Title

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MULTIPLE ABSTRACTED REPRESENTATIONS IN PROBLEM SOLVING AND DISCOVERY IN PHYSICS

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ABSTRACT

We discuss the process of mathematization in science, focussing on uses that theorists make of physical representations that we refer to as abstracted models. We review abstracted models constructed by Faraday and Maxwell in the mathematization of electromagnetic phenomena, including Maxwell's use of an analogy between continuum dynamics and electromagnetism. We discuss ways in which this example requires major modifications of current cognitive theories of analogical reasoning and scientific induction, especially in the need to understand the use of abstracted models containing theoretically meaningful objects that can be manipulated and modified in the development of new concepts and mathematized representations.

INTRODUCTION

Problem solving and discovery in physics involve the application of appropriate mathematical formalisms to specific configurations of physical phenomena. In the history of science, the process of figuring out the fit between mathematics and the physical world has been called "mathematization" (Koyre). That process consists of grasping the relational structure underlying phenomena; abstracting that structure; expressing the abstraction in mathematical formulae; and applying the formalism back to a wider class of phenomena. For example, "mathematization" of the motion of projectiles required formulating a relationship between the component forces acting on the body by abstracting the motion of an object in air to that of an idealized point mass in an empty three-dimensional, homogeneous, infinite space; and, as ultimately formulated in Newton's laws, this analysis showed the motions of projectiles, planets, and pendula to have the same formal structure.

Historically, the process of mathematization has often had as a central component the construction of a physical representation -- a model, a schematic representation, a diagram, etc. -- embodying tentative assumptions about the structural relationships under investigation. These representations are like equations and other descriptions that are physical objects in their own rights, enabling reflection and investigation of their properties separated from the phenomena they are meant to represent. We use the term "abstracted models" to refer to such models, schematic representations, diagrams, etc. Abstracted models allow scientists to manipulate familiar structures, observe consequences of adjusting relationships to satisfy domain constraints, and, ultimately, to generate equations that express the assumed relationships in formal terms. Because abstracted models reify the hypotheses of the investigator, he or she can reason with and about the abstraction rather than the represented phenomena.

One advantage of reasoning with a model is that reasoning about the represented phenomena in all their complexity can create a cognitive "overload," since what is or isn't relevant is often not evident. Another advantage is that the abstracted model provides support for productive situated reasoning about interactions among hypothesized properties and relations. The objects in a model representation that correspond to properties and relations in the phenomena can be examined in different arrangements to allow exploration of interactions that cannot be produced and observed as directly in the domain of the phenomena. (For a discussion, see Greeno, 1989.) This paper is particularly concerned with an example of discovery that came from formulating and modifying a causal model with objects corresponding to properties and relations in phenomena of electricity and magnetism.

MATHEMATIZING ELECTROMAGNETIC PHENOMENA

The history of science is replete with examples. The derivation of the electromagnetic field equations provides a particularly salient case study. The standard textbook account at both the undergraduate and graduate levels (e.g., Jackson, 1962; Feynman *et al.*, 1963; Panofsky & Phillips, 1962) all present Maxwell as starting from a set of field equations for closed circuits plus the equation for continuity of charge shown in Table 1. Maxwell's problem is portrayed as that of reconciling these equations for the case of open circuits. According to this account, considerations of formal consistency required that he add a term to Ampere's Law to represent the contribution of electrostatic polarization to current.

Table 1 about here

Space limitations require that we present an exceedingly compressed version of the actual process through which the field equations were derived. Nersessian (1984, 1986, in press) has presented fuller analyses. What we hope to do here is to show how different the actual process was from that presented in textbooks and, in particular, to demonstrate how important various abstracted models were in the mathematization process.

Figure 1 about here

It was Maxwell who generated the field equations for electromagnetism, but his analysis of the problem began with specific abstracted models created by Faraday. In opposition to the mathematical representation of electric and magnetic actions as Newtonian actions at a distance by Ampère, Faraday hypothesized that the lines of force that form when iron filings are sprinkled around magnets and charged matter indicate that some real physical process is going on in the space surrounding these objects and that this process is part of the transmission of the actions. Figure 1(a) shows the actual lines as they form around a magnet and Figure 1(b) shows an abstracted representation of these lines in geometrical and dynamical form. That this abstraction played a central role in his reasoning about electric and magnetic phenomena can be seen in the many line-like features that he incorporated into his descriptions of the actions and that guided his attempts to detect them experimentally. For our purposes, it is most notable that the only quantitative measure he introduced is between the number of lines cut and the intensity of the induced force. This relationship is incorrect, because "number of" is an integer while "field intensity" is a continuous function. The "mistake" occurs because in the abstracted model the lines are taken as discrete objects, while they actually spiral indefinitely in a closed volume.

Figure 2 about here

Near the end of his research, Faraday introduced another abstracted model representing the dynamical balance between electricity and magnetism. Figure 2(a) is Faraday's actual abstracted model. Figure 2(b) shows how the picture of interlocking curves is abstracted from the earlier abstracted model involving lines of force. For example, a lateral repulsion of the magnetic lines (outer lines) has the same effect as a longitudinal expansion of the current lines (inner lines).

Maxwell used both of Faraday's abstracted models in his first attempt to mathematize electromagnetism (Maxwell, 1890, pp. 155-229). In this analysis he replaced Faraday's relationship between the number of lines cut and the intensity of the induced force with a continuous measure by representing the lines of force as the flow of an incompressible fluid through fine tubes of variable section, filling all space. The interlocking curves, called "mutually embracing curves" by Maxwell (p. 194), formed the basis of his reciprocal dynamics. The effect of this abstraction on his thinking can be seen most directly in his complicated use of two fields -- one for a longitudinal measure of force and one for a lateral measure -- where we would now only use one.

Figure 3 about here

The abstractions that Maxwell took from Faraday were useful primarily for Maxwell's kinematical analysis. A dynamical analysis of the underlying forces that could produce the lines required the construction of a quite different abstracted model: one that would embody the dynamical relations between electric and magnetic forces. The abstraction that Maxwell constructed is an analogy between electromagnetism and continuum mechanics (fluids, elastic media, etc.). Maxwell first constructed a primitive abstracted model, shown in Figure 3(a), consistent with a set of constraints: a fluid medium composed of elastic vortices and under stress. With this form of the abstraction he was able to provide a mathematical representation for various magnetic phenomena. Analyzing the relations between current and magnetism required alteration of this abstraction. In Figure 3(a) all the vortices are rotating in the same direction, which means that if they touch, they will stop. Mechanical consistency, thus, requires the introduction of "idle wheels" surrounding the vortices, and Maxwell argued that their translational motion could be used to represent electricity. Figure 3(b) shows a cross section of this altered abstracted model. For purposes of calculation, Maxwell had to make the elastic vortices into rigid pseudospheres. He next formulated the mathematical relations between currents and magnetism. It then took him nine months to figure out how to represent the final (and most critical) piece of the problem: electrostatic actions. He found that if he made the vortices elastic once again, and identified electrostatic polarization with elastic displacement, it was possible to calculate the wave of distortion produced by polarization. That is, adding elasticity to the abstraction enabled him to show that electromagnetic actions are propagated with a time delay, i.e., they are field actions and not Newtonian actions at a distance. At this point Maxwell had achieved a fully mathematized representation of the electromagnetic field.

DISCUSSION

The main point of this example is that it was through a process of embodying the structural relations between electric and magnetic actions in a series of abstracted models, reasoning with and about these, and manipulating them in various ways, that Maxwell generated the field equations for electromagnetism. Considerations of formal consistency, as presented in textbooks, played no significant role in this analysis. At the same time, the known mathematical structure of continuum dynamics motivated the analysis and provided the basis for achieving the goal of a mathematized representation of the dynamics of electromagnetic fields.

The analysis illustrates reasoning that is concretely situated, yet relies on a process of abstraction that is crucial to the success of the reasoning effort. The construction of an abstracted model removes a set of properties and relations from its initial context, creating objects in a new situation that can be analyzed and manipulated. Analysis of the properties of the abstracted model is situated in the context that the model provides. The effort to use that model as a representation for a different domain proceeds by considering various requirements that arise from features of the second domain that may be known or emerge in the problem-solving process. These requirements act as constraints on the model, as the theorist seeks a modified model that behaves in accordance with them. Modifications are sought with the condition that the modified model should have a mathematical structure that can be understood and expressed in formal terms.

The process of arriving at Maxwell's equations according to this analysis is significantly different from the processes of formula induction of the kind that Simon and his colleagues have studied (Langley, Simon, Bradshaw, & Zitkow, 1987). That process is situated in a context of numerical data and symbolic expressions that specify numerical operations between variables. The search takes place primarily in a space of formulas, with theoretical entities introduced as needed to account for invariant relations among numbers. In Nersessian's interpretation of Maxwell's discovery, the search is primarily a search for a model with a coherent causal structure that originates in one domain but is changed so that its behavior fits constraints that hold in a different domain. After the change, of course, it no longer provides an accurate representation of the phenomena in the domain from which it was abstracted.

Analogical reasoning of the kind identified in this analysis also differs significantly from the analyses that have been studied by cognitive psychologists such as Gentner (1983), by Gick and Holyoak (1983; 1980). In these analyses, an attempt is made to match the relational structures of two or more domains, with those analogies considered best that provide the closest match between patterns of relations. In contrast, the analogy between continuum dynamics and electromagnetic fields was productive because of the possibility of constructing an abstracted model of continuum dynamics that could be changed to fit the constraints of electromagnetism.

The success of the analogy depended upon having a representation that could be analyzed and manipulated as an object separate from its role as a representation of continuum dynamics. The representation included drawings, but clearly those notations were not sufficient to support Maxwell's reasoning. The model includes notations along with their interpretations as hypothetical objects that can be considered, combined, and modified to have different properties and interactions. Study of the abstracted model, including exploration of variations of its basic structure, was possible and necessary for it to provide the basis of Maxwell's theoretical achievement. Models in which analogical thinking depends only on a hypothesized cognitive representation of a mapping or schema that connects two domains have not addressed the role of the representation as providing an object of analysis and hypothesis construction. We expect to learn more about these issues from a thorough comparison of the Maxwell example and others like it with models of analogical reasoning by schemata and structure-mapping. Our analysis should also provide a deeper understanding of the reciprocal processes through which formal representations and abstracted models are constructed in scientific discovery. We hope additionally that this exploration will provide helpful suggestions about ways in which abstracted models can be used more productively in the practice of science education.

ACKNOWLEDGEMENTS

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Coulomb's Law: $\mathrm{div}\ \vec{D} = 4\pi\rho$

Ampère's Law: $\overset{\rightarrow}{\mathrm{curl}}\ \vec{H} = 4\pi\vec{J}$

Faraday's Law: $\overset{\rightarrow}{\mathrm{curl}}\ \vec{E} = -\partial\vec{B}/\partial t$

Absence of Free
Magnetic Poles: $\mathrm{div}\ \vec{B} = 0$

Equation of Continuity: $\mathrm{div}\ \vec{J} + \partial\rho/\partial t = 0$

TABLE 1: Equations Maxwell began with according to standard textbook account

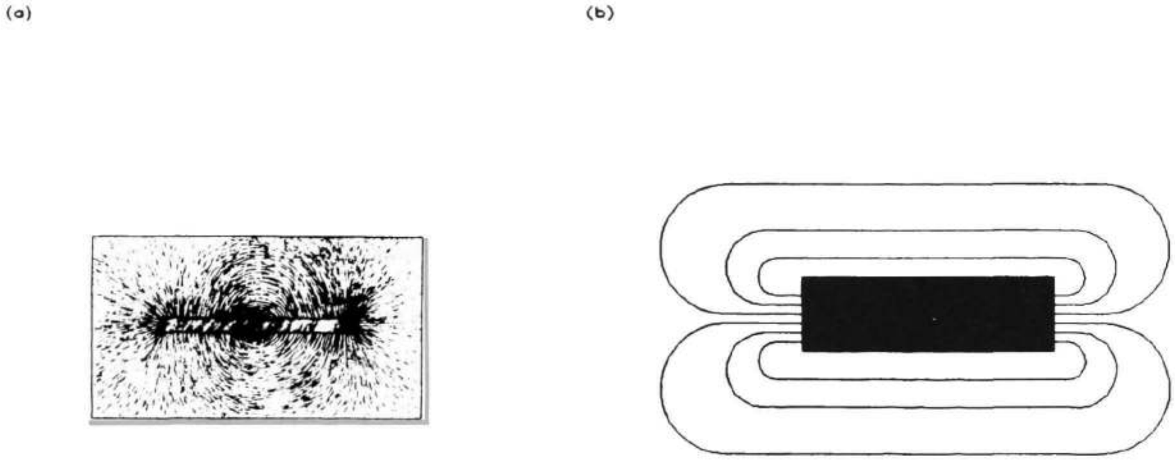

Figure 1: (a) Actual pattern of lines of force surrounding a bar magnet (from Faraday (1839–55), vol. 3); (b) Schematic representation of lines of force surrounding a bar magnet.

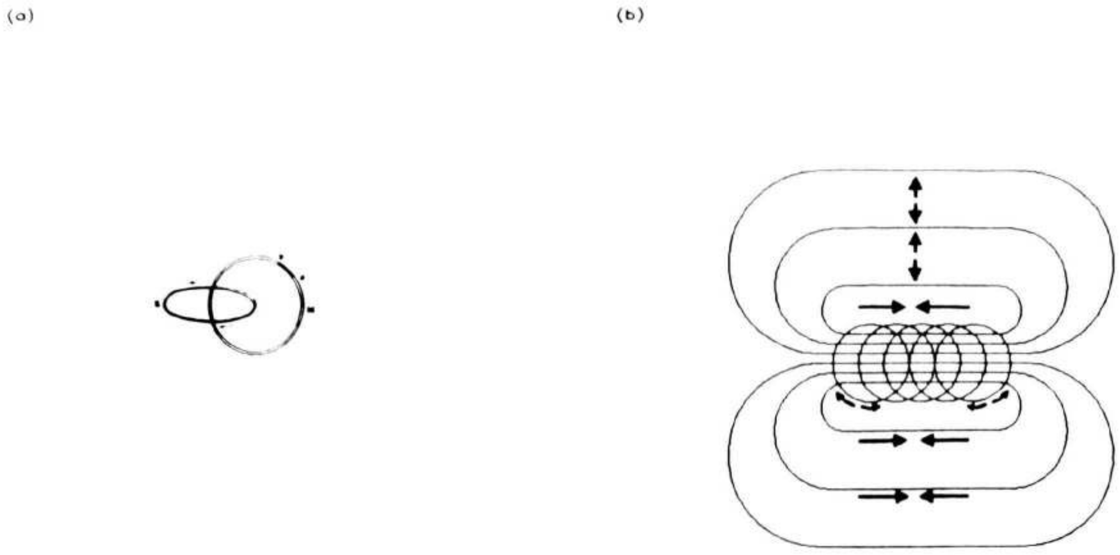

Figure 2: (a) Faraday's representation of the interconnectedness of electric currents and magnetic force (from Faraday (1839–55), vol. 3); (b) Schematic representation of the reciprocal relationship between magnetic lines of force and electric current lines.

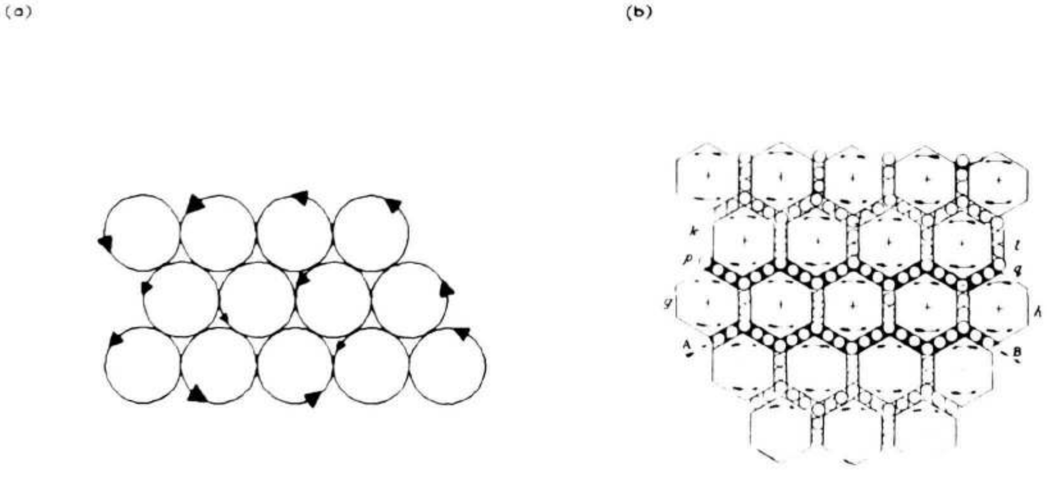

Figure 3: (a) Schematic representation of initial crude source retrieved by Maxwell; (b) Maxwell's representation of his fully elaborated "physical analogy" (from Maxwell (1881–2)).