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Price-Induced Technical Progress in 80 years of U.S. Agriculture

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Abstract
This paper presents a theory of technical progress that interprets the price-induced conjecture of Hicks. It provides also an exhaustive set of comparative statics conditions that constitute the scaffolding for an empirical test of the theory. A crucial assumption is that entrepreneurs make decisions about techniques on the basis of expected information about prices and quantities. Another assumption is that these decisions are made in order to fulfill a profitability objective. The novelty of our approach is that expected relative prices enter the production function as shifter of the technology frontier. The consequence of this assumption is an expansion of the traditional Shephard lemma that is useful for identifying the portion of input quantities that have been determined by the conjecture of price-induced technical progress (PITP). The theory is applied to a sample of 80 years of US agriculture. Three versions of the general model are presented. The first version deals only with expected relative prices. The empirical results do not reject the PITP hypothesis. The second and third versions introduce lagged expected relative prices, lagged R&D expenditures and lagged extension expenditures as explanatory variables of the portion of the input quantities that may be attributable to technical progress.

JEL Classification: C60, D21.
Keywords: Price-induced technical progress, Comparative statics, Primal-dual, Expanded Shephard lemma, Nonlinear errors-in-variables, Translog, US agriculture.

Use a color printer for a better rendering of the figures.

Quirino Paris is a professor of agricultural economics at the University of California, Davis. I acknowledge invaluable discussions on this subject held over several years with Michael R. Caputo. I also acknowledge the use of the theory and its justifications that were presented in previous papers by Paris and Caputo (2001) and by Caputo and Paris (forthcoming). All the errors are mine. I dedicate this paper to my wife, Carlene, who died of a rare cancer on May 5, 2001.

Price-Induced Technical Progress in 80 years of US Agriculture

1. Introduction

John Hicks (1932) is credited with advancing the conjecture that changes in relative prices induce technical progress (TP). This conjecture implies that relative factor prices serve a dual function, as signals of resource scarcity and as determinants of the firm's technology choice. Hayami and Ruttan (1971) revitalized Hicks' conjecture and made important contributions to the explanation of the magnitude and direction of TP in the American and Japanese agricultural sectors using the relative price hypothesis. Over the past thirty years, many authors have attempted to test this hypothesis using aggregate data and obtaining mixed results. In these studies, the consensus approach to the econometric estimation and testing of the hypothesis that technical progress is induced by relative prices has been to regress the ratio of some factors of production over a distributed lag series of their price ratios and other similar series of extension, public and private R&D expenditures. Thirtle, Schimmelpfennig and Townsend (2002) summarized several significant studies of this kind and produced one of their own. The sample information about output quantity and output price is remarkably absent in many of these studies. This omission seems in contrast to the conjecture advanced by several economists according to whom the choice of techniques is determined, to a large extent, by profitability considerations. In this paper, therefore, we attempt to recast the price-induced technical progress (PITP) hypothesis into a framework that utilizes all the available theoretical and sample information, including output price and quantity. This approach leads to a novel set of comparative statics conditions of the economic theory of the firm undergoing technical progress that provides an exhaustive scaffolding for testing the PITP conjecture.

When dealing with technical progress, it is convenient to distinguish the innovation phase from the adoption phase. The majority of price-taker firms self select into the adoption phase. In general, the choice of available techniques made by those firms is guided mainly by expected profitability considerations. When price-taker firms are aggregated into an industry, such as the US agricultural sector, the R&D and extension expenditures may become determinants of the industry technical progress. Griliches (1957, p. 519), Arrow (1969, p. 29), Hirsch (1965, p. 38) and other economists have suggested that expected profitability objectives may be a determinant of adoption rates.

The expected profitability conjecture relating expected profits to TP leads to a model where expected output and input prices enter the production function as shifters of the technology frontier. As originally suggested by Paris (1993) and re-elaborated more recently by Paris and Caputo (2001) and by Caputo and Paris (forthcoming), we incorporate expected relative factor prices (expected input prices normalized by the single expected output price) explicitly into the production function and assume a cost-minimizing behavior of the individual entrepreneur.

The introduction of expected relative prices into the production function invalidates the traditional comparative statics relations of the competitive firms but leads---by necessity---to a more general model of the cost-minimizing/profit-maximizing entrepreneur. The novel set of comparative statics conditions depends on both primal and dual relations and is expressed in the form of a symmetric and negative semidefinite matrix of estimable terms. It follows that the empirical implementation of the PITP conjecture developed in this paper requires the joint estimation of the derivatives of the cost function with respect to relative input prices, the production function and the first order necessary conditions.

2. The Theory of Price-Induced Technical Progress

We assume that cost-minimizing firms are risk neutral and make their decisions on the basis of expected quantities and prices. The process of expectation formation is characteristic of every firm but is unknown to the econometrician.

Given the expected profitability conjecture, we postulate a production function $f(\cdot)$ for a price-taker, risk-neutral and cost-minimizing firm with values

$$y^e \le f(\mathbf{x},\mathbf{w}^e,t) \tag{1}$$

where y^e is the expected level of output for any strictly positive $(J \times 1)$ vectors $\mathbf{x}$ and $\mathbf{w}^e$ of input quantities and expected relative input prices. The expected relative input prices are defined as the ratio of expected input prices to the expected output price. In this paper, we assume a single output. The symbol t represents the index of traditional, exogenous technical progress. With respect to the production function in relation (1) we assume only its existence and differentiability of order 2.

The price-taking risk-neutral cost-minimizing model of the firm operating under the influence of price-induced TP is stated as

$$c(y^e,\mathbf{w}^e,t) \stackrel{\text{def}}{=} \min_{\mathbf{x}}\{\mathbf{w}'^e\mathbf{x} \ \text{ s.t. } \ y^e - f(\mathbf{x},\mathbf{w}^e,t) \le 0\} \tag{2}$$

where the symbol $'$ is the transpose operator. We assume that problem (2) possesses a unique interior $c^{(1)}$ solution $\boldsymbol{\alpha} \mapsto \mathbf{h}^e(\boldsymbol{\alpha})$ for all $\boldsymbol{\alpha} \in B(\boldsymbol{\alpha}^\circ;\delta)$, where $B(\boldsymbol{\alpha}^\circ;\delta)$ is an open $(J+2)$-ball centered at the point $\boldsymbol{\alpha}^\circ \in \Re^{J+2}_{++}$ with radius $\delta > 0$, and where $\boldsymbol{\alpha} \stackrel{\text{def}}{=} (\mathbf{w}^e, y^e, t)$ is the given parameter vector of the problem. The Lagrangean function corresponding to the minimization problem (2) can be stated as $L(\mathbf{x},\lambda;y^e,\mathbf{w}^e,t) = \mathbf{w}'^e\mathbf{x} + \lambda[y^e - f(\mathbf{x},\mathbf{w}^e,t)]$ and, assuming that a nondegenerate constraint qualification holds at the solution (i.e., $f_{x_j}(\mathbf{h}(\mathbf{w}^e, y^e, t); \mathbf{w}^e, t) \neq 0$ for at least one value of the index *j*), the first order necessary conditions are given by

$$L_{x_j} = w_j^e - \lambda f_{x_j}(\mathbf{x}, \mathbf{w}^e, t) = 0, \quad j = 1, \ldots, J \tag{3}$$

$$L_\lambda = y^e - f(\mathbf{x}, \mathbf{w}^e, t) \leq 0, \;\; \lambda \geq 0, \;\; \lambda L_\lambda = \lambda[y^e - f(\mathbf{x}, \mathbf{w}^e, t)] = 0. \tag{4}$$

since $w_j^e > 0$, $j = 1, \ldots, J$ equations (3) and (4) imply $\lambda(\mathbf{w}^e, y^e, t) > 0$. In turn, this fact and equation (4) imply that the marginal product of each input is positive at the optimum, that is, $f_{x_j}\left(\mathbf{h}(y^e, \mathbf{w}^e, t), \mathbf{w}^e, t\right) > 0$, $j = 1, \ldots, J$, where $\mathbf{x}^e = \mathbf{h}(y^e, \mathbf{w}^e, t)$ is the optimum vector of input derived demand functions.

The properties (or lack of them) of the cost function $c(\cdot)$ defined in equation (2) can be listed as follows. The presence of the expected relative prices in the production function induces a property of non-concavity with respect to the same prices on the cost function. Hence, the traditional comparative statics conditions are violated. Secondly, the prototype Shephard's lemma must be modified to assume a functional form that involves also the derivatives of the production function with respect to expected relative prices and the Lagrange multiplier. In fact, the application of the envelope theorem to problem (2) results in

$$c_{w_j}(y^e, \mathbf{w}^e, t) = h_j(y^e, \mathbf{w}^e, t) - \lambda(y^e, \mathbf{w}^e, t) f_{w_j}\left(\mathbf{h}(y^e, \mathbf{w}^e, t), \mathbf{w}^e, t\right), \quad j = 1, \ldots, J. \tag{5}$$

Thirdly, the cost function $c(\cdot)$ in relation (2) is not homogeneous of degree one in the expected relative prices because of the dependence of the production function upon those same prices. Furthermore, the cost function is not necessarily increasing in the expected input prices because nothing was assumed regarding the derivatives of the production function with respect to the expected relative prices. Finally, the cost function in relation (2) is increasing in output. This is the result of combining the envelope theorem with $\lambda(\mathbf{w}^e, y^e, t) > 0$, since

$c_{y^e}(y^e,\mathbf{w}^e,t) = \lambda(y^e,\mathbf{w}^e,t) > 0$, the marginal cost function. Hence, the extended Shephard's lemma in equation (5) can be rearranged to read

$$x_j^e = c_{w_j}(y^e,\mathbf{w}^e,t) + c_{y^e}(y^e,\mathbf{w}^e,t) f_{w_j}\left(\mathbf{h}(y^e,\mathbf{w}^e,t),\mathbf{w}^e,t\right), \qquad j = 1,...,J. \qquad (6)$$

Therefore, both primal and dual relations are required to recover the input demand functions under the cost-minimizing price-induced TP hypothesis.

The extended Shephard Lemma in equation (6) provides the structure for a decomposition of the cost-minimizing input quantities into an amount due to input substitution, $x_{Sj}^e \stackrel{\text{def}}{=} c_{w_j^e}$, and a complementary amount due to the PITP conjecture, $x_{PIj}^e \stackrel{\text{def}}{=} c_{y^e} f_{w_j^e}$. Notice that our theory does not require the simultaneous nonnegativity of the substitution and the PITP components of the input demand function. This decomposition provides a natural setting for introducing the dependence of the input quantities upon lagged expected relative prices, public and private R&D and extension expenditure levels, as suggested, for example, by Thirtle, Schimmelpfennig and Townsend (2002).

Theorem 1 generalizes the comparative statics conditions of the traditional production and cost theory in order to account for the price-induced TP hypothesis. The theorem provides an empirically verifiable, symmetric, negative semidefinite matrix and an upper bound for the rank of that matrix. The proof uses the primal-dual formalism of Silberberg (1974) and is presented in the appendix.

Theorem 1. *The curvature properties of the price-taking, cost-minimizing model of the risk-neutral firm operating under the price-induced technical progress hypothesis are summarized by the statement that the* $J \times J$ *matrix* $\mathbf{S}(y^e,\mathbf{w}^e,t)$, *defined as*

$$\mathbf{S}(y^e,\mathbf{w}^e,t) \stackrel{\text{def}}{=} \mathbf{C}_{\mathbf{w}^e\mathbf{w}^e} + \mathbf{c}_{\mathbf{w}^e y^e}\mathbf{f}'_{\mathbf{w}^e} + c_{y^e}\mathbf{F}_{\mathbf{w}^e\mathbf{w}^e} + \mathbf{f}_{\mathbf{w}^e\mathbf{w}^e}\mathbf{c}'_{y^e\mathbf{w}^e} + \mathbf{f}_{\mathbf{w}^e} c_{y^e y^e}\mathbf{f}'_{\mathbf{w}^e} \tag{7}$$

is negative semidefinite, symmetric, and the $\operatorname{rank}\left(\mathbf{S}(y^e,\mathbf{w}^e,t)\right) \leq J-1$.

Theorem 1 provides a generalization of the traditional cost theory based upon a neoclassical production function in the sense that the curvature property of the traditional cost-minimizing model of the firm is contained in Theorem 1 as a special case. When $\mathbf{f}_{\mathbf{w}^e}(\mathbf{x},\mathbf{w}^e,t) \equiv \mathbf{0}_J$, problem (2) collapses to the traditional model of the cost-minimizing firm, that is $\mathbf{S}(y^e,\mathbf{w}^e,t) = \mathbf{C}_{\mathbf{w}^e\mathbf{w}^e}$, a symmetric and negative semidefinite matrix, which is equivalent to the concavity of $c(\cdot)$ in $\mathbf{w}^e$, the neoclassical result.

A novel feature of Theorem 1 is the appearance of the derivatives of both the production and cost function in the comparative statics matrix of equation (6). This property is absent from any prototype model of the firm and it is the distinguishing feature of our model of price-induced TP. It can be viewed as the scaffolding by which one can erect the estimating framework of the price-induced TP hypothesis. In other words, in general, one must always estimate the production function and first order necessary conditions jointly with dual relations, namely the derivatives of the cost function, when carrying out an empirical test of the price-induced TP theory presented here. This is called the primal-dual approach.

Although the above theory was formulated using the individual firm as the target agent, we will assume that similar relations carry over to the agricultural sector, assuming that the aggregation over firms will hold.

3. Specification of the Error Structure

The theoretical model is defined in terms of expected quantities and prices, given that it represents the planning process of a price-taking, risk-neutral entrepreneur. The econometric formulation of the same model sees the intervention of the econometrician sometime after the planning process was carried out. If the expected quantities and prices used by the entrepreneur for making her decisions were recorded at planning time, the recovery of the underlying production and economic relations would be greatly simplified. Unfortunately, these expected quantities and prices are not in general available and the econometrician must undertake the painstaking job of measuring them. In so doing, he commits measurement errors on every variable. We assume, therefore, that all quantities and prices involved in the production and cost system are observed by $y, \mathbf{x}$ and $\mathbf{w}$ which bear an additive error relation to the corresponding expected counterparts, that is, $y = y^e + \varepsilon_0$, $\mathbf{x} = \mathbf{x}^e + \boldsymbol{\varepsilon}$ and $\mathbf{w} = \mathbf{w}^e + \boldsymbol{\nu}$. Hence, the combination of the theoretical relations and the additive error structure postulated above produces a nonlinear errors-in-variables model with generalized additive errors that poses well-known estimation challenges.

We summarize below the econometric model subject to the theoretical restrictions of the cost-minimizing firm operating under price-induced TP that is given by the following primal and dual relations:

Error structure

$$y = y^e + \varepsilon_0 \tag{8}$$

$$\mathbf{w} = \mathbf{w}^e + \boldsymbol{\nu} \tag{9}$$

$$\mathbf{x} = \mathbf{x}^e + \boldsymbol{\varepsilon} \tag{10}$$

Primal relations

$$y^e = f(\mathbf{x}^e, \mathbf{w}^e, t) \qquad \text{production function} \tag{11}$$

$$\mathbf{w}^e = c_{y^e}(y^e, \mathbf{w}^e, t)\mathbf{f}_{\mathbf{x}^e}(\mathbf{x}^e, \mathbf{w}^e, t) \qquad \text{input price functions} \tag{12}$$

Dual relations

$$\mathbf{x}^e = \mathbf{h}(y^e, \mathbf{w}^e, t) = \mathbf{x}_S^e + \mathbf{x}_{PI}^e \qquad \text{input demand functions.} \quad (13)$$

In case the first-order necessary conditions have no explicit analytical solution, the input demand functions exist via the duality principle. The vector of error terms $\mathbf{e}' \stackrel{\text{def}}{=} (\varepsilon_0, \mathbf{v}', \boldsymbol{\varepsilon}')$ is assumed to be distributed according to a multivariate normal density with zero mean vector and variance matrix Σ.

Traditionally, aggregate models of TP based upon time series data have been specified using a distributed lag representation of either quantities or prices, or both. This approach seems to have been taken for two main reasons: (a) to capture, somehow, a dynamic aspect that is assumed to be inherent in a process of technical progress, and (b) to represent some process of expectation formation of the entrepreneur about quantities and prices. Often, the two aspects are confounded. With respect to the PITP model presented above, we would like to point out that the expectation process is taken into consideration explicitly and there is no need to formulate a distributed lag representation of expected quantities and prices. We acknowledge that the dynamic aspect of TP requires an explicit theory, akin to the static theory formulated above: a distributed lag specification without theory is only an *ad-hockery*. A dynamic theory of PITP will be the subject of another effort.

In general, it will be wise to postulate that the theoretical relations expressed in equations (11)-(13) are represented by flexible functional forms. Such forms are not self-dual in the way that the Cobb-Douglas and the CES functions are. Hence, the implementation of the above model requires the statement of a cost function that has entirely different parameters from those of the production function. The coherent link between the primal and the dual frameworks is represented by the unknown expected quantities and prices that must be estimated along with the parameters.

The discussion of how to estimate the model given by equations (8)-(13) will be the subject of the following sections. We would like to advance here that, in principle, a Bayesian approach along the lines presented by Zellner (1969, ch. 5) would produce consistent estimates. But, as we are not comfortable with elaborate and multi-dimensional integration techniques, we will propose a two-phase approach based upon a nonlinear least-squares estimator.

In phase I, the objective is to obtain estimates of the expected quantities and relative prices. That is, assuming a sample of dimensions $t = 1,...,T$, the explicit representation of the phase I model can be stated as

$$\min_{\boldsymbol{\beta}, y_t^e, x_{tj}^e, w_{tj}^e, \mathbf{e}_t} \sum_{t=1}^{T} \varepsilon_{0t}^2 / \sigma_{\varepsilon_0}^2 + \sum_{j=1}^{J} \sum_{t=1}^{T} \nu_{tj}^2 / \sigma_{\nu_j}^2 + \sum_{j=1}^{J} \sum_{t=1}^{T} \varepsilon_{tj}^2 / \sigma_{\varepsilon_j}^2 \tag{14}$$

or

$$\min_{\boldsymbol{\beta}, y_t^e, x_{tj}^e, w_{tj}^e, \mathbf{e}_t} \sum_{t=1}^{T} \varepsilon_{0t}^2 + \sum_{j=1}^{J} \sum_{t=1}^{T} \nu_{tj}^2 / \lambda_{\nu_j} + \sum_{j=1}^{J} \sum_{t=1}^{T} \varepsilon_{tj}^2 / \lambda_{\varepsilon_j} \tag{15}$$

where $\sigma_{\varepsilon_0}^2, \sigma_{\nu_j}^2, \sigma_{\varepsilon_j}^2$ are the variances of the respective error terms, $j = 1,...,J$. The weights of the objective function (15) are specified as the ratios of the error variances using the variance of the output quantity as the normalizing factor

$$\lambda_{\nu_j} = \frac{\sigma_{\nu_j}^2}{\sigma_{\varepsilon_0}^2}, \lambda_{\varepsilon_j} = \frac{\sigma_{\varepsilon_j}^2}{\sigma_{\varepsilon_0}^2}.$$

In version 1 of the primal-dual model developed in this study, the minimization of the objective function (15) is subject to the error structure and primal-dual constraints given in equations (8)-(13).

Our theory, however, provides a natural decomposition of the expected input quantities into complementary components called the substitution and the PITP counterparts. As noted by Thirtle, Schimmelpfennig and Townsend (2002, p. 608), "… *when factor substitution has been accounted for, the major proportion of the change in factor ratios… can be explained by the lagged effect of relative prices, … private R&D expenditures …*" public R&D and extension expenditures. In our specification, this conjecture can be articulated as follows:

$$y_t^e = f^e(\mathbf{x}_t^e, \mathbf{w}_t^e, t) = f^e\Big((\mathbf{x}_{St}^e + \mathbf{x}_{PIt}^e), \mathbf{w}_t^e, t\Big) \tag{16}$$

$$\mathbf{w}_t^e = c_{y^e}^e(y_t^e, \mathbf{w}_t^e, t)\mathbf{f}_{\mathbf{x}^e}^e\Big((\mathbf{x}_{St}^e + \mathbf{x}_{PIt}^e), \mathbf{w}_t^e, t\Big) \tag{17}$$

$$\mathbf{x}_t^e = \mathbf{h}^e(y_t^e, \mathbf{w}_t^e, t) = \mathbf{x}_{St}^e + \mathbf{x}_{PIt}^e \tag{18}$$

$$\mathbf{x}_{PIt}^e = \mathbf{g}(lag\mathbf{w}_t^e, lagRD_t, lagExt_t), \tag{19}$$

where $\mathbf{x}_{St}^e$ and $\mathbf{x}_{PIt}^e$ are the substitution and PIPT components, respectively, of the $\mathbf{x}_t^e$ decomposition. Hence, a second version of the PITP model can be thought of as minimizing equation (15) subject to the error structure given in equations (8)-(10) and the theoretical restrictions given by equations (16)-(19). If warranted, the PITP model can be further specified to account for autocorrelation.

We assume that an optimal solution of the phase I problem exists and can be found using a nonlinear optimization package such as GAMS (see Brooke *et al*. [1988]). With the estimates of the expected quantities and prices obtained from phase I, a traditional NSUR problem can be stated and estimated in phase II using conventional econometric packages such as SHAZAM (Whistler *et al*. [2001]). For clarity, this phase II estimation problem can be stated as the maximization of the concentrated log-likelihood function of the nonlinear seemingly unrelated regression (NSUR) problem

$$Loglik = -\frac{TM}{2}\log(2\pi) - \frac{TM}{2} - \frac{T}{2}\log\left(|\mathbf{MSR}|(1/T)^{M}\right) \tag{20}$$

where T and M are the number of sample observations and the number of equations, respectively, $\mathbf{MSR}$ is the $(M \times M)$ matrix of sums of squared residuals and their cross products of the following equations

$$y_t = f(\hat{\mathbf{x}}^e, \hat{\mathbf{w}}^e, t; \boldsymbol{\theta}_y) + \varepsilon_{0t} \tag{21}$$

$$w_{jt} = c_{y^e}(\hat{y}_t^e, \hat{\mathbf{w}}_t^e, t; \boldsymbol{\theta}_c) f_{\mathbf{x}^e}(\hat{\mathbf{x}}_t^e, \hat{\mathbf{w}}_t^e, t; \boldsymbol{\theta}_y) + \nu_{jt} \tag{22}$$

$$x_{jt} = h(\hat{y}_t^e, \hat{\mathbf{w}}_t^e, t; \boldsymbol{\theta}_c) + \varepsilon_{jt} \tag{23}$$

where $\hat{y}_t^e, \hat{\mathbf{w}}_t^e$ and $\hat{\mathbf{x}}_t^e$ are the phase I estimates of the expected quantities and relative prices and the dimension of $M = 2J + 1$, where J is the number of inputs. The parameter vectors $\boldsymbol{\theta}_y$ and $\boldsymbol{\theta}_c$ belong to the production and the cost function, respectively. The objective in equation (20) is to maximize the negative logarithm of the determinant of the $\mathbf{MSR}$ matrix. A second version of the phase II specification deals with equations (16)-(19).

After estimating the PITP model, a measure of the input biases of technical change can be assessed. For brevity, we follow Antle and Capalbo's discussion of the subject (1988, ch. 2, p. 38-39) and define a primal measure of the bias of technical progress between input *j* and input *k* as

$$B_{jk.w^e=c} = \frac{\partial \log(f_j / f_k)|_{w^e=c}}{\partial t} = \frac{\partial \log f_j(\hat{\mathbf{x}}^e, \hat{\mathbf{w}}^e, t)|_{w^e=c}}{\partial t} - \frac{\partial \log f_k(\hat{\mathbf{x}}^e, \hat{\mathbf{w}}^e, t)|_{w^e=c}}{\partial t}, \quad j \neq k$$

that reflects the original formulation by Hicks involving the invariance (to technical change) condition of the expansion path, and where f_j represents the marginal product of the *j*-th input. The condition that the input prices be constant guarantees the invariance of the expansion path. As Antle and Capalbo state (1988, p.38): "…this measure of the bias is defined at a given point in input space." It is an open question, then, whether the biases should be evaluated at the same point in input space for the entire sample period. An overall measure of the bias associated with input j is stated as

$$B_{j|w^e=c} = \frac{\partial \log f_j \mid_{w^e=c}}{\partial t} - \sum_{k=1}^{J} S_k^e \frac{\partial \log f_k \mid_{w^e=c}}{\partial t} \tag{24}$$

where S_k^e is the expected cost share of the k-th input. According to Antle and Capalbo (1988, p. 40), the condition $B_j > 0$ characterizes input-using technical progress, implying that the marginal product of input j is increasing relative to all other inputs, while $B_j < 0$ indicates input-saving TP. Hicks neutrality requires $B_{jk} = 0$ for all j and k which, in turn, results in $B_j = 0$ for all $j = 1,...,J$.

As will become clearer in the empirical sections, the meaning of input-using (input-saving) technical progress associated with the sign of the B_j coefficient is rather arbitrary in the sense that the bias coefficients measure simply how the marginal products vary with a change of t. It is difficult, then, to specify in what sense a particular bias is either input-using or input-saving. In other words, the definition of input bias in technical progress is simply a descriptive measure of the change of the marginal product with respect to the exogenous technical progress index t which, in a time series sample, is confounded with the discrete time associated with the sample observations. Consequently, the measure of input biases cannot constitute an empirical test of technical progress.

4. The Data of US Agriculture

The sample input data for the present analysis were made available by Thirtle, Schimmelpfennig and Townsend (2002) and are described in their paper. The time series consist of four input quantity and price indices relating to machinery, labor, fertilizer and land, from 1880 to 1990; public and private R&D and extension expenditures are also from 1880 to 1990. Additionally, aggregate output quantity and price indices from 1910 to 1990 were derived from the US Historical Statistics and USDA databases and provided by Spiro Stefanou. All the index series are defined with base 1967 = 100. Because the primal-dual model of PITP developed in this paper uses also the output quantity and price series, the usable sample data range from 1910 to 1990 with 81 observations. In this paper we chose to deal with the single aggregate of output for the US agriculture. All the data were scaled by a factor of 100 so that the average of most series is close to 1.

5. A Translog Primal-Dual Model of PITP: Version 1

The implementation of the primal-dual model of price-induced technical progress presented in previous sections was realized with the choice of a translog production function and a translog cost function. In particular, the production function is stated as

$$\begin{aligned}\log y_t^e &= \alpha_0 + \sum_{j=1}^{4}\alpha_j \log x_{jt}^e + \sum_{j=1}^{4}\gamma_j \log w_{jt}^e + \sum_{j=1}^{4}\sum_{k=1}^{4}\beta_{jk}\log x_{jt}^e \log x_{kt}^e/2 \\ &+ \sum_{j=1}^{4}\sum_{k=1}^{4}\delta_{jk}\log w_{jt}^e \log w_{kt}^e/2 + \sum_{j=1}^{4}\sum_{k=1}^{4}\eta_{jk}\log x_{jt}^e \log w_{kt}^e \\ &+ \sum_{j=1}^{4}\alpha_{Tj}\log x_{jt}^e \log t + \sum_{j=1}^{4}\gamma_{Tj}\log w_{jt}^e \log t + \theta_1 \log t + \theta_2(\log t)^2/2\end{aligned} \tag{25}$$

with symmetric $\beta_{jk} = \beta_{kj}$ and $\delta_{jk} = \delta_{kj}$.

The corresponding cost function is stated as

$$\begin{aligned}\log c_t &= \phi_0 + \sum_{j=1}^{4}\phi_j \log w_{jt}^e + \sum_{j=1}^{4}\sum_{k=1}^{4}\varphi_{jk}\log w_{jt}^e \log w_{kt}^e/2 + \sum_{j=1}^{4}\varphi_{yj}\log w_{jt}^e \log y_t^e \\ &+ \phi_y \log y_t^e + \varphi_{yy}(\log y_t^e)^2/2 + \sum_{j=1}^{4}\phi_{Tj}\log w_{jt}^e \log t + \phi_{Ty}\log y_t^e \log t \\ &+ \rho_1 \log t + \rho_2(\log t)^2/2\end{aligned} \tag{26}$$

with symmetric $\varphi_{jk} = \varphi_{kj}$.

The input price functions in equations (12) (first order necessary conditions), are given by the product of the marginal cost function and the marginal product function of each input, for $j = 1,...,J$,

$$w^e_{jt} = c_t(\phi_y + \varphi_{yy} \log y^e_t + \sum_{k=1}^{4} \varphi_{yk} \log w^e_{kt} + \phi_{Ty} \log t)(\alpha_j + \sum_{k=1}^{4} \beta_{jk} \log x^e_{kt} + \sum_{k=1}^{4} \eta_{jk} \log w^e_{kt} + \alpha_{Tj} \log t)/x^e_{jt} \quad (27)$$

The first term in parenthesis is the marginal cost (without the y^e_t variable as a divisor) while the second term in the second parenthesis is the marginal product (without the y^e_t variable as a multiplier) of the *j*-th input . The total cost is, by definition, $c_t \overset{\text{def}}{=} \sum_j x^e_{jt} w^e_{jt}$.

The input demand functions in equations (13) assume the structure of the expanded Shephard lemma discussed in section 2 which produces the following expressions, for $j = 1,...,J$,

$$\begin{aligned} x^e_{jt} &= c_t(\phi_j + \varphi_{yj} \log y^e_t + \sum_{k=1}^{4} \varphi_{jk} \log w^e_{kt} + \phi_{Tj} \log t)/w^e_{jt} \\ &+ c_t(\phi_y + \varphi_{yy} \log y^e_t + \sum_{k=1}^{4} \varphi_{yk} \log w^e_{kt} + \phi_{Ty} \log t)(\gamma_j + \\ &\sum_{k=1}^{4} \delta_{jk} \log w^e_{kt} + \sum_{k=1}^{4} \eta_{kj} \log x^e_{kt} + \gamma_{Tj} \log t)/w^e_{jt} \end{aligned} \quad (28)$$

The first line of equation (28) is the traditional derivative of the cost function with respect to the input price, $\frac{\partial c_t}{\partial w^e_{jt}}$, and the two other lines correspond to the novel term $\frac{\partial c_t}{\partial y^e_t} \frac{\partial f_t}{\partial w^e_{jt}}$ which expresses the conjecture of price-induced technical progress in the expanded Shephard lemma.

The modified Slutsky matrix of the PITP model $\mathbf{S}(y^e, \mathbf{w}^e, t)$, as given by equation (7), requires the specification of five matrices involving the parameters of both the production and the cost function as well as the level of all the output and input quantity as well as the input price variables. In order to make the testing of the PITP conjecture a manageable enterprise, we evaluate the $\mathbf{S}(y^e,\mathbf{w}^e,t)$ matrix at the level of each variable equal to 1 (recall that we scaled all the variables so that their average values are close to 1). In turns, the logarithm of each variable evaluated at 1 is equal to zero and the matrix $\mathbf{S}(y^e,\mathbf{w}^e,t)$ is defined only in terms of parameters of the production and cost functions. Theorem 1 defines necessary and sufficient conditions for the PITP conjecture and, therefore, those conditions must hold also at the unit level of all the variables.

To make the computations minimally intelligible, we reproduce below the assembly of the matrix $\mathbf{S}$ as implemented in the programming of the PITP model.

The $\mathbf{S}$ matrix in question is given by

$$\begin{aligned}\mathbf{S} &\overset{\text{def}}{=} \mathbf{C}_{\mathbf{w}^e\mathbf{w}^e} + \mathbf{c}_{\mathbf{w}^e y^e}\mathbf{f}'_{\mathbf{w}^e} + c_{y^e}\mathbf{F}_{\mathbf{w}^e\mathbf{w}^e} + \mathbf{f}_{\mathbf{w}^e\mathbf{w}^e}\mathbf{c}'_{y^e\mathbf{w}^e} + \mathbf{f}_{\mathbf{w}^e}c_{y^e y^e}\mathbf{f}'_{\mathbf{w}^e} \\ &= \mathbf{A}_1 + \mathbf{A}_2 + \mathbf{A}_3 + \mathbf{A}_4 + \mathbf{A}_5\end{aligned} \tag{29}$$

with each of the five component matrices defined as

$$\mathbf{A}_1(j,k) \equiv 4(\phi_j\phi_k + \varphi_{jk}), \quad j \neq k$$

$$\mathbf{A}_1(j,j) \equiv 4\left(\phi_j(\phi_j - 1) + \varphi_{jj}\right)$$

$$\mathbf{A}_2(j,k) \equiv 4(\phi_y\phi_j + \varphi_{yj})\gamma_k$$

$$\mathbf{A}_3(j,k) \equiv 4\phi_y(\gamma_j\gamma_k + \delta_{jk}), \quad j \neq k$$

$$\mathbf{A}_3(j,j) \equiv 4\phi_y\{\gamma_j(\gamma_j - 1) + \delta_{jj}\}$$

$$\mathbf{A}_4(j,k) \equiv 4\gamma_j(\phi_y\phi_k + \varphi_{yk})$$

$$\mathbf{A}_5(j,k) \equiv 4\gamma_j\{\phi_y(\phi_y - 1)\}\gamma_k$$

If the PITP conjecture holds, the $\mathbf{S}$ matrix should be a symmetric negative semidefinite matrix with rank less than *J*. The number 4 results from $c_t \overset{\text{def}}{=} \sum_{j=1}^{4} x_{jt}^e w_{jt}^e$ evaluated at the unit level of all the variables involved.

Given the translog PITP model, the input biases of technical progress in equation (24) translate into

$$B_j(t)|_{w^e=c} = \alpha_{Tj} / \{\alpha_j + \sum_{k=1}^{4} \beta_{jk} \log x_{jt}^e + \sum_{k=1}^{4} \eta_{jk} \log CP_k + \alpha_{Tj} \log t\} t$$
$$- \sum_{k=1}^{4} S_k^e \{\alpha_{Tk} / (\alpha_k + \sum_{kk=1}^{4} \beta_{k,kk} \log x_{kkt}^e + \sum_{kk=1}^{4} \eta_{k,kk} \log CP_{kk} + \alpha_{Tk} \log t) t\} \quad (30)$$

where S_k^e is the expected cost share and CP_k stands for constant prices of the k-th input. We have chosen to let the input quantities vary throughout the sample period, so that the bias measures of TP acquire the meaning stated in the definition (equation (24), "…this measure of the bias is defined at a given point in input space.") for each sample observation.

6. Empirical Results of the Translog Model of PITP: Version 1

Phase I of the PITP model was estimated using the GAMS programming package and unitary λ weights for the objective function (15). This choice was dictated by a lack of knowledge of the true weights. The selection of these weights transforms the given problem into a nonlinear Total Least Squares model, originally described by Gulob and Van Loan (1989, p. 576), and by a vast literature since then. The model constraints, represented by equations (25), (27) and (28), are highly nonlinear and non-convex. Hence, the solution achieved is only locally optimal. The problem was solved several times with different initial values. A serial correlation of order 1 was implemented during the estimation procedure.

The use of the GAMS 21.6 programming package requires a careful choice of upper and lower bounds for all parameters. Still, the solution of the problem is a non-trivial enterprise. The phase I PITP model has 1495 constraints (most of them nonlinear) and 1721 unknown parameters. In a typical run, the CPU time to achieve a locally optimal solution was about 20-30 minutes on a Supermicro machine (Intel dual processor Xeon, 3.0 Ghz, Linux Redhat AS3 OS).

We can report with confidence that the land input caused considerable headaches in all the computations and may have been the cause of the extraordinary number of iterations (between 5,000 and 10,000) required to achieve an optimal solution, perhaps because its quantity index is rather flat and exhibits very little variability.

The estimates of the expected quantities and prices obtained from the phase I estimation problem are neither unbiased nor consistent. This is due to our ignorance of the true λ ratios that weigh the objective function (15). We have already suggested that a Bayesian approach to the errors-in-variables problem may produce consistent estimates, albeit with a much more complex estimator. Hence, we are willing to accept some level of non-consistency of the estimates in exchange for a manageable estimator that can be implemented by normal practitioners. The problem, of course, is how to gauge what is an acceptable level of inconsistency. We do not have an analytical answer to this question. We only suggest that a small residual error may be an indication of the smallness of inconsistency. We proceed under this conjecture.

A measure of the estimates obtained from the phase I model can be viewed in Figure 1 and Figure 2 that report a comparison between the sample and the estimated quantities and prices.

Figure 1. Expected quantities (Series 1) and measured quantities (Series 2)

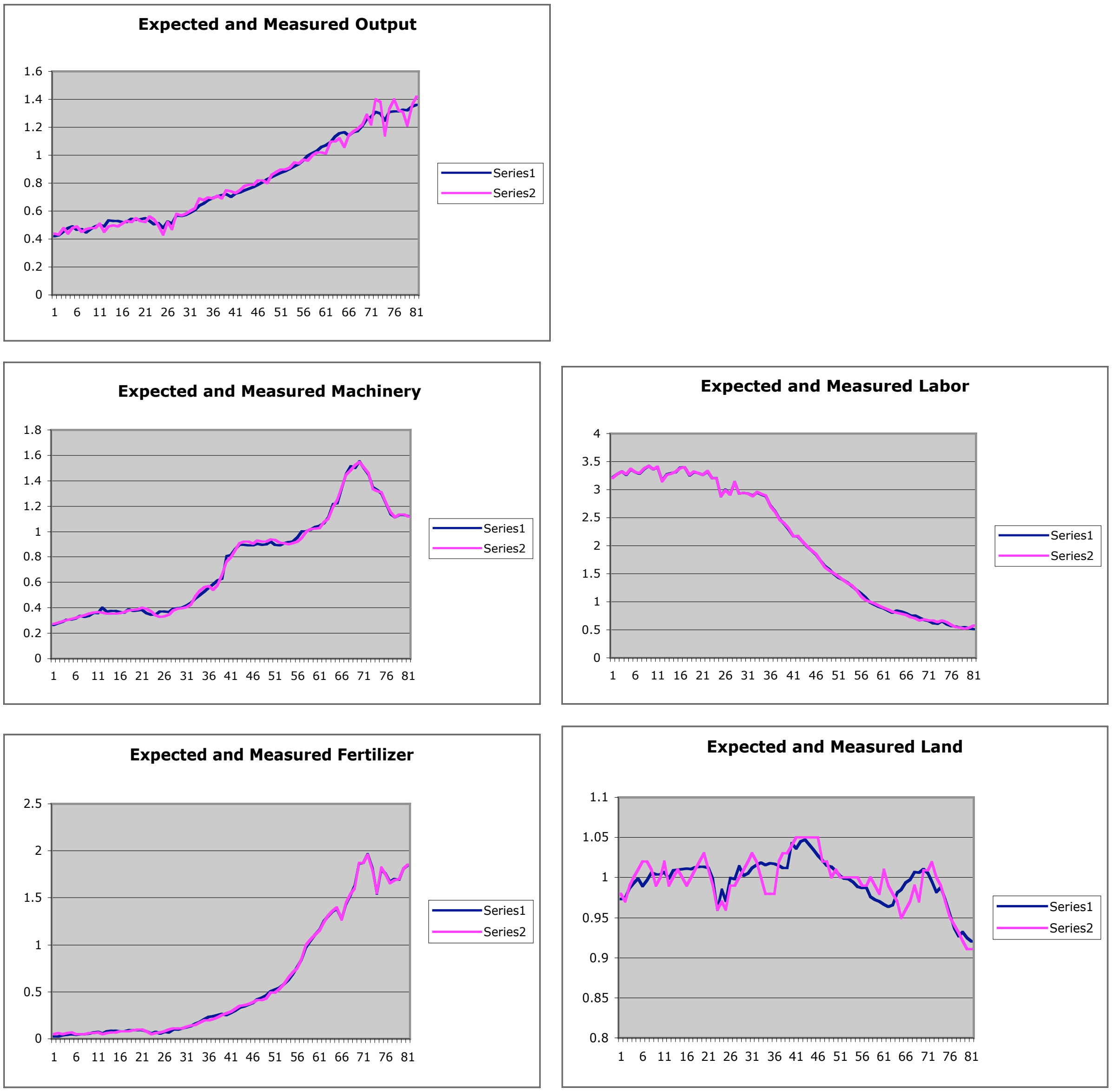
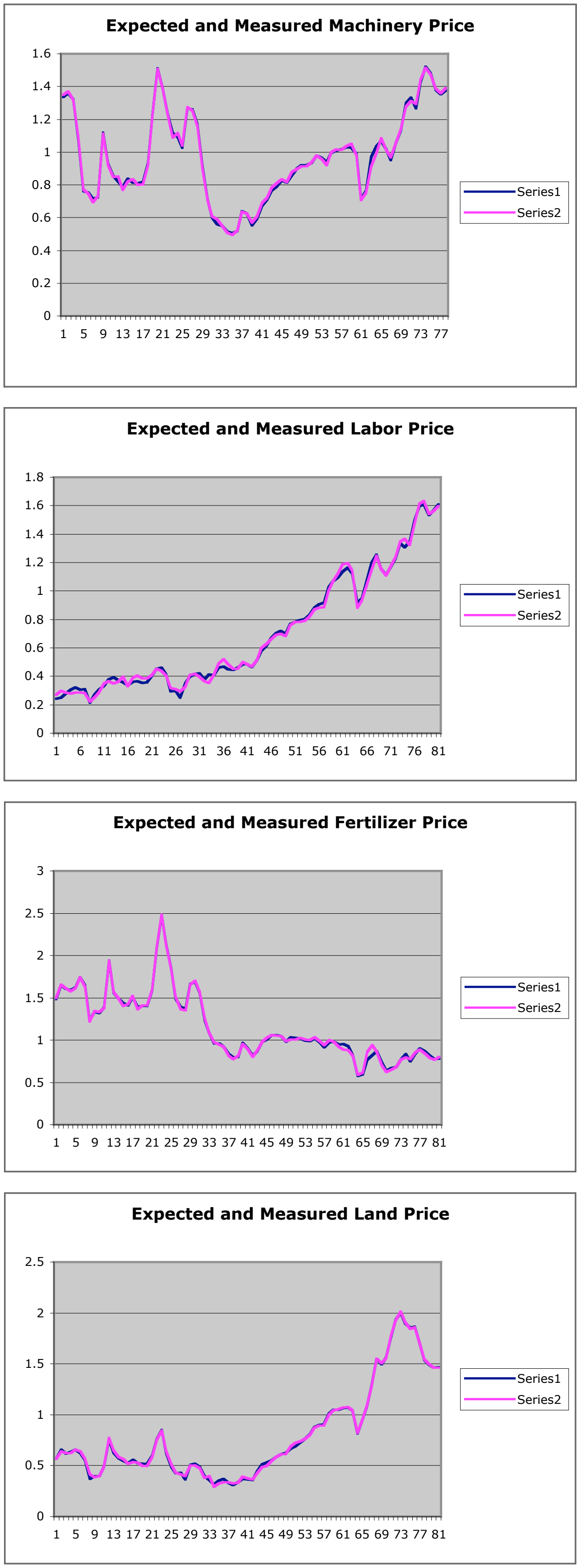


Figure 2. Expected relative prices (Series 1) and measured relative prices (Series 2)



In general, the estimated expected quantities and relative prices track the measured counterparts pretty closely. An exception is represented by the land input quantity index that has fluctuated around the value of 1---in a suspicious saw-tooth pattern---during the sample period.

Another synthetic view of the phase I results can be gleaned by the trend of the expected and measured input shares as reported in figure 3. Overall, the estimated expected series track the measured series rather closely.

Figure 3. Expected (Series 1) and measured (Series 2) cost shares of inputs

With the estimates of the expected quantities and relative prices from phase I, a NSUR model such as described by equations (20)-(23) was estimated using the NL option of the SHAZAM package. Unfortunately, this SHAZAM option does not allow the imposition of parameter constraints that cannot be directly incorporated into the definition of the various equations. Hence, we were not able to test the negative definiteness of the **S** matrix using the SHAZAM package. An autocorrelation scheme of order 1 was implemented in this phase II of the estimation procedure.

In order to gauge the validity of the PITP model, a translog model of the traditional theory (without prices in the production function) was estimated using the same primal-dual approach and using the same estimated expected relative quantities and prices. This traditional model, therefore, is nested into the PITP model and the difference in the level of the two log-likelihood functions could determine whether or not the PITP conjecture ought to be rejected. The PITP model has 89 parameters versus 55 of the traditional model. The results of this comparison are reported in Table 1.

Table 1. Results of the PITP and the traditional translog models, phase II, version 1

Parameter	**PITP translog model, vers. 1**		**Traditional primal-dual model**	
LogLikelihood	1840.040		1517.200	
	Coeff. Value	T-Ratio	Coeff. Value	T-Ratio
Production function				
α_0	-542.75	-1.2305	7.14E-02	0.13204
α_1	78.380	2.5480	1.17400	2.7579
α_2	85.273	2.4471	0.64269	1.7173
α_3	66.067	2.6086	0.88738	2.8188
α_4	75.710	2.3662	1.00920	2.4462
γ_1	153.77	2.4168		
γ_2	-24.992	-0.68515		
γ_3	-9.4141	-0.41724		
γ_4	-86.486	-2.5287		
β_{11}	47.893	2.1649	1.32E-02	0.11344
β_{12}	-20.078	-2.4408	-0.23754	-2.2158
β_{13}	-6.1852	-1.7870	6.83E-02	2.0505
β_{14}	-21.568	-0.40212	2.56150	2.5078
β_{22}	70.235	2.4722	5.30E-02	0.31294
β_{23}	-18.511	-2.4790	-8.28E-02	-1.4088

β_{24}	-41.546	-2.2446	1.4473	2.3793
β_{33}	19.105	2.5155	0.19171	2.4714
β_{34}	-9.6735	-0.92436	-0.38552	-1.9118
β_{44}	100.42	0.35775	-16.860	-2.4737
δ_{11}	1.4899	4.49E-02		
δ_{12}	-29.164	-1.2557		
δ_{13}	-15.443	-0.54165		
δ_{14}	61.155	2.6697		
δ_{22}	12.281	0.57925		
δ_{23}	70.958	1.7158		
δ_{24}	-18.728	-2.1144		
δ_{33}	-13.472	-0.51132		
δ_{34}	-2.5234	-0.1991		
δ_{44}	-35.332	-2.2147		
η_{11}	41.761	2.3601		
η_{12}	-25.255	-2.1036		
η_{13}	-3.6551	-0.9069		
η_{14}	-7.5685	-1.5569		
η_{21}	-14.651	-1.6328		
η_{22}	61.975	2.5563		
η_{23}	-31.937	-2.3454		
η_{24}	3.9305	1.2869		
η_{31}	-7.4586	-2.1431		
η_{32}	-6.8758	-2.1197		
η_{33}	19.662	2.4092		
η_{34}	-1.648	-0.92095		
η_{41}	-24.018	-1.0654		
η_{42}	-33.081	-1.2830		
η_{43}	6.68E-03	4.30E-04		
η_{44}	53.01	1.7911		
θ_T	588.88	1.3124	-0.69034	-1.7026
θ_{TT}	-168.5	-1.4616	-0.15380	-1.5365
α_{T1}	-6.7237	-2.0871	-0.10693	-2.3523
α_{T2}	-10.439	-1.6569	-5.62E-03	-3.23E-02
α_{T3}	-5.6783	-2.2765	-1.76E-02	-1.3936
α_{T4}	-4.9283	-0.90387	-0.10977	-0.91835
γ_{T1}	10.296	1.1246		
γ_{T2}	7.3092	0.95815		
γ_{T3}	-42.574	-2.0586		
γ_{T4}	11.215	2.2295		
Cost function				
ϕ_y	2.76E-03	2.5456	-8.87E-02	-0.93988
φ_{yy}	-1.82E-04	-0.12041	-1.146	-2.2298
φ_{y1}	1.14E-04	0.91699	0.18881	2.9218
φ_{y2}	-3.43E-05	-6.99E-02	0.29347	2.3267
φ_{y3}	7.14E-07	1.41E-03	-0.39884	-2.1151
φ_{y4}	-5.18E-04	-1.9047	0.12406	2.229
ϕ_{Ty}	5.90E-04	2.1537	0.14239	2.3238
ϕ_1	-0.31422	-5.6443	0.10826	2.8702
φ_{11}	0.17457	1.3961	5.74E-02	2.9728
φ_{12}	-5.87E-03	-8.84E-02	-0.10012	-4.9457
φ_{13}	2.81E-02	0.31727	5.37E-02	2.3745
φ_{14}	-0.20204	-3.9016	-1.33E-03	-0.14699
ϕ_{T1}	-4.88E-02	-1.5540	8.43E-02	3.8237
ϕ_2	0.41121	6.3756	0.34966	5.5693
φ_{22}	0.17529	2.0367	-8.82E-02	-2.0802
φ_{23}	-0.30891	-2.7852	0.18173	3.6021
φ_{24}	4.92E-02	1.6575	-9.64E-02	-5.0747
ϕ_{T2}	-5.95E-02	-2.1169	-6.59E-03	-0.35408
ϕ_3	0.32825	4.4649	0.13111	2.1989
φ_{33}	0.15536	1.5236	-0.19042	-2.699
φ_{34}	-6.86E-02	-1.5074	4.07E-02	1.8226
ϕ_{T3}	0.12919	2.9768	4.61E-02	1.5981

ϕ_4	0.38405	9.7842	9.94E-02	3.4525
φ_{44}	0.19859	4.1124	4.88E-02	2.6712
ϕ_{T4}	-1.27E-02	-0.96841	-1.39E-02	-1.8384
Autocorrelation Coefficients				
ρ output	1.00930	183.42	1.02670	97.086
ρ machinery rel price	0.64920	9.3881	0.99020	127.31
ρ labor rel price	0.47907	5.3557	1.02000	137.16
ρ fertilizer rel price	0.81591	6.9074	0.98851	95.595
ρ land rel price	0.79549	10.242	0.96988	30.997
ρ machinery	0.83900	8.0537	0.86879	39.41
ρ labor	0.94764	56.311	0.96718	88.499
ρ fertilizer	0.93131	38.461	0.99036	32.385
ρ land	0.99458	274.13	1.00210	319.42

The difference between the values of the logarithm of the two likelihood functions is equal to 322.84 for a number of restrictions equal to 34. Hence, the likelihood ratio test, which gives a chi-squared variable constructed as twice the difference of the logarithm of the two likelihood functions, is equal to 645.68, well above any imaginable critical value. This preliminary test, therefore, does not reject the null hypothesis that the PITP model is suitable for interpreting 81 years of technical progress in US agriculture.

The relevant test, however, is given by the negative semi-definiteness of the expanded Slutsky matrix **S** defined in equation (29). Three of the four eigenvalues of the **S** matrix corresponding to the estimated PITP model of Table 1 are negative and one is positive (0.9303974 -0.4759490E-01 -0.1932879 -0.3887957) indicating that the matrix is indefinite. We were not able to test (using SHAZAM) whether the PITP model, subject to the restriction that the **S** matrix in equation (29) be negative semi-definite, is rejected by the sample data.

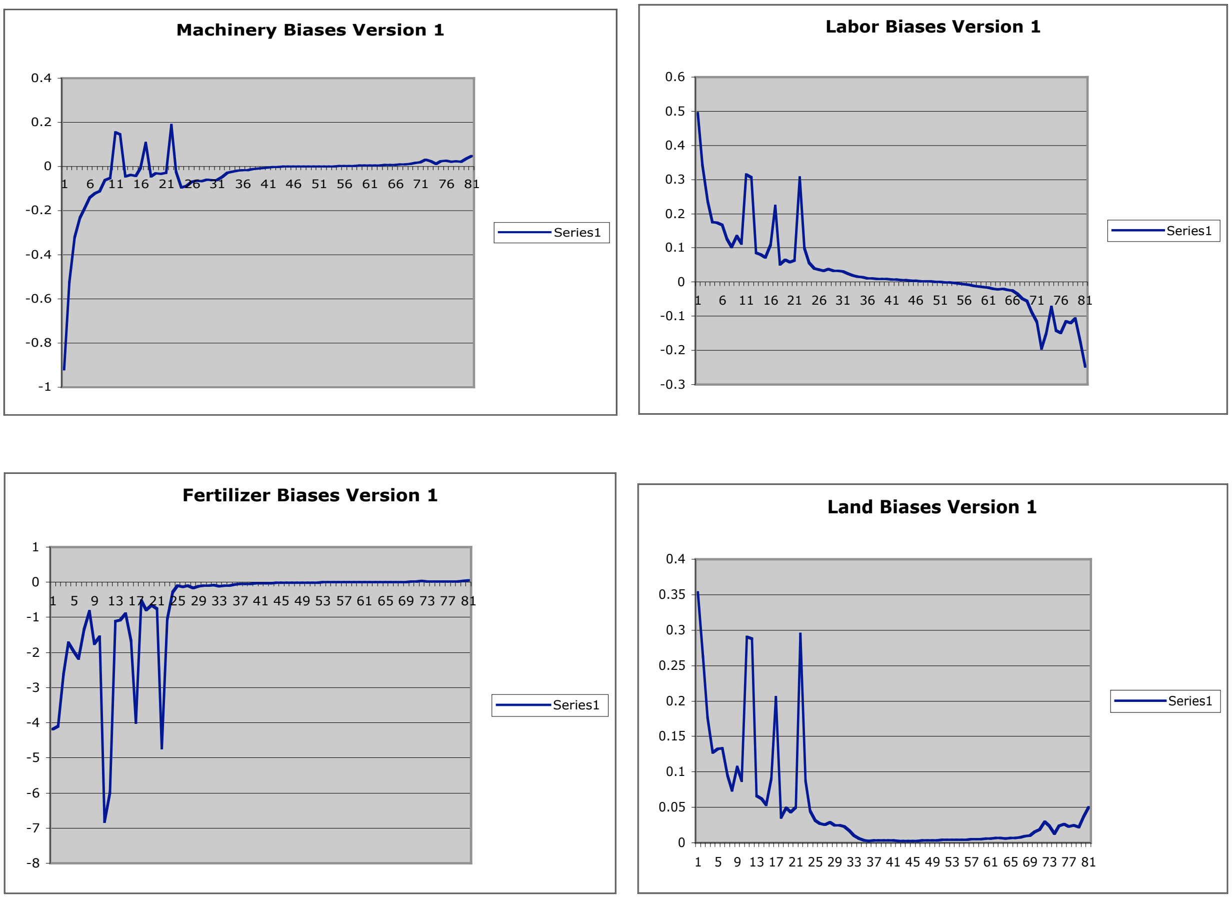
In order to pursue this objective from a different angle, however, we coded the NSUR problem in GAMS achieving a level of the log-likelihood function that is close to, but not exactly equal to the value achieved with the SHAZAM package. This event is undoubtedly due to the highly nonlinear and non-convex problem at hand, and to the different optimization algorithms used in the two packages (or to our programming errors). Another shortcoming of this approach is that we did not compute the standard errors of the estimates, as their programming in GAMS is beyond our limited ability. In any event, the value of the unrestricted concentrated log-likelihood function (as in equation (20)) obtained with GAMS was 1831.820 versus 1840.040 of SHAZAM. The determinant of the **MSR** matrix was computed internally to the maximization program by the LU decomposition, with the determinant defined as the product of the diagonal terms of the U matrix. When the negative semi-definiteness condition of the modified Slutsky matrix given in equation (29) was imposed on the problem (by means of the Cholesky decomposition), the value of the log-likelihood function was 1829.505. A chi-squared test of the negative semidefinite condition, constructed as twice the difference between the values of the two log-likelihood functions (computed in GAMS), gives a measure of 4.630 with 46 degrees of freedom (the parameters of the 5 component matrices of **S**), indicating that the null hypothesis is not rejected even at a very small level of significance. The Cholesky values of the **S** matrix estimated under constraint are (-27.78623 -0.13242 -0.33393 0.00000) and the rank condition is satisfied.

On the strength of this result and of the likelihood ratio test reported above, we will continue the discussion of the empirical results assuming that the PITP model presented in Table 1 was not rejected by the sample data. It is interesting to notice that the conventional **S** matrix for the traditional primal-dual model of Table 1 (represented by the $\mathbf{A}_1$ matrix of equation (29)) is indeed negative definite without imposing such a condition, with eigenvalues (-0.8739866E-02 -0.1921675 -0.3348949 -2.263713). In this case, however, the rank condition is not satisfied.

The biases induced on input quantities by a price-induced technical progress of the type described in this paper were computed according to equation (30) and are reported in Figure 4.

The spikes are due to peculiar combinations of parameters and logarithmic values in the complex formula of equation (30). For example, by changing the level of constant input prices, it is possible to reduce (or increase) those spikes, while maintaining the general pattern of the diagrams. Abstracting from the spikes, the common characteristic of three out of four inputs biases is a trend toward a zero level, with a substantial amount of PITP bias at the beginning of the last century. The bias of machinery input is negative until soon after WWII, indicating an input-saving PITP, and then becomes slightly positive. The bias of the labor input has the opposite trend, remaining an input-using PITP until 1960 for, then, becoming an input-saving PITP. The bias of the fertilizer input was negative prior to 1950, indicating an input-saving TP, and then became slightly positive after that date. The land bias indicates a rapidly diminishing input-using PITP.

Figure 4. Input biases of price induced technical progress



7. Empirical Results of the Translog Model of PITP: Version 2

A second version of the PITP model includes the public and private R&D and extension expenditures as explanatory variables of the portion of inputs attributable to the PITP hypothesis. A synthetic representation of this specification is given in equations (16)-(19). Before reporting on the empirical results, we present the series of public and private R&D and extension expenditures in Figure 5. All three series show a very similar trend, a fact that may lead to multicollinearity and/or to nonsignificant estimates.

As anticipated in a previous section, we took inspiration from the empirical results of Thirtle, Schimmelpfennig and Townsend (2002) who attributed the explanation of the non-substitution portion of their input ratios to a distributed lag specification of relative prices, along with private and public R&D. More accurately, in their machinery/labor factor ratio (equation (5)), they reported that only a series of annual private R&D expenditures was significant, together with the lagged machinery/labor price ratio. In their fertilizer/land factor ratio (equation (6)), the lagged public R&D series was significant. The extension series was reported as being not significant in either factor ratio equation. While the price ratios were specified with a maximum lag of order 2, the private and public R&D series took on lags of 15 and 25 periods, respectively.

Version 2 of the model stated in equations (16)-(19) specifies a lagged relationship between the portions of expected inputs attributed to the PITP hypothesis and expected relative prices, R&D and extension expenditures as explanatory variables. This relationship, then, feeds into the production function and the input price equations in the joint determination of the parameters of interest. In figure 6, we present the decomposition of the estimated expected inputs into their complementary portions attributable to a substitution effect and a PITP effect, as they resulted from version 1.

Figure 5. Public (Series 1) and Private (Series 2) R&D and Extension expenditures

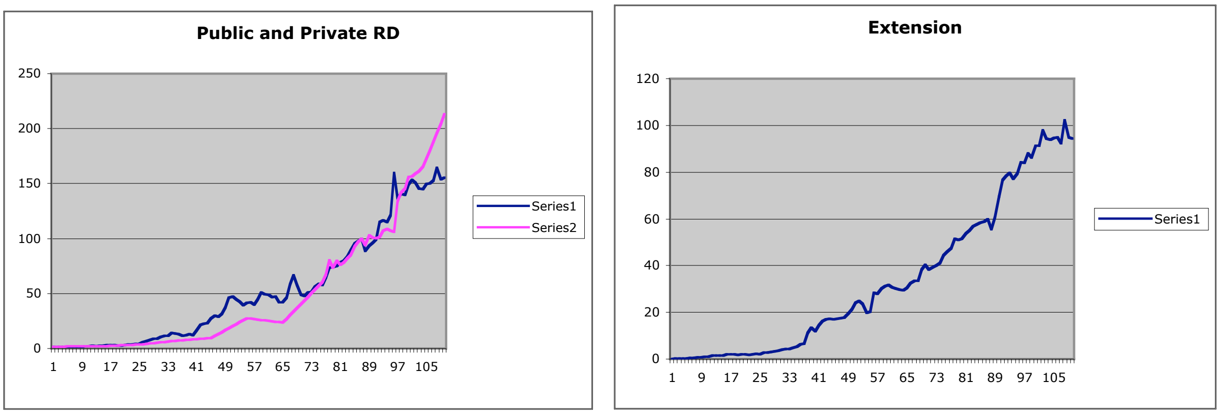
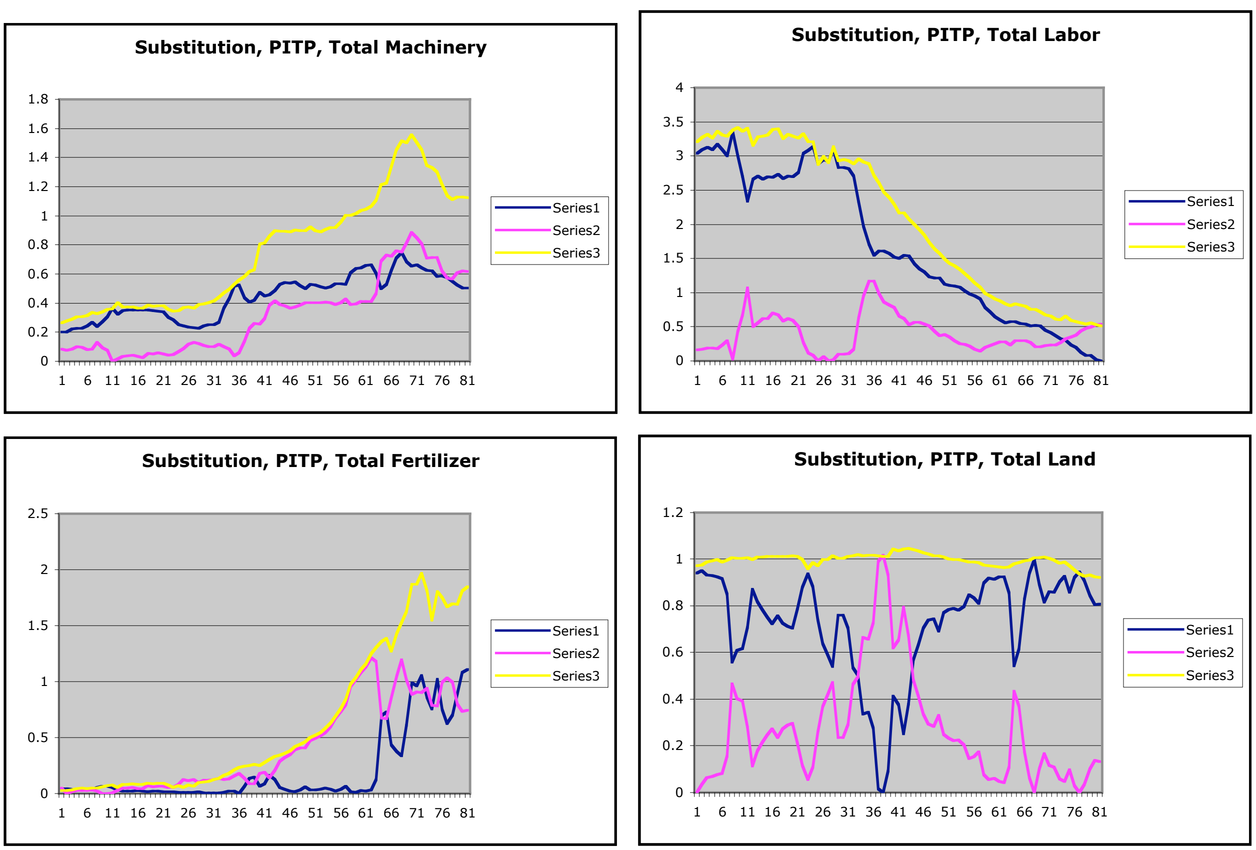


Figure 6. Substitution (Series 1), PITP (Series 2), Total (Series 3) of expected inputs: Model Version 1



The machinery diagram shows almost similar trends of the substitution and PITP components suggesting that, throughout the last century, the machinery substitution effect had about the same strength than the price induced TP effect. The labor diagram, on the contrary, shows that the PITP component of labor was rather minimal throughout the sample period except during the two war periods (including the recession of 1929). The fertilizer diagram indicates that the PITP component is similar to the machinery pattern, with a substantial effect from the early part of the century. Finally, the substitution and the PITP components in the land diagram have an almost mirror-symmetric trend because the total land is roughly constant (around 1), as already pointed out. It is intriguing to notice that the most pronounced substitution effect of the land input took place in a period that begins with world war II (when the structure of agricultural labor had to change in view of the war efforts, as indicated also in the labor diagram) and catches up with the general trend by the middle of the fifties.

At this stage, the problem is to specify the type and the length of the distributed lag series that can plausibly explain the variation of the PITP component of the estimated expected input quantities. As there is no theory that can guide the choice of explanatory variables and the length of their lags, some data mining is inevitable. In Table 2 we present the variables and their lags that were selected in the explanation of the PITP component of the estimated expected input quantities. The information of Table 2 refers to OLS estimates. The symbols for the variables should be read as: Exp = expected, MA = machinery, LB = labor, FR = fertilizer, LA = land, P = price. The lag is indicated explicitly and was restricted to a maximum of 6 periods for the expected input prices and of 7 periods for the R&D and extension variables. These cut-off periods were selected arbitrarily but with the goal of limiting the loss of degrees of freedom in a sample of only 81 observations.

Table 2. Variables and lags for the PITP components of input quantities: version 1

Parameter	**Machinery PITP component**		**Labor PITP component Semi-log regression**		**Fertilizer PITP component**		**Land PITP component Semi-log regression**	
	Coefficient	t-Ratio	Coefficient	t-Ratio	Coefficient	t-Ratio	Coefficient	t-Ratio
Constant	0.1889	2.180	0.1902	5.025	-0.4665	-3.248	0.0784	2.669
ExpMAP-1	0.4735	4.185	-0.7845	-5.873				
ExpMAP-5							-0.2475	-1.852
ExpMAP-6	-0.1042	-2.591	0.1998	2.279			0.3110	2.858
ExpLBP-1	-0.3263	-2.866	0.9706	5.123	1.3677	5.991		
ExpLBP-3	0.4913	4.674	-0.6851	-3.614				
ExpLBP-6	0.5224	4.701	-0.8356	-5.180			-0.5349	-5.033
ExpFRP-1	-0.4498	-5.389			-0.2155	-3.077	-0.2311	-2.359
ExpFRP-4			-0.5227	-3.288	0.3803	4.395	-0.3373	2.714
ExpLAP-1	0.2341	4.863	-0.3188	-2.339	0.4034	3.561	-0.4894	-5.401
ExpLAP-4	-0.2494	-4.963	0.6854	5.457	-0.7548	-5.921	0.2489	2.828
Exten-3	0.5585	3.345						
PriR&D-3					-0.5351	-1.851		
PriR&D-5	-0.6611	-4.460			0.6772	2.205		
PubR&R-4					0.5552	3.055	0.1989	4.978
PubR&D-7					-0.7213	-3.930		
R-square	0.9579		0.7464		0.9175		0.8453	

The machinery and the fertilizer equations, with all variables in natural units, fit the respective PITP components fairly well, with R-square measures of 0.96 and 0.92, respectively. The labor and the land equations, in semi-log specification, fit the respective PITP component less well, with an R-square measure of 0.75 and 0.85, respectively. The *a-p*riori selection of the maximum lags may be responsible, at least in part, for the relatively low fit of these equations. The extension-expenditures variable enters only the machinery equation; no R&D and extension expenditure variables enter the labor equation; both private and public R&D expenditures enter the fertilizer equation jointly.

In spite of the imperfect fit of the PITP equations, the overall information gleaned from the results of Table 2 suggests that a proper combination of lagged expected prices, R&D and extension expenditures may indeed explain (may be thought of as determinant of) the PITP components of the input quantities. We reproduce here equations (16)-(19) for ease of reference in the phase I estimation process of the PITP model that assumes the following structure:

$$\min_{\beta, y_t^e, x_{tj}^e, w_{tj}^e, \mathbf{e}_t} \sum_{t=1}^{T} \varepsilon_{0t}^2 + \sum_{j=1}^{J}\sum_{t=1}^{T} \nu_{tj}^2 + \sum_{j=1}^{J}\sum_{t=1}^{T} \varepsilon_{tj}^2 + \sum_{j=1}^{J}\sum_{t=1}^{T} u_{tj}^2 \tag{15'}$$

subject to

$$y_t = f^e\Big((\mathbf{x}_{St}^e + \mathbf{x}_{PIt}^e)), \mathbf{w}_t^e, t\Big) + \varepsilon_{0t} \tag{16'}$$

$$\mathbf{w}_t = c_{y^e}^e(y_t^e, \mathbf{w}_t^e, t)\mathbf{f}_{\mathbf{x}^e}^e\Big((\mathbf{x}_{St}^e + \mathbf{x}_{PIt}^e), \mathbf{w}_t^e, t\Big) + \boldsymbol{\nu}_t \tag{17'}$$

$$\mathbf{x}_t = \mathbf{h}^e(y_t^e, \mathbf{w}_t^e, t) + \boldsymbol{\varepsilon}_t = \mathbf{x}_{St}^e + \mathbf{x}_{PIt}^e + \boldsymbol{\varepsilon}_t \tag{18'}$$

$$\mathbf{x}_{PIt}^e = \mathbf{g}(lag\mathbf{w}_t^e, lagRD_t, lagExt_t) + \mathbf{u}_t. \tag{19'}$$

The translog specification of equations (16')-(18') is similar to equations (25), (27) and (28) except that the logarithm of the expected input quantities in equations (25) and (27) must now be defined by the two complementary components of the input quantities. Equation (19') expresses the lagged relation between the portion of the input quantities that is attributed to the price induced technical progress and a series of relative prices, R&D and extension expenditures. The structure of the lagged relations follows the pattern of Table 2.

After estimating the phase I specification of the PITP model (version2), the NSUR phase II model was estimated using Shazam. The results are reported in Table 3. A significant autocorrelation coefficient is present in every equation.

Table 3. Results of the PITP and the traditional translog models, phase II, version 2

Parameter	**PITP translog model, vers. 2**		**Traditional primal-dual model**	
LogLikelihood	1829.693		1625.289	
	Coeff. Value	T-Ratio	Coeff. Value	T-Ratio
Production function				
α_0	-48.4660	-3.094	-0.6532	-11.087
α_1	2.4217	9.041	0.1640	6.585
α_2	1.0737	1.321	0.1084	2.156
α_3	2.4969	12.608	0.1228	7.163
α_4	2.3115	4.199	0.2112	5.557

γ_1	8.0440	1.836		
γ_2	-4.2788	-1.227		
γ_3	9.2644	4.043		
γ_4	6.4685	2.932		
β_{11}	1.4991	6.724	0.0279	2.039
β_{12}	-0.7519	-5.679	-0.0371	-3.598
β_{13}	0.0884	-1.554	-0.0099	-1.828
β_{22}	2.1115	4.784	0.0202	1.166
β_{23}	-0.3959	-4.388	-0.0208	-3.769
β_{24}	1.0249	1.747	0.0472	1.923
β_{33}	0.6655	8.845	0.0274	6.369
β_{34}	0.0897	0.324	-0.0115	-0.713
β_{44}	-2.6101	-0.629	-0.3332	-1.316
δ_{11}	14.0650	6.525		
δ_{12}	-0.0761	-0.071		
δ_{13}	-3.8508	-3.429		
δ_{14}	-6.9204	-6.459		
δ_{22}	3.5488	2.071		
δ_{23}	0.7908	0.969		
δ_{24}	0.9697	0.829		
δ_{33}	-2.0441	-1.718		
δ_{34}	4.9415	5.031		
δ_{44}	-0.0555	-0.047		
η_{11}	2.6353	9.287		
η_{12}	-1.1998	-7.152		
η_{13}	-0.1574	-1.645		
η_{14}	-0.4741	-3.820		
η_{21}	-0.1488	-0.453		
η_{22}	1.3583	4.708		
η_{23}	-0.2110	-0.970		
η_{24}	-0.7076	-4.867		
η_{31}	-0.0551	-0.748		
η_{32}	0.0325	0.237		
η_{33}	0.6271	7.458		
η_{34}	-0.3476	-3.467		
η_{41}	0.5633	1.792		
η_{42}	-1.2737	-2.292		
η_{43}	-0.4185	-1.138		
η_{44}	1.8762	9.100		
θ_T	5.8207	1.953	0.0313	0.484
θ_{TT}	0.1835	0.162	0.1943	7.607
α_{T1}	0.1556	2.933	-0.0079	-0.967
α_{T2}	1.0885	2.381	-0.0051	-0.184
α_{T3}	-0.0519	-1.909	-0.0019	-0.762
α_{T4}	0.4060	1.216	-0.0358	-2.088
γ_{T1}	-1.9502	-1.185		
γ_{T2}	0.2379	0.298		
γ_{T3}	-1.5531	-1.244		
γ_{T4}	-3.6993	-3.451		
Cost function				
ϕ_y	0.0981	17.292	1.8194	6.600
φ_{yy}	0.0274	4.448	1.3306	4.240
φ_{y1}	-0.0162	-5.590	0.0861	1.549
φ_{y2}	0.0143	6.208	-0.2577	-3.768
φ_{y3}	0.0058	2.827	0.4450	7.300
φ_{y4}	-0.0034	-1.972	0.0022	0.056
ϕ_{Ty}	-0.0089	-5.567	-0.0529	-0.635
ϕ_1	-0.7028	-1.967	0.1525	2.598
φ_{11}	-0.9831	-4.745	0.1009	3.518
φ_{12}	-0.1544	-1.543	-0.0511	-3.191
φ_{13}	0.2966	3.076	-0.0376	-1.861
φ_{14}	0.5641	5.538	0.0123	1.083
ϕ_{T1}	0.2106	1.563	0.0071	0.249

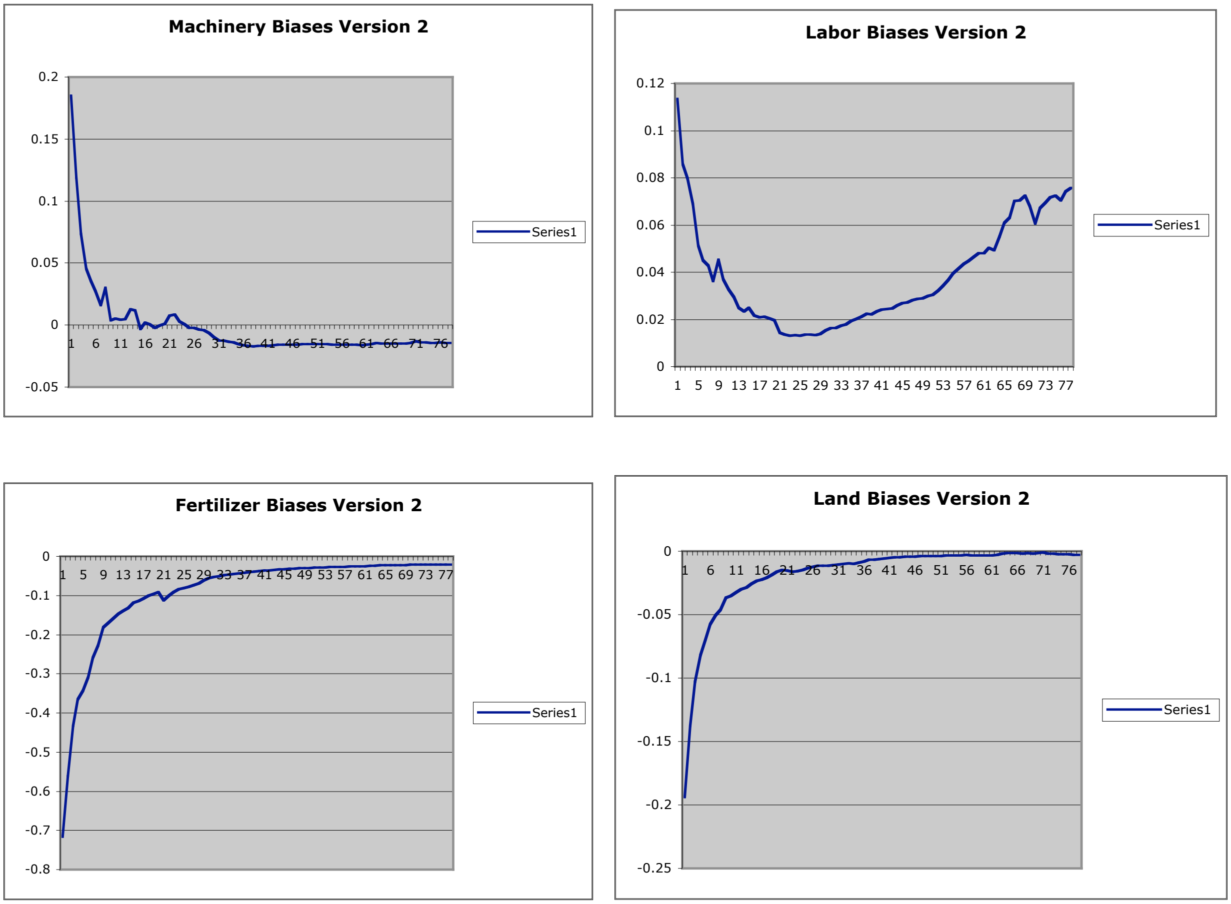
ϕ_2	0.6067	2.129	0.1493	4.797
φ_{22}	-0.1734	-1.070	-0.1006	-3.883
φ_{23}	0.0150	0.228	0.0516	2.937
φ_{24}	-0.1347	-1.373	-0.0258	-1.747
ϕ_{T2}	-0.0001	-0.001	0.0847	6.801
ϕ_3	-0.4769	-2.515	0.3025	5.202
φ_{33}	0.2024	2.086	0.0221	0.733
φ_{34}	-0.4510	-5.126	-0.0207	-1.497
ϕ_{T3}	0.0881	1.039	-0.0899	-3.169
ϕ_4	-0.3912	-2.319	-0.0573	-1.208
φ_{44}	0.0947	1.097	0.0098	0.644
ϕ_{T4}	0.2833	3.528	0.0154	1.333
Autocorrelation Coefficients				
ρ output	1.0112	141.200	0.2805	3.974
ρ machinery rel price	0.9791	90.234	0.8647	21.293
ρ labor rel price	0.9544	29.722	0.9985	64.527
ρ fertilizer rel price	0.9478	22.609	0.8908	15.373
ρ land rel price	0.8518	15.858	0.8308	20.609
ρ machinery	0.9973	259.550	0.9920	84.621
ρ labor	0.9508	28.877	0.9680	42.762
ρ fertilizer	0.4719	5.804	0.9987	59.736
ρ land	0.9941	253.310	0.9980	512.200

A traditional model (without prices in the production function) was also estimated and reported in Table 3. The difference between the logarithmic value of the two likelihood functions is equal to 204.404, which translates into a likelihood ratio test of 408.808, well above any imaginable critical value for a chi-square statistics with 34 degrees of freedom. This preliminary test, therefore, does not reject the hypothesis that a price induced technical progress prevailed during 80 years of US agriculture.

As for the previous version 1 of the PITP model, we used the GAMS package to impose the comparative statics condition represented by equation (29). The implementation of the NSUR program gives a value of the unrestricted and concentrated log-likelihood function equal to 1864.147, while the restricted value is equal to 1858.630. The likelihood ratio test corresponds to a chi-square variable of 11.034 for 46 degrees of freedom, well below the critical value for any reasonable significance level. The Cholesky values of the constrained model are equal to (-29.96039 -0.28846 -0.29598 0.00000) indicating that the extended Slutsky matrix **S** of equation (29) is negative semi-definite and satisfies the rank condition of theorem 1. Hence, the PITP hypothesis is not rejected also in version 2 of the model. We notice that, in this case, the value of the log-likelihood function obtained using the GAMS program is higher than the one computed by SHAZAM. Again, this event may be due to the highly non-convex and nonlinear model and to the different algorithms used by the two programming packages.

With the results of Table 3, the input biases were measured using equation (30). The results are reported in figure 7. The machinery biases indicate a factor-using TP prior to 1935 and then a constant level of factor-saving TP for the rest of the sample period. The labor biases exhibit a factor-using TP that decreases until WWII and then increases steadily for the rest of the sample period. The fertilizer biases show a factor-saving TP for the entire period. The land biases are factor-saving prior to world war II and the hover around a zero bias for the rest of the period. The different trends of input biases in the two sets of diagrams (Figure 4 and Figure 7) reveal the heavy dependence of these measures upon the estimated coefficients. Both patterns, however, are consistent with our PITP theory.

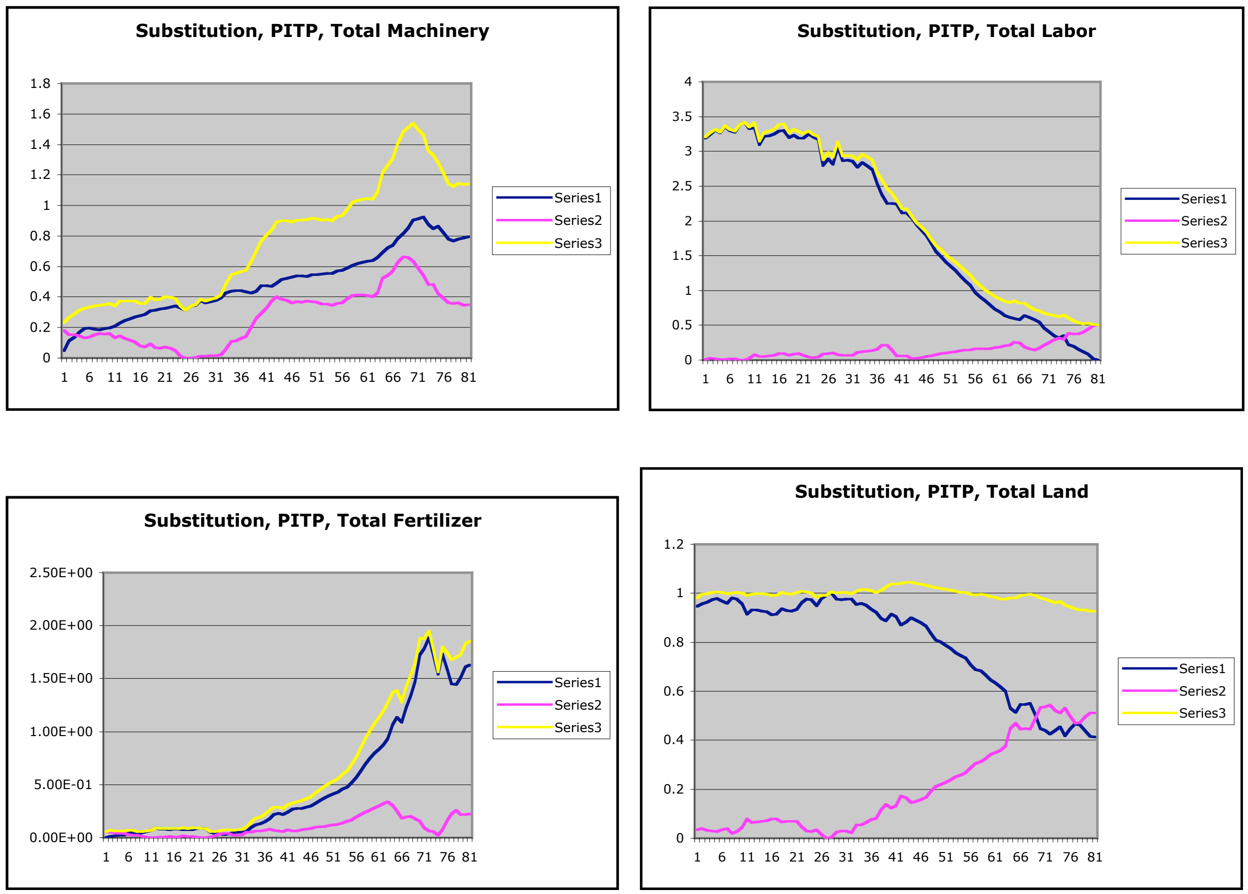
Figure 7. Input biases of price induced technical progress, version 2



A final aspect of the empirical results associated with version 2 of the PITP model deals with the pattern of substitution and PITP components of the input quantities obtained from the application of the extended Shephard lemma as reported in Figure 8. The machinery decomposition is similar to that one of Figure 6. The labor PITP component in Figure 8 is flatter than in Figure 6 but has a similarly rising end portion. The fertilizer PITP component of Figure 8 exhibits a much smaller size than its counterpart in Figure 6. A radical difference lies with the PITP components in the land input of the two Figures. In Figure 8, the PITP component of the land input remains insignificant until the world war II years and then rises steadily until the year

1980. The substitution component has a mirror pattern. Intuitively, the pattern of the land input in Figure 8 is more plausible than the one in Figure 6.

Figure 8. Substitution (Series 1), PITP (Series 2), Total (Series 3) of expected inputs: Model Version 2



There remain to comment upon the distributed lag relationships that explain the PITP components of the various inputs. We recall that the estimation of the version 2 model was carried out according to problem (15')-(19') with the distributed lag pattern for the various explanatory variables as indicated in Table 2. The four equations and their distributed lag pattern expressed a rather high level of fit with R-square measures of 0.90, 0.83, 0.95 and 0.98, respectively. It is apparent, however, that several alternative combinations of lags can achieve high levels of fit. In Table 4, therefore, we report a more refined exploration of fit that reveals a different pattern of distributed lags. Now, all the four relationships exhibit a high measure of fit, as indicated by the R-square, while maintaining a parsimonious specification (in terms of lags). We note that in Table 4 the labor equation now contains significant lags of the extension and public R&D explanatory variables. The extension expenditures and the public R&D variables enter every equation. The private R&D expenditure enters only the fertilizer equation. It is difficult to attach any intuitive meaning to the individual coefficients and we refrain from it.

Table 4. Variables and lags for the PITP components of input quantities: version 2

Parameter	**Machinery PITP component Semi-log regression**		**Labor PITP component Semi-log regression**		**Fertilizer PITP component Natural units**		**Land PITP component Semi-log regression**	
	Coefficient	t-Ratio	Coefficient	t-Ratio	Coefficient	t-Ratio	Coefficient	t-Ratio
Constant	0.2424	14.72	0.3163	27.84	-0.0287	-1.702	0.3605	48.54
ExpMAP-3			-0.1745	-4.914	0.0296	2.447		
ExpMAP-5			-0.0870	-2.459				
ExpMAP-6	-0.2970	-11.59	0.4218	10.14				
ExpLBP-1					0.7523	24.55	-0.1245	-3.796
ExpLBP-3			0.3559	6.397				
ExpLBP-4							0.2295	6.508
ExpLBP-6	0.4263	10.36	-0.6321	-9.835				
ExpFRP-1	-0.3525	-13.81			-0.0629	-5.718	-0.1179	-4.620
ExpFRP-4							-0.1457	-5.519
ExpFRP-6			-0.0913	-2.874				
ExpLAP-1	0.1331	5.369			-0.2203	-15.86	0.1532	11.46
ExpLAP-4	-0.2079	-9.551	0.0688	2.226				
ExpLAP-6			0.2037	5.668				
Exten-3	-0.1023	-3.052					0.0241	1.959
Exten-4			0.1565	5.857				
Exten-7	-0.2025	-6.203	0.0985	3.511	-0.2782	-4.050		
PriR&D-3					-0.2841	-8.403		
PubR&R-3	0.1551	4.766	-0.1089	-4.094				
PubR&R-4					0.1525	4.431	-0.0433	-3.440
R-square	0.9689		0.9578		0.9622		0.9908	

On the basis on the results of Table 4, it is tempting to examine a third version of the PITP model where the distributed lag specification of the PITP input components in phase I is represented by the structure revealed in Table 4. Such an exploration could shed some light upon the stability of the input biases and the decomposition of the input quantities into their substitution and PITP components.

8. Empirical Results of the Translog Model of PITP: Version 3

Table 5 exhibits the empirical results of phase II estimation of the PITP model with a structure of lags as defined in Table 4. The chi-square variable defined as twice the difference between the values of the two log-likelihood functions in Table 5 is equal to 371.600, with 34 degrees of freedom. Once, again, the null hypothesis of a traditional TP model (without prices in the production function) is soundly rejected.

Table 5. Results of the PITP and the traditional translog models, phase II, version 3

Parameter	**PITP translog model, vers. 3**		**Traditional primal-dual model**	
LogLikelihood	1842.164		1656.364	
	Coeff. Value	T-Ratio	Coeff. Value	T-Ratio
Production function				
α_0	-238.38	-1.9259	-0.64293	-10.346
α_1	-2.6941	-2.2693	0.15666	4.941
α_2	-1.2387	-1.2102	0.05856	1.385
α_3	-2.1895	-2.2606	0.09353	4.721
α_4	-3.6955	-2.1654	0.20699	4.205
γ_1	-0.5217	-0.1519		
γ_2	1.8853	0.6480		
γ_3	-4.6907	-1.8148		
γ_4	-2.8925	-1.8454		
β_{11}	-1.3012	-2.2030	0.03367	4.632
β_{12}	0.4325	1.8136	-0.0076	-1.321
β_{13}	0.1490	1.5956	0.00068	0.356
β_{14}	0.0641	0.2144	-0.01599	-0.988
β_{22}	-3.3801	-2.1345	0.04938	2.572
β_{23}	0.2211	1.1372	-0.01386	-2.612
β_{24}	6.2815	1.8504	-0.29913	-3.416
β_{33}	-0.8720	-2.1845	0.01989	3.694
β_{34}	3.0730	1.9108	-0.10367	-3.016
β_{44}	-26.10	-1.4999	2.1069	3.257
δ_{11}	-3.3219	-1.4261		
δ_{12}	1.4447	1.1960		
δ_{13}	-1.2807	-1.1103		
δ_{14}	0.1418	0.1550		

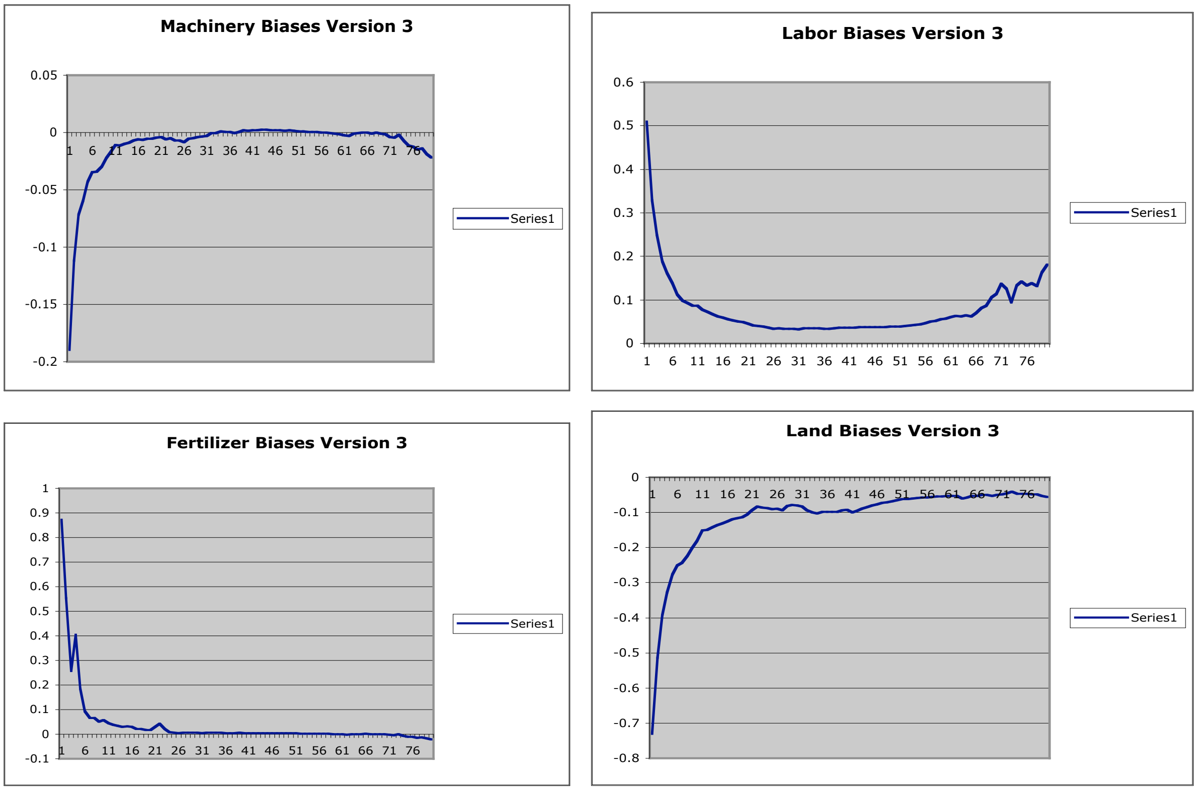
δ_{22}	7.0182	2.5399		
δ_{23}	0.4197	0.5142		
δ_{24}	-3.7315	-2.6597		
δ_{33}	1.2127	0.9158		
δ_{34}	-2.1055	-2.0518		
δ_{44}	3.0563	2.4887		
η_{11}	-1.7923	-2.1750		
η_{12}	0.7285	1.9601		
η_{13}	0.3505	1.8822		
η_{14}	0.1840	1.3521		
η_{21}	-0.6219	-1.7203		
η_{22}	-1.6617	-2.1683		
η_{23}	1.3989	2.1641		
η_{24}	0.3685	1.9939		
η_{31}	-0.0495	-0.8547		
η_{32}	0.5159	1.8983		
η_{33}	-0.5595	-2.2044		
η_{34}	0.0088	0.1350		
η_{41}	0.7146	1.4700		
η_{42}	1.5437	1.8620		
η_{43}	0.6179	1.6974		
η_{44}	-1.9715	-2.1553		
θ_T	232.78	1.8658	-0.03587	-0.611
θ_{TT}	-57.75	-1.8340	0.22912	10.248
α_{T1}	0.0149	0.1170	-0.01835	-3.330
α_{T2}	-0.6679	-1.2151	0.00938	0.404
α_{T3}	-0.0104	-0.2841	0.00055	0.457
α_{T4}	0.6818	1.5639	-0.04385	-2.713
γ_{T1}	0.2569	0.4151		
γ_{T2}	-0.5503	-0.8616		
γ_{T3}	-0.7168	-0.8528		
γ_{T4}	0.9091	1.8341		
Cost function				
ϕ_y	-0.0905	-2.2962	2.1682	5.269
φ_{yy}	0.0036	0.6723	1.1837	4.372
φ_{y1}	0.0199	2.4076	0.06244	1.319
φ_{y2}	-0.0157	-2.4529	-0.36456	-6.216
φ_{y3}	-0.0059	-1.9718	0.45227	7.581
φ_{y4}	0.0058	2.2783	-0.05976	-1.847
ϕ_{Ty}	-0.0031	-1.3014	0.0479	0.606
ϕ_1	-0.0035	-0.0118	2.1682	5.269
φ_{11}	-0.3632	-1.9686	0.19748	4.667
φ_{12}	0.1768	1.7129	0.01674	0.763
φ_{13}	-0.1094	-1.1333	0.00664	0.477
φ_{14}	0.0408	0.4935	0.01943	1.103
ϕ_{T1}	0.0134	0.3213	0.00952	1.074
ϕ_2	0.4542	1.7170	0.11056	4.982
φ_{22}	0.7065	3.4064	-0.10965	-4.209
φ_{23}	-0.0463	-0.6400	-0.00134	-0.084
φ_{24}	-0.3910	-3.8186	-0.04368	-3.591
ϕ_{T2}	-0.0635	-1.1757	0.09403	7.718
ϕ_3	-0.2414	-1.4041	0.4447	5.787
φ_{33}	0.1760	1.4360	0.01909	1.012
φ_{34}	-0.1924	-3.2749	-0.02893	-3.340
ϕ_{T3}	-0.0570	-0.8290	-0.14109	-3.267
ϕ_4	-0.2637	-2.5160	-0.023	-0.552
φ_{44}	0.2934	2.9785	0.0445	3.363
ϕ_{T4}	0.0810	3.0058	0.01878	1.994
Autocorrelation Coefficients				
ρ output	1.0060	163.78	0.3295	8.373
ρ machinery rel price	0.8483	18.2900	0.97418	51.213

ρ labor rel price	0.9296	22.9460	0.99764	60.939
ρ fertilizer rel price	0.8872	19.2760	0.83179	26.906
ρ land rel price	0.9598	41.4410	0.84079	25.791
ρ machinery	1.0052	368.350	0.85737	26.723
ρ labor	0.6978	8.2933	0.76469	15.371
ρ fertilizer	0.9990	445.02	0.99974	487.250
ρ land	0.9941	253.310	0.9980	512.200

We recall that this third version of the model was performed with the objective of evaluating the robustness of the input biases to a variation in the lag distribution. The input biases corresponding to the empirical results of Table 5 are presented in Figure 9. The pattern of the labor and land diagrams is substantially similar to the pattern presented in Figure 7. The land input is clearly characterized by a factor-saving TP throughout the sample period. Labor remains a factor-using input. This counterintuitive result is mitigated by our previous discussion about the difficulty of assigning a clear meaning of input-using (input-saving). The machinery and fertilizer input diagrams of Figure 9 exhibit trends which are opposite to those in Figure 7. Now, the machinery bias is factor-saving until WWII, becomes factor-using until 1980, and then returns to be factor-saving. The fertilizer bias is factor-using until WWII and then hovers around a zero bias for the rest of the sample period.

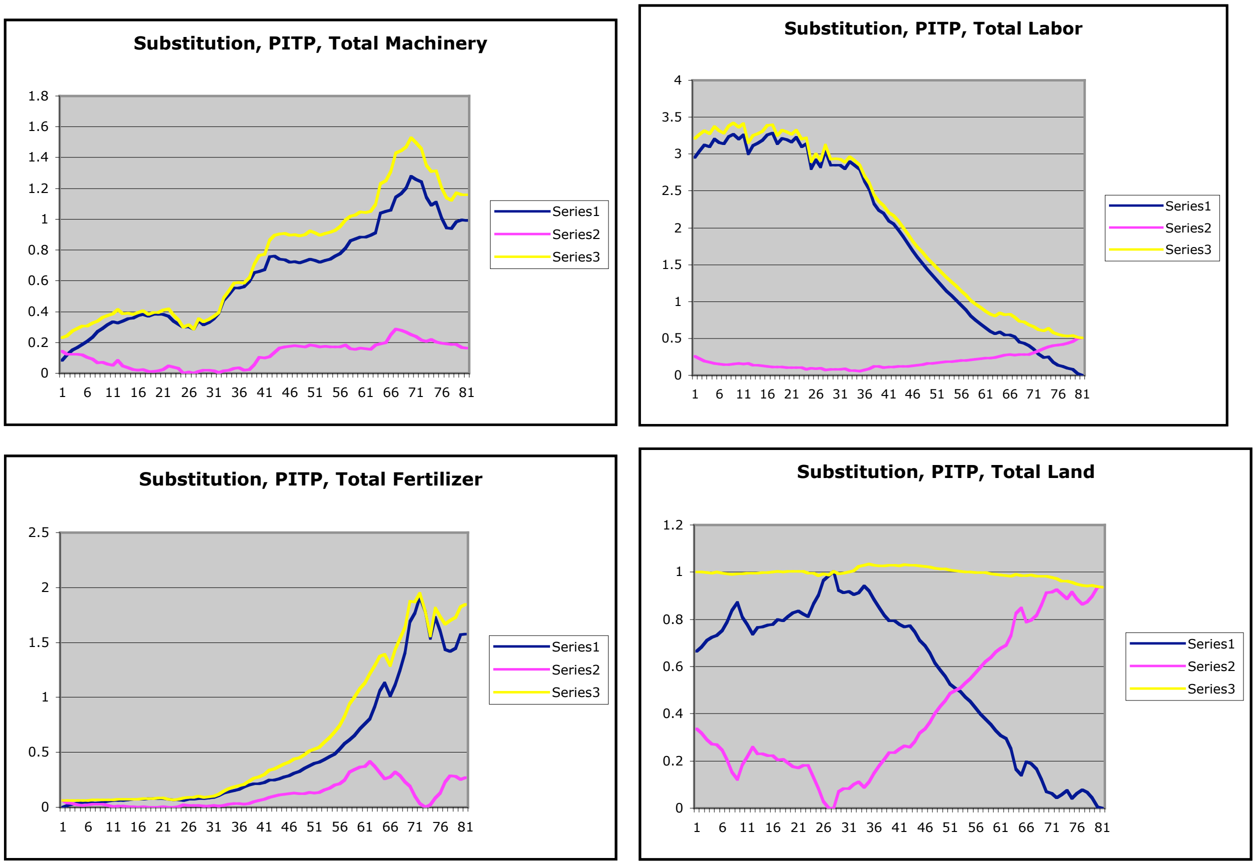
These empirical explorations suggest that the biases of TP are very sensitive to the model specification and the values of the estimated parameters. This conclusion reduces the importance of the notion of input bias in evaluating technical progress, since all the patterns of biases exhibited in Figures 4, 7 and 9 are admissible under our PITP theory. Without a formal test of a null-hypothesis pattern of input bias, it is exceedingly difficult to make sense of any pattern, merely on the basis of “intuition.”

Figure 9. Input biases of price induced technical progress, version 3



The input decomposition for version 3 of the model is given in Figure 10. Although the pattern of decomposition is roughly similar to the pattern depicted in Figure 8, we must point to the quantitative aspect of machinery and land decomposition. The PITP machinery component acquires a substantial magnitude after World War II in both pictures, but its level is halved in Figure 10. The PITP land component in Figure 10 exhibits a trend that exhausts the entire amount of input by the end of the sample period. The land input, with its low variability, may admit many alternative patterns of decomposition.

Figure 10. Substitution (Series 1), PITP (Series 2), Total (Series 3) of expected inputs: Model Version 3



Conclusion

The essential points of the paper can be listed as follows: A) a novel theory of technical progress, complete of its comparative statics conditions, that re-interprets the relative price hypothesis of Hicks; B) within this theory, an extended Shephard lemma that provides a natural decomposition of the input quantities between a purely substitution component and a complementary amount attributable to the price-induced conjecture; C) an empirical application of the theory that requires a primal-dual approach to the corresponding econometric specification because of the necessity to estimate both the production function and the cost function jointly.

The data dealt with in this paper involve a sample of 81 years of US agriculture with one aggregate output and four inputs, machinery, labor, fertilizer and land. Furthermore, private and public R&D series and extension expenditures were available. The sample data analyzed in this paper constitute an unusual amount of information with prices and quantities for every commodity. We attempted to utilize all the available information because this condition is a fundamental requirement toward achieving efficient estimates.

Three versions of the general model were formulated using a translog specification for both the production and its associated cost function. The first version dealt exclusively with expected relative prices and the results indicated that the conjecture of price-induced technical progress could not be rejected based upon a test of the comparative statics conditions that characterize our PITP theory. The analysis of the input biases associated with this version shows that three of the four inputs have minimal biases at the end of the sample period. Only the labor input exhibits a significant level of bias at that point. This version of the PITP model allowed a preliminary analysis of the conjecture that a distributed lag of relative prices, R&D and extension expenditure could explain the portion of inputs attributable to technical progress in the extended Shephard decomposition.

A second version of the model incorporated lagged R&D private and public expenditures as well as lagged extension expenditures. The lags were suggested by the regression analysis of Table 2 and produced estimates of the PITP model that cannot reject the price-induced hypothesis of expected relative prices entering the production function. The pattern of input biases of version 2 differs from that of version 1 in ways that are both satisfying and against intuition. In either case, however, those patterns do not contradict the necessary and sufficient conditions of our theory.

A third version of the model incorporated the lag structure presented in Table 4 and was carried out mainly to assess the robustness of the input biases to a variation of the lag distribution. With a truly dynamic theory of TP, this *ad-hoc* sensitivity analysis can be avoided. The translog functional form may have a determinant role in the shape of the input biases, but the evaluation of this conjecture is left for another occasion.

Two aspects of this paper should be kept distinct: the PITP theory and its empirical implementation. The theory generalizes many traditional specifications of models dealing with technical progress and provides its own specific comparative statics conditions. The particular implementation of the PITP theory that was executed in this paper is certainly imperfect. Yet, the empirical results have given more than a glimpse of the ability of the primal-dual approach to interpret the available information.

Appendix

Proof of Theorem 1. Because there are J decision variables and one constraint in problem (2), and the classical constraint qualification holds at the optimum due to the fact that $f_{x_j}(\mathbf{x},\mathbf{w},t) > 0$, $j = 1,\ldots,J$, at the optimum, the dimension of the decision space is $J-1$. This implies that any comparative statics matrix derived from problem (2) cannot have a rank greater than $J-1$, since any complete comparative statics characterization of problem (2) cannot contain any more information than that contained in the primal second-order necessary conditions. This fact implies that $\operatorname{rank}\big(\mathbf{S}(\boldsymbol{\alpha})\big) \leq J-1$ for all $\boldsymbol{\alpha} \in B(\boldsymbol{\alpha}^{\circ};\delta)$.

Given the above rank property, we are permitted to fix $t = t^{\circ}$ for the purpose of deriving the qualitative properties of problem (2). We therefore focus on the parameters $(\mathbf{w}, y)$. Consequently, let $\mathbf{x}^{\circ} = \mathbf{h}(\mathbf{w}^{\circ}, y^{\circ}, t^{\circ})$ and suppress $t = t^{\circ}$ from the arguments of the ensuing equations for notational clarity. Then the primal-dual optimization problem associated with problem (2) is defined as

$$0 \overset{\text{def}}{=} \min_{\mathbf{w}}\{\mathbf{w}'\mathbf{x}^{\circ} - C(\mathbf{w}, y) \text{ s.t. } y - f(\mathbf{x}^{\circ};\mathbf{w}) = 0\}. \tag{A.1}$$

Problem (A.1) may be rewritten as an equivalent unconstrained minimization problem by using the constraint to eliminate y from it, thereby yielding

$$0 \overset{\text{def}}{=} \min_{\mathbf{w}}\{\mathbf{w}'\mathbf{x}^{\circ} - C(\mathbf{w}, f(\mathbf{x}^{\circ};\mathbf{w}))\}. \tag{A.2}$$

The necessary conditions, which hold at $\mathbf{w}^{\circ}$ by construction of problem (2), are given by

$$(\mathbf{x}^{\circ}) - C_{\mathbf{w}}\big(\mathbf{w}, f(\mathbf{x}^{\circ};\mathbf{w})\big) - C_{y}\big(\mathbf{w}, f(\mathbf{x}^{\circ};\mathbf{w})\big) f_{\mathbf{w}}(\mathbf{x}^{\circ};\mathbf{w}) = \mathbf{0}_J, \tag{A.3}$$

$$\begin{aligned}
&\mathbf{g}'\{-C_{\mathbf{ww}}\big(\mathbf{w}, f(\mathbf{x}^{\circ};\mathbf{w})\big) - C_{\mathbf{w}y}\big(\mathbf{w}, f(\mathbf{x}^{\circ};\mathbf{w})\big) f'_{\mathbf{w}}(\mathbf{x}^{\circ};\mathbf{w}) - C_{y}\big(\mathbf{w}, f(\mathbf{x}^{\circ};\mathbf{w})\big) f_{\mathbf{ww}}(\mathbf{x}^{\circ};\mathbf{w}) \\
&- f_{\mathbf{w}}(\mathbf{x}^{\circ};\mathbf{w}) C'_{\mathbf{w}y}\big(\mathbf{w}, f(\mathbf{x}^{\circ};\mathbf{w})\big) - f_{\mathbf{w}}(\mathbf{x}^{\circ};\mathbf{w}) C_{yy}\big(\mathbf{w}, f(\mathbf{x}^{\circ};\mathbf{w})\big) f'_{\mathbf{w}}(\mathbf{x}^{\circ};\mathbf{w})\}\mathbf{g} \geq 0,\ \forall \mathbf{g} \in \Re^{J}
\end{aligned} \tag{A.4}$$

Now observe that the choice of $(\mathbf{w},y)$ used in holding $\mathbf{h}(\mathbf{w},y)$ fixed in the construction of problems (A.1) and (A.2) is arbitrary, so long as $(\mathbf{w},y)\in B\big((\mathbf{w}^\circ,y^\circ);\delta\big)$. Hence the necessary conditions (A.3) and (A.4) hold for all $(\mathbf{w},y)\in B\big((\mathbf{w}^\circ,y^\circ);\delta\big)$. Using this observation in equation (A.4), multiplying it through by minus unity, and then employing the constraint in identity form, namely $y\equiv f\big(\mathbf{h}(\mathbf{w},y);\mathbf{w}\big)$ for all $(\mathbf{w},y)\in B\big((\mathbf{w}^\circ,y^\circ);\delta\big)$, establishes that $\mathbf{S}(\mathbf{w},y)$ is negative semidefinite for all $(\mathbf{w},y)\in B\big((\mathbf{w}^\circ,y^\circ);\delta\big)$. Symmetry of $\mathbf{S}(\mathbf{w},y)$ for all $(\mathbf{w},y)\in B\big((\mathbf{w}^\circ,y^\circ);\delta\big)$ follows from the $C^{(2)}$ nature of $f(\cdot)$ and $C(\cdot)$. *Q.E.D.*

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