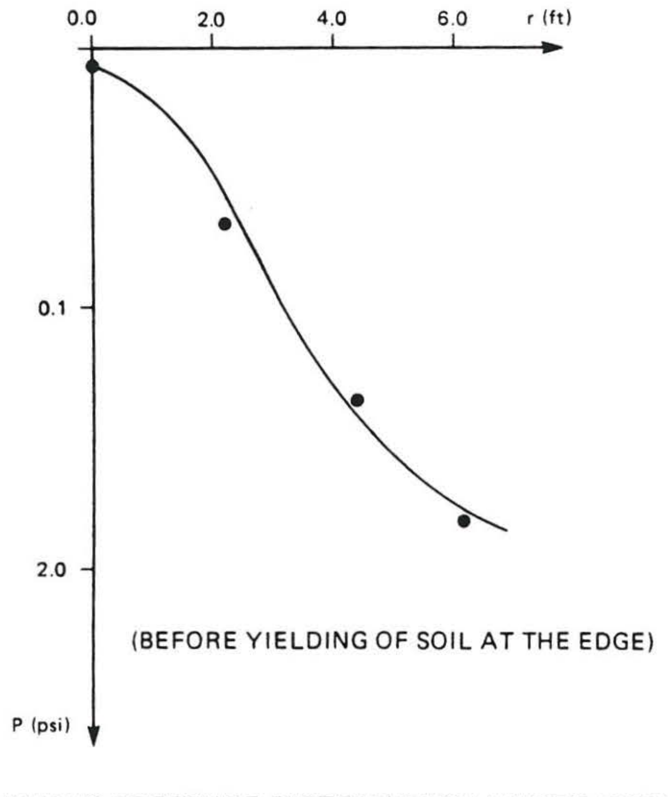
FIGURE 6

suggested by Meyerhof (1951). Both theoretical and observed distributions indicate that because of low confining pressure in the sand under the edges of the footing, it can sustain very little stress, and strength and stiffness increase towards the center where the sand is more confined. The addition of the rocking moment during dynamic test will cause further yielding, densification and settlement of the soil along the leading edge in addition to the initial statically yielded zone and results in more separation of the footing and soil along the trailing edge due to foundation lift-off and deformed outer zone (Figure 6). Since the vertical load on the footing remains constant during the test, for equilibrium the volume under the contact stress diagram should vary continuously until the contact soil surface assumes a stable configuration. Figure 7 shows the static pressure distribution before and after applying the dynamically varying moment and going through resonance during a test. It is observed that because of reduction in contact width the stress diagram has reverted to a much narrower wedge shape configuration. Two distinct dynamic pressure distribution patterns were observed depending on the amplitude of vibration. In the case of low amplitude vibration the dynamic pressure amplitude increases from a minimum value at the footing center to a large amount at the footing edge (Figure 8). This configuration of pressure distribution remains unchanged until there is some yielding and lift-off around the foundation edge, where, a progressive change of dynamic pressure distribution along the footing diameter occurs as the amplitude increases (Figure 9).

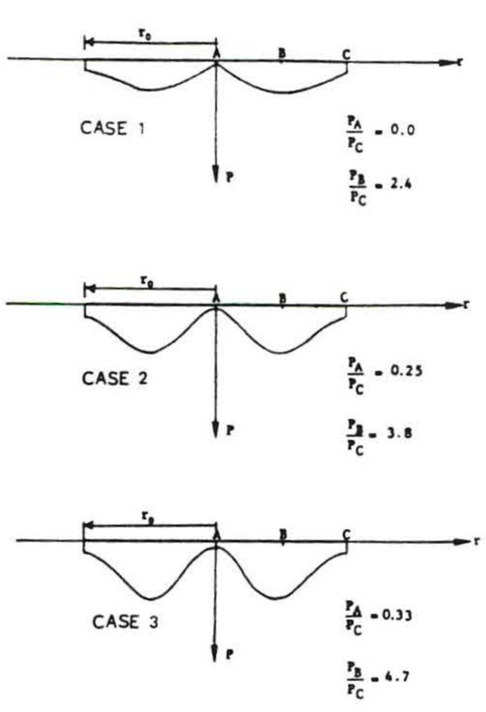
In two series of tests on model structures with different masses the size of the circular foundation was varied keeping all other model parameters constant. The resonant frequency of the model increases approximately linearly as the footing radius increases (Figure 10). Figure 11 presents similar results derived for square footings. Natural frequency of the square footings with equivalent semi-dimensions (equal to radius of a circular footing having area equal to that of the square footing) are very close to the values for equivalent circular footings. However, the resonant frequencies of square footings are slightly bigger than the ones for equivalent circular foundations over the entire range of frequencies of interest. The effect of foundation shape on resonant frequency of vibration is shown in Figure 12. The length-to-width ratio for rectangular footings was varied while other parameters were kept constant. As expected, narrow footings rocking around an axis parallel to their longer side have low values of rocking frequencies and



STATIC PRESSURE DISTRIBUTION BEFORE AND AFTER TEST
FIGURE 7

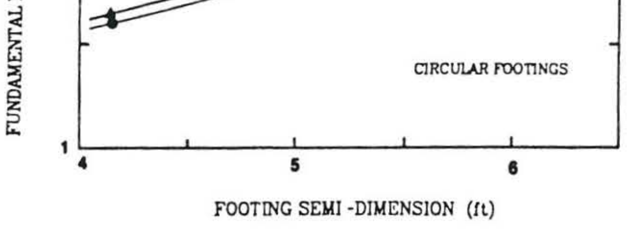


DYNAMIC PRESSURE DISTRIBUTION UNDER ROCKING FOUNDATION DIAMETER
FIGURE 8



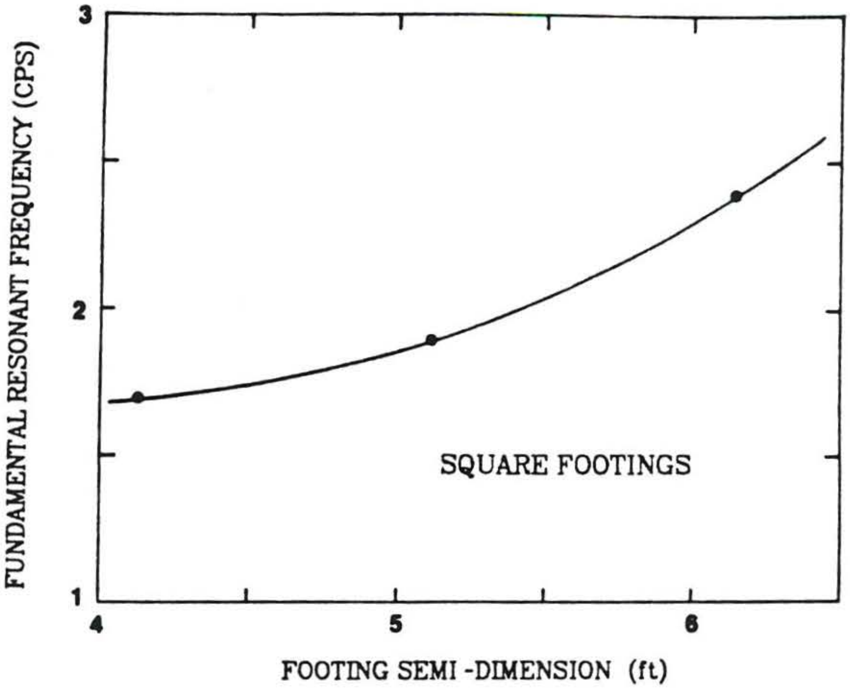
DYNAMIC PRESSURE DISTRIBUTION UNDER ROCKING FOUNDATION ALONG DIAMETER AND AT INCREASING ROCKING AMPLITUDES FROM CASE (1) TO (3) IN FIGURE (AFTER YIELDING OF SOIL AT THE EDGE)

FIGURE 9

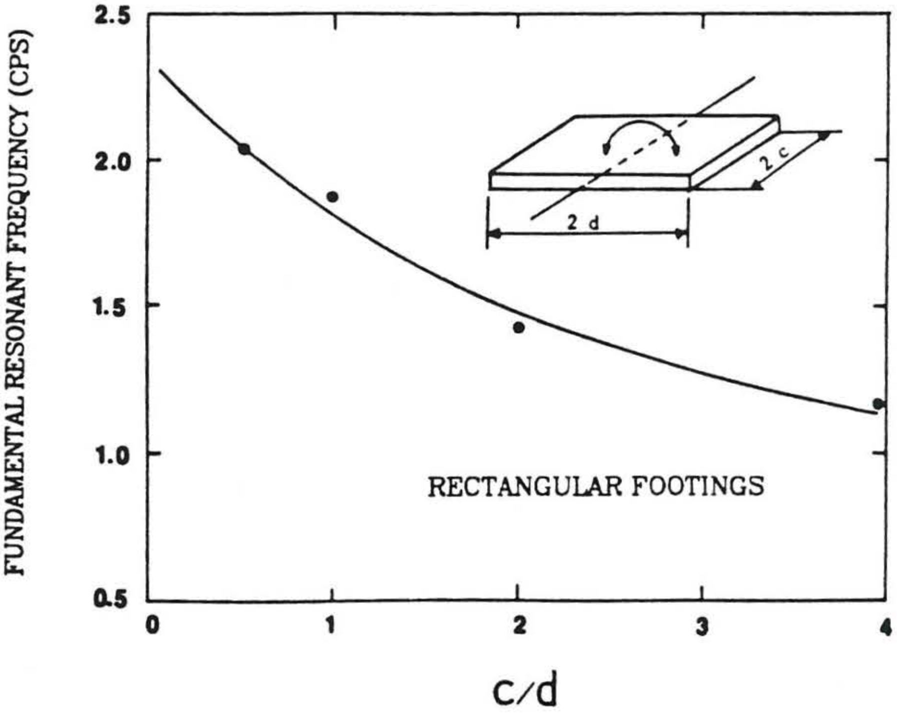


EFFECT OF FOUNDATION SIZE ON ROCKING-SLIDING RESONANT FREQUENCY

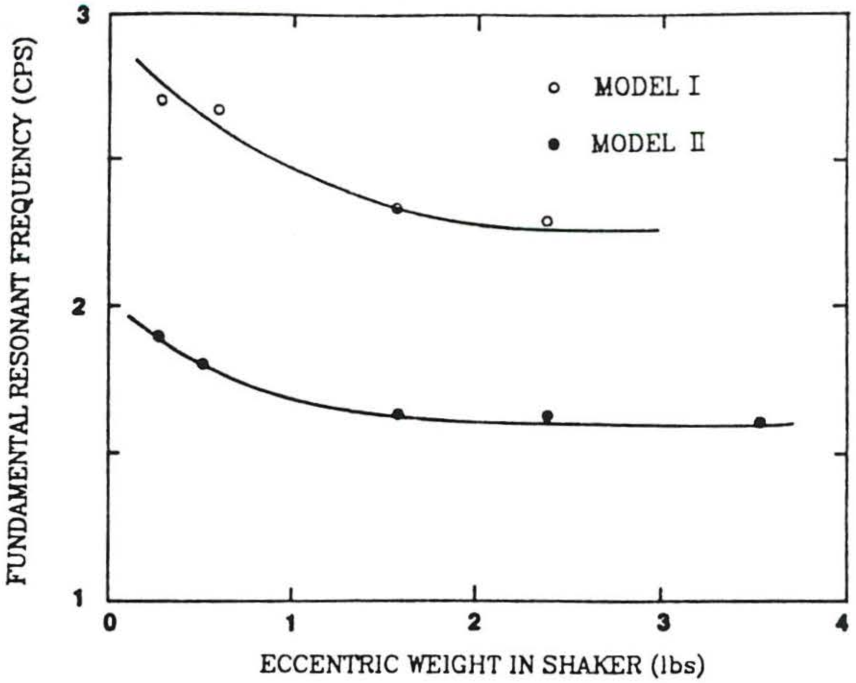
FIGURE 10



EFFECT OF FOUNDATION SIZE ON ROCKING-SLIDING RESONANT FREQUENCY
FIGURE 11

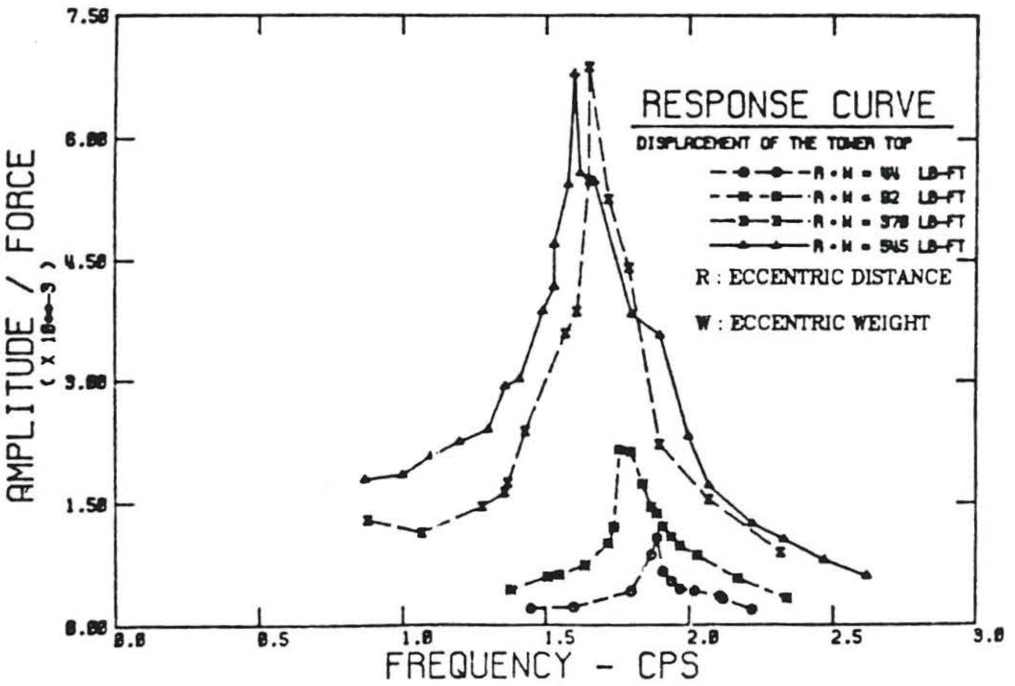


EFFECT OF FOUNDATION SHAPE ON ROCKING-SLIDING RESONANT FREQUENCY
FIGURE 12



EFFECT OF ECCENTRIC WEIGHT OF SHAKER ON ROCKING-SLIDING RESONANT FREQUENCY

FIGURE 13a



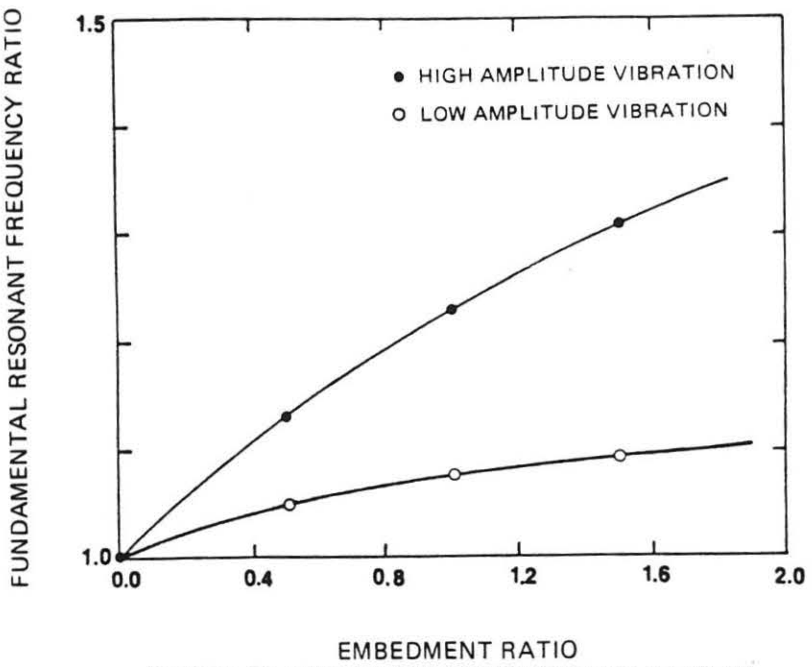
RESPONSE CURVES OF TOWER DISPLACEMENT UNDER DIFFERENT FORCE LEVELS

FIGURE 13b

can be excited very easily in their rocking mode of vibration with larger amplitudes of motion compared with footings rocking around an axis parallel to the shorter side of footing.

Increasing the eccentric weight in the shaker increases the force amplitude at a particular frequency of shaker rotation. This causes higher shear strain amplitudes in the soil under the foundation which results in more softening and nonlinear behavior of the soil. As a result the resonant frequency of the system decreases while damping and amplitude of vibration increase. These phenomena were observed in the experiments showing a clear trend of decrease in resonant frequency with increase in the eccentric mass of the shaker (Figure 13a). A better physical picture of this phenomenon is presented by depicting the response curves for four tests with different shaker masses (Figure 13b).

Increasing the embedment depth of the foundation increases the stiffness of the soil-structure system and therefore results in an increase in natural frequency of the structure. This phenomenon was observed in a series of tests changing the depth of embedment from 0 to 1.5 times the radius of the tower base for both low and high amplitude vibration (Figure 14). As was expected,



EFFECT OF DEPTH OF EMBEDMENT ON FUNDAMENTAL ROCKING-SLIDING FREQUENCY OF TOWER

FIGURE 14

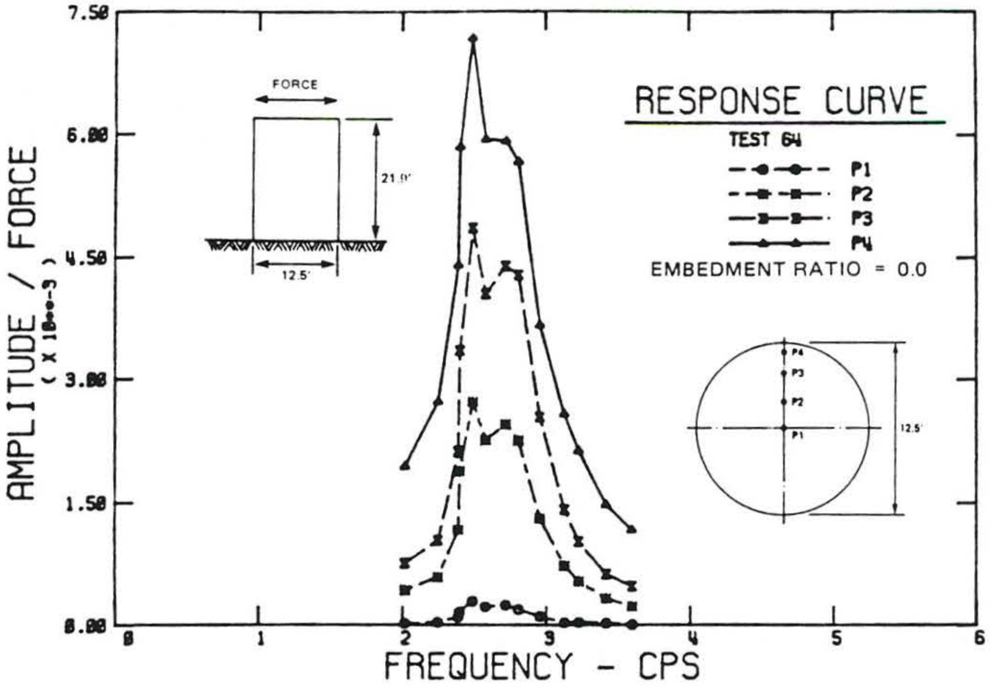
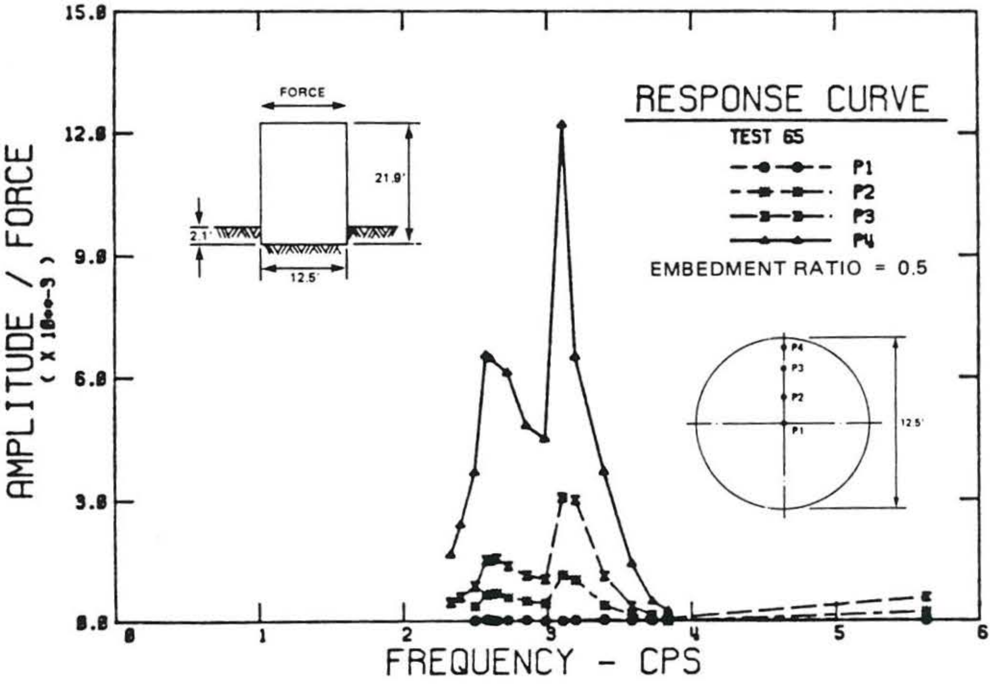


FIGURE 15a



RESPONSE CURVES FOR PRESSURE SIGNALS ALONG DIAMETER AT DIFFERENT EMBEDMENT RATIOS

FIGURE 15b

increase of resonant frequency with embedment is not very large for high amplitude vibration. This is because of ineffective contact between the circular tower side walls and the soil mass. Stokoe and Richart (1974) in an experimental study on model circular footings (8 inch diameter) embedded in a dense, dry sand subjected to rocking excitation showed that embedment without adequate lateral support was essentially ineffective. The contact pressure distribution changes considerably because of embedment. The confining pressure in the soil at the footing edge changes from zero to a finite value over the range from no embedment to an initial depth of embedment. Figures 15a and 15b show variation of pressure distribution with increase of embedment depth by comparing the response amplitudes of pressure signals.

Further parametric studies including tests of foundations on the centrifuge and comparison with theory have been performed (Hushmand, 1983).

CONCLUSIONS

1. Static pressure distribution for a rigid footing on a dense, dry sand has a parabolic shape.
2. Dynamic pressure at low amplitude vibration increases from center to edge, while in case of high amplitude vibration it reduces to zero at the edge because of yielding of the soil.
3. Resonant frequency of footings increases approximately linearly as the size of foundation increases.
4. Resonant frequency of rectangular footings decreases with an increase in length of foundation side parallel to rocking axis.
5. Increasing eccentric mass of the shaker increases nonlinearity of the system, results in decrease of resonant frequency, while damping ratio and amplitude of vibration increase.
6. Effect of embedment under high amplitude vibration reduces considerably because of loss of effective contact between foundation side walls and soil mass.
7. Results of centrifuge model tests are in reasonable agreement with prototype results in high amplitude vibration tests.
8. Incorporation of permanent deformation of the contact soil and foundation-soil separation in the dynamic analysis and design of shallow foundations is recommended.

ACKNOWLEDGEMENT

This work was performed during graduate study of author in California Institute of Technology, Pasadena under the supervision of Professor Ronald F. Scott. The investigation was partially sponsored by Grant No. CEE82-19068 from the National Science Foundation.

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