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I. INTRODUCTION

Although the study of kaon decays has been a subject of research for many years, only relatively recently have enough quantitative results become available to permit detailed comparisons with theoretical models. It is the purpose of the present paper to review the state of our knowledge on kaon decay.

In attempting to collect into coherent form the results of the various experimental groups, one rapidly finds that the error-assignment practices adopted by experimenters vary so widely that the usual method of weighting results inversely as the squares of their quoted errors is often more closely related to the optimism or pessimism of the various investigators than to the informational content of their data. Therefore I have adopted the following procedures with respect to the treatment of the data:

A. In combining results of experiments involving substantially equivalent experimental technique and analysis, I have assigned weights proportional to the statistics rather than related to the stated errors. In arriving at an estimate of the uncertainty for the averaged result, I have generally adopted the relation between error and number of events given by the most conservative of the groups. Where different experimental methods have been used to arrive

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at independent values of particular quantities, I have relied more heavily on the estimated errors to obtain appropriate weights.

B. I have in certain cases revised the quoted errors, usually upwards, if in the light of the statistical and systematic uncertainties they appeared underestimated. Hence the errors quoted in this paper may differ from those given in the original references, and any mistakes arising from these revisions should be blamed on me and not on the experimenters whose results are quoted.

C. I have tended to disregard old measurements with large uncertainties in my present compilations. Systematic errors in early experiments are often grossly underestimated, and I believe that there is much virtue in giving more weight to a single recent measurement than to the average of a large collection of old observations, even though the estimated errors of these two contributions may on the surface appear the same.

In the discussion that follows, I have omitted any detailed consideration of the $\Delta S = \Delta Q$ selection rule, K_1-K_2 mass difference, and CP violation, because these are the subjects of other review papers at this Conference.

II. K^+ RATES

A. Lifetime

A recent precision measurement of the K^+ lifetime by Fitch et al.[1] has been combined with another fairly recent determination[2] to give the current best estimate,

$$\tau = (1.243 \pm 0.004) \times 10^{-8} \text{ sec.}$$

It is interesting to note that this value is significantly larger than that previously accepted.

B. Branching Ratios

Table I shows a compilation of the available data on the K^+ branching ratios and corresponding rates. A few detailed comments concerning some of these numbers may be appropriate.

1. $K^+_{\pi 2}$ Mode

There has been a wide variation in the results of $K^+_{\pi 2}$ branching-ratio measurements, which has at various times led to suggestions that some of the differences arise from real physical effects.[3] To add to the confusion, two successive xenon-bubble-chamber measurements of this branching ratio give 18.6%[4] and 22.4%[5] respectively, each with a quoted error under 1%. The major problem of the xenon experiments lies in the separation of the $K^+_{\pi 2}$ and the $K^+_{\mu 3}$ modes. Consequently for the purposes of this review, I have taken from the xenon data the sum of the $K^+_{\pi 2}$ and $K^+_{\mu 3}$ branching ratios whose average value is 24.6 ±0.7% (the two xenon experiments agree fairly well on this sum) and subtracted the $K^+_{\mu 3}$ branching ratio from Table I, 3.2 ±0.3%, to obtain a $K^+_{\pi 2}$ branching ratio of 21.4 ±0.8%. This figure agrees very well with the independent determination in a freon bubble chamber by Callahan and Cline,[6] namely 21.8 ±0.9%. Since the xenon-chamber experiment relied on observing the π^0-decay photons, whereas that of Callahan and Cline did not, the satisfactory agreement between these should set to rest any further doubts that pions from the $K^+_{\pi 2}$ mode are indeed always accompanied by the decay photons from simultaneously produced π^0.

2. K^+_{e2} Mode

In an experiment reported to the Conference, Bowen et al.[7] have looked for the $K^+ \rightarrow e^+ + \nu$ (K^+_{e2}) mode under conditions in which the background is no larger than the rate expected from the V-A coupling

$$\frac{\Gamma(K^+_{e2})}{\Gamma(K^+_{\mu 2})} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{M_K{}^2 - m_e{}^2}{M_K{}^2 - m_\mu{}^2}\right)^2 = 2.6\times 10^{-5}.$$

Their result is completely compatible with the V-A prediction, and rules out with better than 90% confidence a rate larger than three times the V-A prediction.

3. K^+_{e3}, $K^+_{\mu 3}$ Modes

The disagreements between various measurements of the K^+_{e3} and $K^+_{\mu 3}$ branching ratios indicate clearly that the small error estimates given by the experimenters are highly optimistic, and that sources of systematic error have been underestimated. Consequently, in Table I rather large errors based on the spread of the experimental results rather than the quoted uncertainties have been assigned to the $K^+_{\mu 3}$ and K^+_{e3} rates. The same is true of the quoted error in the τ rate although the discrepancies between various measurements are less serious than for the three-body leptonic modes. It is to be hoped that in future K^+ decay experiments, the preoccupation with large statistics will be somewhat tempered by a careful consideration of ways of minimizing systematic errors, at least insofar as branching ratios are concerned.

4. K^+_{e4} and $K^+_{\mu 4}$ Modes

Although both the $\Delta S = \Delta Q$ $K^+_{e4}(\rightarrow \pi^+ + \pi^- + e^+ + \nu)^8$ and $K^+_{\mu 4}(\rightarrow \pi^+ + \pi^- + \mu^+ + \nu)^9$ modes have been observed, there has been no evidence for the corresponding $\Delta S = -\Delta Q$ modes $K^+_{e4}(\rightarrow \pi^+ + \pi^+ + e^- + \overline{\nu})$ and $K^+_{\mu 4}(\rightarrow \pi^+ + \pi^+ + \mu^- + \nu)$. However, since the final-state $\pi\pi$ interaction may tend to favor the $\Delta S = \Delta Q$ modes, it is difficult to obtain any strong limit on the degree of forbiddenness of $\Delta S = -\Delta Q$ amplitudes with respect to $\Delta S = \Delta Q$ amplitudes.

5. Neutral Lepton Currents

The rates of the decay modes $K^+ \to \pi^+ + e^+ + e^-$ [10] and $K^+ \to \pi^+ + \mu^+ + \mu^-$ [11] are less than 5×10^{-5} of the corresponding $K^+ \to \pi^0 + e^+ + \nu$ and $K^+ \to \pi^0 + \mu^+ + \nu$ rates.

6. Radiative Decay Modes

The rates for the modes $K^+ \to \pi^+ + \pi^0 + \gamma$ [12] and $K^+ \to \pi^+ + \pi^+ + \pi^- + \gamma$ [13] are compatible with those expected from inner bremsstrahlung, without requiring large direct-emission amplitudes.

III. K^0 RATES

A. $K_1^0 \to 2\pi$ Mode

1. Mean Life

A new precise determination of the K_1^0 mean life[21] has been combined with some of the more recent measurements to give a new best estimate,

$$\tau = (0.866 \pm 0.014) \times 10^{-10} \text{ sec.}$$

2. $K_1^0 \to 2\pi$ Branching Ratios

The K_1^0 branching ratios suffer from the same malady that afflicts the three-body leptonic K^+ decay modes, namely the existence of three "precise" measurements which are not compatible with each other. A weighted mean of these three measurements is given in Table III. The error that has been assigned reflects the spread in values rather than the quoted errors. The result, of course, completely agrees with the $|\Delta \vec{I}| = 1/2$ prediction $\Gamma(\pi^0\pi^0)/\Gamma(\pi\pi) = 1/3$ or its weakened form based on an admixture of $|\Delta \vec{I}| = 3/2$ compatible with the $K^+_{\pi 2}$ rate.

B. K_2^0 Rates

Existing data on the rates for various K_2^0 modes have been obtained in two ways:

1. In experiments in which K^0 are made in or close to the detector by incoming beams of pions, charged kaons, or antiprotons, the numbers of $K^0 \rightarrow$ leptons and $K^0 \rightarrow 3\pi$ decays are determined, and the total number of K^0's produced is obtained directly by observation of the usual $K_1^0 \rightarrow \pi^+ + \pi^-$ events. From the measured momenta and available path lengths, the rates for the observed K_2^0 modes are then readily computed. For decays close to the K^0 production point, the problem of K_1^0 leptonic modes and the related question of the validity of the $\Delta S = \Delta Q$ selection rule comes into the analysis. However, since the detectors are usually large compared to a K_1^0 mean life, the results for the K_2^0 rates are not very sensitive to the $\Delta S/\Delta Q$ problem.

The major difficulty connected with this technique lies in the prodigious K_1^0 background which must be effectively prevented from simulating the decay modes of interest. Thus, in the experiment of Franzini et al.[21] about 100 leptonic and 3π decays had to be separated from a background of some 10 000 K_1^0 decays. The problem is probably greatest if the detector is a hydrogen bubble chamber, being somewhat reduced in a heavy-liquid chamber by the distinctive signature of some of the decay modes of interest. It is worth noting that the one spark-chamber experiment that has contributed to this method of obtaining rates solved this problem very satisfactorily by triggering only on decays separated by some two K_1^0 mean lives from the production point and by using a rather large spark chamber to provide considerable decay path length.[22]

2. Detectors have also been set up directly in K_2^0 beams, thus completely avoiding the K_1^0 background. Since the absolute K_2^0 flux in such experiments is generally not obtainable with any useful precision, only branching ratios into modes to which the detector is sensitive can be determined. These branching ratios can be combined with independent determinations of the K_2^0 mean life, as obtained from attenuation measurements, to calculate the absolute rates for the decay modes. These considerations are of course only valid if there is no "completely undetected mode" which would contribute to the mean life but not to the observed branching ratios.

Most of these branching-ratio determinations have been made either with cloud chambers or more usually with hydrogen bubble chambers as the detectors. It must be emphasized that these determinations are far more difficult than the corresponding ones for K^+ decay modes, and in a certain sense less satisfying. Whereas in the case of the K^+ modes the individual events are identified as belonging to one mode or another, this is usually not possible for the K_2^0 events. In the latter case, one must carry out a statistical analysis somewhat sensitive to the energy and angular distributions of the secondaries of the decay, and, more seriously, quite sensitive to the measurement errors.

In the present analysis, the various results from both direct rate observations and branching-ratio determinations have been subjected to a least-squares fitting program. The input data and its sources are given in Table II, and the results of the fit are listed in Table III. It is an assumption of the analysis that the $K_2^0 \to \pi^+\pi^-\gamma$ mode on which there exists only fragmentary information can be neglected insofar as its contribution to the total rate is concerned. The χ^2 for the overall fit is 7.23 for five degrees of freedom, coresonding to a probability level of 20%. Thus the various experimental data are reasonably self-consistent.

IV. K → 3π DECAYS

A. Rate Comparisons

The well-known $|\Delta\vec{I}| = 1/2$ rule for nonleptonic decays leads to several predictions for relations between the rates for the various K → 3π modes. Because the pion and kaon mass differences are not negligible in comparison to the energy release in the decay, it is essential to compare not the rates Γ but the ratios of rates to available phase space,

$$\gamma = \frac{\Gamma}{\Phi} ,$$

where Φ is the appropriate Lorentz-invariant phase space. Several sets of these phase-space factors have appeared in the literature, but insofar as I have been able to tell, none of them are quite correct. A new set of values, believed to be accurate to about 1%, are shown in the second column of Table IV with the corresponding values of γ shown in the third column.

The $|\Delta\vec{I}| = 1/2$ rule requires that the final 3π states in both K^+ and K_2^0 decay have I = 1 (for K_2^0 decay the I = 0 is forbidden by CP conservation). This leads to the following predictions:

$$\gamma\,(\pi^+\pi^+\pi^-) = 4\gamma\,(\pi^+\pi^0\pi^0) \tag{1a}$$

$$\gamma\,(\pi^0\pi^0\pi^0) = 3/2\gamma\,(\pi^+\pi^-\pi^0). \tag{1b}$$

Inspection of Table IV shows that both of these predictions are very well fulfilled by the experimental data. However these really test mainly whether the I = 3 symmetric final state is present to any significant degree, and hence rule out sizable $|\Delta\vec{I}| = 5/2,\ 7/2$ contributions, but not $|\Delta\vec{I}| = 3/2$ contributions. However for a pure $|\Delta\vec{I}| = 1/2$ transition one also has the relations

$$\gamma\ (\pi^+\pi^-\pi^0) = 2\gamma\ (\pi^+\pi^0\pi^0) \tag{1c}$$

and

$$\gamma\ (\pi^0\pi^0\pi^0) = \gamma\ (\pi^+\pi^-\pi^-) - \gamma\ (\pi^+\pi^0\pi^0), \tag{1d}$$

which are significantly affected by the presence of $|\Delta\vec{I}| = 3/2$ components. According to Table IV, relation (1d) is satisfied within the errors of measurement. Although relation (1c) fails by about 1.7 times the rather small standard deviation, the discrepancy is not sufficiently large to be considered a significant piece of evidence against the validity of the $|\Delta\vec{I}| = 1/2$ rule in describing the K → 3π decays. It is of interest in this connection to point out that the direct rate measurements of the $K_2^0 \to \pi^+ + \pi^- + \pi^0$ mode agree very well with Eq. (1c),[27] and that the discrepancy arises completely from the experimental data on the branching ratio $\Gamma_\pi(+ - 0)/\Gamma_{\text{charged}}$. If one examines the various measurements from which the average branching ratio given in Table II is obtained, it can be seen that the overall consistency is not very good, and, in fact, that the two measurements presented to this Conference, namely 0.178 ±0.017[33] and 0.144 ±0.004,[32] differ by two standard deviations. In view of these considerations, one must conclude that the $|\Delta\vec{I}| = 1/2$ rate predictions are all satisfactorily fulfilled by the present experimental data.

B. Pion Energy Spectra

1. Linear Representation

It is well known that the pion energy spectra in K → 3π decay do not precisely follow phase space. Weinberg has suggested that the matrix element be expanded in the form:[41]

$$M = 1 - \frac{a}{m_\pi^2}\ (S_3 - S_0),$$

where a is a constant,

$S_i = (P_K - P_{\pi i})^2 = (M_K - m_\pi)^2 - 2M_K T_i$,

i denumerates the three pions, with i = 3 representing the odd pion in K^+ decay or the π^0 in $K_2^0 \to \pi^+\pi^-\pi^0$ decay,

$S_0 = 1/3\ (S_1 + S_3 + S_3) = (M_K - m_\pi)^2 - 2/3\ QM_K$,

and T_i = kinetic energy of ith pion in the K rest frame.

The spectrum is then given by the product of the phase space and the squared matrix element

$$|M|^2 \approx 1 - \frac{2a}{m_\pi^2} (S_3 - S_0)$$

or

$$|M|^2 \approx 1 + \frac{2a}{m_\pi^2} M_K T_{max} \left[\frac{2T_3}{T_{max}} - 1\right],$$

where the quadratic term in $|M|^2$ has been neglected. If the 3π final state has I = 1, the "a" values for the $\pi^+\pi^+\pi^-$ and $\pi^+\pi^0\pi^0$ modes obey the relation[42]

$$a\ (+00) = -\ 2a\ (++-). \qquad (2a)$$

Furthermore, if the decay obeys the $|\Delta \vec{I}| = 1/2$ rule, the slopes of the energy distributions in $K^+ \to \pi^+\pi^0\pi^0$ and $K_2^0 \to \pi^+\pi^-\pi^0$ must obey the relation[42]

$$a\ (+-0) = a\ (+00). \qquad (2b)$$

The experimental results of various groups for the three decay modes of interest are listed in Table V. It is evident that the data show very good agreement with (2b). On the other hand, the slope for the τ decay mode shows a tendency to be somewhat smaller than that expected from relation (2a). This disagreement of about 1.7 standard deviations is again not sufficiently large to be considered as a significant disagreement with the prediction (2a).

In conclusion, it can be said that the increased precision made possible by recent experiments with improved statistics has shown up no significant violation of the $|\Delta\vec{I}| = 1/2$ rule in $K \to 3\pi$ decay either in relation to the total rates or in relation to the pion energy spectra. It is, of course, desirable to check by additional studies that, with still further increases in data, those deviations now on the edge of being significant do decrease.

2. The σ Resonance

An $I = 0$, $J = 0$ pion-pion resonance with mass about 400 MeV has been suggested by Brown and Singer[49] to account for the large three-pion branching ratio of the η. These authors have further suggested that one could account for the departures of $K \to 3\pi$ spectra from pure phase space by supposing that these decays proceed through the intermediate state $K \to \pi + \sigma \to 3\pi$. The application of the Brown-Singer theory to experimental data on η decays gives as best fit values:[50]

$$m_\sigma = 395^{+17}_{-9};\quad \Gamma_\sigma = 100^{+21}_{-17}\ \text{MeV}.$$

Unfortunately the $K \to 3\pi$ data provide relatively little information on the possible existence of the σ meson, principally because the highest possible π-π invariant mass in the K decay is 40 MeV lower than the above value of m_σ. Indeed, as pointed out by Taylor et al.,[51] as long as the 3π data require no quadratic terms in the matrix element, one can only use this K-decay information to derive a relation between m_σ and Γ_σ such that the predicted mean slope of the spectrum agrees with the experimental data. The above pair of values for m_σ and Γ_σ is not in very good agreement with the data from either the τ or the τ' mode.[46, 47]

A stronger item of evidence against the σ hypothesis is provided by the $\pi^+\pi^-$ spectrum in the K^+_{e4} decay ($K^+ \to e^+ + \pi^+ + \pi^- + \nu$).[8] Figure 1 shows the experimental dipion mass distribution for the 69 events obtained in the Wisconsin-Berkeley collaboration. Curve No. 4 which corresponds to the above σ parameters is in total disagreement with the data.

V. $K \rightarrow \pi + (e, \mu) + \nu$ DECAY

A. Rate Comparisons

It has been proposed that the strangeness-changing current, in the current-current interaction scheme, has the properties of an isotopic spinor in charge space.[52] The $|\Delta \vec{I}| = 1/2$ rule for leptonic decays implies the following relations between K^+ and K_2^0 rates of decay into the three-body leptonic modes:

$$\Gamma_{2e} = 2\Gamma_{+e} \tag{3a}$$

$$\Gamma_{2\mu} = 2\Gamma_{+\mu}. \tag{3b}$$

Furthermore, the angular and energy spectra for the K_2^0 and K^+ decays are predicted to be precisely the same. Rather than checking directly whether the data satisfy Eq. (3), it has been found more convenient to consider the equivalent relations

$$\Gamma_{2e} + \Gamma_{2\mu} = 2\ (\Gamma_{+e} + \Gamma_{+\mu}) \tag{4a}$$

and

$$\Gamma_{2\mu}/\Gamma_{2e} = \Gamma_{+\mu}/\Gamma_{+e}\ . \tag{4b}$$

The advantage of using (4) is that almost independent sets of experimental data are used to check (4a) and (4b); hence the experimental errors in the two tests are almost uncorrelated. From Tables I and III we find

$$\frac{\Gamma_{2e} + \Gamma_{2\mu}}{2(\Gamma_{+e} + \Gamma_{+\mu})} = 1.05 \pm 0.08$$

and

$$\frac{\Gamma_{2\mu}/\Gamma_{2e}}{\Gamma_{+\mu}/\Gamma_{+e}} = 0.98 \pm 0.14.$$

Agreement with the $|\Delta\vec{I}| = 1/2$ predictions is excellent.

B. Angular and Energy Spectra

1. Theoretical Preliminaries

If one assumes vector coupling and locality of the lepton current, the matrix element can be written in the form[53]

$$M \propto \langle \pi | J_\lambda | K \rangle \, u(P_\nu)\, \gamma_\lambda \, (1 + \gamma_5)\, v(P_\ell),$$

where $$\langle \pi | J_\lambda | K \rangle = 1/2\, f_+ \,(P_K + P_\pi)_\lambda + 1/2\, f_- \,(P_K - P_\pi)_\lambda.$$

The form factors f_+, f_- are functions of only $q^2 = (M_K - m_\pi)^2 - 2M_K T_\pi$, the square of the invariant four-momentum transfer, and are relatively real if time-reversal invariance holds in the decay. Furthermore, if the muon differs from the electron only through its mass, the functions f_+ and f_- will be the same for either lepton. However, since the f_- term leads to factors proportional to the lepton mass, m_ℓ, which are negligible for electron decay, only the form factor f_+ comes into play in the K_{e3} mode. Finally, if the leptonic $|\Delta\vec{I}| = 1/2$ rule is valid, the functions f_+ and f_- must be the same for K^+ and K_2^0 decays except for a multiplicative factor of $\sqrt{2}$.

If one considers the possibility of other than vector couplings, one can write two other matrix elements corresponding to scalar and tensor coupling respectively each multiplied by a single form factor.

2. K_{e3}^+ Decay

Data on K_{e3}^+ decay have come principally from a xenon-bubble-chamber experiment[54, 55] and more recently from the Wisconsin-Berkeley freon-bubble-chamber run.[56] Both experiments have established that the scalar and tensor matrix elements fail completely to account for the experimental information, whereas the vector matrix element gives a satisfactory representation of the data. **This** point is most convincingly demonstrated in Fig. 2 taken from the freon experiment which shows the distribution of the angle $\alpha_{\pi\ell}$ between pion and dilepton lines of flight in the dilepton rest frame. The predictions for this distribution are $\sin^2\alpha_{\pi\ell}$, constant, and $\cos^2\alpha_{\pi\ell}$ for the vector, scalar, and tensor matrix elements respectively, independently of the q^2 dependence of the form factors. The fit to the vector prediction is excellent, whereas the others are completely ruled out.

The energy dependence of the form factor f_+ has been studied in two independent analyses of xenon-chamber film, in the freon-chamber run, and in a study of the positron energy spectrum observed in a hydrogen bubble chamber.[14] The results expressed as expansions of the form

$$f_+ = 1 + \lambda\, q^2/m_\pi^2$$

are summarized in the left column of Table VI. A histogram of the experimental pion-energy distribution, compared with that expected for a constant form factor, is shown for the freon-chamber data in Fig. 2. It is clear that there is no evidence whatsoever for any energy dependence of the form factor f_+, the average value of λ being 0.00 ±0.02.

3. K_{e3}^0 Decay

Energy and angular distributions in K_{e3}^0 decay were first investigated in a hydrogen chamber by Luers et al.,[28] and more recently were the subject of both another bubble-chamber experiment by Eisler et al.[37] and a spark-chamber experiment reported by Fisher et al.[57] All the data strongly favor the vector matrix element as expected.

Surprisingly enough, the energy dependence of the form factor f_+ in K_{e3}^0 decay appears to be substantial and, in fact, to disagree with the previously quoted K^+ result. The behavior of the form factor as determined in the spark-chamber experiment[57] is shown in Fig. 3. Although this energy dependence is primarily suggested by the data of Fisher et al., it is not inconsistent with the Luers data. The average value of λ for the combined Luers and Fisher experiments is $\lambda = 0.12 \pm 0.04$, which, if the stated errors are taken seriously, is incompatible with the K^+ value. These results can also be expressed in a different form for f_+,

$$f_+ = \frac{1}{M^2 - q^2},$$

where M is the mass of an appropriate J = 1 intermediate $K\pi$ state. The values of λ and M for the K_2^0 experiments are summarized in the right column of Table VI.

In the course of a hydrogen bubble chamber K_2^0 experiment, Hopkins[58] et al. have identified by bubble density some 737 K_{e3}^0 decays. Because of the quadratic ambiguity in calculating the primary momentum for each event, they obtain two possible values for the center-of-mass energies of both pion and electron (the neutrino energy is obviously unambiguous). Figure 4 gives their pion-energy spectrum, with each event plotted as one-half for one value and one-half for the other possible value of the pion energy. The histogram is the result of a Monte Carlo calculation using an energy-independent form factor. It is seen that the agreement is in fact very good, and particularly that there is no systematic tendency for the data to fall significantly below the predictions at high values of the pion energy, as would be expected from the form-factor variations obtained in the experiment of Fisher et al. On the other hand, the agreement between theory and experiment for the neutrino energy distribution obtained by Hopkins et al. is not good; consequently their results cannot now be considered an unqualified endorsement of the present theory with constant form factors.

It is important to emphasize that the errors and confidence levels quoted in Table VI are purely statistical, and what conclusion one draws from the apparent disagreements between the K^{+}_{e3} and K^{0}_{e3} results concerning the energy variation of the form factors depends upon one's optimism concerning systematic errors. Because of the greater technical problems associated with K^{0}_{2} experiments, it seems probable that they are more subject to large systematic errors. It is clear that further study, with great attention paid to experimental biases, is needed to verify whether or not the K^{+}_{e3} and K^{0}_{e3} form factors behave differently and the $|\Delta\vec{I}| = 1/2$ leptonic rule is thereby significantly violated.

4. $K^{+}_{\mu 3}$ Decay

Although several experiments to investigate various aspects of $K^{+}_{\mu 3}$ decay have been carried out, I will mostly confine myself here to a discussion of the results obtained by the Wisconsin-Berkeley-Riverside-Bari (WBRB) collaboration from a stopping K^{+} run in the Berkeley heavy-liquid chamber filled with freon.[15] Apart from some early disagreements, the conclusions of the other experiments agree satisfactorily with those of the WBRB group; however, the latter experiment has far more numerous statistics than the others.

Analysis of the $K^{+}_{\mu 3}$ decay is more complicated than that of the K^{+}_{e3} because the theory presents us with two rather than a single form factor. Furthermore, the possible violation of time-reversal invariance implies that the form factor ratio $\xi = f_{-}/f_{+}$ is not necessarily real, and hence there is yet another possible parameter. To limit the number of variables one can reasonably attempt the analysis with the assumption that f_+ and ξ can be treated as constants, a supposition which in fact agrees well, insofar as f_+ is concerned, with the K^+_{e3} data and e-μ universality. If we then accept this assumption, all experimental information concerning energy spectra, angular correlations, and polarization will depend only on ξ, the value of f_+ entering only in the magnitude of the absolute rate.

Information concerning Re ξ and Im ξ have been obtained from four sources within the same WBRB experiment. These are independent from each other in the sense that either different subsets of events are used or the quantities measured are kinematically independent. The four sources and the results obtained from them are listed in Table VII. A few additional clarifying comments may be helpful here:

(a) The $\mu+$ energy spectrum (based on 2650 events) was studied between 42 and 94 MeV to remove background from τ' and $K^+_{\pi 2}$ decays. All muon energies were determined by range with suitable chamber-geometry corrections applied to the distributions.

(b) The longitudinal polarization was determined for 2950 events with muons in the energy range between 40 and 100 MeV. It is worth noting that Borreani et al. have recently measured the muon longitudinal polarization in the 6- to 27-MeV energy range.[59] Their result is completely consistent with the other polarization data, but eliminates, on the basis of polarization measurements alone, alternative values of ξ that for the earlier data can also provide an adequate fit.[60] A summary of all longitudinal polarization data taken from the Borreani paper is shown in Fig. 5.

(c) The π^0 energy spectrum at known muon energies between 40 and 90 MeV was studied for some 444 events in which both gamma rays materialized.

The dependence of population on muon energy was not used here because of possible biases arising from the requirement of double photon conversion. As indicated in Table VII, this analysis gives rather little information on $|\mathrm{Im}\,\xi|$, but a rather good value for Re ξ.

(d) The $\mu+$ total polarization was studied in the muon energy range $40 < T_\mu < 90$ MeV for 397 events where again both photons convert, permitting a determination of the decay plane.

The results of the analysis can be summarized as follows:

(1) The vector nature of the interaction is well established. This is clearly seen in Fig. 6 which compares the experimental π^0 spectrum at fixed muon energies for the 444 events discussed in (c) above with the expected spectra for the scalar, tensor, and vector couplings. Furthermore, the consistency of the values of ξ obtained from the various measurements gives further confidence in this conclusion.

(2) At the level of precision of the experiment there is no evidence for violation of time-reversal invariance.

(3) Comparison of the measured ratio of $K^+_{\mu 3}$ and K^+_{e3} rates with that expected from the value of ξ determined from the $K^+_{\mu 3}$ data permits verification that $|f_+(\mu)| = |f_+(e)|$ within the uncertainty of a few percent.

(4) The data permit some study of the energy dependence of f_+, but give almost no information on this point for f_-. Indeed for f_+, the value of λ obtained is $\lambda = 0.00 \pm 0.05$. Again this result is in excellent agreement with that for K^+_{e3} decay, but not with the spark-chamber data on K^0_{e3} decay.

5. $K^0_{\mu 3}$ Decay

Carpenter et al. have made the first detailed study of $K^0_{\mu 3}$ decay in a spark-chamber experiment.[40] As always, problems arising from the quadratic ambiguities in reconstruction of the kinematics make the analysis much more difficult than for K^+ decays. From an analysis of the Dalitz plot from some 1371 events with suitable corrections for this ambiguity problem they arrive at the following conclusions.

(i) Their data agree with the vector interaction and strongly disagree with tensor or scalar couplings regardless of form-factor variations. This result is evident from the neutrino energy distributions at fixed pion energies shown in Fig. 7.

(ii) The data are compatible with constant form factors and yield for ξ, on the assumption that $\mathrm{Im}\,\xi = 0$,

$$\xi = 1.2 \pm 0.8$$

where the quoted error is largely systematic. This result is of course fully consistent with the K^+ data.

(iii) The data are also compatible with substantial energy variations in the form factors. Thus if one assumes a dominant K–π intermediate state of spin 1 and mass M,

$$f_+(q^2) = \frac{f_+(0)}{M^2 - q^2}$$

and

$$f_-(q^2) = f_+(q^2)\,\frac{M_K^2 - M_\pi^2}{M^2},$$

a good fit is obtained with

$$M = 540^{+140}_{-70}\ \mathrm{MeV}.$$

This result is consistent with that obtained for f_+ in the K^0_{e3} experiment and with the measured ratio of the K_{e3} and $K_{\mu3}$ rates. The quality of the fits for both assumptions (ii) and (iii) are shown in Fig. 8.

(iv) There is no evidence for violation of time-reversal invariance in that none of the theoretical fits to the data are improved by considering a complex ξ.

VI. FINAL REMARKS

The conclusions from the foregoing exposition are basically that the theoretical models, namely the $|\Delta \vec{I}| = 1/2$ rule for leptonic and nonleptonic decays, and the vector interaction with μ-e universality agree very will with the totality of the experimental data. The sole exception is that the form factor f_+ for K_{e3} decay appears to have a much stronger energy dependence in K_2^0 than in K^+ decay. The performance of further K_2^0 experiments with, if possible, a good understanding of systematic errors will clearly be of great interest in clarifying this point.

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Table I. K^+ rates

Mode	Branching ratio	Rate (sec^{-1})	Remarks
All Modes[a]		$(8.045 \pm 0.027) \times 10^7$	
$K^+ \rightarrow \mu^+ + \nu$ [b]	$63.5 \pm 0.7\%$	$(5.11 \pm 0.06) \times 10^7$	
$K^+ \rightarrow \pi^+ + \pi^0$ (a)	$21.6 \pm 0.6\%$	$(1.74 \pm 0.05) \times 10^7$	
$K^+ \rightarrow e^+ + \pi^0 + \nu$ [c]	$4.49 \pm 0.25\%$	$(3.61 \pm 0.20) \times 10^6$	
$K^+ \rightarrow \mu^+ + \pi^0 + \nu$ [d]	$3.17 \pm 0.35\%$	$(2.55 \pm 0.28) \times 10^6$	
$K^+ \rightarrow \pi^+ + \pi^+ + \pi^-$ (e)	$5.59 \pm 0.11\%$	$(4.50 \pm 0.09) \times 10^6$	
$K^+ \rightarrow \pi^+ + \pi^0 + \pi^0$ (f)	$1.68 \pm 0.06\%$	$(1.35 \pm 0.05) \times 10^6$	
$K^+ \rightarrow e^+ + \nu$ [a]	$\sim 1.6 \times 10^{-5}$	$\sim 1.3 \times 10^3$	
$K^+ \rightarrow \pi^+ + \pi^- + e^+ + \nu$ [a]	$(3.6 \pm 0.8) \times 10^{-5}$	$(2.9 \pm 0.6) \times 10^3$	
$K^+ \rightarrow \pi^+ + \pi^- + \mu^+ + \nu$ [a]	$(7.7 \pm 5.2) \times 10^{-6}$	$(6.2 \pm 4.2) \times 10^2$	
$K^+ \rightarrow \pi^+ + \pi^0 + \gamma$ [a]	$(2.2 \pm 0.7) \times 10^{-4}$	$(1.8 \pm 0.6) \times 10^4$	$55 MeV < T_{\pi^+} < 80 MeV$
$K^+ \rightarrow \pi^+ + \pi^+ + \pi^- + \gamma$ [a]	$(1.0 \pm 0.4) \times 10^{-4}$	$(8.0 \pm 3.2) \times 10^3$	$E_\gamma > 10 MeV$
$K^+ \rightarrow \pi^+ + \pi^+ + e^- + \bar{\nu}$ [a]	$< 2 \times 10^{-6}$	$< 1.6 \times 10^2$	$\Delta S/\Delta Q = -1$ transition
$K^+ \rightarrow \pi^+ + \pi^+ + \mu^- + \nu$ [a]	$< 3 \times 10^{-6}$	$< 2.4 \times 10^2$	
$K^+ \rightarrow \pi^+ + e^+ + e^-$ (a)	$< 1.1 \times 10^{-6}$	$< 0.8 \times 10^2$	Involves neutral lepton currents
$K^+ \rightarrow \pi^+ + \mu^+ + \mu^-$ (a)	$< 3 \times 10^{-6}$	$< 2.4 \times 10^2$	

(a) See text for discussion.

(b) Calculated from 1 - sum (other branching ratios).

(c) Input data on branching ratio:

4.7 ±0.3%	(Ref. 5)	5.12 ±0.36%	(Ref. 14)
5.0 ±0.5%	(Ref. 4)	4.04 ±0.24%	(Ref. 15)

Values measured relative to the τ mode have been renormalized to the τ rate quoted in the Table.

Table I. (continued)

(d) Input data	3.0 ±0.5% (Ref. 5)
	3.52 ±0.20% (Ref. 62)
	2.82 ±0.19% (Ref. 15)
(e) Input data	5.54 ±0.12% (Ref. 17)
	5.71 ±0.15 % (Ref. 18)
	5.10 ±0.2% (Ref. 5)
	5.7 ±0.3% (Ref. 4)
	5.2 ±0.3% (Ref. 19)
(f) Input data	1.8 ±0.2% (Ref. 5)
	1.5 ±0.2% (Ref. 19)
	1.7 ±0.2% (Ref. 4)
	1.71 ±0.07% (Ref. 20)

Table II. Input Data for K_2^0 Rate Determinations

Γ_{total}	=	$(1.85 \pm 0.18) \times 10^7\ \text{sec}^{-1}$ (a)
Γ_{charged}	=	$(1.47 \pm 0.18) \times 10^7\ \text{sec}^{-1}$ (b)
Γ_e	=	$(0.81 \pm 0.10) \times 10^7\ \text{sec}^{-1}$ (c)
$\Gamma_e + \Gamma_\mu$	=	$(0.94 \pm 0.13) \times 10^7\ \text{sec}^{-1}$ (d)
$\Gamma_\pi(+-0)$	=	$(0.254 \pm 0.025) \times 10^7\ \text{sec}^{-1}$ (e)
$\Gamma_\pi\ (000)$	=	$(0.53 \pm 0.09) \times 10^7\ \text{sec}^{-1}$ (f)
Γ_μ/Γ_e	=	0.70 ± 0.05 (g)
$\dfrac{\Gamma_\pi(+-0)}{\Gamma_{\text{charged}}}$	=	0.152 ± 0.005 (h)
$\dfrac{\Gamma_\pi(000)}{\Gamma_{\text{charged}}}$	=	0.25 ± 0.06 (j)

(a) Input data on mean life: $(5.3 \pm 0.6) \times 10^{-8}$ sec (Ref. 23)
$(6.1^{+1.5}_{-1.2}) \times 10^{-8}$ sec (Ref. 24)

(b) Ref. 22 with correction due to the new value of the K_1^0 mean life.

(c) Ref. 25.

(d) Ref. 21.

(e) Input data: $(1.4 \pm 0.4) \times 10^6\ \text{sec}^{-1}$ (Ref. 21)
$(3.26 \pm 0.77) \times 10^6\ \text{sec}^{-1}$ (Ref. 26)
$(2.57 \pm 0.30) \times 10^6\ \text{sec}^{-1}$ (Ref. 27).

(f) Ref. 27.

(g) Input data: 0.73 ± 0.15 (Ref. 28)
0.81 ± 0.19 (Ref. 29)
0.85 ± 0.18 (Ref. 30)
0.680 ± 0.053 (Ref. 65).

Table II. (continued)

(h) Input data:	0.157 ± 0.03	(Ref. 28)
	0.151 ± 0.02	(Ref. 29)
	$0.15^{+0.03}_{-0.04}$	(Ref. 30)
	0.159 ± 0.015	(Ref. 31)
	0.144 ± 0.006	(Ref. 32)
	0.178 ± 0.017	(Ref. 33).
(j) Input data:	0.24 ± 0.08	(Ref. 34)
	0.25 ± 0.08	(Ref. 35).

Table III. K^0 Rates

Mode	Branching Ratio	Rate (sec^{-1})	Comments
All K_1^0 Modes [a]		$(1.155 \pm 0.019) \times 10^{10}$	
$K_1^0 \to \pi^0 + \pi^0$ [b]	30.9 ±2.2%	$(0.357 \pm 0.025) \times 10^{10}$	
$K_1^0 \to \pi^+ + \pi^-$ [b]	69.1 ±2.2 %	$(0.798 \pm 0.025) \times 10^{10}$	
All K_2^0 Modes [c]		$(19.9 \pm 1.0) \times 10^6$	
$K_2^0 \to \pi^\pm + e^\mp + \nu$ [c]	38.4 ±1.4%	$(7.64 \pm 0.44) \times 10^6$	Normalized so that total branching ratio for these modes = 100%
$K_2^0 \to \pi^\pm + \mu^\mp + \nu$ [c]	26.6 ±1.3%	$(5.30 \pm 0.38) \times 10^6$	
$K_2^0 \to \pi^+ + \pi^- + \pi^0$ [c]	11.8 ±0.5%	$(2.34 \pm 0.13) \times 10^6$	
$K_2^0 \to \pi^0 + \pi^0 + \pi^0$ [c]	23.2 ±2.0%	$(4.60 \pm 0.50) \times 10^6$	
$K_2^0 \to \pi^+ + \pi^- + \gamma$ [d]	<0.3%	$<5 \times 10^4$	
$K_2^0 \to \pi^+ + \pi^-$ [e]	$(1.58 \pm 0.12) \times 10^{-3}$	$(3.15 \pm 0.17) \times 10^4$	CP violating
$K_2^0 \to \mu^+ + \mu^-$ [f]	$<10^{-4}$	$<2 \times 10^3$	Involves neutral lepton currents
$K_2^0 \to e^+ + e^-$ [f]			
$K_2^0 \to e^\pm + \mu^\mp$ [f]			
$K_2^0 \to 2\gamma$ [g]	$<10^{-3}$	$<2 \times 10^4$	

(a) Input data on lifetimes:
$(0.90 \pm 0.05) \times 10^{-10}$ sec (Ref. 36)
$(0.94 \pm 0.05) \times 10^{-10}$ sec (Ref. 36)
$(0.885 \pm 0.025) \times 10^{-10}$ sec (Ref. 36)
$(0.85 \pm 0.04) \times 10^{-10}$ sec (Ref. 36)
$(0.87 \pm 0.05) \times 10^{-10}$ sec (Ref. 36)
$(0.86 \pm 0.04) \times 10^{-10}$ sec (Ref. 36)
$(0.848 \pm 0.014) \times 10^{-10}$ sec (Ref. 21)

(b) Input data for $[\Gamma(2\pi^0)]/[\Gamma(2\pi)]$:
33.5 ±1.4% (Ref. 66)
28.8 ±2.1% (Ref. 67)
26.0 ±2.4% (Ref. 68)

(c) From fit of data in Table II.

(d) Ref. 63.

(e) Compilation by J. Cronin, presented at Argonne Weak Interactions Conference.

(f) Refs. 40 and 61.

(g) Ref. 64.

Table IV. Rate comparisons for $K \rightarrow 3\pi$ Modes

Mode	Phase-space factor, Φ	γ = Rate/Φ (sec^{-1})
$K^+ \rightarrow \pi^+ + \pi^+ + \pi^-$	1.00	$(4.50 \pm 0.09) \times 10^6$
$K^+ \rightarrow \pi^+ + \pi^0 + \pi^0$	1.24	$(1.09 \pm 0.04) \times 10^6$
$K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0$	1.22	$(1.92 \pm 0.11) \times 10^6$
$K_2^0 \rightarrow \pi^0 + \pi^0 + \pi^0$	1.49	$(3.09 \pm 0.34) \times 10^6$

Tests of $|\Delta \vec{I}| = 1/2$

	Experimental	Predicted
$\dfrac{\gamma\,(\pi^+\pi^-\pi^0)}{2\gamma\,(\pi^+\pi^0\pi^0)}$	0.88 ±0.07	1.00
$\dfrac{\gamma\,(\pi^0\pi^0\pi^0)}{\gamma(\pi^+\pi^+\pi^-) - \gamma(\pi^+\pi^0\pi^0)}$	0.91 ±0.12	1.00
$\dfrac{\gamma(\pi^+\pi^+\pi^-)}{4\gamma(\pi^+\pi^0\pi^0)}$	1.03 ±0.04	1.00
$\dfrac{\gamma(\pi^0\pi^0\pi^0)}{\frac{3\gamma}{2}(\pi^+\pi^-\pi^0)}$	1.07 ±0.12	1.00

Table V. Measurements of odd-pion spectra in K → 3π decay values of the parameter a from the fit

$$|M|^2 \propto 1 - 2a\left(\frac{S_3-S_0}{m_\pi^2}\right) = 1 + 2a\,\frac{M_K\,T_{max}}{m_\pi^2}\left(\frac{2T_3}{T_{max}} - 1\right)$$

	$K^+ \to \pi^+ + \pi^+ + \pi^-$	$K^+ \to \pi^+ + \pi^0 + \pi^0$	$K_2^0 \to \pi^+ + \pi^- + \pi^0$
	0.102 ±0.028 (a)	-0.24 ±0.02 (e)	-0.24 ±0.09 (g)
	0.114 ±0.02 (b)	-0.30 ±0.05 (f)	-0.24 ±0.09 (h)
	0.083 ±0.028 (c)		-0.24 ±0.04 (i)
	0.083 ±0.015 (d)		-0.27 ±0.05 (j)
			-0.24 ±0.05 (k)
			-0.17 ±0.06 (l)
			-0.26 ±0.06 (m)
Averages:	0.093 ±0.011	-0.25 ±0.02	-0.24 ±0.02

Comparisons with $|\Delta\vec{I}| = 1/2$

	Experimental value	Predicted value
$\frac{a\,(\pi^+\pi^0\pi^0)}{a\,(\pi^+\pi^+\pi^-)}$	-2.7 ±0.4	-2
$\frac{a\,(\pi^+\pi^0\pi^0)}{a\,(\pi^+\pi^-\pi^0)}$	1.0 ±0.11	1

(a) Ref. 43
(b) Ref. 44
(c) Ref. 45
(d) Ref. 46
(e) Ref. 47
(f) Ref. 20
(g) Ref. 28
(h) Ref. 29
(i) Ref. 31
(j) Ref. 32
(k) Ref. 48
(l) Ref. 33
(m) Ref. 27

Table VI. Ke_3 decay

$$f_+ = 1 + \frac{\lambda q^2}{m_\pi^2}$$

Measured values of λ (±1 standard deviation)

	Ke_3^+ decay	Ke_3^0 decay
	0.038 ± 0.045 (a)	0.07 ± 0.06 (e)
	-0.01 ± 0.029 (b)	0.15 ± 0.04 (f)
	-0.04 ± 0.05 (c)	
	$0.02\,^{+0.04}_{-0.03}$ (d)	
Average:	0.00 ± 0.02	0.12 ± 0.04

$$f_+ = \frac{1}{M^2 - q^2}$$

95% confidence levels for M

Ke_3^+ decay	Ke_3^0 decay
$M > 700$ MeV	$M = 600\,^{+\infty}_{-150}$ MeV (e)
	$M = 480\,^{+110}_{-50}$ MeV (f)

(a) Ref. 54

(b) Ref. 55

(c) Ref. 14

(d) Ref. 56

(e) Ref. 28

(f) Ref. 57

Table VII. $K^{+}_{\mu 3}$ decay. Data are from Ref. 15.

Quantity Measured	Result
μ^{+} energy spectrum	$\mathrm{Re}\ \xi = 0.0 \,{}^{+1.1}_{-0.9}$
	$\lvert \mathrm{Im}\ \xi \rvert = 0.0 \pm 1.0$
μ^{+} longitudinal polarization	$\mathrm{Re}\ \xi = -0.7 \,{}^{+0.9}_{-3.3}$
	$\lvert \mathrm{Im}\ \xi \rvert = 0.5 \,{}^{+1.4}_{-0.5}$
π^{0} energy spectrum at fixed μ^{+} energies	$\mathrm{Re}\ \xi = 0.72 \pm 0.37$
	for $\lvert \mathrm{Im}\ \xi \rvert \lesssim 1$
Total μ^{+} polarization	$\mathrm{Re}\ \xi = -1.4 \pm 1.8$
	$\lvert \mathrm{Im}\ \xi \rvert = 1.6 \pm 1.3$
Overall result	$\mathrm{Re}\ \xi = 0.34 \pm 0.35$
	$\mathrm{Im}\ \xi = 0.69 \,{}^{+0.08}_{-1.0}$
If one assumes Im $\xi = 0$ and μ - e Universality	
Combination of above measurements	$\xi = 0.47 \pm 0.30$
Branching ratio $[\Gamma\ (K^{+}_{\mu 3})]/[\Gamma\ (K^{+}_{e3})]$	$\xi = 0.42 \pm 0.63$
Average of these values	$\xi = 0.46 \pm 0.27$
If one assumes Im $\xi = 0$ but not μ - e Universality	
Combination of branching ratio and above measurements	$\left\lvert \dfrac{f_{+}\ (\mu)}{f_{+}\ (e)} \right\rvert = 1.00 \pm 0.06$

FIGURE LEGENDS

Fig. 1. $\pi^+\pi^-$ invariant-mass plot for 69 events of the type $K^+_{e4} \rightarrow \pi^+ + \pi^- + e^+ + \nu$. The histogram shows the experimental data. The plotted curves indicate predictions for various models described in Ref. 8. Curve No. 4 corresponds to the σ parameters which fit the η data.

Fig. 2. Upper figure: Distribution of $\cos\alpha_{\pi\ell}$, where $\alpha_{\pi\ell}$ is the angle between the dilepton line of flight and the pion in the dilepton rest system, in K^+_{e3} decay (Ref. 56). Lower figure: Pion kinetic-energy spectrum for the same data. The curve refers to $\lambda = 0.02$ in the formula for f_+.

Fig. 3. Energy dependence of the form factor f_+ in K^0_{e3} decay from Ref. 57.

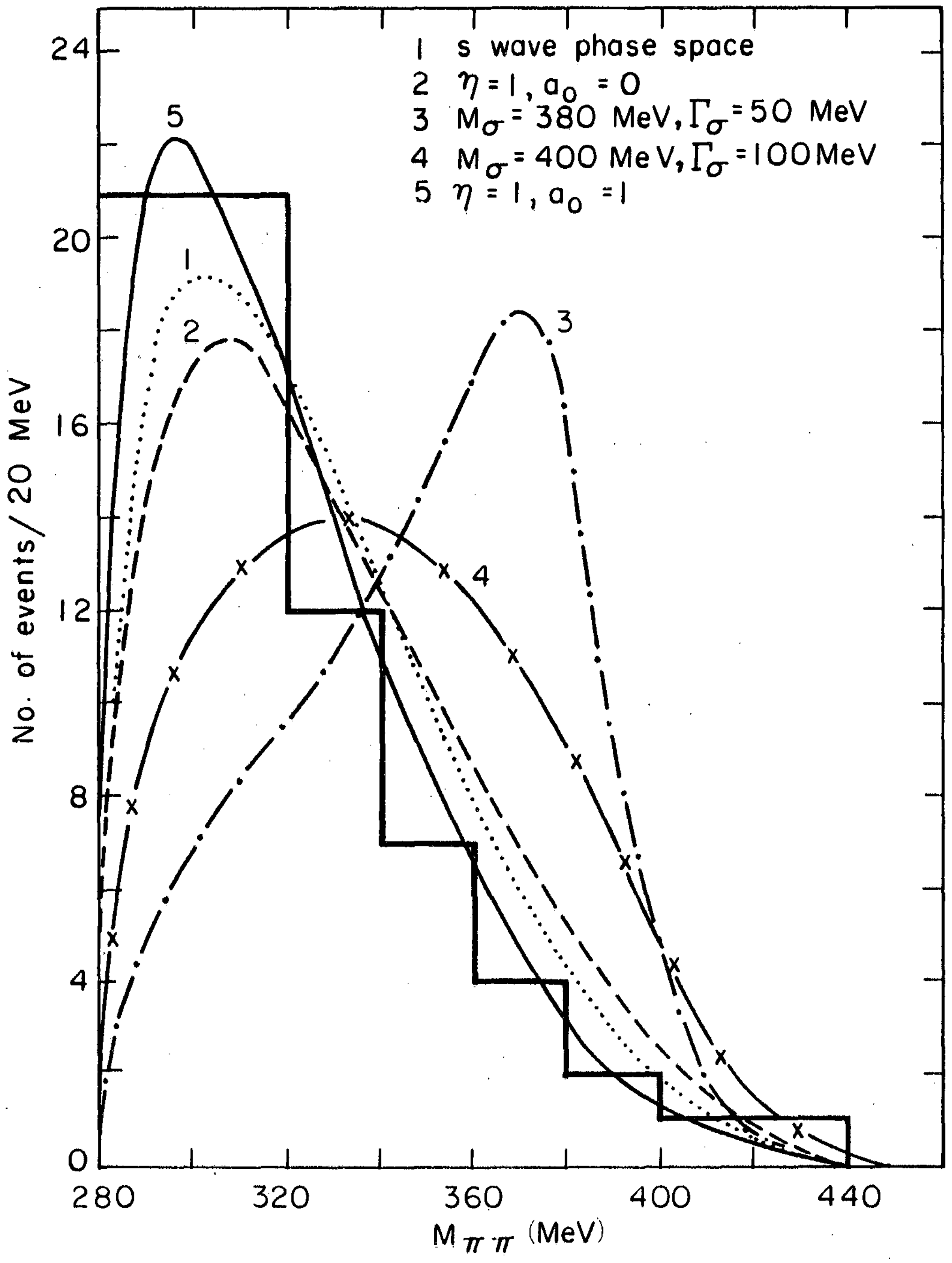
Fig. 4. Pion energy spectrum in K^0_{e3} decay from Ref. 58.

Fig. 5. Muon polarization data as a function of muon energy for the $K^+_{\mu 3}$ decay from Ref. 59.

Fig. 6. Pion energy spectrum of fixed muon energies from 444 $K^+_{\mu 3}$ events (Ref. 15).

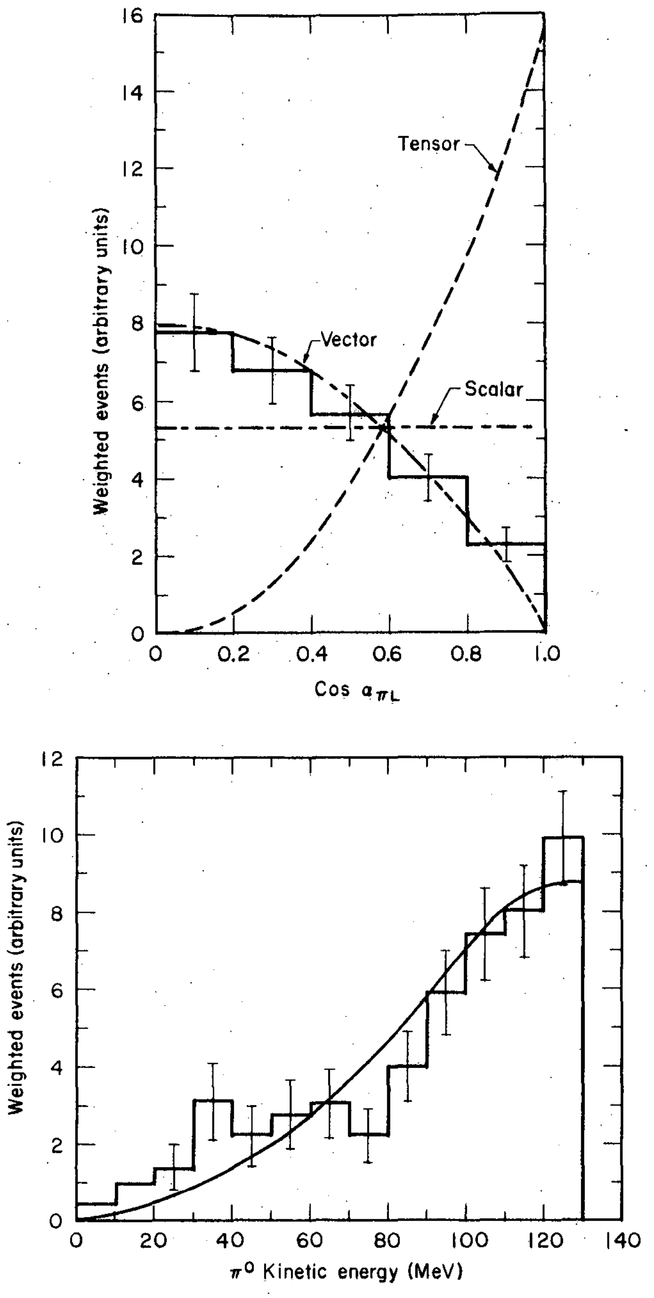
Fig. 7. Neutrino energy spectrum for various bands of pion kinetic energy for $K^0_{\mu 3}$ events (Ref. 40).

Fig. 8. Likelihood curves for fits to $K^0_{\mu 3}$ data, for ξ (constant form factor), and M (intermediate $J = 1$ K-π state) (Ref. 40).



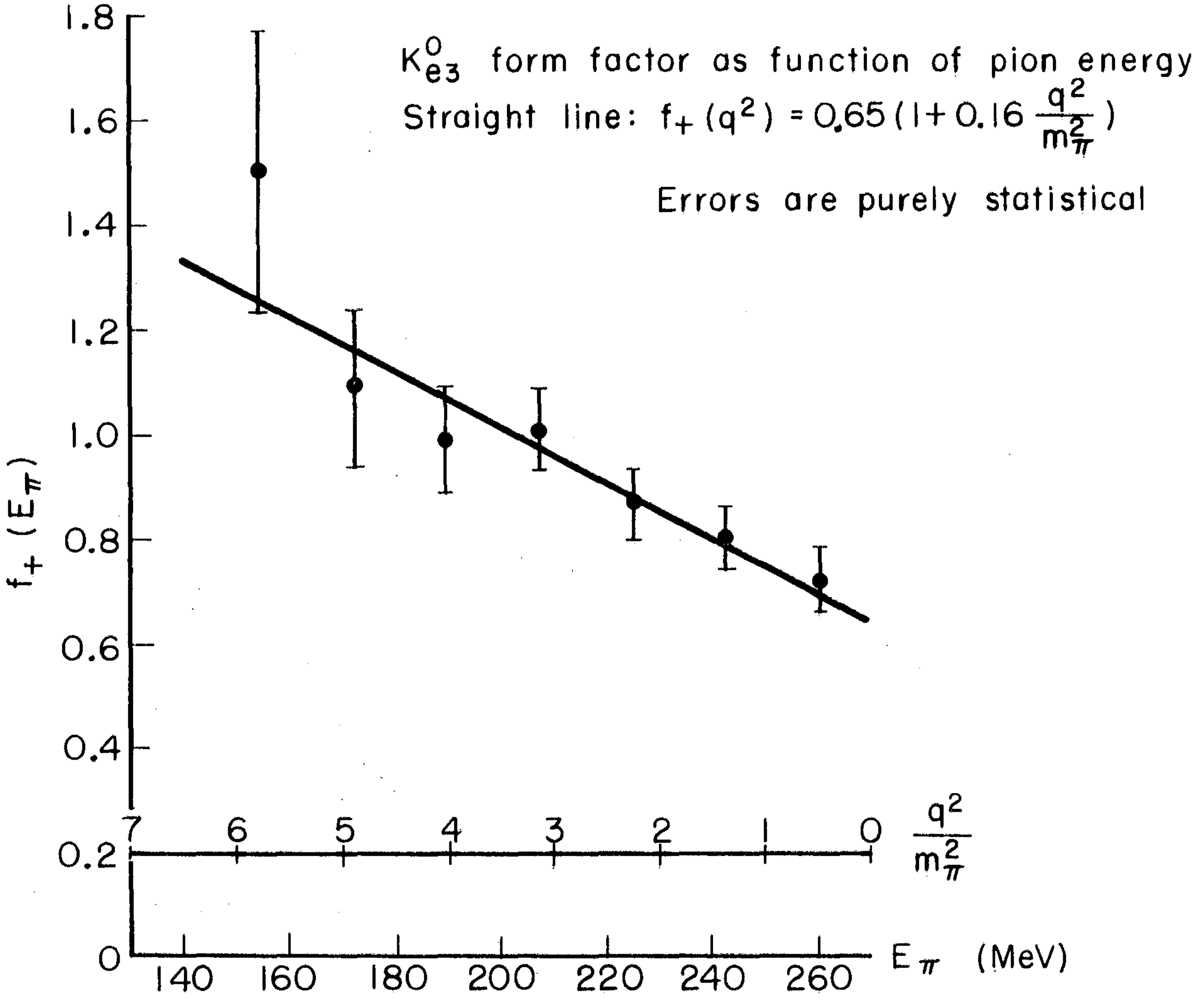
MUB-6080

Fig. 1



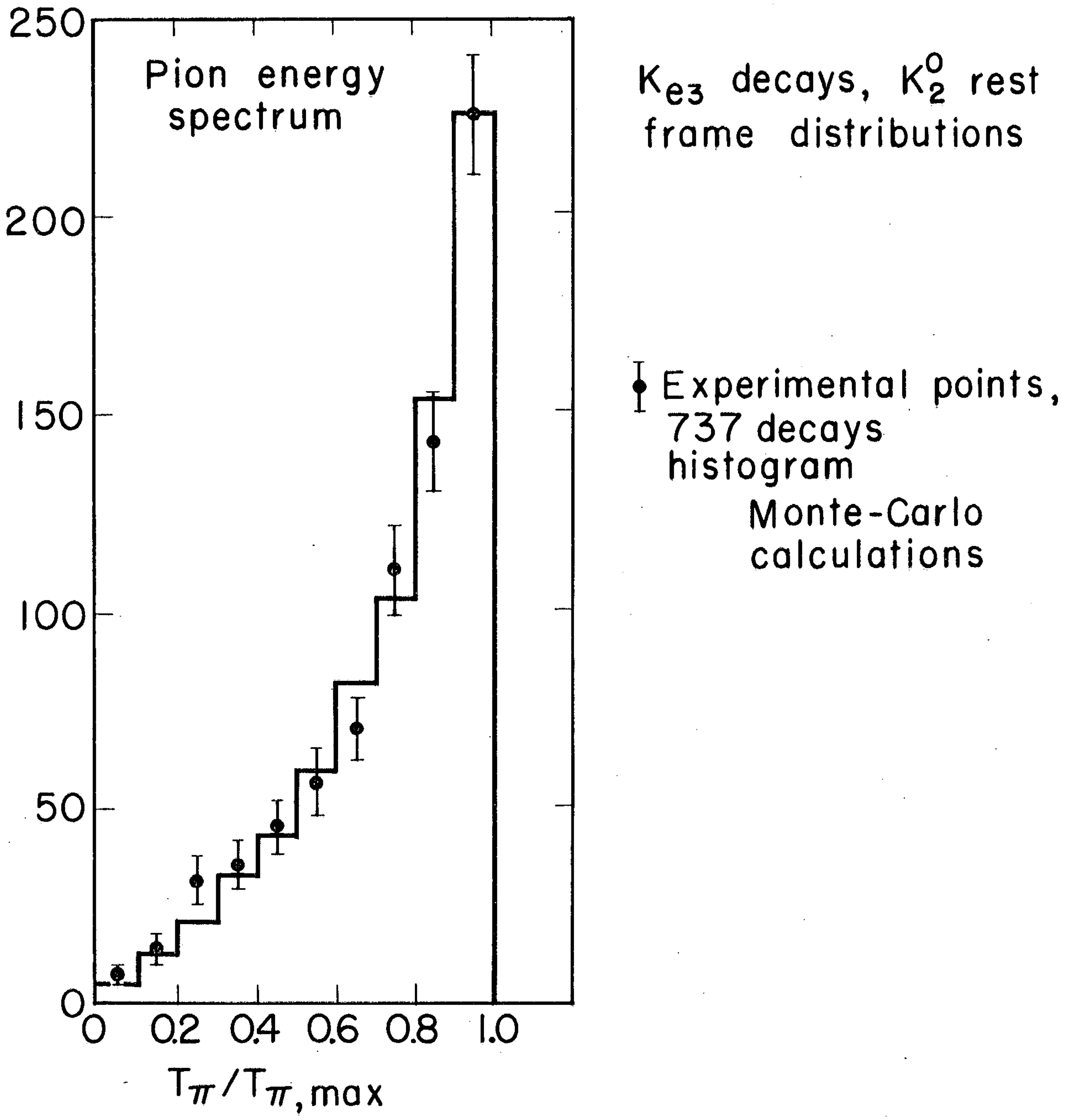
MUB-3412

Fig. 2



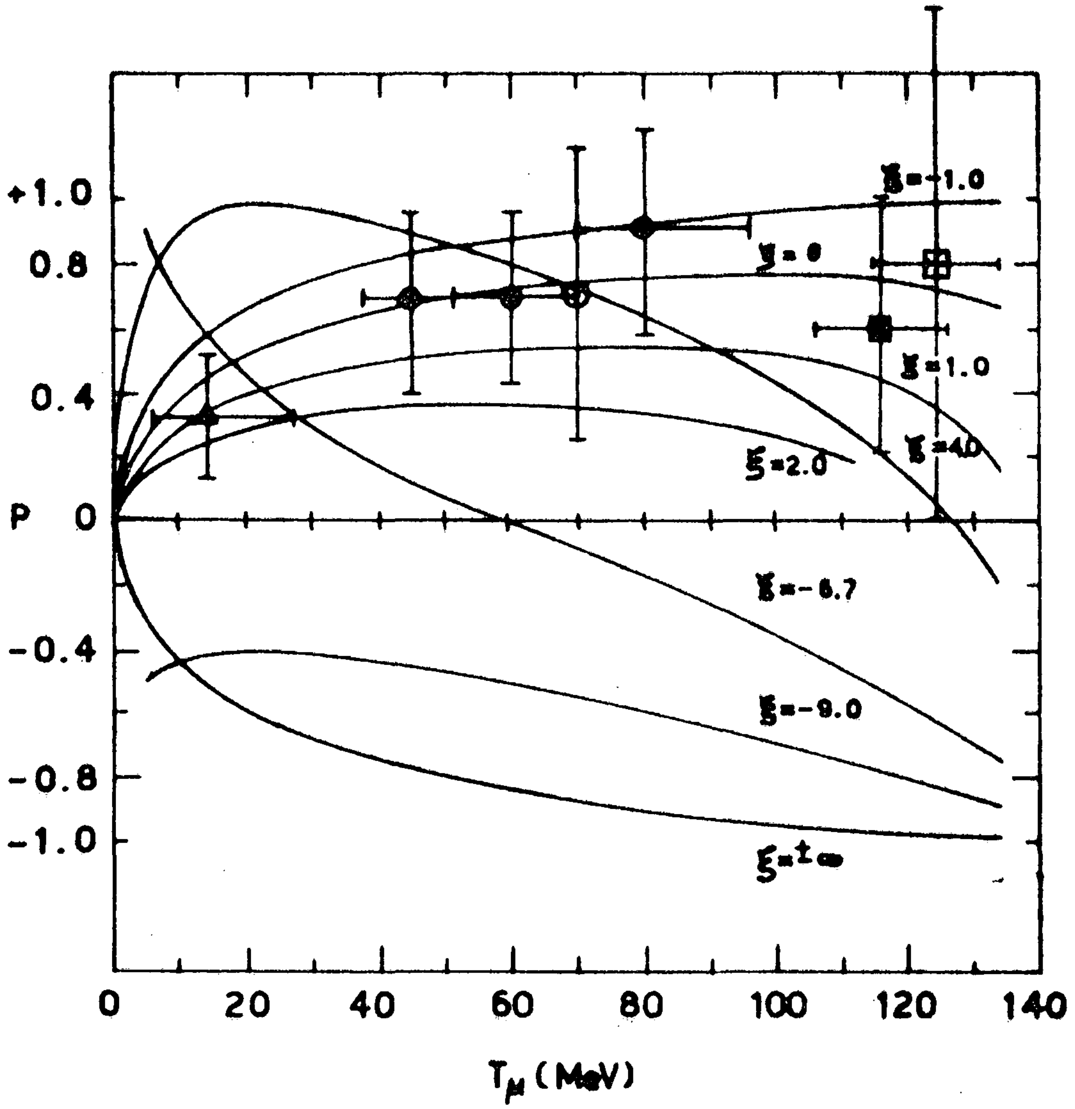
MUB-8398

Fig. 3



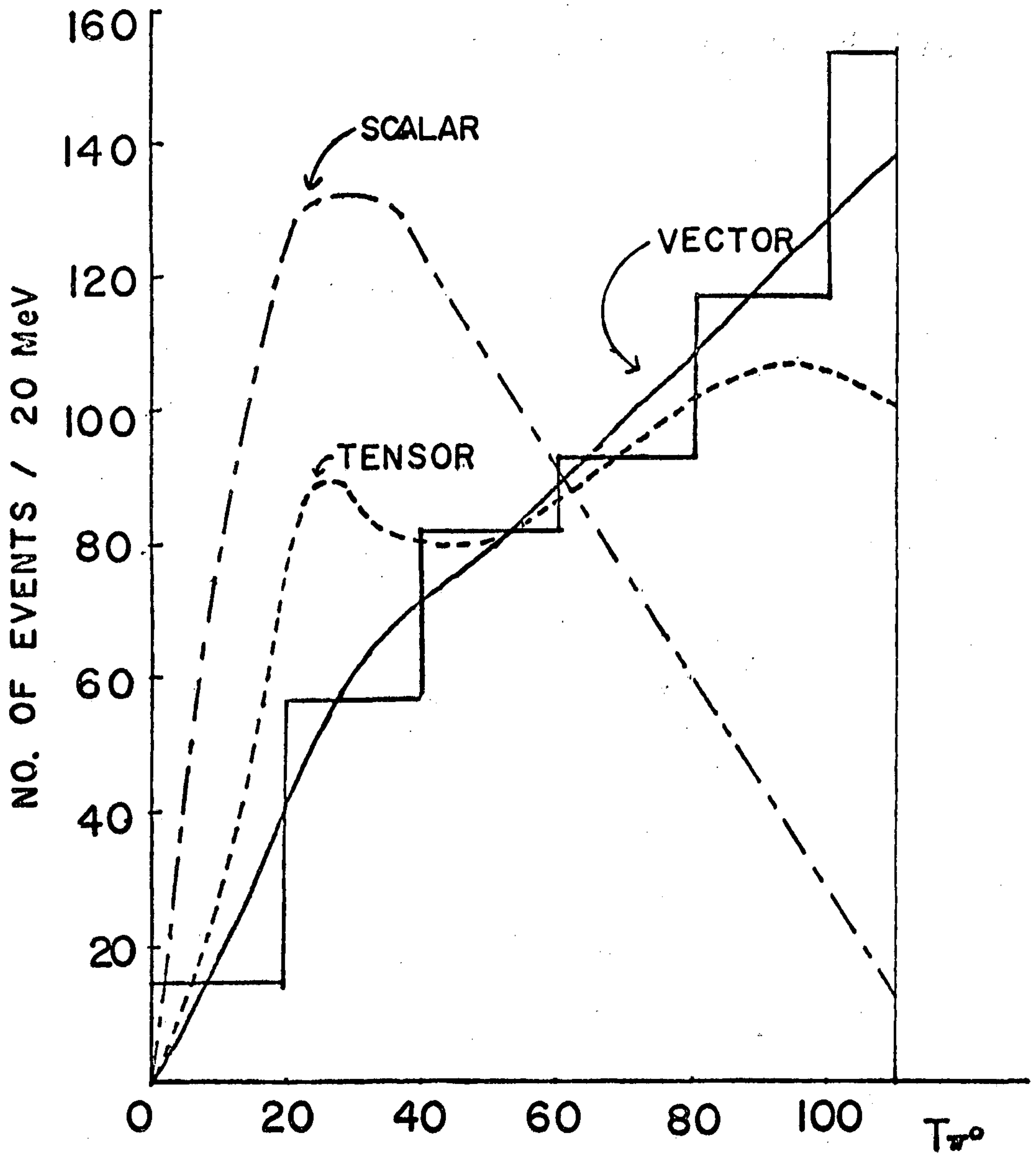
MUB-8399

Fig. 4



MUB-8339

Fig. 5



MUB-8400

Fig. 6

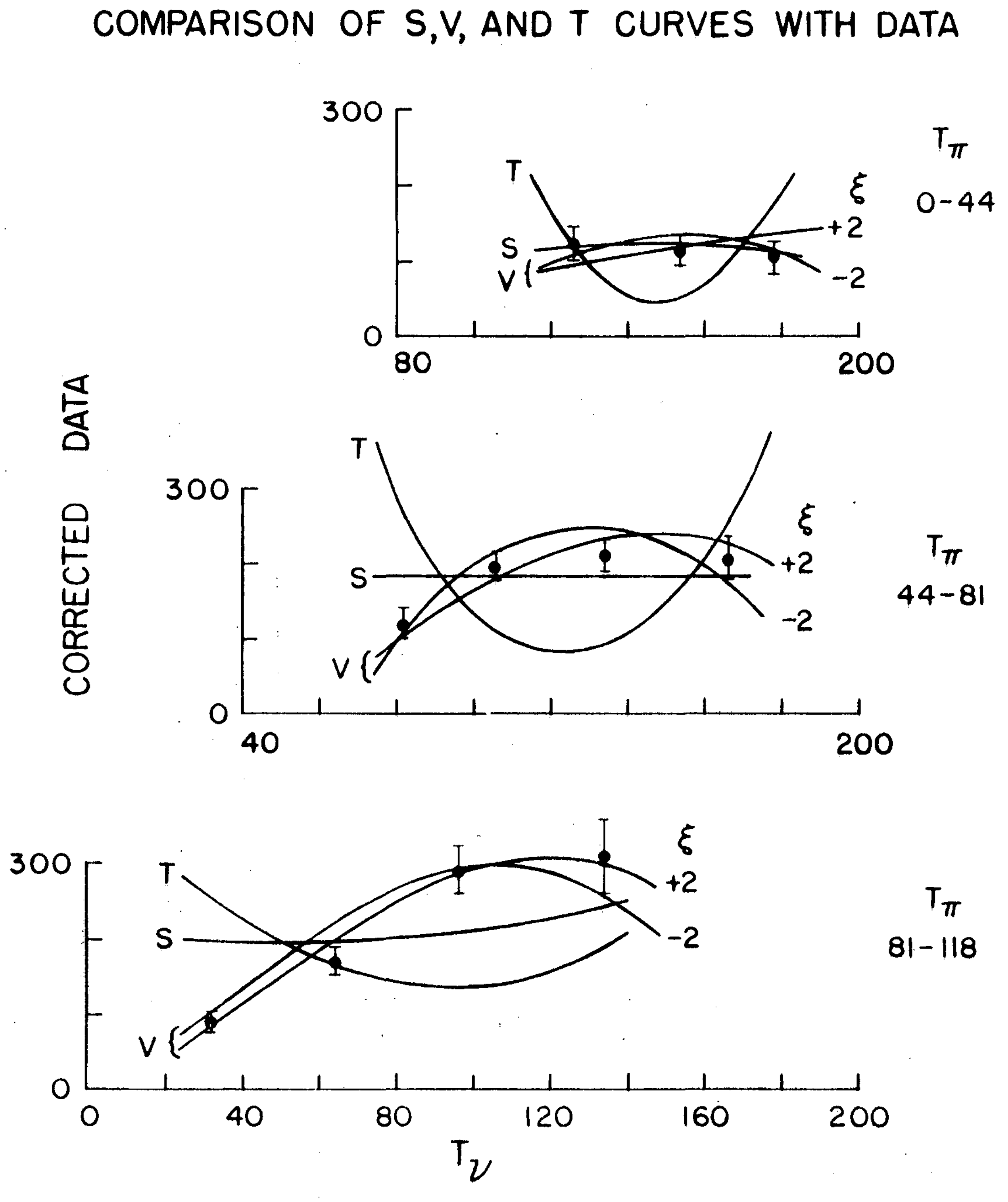


Fig. 7

MUB-8340

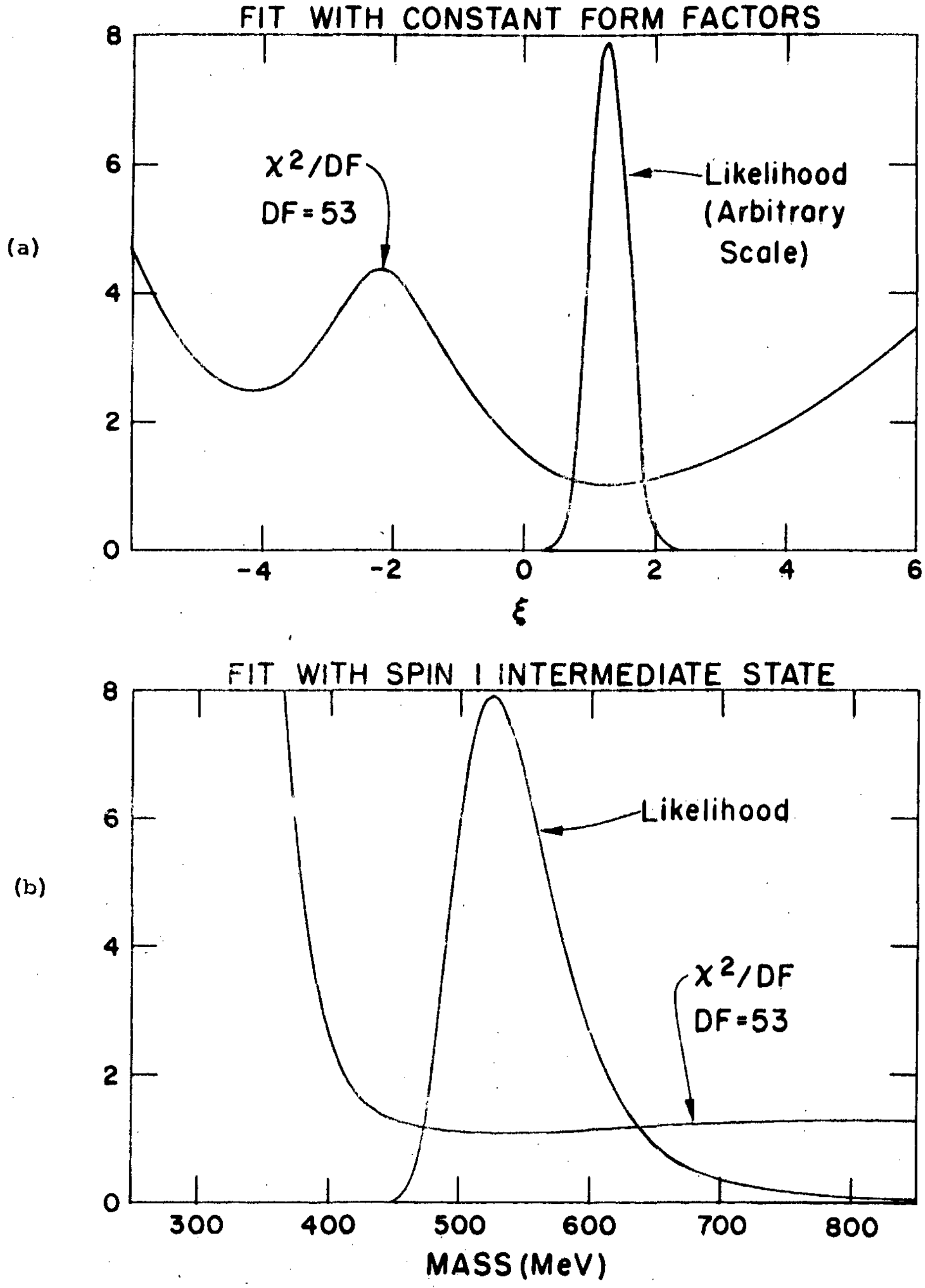


Fig. 8

MUB-8341