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Evidence for Reynolds-Stress Driven Shear Flows Using Bispectral Analysis: Theory and Experiment

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Abstract

Spontaneous shear flow generation in magnetized fusion plasmas is thought to occur by an interaction of the turbulent Reynolds stress with the shear flow. This interaction can be viewed as a transfer of turbulent energy to the shear flow scales via a 3-wave coupling process, which suggests that investigations of bispectral quantities may be of interest. In this paper we discuss the theory of mean flow generation when described via the bispectrum. Simple theoretical analysis is used to demonstrate the connection between the bispectrum and the nonlinear amplification mechanism for zonal shear flows. We also discuss results from analysis of edge fluctuations from DIII-D during an L-H transition. These results indicate a transient rise and fall of three-wave coupling during the transition, which is spatially localized to inside the separatrix. Future efforts to determine the direction and rate of turbulent energy transfer during shear-flow formation will also be discussed.

I. Introduction

One of the most important results in magnetic fusion research of the last two decades is the discovery of improved confinement regimes ("H mode", "VH mode", internal transport barriers, etc), in which cross-field particle and heat transport are greatly reduced [1]. These reductions in transport are attributed to E x B shear flows [2]; however, the origin of these flows has yet to be experimentally determined. Theory suggests that such flows can be nonlinearly generated via the electrostatic Reynolds stress [3-5], and that these very weakly damped shear flows (termed zonal flows), represent a saturation mechanism for the underlying turbulence [5]. While a number of predictions for the response of the plasma to shear flows have been studied [6], experimental investigations of the underlying interaction and dynamics of the shear flows and turbulence have proved more difficult. These difficulties in experimental verification suggest that non-traditional approaches and analysis may be needed. To this end, it is suggested that the use of bispectral analysis may facilitate progress in understanding this crucial phenomenon. In this paper, the theory of zonal flow generation is revisited, and discussed in terms of bispectral quantities. Initial results from analysis of edge fluctuations in the DIII-D tokamak are presented and discussed. These results indicate a transient rise and fall in three-wave coupling (shown to be related to nonlinear flow generation), and which is localized to inside of the separatrix.

The paper is organized as follows: in Section II, a brief overview of bispectral analysis is given. Section III discusses the theory of zonal flow generation in terms of bispectra, while Section IV presents analysis of data from DIII-D. Section V offers conclusions and discusses future work.

II. Bispectral Analysis

Bispectral analysis is a well known (if not widely known) technique which is useful for studying triple moments of systems. Ritz, Powers, Kim, and co-workers have extensively documented its use in and application to plasma physics in a number of papers (see, for instance [7]). If one considers the Fourier transform $\hat{Y}(\omega)$ of a signal $Y(t)$, the (auto-) bispectrum is defined as:

$$S(\omega_1,\omega_2) = \left\langle \hat{Y}^*(\omega_3)\hat{Y}(\omega_1)\hat{Y}(\omega_2)\right\rangle, \quad \omega_3 = \omega_1 + \omega_2$$

The angular brackets represent an ensemble average. The bicoherence is then defined as:

$$b(\omega_1,\omega_2) = \frac{\left|S(\omega_1,\omega_2)\right|}{\sqrt{\left|\hat{Y}(\omega_3)\right|^2}\sqrt{\left|\hat{Y}(\omega_1)\hat{Y}(\omega_2)\right|^2}}$$

Thus the bicoherence represent the phase coherence between three Fourier components of a signal satisfying a sum rule (a three-wave coupling process). Note that as the bispectrum is complex, there is also a biphase associated with it.

The utility of bispectral analysis is best illustrated via an example. Consider a very basic model equation, which includes a linear and a quadratic term:

$$\frac{\partial \phi_k}{\partial t} = \gamma_k \phi_k + \sum_{k'+k''=k} \Lambda_{k',k''}\phi_{k'}\phi_{k''} \,.$$

One can then write an equation for the ensemble-averaged energy in a particular mode:

$$\frac{\partial\left\langle|\phi_k|^2\right\rangle}{\partial t} = 2\gamma_k\left\langle|\phi_k|^2\right\rangle + \sum_{k'+k''=k}\Lambda_{k',k''}\left\langle\phi_k^*\phi_{k'}\phi_{k''}\right\rangle \; + \text{c. c.}$$

Note that the nonlinear term can be expressed as the product of the coupling coefficient and the real part of the bispectrum. This result can be rewritten as:

$$\frac{\partial\langle|\phi_k|^2\rangle}{\partial t} = 2\gamma_k\langle|\phi_k|^2\rangle + \sum_{k'+k''=k}\Lambda_{k',k''} b(k',k'')\langle|\phi_k|^2\rangle^{1/2}\langle|\phi_{k'}\phi_{k''}|^2\rangle^{1/2}\cos\Theta(k',k'').$$

Observe the similarity of the nonlinear term to traditional expressions for fluxes when written in terms of coherence, cross-phase, and amplitude. For example, the particle flux can be written as (in 1-D for simplicitiy):

$$\frac{\partial\langle n\rangle}{\partial t} = \sum_k k\gamma(k)\langle|n_k|^2\rangle^{1/2}\langle|v_k|^2\rangle^{1/2}\sin\Phi(k),$$

where $\gamma(k)$ is the coherence, and the $\Phi(k)$ cross-phase. As the nonlinear term represents a flux of energy into a given mode, it is similar in form to the traditional quadratic expression, with a coupling coefficient $\Lambda_{k',k''}$, the bicoherence $b(k',k'')$, and a biphase instead of k, a coherence, and a cross-phase. The primary difference is that in the bispectral case, individual mode amplitudes do not appear independently as they do in the quadratic case ($\langle|\phi_{k'}\phi_{k''}|^2\rangle^{1/2}$ instead of $\langle|\phi_{k'}|^2\rangle^{1/2}\langle|\phi_{k''}|^2\rangle^{1/2}$). Finally, it should be clear from this discussion that bispectral analysis represents a fundamentally different approach than the random-phase approximation [8].

III. Theory of Zonal Flow Generation

Having established that bispectral analysis can be a useful tool for investigating nonlinear dynamics, it is enlightening to reconsider the theory of zonal flow generation when cast in terms of bispectral quantities. The simplest model for a zonal flow is to consider the divergence of the polarization current

$$\vec{\nabla}\cdot\vec{v}_{pol} = 0 \rightarrow \frac{\partial}{\partial t}\nabla_\perp^2\phi = \left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial\phi}{\partial x}\frac{\partial\phi}{\partial y}\right) - \frac{\partial^2}{\partial x\partial y}\left(\left(\frac{\partial\phi}{\partial x}\right)^2 - \left(\frac{\partial\phi}{\partial y}\right)^2\right)$$

We define a zonal flow has having a wavevector $\vec{q} = q\hat{x}$, with q much less than the average wavenumber of the turbulence. Averaging over a flux surface, and noting that as the primary contribution to the zonal flow growth comes from modes with $|k| >> q$, one can rewrite the above equation as:

$$\frac{\partial \phi_{ZF}}{\partial t} \approx \iint_{k>k_{\min}} d^2k \Lambda_k |\phi_k|^2, \quad \Lambda_k = k_x k_y.$$

As the Reynolds stress is a quadratic nonlinearity (making it suitable for description in terms of bispectral quantities), one can find an equation for the zonal flow growth rate:

$$\gamma_{ZF} = \iint_{k>k_{\min}} d^2k \Lambda_k b(k,-k) \cos\Theta(k,-k) \frac{\left\langle |\phi_k|^2 \right\rangle^{1/2}}{\left\langle |\phi_{ZF}|^2 \right\rangle^{1/2}}.$$

The most interesting aspect of this expression is that the use of the bicoherence introduces a dependence on the zonal flow amplitude quite different from the traditional expressions (see e.g. [4,5]). Of course, one can view this result as merely signifying that there must be a pre-existing "seed" flow to be amplified by the turbulence.

One should note that the above model is the simplest possible one which could describe zonal flow generation. Additional effects (such as diamagnetic contributions, magnetic effects, higher order instabilities, and neoclassical damping) may complicate the picture, especially in the edge region. Finally, it seems reasonable that at least in principle, one should be able to test the above prediction in simulations.

IV. Experimental Results

We briefly describe here initial experimental results obtained from analysis of Langmuir probe data on the DIII-D tokamak. For a more detailed discussion of these results, the reader is referred to [9]. Bicoherence is calculated from data from 3 mm inside the separatrix, as well as 7 mm outside, during a spontaneous L-H transition. Inside the separatrix, a strong rise in total bicoherence is observed just before the transition, with a return to initial levels as the H-mode establishes itself; no such rise is observed outside the separatrix [Fig. 1]. Examination of the bicoherence shows that this increase in coupling comes from couplings of the form $f_1 + f_2 \approx 0$, consistent with the notion that a low-frequency shear flow is generated via (relatively) high frequency turbulent modes. It is also very interesting to note that the increase coupling disappears as the H-mode establishes itself. If one considers the equation for radial momentum, it is easy to see that a radial electric field can be driven by a mean flow via the Lorentz force, as well as by a mean pressure gradient. These results suggest that the evolution of the mean sheared E_r of the H-mode is triggered via a nonlinearly generated shear flow, but its structure is sustained in steady-state by the mean pressure gradient which arises during the flow's lifetime. It is thus important to distinguish between the nonlinearly generated, finite lifetime shear flows termed "zonal flows" in the literature, and the mean sheared E x B flow observed in H-modes (which could also be termed a zonal flow because of its polodial symmetry).

V. Conclusions and Future Work

Experimental verification of the underlying mechanisms of spontaneous shear flow generation in magnetic confinement devices represents an important goal for the fusion program. The complex, non-linear nature of these mechanisms presents great difficulties for traditional analysis techniques, and suggests that new approaches are needed. In this paper, it has been argued that bispectral analysis represents just such an approach. Simple analysis shows that the growth rate of such features can be simply related to the bispectrum, and other associated statistics, which in principle are measurable. Initial experimental analysis shows a localized rise **and** fall of three-wave coupling between well-separated frequencies, which is spatially localized to inside the separatrix. This result is qualitatively consistent with current theories of shear flow generation.

The promise of, and initial results from, this approach suggests that it should be pursued further, and a variety of issues present themselves. First, it should be noted that by no means has it been established that zonal flow generation has been definitively observed. Such a claim would require examination of power transfer issues, as well as confirmation of the interacting wavenumbers. More generally, quantitative comparisons to analytical predictions of structure and magnitude of bicoherence would be very useful, but perhaps rather difficult; the reader is referred to the work of Diamond et. al [10] in this area. It would also be very interesting to extend this analysis to both numerical simulations, and additional diagnostics A goal here would be to investigate dynamics of zonal flows in the core and internal transport barriers, where said dynamics may be simpler than in the edge [11].

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Figure Captions

Figure 1: Evolution of the total bicoherence of I_{sat} 3 mm inside (●) and 7 mm outside (Δ) the separatrix.